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No. 2220: May 22, 1922

The Texas Mathematics Teachers' Bulletin

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

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MATHEMATICS FACULTY OF THE UNIVERSITY OF TEXAS

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A METHOD OF FACTORING TRINOMIALS

BY AN EX-HIGH SCHOOL INSTRUCTOR

May I offer a protest against a method of factoring given in Wentworth's *New School Algebra* page 98? I have long been faintly conscious that the method on these pages was undesirable, but I think never until lately have I really known that any teacher of the text took seriously the suggestions there given for factoring a trinomial like $8x^2+14x-15$. The method is not only of needless length and complexity, but it seems to me to defeat as much as lies in its power one of the purposes for which mathematics is taught. If algebra does not develop the student's judgment and enable him to acquire mental acumen, little reason for teaching the subject remains. So why use a machine to factor the trinomial and acquire nothing but its factors, when another process would obtain the factors, often more quickly, and would give as a by-product greater facility in dealing with numbers and keener insight into factoring itself?

But to illustrate this method, take the trinomial $8x^2+14x-15$. Instead of noticing the factors of 8 and 15 and considering mentally or even in writing the possibilities of arranging them in cross products (as $\begin{matrix}4 & 5\\ 2 & 3\end{matrix}$ and $\begin{matrix}4 & 3\\ 2 & 5\end{matrix}$) and selecting the arrangement that gives 14 for the sum of the cross products, we must proceed as follows:

$$64x^2+8\times14x-8\times15.$$

For $8x$ put in z. We have then

$$z^2+14z-120.$$

This we see equals $(z+20)(z-6)$.

Now replace z by $8x$ and we get

$$(8x+20)(8x-6).$$

Next divide this by 8, broken into factors:

$$\frac{(8x+20)}{4}\quad\frac{(8x-6)}{2}$$

And at last, after a mechanical process, we get the factors of $8x^2+14x-15$ to be

$$(2x+5)(4x-3).$$

Can't you imagine the student who has been carried so needlessly far away from his original starting point now reaching after his ever accessible "answer book" to see if his long pursued factors are correct? His mind must have been rather on the process than on the original trinomial. I think and I very much fear that he will not after such a proceeding check his work himself.

The method would not be quite so bad had we been told to use 2 instead of 8 as a multiplier. This process is sure to produce numbers unnecessarily large, and these large numbers are a fertile source of errors. Then the longer the process, the more likely a student is to make mistakes. In the third place, errors increase just in proportion as mechanical processes are substituted for thought provoking methods. Moreover, I am convinced that students will find more pleasure in arranging the factors of the first and last terms to obtain the correct middle term than in the many arbitrary steps above. I have seen classes factor trinomials by these cross products, obtaining the factors with increasing speed. Imagine one factoring $19x^2+2x—17$ by multiplying by 19 and proceeding as above, when any fairly intelligent child can see at once that $19—17=2$ and hence the factors are $(19x—17)(x+1)$.

I am almost ready to define a good method in elementary mathematics as one which eventually leads to performing the process mentally without recourse to pencil and paper. Viewed from this standpoint, the method rehearsed above certainly stands condemned.

Lastly, perhaps one of the strongest reasons for objecting to the above method is that when the student reaches quadratics, the same lengthy process will be applied in solving quadratics by completing the square as well as in solving them by factoring.

"It takes me so long to get my math." Oh sad confession! It would not take so long if the student were trained to employ short, direct methods calling largely for mental performance.

MARY E. DECHERD,
University of Texas.

A FEW OBSERVATIONS ON TEACHING GEOMETRY TO BEGINNERS

When the high school pupil takes up the study of plane geometry for the first time, he is probably more nearly projected into the realm of the unknown than at any other time in his whole life. The material has little connection with anythng he can recall in his past experience. For the first time in his life he is called upon to think connectedly and rigorously, and he must be brought to a consciousness of what really constitutes a proof. The wise teacher will take advantage of this fact and will try to connect the new subject with whatever knowledge of geometric concepts the pupil already has, for he will realize that his success depends upon his ability to keep his pupils where they have "their feet on the ground and their heads above water." The experienced teacher knows it is quite difficult to analyze just how he does this, and even more difficult to direct others how to do it. If anything in this article suggests to any young teacher of geometry how he may profit by my experience the article will not have been in vain.

I will say that the first thing to realize is that we are teaching pupils, not geometry. One may present the geometry ever so logically and fail entirely to connect with his pupils. In fact, pupils learn psychologically, not logically. Some teachers can not see that they can afford to pass up a proof that is too difficult or beyond the capacity of their pupils at that particular time, and have them "accept it on faith" until such time as they have strength enough for it to have a real appeal.

I find it a good plan to begin with the angle, as its numerical measure makes it possible to throw the pupils into the realm of arithmetic and algebra, on which they have spent many years of their school life. Right away the pupils are able to figure out complements and supplements, complements of supplements, etc., and really enjoy doing so. Next I teach them how to bisect, construct perpendiculars, angles of sixty degrees, and angles equal to given angles (saying nothing as to why the process results in bisection, perpendiculars, etc.). Soon they are handling such questions as: If the angles of a triangle are together equal to 180 degrees and one angle is 50 degrees, find the size of the angle made by the bisectors of the other two angles. This starts them to thinking geometry and talking geometry and they know what they are talking about.

Soon the vertical angles theorem is assigned and of course all will understand it. Other theorems are assigned in order and we see to it that the pupils understand the facts as stated, and understand what these facts mean in terms of accurately drawn figures, whether they really understand the proofs or not. Some, namely those that involve arithmetic and algebra, will be readily understood, as for instance the one concerning the sum of two lines drawn from an external point to the extremities of a given line, compared to the sum of two other lines similarly drawn, etc. Others, such as the superposition propositions, will probably mean little. In fact these had best be treated graphically. Have the pupils draw a triangle with two sides and the included angle equal to two sides and the included angle of a given triangle, and let them find by actual measurement that the other corresponding parts are equal. Then they are made to understand that if ever they can find two triangles with the required parts equal, the remaining parts will be equal respectively. This places them in a position to attack and prove other theorems, while if they had lost sight of these facts because they did not understand the proofs of the theorems, they would have lost the most essential part of the whole study, which is the perfect continuity of the subject.

In three or four weeks all will be able to prove such propositions as this: "If the bisector of an angle of a triangle is perpendicular to the opposite side, the triangle is isosceles." I would have them draw the angle first, then the bisector, and then at any point on the bisector draw the perpendicular and produce it both ways till it cuts the sides of the angle. The idea of having the figure drawn according to the statement must be emphasized, for it is fundamental. At first special exercises in so drawing the figures are given, and more credit is given for accurately drawn figures than for anything else. The following is a suggestion for such an exercise. Have the pupils draw figures for the following statements: "The bisector of the vertical angle of an isosceles triangle is the perpendicular bisector of the base"; "If the bisector of an angle of a triangle is perpendicular to the opposite side, the triangle is isosceles"; "If the bisector of an angle of a triangle bisects the opposite side, the triangle is isosceles"; "If a line drawn from the vertex of a triangle to the mid-point of the opposite side is perpendicular to that side, the triangle is isosceles." Many others could be stated in this connection. Such exercises are intended to drill the pupils in the drawing of the figure according to the statement, and they can all see that different things are given in each and that different things are to be proved in each. We sometimes give certain rules concerning the statement itself that are helpful, such as: In simple sentence statements the modified subject tells what we are talking about, and in "if" and "when" propositions the dependent clause tells how to draw the figure.

Few pupils during the first few weeks of their experience in geometry can see the sense of drawing hypothetical lines such as are used in the parallel lines theorems. Some textbook writers try to overcome this difficulty by defining parallel lines as lines in the same plane so situated that when they are cut by a transversal, the alternate-interior angles are equal. It is probably a better plan to let those who find trouble here accept both the direct and the converse of the alternate-interior angles proposition "on faith." They can then easily work out the others and will be delighted to do so.

In this way we cover Book I in from two to three months, during which time the pupils have worked out practically all the originals in the text and several supplementary ones suggested by the teacher. By this time the pupils have developed considerable geometric power and could now, if it be thought necessary, go back and prove any of those propositions "accepted on faith" without much trouble. Above all else, however, is the outstanding fact that during all this time the pupils have kept their feet on the ground and talked sense.

Probably the two greatest obstacles in the way of pupils' acquiring a working knowledge of geometry are the failure to build up the new concepts, both of geometric figures and of what it really takes to constitute a proof; and failure to realize that new material must be presented to the pupils in terms of what they already know so that it can be grasped, and that immediate use should be made of all new material so as to stamp it indelibly on the mind.

W. O. DeWees,
Temple High School.

OUR TEXAS GIRLS AND MATHEMATICS

"What's the use of studying mathematics that will be forgotten in a year's time?" or "I'll never make any use of it." These questions are raised among girl students that are taking mathematics not because they want to take it, but because they have to take it in order to graduate. It is almost alarming to find how many girls are trying to avoid this subject. In advanced arithmetic, plane and solid geometry, and trigonometry, it is quite noticeable how the boys outnumber the girls, and how the girls "outquestion" the boys as to the purpose of the subject.

Is this condition due to the inherent weakness of the student, to the individual teachers, or to the school curriculums of the state? We mathematics teachers must not be too greatly alarmed over this situation whatever the cause may be; there are probably as many students in high schools and colleges that would never study English or foreign languages or history, if it were not required. Yet this is not a consolation; we would like to see the girls take more interest in mathematics. No one will deny that the fundamental principles of mathematics are so essential in practical life that no one can get along without them. Besides, we are so built by nature that a person, even though unconscious of any mathematical knowledge within him, will group things together, measure distances, think in terms of breadth and width, without ever having received any training. In some this is more noticeable than in others, but all show some mathematical instinct.

In geometry, we only put into words and analyze what the boys and girls knew long before they studied the subject, as for instance: a straight line is the shortest distance between two points; quantities equal to the same quantity are equal to each other. In many instances the pupils will ask the question: "Why do we have to give authority for the statement that a straight line is shorter than a broken line?" These, and many other things that seem obvious, yet require proof, are inherent in the students. With this fact fixed in mind, the teacher can usually make geometry a very interesting subject. To boys, at least, there is no other subject that can serve any more as an eye-opener than geometry. You set out to prove a certain thing, and in your conclusions you have the exact thing that you set out to prove, because of correct reasoning. This is an accomplishment and the student will soon realize it, but still some of the girls are not struck by its "charm."

As has been indicated, it is a fact that more girls in our Texas schools dread mathematics than boys. Of all the girls in the different high schools, only about fifty per cent would ever take any kind of mathematics, if it were elective, and eighty per cent of that fifty per cent would take algebra; the other fifteen per cent would be found taking advanced arithmetic, plane geometry, solid geometry, or trigonometry. Again, if it were left to the girls to elect, only four per cent would choose plane geometry. On the other hand, approximately eighty-five per cent of the boys would take mathematics, and of those, seventy per cent would prefer geometry to algebra.

Since my work has been exclusively with Texas students, this subject has strikingly come to my attention. My purpose is not to raise the question: "Are men superior to women mentally?" and discuss it. That is a worn subject, though ever intensely interesting, and would be a subject worth while. You do not want to say that intelligence is an attribute of sex. In a varying degree it is the possession of all human beings, and the degrees vary mostly according to individuals. As our table shows, the girls make as good a showing as do the boys, and we attempt to give a few reasons for it.

In the first place, the girls apply themselves to their work with greater concentration; it seems that they are less given to outside attractions when they prepare a lesson. Another reason is their greater conscientiousness. Here someone may ask: "Is their sense of duty keener than man's?" I don't know, maybe so. At any rate, it seems to be more conservative. Again, a girl seems more likely to submit her original ideas to the established ones. A boy wants to see independently. A girl, if she is expected to study, will try more to do what's expected of her than a boy, for he will invariably try to follow his personal inclination. Then too, girls seem to have more pride than boys. What a stimulus pride is to work! Further, girls are more sensitive to criticism and censure. They try to avoid incurring it. A good deal of responsibility for the poor scholarship record of boys is, therefore, due to a difference in the ethical standards of men and of women. Finally, perhaps the most interesting cause is the fact that girls mature earlier than boys do, mentally as well as physically. This is generally agreed to by all high school teachers. Let me venture here to assert that among high school freshmen the girls are much more clever than the boys. It is quite noticeable that this characteristic grows less perceptible from year to year.

Dr. Marion Le Roy Burton, for seven years the head of Smith College, the largest woman's college in the world, and now the president of the University of Michigan, says that "by the time a young man has reached mental maturity, he has also as a rule reached a decision as to his future. The result is that he concentrates on things he believes to be of use to him. With him it is much easier to do this definitely, than with the young woman. Back in her mind there must be a good deal of doubt as to just how much practical benefit she is going to derive as a homekeeper from what she is learning from her textbooks. The man is finding his spur to effort. The woman is losing hers. That is the crux of the whole matter."

In studying the subject under consideration, I sent out information blanks to a number of schools in Texas, asking for the number of boys and girls in the different mathematics classes and the failures of each. On finding the per cent there was noticeable only a very slight difference in failures. But the number of girls in elective mathematics classes is strikingly smaller than that of boys. In algebra and plane geometry there were seemingly more girls failing than boys, even though the average grade of girls runs higher than that of boys.

The following are figures obtained from some of our most representative schools and may therefore be considered as a rather safe index:

No. in Classes Boys	Girls	Failures Boys	Girls	Per Cent Boys	Girls
		Algebra			
574	537	113	177	20	32
331	202	41	49	12	24
413	393	65	70	16	18
		Geometry			
113	102	17	28	15	27
125	108	19	31	15	28
85	135	15	13	17	10
		Solid Geometry			
17	0	2	0	12	0
22	5	1	0	5	0
25	5	2	1	8	20
		Advanced Arithmetic			
7	2	0	0	0	0
17	4	0	0	0	0
14	5	1	0	7	0
1743	1498	275	369	16	25

Taking all things into consideration, the question might be in order: "Should mathematics be elective in our Texas schools?" Have conditions so changed in our present age that we are forced to consider this question?

C. E. DANHEIM,
Wichita Falls, Texas.

"THE ETERNAL WHY"

Perhaps no query is heard more often than this: "What good will this do me?" And perhaps no subject is the bone of contention more often than geometry. Geometry is a subject that can claim strong likes and dislikes at the same time. And because it is impossible to ignore the question that is asked us daily, it is my purpose to suggest, if I can, some means of giving adequate replies. It is true that to a certain extent this question must not be taken too seriously. The student is very liable to ask the same question about everything which does not hold an immediate reward. At his age he cares little about the distant good; he has a lofty contempt for everything which smacks of self-denial or tedious development. And yet we can not dismiss his questions with a shrug, for practically the same questions are being asked in educational circles the country over. Every age has had those who have proclaimed that the old order changeth and that the new methods shall be ushered in. And who shall say that we have not profited by their labors?

Some of the main questions which have been and are being asked are:

1. Shall geometry continue to be taught as an application of logic, or shall it be treated solely with reference to its applications?
2. Shall geometry be fused with algebra in the form of a combined course in mathematics?
3. Shall the basal propositions be proved in full or be left for the pupil?
4. Shall geometry be made an elective subject, to be taken by those whose minds are capable of serious work; or shall it remain a required subject and be diluted to the comprehension of the weakest minds?

The standard of utility has been applied to geometry, as well as to other subjects; there is no little activity even now in favor of the utilities of geometry. In my judgment there is only disappointment in store for that teacher who tries to yield to this demand. Geometry is not studied, and never has been studied, because of its positive utility in commercial life or even in the workshop. All the facts that a skilled mechanic or an engineer would ever need to know could be taught in a few lessons. And yet we can not ignore entirely the demand for applications; in fact, if properly handled, therein is one of the strongest factors in the development of vital interest in the subject. But therein also lies a grave danger. Many of the so-called applications are in reality nothing more than mere artificialities, and as such do much to defeat the teacher's aim and purpose. Not more than 25 per cent of the propositions have any genuine applications outside of geometry. I have found it helpful to meet the issue boldly; to admit from the outset that we do not study geometry for the so-called practical utility, and then to develop gradually the idea that our conception of practical values needs to be readjusted. After all, the practical ends we seek are in a sense ideal practical ends, that nevertheless have an eminently utilitarian value in the intellectual sphere. Vision and perspective come only after great patience and effort have been expended, but the results are more than worth while. In this day, many men are prone to scorn the real foundations of success and give full credit only to the direct facts gained in specialization. As teachers we are fully acquainted with the necessity for a broad, fundamental education upon which to build any above-the-average success.

The second and third questions may be passed rapidly by. At present in America we have a fairly well-defined body of matter in geometry and this occupies a fairly well-defined place in the curriculum. There are many teachers who would change both the matter and the place in a very radical manner. There are many also who are content with things as they find them. It is my belief, however, that the great majority of teachers welcome the natural and gradual evolution of geometry towards better things, glad to know the best that others have to offer, receptive of ideas that make for better teaching. It is the duty of the open-minded, progressive teacher to seek for the best in the way of improvement.

There is another question, however, which we should face fairly and squarely. As far as my observation goes, the conditions in the teaching of geometry are very much the same everywhere. We must recognize that the recent growth in popular education has brought into the high school a less carefully selected type of mind than perhaps was formerly the case. There are many pupils who seem to lack entirely the ability to grasp the fundamentals of the subject. In view of this fact, there is to some extent the tendency to believe that geometry should be made an elective subject. It would seem to me that to make such a change would be a grave mistake. True, much of the anxiety and worry, much of the toil and patience of the teacher would be eliminated. But at the same time the pupils themselves would suffer because of the change; many of the pupils who are least inclined towards the subject are the very ones who need the subject most. Hard work and patience will work wonders with that same class of pupils. Where it is possible, a careful grouping of the pupils will obviate much of the difficulty. Geometry should continue to be taught as a vigorous thought-compelling subject; soft mathematics is not interesting to the average mind and a sham treatment will never appeal to the pupil. As teachers, we should stand for "vitalizing geometry in every legitimate way; for improving the subject-matter in such manner as not to destroy the pupil's interest; for so teaching geometry as to make it appeal to pupils as strongly as any other subject in the curriculum; but for the recognition of geometry for geometry's sake and not for the sake of a fancied utility that hardly exists." That, in the language of D. E. Smith, should well summarize our attitude. After all, the problem of teaching any subject comes down to this: get a subject worth teaching and then make every minute of it interesting. Geometry is certainly such a subject, and the rest is our task.

E. G. HAMILTON,
Waxahachie High School.

SOME SUGGESTIONS FOR THE TEACHING OF PLANE GEOMETRY

Most teachers of mathematics in Texas, I am sure, are of the opinion that the task assigned to them of teaching to a tenth grade class the contents of Wentworth and Smith's *Plane Geometry* with all of its five books of propositions and exercises is the hardest they have to face. Yet it can be done by real, live teachers of mathematics.

Some so-called teachers of mathematics seem to think that when they have taught the propositions and theorems worked out in the text and a few of the original exercises they have deserved the "well done" commendation. In reality they have done very little in developing in their students a real mastery of the fundamentals of geometric reasoning. This is attained only through skilful guidance on the part of the teacher in making the students by themselves dig out most of the original exercises at the end of each of the five books.

The propositions worked out in the text are primarily patterns, guides, tools by which the student may test and develop his reasoning powers in solving the originals. Tools are of very little value unless they are put to use; hence, a knowledge of the theorems is of doubtful value unless they can be used in solving practical problems such as the original exercises.

What then, you ask, is a fundamental test by which you may know whether or not a student has a comprehensive grip on the facts of geometry? The answer to this question lies in the ability of the student by himself to make such use of the theorems solved in the text for him as will enable him to solve a high percentage of the original exercises in the five books during a period of nine months. Can the student through self-activity—by his own mental processes—think through to a solution half a dozen original exercises each class day? If he can he has acquired a real mastery of the subject. His knowledge in this regard is not half-baked like the knowledge of those who can not use what they learn; it is solid, thorough, and profound. By far, then, the most important thing is to train students to the point where they can solve the original exercises; this is the real test.

I shall now make a few suggestions to teachers of geometry as to how this goal may be attained. A certain amount of routine is always essential as a time saver. Each student should have a certain place at the board and should be given work there nearly every day. The student must be made to feel that he is expected to contribute something to the work of the class, and should be called upon to recite every day, if possible, either to explain a problem or to answer a question, and his daily work should be graded.

The student should be taught to draw figures quickly and accurately. It is to my mind a waste of very valuable time to have students write out the complete proof at the board. Practice in writing out proofs can be obtained by requiring that complete proofs of all original exercises be kept in a geometry note-book. Only the first ten minutes of the forty-five should be given to drawing the figures at the board, and the other thirty-five minutes should be reserved for the oral proof by the students of the problems of the lesson. Students should not be permitted to memorize verbatim any part of the problem except the bare statement of the problem as given in the text in *italics*. This statement of all the important theorems and corollaries should be memorized for the reason that these statements, such as: "two triangles are congruent if three sides of one triangle are equal respectively to the corresponding three sides of the other triangle," are used in the proof of other problems. These are the tools and must be ready for use in support of all important statements in the proof of problems to follow.

A student called on to prove a problem at the board, where he has already drawn the figure, should leave his book at the seat and take his position at the board so that all the class can see the figure. He should then state his problem from memory, then what is given and what is to be proved, and finally give the proof completely and clearly, backing up each statement with a reason in such a manner as will convince the class of the soundness of his solution. By wise questioning on the part of the teacher it can be quickly determined whether or not the students thoroughly understand the solution.

It is very important to require students to keep a note-book of all the original exercises solved in class. In this connection form should be stressed. The figure should be neatly drawn in the upper left half of the page. The upper right side should be reserved for what is given and what is to be proved. The lower left half should be reserved for the proof, and the lower right half for the reasons given in the proof. The note-books should be corrected and graded frequently and conferences held with students in regard to their work. Shoddy work should never be accepted. Students should be required to do their own work without outside assistance.

Teachers should never permit the recitation to drag. Quickness, skill, and accuracy should always be stressed. Students should be trained to have the problems thought out before coming to class. From four to ten problems should be explained completely at each recitation, the number depending on their length and relative difficulty. In that way nearly all of the exercises and many practical problems drawn from every-day experience should be mastered in one session of school. Problems that are constantly coming up for use in proving other problems are the most important, and great stress should be laid on them, while the less important should receive less attention.

C. D. EADES,
Superintendent, Santa Anna Public Schools.

SOME DEVICES USED IN TEACHING PLANE GEOMETRY

"To measure is to know."—*Kepler*.

In teaching plane geometry to beginners, it is sometimes difficult for the student to get the construction correct. A good plan is to have the pupil construct or make the figures from cardboard or strips of white pine. To construct a figure requires more time and thought than mere drawing, and the student measures every line exactly. In this he is taught accuracy and exactness, and this is one of the many reasons for studying geometry.

In teaching the congruency of triangles, it is sometimes difficult for the student to suppose one figure moved from one place to another and made to coincide with another figure; this difficulty is easily eliminated when the two like triangles are made.

In teaching the loci of points, this plan works well: all auxiliary lines are made of heavy cord of different colors. In some originals, such as: "To construct a triangle, having given the perimeter, one angle, and the altitude from the vertex of the given angle," where the construction is difficult, the auxiliary lines may be made from colored construction paper to distinguish them from the given lines. When a figure is once made, the student understands exactly what is given and what is required. The proof is then easily obtained. These figures, when made, may be used from time to time in review work.

Another device which interests the student and provokes thought in review work may be carried out in the following manner: have some student state and prove a proposition; in the proof of this, several other propositions are used; when the proof of the first is finished, take up the propositions used in this proof, and prove them; from these, others are derived. This may be continued indefinitely and serves as a splendid drill and review.

The classification of propositions, theorems, and examples according to methods of proof will be a great help to both students and teacher.

Make the study of geometry a chain—each proposition, corollary, or example another link in the chain, and see that no link is weak. Remember, "No chain is stronger than its weakest link!"

MATTIE G. ROBERTS.
Principal, Somerville High School.