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BASIC MODELS FOR ENGINEERING DRAWING AND DESCRIPTIVE GEOMETRY

By

CHARLES ELMER ROWE

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CHARLES ELMER ROWE
Professor of Drawing,
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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of Democracy, and while guided and controlled by virtue, the noblest attribute of man. It is the only dictator that freemen acknowledge, and the only security which freemen desire.

Mirabeau B. Lamar

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INTRODUCTION

The value of visual aids in teaching certain subjects is recognized by leading educators. Years of experience in teaching drawing and descriptive geometry have demonstrated that effective results can be obtained by the use of models designed for the explanation of the principles involved. Ten years ago the author began designing and constructing basic models for this purpose, and has continued this work to the present date. The Department of Drawing has at least one model for use in explaining almost any principle or problem in drawing or descriptive geometry. This bulletin describes and illustrates about sixty of them, including some of both the simplest and the most elaborate. Many of these models can be operated. The value of a model for demonstrating a principle depends largely upon the explanation given by the instructor. Space does not permit of such explanations if, indeed, they could be made satisfactorily with the photograph instead of the model itself. Neither does space permit of drawings to show details of construction, which in some cases involved mechanism and machine design. Most of the models were built according to full size detail drawings, some of which were used as templates for drilling holes and obtaining angles.

ACKNOWLEDGMENTS

The Department of Mechanical Engineering, in a fine spirit of coöperation, has permitted the use of the facilities of their shops for the construction of the models. Most of the work was done by students working under the National Youth Administration. Each year a young man skilled in wood and metal work has been supplied by this agency until the past year, during which the Bureau of Engineering Research supplied an efficient student mechanic. The author is indebted to Professors W. H. McNeill, J. D. McFarland, W. F. Cottingham, and Jack Lenhart, all of the Department of Drawing, for suggestions from time to time, especially in the case of some of the more complicated models which have reached the final stage through a process of development. Professor Lenhart has shown great skill in making the photographs used for the illustrations. Appreciation is expressed to Mr. James R. Holmes, Instructor in Drawing, for the excellent and very careful work done in preparing the photographs for reproduction.

CONSTRUCTION

The rectangular base or sub-base adopted as a standard is made of ¾″ plywood, bound by a ½″ x 1⅛″ strip of wood mitered at the corners. This design provides a hidden space for nuts and other fastenings. Plywood is desirable for this and other constructions because it does not shrink or warp to a noticeable extent. It is an inexpensive and satisfactory material for making the ends of string models. It is easy to work and drill.

For the string models a different color of string is used for the elements of each surface or generation. In a few cases elastic hat cord is used for the elements if the model operates and requires material that stretches. The cord deteriorates, but, ordinarily, replacement is not serious.

In most cases the wires or metal rods are made of ⅛″, 3/16″ or ¼″ steel or Tobin bronze stock. The sheet metal used is tin plate or heavy gage steel. Galvanized metal is undesirable on account of the difficulty of painting it.

On models which require transparent planes, celluloid is the most common material. In other cases, plate glass, Vinylite or Plexiglas was preferred. These materials have different properties. Plate glass is transparent, hard to work, and easy to break. Celluloid has a fair degree of transparency when new, but in this climate is a treacherous material which deteriorates in a few years and causes corrosion of all metal parts near it. Vinylite seems to be more nearly stable. As far as our experience goes, Plexiglas seems to be the most desirable material. It is transparent, unbreakable, light, and easy to work. It is said to be stable.

For some of the models, special hinges were designed. On the transparent plane models, the hinges are of brass, ¾″ x 1″, attached by No. 6–32 brass machine screws.

All models are finished with varnish or enamel, usually rubbed. Colors are used to make the models more attractive and effective. A technique was developed for using special enamels in a ruling pen for drawing lines and lettering.

DESCRIPTION OF THE MODELS

MODEL FOR DEMONSTRATING ORTHOGRAPHIC PROJECTION

There are four glass picture planes in Fig. 1 for the following views: top view, front view, left-side view, and right-auxiliary view. The top, left-side, and

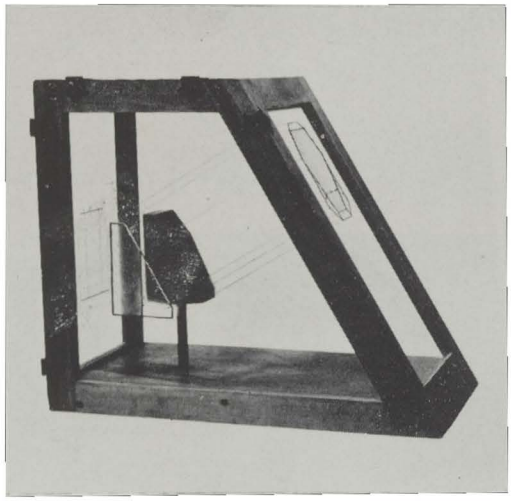

Fig. 1. Orthographic Projection.

auxiliary planes are hinged to the frontal plane and can be revolved into this plane to show the proper relationship of these views on the drawing board.

THE FIRST AND SECOND POSITIONS FOR A RIGHT-SIDE VIEW

In Figs. 2 and 3 the plane on which the right-side view is drawn revolves about a corner so that the right-side view can be placed adjacent to the front view or the top view. This is intended to show clearly the use of a frontal reference plane *FRP* and the depth dimension *d* for constructing the right-side view adjacent to the front view; and to show the use of a horizontal reference plane *HRP* and the height dimension *h* for constructing the right-side view adjacent to the top view. The object represented is a quadrilateral *ABCE* folded along *BE*. A separate model of the object is used to assist in visualization. The numbers shown in the three views are used in working out the visibility of

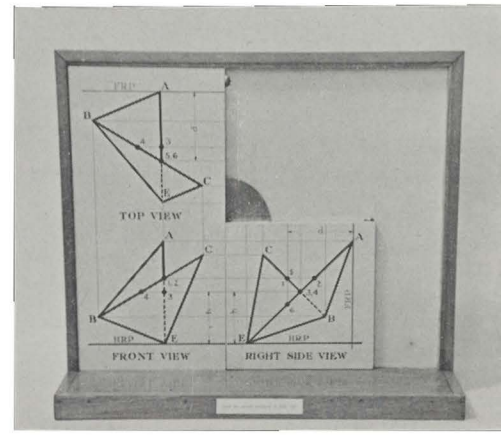

Fig. 2. First Position for a Right-Side View.

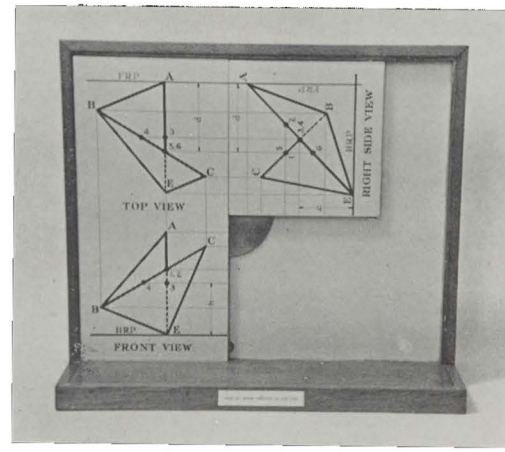

Fig. 3. Second Position for a Right-Side View.

the edges of the object. The plywood plane for the side view is attached to a metal disk which is rotated by means of a crank at the rear, and is held in place by friction.

AN ALL-VIEW MODEL FOR DEMONSTRATING ORTHOGRAPHIC PROJECTION

At the center of the model, Fig. 4, there is a small block in the form of a house. Small celluloid planes are set up in the correct positions for the following views:

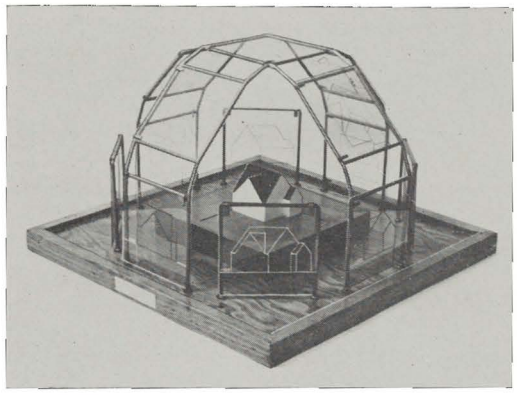

Fig. 4. All-View Model for Orthographic Projection.

top view, front view, right-side view, left-side view, rear view, four auxiliary elevations, right-auxiliary view, left-auxiliary view, front-auxiliary view, and rear-auxiliary view. These planes are hinged onto the supporting framework along one edge in each case, so that certain planes can be revolved until they are parallel to each other to show the proper arrangement of views on the drawing board. In Fig. 5 the framework of an attachment is shown properly related to the house which has been removed from the main model so as to make a clearer photograph. The picture planes are revolved so as to be parallel to an auxiliary elevation (shown on the main model) taken at the left front corner of the building. Figure 5 shows how the top view and two oblique views are related to this auxiliary elevation. Incidentally, two other auxiliary elevations have been drawn as indicated in this band of views.

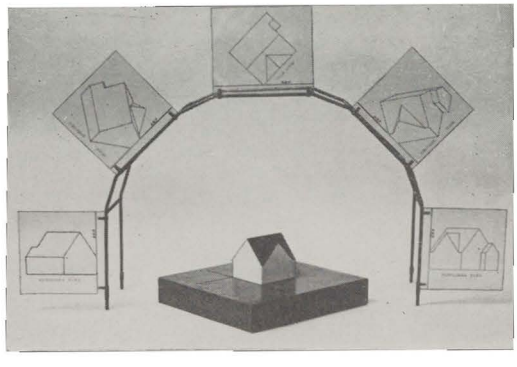

Fig. 5. Attachment for All-View Model.

MODEL FOR DEMONSTRATING THE THEORY OF ISOMETRIC PROJECTION

Figure 6 shows the picture plane in a frontal position with the cube tilted forward so that a body diagonal is horizontal.

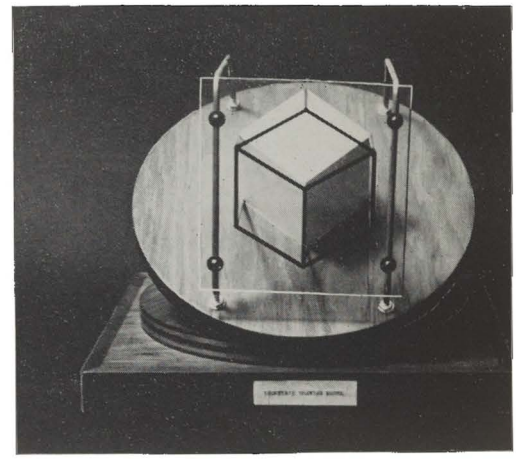

Fig. 6. Isometric Projection.

This is the final position of a cube in the usual explanation of isometric projection. Upon a square sub-base there is mounted a circular base which is in two parts that are hinged together at the front of the picture. The circular base rotates upon the sub-base, and the upper half of it can be let down to a horizontal position. In the usual explanation of isometric projection, the cube is placed in such a position that one face is frontal and the base is horizontal. Then the first operation is to revolve the cube 45° about a vertical axis. The second operation is to tip the cube (and the upper part of the base) forward through an angle of 35° 16′. Two other

ideas of obtaining an isometric projection may be demonstrated. Perhaps the best one is not to tip the cube forward, but to look downward in the direction of a body diagonal. A third explanation can be demonstrated without moving the cube at all, but by using an auxiliary elevation and an oblique view. This means that the observer moves around through an angle of 45° for the auxiliary elevation, and then looks downward along the body diagonal.

FIXED MODEL FOR DEMONSTRATING THE THEORY OF OBLIQUE PROJECTION

In Fig. 7 a plate glass picture plane is parallel to the front face of a white rectangular block. A set of parallel colored

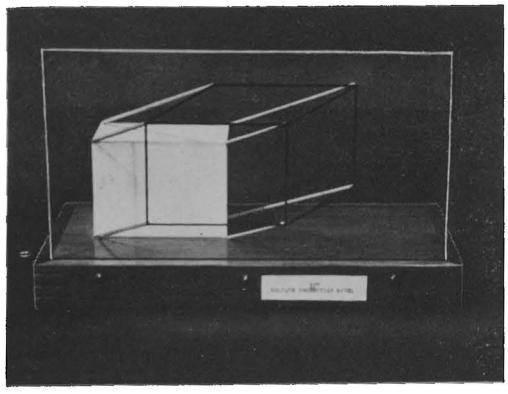

Fig. 7. Fixed Model for Oblique Projection.

wires project obliquely from seven corners of the block to the picture plane upon which the oblique drawing of the block is made in black enamel.

OBLIQUE PROJECTION (MOVING PICTURE)

The background for this model, Fig. 8, is plywood painted black, and the plane which represents the front face of a block is sheet metal painted black. The outline of the front face is shown by lines of white enamel. The lines representing the receding edges, perpendicular to the front face, are painted on narrow black strips of metal. The outline for the back is likewise painted on a narrow piece of metal and loosely riveted to the strips previously mentioned. The receding edges can be rotated around the corners of the front face so that they will appear nearly

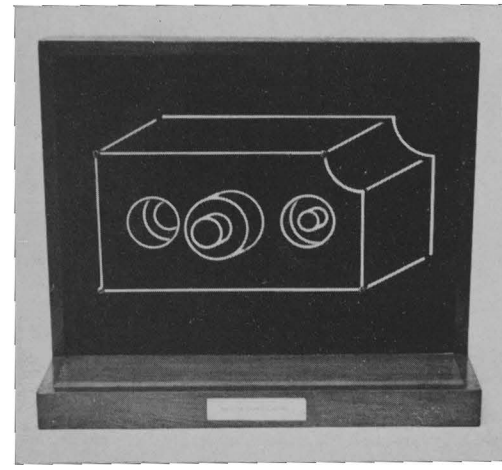

Fig. 8. Oblique Projection (Moving Picture).

horizontal or nearly vertical in the picture, thus showing much of the right side and little of the top, or showing much of the top and little of the right side of the object. The drawing or picture actually seems to move, and is most effective when the mechanism is operated slowly. The model is useful in showing which position of the receding edges will produce the most satisfactory picture. The metal for the front face has two holes through it, right and left, and there are rotating disks immediately behind the front plate. Also, there is an elongated disk at the center of and in front of the front plate. The circles drawn on these disks represent cylindrical holes and projections. The axes on which these disks are mounted are connected with the receding edges by a suitable linkage at the back so that they all revolve in unison, thereby keeping the axes and elements of the cylinder parallel to the receding edges, and showing the changing position of the points of tangency.

MOVING MODEL FOR DEMONSTRATING THE THEORY OF OBLIQUE PROJECTION

This model, Fig. 9, is a combination of the two models previously described. The picture plane is a thick sheet of Plexiglas.

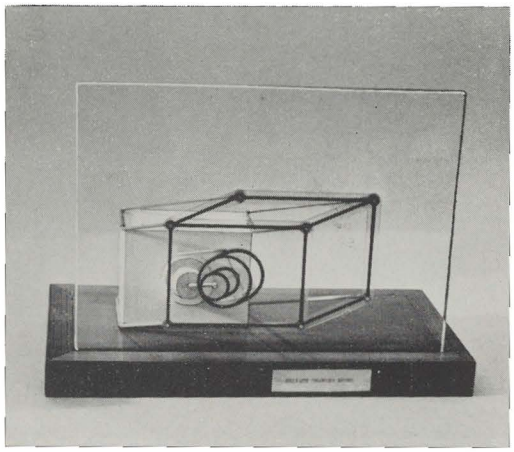

Fig. 9. Moving Model for Oblique Projection.

The plane for the front face and the strips for the receding and rear edges are of thin Plexiglas. The projectors revolve about special hinges mounted along the four receding edges of the block and about a hole in its front face. The form of the picture varies as in Fig. 8, but it is projected from the block as in Fig. 7. This model demonstrates that the projectors remain parallel and make a constant angle with the picture plane, but may make a variable angle with the horizontal. The whole mechanism is operated by a crank formed at the rear end of the wire at the lower left edge of the block. The picture swings around almost in contact with the frontal picture plane and appears to be moving upon it.

MODEL FOR DEMONSTRATING THE DIRECTION OF SIGHT AND THE ORIENTATION OF ANY VIEW OF AN OBJECT

Figures 10, 11, and 12 show a model in which a block is supported on a vertical column. The block has two or more faces

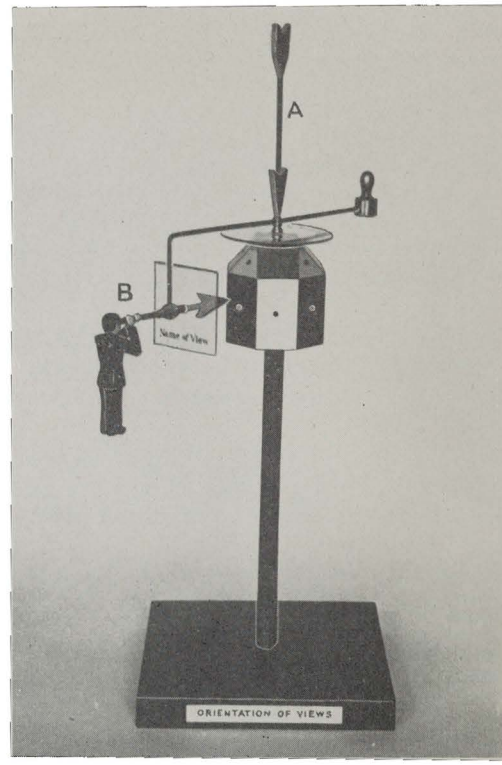

Fig. 10. Direction of Sight for Auxiliary Elevation.

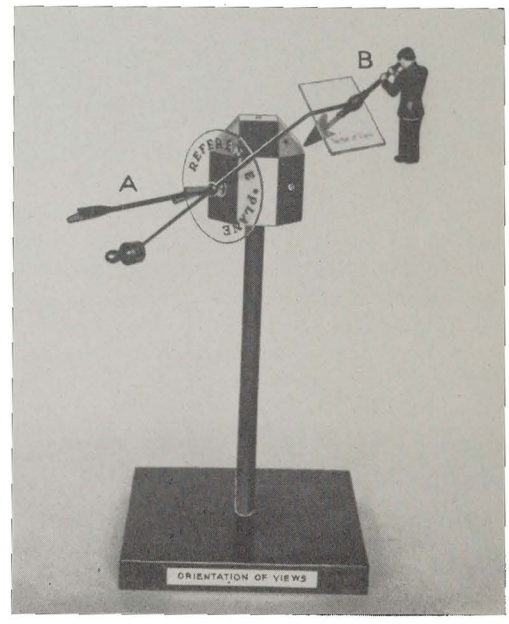

Fig. 11. Direction of Sight for Oblique View.

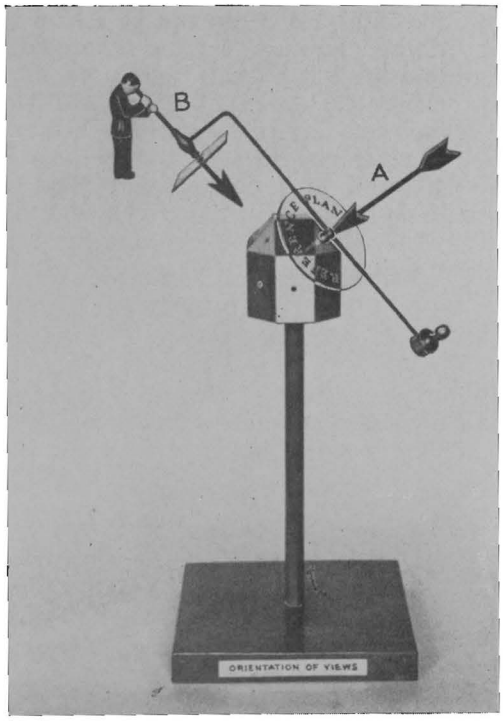

Fig. 12. Direction of Sight for Second Oblique View.

for each of the seven typical directions of a plane. A hole is drilled perpendicular to each face at its center (except the bottom face). The wire frame for holding the arrows, *A* and *B*, at right angles to each other has a short axis which fits into any of these holes and allows the whole combination to revolve about arrow *A* as an axis. The small manikin attached at the end of arrow *B* represents the observer. This arrow rotates in its supporting bearing so that the manikin's body is always in a vertical position. There is a circular transparent reference plane perpendicular to the arrow *A*, and a square picture plane perpendicular to the direction of sight, arrow *B*. The operation of this model is used to show how successive views are made in which there is always a 90° change in the direction of sight. It is used for principal views, auxiliary elevations, right-auxiliary and left-auxiliary views, front-auxiliary and rear-auxiliary views, oblique views, and second oblique views.

Figure 10 shows the location of arrow *A* for any view which may be drawn adjacent to a top view, and the manikin is shown looking in the required direction for an auxiliary elevation. Before moving to this position he was looking in the direction for a front view. Figure 11 shows arrow *A* in the previous position of arrow *B*, and the manikin is looking in the direction shown for a first oblique view, which is one of an infinite number of views which it is possible to obtain during a complete revolution. Figure 12 shows arrow *A* moved to the previous position of arrow *B*, and the manikin in position for a second oblique view, one of an infinite number. The operations described are only a few of the possible combinations which may be demonstrated. Since the axis of the manikin's body always remains in a vertical position he is always properly orientated with reference to any view of the object. The vertical edges of the object always appear to him to be vertical. Orientation of all the views of the object on the drawing board can be easily accomplished by the use of this fact.

AN OBLIQUE DIRECTION OF SIGHT

The large arrow of Fig. 13 is mounted on a stand which is adjustable in height. This arrow can revolve about the horizontal arm of the stand as an axis, and the horizontal arm can revolve on its vertical support. Thus, the large arrow may

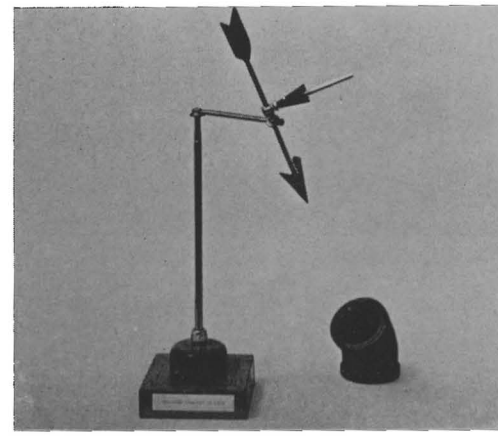

Fig. 13. Oblique Direction of Sight.

be pointed in any direction. There is a shorter arrow attached to the larger one, and it can revolve in a plane perpendicular to the large arrow, so that if the large arrow is placed in an oblique position, the small arrow, which is always perpendicular to it, can be revolved so as to be horizontal, frontal or profile. This combination is used to demonstrate that an auxiliary view must be made before it is possible to construct an oblique view of an object as seen in the oblique direction of the large arrow. Also, it is used to fix the attention upon the particular auxiliary view which is to be used in the solution of the problem.

UNIVERSAL MODEL HOLDER WITH MODEL

The pyramid shown in the holder of Fig. 14 is one of a group of models of the same altitude or length which can be held in any desired position by means of the pivot point at its base and the screw point at its vertex. The axis or center line of the pyramid can be placed in any desired direction by revolving about the vertical

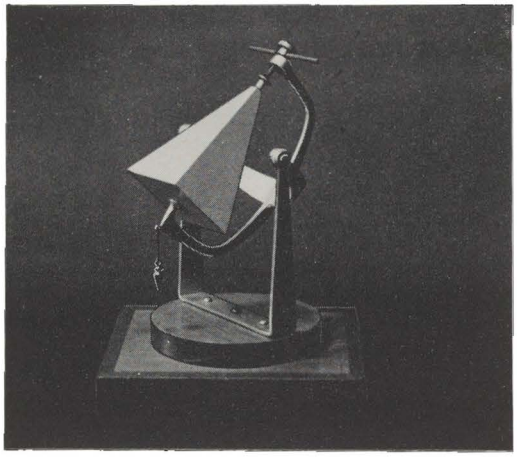

Fig. 14. Universal Model Holder.

axis connecting the circular base of the holder with the square sub-base, and can be made to slope any amount by revolving about the horizontal axis of the holder. The pyramid can be rotated on its axis to make an edge of the base horizontal, frontal, profile or otherwise. Therefore, any model of the group can be set up in any desired position. At the base of the pyramid there is a small plumb bob used for orientation. At the other end and perpendicular to the screw there is a small rod which can be revolved around the axis of the screw to a horizontal, frontal or profile position.

IMPROVED UNIVERSAL MODEL HOLDER WITH LOCUS PLANE ATTACHMENT

The photograph of Fig. 15 was taken to show clearly the side support construction and the detachable bracket for the

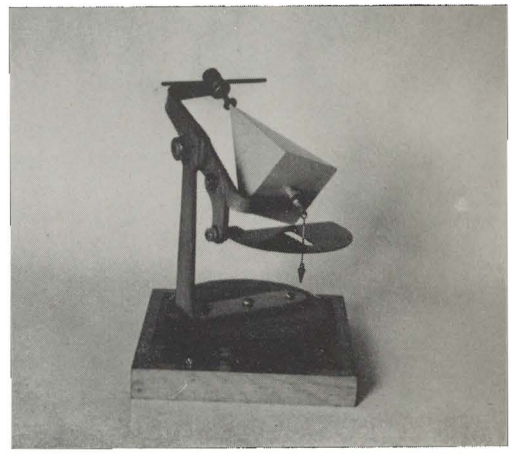

Fig. 15. Improved Model Holder with Locus Plane Attachment.

locus plane, but the other side is always presented to the class so that they may have an unobstructed view of the model. The axes of rotation are the same as in Fig. 14. The locus plane for a corner of the base of the pyramid can be turned about a horizontal axis perpendicular to the bracket which is attached to the arm of the model holder by means of a thumb screw. For any position of the model, the locus plane can be made horizontal. There is a slot in it through which the plumb line may pass.

There is a frame for this model holder which will hold the square sub-base in a frontal or profile position so that the locus

plane may be set up either frontal or profile if the conditions of the problem require it.

ALL-VIEW UNIVERSAL MODEL HOLDER AND ORIENTATOR

Figure 16 shows a combination of the models in Figs. 10 and 15. At the top of the tall, improved universal model holder there is a counter-weighted arm which may be revolved on a vertical axis at *C*. At the lower end of this arm a horizontal

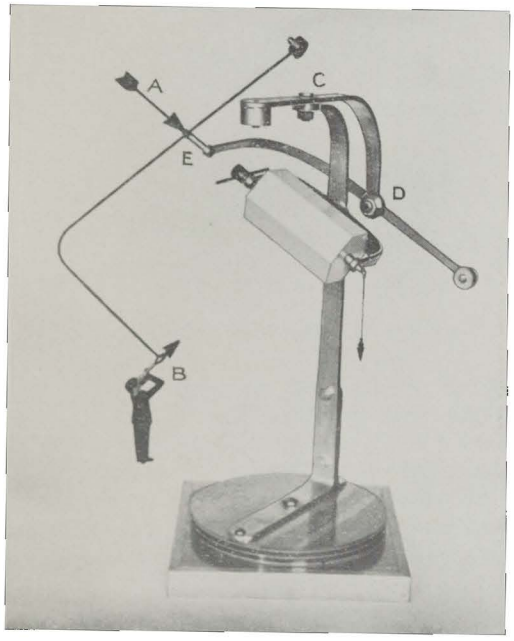

Fig. 16. All-View Universal Model Holder and Orientator.

bolt *D* serves as the axis of revolution for a second counter-weighted arm with a socket at *E* for the short axis at the end of arrow *A*. The bolt at *D* is drilled for a socket of the same size. By revolving these two counter-weighted arms about their axes, it is possible to bring these two sockets into the correct positions for any series of views with a right-angle sequence of changes in the direction of sight.

In the model of Fig. 10 there is one fixed object, and there are 17 sockets, so that it is possible to locate the axis of arrow *A* in 17 positions. The point of arrow *B*, therefore, can describe 17 great circles on an imaginary sphere. In this all-view model a variety of objects can be placed in any position in space, and the axis of arrow *A* can be located in any position whatever in regard to the object. Therefore, the point of arrow *B* can be made to travel in any great circle on an imaginary sphere. The choice of position of the object and the direction of sight for a view is unlimited.

To illustrate a sequence of operations, start with arrow *A* in position for the top view and arrow *B* for the front view, the axis of arrow *A* being in socket *E*, directly above *C*; revolve arrow *B* to a desired direction of sight for an auxiliary elevation; revolve socket *D* around to the point of arrow *B*, transfer arrow *A* to socket *D*, and revolve arrow *B* to any desired position for an oblique view; then move socket *E* to the point of arrow *B*, transfer arrow *A* to this position of the socket and revolve arrow *B* to any desired position for a second oblique view, as shown in the figure.

NORMAL VIEWS AND END VIEW OF AN OBLIQUE LINE

The oblique line *AB* in Fig. 17 is shown as a small rod with the knobs representing *A* and *B*. The following views are shown: top view, front view, right-auxiliary view, auxiliary elevation and oblique view (end

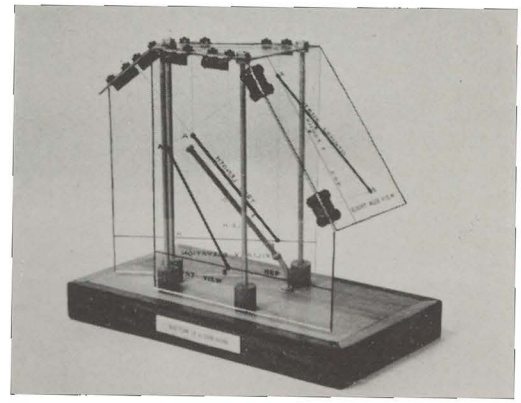

Fig. 17. Normal Views and End View of a Line.

view of *AB*). The right-auxiliary view shows a normal view of *AB* and the angle *F*. The auxiliary elevation at the back of the model shows another normal view of *AB* and the angle *H*. The end view of the line is shown on a small plane which is perpendicular to *AB* at the upper left end. The picture planes are of transparent material. Small brass hinges are used at the right-angle intersections of the planes. All of these planes can be opened into the plane of the drawing board so that all views appear properly related on the flat.

A RIGHT PRISM WITH AN OBLIQUE CENTER LINE

This photograph, Fig. 18, was taken from the rear of the model to show more clearly the auxiliary elevation in which

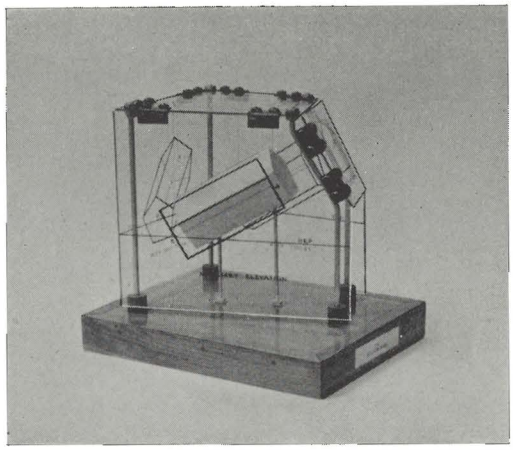

Fig. 18. Right Prism with Oblique Center Line.

the center line *AB* is shown as a normal view. The end view of *AB* is shown on a plane normal to *AB*. The end view of the required prism was drawn on this plane. Then the auxiliary elevation was completed. From these two views, the top view, front view and right-side view were obtained by the usual procedure. All of the picture planes of this model open into the plane of the top view, so that the various views are properly related to each other.

EDGE VIEWS OF A PLANE AND ANGLES WITH HORIZONTAL AND FRONTAL PLANES

An oblique plane *ABC* of Fig. 19 has a horizontal line *AH* and a frontal line *AF* drawn upon its surface. An auxiliary

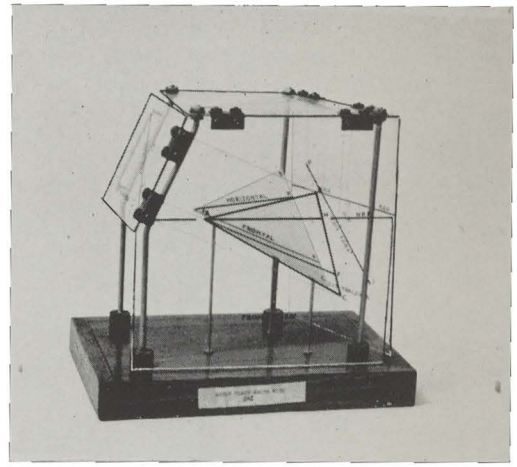

Fig. 19. Edge Views of a Plane.

elevation showing an edge view of the plane and the true size of angle *H* is shown on a transparent picture plane at the right of the model. A left-auxiliary view showing an edge view of the plane and the true size of the angle *F* is shown on a small plane at left of the model. The transparent box can be opened into the plane of the top view so that all the views appear properly related on the flat.

MODEL FOR DEMONSTRATING THE PRINCIPLE OF PERPENDICULAR LINES

In Fig. 20 three metal planes, each having two perpendicular lines painted on it, are mounted on supports. The left plane may be revolved about the frontal line as an axis, and in every position the two lines will appear to be perpendicular in the front view. The second plane may be revolved about a vertical axis, and the right angle will appear to decrease in size to a value of zero. The right plane may

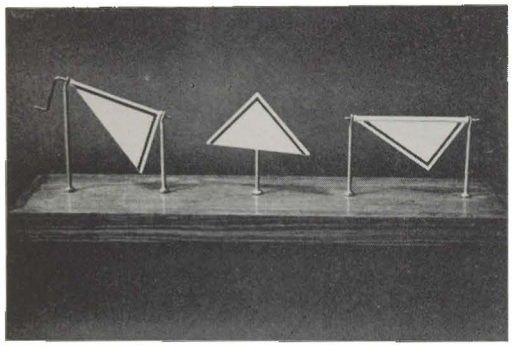

Fig. 20. Cases of Perpendicular Lines.

be revolved about a horizontal axis, and the right angle will appear to increase in size to 180°. Therefore, two perpendicular lines appear perpendicular only in a view which is a normal view of one or both of the lines.

THE COMMON PERPENDICULAR BETWEEN TWO SKEW LINES

In Fig. 21 the skew lines are represented by two pieces of painted hardwood rods. The common perpendicular is a small steel rod which fits very snugly into holes

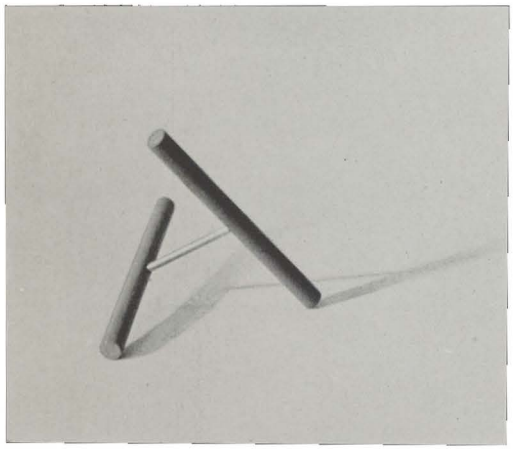

Fig. 21. Common Perpendicular Between Skew Lines.

drilled perpendicularly through the wooden rods. The angle between the skew lines can be varied as desired, and the whole combination can be set up in any position in space. This model is useful for demonstrating the principles involved in finding the common perpendicular by making a normal view and an end view of one of the skew lines.

THE COMMON PERPENDICULAR BETWEEN TWO SKEW LINES BY THE EDGE VIEW METHOD

The common perpendicular of Fig. 22 is a small steel rod connecting two wooden rods representing the skew lines *AB* and

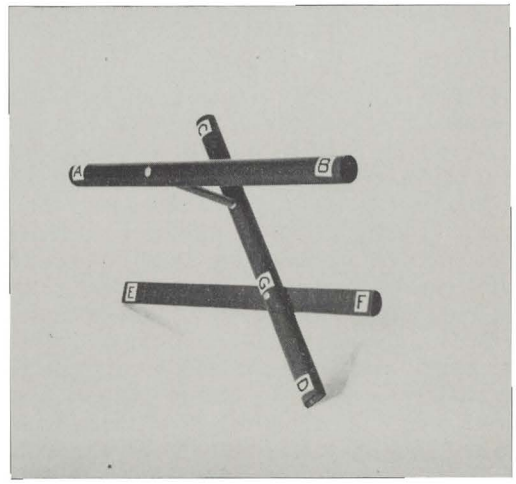

Fig. 22. Common Perpendicular by the Edge View Method.

CD. *EF* and *CD* are split rods riveted together at *G*, so that *EF* can be revolved to coincide with *CD*, or can be made parallel to *AB*. In the illustration *EF* represents a line drawn through the point *G* on *CD* and parallel to *AB*, to set up a plane *CDEF* through *CD* parallel to *AB*. The common perpendicular is found by obtaining an edge view and a normal view of this plane.

MODEL FOR DEMONSTRATING THE STRIKE AND DIP OF ANY PLANE

In Fig. 23 a disk showing the points of the compass is attached rigidly to the base. The supporting *Y* frame for holding the white painted metal plane has a horizontal pointer rigidly attached to its vertical axis.

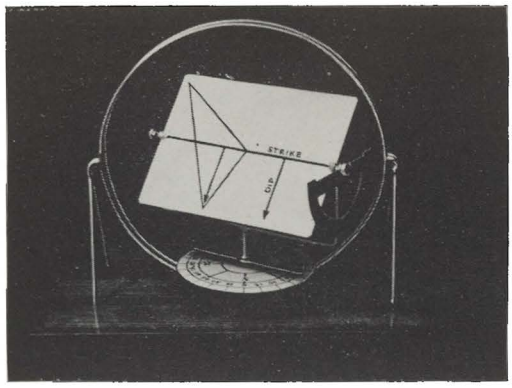

Fig. 23. Strike and Dip of a Plane.

The *Y* frame revolves on its vertical axis so that the strike line, parallel to the pointer, can be made to assume any direction, which may be read as a bearing on the compass disk. The plane revolves on a horizontal axis which is the strike line, and can be made to dip any amount from the horizontal toward the front or toward the rear. A protractor fixed to the frame at one end of the axis shows the dip angle. Since an oblique plane is commonly represented by a triangle, a colored triangle is drawn on the white plane. There is a frontal double ring which may hold a wire in a frontal position and in contact with the plane, so as to show a frontal line of the oblique plane.

ADJUSTABLE PLANE WITH REVOLVING LINES

In Fig. 24 a vertical column with a circular base revolves upon a rectangular sub-base. The support for the plane is attached to the upper end of the column by means of a horizontal bolt with a spring washer. The triangular plywood plane is attached to its support by a short bolt with a spring washer. This bolt is perpendicular to the plane and does not show on its face. The construction of the model makes it possible to revolve the plane of the triangle about the vertical axis and the horizontal axis to any required oblique position. The triangle may be revolved about the axis, perpendicular to itself, as desired. A small rod *AB* which represents a line, suggested by Professor

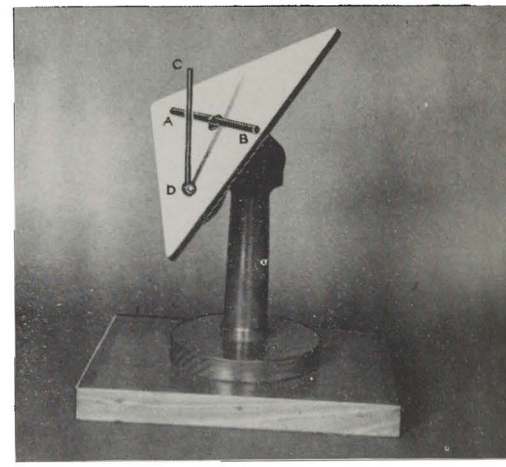

Fig. 24. Adjustable Plane with Revolving Lines.

H. E. Grant, revolves almost in contact with the plane, and can be placed in any position, namely: horizontal, frontal, profile, or oblique.

Another rod *CD* is attached by a pin to a plug which turns in a socket in the plane. This rod can be revolved about two axes at *D* so as to make any angle with the plane, in any direction.

THE POINT AT WHICH A LINE PIERCES A PLANE

In Fig. 25 the line is represented by a painted wire *AB*, and the plane is represented by a painted triangle of sheet metal

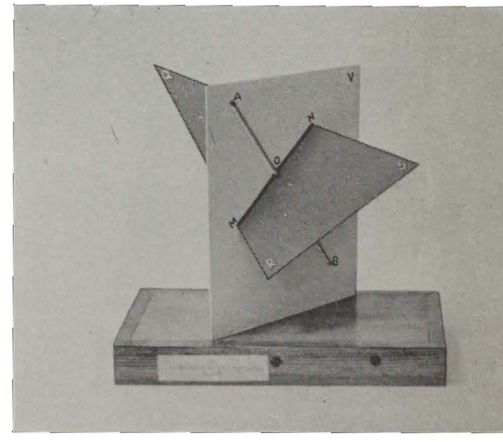

Fig. 25. Point at which a Line Pierces a Plane.

QRS. The cutting plane *V* is vertical and coincides with *AB*. *MN* is the line of intersection of the cutting plane and *QRS*. *O* is the point of intersection of *MN* and *AB*. Therefore *O* is the point at which *AB* pierces *QRS*. By placing the wooden base in a frontal or in a profile position, the use of an orthofrontal or an orthoprofile cutting plane may be demonstrated.

THE INTERSECTION OF PLANES BY THE CUTTING PLANE METHOD

In Fig. 26 the planes are given as the triangles *ABC* and *QRS* shown on the white plywood planes. Two orthofrontal

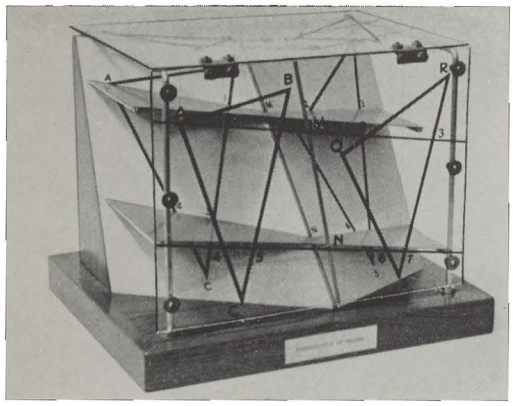

Fig. 26. Intersection of Planes by Cutting Plane Method.

cutting planes are used. The top view and the front view are shown on transparent planes hinged together so that the horizontal plane may be revolved upward and forward into a frontal position to show the relationship of the views and the lines cut from the triangles by the cutting planes. The complete drawing for the determination of the line of intersection *MN* is shown.

THE INTERSECTION OF PLANES BY THE STRIKE AND DIP METHOD

In Fig. 27 the two oblique planes, which represent two veins of ore, are differently colored pieces of sheet metal. Two horizontal cutting planes represent two levels in a mine. There are transparent planes for the top view, for an edge view as an

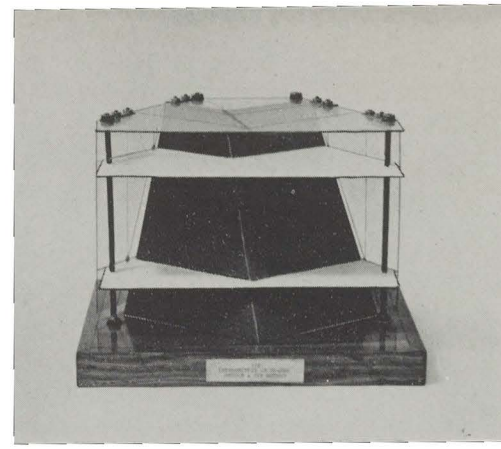

Fig. 27. Intersection of Planes by Strike and Dip Method.

auxiliary elevation at the left, and for an edge view as an auxiliary elevation at the right. The right and left planes are hinged to the top plane so that they can be revolved into a horizontal position to show the solution of the problem.

THE REVOLUTION OF A LINE ABOUT AN AXIS PERPENDICULAR TO A PLANE

This simple but useful model, Fig. 28, consists of a block of wood and a small

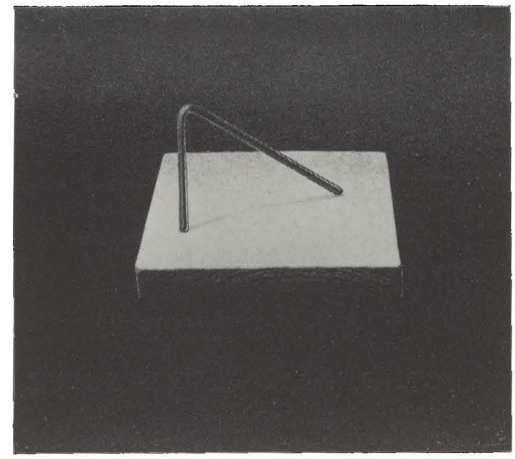

Fig. 28. Revolution of a Line about an Axis Perpendicular to a Plane.

rod bent so that one part of the rod is perpendicular to the white plane face and fits tightly into a hole which passes nearly through the block. The portion of the rod which is inclined to the plane is painted red and represents a line. The white face of the block can be placed in a horizontal, a frontal, or a profile position. If the line is placed in an oblique position, it can be revolved about the axis perpendicular to the plane of the base into a position so that its true length and true angle with the plane may be seen in one of the principal views.

This model may be used to demonstrate the direct method of finding the angle that an oblique line makes with an oblique plane. This angle may be seen in a view which is a normal view of the line and the edge view of the plane without using revolution.

THE REVOLUTION OF A PLANE ABOUT A PRINCIPAL LINE

In Fig. 29 the triangular white metal plane has on its face two intersecting yellow lines, a horizontal blue line and a frontal red line. All these lines are represented by wires attached to the plane.

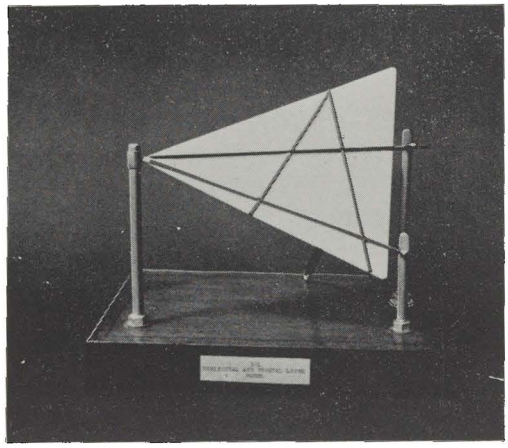

Fig. 29. Revolution of a Plane about a Principal Line.

The wires for the horizontal and frontal lines rest in forks in the supports at the right, and terminate at a ball and socket joint at the top of the left support. A coil spring attached at the back of the plane holds the wires in the forks. This construction makes it possible to revolve the plane upward into a horizontal position or to revolve it forward into a frontal position, so as to show its true shape in either case, and therefore, the true angle between the intersecting lines.

A TRUE ANGLE BETWEEN A LINE AND A PLANE BY THE DIRECT METHOD WITH REVOLUTION

In Fig. 30 a wire *AB* representing an oblique line is bent at its lower end *A* and passes perpendicularly through a white oblique plane so that the wire may be

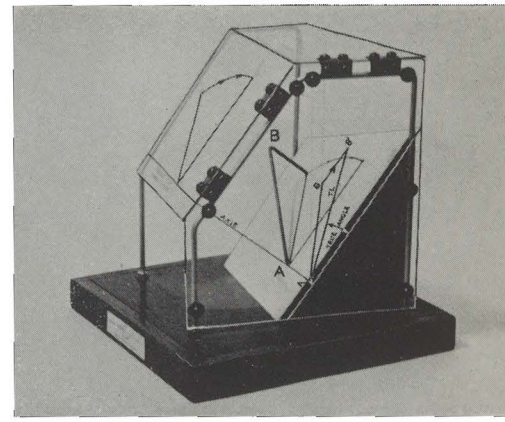

Fig. 30. The Angle between a Line and a Plane.

revolved about an axis at *A*. There are transparent planes for the top view, the auxiliary elevation, and the oblique view. The auxiliary elevation is an edge view of the plane, and the oblique view is a normal view of it. In the oblique view the revolution of the line is indicated, the line having been moved so that it appears in true length in the auxiliary elevation, in which the true angle between the line and the plane is shown only after this revolution.

THE TRUE SIZE OF A DIHEDRAL ANGLE BY THE USE OF A CUTTING PLANE AND REVOLUTION

In Fig. 31 the faces of the dihedral angle are made of plate glass. The edge of the dihedral angle may be considered to be

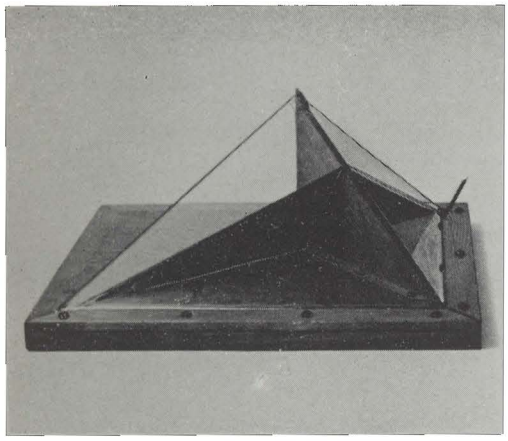

Fig. 31. True Size of a Dihedral Angle.

the hip line of a roof. This line is the upper edge of a vertical piece of sheet metal. Inside the dihedral angle and perpendicular to this upper edge there is a triangular sheet metal plane which practically touches lines on the glass and is split at the vertical plane. It has a horizontal axis at the base of the model, and by means of a small lever at the right end, it can be revolved into a horizontal position, either forward or backward. After revolution, the true size of the dihedral angle will be shown in the top view.

LINES MAKING GIVEN ANGLES WITH HORIZONTAL AND FRONTAL PLANES

In Fig. 32 there are two intersecting right cones of tin with a common vertex and equal slant heights. The four wires represent the possible solutions of this problem. This model shows the right cones which may be generated by the revolutions of the lines shown in Fig. 33.

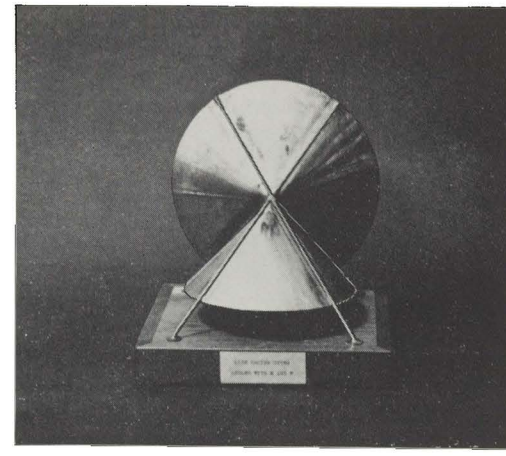

Fig. 32. Lines Making Given Angles with *H* and *F*.

Obviously, the base may be held in various positions to illustrate more clearly several desired setups.

REVOLVING LINES MAKING GIVEN ANGLES WITH HORIZONTAL AND FRONTAL PLANES

In Fig. 33 the base represents a horizontal plane and the back a frontal plane. An oblique line is represented by two wires which are in contact. At the end of one wire there is an axis perpendicular to the horizontal plane, and at the end

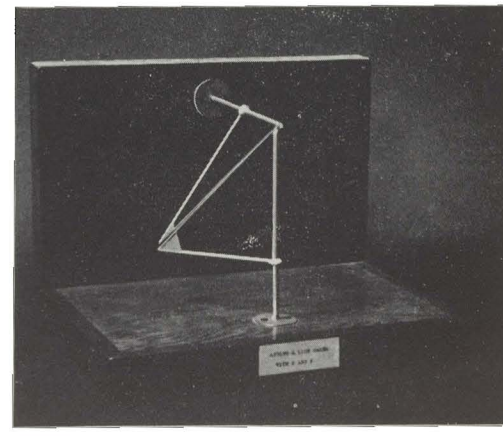

Fig. 33. Revolving Lines Making Given Angles with *H* and *F*.

of the other an axis perpendicular to the frontal plane. One of these wires which represents the oblique lines can be revolved about its vertical axis to a position parallel to the frontal plane, thereby showing its true length and the angle with any horizontal plane. The other wire which represents the oblique line can be revolved about its orthofrontal axis to a horizontal position to show the same true length and the angle with any frontal plane.

This model is useful also in demonstrating the reverse problem: having given the angles a line makes with a horizontal and a frontal plane, to find the position of the line in space. Select two suitable wire-forms, and place in a horizontal position the one having the given angle F, and in a frontal position the other having the given angle H; then revolve them so that they will come into coincidence in the position shown in Fig. 33, or in another position in which the resulting oblique line slopes downward, backward to the right. The axes of these wires are held in their holes by friction. There are four of these wire-forms which are made with the following angles: 30°, 45°, 60°, and 75°. Any two of them may be placed in the model. If a pair is selected for which the sum of the angles is greater than 90° the operation of the model will demonstrate that the assumed conditions are impossible.

THE INTERSECTION OF TWO CYLINDERS BY THE CUTTING PLANE METHOD

In Fig. 34 the intersecting cylinders are made of tin and painted in different colors. The slice made by the cutting plane, parallel to both axes, is painted in a third color. The plane cuts two elements from each cylinder, and the intersections of the elements determine four points on the line of intersection of the cylinders.

THE INTERSECTION OF TWO CONES BY THE CUTTING PLANE METHOD

In Fig. 35 the intersecting cones are of tin and are painted different colors. A cutting plane, which is painted a third color, cuts two elements from each cone, and their intersections determine four points on the curve of intersection of the cones.

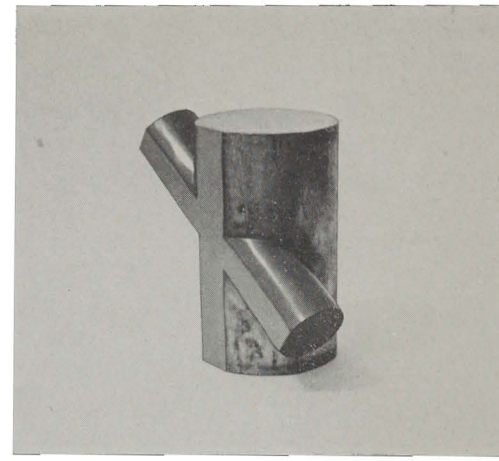

Fig. 34. Intersecting Cylinders and Cutting Plane.

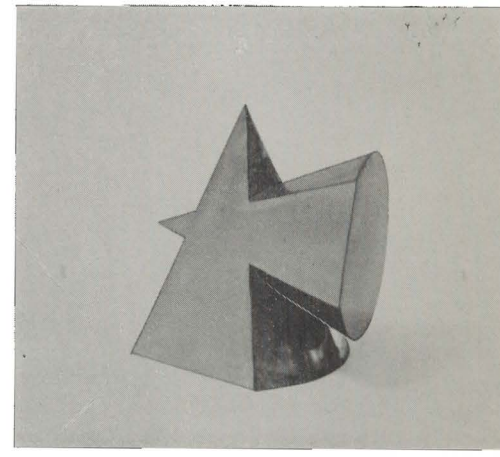

Fig. 35. Intersecting Cones and Cutting Plane.

ADJUSTABLE MODEL FOR A CYLINDER

In this string model, Fig. 36, the upper and lower bases of the cylinder are circles in parallel planes. These planes are attached to a central column by means of hinges, the pins of which pass through the centers of the circles. The column, which

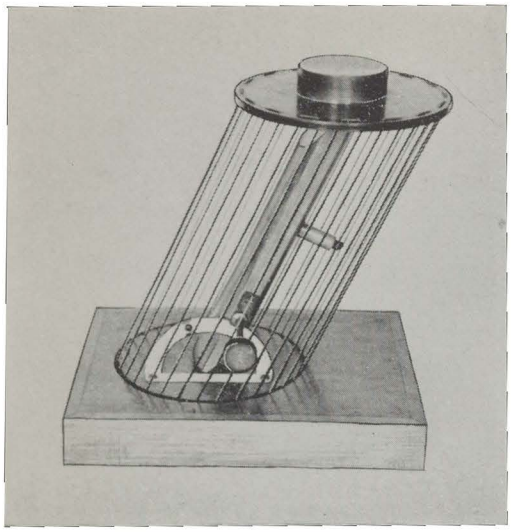

Fig. 36. Adjustable Model for a Cylinder.

is at the axis of the cylinder, can be inclined to the base at any angle from 20° to 90°. The elements of the cylinder all have the same constant length; therefore, the construction is such that the bases remain parallel.

ADJUSTABLE MODEL FOR A CONE

The axis of this cone, Fig. 37, is a rod perpendicular to a short shaft which turns

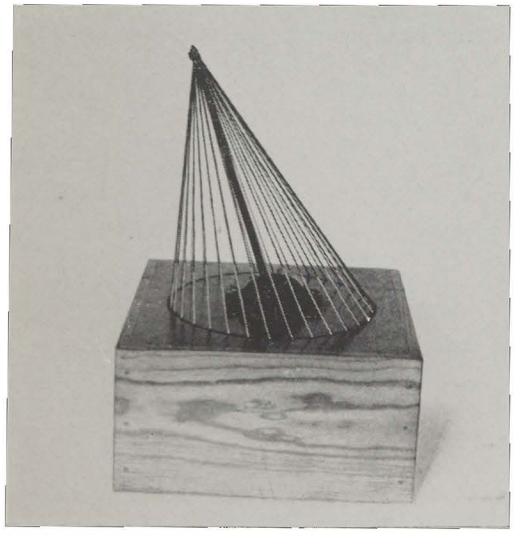

Fig. 37. Adjustable Model for a Cone.

in a horizontal bearing in the base of the cone. The elements are strings which pass through an eye at the tapered end of the rod and are tied to form the vertex of the cone. The strings pass through small holes in the base, and are kept taut by small lead weights in the base of the model. The axis can be set at any angle with the base from 90° to 30° so as to demonstrate a right cone or any oblique cone within its range of adjustment.

THE INTERSECTION OF A CYLINDER AND A CONE

In Fig. 38 cutting planes which will cut elements from both surfaces must be parallel to the axis of the cylinder and

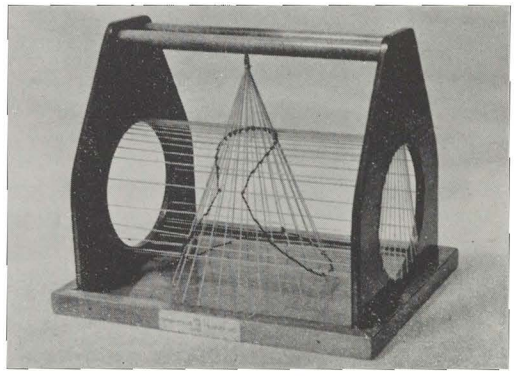

Fig. 38. Intersection of Cylinder and Cone.

pass through the vertex of the cone. The edge views of the cutting planes are indicated by strings at the right end of the cylinder. Most of the strings representing the elements are those used in the solution of the problem. The elements intersect each other at the points shown. The double-curved line of intersection is indicated by a string which passes over these points.

INTERSECTION OF INCLINED CYLINDER AND CONE

In Fig. 39 cutting planes were selected so as to cut elements from both surfaces. Therefore, the one line common to all the cutting planes to be used passes through the vertex of the cone and is parallel to

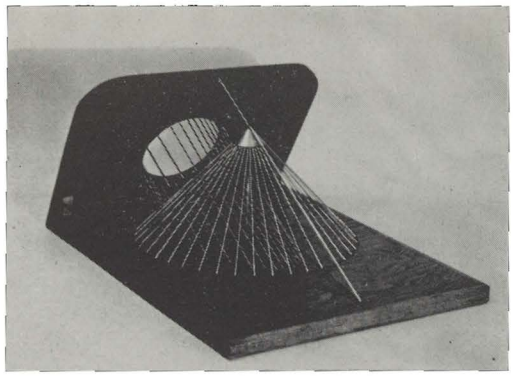

Fig. 39. Intersection of Inclined Cylinder and Cone.

the axis of the cylinder. This line appears as a point on the inclined plane at the end of the cylinder. The edge view of each cutting plane, shown as a line on this inclined plane, passes through this point. The third line of each of these planes is shown on the horizontal base. For any cutting plane the line on the inclined plane and the line on the base determine elements on the cylinder and on the cone which intersect to determine points on the line of intersection.

THE INTERSECTION OF TWO OBLIQUE CYLINDERS

In Fig. 40 one of these cylinders has a circular base, and the other has an elliptical base and a circular right section. The construction is indicated for setting up cutting planes which will cut elements from each cylinder. The cutting planes are shown by parallel lines drawn in the planes of the bases, and the elements shown by strings are those obtained in the solution of the problem. The double-curved line of intersection is shown by a string which passes through the points of intersection of the straight strings.

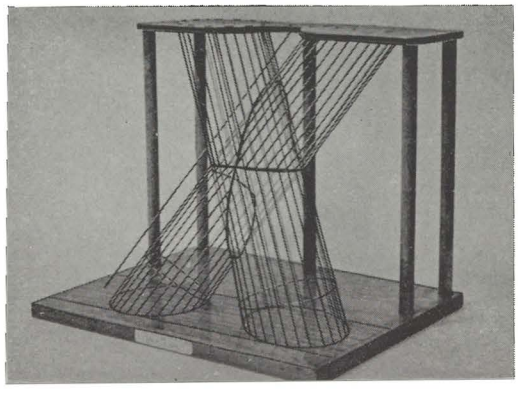

Fig. 40. Intersection of Oblique Cylinders.

A CONNECTING ROD END AND TORUS

Figure 41 shows a wooden model of a rectangular connecting rod end which was left after turning a prismatic bar down to a small cylinder with a large fillet. The surfaces of the fillet and the large annular torus were generated under the same conditions. They can be brought into contact to show that the curves on the faces of the connecting rod end represent the intersection of the faces of the prism and the torus.

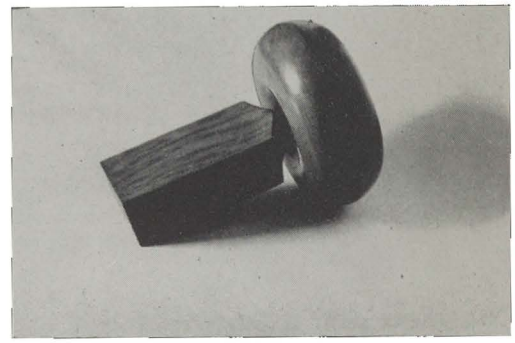

Fig. 41. Connecting Rod End and Torus.

THE INTERSECTION OF SURFACES BY THE CUTTING CYLINDER METHOD

Figure 42 shows the intersection of a paraboloid of revolution and a right cylinder whose axes are neither parallel nor intersecting. The right cylinder is continued upward from its circular base to a horizontal plane, where the section is an ellipse. Two transparent cutting cylinders are shown, one of them being tinted.

These cylinders have their circular bases on the surface of the paraboloid, and their axes parallel to the axis of the cylinder. They are oblique cylinders whose upper bases are circles in the plane of the elliptical section of the right cylinder. The

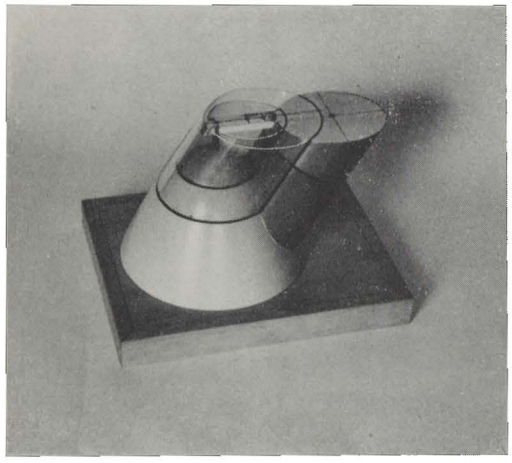

Fig. 42. Intersection by Cutting Cylinder Method.

intersection of each circle with the ellipse determines the upper end of two elements common to the cutting cylinder and the given cylinder. The lower ends of these elements are points on the curved intersection of the surfaces.

A PRACTICAL CONVOLUTE SURFACE

This simple model, Fig. 43, which represents a convolute type of surface, is made of a tin can, both ends of which have been removed. One end remains nearly a circle, but the other end has been changed to an elliptical form. By the method of tangents to the plane curves near the ends, the elements of the convolutes were determined. These are drawn on a white sheet of paper which is wrapped around the tin, and fits perfectly. Unwrapping the paper cover illustrates the development of the convolute surface. The surface is made up of several convolutes which blend into each other to form a single curved surface.

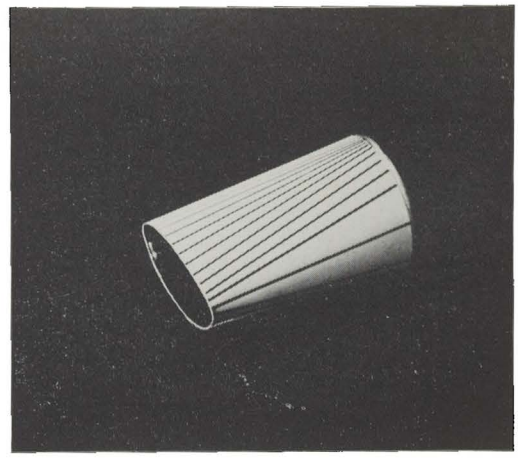

Fig. 43. A Practical Convolute Surface.

A COMBINATION OF SINGLE-CURVED SURFACES

The plywood ends of this model, Fig. 44, are mounted on a horizontal shaft which can be turned in its supports so that all parts of the surface can be examined.

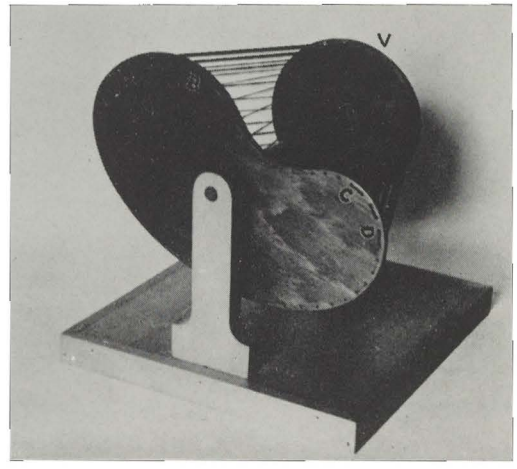

Fig. 44. Combination of Single-Curved Surfaces.

Various portions of the edges of the plywood are painted red, yellow, green, and blue to match the color of the strings used for the different kinds of surfaces which blend into each other so as to form a single developable surface, which is similar to

that described by Hachemeister and Cefola. One end is circular and the other end is a kidney-shaped curve which is made up of elliptical and circular arcs. The elements for the various single-curved surfaces were found by the method of parallel tangents to the end curves. The portion from A to B is an oblique cylindrical surface; that from B to C is a conical surface with V as its vertex and the reverse curve BC as its base; that from C to D is the frustum of an oblique cone with its base at the circular end. The portion for the ellipse from A to D is a convolute surface made up of several convolutes whose edges of regression are not determined. There is a paper cover for this surface which can be unrolled to show its development.

A CONICAL ELBOW

Figure 45 shows a wooden model of a conical elbow, and Fig. 46 shows the same model after each piece has been revolved a half-turn with reference to the adjoining piece to demonstrate that an elbow of this type can be made by cutting the

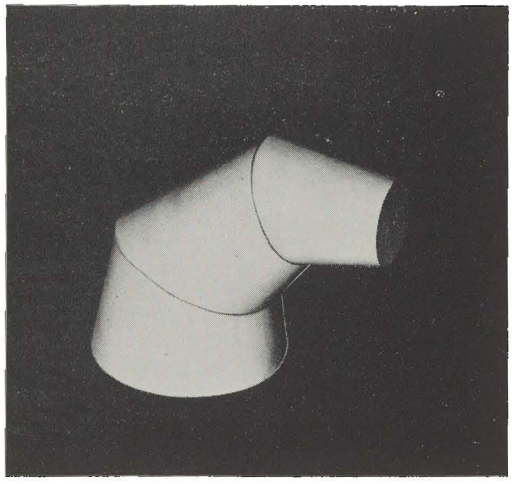

Fig. 45. A Conical Elbow.

parts from a right circular cone. Since the sections are ellipses, the two faces in contact will fit together perfectly if turned 180° with reference to each other. Adjoining pieces are held together by a hidden screw, set perpendicular to the

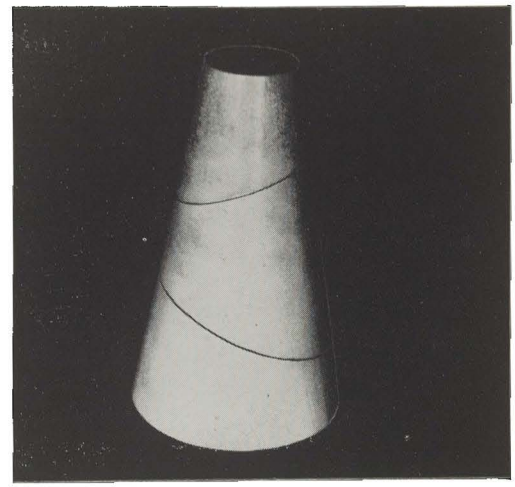

Fig. 46. Cuts for a Conical Elbow.

elliptical sections at their centers, the screw having a short coil spring under its head.

AN ADJUSTABLE MODEL FOR A HYPERBOLIC PARABOLOID OF ONE GENERATION

In this model, Fig. 47, a plane can be twisted so as to become a warped surface. Yellow elastic hat cord is used for all the elements except the central one, which is a brass wire. During the twisting of one

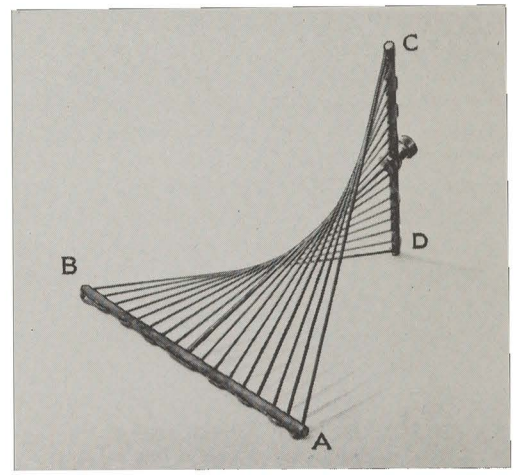

Fig. 47. Adjustable Hyperbolic Paraboloid of One Generation.

directrix with reference to the other, the hat cord stretches to take care of the various increases in length of the elements. The wire is attached rigidly to one directrix, *AB*, and has a clamping device at the other end.

AN ADJUSTABLE MODEL FOR A HYPERBOLIC PARABOLOID OF TWO GENERATIONS

In Fig. 48 the two halves are hinged to a rod so that the upper half can be rotated into the plane of the lower half, in which case the warped surface becomes the plane.

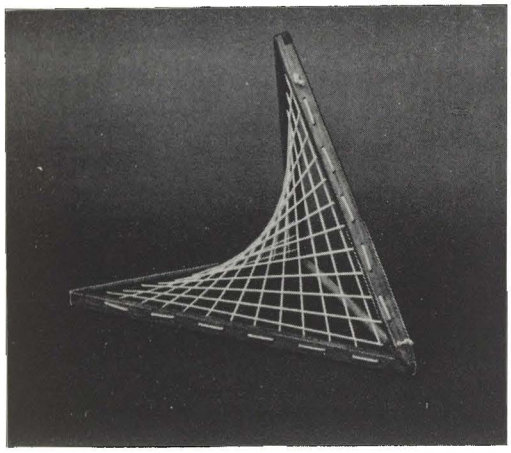

Fig. 48. Adjustable Hyperbolic Paraboloid of Two Generations.

The various positions between these extremes will show different hyperbolic paraboloids, each having elements of two generations. The elements are of elastic hat cord.

CARD MODEL OF A HYPERBOLIC PARABOLOID

Figure 49 shows a preliminary model in which the upper edges of a series of equally spaced parallel cards represent the warped surface. It may be used to demonstrate that the lines which join corresponding equally spaced points on the directrices *AB* and *CD* are the elements of a hyperbolic paraboloid. The equally spaced planes cut the directrices at equally

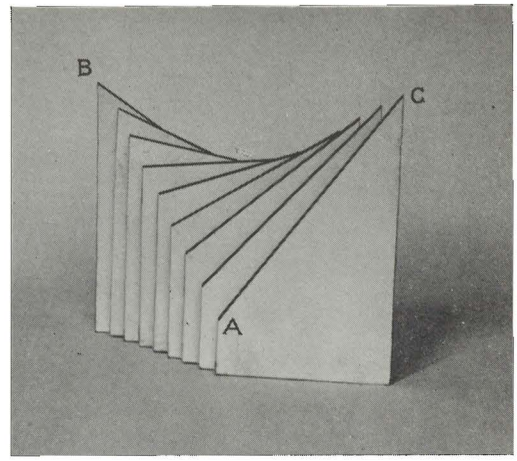

Fig. 49. Card Model of Hyperbolic Paraboloid.

spaced points. Any plane of the series may be considered to be the plane director of the surface. In an edge view of the cards the elements appear to be parallel.

STRING MODEL OF A CYLINDROID

In Fig. 50 the curves on the plywood ends are a semicircle and a semiellipse of the same height. These are the directrices for the surface, and the base represents the horizontal plane director, to which all the elements represented by the strings are parallel.

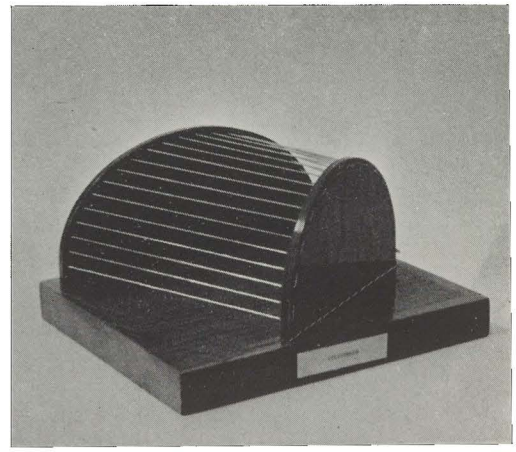

Fig. 50. Cylindroid.

A RIGHT CONOID

Figure 51 shows a horizontal circle as one directrix and the upper edge of a horizontal bar as the other directrix of a

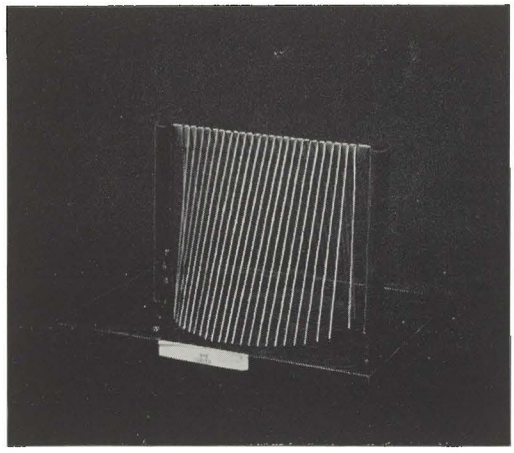

Fig. 51. Right Conoid.

right conoid. The plane director (not shown) is perpendicular to the horizontal bar. The elements are represented by strings which are parallel to the plane director.

A RIGHT-HAND RIGHT HELICOID

This helicoid, Fig. 52, is represented by two helices and 24 elements for one turn.

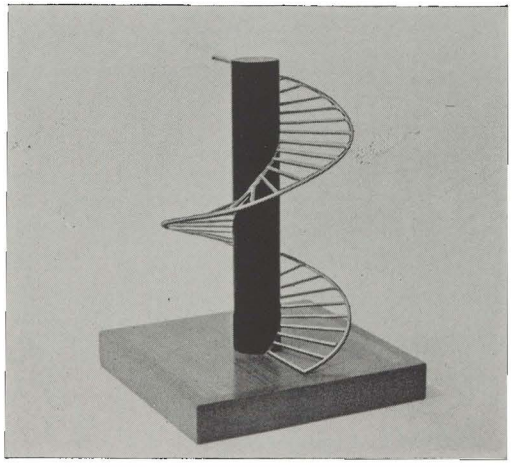

Fig. 52. Right Helicoid.

One and one-half turns of the helicoid are shown. The central core is made of wood, one and one-half inches in diameter, which is painted black. The small helix is a stripe of aluminum paint, and the large helix is a wire. The diameter of the large helix is six inches, and its lead is six inches. The elements are represented by straight wires which were driven into holes carefully drilled into the core at the proper spacing.

OBLIQUE HELICOIDS (A GIANT SCREW THREAD)

Figure 53 shows one and one-half turns of a 60° double V-thread. One thread is shown completely by the elements of two oblique helicoids. The beginning of the other thread is shown. Each helicoid is represented by 24 elements to the turn. The central core is made of wood one and one-half inches in diameter, painted black, on which the small helices are stripes of aluminum paint. The lead is six inches. The large helix is about six and three-quarters inches in diameter, and is made of wire, attached to the elements. Each

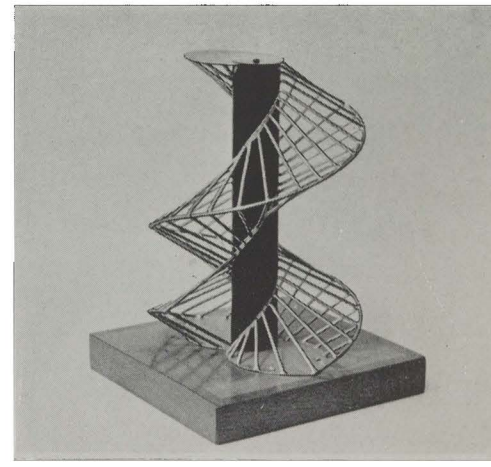

Fig. 53. Oblique Helicoids (Giant Screw Thread).

wire for an element of the upper and the lower helicoids is bent to form a large staple with an angle of 60°. The ends of these staples are driven into holes carefully drilled in the central core. Pieces of metal at the top and the bottom of the

core show right sections of the screw thread, each curve being part of a spiral of Archimedes.

THE COW'S HORN (CORNE DE VACA)

Figure 54 shows two parallel semi-ellipses as the curved directrices. The third directrix is a straight line, *AB*, perpendicular to the planes of the ellipses,

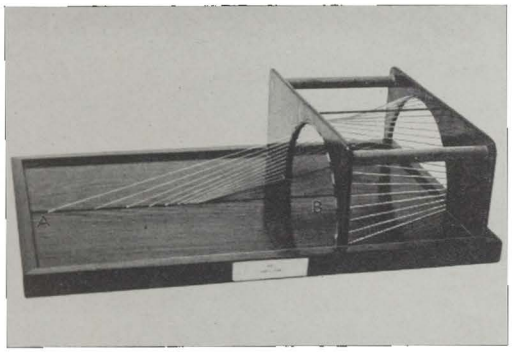

Fig. 54. Cow's Horn (Corne De Vache).

and in the plane of the base. Eleven of the elements are represented by strings which touch all three directrices showing clearly the twist in the surface at the left portion of the model. That portion of the surface between the two ellipses is the only part of any practical use.

ADJUSTABLE HYPERBOLOID OF REVOLUTION

Figure 55 shows a string model having 32 elements. The axis is a small rod which is surrounded by a compression coil spring which keeps the strings taut. By rotating the upper base, the inclination of the strings is made to vary, and consequently the gore diameter of the hyperboloid is reduced and the total height is decreased, the length of the strings being constant. The range of operation is from a cylinder through various hyperboloids to practically a right cone of two nappes. Counter-clockwise rotation of upper base produces hyperboloids of the generation shown. Clockwise rotation produces surfaces of the other generation. The clamping screw

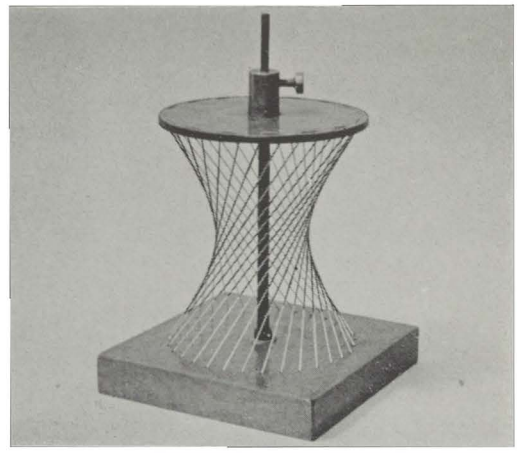

Fig. 55. Adjustable Hyperboloid of Revolution.

is used to hold the upper base in any desired position, but the most impressive demonstration is obtained by slowly rotating the upper base in both directions through the full range.

ADJUSTABLE HYPERBOLOID OF REVOLUTION WITH TANGENT SURFACE

Figure 56 shows an elaboration on the previous model. When the upper base is in its highest position, the 32 strings connecting the circles represent the elements

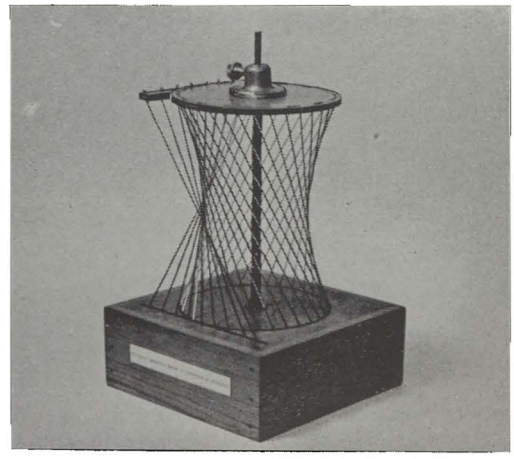

Fig. 56. Adjustable Hyperboloid of Revolution with Tangent Surface.

of a cylinder. The nine strings connecting the two straight lines tangent to the circles represent a plane tangent to the cylinder, suggested by Professor F. M. Porter. Small weights are attached to the lower ends of eight of these strings to provide for an increase in the lengths of the visible portions. Two colors of string are used. During the operation of the model by twisting the upper base clockwise or counterclockwise the following conditions are obtained. First, a plane tangent to a cylinder. Second, a variable hyperbolic paraboloid tangent to a hyperboloid of revolution of variable gorge diameter. Third, practically a plane tangent to the two nappes of a cone when the upper base has been revolved through an angle of 180°.

HYPERBOLOIDS FOR SKEW GEARING

In Fig. 57 two equal hyperboloids of revolution, which represent the pitch surfaces, are shown tangent to each other

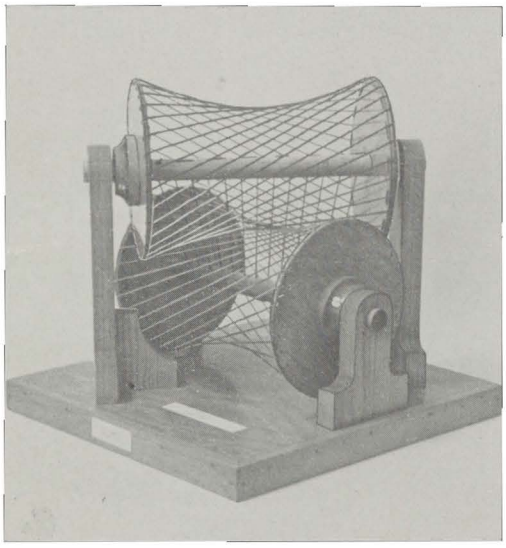

Fig. 57. Hyperboloids for Skew Gearing.

along an element of each. Thirteen normals to these surfaces equally spaced along the elements of tangency are shown intersecting the two axes. These common normals represent a hyperbolic paraboloid. Although the hyperboloids are fixed, the elements on each side of the line of tangency indicate very clearly the endwise slipping inherent to this form of gearing.

In order to bring the hyperboloids into contact it was necessary to bring the strings over the outside edges of the disks by sawing small slots adjacent to the holes. A special jig was constructed for this operation. Three colors of strings were used.

AN ELLIPTICAL HYPERBOLOID

Figure 58 shows the double generation of a hyperboloid for which the bases and the gorge section are ellipses. This is the

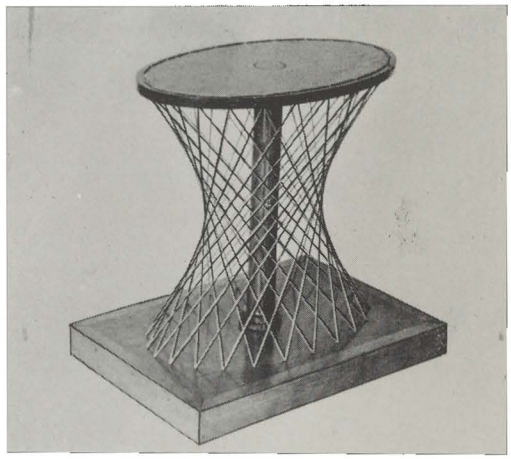

Fig. 58. Elliptical Hyperboloid.

general type of a hyperboloid, which can be generated by moving the generatrix so that it continuously touches three straight-line directrices, no two of which are parallel or intersecting. However, the model was actually designed by using the three ellipses as the directrices.

A PORTION OF AN ELLIPTICAL HYPERBOLOID

The strings in Fig. 59 represent the elements of a hyperboloid generated by moving a straight line so that it remains in contact with three straight-line directrices, *AB*, *CD*, and *EF*, no two of which are

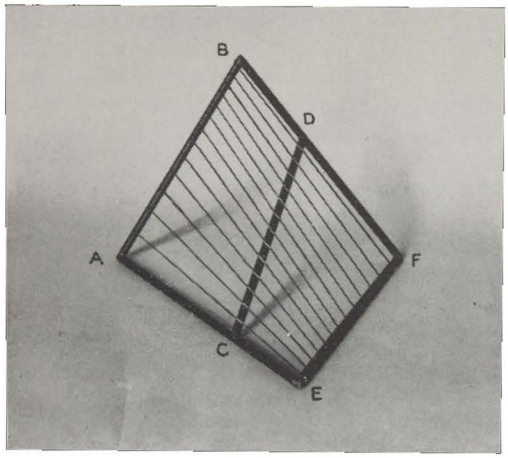

Fig. 59. Portion of Elliptical Hyperboloid.

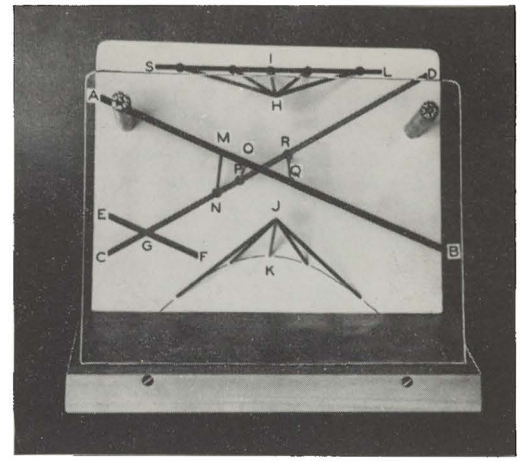

Fig. 60. Shortest Grade Lines between Parallel Planes.

parallel or intersecting. The warped surface is a small portion of an elliptical hyperboloid. Upon casual inspection, it may appear to be a hyperbolic paraboloid, but closer examination will reveal that all the elements cannot be parallel to any plane which could be used as a director for the surface.

SHORTEST GRADE LINES BETWEEN THE PARALLEL PLANES OF SKEW LINES

The model, Fig. 60, has a painted plywood oblique plane at the rear, and a plate glass plane parallel to and three inches in front of it. At point *H* there are five horizontal wires representing lines connecting these two planes. The shortest one, *HI*, is perpendicular to the strike-line *SL*. At point *J* there are five wires making an angle of 45° with the horizontal. The shortest one, *JK*, is also perpendicular to the strike-line *SL*. *AB* is a line in the glass plane and *CD* is a line in the plywood plane. The horizontal line *QR* connecting these skew lines is parallel to *HI*, and the 45° line *MN* is parallel to the line *JK*. There is also a common perpendicular *OP* connecting the skew lines.

For a general setup, the base of this model should be placed so that the parallel planes will be in an oblique position.

THE SHORTEST CONNECTIONS ON GRADES BETWEEN MINE WORKINGS

In Fig. 61 the oblique lines *AB* and *CD* represent two workings in a mine. The white oblique plane passes through *CD* and is parallel to *AB*. This plane was established by the line *CD* and the intersecting line *EF* which is parallel to *AB*. There are four equally spaced lines connecting the skew lines, and each one is perpendicular to the strike-line or upper

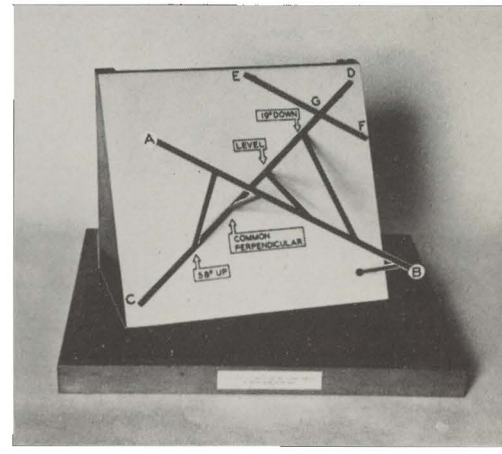

Fig. 61. Shortest Connections on Grades between Mine Workings.

edge of the oblique plane. Going from *CD* toward *AB*, the left line slopes upward 58°, the second slopes upward and is the common perpendicular, the third is level (zero grade), and the fourth slopes downward 19°. It is interesting to note that all of the shortest grade lines connecting two skew lines form the elements of a hyperbolic paraboloid, and in an auxiliary elevation which is an edge view of the plane *CDEF*, they will appear to intersect each other at one point.

A QUARTER-TURN BELT DRIVE WITH AND WITHOUT A GUIDE PULLEY

Figure 62 shows a large pulley on a vertical shaft and a small pulley on a horizontal shaft with a guide pulley properly placed so that the belt drive operates

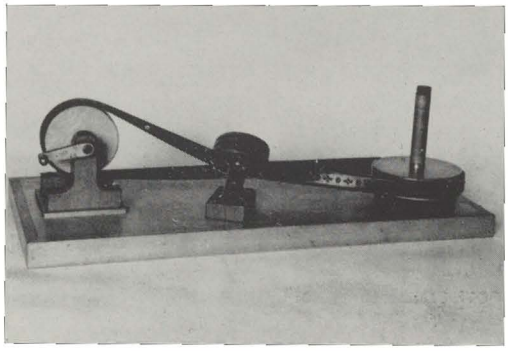

Fig. 62. Quarter-Turn Belt Drive with Guide Pulley.

satisfactorily with either direction of rotation. The guide pulley bracket is attached to the base of the model by two pins and a locking device so that it can be removed easily from the base. The block at the bottom of the vertical shaft is attached to the base in the same manner. The vertical shaft is nearer to one of the pins than to the other, so that if the block is turned around 180°, the distance between the two pulleys is increased sufficiently to use the same belt for a quarter-turn drive without the guide pulley. In this case only one direction of rotation is possible. For a second set-up the large pulley can be placed above the spacer on the vertical shaft. By moving the vertical shaft to the holes toward the rear of the base, a belt drive with a third and fourth position of the large pulley can be demonstrated. There are four combinations of relative directions of rotation of the two pulleys without using the guide pulley.

MODEL FOR COMPARING THE SHAPES OF CURVES BY PERSPECTIVE

At the top of this model, Fig. 63, there is a lantern slide of a parabola. At the bottom of the model there is a drawing of a shorter portion of a parabola to three times the scale of that on the lantern slide.

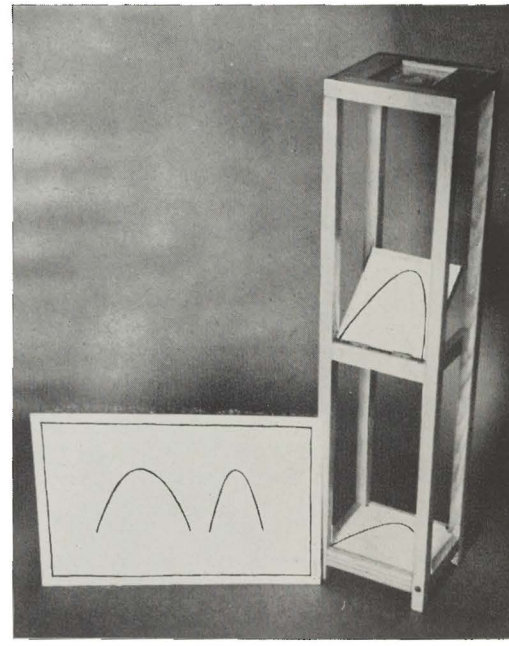

Fig. 63. Comparison of Curves by Perspective.

These shapes probably do not appear to be the same on the drawing shown in the photograph. If the eye is placed about a foot above the lantern slide, the two curves can be made to coincide in perspective. At the center of the model, a portion of a parabola of intermediate size is mounted on a hinged trapdoor which can be brought

into position to be checked with the parabola on the lantern slide in the same manner. This model indicates that all parabolas have the same shape, differing only in scale and length. Other curves may be placed in perspective and checked in the same manner to determine whether they are of the same shape, but to a different scale. For example, all circles have the same shape, but all ellipses do not.

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