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Comparing Students with Mathematics Learning Disabilities and Students with Low Mathematics Achievement in Solving Mathematics Word Problems

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by

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Comparing Students with Mathematics Learning Disabilities and Students with Low Mathematics Achievement in Solving Mathematics Word Problems

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This study identified factors related to solving mathematical word problems and then examined the differences in characteristics between students with low achievement in mathematics who were likely to have a learning disability and students with low achievement in mathematics who were unlikely to have a learning disability. Factoral analysis identified two significant factors: abstract thinking and long term retrieval from memory. Results indicated qualitative differences between sixth grade students with achievement in mathematics at or below the 25^{th} percentile with indications of learning disabilities (MLD) and students with achievement in mathematics at or below the 25^{th} percentile without an indication of a learning disability (Low Math/NLD). The *Learning Disabilities Diagnostic Inventory,* which measures intrinsic processing disorders indicative of learning disabilities, was used to differentiate between students with MLD (n = 13) and students with Low Math/NLD (n = 16). The *Woodcock-Johnson III Tests of Achievement*, *Clinical Evaluation of Language Fundamentals-Fourth Edition*, and the *Informal Mathematics Assessment* (IFA) were used to compare the two groups. In contrast to students with MLD, students with Low Math/NLD had a higher mathematical performance and had more difficulties with math fluency. When solving mathematics word problems on the IFA, a test composed of word problems, student interview, and error analysis, students with Low Math/NLD had more correct answers, more computational errors, and fewer translation errors than students with MLD did. Students with MLD had conceptual difficulties in the areas of analyzing, reasoning, and abstract thinking.

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CHAPTER I

Introduction

In today's global economy, an educated workforce is essential for a country to be successful on an international scale. To obtain that educated workforce, all students including those with learning disabilities (LD) must be prepared to compete globally. It is not enough to have a citizenry that can read. Jobs requiring only the mastery of basic skills are disappearing from the United States. Today more and more American workers are being required to think for a living. They are expected to process challenging material and solve complex problems (McClure, 2005). Ever more frequently, those complex problems require an understanding and use of mathematics (Furner & Duffy, 2002). In the past, mathematics was necessary in the realm of engineers, scientists, and architects. Today, auto mechanics, cashiers, and assembly line workers in the technology field require mathematical skills. In today's technological world, failure in the area of mathematics and problem solving can prevent an individual from obtaining a job or being able to function in the work place (Bottge & Hasselbring, 1993). According to the National Council of Teachers of Mathematics (NCTM) in their *Principles and Standards for School Mathematics* (2000), "those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. A lack of mathematical competence keeps those doors closed" (p. 5). Today the ability to use mathematics is seen as a survival skill for individuals negotiating grocery shopping through price comparison, selecting interest rates for loans, and computing taxes. Mathematics pervades most facets of modern life. The NCTM stated in the *Standards,* “the need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase” (p. 4). It is solving these everyday problems of modern living, otherwise known, as mathematical word-problem solving that appears to be especially difficult for students of all ages and ability levels (National Assessment of Educational Progress, 1992). This difficulty is even more pronounced for students with learning disabilities in mathematics (Bryant, Bryant, & Hammill, 2000).

The need for an educated internationally competitive American workforce in a knowledge-based economy has driven the demand for more accountability from the schools to ensure that a highly trained future workforce is being produced (Chapman, 2004). In comparisons of achievement between American students and that of students in Europe and Asia, American students do not perform as well as their overseas counterparts. They are especially lacking skills in mathematics beyond the fourth grade level (Third International Mathematics and Science Study, 2003).

As a result, the field of mathematics education has undergone major reform to improve mathematics instruction and enable all students to experience a robust mathematics education in order to fulfill "the nation's need for a competent work force and an informed society" (Conference Board of the Mathematical Sciences, 1995, p. 1). Due to technological advancements (Office of Technology Assessment, 1988), national movements (e.g., Goals 2000: The Educate America Act), and mathematical work force expectations (Johnston, & Packers, 1987), the mathematics professionals (e.g., National Council of Supervisors of Mathematics, 1988; [NCTM, 1989, 1991, 1995, 2000) have produced instructional and curricular restructuring recommendations of mathematical programs. The NCTM led the reform by developing standards-based curricula. The NCTM developed the *Principles and Standards for School Mathematics* (2000) which guides states and districts in their development of mathematics curriculum, instruction, and assessment. Curriculum and content standards were developed to describe what students should know at each grade level. Their performance standards described activities that would demonstrate whether a student had mastered the expected material at a particular grade level. State assessments aligned with content and performance standards were expected to measure a student's performance against the standards (McClure, 2005). Under the influence of the No Child Left Behind Act of 2001 (NCLB), states began requiring students to take more rigorous courses and to pass proficiency tests for promotion and graduation (Ding & Navarro, 2004).

Unfortunately, longitudinal studies have shown that these accountability assessments have little impact on students' yearly achievement (Bolon, 2001; Ding & Navarro, 2004; Marchant, Paulson, & Shunk, 2006). The growth rate in mathematics actually tends to slow when students enter middle school (Ding & Navarro, 2004). Using the Stanford Achievement Test Ninth Edition (SAT-9), math scores researchers followed the progress of a fifth grade cohort for three years and found that 58% of the growth occurred from fifth to sixth grade, 24% of the growth occurred from sixth to seventh grade, and 18% of the growth occurred from seventh to eighth grade. Examination of the individual growth profiles of students revealed uneven growth in mathematics test scores. It cast doubt on the assumption that students should show linear progress in norm-referenced tests in the content areas, even though middle school teachers are expected to demonstrate that their students are showing Adequate Yearly Progress (AYP) as required by NCLB (2001, which assumes yearly linear growth.

The difficulties that middle school teachers face are tremendous. NCLB (2001) requires middle school teachers to assure that all their students perform well on increasingly rigorous state assessments regardless of the uneven way in which student mastery of new concepts is achieved across developmental ages. Teachers' efforts are further impeded by the apparent decrease in growth rate in mathematics that appears as students enter adolescence (Ding & Navarro, 2004). Additionally, this expectation is made even more difficult by the complex conceptual material that middle school students must master.

Middle School Mathematics

As a result of the changing curriculum, which follows the *Principles and Standards for School Mathematics* (NCTM, 2000) developed by the National Council of Teachers of Mathematics (NCTM), mathematics teachers have had to replace the traditional didactic skills-based models of instruction with constructivist inquiry-based learning emphasizing problem solving, conceptual understanding, and mathematics communication. Middle school mathematics students are now expected to investigate and develop multiple problem-solving solution strategies for real-world problems, explain their mathematical reasoning to others as well as listen to and understand their peers' explanations, and develop a deep understanding of mathematical concepts and skills through interactions with other students and their teachers (Bryant, Kim, Hartman,

& Bryant, 2006). Under the assumption that their students have previously acquired foundation knowledge, possess mastery of basic skills and problem-solving strategies, and understand mathematical concepts, middle school mathematics teachers present increasingly complex mathematics to their students (Montague & Jitendra, 2006). Middle school mathematics is difficult. It requires students to engage in multiple operational steps, when using whole number computation, fractions, word problem solving, and algebra, as part of the process when seeking the solution. Students in the sixth through eighth grades are taught increasingly difficult curriculum that includes number, operation, and quantitative reasoning; measurement; probability and statistics; as well as underlying processes and mathematical tools (NCTM, 2000). The acquisition of this mathematical knowledge and these skills is necessary in order for students to meet the continually rising stakes on accountability assessments used to determine promotion and high school graduation as well as to be successful in the advanced high school mathematics classes that are to prepare them for postsecondary education.

Algebra and geometry are especially difficult for secondary students. Understanding algebra is crucial because of its connection to higher-level mathematics and a post secondary education (Bryant, 2007). But to understand algebra, the student must already possess a mastery of arithmetic combinations and an understanding of mathematics vocabulary. Algebra requires students to understand the concepts of variables, equality, equations, and functions and to utilize patterns, symbolism, relations, and representations when solving for solutions (Allsopp, Kyger, & Lovin, 2006; Van de Walle, 2004). In addition students must understand arithmetic properties which are essential to algebraic reasoning (Bryant, 2007). Students who have experienced difficulty with remembering and retrieving arithmetic combinations seem to display an inability to grasp the more complex algebraic concepts that they encounter in higher mathematics including word problems (Geary, 2004; Jordan, Hanich, & Kaplan, 2003).

Word Problems

Importance of Word Problem Solving

Solving word problems is a major component of mathematics curriculum (NCTM, 2000) and a basic life skill that students need in order to solve the real-world problems that they will encounter in their everyday life (Bottge & Hasselbring, 1993). Solving word problems demonstrates the utility of mathematics to students (NCTM, 2000) by presenting them with everyday situations requiring probability, statistics, algebra, and geometry for solutions. Solving word problems allows students to see that there can be more than one "right" way to solve a problem and that the information they gain from an incorrect solution can provide valuable clues for eventually finding a correct solution (NCTM, 2000).

The problem solving strategies that students utilize when solving word problems can be expanded when they analyze and solve problems occurring in their own lives. Solving word problems encourages students to practice logical thinking as they strategize and reflect. Problem solving encourages language and vocabulary development not only in the students' receptive language, as they attempt to understand the meaning of the word problem, but also in their expressive language when they present their results and their thinking orally and in writing (Cobb, 2004).

Teachers agree that word problems are the most difficult type of mathematics problem for students to solve (Bryant, et al., 2000). Solving word problems is problematic for many students, in general, but especially for students with mathematics deficiencies (Bryant, 2007). It is particularly difficult for students with learning disabilities at the elementary (Englert, Culatta, & Horn, 1987; Parmar, Cawley, & Frazita, 1996) and secondary levels (Montague & Applegate, 1993; Parmar, et al., 1996).

Skills Needed for Word Problem Solving

Solving word problems is difficult due to the complexity of the presentation of the word problem. The student must have adequate linguistic, cognitive, and reading abilities (Englert, et al., 1987; Parmar, 1992; Parmar, et al., 1996; Rivera, 1997).

Language. Before a student can understand what is to be solved, the student must first comprehend the problem statement whether it is given in oral or written form (Laborde, 1990). The problem must be presented according to the rules of the language, and the student, using his/her conceptual and linguistic knowledge base, must process the language and interpret the problem (Fayol, 1992; Jitendra, 2002; Kintsch, 1988). The student's ability to understand the problem is influenced by the way in which the relationship between the given and unknown quantities are verbally expressed, the amount of explicitness in the language used (Bachor, 1987; De Corte, Vershaffel, & De Win, 1985), the order in which the information is presented (Mestre, 1988), and the complexity of the vocabulary and the language syntax (Bachor, 1987; Spanos, Rhodes, Dale, & Crandall, 1988). Because mathematics is conceptionally dense it requires students to understand the meaning of each mathematical symbol and word (Wiig & Semel, 1984). An understanding of language is needed to express numerical language concepts into mathematics symbols (Miller & Mercer, 1997) and to convert linguistic and numerical information through paraphrasing and visualization into a representation of the problem in the form of mathematical equations or graphic representations (Jitendra, 2002) in order to construct a mental model which will expedite the solving of the problem (Greer, 1997; Jitendra, 2002; Johnson-Laird, 1983).

Cognitive. Proficient cognitive processing is essential for successfully solving mathematics computations and word problems. Cognitive processing refers to the operations involved in the intelligence processes, including problem solving, thinking, and memory (Smith, Polloway, Patton, & Dowdy, 1998). Long-term memory brings the student's background knowledge to the task and facilitates the student's understanding of the written text and the word problem (Schunk, 1996). When a student reads a mathematical word problem, working memory and long-term memory interact to influence comprehension (Fayol, 1992). Long term memory and language ability facilitate fact retrieval and speed of processing as well as the recall and use of the many steps, rules, and mathematics facts (Strang & Rourke, 1985), which permits working memory to be utilized on problem solving (Mercer, 1997). Sufficient working memory capacity is essential for successful problem-solving. If working memory is expended on solving basic mathematics facts that amount of memory becomes unavailable to solve problems (Just & Carpenter, 1992).

Reading. When a student must read a written word problem, all of the language processes that are required for reading comprehension are needed for an understanding of the mathematical word problem. General intelligence, memory functions, language skills, and especially concept information are activated with reading comprehension, which is strongly tied to language (Vidal-Abarca, 1992; Winograd, 1984). Thus, in order to solve a mathematical word problem, all of the language processes associated with mathematics in addition to the language processes associated with reading are needed in order to understand and solve the mathematics problem. Therefore, in order to gain a better understanding of the factors associated with a specific mathematics ability performance, it is necessary also to examine the student's reading performance.

In order to be a proficient problem solver, the student must have reading comprehension skills, linguistic skills for translating the problem, computation skills, basic fact memorization, procedural knowledge, and cognitive and metacognitve competencies (Englert, et al., 1987; Mayer, 1992; Parmar, 1992; Parmar, et al., 1996). A weakness in any of these areas will create difficulties for the student.

Mathematics Learning Disabilities

Students whose low achievement in mathematics is due to a specific learning disability are of special concern to educators because of their frequent failures throughout their school experience despite their average or better intelligence (Carnine, Jones, & Dixon, 1994; Nuzum, 1987; Zentall & Ferkis, 1993). A specific learning disability refers to a disorder in basic psychological processes involved in the understanding or using of language (IDEA, 2004) and related to intrinsic processing disorders which are "the specific mental operations believed to be essential for competent spoken or written language, computation, and reasoning" (Hammill & Bryant 1998, p.13). Mathematics learning disabilities (MLD) are due to difficulties with language that interfere with a student's learning ability (Kavale & Forness, 1995). They manifest themselves as difficulties in mathematics calculation or mathematics reasoning and are not due to other causes such as poor instruction, lack of motivation, or behavior (IDEA, 2004).

Mathematics calculation is associated with an inability to perform mathematical computations (Garnett & Fleischner, 1983). Mathematics reasoning is usually associated with difficulty solving word problems and may appear as an inability to conceptualize and develop a strategy for solving the problem (Geary, 1993). In other words, the student experiences difficulty translating the problem into mathematical language.

Because students with MLD form a heterogeneous group (Ackerman & Dykman, 1995; Geary, Hoard, & Hamson, 1999; Geary, Hamson, & Hoard, 2000; Rourke, 1998), different students with MLD show different patterns of numerical and arithmetical deficits. About half of the differences in mathematics achievement by students with MLD can be accounted for by individual differences in basic number, counting, arithmetic skills, and working memory (Geary et al., 1999). The achievement of other students with MLD is affected by their difficulties with language and their ability to understand the meaning of word problems, to follow the teacher's instructions, and to obtain automaticity with basic mathematics facts (Hanich, Jordan, Kaplan, & Dick, 2001). Some students with MLD experience difficulties with visual-spatial perception, which affects the ability of the student with MLD to deal with the graphic nature of mathematics such as solving calculations involving columns and determining the difference between a “6” and a “9” (Geary, 2004; Kinsbourne & Warrington, 1963; Rourke, 1998; Spiers, 1987). A learning disability in mathematics, due to an intrinsic processing deficit in cognitive, language, or visual-spatial areas, can result in low mathematics achievement and will cause the student with mathematics learning disabilities to exhibit developmental differences from a typical student with the same chronological age. The difficulties of students with LD are due to cognitive processing factors (Ginsburg, 1997). and are associated with memory, language processing, cognitive development, and visual-spatial areas (Bryant, et al., 2006).

Types of Difficulties

Cognitive. Studies have shown that students with MLD exhibit cognitive deficits and limited proficiency related to fact (memory) retrieval (Garnett & Fleischner, 1983; Ostad, 1997, 1998), problem conceptualization, speed of processing (Geary, 1993), calculation strategies (e.g., counting on, counting all, counting fingers) (Geary, 1990; Ostad, 1998), and procedural errors (Geary, Brown, & Samaranayake, 1991). The low achievement in mathematics of students with MLD is related to difficulties with language and their one or more processing deficits.

Language. Language processing problems also affect the development of number skills and the concept of number or number sense (Manor, Shalev, Joseph, & Gross-Tsur, 2000). The act of counting uses language to name a mathematics concept of quantity. Students who have difficulty with number sense lack an understanding of the magnitude of numbers, number relationships, and arithmetic combinations (Gersten, Jordan, & Flojo, 2005; Case & Okamoto, 1996). Students with MLD appear able to count small quantities but unable to sustain a count over large magnitudes due to a lack of conception of quantity and seriation (Ta'ir, Brezner, & Ariel, 1997). Students with MLD understand counting as a rote, mechanical activity but do not necessarily have a concept of number. Students may not realize that the number word they say rotely is associated with a mathematical concept of quantity (Geary, et al., 1999). This lack of realization impacts their ability to do mental computations and select appropriate representations for quantities (Gersten, et al., 2005; Case & Okamoto, 1996). Language processing disorders also are thought to be associated with difficulty conceptualizing number quantity and the ability to recognize and produce number and operator symbols, which results in poor sequencing skills and shorter auditory memory (O'Hare, Brown, & Aitken, 1991). A difficulty with number awareness affects the student's ability to interact with numbers, and remediation becomes problematic.

Young students with MLD often experience difficulties with language and conceptual understanding as related to counting. These students seem to understand the counting rules of stable order and cardinality (Gelman & Gallistel, 1978), but they consistently make errors on tasks assessing order-irrelevance (Briars & Siegler, 1984). Students' poor counting knowledge appears to contribute to their difficulty in using counting to solve arithmetic problems and to detect and correct counting errors (Ohlsson & Rees, 1991). Students with MLD commit more counting errors and use developmentally immature procedures (e.g., counting-all rather than counting-on) more frequently and for more years than do their same aged peers (Geary, et al., 2000; Jordan & Montani, 1997). These same students who have difficulty understanding the concept behind counting also have difficulty later on learning basic mathematics facts. They tend to use the strategy of counting all when their typically developing same age peers have moved to counting-on. Later, after students with MLD have started using counting-on, their same age peers are already beginning to commit basic facts to memory. Long after their typically developing same age peers have memorized their basic facts and acquired automaticity, students with MLD will continue to rely on counting-on rather than memorizing addition facts (Geary et al., 2000). These early strategies, used typically by younger children, are based on using counting as a procedural or back-up strategy, whereas the strategies that appear later are deductive or retrieval strategies based on the student's known knowledge of basic facts. The student with MLD will continue to use procedural strategies (Geary et al., 2000) even after his/her same age peers have begun to use deductive strategies.

Automaticity. Learning basic mathematics facts is typically difficult for students with MLD (Ackerman & Dykman, 1995; Geary, 1993; Greene, 1999; Kulak, 1993; Rasanen & Ahonen, 1995). A student must use language in either written or spoken form when retrieving the answer to a basic mathematics fact. Students with MLD consistently have difficulties with memory and automaticity or rapid recall of basic facts. The student with MLD is much more likely to make an error when trying to retrieve an answer from memory than the typical same aged peer experiences (Rourke & Conway, 1997). To compensate for their lack of accurate memory for answers to basic facts, students with MLD attempt to calculate the answer for a basic fact each time it is encountered. Their lack of automaticity of basic mathematics facts increases the amount of time they take to solve simple problems and makes solving complex problems much more difficult for them (Garnett & Fleischner, 1983; Jordan & Montani, 1997). Because they do not have the basic facts memorized, it is necessary for them to solve separately each step that requires knowledge of a basic mathematics fact when they are confronted with a more complex problem such as a double-digit multiplication problem. This results in additional steps taken by the student with MLD that the typical same aged peer does not encounter (Russell & Ginsburg, 1984). Because the student with MLD needs to solve more steps, this student requires additional time to solve each problem and also has more opportunities to make errors. The additional time required to solve basic mathematics facts for each problem results in less time being available for the student with MLD to practice and become more proficient in other mathematics skills. Even when additional time is available, this lack of automaticity in basic mathematics facts not only affects the ability to solve computations with paper and pencil but also affects the ability of the student with MLD to problem solve word problems (Hitch & McAuley, 1991). Lack of knowledge of basic mathematics facts prevents the student with MLD from successfully using estimation to determine if an answer is reasonable for a word problem. Lacking knowledge of basic mathematics facts, the student with MLD is unable to recognize number patterns that appear in some mathematical word problems which makes solving such problems more difficult for the student with MLD.

Working memory. The speed of processing, as measured by written work or by verbal answers from students with MLD, is affected not only by the students' lack of automaticity of basic mathematics facts but also by their difficulties with working memory as well (Barrouillet, Fayol, & Balluliere, 1997; Garnett & Fleischner, 1983; Geary & Brown, 1991). Limited working memory capacity constrains the comprehension of mathematics word problems (Just & Carpenter, 1992). According to Mercer (1997), there is a limited amount of memory that can be utilized to problem solve. Consequently, difficulty with language as demonstrated through attempts at verbal or symbol retrieval, which is a cognitive processing disorder, affects their ability to use mathematical operations and to problem solve. Cognitive processing problems also appear as a difficulty with language in the area of listening and affect students' ability to remember and retrieve teachers' explanations and instructions and follow teacher's directions in the classroom.

Long Term Memory. Besides working memory problems, these students also have long-term memory or permanent information storage difficulties (Barrouillet et al., 1997; Garnett & Fleischner, 1983; Geary & Brown, 1991). Students with long-term memory difficulties can lack automaticity of basic facts (Geary, 1993), are unable to identify mathematical symbols, and are unable to recall steps in operations or in solving algebraic equations (Bryant, 2007). Alternatively, other students with MLD can preserve symbol recognition and production but have difficulty conceptualizing number quantity (O'Hare et al., 1991). Students with MLD are weak in the area of language as shown through encoding and recalling nonverbal material such as mathematical symbols in long-term memory (Brandys & Rourke, 1991). Encoding difficulties that can be due to language problems in written form (visual) or listening (auditory) errors when first exposed to the information could result in the inaccurate recall of information. Students with MLD affecting the area of arithmetic often have difficulty with encoding when confronted with a novel task regardless whether the task is verbal or nonverbal (Brandys & Rourke, 1991). They have difficulty generating a new strategy that can be used in an unfamiliar learning situation. The way in which the memory is affected appears related to the type of language processing disorder experienced by the student. Students with learning disabilities appear to demonstrate more than one type of language processing disorder in the area of mathematics. These multiple types of language processing disorders reinforce the realization that students with MLD do not come from a homogenous group (Rourke, 1998).

Reading. Students with learning disabilities and reading difficulties find that their poor reading proficiency interferes with their ability to solve word problems (Smith, 1994). Students with MLD can be students with low achievement in only mathematics or students with low achievement in mathematics and reading. It is not unusual for students with MLD to have reading disabilities also (Badian, 1983). This has resulted in the further examination of students with MLD and low achievement in reading (Robinson, Menchetti, & Torgesen, 2002). Both groups of students have low achievement in math, but the areas and degrees to which students’ difficulties in mathematics are apparent vary between the two groups. Those students with a learning disability who have low achievement in mathematics but not in reading have verbal abilities that appear to be stronger than their nonverbal abilities as shown by frequently higher Verbal than Performance IQ scores (Rourke, 1988, 1989; Rourke & Tsatanis, 2000). They consistently perform better on memory tasks when information is presented verbally rather than nonverbally. They frequently have above average scores in early reading and spelling skills but may struggle with skills such as tracing cutting, coloring, and handwriting (Rourke, 1988). They may excel on reading/decoding and spelling tasks but may experience confusion when faced with mathematical symbols and procedures (Gross-Tsur, Shalev, Manor, & Amir, 1995; Rourke, 1988). They demonstrate good mastery on early rote skills but experience difficulty on tasks that are novel and complex such as making inferences and synthesizing and integrating content. They have difficulties with written language as shown by their problems with symbols as well as difficulty understanding language when it is used in novel and unrehearsed ways especially those requiring higher order thinking skills.

Students with MLD and low achievement in reading consistently score lower in the areas of number facts, arithmetic, place value, and written calculation than students with MLD only (Jordan & Hanich, 2000; Rourke & Conway, 1997). Students with MLD and low reading achievement frequently have difficulty acquiring basic mathematics facts (Robinson et al., 2002). A student's success at solving mathematical word problems is related to the student's ability to read. The student who has poor skills in reading and mathematics experiences more pervasive and severe difficulties in mathematical problem solving than those students whose weakness is in mathematics only (Rourke & Conway, 1997).

Identification of Mathematics Learning Disabilities

There have been numerous studies in reading and dyslexia, including studies that have focused on determining if there is a difference between students with a learning disability in the area of reading and those students who could be considered "garden variety" slow learners (Spear-Swerling & Sternberg, 1996) also known as developmentally delayed. However, there are relatively few studies exploring differences between students with a learning disability in the area of mathematics and students with low achievement in mathematics without learning disabilities (Geary, 1993). Research in the area of mathematics learning disabilities in the field of special education has only recently begun to increase (Brandys & Rourke, 1991).

When examining standardized mathematics achievement scores for students with developmental delays in mathematics and students with mathematics learning disabilities, there is no discernable difference. Both groups of students are included within the 50% of the population that falls below the mean in a normal distribution and is found in normal variation (Shaywitz, Escobar, Shaywitz, Fletcher, & Makuch, 1992). Such low achievement can result from poor schooling, lack of educational opportunities, cultural disadvantages as well as other causes such as disabilities or developmental delays (Kavale & Forness, 1995). The student with developmental delays has intelligence sufficiently high enough to prevent the student from being identified as having mental retardation but is unable to maintain the same progress as a typical peer with the same chronological age (Burt, 1937; Horn, 1924; Gaddis, 1971). Similar to the student with a learning disability, the student with developmental delays demonstrates low mathematics achievement, but unlike the student with a learning disability, the student with developmental delays does not have a processing disorder (Kavale & Forness, 1995).

In the area of math, students with developmental delays display low achievement and acquire mathematical knowledge and skills at a rate slower and more immature than the typical student demonstrates. Students in elementary school with low achievement in mathematics (Low Math/NLD) demonstrate a developmental delay through their immature and error-prone procedures in mathematics (Geary & Brown, 1991). However, because these students are experiencing developmental delays and not a long-term cognitive deficit, such as students with MLD experience; after receiving intensive and appropriate instruction, their developmental delays will disappear with age (Geary, 1993; Fuchs & Fuchs, 1998). Although students with developmental delays in mathematics acquire mathematics skills more slowly than their typical peers acquire them, they are expected eventually to develop appropriate levels of arithmetic skills (Stanley, 1978).

Researchers have recognized the need for a method for discriminating between students with learning disabilities and those without in order to provide early identification and early intervention for students with learning disabilities (Fuchs, 2005). For mathematics learning disabilities (MLD) to be recognized as a disorder that relates to intrinsic processing in a pathological sense, researchers must be able to identify accurately individuals who manifest deficit or disordered processing that interferes with their ability to acquire mathematics skills and concepts commensurate with their peers.

Moreover, this group of individuals must be distinguishable from individuals who exhibit low achievement in mathematics without a learning disability (Ginsburg, 1998). Therefore, the mathematics performance of students with MLD must be identifiable as disordered or deficit rather than developmentally different from their same-aged peers with low achievement in mathematics without a learning disability (Low Math/NLD).

Research in mathematics disabilities has begun to explore differences between students with MLD and those with Low Math/NLD. Assessment practices utilizing norm-referenced instruments, criterion-referenced assessment, and nonstandardized procedures, such as clinical interviews, error analysis, and portfolios (Bryant & Rivera, 1997) have increased the focus on the processes and conceptual understanding students use to arrive at their solutions as well as the correctness of those solutions in an attempt to document student progress and the existence of mathematics learning disabilities. Studies that have examined the mathematics characteristics of students who are performing poorly in mathematics have yielded conflicting results. Whereas, some findings support no differences between students with developmental mathematics delays and students with mathematics learning disabilities, other studies have shown that within a group of students with low mathematics performance there are students who demonstrate characteristics indicative of intrinsic processing disorders in math. One possible reason to explain the conflicting results is the lack of a common agreed upon method of identification for participants in research projects (Hammill & Bryant, 1998). The criteria used to determine the presence of MLD varies with each study and this lack of consistent criteria means that the studies cannot be easily compared (Kavale &

Forness, 1995). Although most studies utilize equations based on chronological age and intelligence to predict an expected achievement level which is then compared to the actual achievement level, the cutoff scores in the various studies vary by as much as a standard deviation. Studies have demonstrated equivocal findings in discerning quantitative differences between students in their studies who are identified as MLD and students with Low Math/NLD (Kavale, Fuchs, & Scruggs, 1994; Ysseldyke, Algozzine, Shinn, & McGue, 1982). This lack of difference might be attributed to sample selection because students, who were identified as participants, were identified as MLD through the use of local school identification procedures or achievement scores (Morris et al., 1994) which vary between districts rather than by a consistent commonly agreed upon criteria. In other studies students who had been identified by the schools as having a learning disability, but did not have a neurological dysfunction were included and students who did have a learning disability but had not been referred by the classroom teacher for special education services were excluded (Zigmond, 1993).

There have been very few tests specifically developed to determine if an individual has a learning disability. When an assessment is conducted for purposes of meeting a discrepancy formula, usually some type of intelligence test and achievement test are given. However, McDermott, Fantuzzo, and Glutting (1990, 1992) suggested that using only ability or IQ measures to create a profile analysis is of limited value for identifying learning disabilities. Such a profile and analysis may provide some information about a student’s verbal skills but is unlikely to provide a definitive identification of a learning disability based on language processing abilities. Because a specific learning disability refers to a disorder in basic psychological processes involved in the understanding or using of language (IDEA, 2004), utilizing assessments to identify the student's possession of an intrinsic processing disorder could be made part of the identification procedure for learning disabilities (Kavale & Forness, 1995; Mercer, 1997). However, historically confirmation of processing problems of students identified as MLD has not been a part of the identification process used by local schools.

Statement of Problem

The world's heavy reliance on the use of mathematics today (Furner & Duffy, 2002) has resulted in the demand for accountability of the public school's performance. With the introduction of high stakes testing, an individual's lack of mathematical competency can result in an inability to pass an exit test required for high school graduation and access to further education (Thompson & Thurlow, 2003) or even result in unemployment (Bottge & Hasselbring, 1993). Students with learning disabilities are at special risk for failure because the instructional methods and materials proposed in the *2000 Principles and Standards* (Woodward & Montague, 2002) do not meet their needs because they learn differently than students without learning disabilities (Carnine, et al., 1994). Accurate identification of students with MLD is crucial in order for them to receive appropriate instruction. Over identification results in funding being used for students without MLD who do not need the specialized instruction provided through special education but can benefit from instruction provided to students without disabilities. The result of under identification is that students, who have MLD and who need special instruction, are denied an appropriate education and given an expectation of failure.

The problem is that initially students with MLD and students with Low Math/NLD (both of whom have low achievement in mathematics) appear similar. Both groups of students are part of the larger group of students who are experiencing low achievement in mathematics. Eventually, with proper instruction the students with Low Math/NLD will show an improvement in their performance (Geary, 1993). The students with MLD will continue to demonstrate low achievement. If a method of accurate early identification was available, students with MLD would be able to receive the appropriate interventions, for which they are legally eligible, much earlier at a point of prevention (Fuchs, 2005). By administering measures that are hypothesized to be associated with mathematics word problem solving factors the characteristics of mathematical word problem-solving behaviors and the types of mathematical word problem errors that are demonstrated by students with mathematics learning disabilities could be identified (Fuchs, 2005).

In respect to the accurate identification of students with mathematics learning disabilities, so far results from research studies have been inconclusive. Studies have not consistently shown a qualitative difference between students with low achievement in mathematics who are not identified as having a mathematics learning disability and students with low achievement in mathematics that are identified as having a mathematics learning disability. Some of this lack of discrimination may be due to differences in definitions of disability as well as differences in the cut-off points used to determine a disability (Kavale et al., 1994; Ysseldyke et al., 1982). This lack of difference may be due to the fact that the students selected for the studies were provided by local school districts that did not use a consistent definition of learning disabilities when identifying students as having a learning disability (Kavale et al., 1994; Ysseldyke et al., 1982). Participating students in research studies might be identified as LD through a discrepancy formula, by achievement scores (Morris, et al., 1994), by a below expected grade academic performance (Parmar, et al., l996; Russell & Ginsburg, l984), or by various percentile scores (Hanich, et al., 2001; Torgesen & Bryant, 1994). Because none of the criteria used to identify students as LD required a processing problem, which is considered to be a characteristic of LD, it is possible that the samples of students identified with LD were actually composed predominantly of students with low achievement without LD. Studying students identified as having LD by the schools will not aid in understanding the fundamental nature of LD (Zigmond, 1993) if the schools' identification process for learning disabilities is flawed (Lester & Kelman, 1997). Because groups of students identified as having LD by the schools contain students with low achievement who do not necessarily have a processing problem, their learning characteristics are not likely to be distinguished from students with low achievement who have not been identified as LD by the schools. The main composition of both groups is students with low achievement without any identified processing problems. These students would be expected to show similarities.

In recent years the usefulness of IQ/achievement discrepancy for identification of a specific learning disability has been increasingly questioned (Bradley, Danielson, Hallahan, 2002). A responsiveness-to-intervention (RTI) model has been proposed to replace the discrepancy model (Kavale, Holdnack, & Mostert, 2006). The RTI model advocates using a student's inadequate response to intervention as a means of identifying a specific learning disability (Vaughn & Fuchs, 2003). Students with low achievement in mathematics would be provided with scientific research-based instruction and their progress monitored. If they do not respond to the instruction, they would be provided with more intensive or different instruction. If continuous monitoring shows that they are failing to respond to these empirically validated treatments, they then might qualify for special education (Fuchs, Moch, Morgan, & Young, 2003). Although the RTI model provides some type of early intervention, it essentially prolongs the process of special education identification. Eventually, the student who does not make adequate progress will need to be directly assessed to determine if the student does possess a processing deficit and has a specific learning disability or is merely a "slow learner" (Torgesen, 2002).

Identification of students with LD becomes even more problematic when the heterogeneous nature of the LD population is considered (Ackerman & Dykman, 1995; Geary et al., 1999; Geary et al., 2000; Robinson et al., 2002; Rourke, 1998). The characteristics of students with mathematics learning disabilities in mathematics reasoning and students with mathematics learning disabilities in mathematics calculation are quite different even though all of these students have a learning disability in mathematics (Jordan & Hanich, 2000; Jordan et al., 1997; Rourke, 1989, 1991; Rourke & Conway, 1997). The students with a learning disability in mathematics calculation and those with a learning disability in mathematics reasoning actually display opposite patterns and cancel out the other's performance in testing situations (Rourke, 1998). Because their differences cancel each other out, the group of students with mathematics learning disabilities as a whole produces a profile similar to that of a group of students without learning disabilities.

The assessment of domain-specific skills and the careful analysis of errors in diagnosing learning disabilities can supply valuable information beyond what is provided by standardized achievement test (Siegel, 1999). Information beyond a simple IQ test or a discrepancy score between IQ and achievement score is needed for identifying an intrinsic processing problem (McDermott, et al., 1990, 1992). The cognitive processes and strategies employed by individuals in various problem-solving situations should be observed to determine if a learning disability exists, and if it does, what type of interrelationships might exist between processes that affect the student's strengths and weaknesses (Meltzer, 1994).

Significance of the Problem

Researchers suggest that 6−7% of students with an average or higher IQ who have received adequate instruction have a cognitive or neuropsychological deficit representative of a learning disability that interferes with their ability to perform mathematics at grade level (Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974; Lewis, Hitch, & Walker, 1994) and that this learning disability will not go away once the student has graduated from high school (McCue & Goldstein, 1991). A recent study through the Mayo Clinic (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen,

2005) suggests that the numbers of students with learning disabilities in mathematics is even higher and actually varies from 5.9%−13.8% with 35%−56.7% of the students with MLD not having a comorbid reading disorder as well. Research indicates that these students, who have characteristics of learning disabilities, as shown by their cognitive or neuropsychological deficits commonly manifested as a processing problem, are not necessarily being identified as having a learning disability (Shaywitz et al., 1992; Ysseldyke et al., 1982). Even though they have average or higher intelligence, without proper identification (which provides funding for services), these students, who have unique learning characteristics (Miller & Mercer, 1997), are unable to access the different instruction, materials, accommodations, and modifications of the curriculum essential to their progress and success (Bateman, 1992; Carnine, 1992; Howell, 1993) and are at risk of failure (Carnine et al., 1994; Zentall & Ferkis, 1993). In particular, further research is needed with older students to identify and explain mathematics learning disabilities through understanding the different types of mathematical disorders, the ways in which children construct mathematical knowledge, and the effects of classroom curriculum as delivered by teachers through textbooks and instructional adaptations on math learning.

Learning disabilities in mathematics usually appear in elementary school in the areas of mathematics fluency and computation. Accurate and early identification is critical for optimum progress even though serious difficulties with mathematics problem solving do not appear until sixth grade. At the end of high school students with mathematics learning disabilities leave with lower levels of mathematics achievement than their peer group demonstrates (Wagner, 1990). Mathematics learning disabilities continue to follow the individual into adulthood and affect performance in the workforce and in daily living (Miller & Mercer, 1997). This lack of ability to calculate and problem solve may limit the ability of an individual to hold a job requiring additional skills and can impact the individual's ability to provide for a home and family through earnings in the workforce and successful use of money management (Bottge & Hasselbring, 1993).

Statement of Purpose

The purpose of this research study was to identify the factors associated with mathematics word problem solving and then to determine if there were differences in problem-solving behaviors, linguistic characteristics, and types of errors displayed by (a) students with low achievement in mathematics who were likely to have a learning disability (MLD) and (b) students with low achievement in mathematics who were unlikely to have a learning disability (Low Math/NLD). (The indication of the student possessing a processing disorder was used to determine whether the student was likely to have a learning disability. A student without an indication of the possession of a processing disorder was considered unlikely to have a learning disability.)

The students' use and understanding of language as they solved word problems was observed through the use of the *Informal Mathematics Assessment*. Students' procedural steps, possible misconceptions, algorithm choices, and errors were analyzed for differences in the ways that student groups approached and attempted to solve word problems. Mathematics and reading achievement was assessed with the *Woodcock Johnson III Tests of Achievement*, indications of a processing disorder was established through the use of the *Learning Disabilities Diagnostic Inventory*, and linguistic processing was examined with the *Clinical Evaluation of Language Functions—Fourth Edition*.

If it could be quickly determined by distinguishing characteristics or a simple assessment whether a student with low mathematics achievement possessed a learning disability, educators could immediately provide appropriate instruction. Students with mathematics learning disabilities would no longer need to demonstrate failure before receiving special education services. Money would also be saved by not providing special education instruction to students who are merely low achieving without a learning disability and would benefit from remedial instruction provided through general education.

Research Questions

The following research questions were addressed:

(1) What are the underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student mathematics and reading achievement from the *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests?

(2) Are there statistically significant differences in mathematics automaticity as shown by Math Fluency scores as measured by the WJ III ACH between (a) students with MLD who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH who were likely to have a

learning disability according to the *Learning Disabilities Diagnostic Inventory* (LDDI) and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(3) Are there statistically significant differences in Calculation scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(4) Are there statistically significant differences in Applied Problems scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(5) Are there statistically significant linguistic differences in cognitive areas measured by the subtest scores of the CELF-4 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability

according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(6) Are there statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(7) Are there differences in mathematical problem-solving error type percentages as measured by the categories of computation, operation, translation, and no attempt on the *Informal Mathematics Assessment* between (a)students with MLD who received a score ≤25^{th} percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25^{th} percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were not likely to have a learning disability according to the LDDI?

Null Hypotheses

From these questions the following null hypotheses were posited:

(1) There are no statistically significant underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student mathematics and reading achievement from *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests.

(2) There are no statistically significant differences in mathematics automaticity as shown by Math Fluency scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(3) There are no statistically significant differences in Calculation scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(4) There are no statistically significant differences in Applied Problems scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(5) There are no statistically significant linguistic differences in cognitive areas measured by the subtest scores of the CELF-4 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(6) There are no statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math

Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(7) There are no differences in mathematical problem-solving error type percentages as measured by the categories of computation, operation, translation, and no attempt on the *Informal Mathematics Assessment* between (a) students with MLD who received a score $\leq 25^{th}$ percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score $\leq 25^{th}$ percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were not likely to have a learning disability according to the LDDI.

CHAPTER II

Literature Review

Research in the area of mathematics learning disabilities has only recently become an area of concern. Previously, most research in the area of LD was focused on the area of reading (Brandys & Rourke, 1991; Geary, 1993; Robinson et al., 2002. The study of mathematics learning disabilities (MLD) as a developmental disorder is a relatively recent occurrence (Rourke & Conway, 1997). In recent years the population of students identified as having a learning disability has grown rapidly. The number of students identified as having LD has increased from 796,000 students or 1.8 percent of school-aged students in the 1976-1977 school year to almost 5.2 percent in 1997-98. In the 1997-98 school year, the 2,726,000 students with LD represented approximately 46 percent of all school-aged children in special education programs (Office of Special Education and Rehabilitative Services, 2000). Research findings have shown that 5% to 8% of school-age children are identified as having a mathematics disability (Geary, 2004). It is suspected that 6-7% of all students with average or higher IQ who have received adequate instruction may have a learning disability in mathematics which prevents them from performing on grade level in mathematics (Badian, 1983; Gross-Tsur et al., 1996; Kosc, 1974; Lewis et al., 1994; Light & DeFries, 1995). A recent study through the Mayo Clinic (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005) suggests that the numbers of students with learning disabilities in mathematics is even higher and actually varies from 5.9%−13.8%.

The purpose of this study was to identify the factors associated with mathematics word problem-solving and to determine if there were differences in these factors: mathematics fluency (automaticity), calculation, linguistic characteristics, reading abilities, and problem-solving behaviors and types of errors in mathematics displayed by students with low achievement in mathematics with learning disabilities (MLD) and students with low achievement in mathematics without learning disabilities (Low Math/NLD). Chapter II examines the research studies that have compared students with MLD to students with Low Math/NLD and the assessment batteries they have used. Such studies are relatively few in number because most studies have compared students with a mathematics learning disability to typically developing students of the same chronological age or typically developing students a year younger (Barrouillet et al., 1997; Geary & Brown, 1991; Garnett & Fleischner, 1983; Rourke & Conway, 1997).

Mathematics Word Problem Solving

Principles & Standards for School Mathematics (NCTM, 2004) promotes the idea that all students must have access to mathematics so that they can learn important concepts and procedures. An ability to think mathematically and problem-solve creatively and resourcefully is crucial for a student's eventual success in the world of work. To understand mathematics and use it to solve novel problems requires that the student combine factual knowledge, procedural facility, and conceptual understanding (NCTM, 2004). For effective problem solving, the student needs an awareness of number sense and operations, which a student acquires by realizing that numbers can be thought about in a variety of ways through decomposition. The student needs computational fluency, which includes mental mathematics, estimation, and paper-and-pencil calculations. The student also needs to analyze the situation and develop a range of strategies for solving problems. Problem solving requires students, when faced with a novel situation, to use their knowledge of concepts, procedures, reasoning, and communication/representational skills (National Assessment Governing Board, 2000).

Student difficulties with word problem-solving. Unfortunately, many students find solving mathematics word problems difficult (National Assessment of Educational Progress, 1992), and students with mathematics learning disabilities find solving word problems extremely difficult (Bryant, Bryant, & Hammill, 2000). Students must have skills in linguistic, computational, memory, cognitive, and metacognitive areas (Englert, Culatta, & Horn, 1987; Parmar, 1992; Parmar, Cawley, & Frazita, 1996; Rivera, 1997) in order to be successful problem-solvers. The student with MLD has difficulty understanding and translating written or verbal language into mathematical symbols (Miller & Mercer, 1997; Montague, 1992) or into visual or graphic representations (Jitendra, 2002). They have difficulty with counting and basic number skills (Geary et al., 1999). They especially have difficulty with fluency of basic mathematics facts (Geary, 1993; Hanich, Jordan, Kaplan, & Dick, 2001). Researchers generally agree that this lack of memorization of basic mathematics facts is the failure most often seen with students with MLD (Swanson & Jerman, 2006). However, because students with Low Math/NLD may demonstrate weaknesses in these areas as well (Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005; Geary, 1993), it has not been possible to discriminate between the students with MLD and students with Low Math/NLD without first providing intensive interventions to both groups.

Students in elementary school with Low Math/NLD demonstrate a developmental delay as shown by their immature and error-prone procedures in mathematics (Geary & Brown, 1991). However, because these students are experiencing developmental delays and not a long-term cognitive deficit, such as students with MLD experience, with intensive and appropriate instruction, their developmental delays disappear with age (Geary, 1993; Fuchs & Fuchs, 1998). Although students with Low Math/NLD acquire mathematics skills more slowly than typical students, they are expected eventually to develop appropriate levels of arithmetic skills (Stanley, 1978). Other students who do not eventually develop appropriate mathematics skills and who exhibit developmental differences that do not disappear with age are seen as having a mathematics learning disability (Geary, 1993; Scheid, 1993). Therefore, although initially students with MLD and students with Low Math/NLD both have low achievement in mathematics, with intensive and appropriate instruction students with Low Math/NLD will develop appropriate skill levels in mathematics while students with MLD will continue to display developmental differences.

This low achievement in mathematics shown by students with MLD is a distinct form of low achievement displayed by students with learning disabilities (Clarizio & Phillips, 1992). Discriminating, between students with MLD and students who are merely low achievers is necessary because instructional strategies that are effective for students with Low Math/NLD have not necessarily been successful strategies for students with MLD (Swanson, 1993a). Even when students with MLD achieved comparably to students with Low Math/NLD in some areas, it cannot be assumed that the individuals in the two groups were using the same learning strategies. Swanson (1993a) indicated that it was quite likely that the students with MLD actually used different strategies even on tasks with which they did not seem to have difficulty. Swanson (1993a) also suggested that when students with MLD did use effective strategies, it could not be presumed that processing differences had disappeared but only that the strategy had helped to compensate for the processing disorder. Students with MLD appear to have definite differences and need different types of instruction than students with Low Math/NLD (Rourke & Conway, 1997; Swanson, 1993a; Swanson, Caron, & Saches-Lee, 1996). The challenge is in finding identifiable and measurable differences between these groups so that students can be correctly identified and their needs appropriately met through early interventions at the primary level for computation (Fuchs et al, 2005) and at the intermediate and middle school level for multiple-step mathematics word problem-solving (Swanson, 1993a).

Identifiable differences of students with MLD. In an attempt to find measurable differences, Kirk's (1963) suggestion that a student with a learning disability would demonstrate a difference between the student's actual low achievement and the student's expected achievement (based on the student's appearance of capability from an assessment of intelligence) was embraced. Kirk's suggestion generated the idea that a student with a learning disability could be identified based on the difference between an IQ score and an achievement score, which was later referred to as the discrepancy formula. However, identifying students as having a mathematics learning disability solely on the basis of IQ and achievement results is too broad (Ginsburg, 1997) and allows for frequent misidentification. By only requiring a discrepancy between IQ and achievement scores, too few students were excluded from the learning disability classification (Reynolds, 1984; Shaywitz et al., 1992). Students with average IQs, who were failing school because of lack of motivation, poor self-concept as a learner, or poor instruction, could be misidentified as having a learning disability (Ginsburg, 1997; Reynolds, 1994; Shaywitz et al., 1992). Inversely, students with a learning disability but without a discrepancy between IQ and achievement were not being identified (Ysseldyke, Algozzine, Shinn, & McGue, 1982).

Geary (1990) attempted to identify students with MLD by a means other than achievement scores. He selected a group of students identified as MLD and followed their academic progress while remediation in mathematics was provided. After receiving remedial mathematics, Geary's single group of students with MLD divided into two groups: (a) students with MLD who had demonstrated progress in mathematics and (b) students with MLD who showed little or no change. Geary suggested that those students who had demonstrated progress did not actually have MLD but were only slow learners who had been misidentified as having MLD. Geary suggested that a way of determining if a student had MLD was to observe whether the student was resistant to treatment provided through remediation. If the student made progress, then the student did not have MLD. Geary's (1990) work supports the idea that there are qualitative differences between students with MLD and students with low achievement without learning disabilities (Low Math/NLD).

The responsiveness-to-intervention (RTI) model is very similar to Geary's (1993) proposal for identification. As the usefulness of IQ/achievement discrepancy for identification of a specific learning disability has been increasingly questioned (Bradley, Danielson, & Hallahan, 2002) there has been a movement to a RTI model (Kavale, Holdnack, & Mostert, 2006). With this model, students with low achievement in mathematics are provided with research-based instruction and their progress is monitored. If they do not respond positively to the instruction, they continue to be monitored while they are provided with more intensive or different instruction. If the students still continue to show no improvement, then the RTI model advocates identifying a specific learning disability as the cause of a student's inadequate response to these interventions (Vaughn & Fuchs, 2003). Eventually the student who does not make adequate progress will need to be directly assessed to determine if the student does possess a processing deficit and has a specific learning disability or is merely a "slow learner" (Torgesen, 2002). When the student's performance does not improve over time, only then is a possibility of learning disabilities considered.

Factors needed for word problem-solving. It is not sufficient to study only the domains of calculation, computation, mathematics reasoning, and word problem solving into which mathematics is commonly divided because these domains are affected by a variety factors (Kavale & Forness, 1995). Such factors include sequential processing and language processing (Rourke & Conway, 1997), which affect information representation, as in articulating number words, information manipulation in working memory, and mathematics fact retrieval or automaticity. Language processing problems affect the development of number skills, the concept of number (Manor et al., 2000), the ability to sequence, and auditory memory. Operations involved in the intelligence processes, including thinking, memory, conceptualization, speed of processing, and problem solving (Geary, 1993; Smith, Polloway, Patton, & Dowdy, 1998) refer to cognitive processing also involved in mathematics problem-solving. Kavale and Forness (1995) promoted studying the intricacies of all factors and the impact that learning disabilities can have on these areas often referred to as processing skills. They suggested that the complexity of learning disabilities (LD) has not been fully recognized, and that currently there is only a superficial understanding of LD and how it can affect factors such as the language related to successful mathematics problem-solving.

Summary. All students must have access to mathematics (NCTM, 2004). To be successful at mathematics problem solving, a student needs an awareness of number sense, and skills in linguistic, computational, memory, cognitive, and metacognitive areas (Engler, Culatta, & Horn, 1987; Parmar, 1992; Parmar, Cawley & Frazita, 1996). Students with MLD have difficulty with understanding and translating language into mathematics symbols (Miller & Mercer, 1997) and with the automaticity of basic mathematics facts (Geary, 1993; Hanich, Jordan, Kaplan, & Dick, 2001). Before intensive interventions, students with Low Math/NLD also display some of these same difficulties so it is difficult to tell the students from the two groups apart. Eventually, however, students with Low Math/NLD will develop appropriate levels of mathematics skills whereas students with MLD will not (Geary, 1993). Because students with MLD do not necessarily respond to the same instructional strategies as students with Low Math/NLD, it is essential to identify students with MLD so that appropriate interventions can be provided (Swanson, 1993a). By administering measures thought to be related to the factors associated with mathematics word problem solving, it is anticipated that the characteristics of mathematics word problem-solving behaviors and the types of mathematics word problem errors that are demonstrated by students with MLD could be identified (Fuchs, 2005). These factors include sequential processing, language processing (Rourke & Conway, 1997), processing speed, working memory, and cognitive processing (Geary, 1993; Swanson, 1993).

Comparing Students with MLD and Students with Low Math/NLD

Identification of participants with MLD. There are substantially fewer research studies into mathematics learning disabilities than studies into reading learning disabilities (Brandys & Rourke, 1991; Swanson, Carson, Cristi, & Saches-Lee, 1996), and the comparison between mathematics studies has been difficult due to the lack of a consistent pool of students (Hammill & Bryant, 1998). There have been numerous attempts to identify students with LD by comparing their achievement to their perceived capability or potential to learn. Various methods have been used for this comparison such as using achievement scores for age-based discrepancy and IQ scores for IQ derived discrepancy. These different methods have resulted in students needing to meet different criteria in different years and in different states in order to be identified as having LD. Further more, students identified as having LD did not form a consistent pool of students with LD because of the difficulties with the discrepancy formulas and the variations of identification procedures. Therefore, researchers began using alternative methods for identifying students with LD when looking for participants for their studies. Geary (2003) found that students identified in research studies as having MLD had mathematical scores ranging from the 8th percentile to the 48th percentile. For example, Swanson (1993a) did not use the discrepancy formula when selecting students for his working memory study, and Geary, Hoard, and Hanson (1999) selected students who had mathematics reasoning scores from the *Woodcock Johnson PsychoEducational Battery-Revised* (Woodcock & Johnson, 1989, 1990) below the 30th percentile instead of using the discrepancy formula. Jordan and associates (Jordan & Hanich, 2000; Jordan & Montani, 1997) selected students who had scores below the 30th percentile on the mathematics section of the *Comprehensive Tests of Basic Skills*. Geary (1993) used the 25th percentile on the mathematics composite score of the *Stanford Achievement Test* to select his participants. In the study of Ysseldyke et al. (1982) the participants were designated as having either low achievement in mathematics without learning disabilities (Low Math/NLD) or mathematics learning disabilities (MLD) based on identification by the local school district. Because the participants in these studies were not selected from the same population sample, and variations in criteria for identification of LD from study to study occurred, it has been difficult to replicate studies and to generalize findings (Hammill & Bryant, 1998; Hammill, Bryant, Brown, Dunn, & Marten, 1989; Rosenberg et al., 1992, 1993).

Most studies have compared students with MLD to typically developing peers of the same age or a year younger (Swanson, Carson, Cristi, & Saches-Lee, 1996; Swanson & Olga, 2006; Xin & Jitendra, 1999). A more recent synthesis by Swanson and Jerman (2006) compared students with MLD to average achievers and to students with learning disabilities in mathematics and reading. Few studies have directly compared students with MLD to students with Low Math/NLD.

Most of the studies on students with MLD have focused on the areas of mathematics calculation and basic mathematics facts Fuchs & Fuchs, 1998; Geary, 1993). Research studies, concentrating on students with MLD, in respect to mathematical word problems and the use of language, are very sparse. For those studies that do exist, results have been inconclusive. Difficulties arise when comparing studies because students identified as having MLD in the various studies are not coming from the same pool of students. Identification of students as MLD varies between studies with some studies using the school district to identify the student as having MLD (Vaughn & Fuchs, 2003, Geary, 1990; Ysseldyke et al., 1982; Ysseldyke et al., 1983), other studies using achievement (Jordan & Hanich, 2000) or IQ scores (Rourke, 1989, 1991, 1998), and other studies looking at the gain in mathematics achievement to determine whether or not the student has MLD (Geary, 1990). Currently there is not an agreed upon standard for identifying students with MLD. This problem of identification is made even more challenging by the inability to discriminate between students with MLD and with Low Math/NLD currently through the use of a testing instrument.

Differentiation by assessments. The oldest and perhaps seminal study, which attempted to identify differences between students with low mathematics performance and students with MLD, was conducted by Ysseldyke, Algozzine, Shinn, and McGue (1982). This is one of the few studies that sought to measure processing disorders. In this study student performance in five domains thought to be affected by processing disorders: cognitive, academic achievement (including mathematics), perceptual-motor, self-concept, and behavior problems was studied. Researchers compared a group of 50 fourth-grade students who had been identified as LD by their school districts with 49 fourth-grade students who had not been identified as LD but who had scored at or below the 25th percentile on the *Iowa Tests of Basic Skills*. To assess the students' performance in the five domains: the students were administered a battery of tests including the *Wechsler Intelligence Scale for Children-Revised* (WISC-R), the *Peabody Individual Achievement Test* (PIAT), the *Stanford Achievement Test*, the *Bender Visual-Motor Gestalt Test* (BVMGT) the *Developmental Test of Visual-Motor Integration* (DTVMI), the *Piers-Harris Self-Concept Scale*, the *Peterson-Quay Behavior Problem Checklist*, and the *Woodcock-Johnson Psycho-Educational Battery* (W-J). A series of t tests were used to compare the mean subtest scores of the two groups on each of the tests. Although the group with LD performed significantly more poorly than the students with low achievement on the PIAT subtests, the researchers found no psychometric differences of practical utility between the groups because the percentage of overlap between the individual scores in the two groups ranged from 82% to 100% on the scores. Reanalyzing the study led to inclusive conclusions and the possibility that there might be differences (Kavale, 1995; Kavale et al., 1994). The study did not attempt to construct groups based on the performance information obtained on the five domains and thought to be affected by processing disorders. Because students with processing disorders could appear in both the group identified by the school districts and the group identified by low scores on the *Iowa Tests of Basic Skills*, it was unlikely that differences based on processing disorders would appear between the two groups.

The results of the Minnesota Institute studies (Ysseldyke et al., 1983) reinforced the original findings of Ysseldyke et al. (1982). These studies formed the basis of the argument that there were few differences between the specific mathematics behaviors possessed by students with MLD and students with Low Math/NLD. Algozzine, Ysseldyke, and McGue (1995) suggested that although students with MLD may be the lowest of the low achievers, they did not necessarily have qualitative differences nor did they require qualitatively different instruction. They asserted that there were not qualitative differences between students with LD and their peers with low achievement without LD. Finding only quantitative differences between the MLD and Low Math/NLD groups is an expected result based on the means of selecting the groups. The MLD group was composed of students with lower achievement scores than the Low Math/NLD group. However, in these studies the identification of a learning disability did not include the possession of a processing disorder.

Differentiation by interventions. Unlike the Ysseldyke, Algozzine, Shinn, and McGue (1982) study, the studies of Geary (1993) and Fuchs and associates (2005) were longitudinal and contained an intervention. Both studies focused on discriminating between students with MLD and students with Low Math/NLD. For his study, Geary (1990) began by selecting first- and second-grade students defined as having learning disabilities (LD) by their schools. These students received remedial education in mathematics for about 20 minutes a day. The investigator used the current year's achievement test scores to separate these students into two groups: LD-improved and LD-no change. The former group's scores had improved to the point where the students placed out of remedial education; the latter group's scores were substantially stable. Furthermore, the results showed that the LD-improved group displayed underlying cognitive processes similar to those of nondisabled students and did not demonstrate processing disorders. The LD-improved group were perhaps developmentally delayed and were slow learners but were not developmentally different. The members of the LD-no change group, however, were developmentally different. Geary (1990) concluded that the "initial poor achievement scores of the LD-improved group were likely due to inadequate preacademic skills and/or the initial misclassification of some of these subjects and not due to an underlying cognitive or meta-cognitive deficit" (p. 378). The implication was that students in the LD-improved group were not really learning disabled and that Geary's design had eliminated false positives. Geary viewed the procedure of providing remediation and then waiting and watching to observe whether the remediation resulted in a positive change as a useful method of determining whether a student had MLD. The usefulness of the method was curtailed by the fact that the method required time to pass before deciding if the remediation was successful. During the time period that the remediation was taking place, the student with MLD was not receiving instruction that was appropriate for a student with MLD but instead was receiving an intervention designed for the student with Low Math/NLD who developed at a slower rate than the typical student but was not developmentally different as the student with MLD was. So although Geary's method effectively identified students with MLD, the inappropriate instruction that students with MLD received while waiting to prove their failure was a loss in instructional time that could not be recovered by the student with MLD whose performance was already lagging behind that of classmates.

A number of variables may be associated with some aspect of mathematics word problem-solving. These variables include mathematics fact fluency (Geary, 1993; Hitch & McAuley, 1991), working memory (Geary, 1993; Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger 2004), processing speed (Bull & Johnston, 1997), language skills (Jordan, Levine, & Huttenlocher, 1995), and reading (Jordan & Hanich, 2000; Rourke, 1989, 1991, 1998). However, studies are generally limited to exploring one or two variables (Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005). Fuchs and associates (2005) investigated cognitive abilities in predicting which first graders would be eventually identified as having MLD. They included assessments for mathematics basic fact fluency, calculation, applied problems, mathematics concept/applications, story problems, word attack, word identification, passage comprehension, and for language (vocabulary, similarities, and listening comprehension), and problem solving (block design, matrix reasoning). After completing a battery of assessments with the students, Fuchs and associates (2005), similarly to Geary (1990, 1993), used a response to intervention model by providing an intervention of tutoring to first grade students with low achievement in mathematics. There was also a control group of students with low achievement in mathematics who did not receive tutoring, a group of typical students who did not receive tutoring, and a group of typical students who did receive tutoring. The students who received tutoring improved more than either of the other groups. After a statistical analysis was completed, it was determined that 2-3 students from each group would meet criteria as having MLD except for the group of tutored students with low mathematics achievement who would not have any members meet criteria as MLD. This result made it difficult to analyze the second part of the study, which examined the cognitive characteristics that underlie the development of mathematics competency and word problem-solving. A further hindrance was the discovery that the *Woodcock-Johnson Tests of Achievement III* was not a sensitive enough measure for mathematics at the first grade level.

Differentiation by reading performance. Other studies also using batteries of assessments explored the affect of reading on students' mathematical abilities (D'Angiulli & Siegel, 2003; Jordan & Hanich, 2000; Rourke, 1989, 1991, 1998). Amedeo D'Angiulli and Linda Siegel (2003) studied three groups selected using *Wide Range Achievement Test Revised* (WRAT-R) scores. Students with typical achievement (TA) had scores on WRAT-R of reading, spelling, and arithmetic >30%. The reading disabilities group (RD) had a reading WRAT-R score < 25%. The group of students with MLD had an arithmetic WRAT-R score < 25% and reading >30%. In Verbal Conceptualization, Knowledge, and Sequential factors the TA group had significantly higher scores than the MLD group and the MLD group had significantly higher scores than RD group. The MLD group had significantly lower Arithmetic, Coding, and Performance IQ scores on WISC-R. WISC-R scores suggested that the students in the MLD group had deficits in memory, attention, and speed. Students in the RD group had deficits in memory, attention (working memory), and language. Students in the RD group and students in the MLD group both had significantly lower scores on subtests that tapped working memory or language. The researchers suggested using assessments that measured skill in achievement areas as being the most appropriate measure for diagnosing disabilities and being more likely to show patterns than the WISC-R. The RD group contained students that had low reading scores as well as students with low reading and arithmetic scores. The mean on the WRAT-R arithmetic score for the RD group was 17.6 with a standard deviation of 17.5 indicating that the RD group contained some students with low mathematics scores.

Other studies, also using batteries of assessments, did not compare students with MLD to students who were typical age peers or younger. Instead based on reading ability, students with MLD were divided into groups (Rourke, 1989, 1991, 1998). Rourke (1989, 1991, 1998) used reading and arithmetic scores from the WRAT to group students with MLD before examining their performance on the WISC for patterns and hints as to their cognitive abilities. When reviewing Rourke's studies, it is important to remember that the reading assessment on the WRAT measures word identification rather than reading comprehension, and the mathematics test is measuring arithmetic mechanics. The tests are assessing memory rather than conceptual understanding. The tests may be useful in identifying students who are having difficulty in mathematics due to problems with retrieval, which may be indicative of a mathematics learning disability in the area of mathematics calculation, but they will not necessarily discern a student's difficulties in the area of mathematics reasoning as demonstrated by difficulties with word problems.

Rourke (1989, 1991, 1998), Jordan, and Hanich (2000) found a difference between students with MLD and students with a disability in mathematics and reading disability (MLD/RD). However, the participants selected in their studies may not actually have had MLD since they were selected based on achievement scores, which were required to be at or below the 30th percentile. Their studies showed that students with MLD/RD performed significantly worse in most areas of mathematics than students with MLD only. Students with MLD outperformed the MLD/RD group on story problems and written calculation, and they showed more specific types of deficits in problem solving. Students with MLD/RD showed more pervasive deficiencies in mathematical thinking and experienced weaknesses with problem conceptualization and execution of calculation strategies. Students with only MLD experienced difficulties with rapid fact retrieval and problem solving efficiency. The research of Jordan and Hanich (2000) and Rourke (1989, 1991, 1998) supports the two-factor theory promoted by Carol Robinson, Bruce Menchetti, and Joseph Torgeson (2002) that students with co-morbid disabilities MLD/RD demonstrate problems that are more serious that students with MLD.

Differentiation by word problem-solving performance. Students with MLD also demonstrated difficulty with multi-step problems, and the language of mathematics.

They failed to verify answers, and settled for first answer. Although most comparison studies have used areas of computation or basic mathematics facts retrieval (Carnine et al., 1994; Geary, 1993) to represent mathematics in general, the study of Diane Bryant, Brian Bryant, and Donald Hammill (2000) found the most frequently observed mathematics difficulties for students with MLD was word problem solving. Even though solving mathematics word problems is extremely difficult for students with MLD, few studies have researched this area. The study by Woodward, Monroe, and Baxter (2001), which also supported Geary's conclusions, was one of the few studies that examined solving word problems. It did not present a battery of tests as in other studies (Fuchs et al., 2005; Ysseldyke, Algozzine, Shinn, & McGue, 1982) but instead emphasized solving mathematics word problems to an even greater degree than Fuchs and associates (2005). In this study there were 102 fourth grade students in the four intervention classrooms and 79 fourth grade students in the three comparison classes. Eleven students identified as MLD received mathematics instruction in the resource room and the general education classroom. Six of the students receiving special education services for LD were in intervention classes and five students with LD were in comparison classes. Based on scores at or below the 34^{th} percentile on the *Iowa Test of Basic Skills* (1996) twenty-five students were identified as at-risk or with Low Math/NLD. A categorical analysis of the performance assessments taken during the school year showed some differences between students with MLD and students with Low Math/NLD. Initially in the fall both groups of students would guess, repeat numbers presented in the problem, or say, "I don't know" as soon as the examiner finished reading the word problem.

However, by spring the students with Low Math/NLD had shifted from saying “I don’t know” to attempting to use the relevant numbers in the word problem although they used them incorrectly. The students with MLD continued saying “I don’t know” or would attempt to solve the word problem with irrelevant information. When students were assessed with the *Individual Mathematics Assessment*, which is a type of student interview, students with MLD who received the interventions of (a) class wide instruction on performance assessment tasks and (b) problem-solving instruction in ad hoc tutoring, fell into two groups. One group began to include strategies for solving a problem, such as making a table, and their explanations for how they solved the problem rose above a literal restatement of the pictures or numbers they had written down. The other group of students with MLD continued to restate what they had written down and their answers showed a lack of conceptual understanding of the mathematical word problem. For students with MLD who did not receive the intervention there was little or no change over time in their answers. Students in the average and high groups, who received the intervention of class wide instruction on performance assessment tasks, but did not receive ad hoc tutoring in problem-solving instruction, showed gains. According to Geary’s definition of MLD, those students with MLD who made progress did not actually have a mathematics learning disability but were merely slow learners. Only those students who had received the intervention and made little or no progress had MLD.

One of the few studies on low mathematics achievement due to difficulties with mathematics word problems was conducted by Sovik, Frostrad, and Heggberget (1999).

They explored the relationship between reading comprehension and solving mathematical word problems by looking at the strategies used to solve mathematical word problems by four groups of nine-year-olds: (a) Group 1 was good at reading and mathematics skills, (b) Group 2 was good at mathematics and poor at reading skills, (c) Group 3 was good at reading and poor at mathematics skills, and (d) Group 4 was poor at mathematics and reading skills. Group 3 was similar to students with MLD only, and Group 4 was similar to a MLD/RD group. Group 1 used deductive strategies to solve all the word problems. Group 2 used mainly deductive strategies but also some procedural approaches for the more difficult word problems. Group 3 applied procedural strategies most of the time when solving the word problems. Group 4 used procedural approaches, relied on counting and concrete objects, and appeared not to have developed general number sense or mathematics concepts. Group 4 took longer than the other groups to work on the problems, had more incorrect answers, and attempted to solve fewer problems than the other groups. This study found that a student's level of reading comprehension was a predictor of the student's mathematics achievement but that the student's IQ was a much stronger predictor of mathematics achievement.

In attempting to build a profile of students with MLD in the area of mathematical problem-solving of word problems, Lynn and Douglas Fuchs (2002) studied the performance of students with MLD and reading disabilities (RD) and students with MLD, as identified by their schools, on a range of mathematics problem solving tasks: arithmetic story problems, complex story problems, and real-world problem solving. Previous research (Siegel & Ryan, 1989) indicated that students with MLD/RD had working memory problems, while students with MLD only had difficulties in visual memory and visual-spatial working memory (Fletcher, 1985; McLean & Hitch, 1999; Siegel & Linder, 1984). Fuchs and Fuchs (2002) found that students with MLD only averaged an accuracy rate of 75% for arithmetic story problems, 14% for complex story problems, and 12% for real world problem solving. Students with MLD/RD received percentages of 55%, 8% and 5%. Their results supported previous studies that indicated that students with MLD/RD experienced more difficulty with mathematical word problems than students with MLD only did.

Summary. Various methods have been used to identify students with MLD for participation in research studies besides the use of the discrepancy formula and identification by the school district (Vaughn & Fuchs, 2003; Ysseldyke et al., 1982). Students with MLD were identified by achievement scores below the 30th percentile (Geary, Hoard, & Hanson, 1999; Jordan & Hanich, 2000; Jordan & Montani, 1997) or below the 25th percentile (Geary, 1993). Students with MLD were sometimes identified by IQ scores (Rourke, 1989, 1991, 1998). This lack of a consistent method for identification makes it difficult to compare studies of students with MLD (Hammill et al., 1989).

Students with MLD have been compared to typically developing peers of the same age or a year younger (Swanson et al., 1996; Xin & Jitendra, 1999) and to students with reading deficits (D'Anguiulli & Siegel, 2003; Jordan & Hanich, 2000; Rourke, 1989, 1991, 1998). There are very few studies that compare students with MLD to students with Low Math/NLD (Ysseldyke et al., 1982; Fuchs et al., 2006).

To assess the performance of students with MLD, research studies have administered batteries of tests (Fuchs et al., 2005; Jordan & Hanich, 2000; Rourke, 1989; Ysseldyke et al., 1982), interventions (Fuchs et al., 2005; Geary, 1993), and word problem solving assessments (Bryant, Bryant, & Hammill, 2000; Sovik, Frostrad, & Heggberget, 1999; Woodward, Monroe, & Baxter, 2001). Consistently students with MLD/RD have scored lower than students with MLD.

Language Ability and Mathematics Performance

It appeared that not only was reading ability tied to mathematics achievement, but also language ability in general affected mathematics achievement. The association of learning disabilities with language disorders is well established (Catts, 1996; Schoembradt, Kumin, & Sloan, 1997), and students with specific language impairment (SLI) in the primary grades are often found to be learning disabled in the upper elementary grades (Schoenbrodt et al., 1997). Fazio's (1999) study of students with SLI was a five year longitudinal study and compared the performance of students with SLI to that of typically developing low-income peers and also to younger, typically developing low-income students. Fazio (1996) found that students with SLI showed signs of delay in counting and basic number fact knowledge especially the rote retrieval of memorized material. In fourth and fifth grade, students encountered the need for numerous steps necessary to solve written calculation problems as well as an increase in mathematical vocabulary (e.g., divisor, addends, regrouping and their corresponding notation) (Bley & Thornton, 1994) for which they also experienced difficulties. Students with SLI appeared to have problems with phonological memory (Gathercole & Baddeley, 1990), verbatim recall of numbers, words, and sentences (Fazio, 1997, Gillam, Cowan, & Day, 1995), working memory capacity (Weismer, 1996; Fazio, 1998), and automatic retrieval of items stored in long-term memory (Lahey & Bloom, 1994). Fazio (1999) surmised that the students' language impairment would inhibit the learning and recall of mathematics facts and procedural knowledge required for recalling the specific steps needed to solve calculation problems beyond single digits. Fazio (1999) found a strong positive correlation between the students' overall performance on the *Clinical Evaluation of Language Function-Revised* (CELF-R) and their arithmetic performance (.76). She also found an inverse relationship between the speed of written calculation and mathematics fact knowledge (-.64). Students who had poorer mathematics fact retrieval took longer to complete written calculation. Fazio's (1999) results suggested that language and other cognitive processes such as memory interact in such a manner as to cause a deficiency in mathematics.

This link that Fazio (1999) discovered between language impairment and arithmetic is supported by the study of Manor, Shalev, Joseph, and Gross-Tsur (2000) in which kindergartners with normal intelligence, exhibiting a developmental language disorder, displayed poor performance on an arithmetic battery. The domains tested were counting, comprehending number words and symbols, reasoning principles, and arithmetic operations.

Error Analysis

To understand fully the affect that language has on the mathematics performance of students with MLD especially in the area of mathematical word problem solving, it is necessary to go beyond recording a student's answer as correct or incorrect. It is necessary to determine the strategies the student uses. Error analysis is central to that effort. Many errors in mathematics, especially in factual or declarative knowledge such as fluency in basic mathematics facts, can be reduced or eliminated through systematic practice and review (Hasselbring, Goin, & Bransford, 1988). However, procedural errors based on misconceptions, frequently held by students with MLD, appear more resistant to increased drill and practice only. Such procedures include those such as the application of algorithms in multi-digit division and solving story problems using a key word strategy (Woodward and Howard, 1994). Research shows that a superficial understanding of mathematics results in students, especially those with learning disabilities in mathematics, continuing in their misconceptions over years (Woodward, Baxter, & Howard, 1994). Frequently, students with mathematics learning disabilities concentrate only on the surface or syntactic features of the problem without understanding the meaning of the symbols and their manipulation at a conceptual or semantic level (Woodward, & Howard, 1994). Although language is usually thought of in terms of its association with reading, proficient language usage is highly interconnected with an understanding of mathematics. Just as sentences rely on rules of syntax and comprehension emerges from semantic understanding, mathematical equations also have their own syntax, and an understanding of the mathematical symbols and their manipulation is derived from an understanding of the semantics (Radatz, 1979).

Typical instruction in mathematics (especially in the area of word problems) is delivered through language. The concrete level ties objects to language, but the abstract level utilizes language or symbols without the assistance of concrete representation. Students with a history of learning disabilities in mathematics have difficulty with the use of symbols and abstract reasoning. Their difficulties are further compounded by the instruction and explanation in the use of these symbols being conveyed from teacher to student through the use of language (Jitendra & Kameenui, 1996).

Several studies have examined students' misconceptions in problem solving. The BUGGY project (Van Lehn, 1988) studied the misconceptions of 22 fourth grade students in a special education setting by having a computer analyze individual student's computational error patterns in subtraction problems. The TORUS computer program (Woodward, Freeman, & Howard, 1992), designed to extend the research of the BUGGY project into the area of special education, analyzed errors made on subtraction and addition computational problems. More recent studies have shown that for students with low performance, the greatest deficiencies were not in computational ability but in logical/linguistic abilities needed to divide complex problems into parts. By examining students' errors, the student's word-problem-solving performance can be described and the student's method for solving mathematical word problems can be determined (Jitendra & Kameenui, 1996). By examining students' written work, when analyzing students' patterns and the types of errors produced when students attempted to solve mathematical word problems, it is possible to gain insight into the students' reasoning processes (Muoneke, 2001).

Classification of Errors

A variety of classification systems exist. Classification of errors made in solving mathematical word problems includes analyzing simple computational errors, performing a miscue analysis of errors (similar to miscue analysis used in reading), and recognizing errors in information processing (Jitendra & Kameenui, 1996). Errors in information processing may appear as translation errors such as the students' difficulty in translating the information from the word problem into mathematics language. Radatz (1979) used an information-processing classification of errors. He classified errors as due to (a) language difficulties, (b) difficulties in obtaining spatial information, (c) deficient mastery of prerequisite skills, fact, and concepts, (d) incorrect association or rigidity of thinking, and (e) the application of irrelevant rules or inadequate strategies. A student's language difficulties included semantic misunderstandings or errors in translating the word problem into mathematical language. The application of irrelevant rules or inadequate strategies by the student resulted from the student not recognizing the differences in the current problem from problems encountered in the past. Radatz's information-processing classification also includes the failure to complete the solution process. Ginsberg (1987) categorized mathematical error patterns into three categories (a) number facts (basic mathematics fact errors made by the student in addition, subtraction, multiplication, and division), (b) slips (slight errors or a careless mistake made by the student when utilizing a known procedure), and (c) bugs (a strategy thought to be correct by the student that results in a procedural error). Babbitt (1990) described four types of errors that occur in mathematics problem solving: (a) computational errors

(errors due to lack of proficiency with basic mathematics facts), (b) operational errors (errors due to selection of wrong operation), (c) miscellaneous errors (errors due to including extraneous information), and (d) no-attempt errors (error caused by student not attempting to solve the problem). However, the limitation of looking only at a student's written response to mathematical word problems is that the examiner might misconstrue the student's errors (Engelhardt, 1977; Muoneke, 2001). The use of student interviews and "talk alouds," can prevent misinterpreting student error types that are inferred from written work alone. However, these procedures are nonstandardized.

Student Interview

There are several types of interviews that have been used to gather information from the student about strategies and procedures that the student is using. The clinical interview is based on the clinical interview method originally used by Piaget (1952), which uses an open-ended and unstructured method of flexible questioning of the individual student designed to uncover basic features of the student's thinking process (Ginsburg, et al., 1992). Clinical interviews are more sensitive than standardized tests and vary as to their complexity. They are also difficult to administer successfully. However, for purposes of measuring cognitive processes, many researchers believe the clinical interview is the method of choice (Ginsburg, et al., 1992). While using this method, there is a continual verbal interaction between the student and the researcher which allows for the possibility that the researcher might influence the student. Through flexible questioning, the researcher attempts to determine the underlying cognitive processes that the student is using in problem solving.

In the “talking aloud” or as it is sometimes known “thinking aloud” method the researcher instructs the student to say everything that s/he thinks when solving the problem (Newell & Simon, 1972). The researcher collects behavioral observations as the student solves the problem but does not intervene beyond giving the initial instruction. The researcher analyzes the student’s statements and makes inferences about the student’s thinking. Based on the student’s responses, the evaluator can determine what type of strategy was used by the student (Kennedy & Tipps, 2000). Jitendra and Kameenui (1996) advocate that researchers use think aloud procedures when investigating the student’s approaches that have led to the student’s errors. By using the think aloud method rather than a clinical interview, the student’s responses should be unbiased because they are generated from the student’s involvement with the problem rather than through an interaction with the researcher (Drueck, 1997).

The Individual Interview also uses talking aloud but here the students are asked to describe what they were thinking and how they derived their answer after they have solved the problem (Woodward, Baxter, & Robinson, 1999) rather than as they are solving the problem. This semistructured individual interview is often used in decimal research (Hiebert, Wearne, & Taber, 1991; Resnick et al., 1989; Smith, 1995). It is also used with other types of mathematics research and has been found to be a valid method of determining how children solve mathematics problems (Siegler, 1995).

The Individual Interview can be an important tool for determining the problem-solving behaviors displayed by students with learning disabilities in mathematics. Through the interview, information is gathered that assists with error analysis and recognition of strategies employed by the student. The interview permits discovery into the role that language takes in mathematics for the student with MLD. The interview also enables the researcher to obtain subtle information, which cannot be gathered through a test of calculation or merely recording answers to word problems. Analyzing information provided by the student in the interview may provide insights into discriminating between students with MLD and students with Low Math/NLD.

Summary

Language abilities and other cognitive processes affect mathematics achievement (Fazio, 1999). In order to understand how language impacts a student's performance in solving mathematics word problems, it is necessary to conduct an error analysis. Error analysis provides an opportunity to discover errors based on the student's misconceptions (Woodward, Baxter, & Howard, 1994) or faulty translation of words to symbols and operations (Jitendra & Kameenui, 1996). Systems for classifying errors have been proposed by Muoneke (2001), Jitendra, Kameenui (1996), Radatz (1979), Ginsburg (1987), and Babbitt (1990). All of the systems include a component related to language. The student interview enhances error analysis by gathering information directly from the student's verbal explanation of the strategy s/he used to solve the mathematics word problem. Flaws in the student's logic and misconceptions are readily exposed (Ginsburg, et al., 1992; Woodward, Baxter, & Robinson, 1999).

Summary

The study by Ysseldyke, Algozzine, Shinn, and McGue (1982) is one of the few studies that compared students with MLD to students with Low Math/NLD and its results were inconclusive. The longitudinal studies (Geary, 1990; Geary et al., 2000) used to distinguish between students with MLD and students with Low Math/NLD were not useful at predicting which students had MLD because identification only occurred after remediation had taken place, at which time students who did not respond positively to the remedial instruction were said to have MLD. The predictive study (Fuchs et al., 2005) used to identify first graders with MLD investigated a wide range of measures but concluded that some of the measures did not discriminate finely enough to be useful at the first grade level and that further research was needed. Other studies examined students with MLD and reading deficits. Investigating language and mathematics reasoning and word problems for students with MLD and students with Low Math/NLD, especially at the middle school level, is an area with few studies and further research is needed (Fuchs et al., 2005). Gathering information through error analysis and interviewing the student can assist in developing a profile of the student with MLD, which may prove useful in early identification, and will be valuable in remediation efforts (Woodward, Baxter, & Robinson, 1999).

Chapter III

Method

The purpose of this study was to determine the underlying factors associated with solving mathematical word problems and possible differences in the problem-solving behaviors for mathematical word problems and the types of mathematical word problem errors between (a) students with mathematics learning disabilities (MLD) who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the *Woodcock-Johnson-Third Edition Tests of Achievement* (WJ III ACH) and who were likely to have a learning disability according to the *Learning Disabilities Diagnostic Inventory* (LDDI) and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI. Differences in mathematics automaticity were assessed with the WJ III ACH Math Fluency score, differences in mathematics computation were assessed with the WJ III ACH Calculation score, and differences in mathematical word problems were assessed with the WJ III ACH Applied Problems score. Linguistic differences that might affect language processing required by mathematical word problems were assessed quantitatively by comparing Concepts and Directions, Word Classes Receptive, Number Repetition Total, and Familiar Sequences subtest scores from the *Clinical Evaluation of Language Fundamentals-Fourth Edition* (CELF-4) between the two groups of students. The *Informal Mathematics Assessment* was used to identify processes and strategies used by students to solve mathematical word problems and the types of mathematical word problem errors that they made (Babbitt, 1990; Jitendra & Kameenui, 1996; Muoneke, 2001). This chapter describes the methodology used in the study including research design, participants and setting, instruments, procedure, and data analysis.

Design

This non-experimental empirical study utilized two research methods: (a) exploratory factor analysis (Fabrigar, Wegener, MacCallum, & Strahan, 1999) and (b) causal comparison (Mertens, 1998). Exploratory factor analysis (EFA) was used (Fabrigar, et al., 1999) to search for underlying factors in the measured variables and to identify commanalities through the use of correlations between variables which were students' subtest scores from the WJ III ACH mathematics and reading subtests and the CELF-4 subtests. The R-Type factor analysis was used to examine dependent variables and group them according to underlying factors. Student subtest scores from the WJ III ACH and CELF-4 were entered as variables into an EXCEL database. SPSS was used for factor extraction. The Cattell scree test was used to determine the number of factors underlying the measured variables. The plot of the eigenvalues was examined and all factors with eigenvalues in the sharp descent part of the plot prior to where the plot levels off were retained. To aid in the interpretation of factors, a varimax rotation (orthogonal rotation) was used.

This non-equivalent comparison group non-experimental empirical study utilized a causal comparison research method (Mertens, 1998) to look for statistical signs of differences between (a) students with mathematics difficulties who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH who were likely to have a learning disability (MLD) according to the LDDI and (b) students with mathematics difficulties who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH who were unlikely to have a learning disability according to the LDDI (Low Math/NLD). A nonexperimental research design was selected because it provided a means for systematic empirical inquiry into inferences about the differences among the variables without any direct intervention (Kerlinger, 1986) through the manipulation of any of the variables or the changing of any conditions to affect the participants' responses.

Two key dimensions for classifying non-experimental research are the research objective dimension and time dimension. Quantitative nonexperimental studies can be classified as descriptive, predictive, or explanatory research (Johnson, 2000). This study qualified as descriptive nonexperimental research because the objective was to describe and document the characteristics of the phenomenon; there was no manipulation. The types of time dimension are cross-sectional research, longitudinal research, and retrospective research (Johnson, 2000). This study qualified as cross-sectional research because the data were collected from the participants during a relatively brief time period. Causal-comparative research and correlational research are both types of nonexperimental design. The third dimension of causal-comparative research and correlational research involves the scaling of the independent variable. Correlational research only includes quantitative variables and looks for relationships within a single group, whereas causal-comparative research includes at least one categorical independent or dependent variable and involves comparing groups to explain the differences between them (Johnson, 2000). Because this study included a categorical independent variable, the nonexperimental causal-comparative research method was employed. Nonexperimental causal-comparative research is a type of descriptive research that describes conditions that already exist and attempts to determine reasons or causes, for that which is being studied. Because the data were collected at a single point in time, the study was considered to be cross-sectional research. Causal-comparative studies attempt to establish cause-effect relationships and involve group comparisons (Gay, 1996).

For this study participants were assigned to groups based on their scores on the mathematics subtests of the WJ III ACH (Woodcock, McGrew, & Mather, 2001) and the LDDI (Hammill & Bryant, 1998). An independent *t*-test was used to analyze differences between groups on the dependent variables to determine whether they were statistically different from each other (Merten, 1998).

Strengths

One of the strengths of the study was that it used nationally normed tests. The *Woodcock-Johnson-Third Edition Tests of Achievement* (WJ III ACH) is a standardized nationally normed test which provides test scores that are independent of each other. Using WJ III ACH test scores assured accurately identifying groups having low achievement in mathematics. The *Learning Disabilities Diagnostic Inventory* (LDDI) is also a standardized nationally normed test, which has been designed to identify students as likely or unlikely to have a learning disability.

The sample size was small and the two groups were not of equal size for this study. However, an independent *t*-test with a small sample size, 15 cases per group, yields fairly accurate *p*-values. With moderate to large sample sizes, the assumption of normal distribution in each of the groups may be violated and the results will be less accurate (Green, Salkind, & Akey, 2000). Because this study used small sample sizes, the *p*-values could be assumed to be accurate. Because the groups for this study were not the same size, the *t*-test for independent samples offered another advantage in that it would compute an approximate value that did not require equal sample sizes or assume that the population variances were equal.

There were seven dependent variables from the subtest scores. These included Math Fluency, Calculation, Applied Problems, Letter-Word Identification, and Reading Fluency on the WJ III ACH and Receptive Language and Working Memory on the CELF-4. For factor analysis it was necessary to have more cases than dependent variables (7) in the cell so that the cell did not become singular and the assumption untestable. Additionally, if the cell had only one or two more cases than dependent variables, the assumption was likely to be rejected. For this study there was greater than a 5:1 ratio of participants to dependent variables for the factorial analysis. For the *t*-test the Receptive Language and Working Memory scores, used in factor analysis, were replaced for purposes of finer discrimination with scores from their component subtests Concepts & Following Directions and Word Classes for Receptive Language and Number Recognition and Familiar Sequences for Working Memory. There was still greater than a 5:1 ratio of participants to dependent variables. Although small, the sample size was considered adequate for this study because all participants were from the same grade level. A larger sample size would have been required if more than one grade level was included in the study. The Hispanic population of one of the schools in the study was 47.44%. Although students whose parents indicated that English was not the home language were dropped from the study, the number of remaining Hispanic students who participated was representative of the total number of Hispanic students attending the school. The demographics of the sample populations for both schools were representative of each school's population.

Variables

The dependent variables were the Math Fluency subtest score from WJ III ACH used to assess mathematics automaticity, the WJ III ACH Calculation subtest score used to assess mathematics computation, and the WJ III ACH Applied Problems subtest score used to assess problem-solving skills. Other dependent variables were from the CELF-4, which was used for the assessment of language processing. The CELF-4 subtest scores Concepts & Following Direction and Word Classes generated the Receptive Language score, and the subtest scores Number Repetition and Familiar Sequences generated the Working Memory score. Other dependent variables were the WJ III ACH Letter-Word Identification subtest score and the WJ III ACH Reading Fluency subtest score used to assess reading. The other dependent variable was the score from the *Informal Mathematics Assessment*.

The independent variables were (a) achievement level in mathematics and (b) the indication of a learning disability. The two levels of the independent variable for the indication of a learning disability assessed by teacher ratings on the *Learning Disabilities Diagnostic Inventory* (LDDI) were: (a) likely to have a learning disability and (b) unlikely to have a learning disability. The two levels of the independent variable for mathematics were (a) low achievement in mathematics as determined by a subtest score which was less than or equal to the 25^{th} percentile on the Math Fluency, Calculation, or Applied Problems subtests from the WJ III ACH and (b) achievement in mathematics which was greater than the 25^{th} percentile as determined by the Math Fluency, Calculation, and Applied Problems subtest score from the WJ III ACH.

The causal-comparative design was used to compare the two levels of each of the two independent variables of the categorical groups which were: (a) students with low achievement in mathematics as assessed by the WJ III ACH and likely to have a learning disability as assessed by teacher ratings on the LDDI and (b) students with low achievement in mathematics as assessed by the WJ III ACH and unlikely to have a learning disability as assessed by the LDDI. Causal-comparative research is used to compare groups in order to explain existing differences between the groups on variables of interest or in this case the subtest scores of the CELF-4 and subtest scores of the WJ III ACH Math Fluency, Calculation, Applied Problems, Letter-Word Identification and Reading Fluency. The groups being compared in causal-comparative research have already been formed, and any treatment has already been applied. The two groups in this study were identified through the rating scale of the LDDI and the Math Fluency, Calculation, and Applied Problems scores on the WJ III ACH. No treatment was involved in the creation of the groups. The records of the two groups were examined by comparing subtest scores to see if a reasonable explanation could be offered as to what caused the existing differences between the two groups.

Causal-comparative research allows reasonable inferences to be made about the causation (Fraenkel, 1998).When examining causality, researchers should address three necessary conditions for cause and effect (Cook & Campbell 1979; Johnson & Christensen, 2000): (a) The variables must be related. (b) The proper time order must be established by determining which variable represents the antecedent condition. (c) The observed relationship must not be due to a confounding extraneous variable (i.e., the lack of alternative explanation condition or the nonspuriousness condition). There must not remain any plausible alternative explanation for the observed relationship if one is to draw a causal conclusion. A theoretical explanation or rationale for the observed relationship is also essential to make sense of the causal relationship and to lead to hypotheses to be tested with new research data. Nonexperimental research is generally useful for identifying relationships, but it is weak on establishing the time order and ruling out alternative explanations. Nonexperimental research is especially weak on condition three (ruling out alternative explanations) because of the problem of spuriousness.

In this study the independent variables were (a) categories from the teacher ratings on the *Learning Disabilities Diagnostic Inventory* (LDDI) which is based upon characteristics of processing disorders associated with learning disabilities and (b) categories from the *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) scores in math. The groups were (a) students with mathematics difficulties who received a score

≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability as assessed by teacher ratings on the *Learning Disabilities Diagnostic Inventory* (LDDI) and (b) students with mathematics difficulties who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability as assessed by teacher ratings on the LDDI.

The dependent variables were the subtests scores on the CELF-4 and the WJ III ACH scores which should have been influenced by whether the student was likely or unlikely to have a learning disability. If students' performance on these tests was influenced by a learning disability, then there should have been a difference of scores when the scores of students with low achievement in mathematics and likely to have a learning disability were compared to the scores of students with low achievement in mathematics and unlikely to have a learning disability. By using ratings from the LDDI to differentiate between the groups of students with low achievement who were likely and unlikely to have a learning disability, rather than relying on the local school district's identification of students as having a learning disability, the possibility of placing students, who had been misidentified by local school district, in the wrong group was eliminated.

Research Questions

The following research questions were addressed:

(1) What are the underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student

mathematics and reading achievement from the *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests?

(2) Are there statistically significant differences in mathematics automaticity as shown by Math Fluency scores measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH who were likely to have a learning disability according to the *Learning Disabilities Diagnostic Inventory* (LDDI) and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(3) Are there statistically significant differences in Calculation scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(4) Are there statistically significant differences in Applied Problems scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the

LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(5) Are there statistically significant linguistic differences in cognitive areas measured by the subtest scores of the CELF-4 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(6) Are there statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(7) Are there differences in mathematical problem-solving error type percentages as measured by the categories of computation, operation, translation, and no attempt on the *Informal Mathematics Assessment* between (a)students with MLD who

received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were not likely to have a learning disability according to the LDDI?

Null Hypotheses

From these questions the following null hypotheses were posited:

(1) There are no statistically significant underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student mathematics and reading achievement from *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests.

(2) There are no statistically significant differences in mathematics automaticity as shown by Math Fluency scores measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(3) There are no statistically significant differences in Calculation scores as measured by the WJ III ACH between (a) students with MLD who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(4) There are no statistically significant differences in Applied Problems scores as measured by the WJ III ACH between (a) students with MLD who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(5) There are no statistically significant linguistic differences in cognitive areas measured by the subtest scores of the CELF-4 between (a) students with MLD who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(6) There are no statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(7) There are no differences in mathematical problem-solving error type percentages as measured by the categories of computation, operation, translation, and no attempt on the *Informal Mathematics Assessment* between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were not likely to have a learning disability according to the LDDI.

Participants and Setting

Students

Participants for this study were 62 students, who were enrolled in inclusive sixth grade general mathematics classrooms in two public middle schools during the 2005-2006 school year who were at least 11 years 0 months old and not more than 13 years 11 months. This grade level and age range was selected because research indicated that the population in this age range was relatively stable in mathematics performance compared with younger children. Older students displayed less variability in test performance and more differentiation of abilities compared to younger students (Geary, 1990). By the intermediate and middle school grades, students exhibit distinct patterns of cognitive and sensorimotor abilities and deficits (Rourke, 1998). Also, for some students with mathematics disabilities their mathematics performance tends to plateau at the fifth or sixth grade level (Cawley, Baker-Kroczynski, & Urban, 1992).

The students' ethnicity included two Asian, 34 Caucasian non-Hispanic, 15 Hispanic, and 11 African-American students. Twenty-six students received free or reduced lunch and 36 students paid full price for lunch. There were 35 females and 27 males in the study. The school district was a large suburban school district in the Southwest. Students in the school district were predominantly Caucasian, non-Hispanic (62%) and only 18.4% of the students received free or reduced lunch. Gender information for the district was unavailable. School district demographic data are presented in Table 3.1.

Table 3.1 Demographics of School District

School District	
Total Number of Students	34,029
Grade level	preK-12
Free or reduced lunch	18.4 %
Ethnicity	
Caucasian, non-Hispanic	62 %
Hispanic	20 %
African-American	9 %
Other	9 %

The two middle schools from which participants were selected had different demographic profiles (Table 3.2) from each other. Middle School #1 was the larger more affluent school with 354 students and only 24.6% students receiving free or reduced lunch. Middle School #2 had 236 students with 61.44% of the students receiving free or reduced lunch. Gender information for these schools was unavailable.

Table 3.2 Demographic Profile of Middle Schools

Demographics of Middle Schools							
	Total Number of Students	Asian	White	Hispanic	African-American	Free or Reduced Lunch	Full Lunch Price
School #1	354	7.4%	61.6%	21.6%	8.8%	24.6 %	75.4%
School #2	236	1.5%	35.3%	47.6%	15.6%	61.44%	38.56%

Participant Selection Criteria

The students selected for the current study, according to teacher reports, had no known emotional or physical problems, and the students' parents indicated on the school district's Home Language Survey that English was the primary language spoken in the home. To participate in the study, a student was asked to volunteer to participate in the study, return a Consent from Parents Form (Appendix A) that had been signed by the parent/guardian giving permission for the student to participate in the study, and sign a student Assent Form (Appendix B) before being assessed. Parents and students had been informed that participation was voluntary and that if a student chose to participate that the student could withdraw from the study at any time without penalty.

Students were required not to have mental retardation in order to participate in the study. According to the legal definition of learning disability, a student with a learning disability cannot have mental retardation (IDEA, 2004). Therefore, it was necessary for the participants in this research study not to have mental retardation, which is generally considered to be an IQ score below 70 (Spruill, 1998). Taking into consideration the ethnicity and SES of the students in the schools participating in the study, it was decided to include the low average IQ range of 80-89 for participants and use a cutoff score of 80 for the FSIQ. Therefore, the participants had to have a composite score of 80 or higher on a standardized test of intelligence such as the *Cognitive Abilities Test* (COGAT), *Otis-Lennon School Ability Test* (OLSAT), *Wechsler Intelligence Scale for Children-Third Edition* (WISC-III), or the *Brief Intellectual Ability* (BIA) from the *Woodcock-Johnson III Tests of Cognitive Abilities* (WJ III COG) in order for the student to be included in the study. An IQ score of 80 or higher was chosen because it would include more African-Americans, Hispanics, Caucasian non-Hispanic, and students with low socioeconomic status who had average performance than would a cutoff score of 90. This decision was justified by previous studies which had used various ranges of IQ scores (Case & Harris, 1992; Kavale & Reese, 1992). Case and Harris (1992) used an IQ score of 75 to identify the participants for their study. Although usually an IQ score below 70 is considered to be the criterion for significant limitation of intellectual functioning (Spruill, 1998), in the Iowa study (Kavale & Reese, 1992) participants with LD had IQ scores ranging from 53 to 132, For this study, students who did not receive a composite score $\geq$80, as measured by a standardized intelligence test such as the COGAT, OLSAT, WISC-III, or BIA were not be eligible to complete the study. To be as inclusive as possible while ensuring the exclusion of students with mental retardation, the cutoff score of 80 was chosen, because it was above the IQ of 70, at which level mental retardation becomes a concern as an exclusionary factor, and yet was low enough to include some African-Americans, Hispanics, Caucasians non-Hispanic, and students with low socioeconomic status whose average performance falls in what would typically be considered to be the low average range.

A chi-square test was used to ascertain that the sample of participants was representative of the two schools in all areas except achievement. Several analyses were conducted to determine if there were significant differences between the sample and the school populations on the demographic dimensions of ethnicity (see Tables 3.3 and 3.5) and socioeconomic status (see Tables 3.4 and 3.6). Chi-square analyses (Pearson χ^2) was used for the variables of ethnicity and socioeconomic status. There were no significant differences between the two samples of participants and the population at the two schools.

Table 3.3 Description of Middle School #1 Participant Ethnicity Variables

Ethnicity of Middle School #1					
	Asian	White	Hispanic	African-American	Number of Students
School Population	7.4%	61.6%	21.6%	8.8%	354
Actual Number of Students in Sample	2	26	8	7	43
Expected Number of Students in Sample	3.18	26.5	9.29	3.78	43

Table 3.4 Description of Middle School #1 Participant SES Variables

SES of Middle School #1			
	Free or Reduced Lunch	Full Lunch Price	Number of Students
School Population	24.6 %	75.4%	354
Actual Number of Students in Sample	12	31	43
Expected Number of Students in Sample	10.58	32.42	43

Table 3.5 Description of Middle School #2 Participant Ethnicity Variables

Ethnicity of Middle School #2					
Middle School #2	Asian	White	Hispanic	African-American	Number of Students
School Population	1.5%	35.3%	47.6%	15.6%	236
Actual Number of Students in Sample	0	8	7	4	19
Expected Number of Students in Sample	.285	6.707	9.044	2.964	19

Table 3.6 Description of Middle School #2 Participant SES Variables

SES of Middle School #2			
Middle School #2	Free or Reduced Lunch	Full Lunch Price	Number of Students
School Population	61.44%	38.56%	236
Actual Number of Students in Sample	14	5	19
Expected Number of Students in Sample	11.67	7.33	19

Participant Assignment to Group

Research studies were reviewed in order to determine what criteria to select for identifying students with mathematics difficulties. When studying mathematics problem solving, Woodward, Monroe, and Baxter (2000) selected students with scores below the 34^{th} percentile on problem-solving and total mathematics on the *Iowa Test of Basic Skills*. Geary, Hoard, and Hanson (1999) selected students who had mathematics reasoning scores from the *Woodcock Johnson PsychoEducational Battery-Revised* (Woodcock & Johnson, 1989, 1990) below the 30^{th} percentile. The criterion of selecting students whose score fell below the 30th percentile on mathematics tests was also used in other investigations of young children with learning difficulties (Jordan & Hanich, 2000; Jordan & Montani, 1997). Fletcher (1985) and Rourke (1991) used the 25^{th} percentile for their studies. Swanson and Beebe-Frankenberger (2004) used the 25^{th} percentile for their study of primary students with severe mathematics difficulties. The 25^{th} percentile has been considered as separating students with learning disabilities from typically achieving students (Rourke & Finlayson, 1978; Shaywitz et al., 1990). For this study, the 25^{th} percentile was chosen as the cutoff score for students with mathematics difficulties because it represented below average functioning on norm-referenced tests (Hammill, 1990). Therefore, students who received a minimum of one score $\leq 25^{th}$ percentile, a cut-point frequently employed for designating disability risk (e.g., Torgesen & Bryant, 1994), on Calculation, Math Fluency, or Applied Problems on the WJ III

ACH were recognized as having mathematics difficulties and included in the group of students with low mathematics achievement.

The *Learning Disabilities Diagnostic Inventory* (LDDI) was used to classify the students into (a) a group of students with low achievement in mathematics and with indications of learning disabilities (MLD) and (b) a group of students with low achievement in mathematics and without indications of learning disabilities (Low Math/NLD). The assignment of students to the two research study groups is shown in Table 3.7. Participants with low achievement in mathematics were assigned to a group based on their score on the LDDI. The LDDI was completed by the classroom teacher for each participant identified as having low achievement in mathematics as shown by a score at or below the 25^{th} percentile on Calculation, Math Fluency, or Applied Problems of the WJ III ACH. According to the LDDI procedures, students who received a stanine score of 1, 2 3, 4, or 5 in one area and at least one score of 7, 8, or 9 in another area were assigned to the group likely to have a learning disability (MLD), and students receiving a stanine score of 7, 8, or 9 in all areas or a stanine score of 1, 2, 3, 4, or 5 in all areas were assigned to the group unlikely to have a learning disabilities (Low Math/NLD).

Table 3.7 Assignment of Participants to Groups

Participant Assignment to Research Study Groups		
	MLD	Low Math/NLD
IQ composite standard score	$\geq$ 80	$\geq$ 80
WJ III ACH Calculation Math Fluency **or** Applied Problems	A minimum of one math score < 25th percentile	A minimum of one math score < 25th percentile
LDDI	A minimum of one stanine is 1, 2, 3, 4, or 5 **and** a minimum of one stanine is 7, 8, or 9	All stanine are 7, 8, or 9 **or** all stanine are 1, 2, 3, 4, or 5

The characteristics of the two research study groups of students are shown in Table 3.8.

Table 3.8 Characteristics of Students by Group

Characteristics of Research Study Groups		
Characteristic	MLD	Low Math/NLD
Total Number (n)	14	16
Females	10	8
Males	4	8
Age (months)		
Grade level	6th	6th
Ethnicity		
Caucasian, non-Hispanic	6	9
Hispanic	4	5
African-American	4	2
Asian	0	0
Native American	0	0
Free or reduced lunch	8	9
IQ Mean	88	94
Special Education	6	2

Setting

The school district was a large suburban school district in the Southwest. The district had a total population of 34,029 students in grades prekindergarten through the 12^{th} grade. Students in the school district were predominantly Caucasian non-Hispanic (62%). The Hispanic population was 20%, and the African-American population was 9%. Only 18.4% of the students received free or reduced lunch. Gender information for the district was unavailable. Demographic information is available in Table 3.1.

Measures

In this study, four measures were used to answer the research questions. The LDDI was used to assess the existence of a learning disability. The *Woodcock-Johnson III Tests of Achievement* was used to assess participants' academic achievement skills of reading and mathematics. The *Informal Mathematics Assessment*, a clinical interview and process assessment, was used to measure characteristics of problem solving behavior and to identify processes and strategies used by students to solve mathematical word problems and their types of mathematical word problem errors. The CELF-4 subtest scores (Concepts & Following Directions, Word Classes, Number Repetition, and Familiar Sequences) were used to assess language skills that might have affected the processing of language used in mathematical word problems. The WJ III ACH Math Fluency was used to assess mathematics automaticity, Calculation was used to assess mathematics computation, and Applied Problems was used to assess problem-solving skills. Letter-Word Identification and Reading Fluency from the WJ III ACH were used to assess reading.

Learning Disabilities Diagnostic Inventory

The LDDI was used to assess the existence of a learning disability in order to generate the two levels of the independent variable (a) with indications of a mathematics learning disability (MLD) and (b) without indications of a mathematics learning disability (Low Math/NLD). Teacher rating instruments have been successfully used to identify students with learning disabilities (Gresham, MacMillan, & Bocian, 1997). This study will use the *Learning Disabilities Diagnostic Inventory* (LDDI), a teacher rating instrument, developed in 1998 by Donald Hammill and Brian Bryant. It has six scales that measure behaviors associated with intrinsic processing problems that result in the specific learning disabilities of listening, speaking, reading, writing, mathematics, and reasoning. Teachers rate the student on each of the 15 items describing a specific behavior associated with learning disabilities in the content area for each scale. The rating is based on the frequency with which the teacher observes the student displaying the specific behavior for the individual item (1=most frequently, 9=most rarely). The LDDI measures deficiencies in specific skills related to spoken and written language, computation, and reasoning that are commonly observed in students diagnosed as having LD but are not so commonly found in students with other kinds of learning problems so that the results can be used to help separate underachieving students into those with and those without LD (Hammill & Bryant, 1998, p. 25-26). The mathematics scale on the LDDI was designed to identify students with and without the characteristics of intrinsic processing disorders in math. It was decided to use the LDDI because after the mathematics scale on the LDDI was intercorrelated with the other scales, the resulting coefficients were .47 listening, .44 spelling, .39 reading, and .45 writing, which suggest that the scales interrelated to a moderate degree and measured related but different abilities. One would expect the scores to be interrelated because they are all measuring types of academic skills with a language component, but the score for mathematics should indicate its difference from the other areas because although it is measuring an academic skill, mathematics is distinctly different from the other academic areas. While solving mathematical word problems requires adequate language skills (similar to those needed for listening, spelling, reading, and writing), proficiency in computation may require less skill with language.

The LDDI was designed to identify a person with a specific learning disability from people with other types of conditions by determining the extent to which a student's observable behaviors in mathematics are consistent with those of individuals known to have a mathematics learning disability (Hammill & Bryant, 1998). The inventory lists specific observable behaviors taken from research and theoretical literature on specific learning disabilities.

The six LDDI scales measure behaviors associated with intrinsic processing problems, which result in the specific learning disabilities of listening, speaking, reading, writing, mathematics, and reasoning. The LDDI measures deficiencies in specific skills related to spoken and written language, computation, and reasoning. Students diagnosed with LD commonly display deficiencies in these skills while students with other kinds of learning problems do not. Thus, the results can be used to help separate students with low achievement into those with and those without LD (Hammill & Bryant, 1998).

The summed scores of teachers' ratings of the students are reported in stanines and percentiles. The student's stanine score on the LDDI, based on the teachers' ratings of the students, are assigned to one of three groups: (a) unlikely has an intrinsic processing disorder (stanine score of 7-9), (b) possibly has an intrinsic processing disorder (stanine score of 6), and (c) likely has an intrinsic processing disorder (stanine score of 1-5). Stanines have a mean of 5 and a standard deviation 1.96. The test was normed on 2,152 students with learning disabilities between the ages of 8-0 and 17-11 residing in 43 states and the District of Columbia. Internal consistency reliability coefficients exceed .90 for all scales. Coefficients for stability are in the .80s and interscorer reliability is in the .90s. The scale for mathematics identifies behaviors as problematic of students with LD who do poorly in mathematics. Behaviors were validated by professionals' judgment (Gresham, 1986; Gresham MacMillan, & Bocian, 1997; Nelson, 1971; Shafer, 1982).

The internal consistency reliability of the LDDI scale items was investigated by using the coefficient alpha procedure. Analyses were conducted at 1-year age intervals. The averaged coefficient for mathematics was .90 which is high. The SEM for mathematics was .52 reflecting a high degree of scale reliability for the LDDI scores. Reliability was checked for Euro Americans, African Americans, Hispanics, Native Americans, Asians, males, and females. Data were reported across ages 8-18 years. Obtained coefficients were corrected by using the partial correlation procedure (Guilford

& Fruchter, 1978). Alpha for Native American was .70. All other alphas ranged from .90 to .94 demonstrating that the inventory contains little or no bias relative to these subgroups. To determine stability over time, raters completed the inventory twice, one week apart on 77 individuals with LD. The internal consistency reliability coefficients for the identified subgroups for mathematics ranged from .90 to .94 with the exception of Native American, which was .70. Scorer differences (amount of assessment error due to examiner variability in scoring) in variability in tallying of scores resulted in a coefficient of .98 for mathematics, which supports scorer reliability. Results of comparison of interpretation of profiles between raters were 97% agreement of the interpretations supporting scorer reliability of the LDDI. The LDDI shows evidence of a high degree of reliability across content, time and scorer.

Evidence was provided for three types of validity: content-description, criterion-prediction, and construct-identification. (a) Content–description validation for the LDDI was supported through the consistency of each scale's content to the consensus of the most widely accepted definitions of LD, relating the literature on LD to the items, validating the items of each scale by a professional panel (raters thought that 4% of the items on the mathematics scale were minimally indicative of a mathematics learning disability, 36% were somewhat indicative, and 61% were considerably indicative), validating the items through conventional item analysis (item discrimination coefficients for mathematics ranged from .63 to .74), using item-bias analyses to study the LDDI items (Delta score procedure found mathematics coefficients .99 for male/female, .96 for African American/Non-African American, and .98 for Hispanic/Non-Hispanic), and using confirmatory factor analysis to examine the items' goodness of fit for each scale (Tucker and Lewis's Index of Fit for mathematics had a coefficient of .90 and Bentler's Comparative Fit Index was .92). (b) Criterion-prediction validity was assessed by having teachers use the LDDI to rate 59 students previously designated as having mathematics problems. Of these, 78% were classified by the LDDI as likely or possibly likely of having an intrinsic processing disorder, and only 22 % were classified as unlikely to have such a disorder. (c) Seven basic concepts were examined for construct-identification validity for the LDDI. (1) Because the behaviors measured by the LDDI reflect the lifelong nature of LD, the LDDI raw scores should not relate highly with chronological age. The LDDI mathematics raw score had a coefficient of .26, which was sufficiently low to verify the hypothesis. (2) Because the LDDI scales are related to each other, the scales' scores intercorrelation should be statistically significantly. The LDDI raw scores for the entire normative sample were intercorrelated, and the resulting coefficients were significant at or beyond the .01 level. The mathematics coefficients ranged in size from .39 to .47. All of the medians for the subgroups are within .05 of the .47 associated with the normative sample. (3) Because the LDDI measures behaviors that are symptomatic of scholastic difficulties, the inventory's scores should correlate significantly with test scores that specifically measure scholastic abilities. The relationship between the mathematics scale LDDI value and the students' Broad Math score on the *Woodcock-Johnson Psych-Educational Battery –Revised* (Woodcock & Johnson, 1989) was .74. (4) Because the LDDI measures aspects of LD, its results should differentiate between groups of people who have specific learning disabilities, and those who do not. Using the profiles of 115 students diagnosed with learning disabilities, the LDDI identified 86% of the of the students diagnosed with learning disabilities as having profiles characteristic or possibly characteristic of LD. Using the profiles of 260 students without LD who were not experiencing learning problems, the LDDI identified 90% of them as having "normal" profiles. (5) To test for gender and ethnic influences on LDDI scores the mean standard scores for the five subgroups within the normative sample was compared to the standard scores for the entire sample. The means were identical for all subgroups in the area of math. (6) Because all of the scales measure some type of learning disability, they loaded on a single factor when the Varimax procedure was used. When the performance on the mathematics items was correlated to the total score for the mathematics scale, an unrotated factor analysis yielded a score of .64 for math, which supports construct validity. (7) Evidence for the LDDI's construct validity was found by correlating performance on the mathematics items with the total score for the mathematics scale. For the mathematics scale the coefficients for the discrimination indexes ranged from .63 to .74.

Woodcock-Johnson III Tests of Achievement

The *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) has sections that measure word identification, reading fluency, mathematics calculation, mathematics reasoning, and mathematics fluency. Although there are other achievement tests that also test these abilities, the WJ III ACH was selected for use in this study because it is one of the achievement tests whose scores are frequently used to determine eligibility for special education services through discrepancy between achievement and IQ scores (Gridley & Roid, 1998).

The WJ III is an updated version of the *Woodcock-Johnson Psycho-Educational Battery-Revised* (WJ-R), published in 1989, and the most recent version of the original battery published in 1977. The WJ III ACH was standardized on more than 8,818 subjects, ages 2 to 90 years of which 4,783 were students in kindergarten through 12th grade. Scores are reported in grade equivalents, percentile ranks, and standard scores. The WJ III ACH shows validity correlations in the range .50 to .80 with corresponding tests on the Kaufman Test of Educational Achievement and the Wechsler Individual Achievement Test. For reliability the internal coefficients are in the mid .90s. The WJ III is considered the premier battery for measuring achievement and cognitive abilities of school-aged children (Sandoval, J., 2002).

Letter-Word Identification. The Letter-Word Identification subtest assesses the student's skill in reading words in isolation. In the 5 to 19 age range Letter-Word Identification has a median reliability of .91.

Reading Fluency. The Reading Fluency subtest is a 3-minute timed test, which assesses the student's ability to quickly read simple sentences and determine if the statements are true. In the 5 to 10 age range Reading Fluency has a median reliability of .90.

Calculation. The Mathematics subtest assesses the student's skill in performing mathematical computations. In the 5 to 19 age range. Calculation has a median reliability of .85.

Applied Problems. The Applied Problems subtest was designed to assess a student's ability to analyze and solve mathematics problems. Many of the problems include visual stimuli such as pictures of objects and questions are presented orally. After listening to the problem the student must recognize the needed procedure and appropriate operation, ignore extraneous information, and perform simple calculations. The Applied Problems subtest is indicative of the language that is needed to solve mathematical word problems. In the 5 to 19 age range Applied Problems has a median reliability of .92.

Math Fluency. The Math Fluency subtest measures automaticity of basic mathematics facts. The Math Fluency subtest assesses the student's ability to solve quickly simple addition, subtraction, and multiplication facts. The test has a 3-minute time limit. In the 7 to 10 age range Math Fluency has a median reliability of .89.

Clinical Evaluation of Language Functions-Third Edition

Previous studies have looked at language (Kavale & Reese, 1992; Ysseldyke et al., 1982). Cohen and his colleagues (Cohen, Hynd, & Hugdahl, 1992; Cohen, Krawiecki, & DuRant, 1987) classified students with dyslexia into subtypes taking into consideration students' abilities with language. The group with the subtype of language disorders had Performance IQs significantly higher than their Verbal IQs. Although this study is targeting students with mathematics difficulties rather than dyslexia, it is possible that language disorders may affect mathematics performance (Jordan, & Hanich, 2000). Barbara Fazio (1999) in her five year longitudinal study of students with specific language impairment (SLI) compared their progress in language arts and arithmetic to their typically developing peers of the same age and a year younger. Although her study was with students with SLI rather than confirmed mathematics learning disabilities (MLD) it is believed that her choice of using the *Clinical Evaluation of Language Functions-Third Edition* (CELF-3) was due to its sensitivity to language disorders. For the purpose of this study an instrument was needed that would discern any relatively subtle differences in language that might occur between students with low mathematics achievement with learning disabilities and students with low mathematics achievement without learning disabilities. The *Clinical Evaluation of Language Functions-Fourth Edition* CELF-4 offered more sensitivity than the *Peabody Picture Vocabulary Test* (PPVT) that was given in the study of Ysseldyke et al. (1992). Students with MLD who have difficulty interpreting mathematical word problems and following the teacher's directions in class may indicate their difficulties with this aspect of language. Instead of using the PPVT that was used in previous studies, the CELF-4 was used because it had a variety of subtests that addressed language processing in expressive and receptive areas as well as providing information related to automaticity of mathematics sequences. The students in the study were given the CELF-4 on the assumption that any intrinsic processing disorders indicative of learning disabilities in the area of language would be seen on the CELF-4.

Both Expressive and Receptive Language Scores yielded standard scores and percentile ranks. The total battery yielded a Total Language score, percentile rank, and age equivalent. The CELF-4 was standardized on a sample of 2,650 students representative of the 2000 population in the United States in age, gender, race/ethnicity, socioeconomic status, and based on the education level of the parent and geographic region. Children with identified language disorders were included in the sample. Using 320 students, the test/retest reliability was evaluated. Stability coefficients ranged from .71 to .86 for subtest scores and from .88 to .92 for composite scores. Using Chronback's alpha range from .69 to .91 for subtests and from .87 to .95 for composite scores, internal consistency data show subtest items to be homogeneous. The split-half reliability ranged from .71 to .92 for subtests and from .87 to .95 for composite scores. Inter-scorer decision agreement for subtests, requiring clinical judgments and interpretation of scoring rules, ranged from .88 to .99. Evidence of validity was based on test content, response processes, internal structure, relationships with other variables, and consequences of testing. Content validity, addressed through item analysis, judged items to adequately measure language of preschool and school age children. Language skills sampled in CELF-4 were those documented through the literature addressing language disorders and competencies. An expert panel reviewed all CELF–4 test items for ethnic and gender bias. Because the CELF-4 has only recently been released, not all information is currently available. However, the CELF-4 has many of the same subtests as the CELF-3.

Concepts and Directions. The Concepts and Directions Receptive subtest evaluated the student's ability to interpret, recall, and execute oral commands of increasing length and complexity and to interpret oral directions that contain linguistic concepts requiring logical operations such as "and," "either…or," and "if…then."

Subtest reliability produced a standard deviation of subtest scores on Concepts and Directions of 3.3.

Word Classes. The Word Classes Receptive subtest evaluated the student's ability to perceive the associative relationships between word concepts. Subtest reliability produced a standard deviation of subtest scores on Word Classes of 2.8.

Number Repetition. Number Repetition measured the student's ability to repeat forward and backward random digit sequences.

Familiar Sequences. Familiar Sequences, which was timed, measured the student's ability to mentally manipulate and sequence auditory or verbal information.

Examining subtest reliability of the CELF-3 through test-retest of 52 students given a second test within 1 week to 1month after the first test produced standard deviation of subtest scores on Concepts and Directions 3.3, Word Classes 2.8.

Informal Mathematics Assessment

The *Informal Mathematics Assessment* (Woodward & Baxter, 1997) is an individually administered test of problem solving abilities, which examines not only the answer to the problem but also the problem solving processes or strategies the student uses to derive the answer. The *Informal Mathematics Assessment* is a type of process assessment. Process assessments may include norm-referenced assessments, comparing a student's performance with representative students of the same age, as well as student interviews. The purpose of the process assessment is to analyze the student's answers and determine the process by which the student obtained those answers. The process assessment may be either passive or active (Rivera & Bryant, 1992). Using a passive process assessment, the evaluator examines the student's answers and based on the types of errors determines the defective strategy that was used (Woodward et al., 1992). The active process assessment involves some type of interview with the student or an analyzation of the self-talk that the student might have employed when solving the problem in order to illuminate the underlying cognitive processes to the student's problem solving. Using either a passive or active process assessment, the evaluator attains information about the student's error patterns. By analyzing and categorizing student errors observed in mathematical word problem solving (Appendix C), researchers can study the problem solving strategies employed by students (Montague & Applegate, 1993).

The *Informal Mathematics Assessment* measure is a semistructured individual clinical interview in which each student is given eight word problems and asked to solve them. The students have the option of using paper and pencil to solve the problems. To prevent fatigue and frustration the examiner reads each item to the student. Having the problems read to the students also eliminates the response of "no attempt" due to a student's inability to read the problem (Jitendra & Kameenui, 1996) and removes the disadvantage that the group of students with learning disabilities in mathematics and weaknesses in reading might encounter. The items are relatively brief although the student may take as much time as he/she wants to complete each item. Timed conditions with word problems are used in studies to measure rapid retrieval (*Jordan et al.*, 1997). Word problems (Appendix D) included in the *Informal Mathematics Assessment* were written to exclude key words (e.g., *each, got more,* and *gave away*). The problems did not contain extraneous information. The mathematics problems were brief and were provided in text form (Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005) to the students as well as being read aloud by the examiner to each student. Students were given as much time as they wanted to complete each item. These individual sessions were audiotaped. The *Informal Mathematics Assessment* used a semistructured individual clinical interview in which students were asked to describe what they were thinking as they solved mathematics problems (Kennedy & Tipps, 1994). Students were given a problem and asked to think aloud as they solved it. After the problem was solved, follow up questions were asked of the student by the evaluator to further investigate strategies that the student had employed and for purposes of clarification. The scores for the Mathematical Word Problem Error Analysis (Appendix C) were generated from the student's verbal and written performance on the *Informal Mathematics Assessment.*

Summary of Instruments

The *Learning Disabilities Diagnostic Inventory (*LDDI), a teacher rating instrument, was utilized to identify indications of mathematics learning disabilities of the students. The *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) was chosen for this study because it is an achievement test frequently used to assess academic achievement of students with learning disabilities. The WJ III ACH subtests selected were Applied Problems, Math Fluency, Calculation, Letter-Word Identification, and Reading Fluency. The *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) was chosen as an instrument for this study because of its validity and reliability in measuring language areas. The CELF-4 (language) subtests selected were Concepts and Directions Receptive, Word Classes Receptive, Number Repetition Total, and Familiar Sequences. The *Informal Mathematics Assessment* (Woodward & Baxter, 1997) was a semistructured individual clinical interview in which each student was individually administered a test of problem solving abilities. The answers to the problems and the problem solving processes or strategies by which the student used to derive the answers were examined. In an attempt to classify different kinds of problem-solving behavior by the students, the Mathematical Word Problem Error Analysis (Appendix C, Muoneke, 2001) was conducted.

Procedure

Consent

This study was reviewed and approved by the Internal Review Board (IRB) at The University of Texas at Austin. With permission from the participating school district, principals were invited to have their schools participate in the project. Two middle schools volunteered to participate in the research study. The researcher met with all sixth grade teams at the schools in a series of meetings and described the study and the teachers' responsibilities in the study. All sixth grade teachers agreed to participate and signed the Teacher Consent Form (Appendix E).

The permission forms Consent from Parents Form (Appendix A) were sent home once with all sixth grade students and two more times with sixth grade students whose Texas Assessment of Knowledge and Skills (TAKS) Mathematics Test scores did not fall in the commended student range, which was a scaled score over 2400, or who did not have a TAKS Mathematics Test score because the student was in special education and exempt from the test, or the student had recently moved to Texas and had not yet taken the TAKS. In order to be eligible to participate in the study, a student was required to volunteer to participate in the study and to return to the classroom mathematics teacher a Consent from Parents Form (Appendix A) that had been signed by the parent/guardian giving permission for the student to participate in the study and to be assessed using subtests from the *Woodcock-Johnson III Tests of Achievement* and *Clinical Evaluation of Language Fundamentals-Fourth Edition* (CELF-4), the *Informal Mathematics Assessment*, and the *Learning Disabilities Diagnostic Inventory.* Parents and students were informed that participation was voluntary and if students did participate that they could withdraw from the study at any time without penalty. Before a student was assessed, the volunteering student was required to sign a student Assent Form (Appendix B).

Background Information

The parent signature on the Consent from Parents Form (Appendix A) allowed the researcher to gather background information, including mathematics achievement scores, status of English proficiency, medical history, parents' level of education, families socioeconomic status, test scores from previous testing such as for dyslexia or special education, scores from the Texas Essential Knowledge and Skills assessment, and grades from report cards from all previous school years as well as the 2004-2005 school year. Students whose home language was not English were dropped from the study. Students who had previously received a score of less than 80 on a cognitive test such as COGNAT, WISC-III, OLSAT, or SAT-9 were dropped from the study. Once an initial group of students had been identified reversal procedures were implemented. Each student was assigned a number, which was used for identification purposes rather than the student's name in order to maintain privacy.

Assessment

Students remaining in the study were assessed individually for approximately 90−120 minutes at their school in a quiet, well-lit room that was free of distractions by a researcher who had nine years of experience administering diagnostic assessments and had certification as an educational diagnostician in the state of Texas. The students were given the Applied Problems, Calculation, Math Fluency, Letter-Word Identification, and Reading Fluency subtests from the WJ III ACH, Concepts & Directions, Word Classes, Number Repetition, and Familiar Sequences subtests from the CELF-4, and the word problems (Appendix D) to the *Informal Mathematics Assessment* which was audio recorded as the student spoke aloud and explained his/her written work for the mathematics word problems. It was unnecessary in this study to use a timed situation because retrieval difficulties by students using overt counting strategies such as counting on fingers and whispering were observed and recorded. In this study the student interview was audio-recorded and an examiner noted if the student reread the problem from the paper and what calculations were made (Woodward & Baxter, 1997; Woodward et al., 1999). To assess basic understanding of the problem, conceptual knowledge, strategy application (*Jordan et al.*, 1997) as well as computation, calculation procedures, and problem solving approaches, the students were asked to describe what they were thinking and how they were deriving their answer as they solved the mathematical word problem (Ginsburg, et al., 1992). The student was asked, "Explain your thinking using words, numbers, or pictures." The student could show work on the paper provided as he/she used talk aloud to describe the process or wrote sentences on the paper. After the student had solved the problem, the student was asked, "Tell me how you got that answer" (Siegler, 1995). Follow up questions or probes were included when necessary, "Can you tell me a little more about how you got the answer?" or "How did you figure out to get those numbers?" (Woodward et al., 2001). The examiner performed an adapted Mathematical Word Problem Error Analysis (Appendix C, Muoneke, 2001) to classify different kinds of problem-solving behavior by the students (e.g., guessing, using numbers provided in the problem in random order, decomposing problems into smaller units). The types of errors students made during their word problem solving and the frequency of error occurrence were calculated. Twenty percent of the protocols were rescored by a second examiner who listened to the audio recording and reviewed the student's written work as a check for reliability and to reduce the chance of error. An interrater reliability greater than .85 was needed for the Mathematical Word Problem Error Analysis (Appendix C). The interrater reliability was .95.

Scoring

The researcher scored all protocols and entered the scores into the assessment software program and the EXCEL database. Based on the results of the WJ III ACH mathematics scores the mathematics teachers were given LDDI forms. Mathematics teachers with the assistance of their grade level teams were asked to complete an LDDI form on each student who received a score at or below the 25th percentile on the Applied Problems, Calculation, or Math Fluency subtests from the WJ III ACH. The teams of teachers received verbal and written instructions from the researcher on completing the LDDI. The score on the LDDI was based on the rating of the student's characteristics of learning disabilities by the mathematics teacher with the assistance of the team. After the mathematics teacher had completed the LDDI protocol for a student, the researcher scored the protocol. Based on the student's resulting stanine score, the student was assigned to one of three groups of students. The first group included students with low achievement in mathematics and without indications of learning disabilities (Low Math/NLD). These students had stanine scores of 7-9 on all sections of the LDDI. The second group included students with low achievement in mathematics and with indications of learning disabilities (MLD). These students had a stanine score of 1-5 on a minimum of one area and a stanine score of 7, 8, or 9 on a minimum of one other area on the LDDI. The third group included students who did not meet either of the previous criteria for the LDDI or who had all scores on Applied Problems, Calculation, Math Fluency on the WJ III ACH greater than the 25th percentile. All three groups were included in the factor analysis portion of the study.

Data Analysis

A chi-square test was used to show that the sample of participants was representative of the school district in all areas except achievement. Several analyses were conducted to determine if there were significant differences between the sample populations and the two middle schools on demographic dimensions including ethnicity and socioeconomic status. Chi-square analyses (Pearson χ^2) was used for the variables of socioeconomic status and ethnicity.

Subtests from the WJ III ACH and the CELF-4 had been selected based on the likelihood that their measures would reflect the different dimensions in solving mathematical word problems. Tests were scored for all students participating in the research project. All the scores from the Mathematical Word Problem Error Analysis, the CELF-4 subtests, and WJ III ACH subtests for each student were entered as variables into an EXCEL database as well as the student's classification as MLD or Low Math/NLD or other. SPSS was used to run an exploratory factor analysis to identify the underlying factors associated with solving mathematical word problems. SPSS was also used to run the independent-samples *t* tests. The independent-sample *t* tests evaluated the differences between the means of the two independent-groups MLD and Low Math/NLD for all the subtest variables from the WJ III ACH and CELF-4 subtests in order to determine if the mean for one group differed significantly from the mean value of the other group.

Reliability

A second researcher rescored 20% of the *Informal Mathematics Assessment*s as a check for reliability and to reduce the chance of error. The second researcher listened to the audio recording and reviewed the student's written work. The interrater reliability was .95. An interrater reliability greater than .85 was needed for the Mathematical Word Problem Error Analysis (Appendix C).

Summary

This non-experimental empirical study utilized two research methods: (a) exploratory factor analysis (Fabrigar, Wegener, MacCallum, & Strahan, 1999) and (b) causal comparison (Mertens, 1998) to determine the underlying factors associated with solving mathematical word problems and possible differences in the problem-solving behaviors for mathematical word problems and the types of mathematical word problem errors between students with MLD and students with Low Math/NLD. Students were assigned to groups based on their scores on the Calculation, Math Fluency, or Applied Problems subtests from the *Woodcock-Johnson-Third Edition Tests of Achievement* (WJ III ACH) and their profile for the likelihood of having a learning disability according to the *Learning Disabilities Diagnostic Inventory* (LDDI)

Differences in mathematics automaticity were assessed with the WJ III ACH Math Fluency score, differences in mathematics computation were assessed with the WJ III ACH Calculation score, and differences in mathematical word problems were assessed with the WJ III ACH Applied Problems score. Linguistic differences that might affect language processing required by mathematical word problems were assessed quantitatively by comparing Concepts and Directions, Word Classes Receptive, Number Repetition Total, and Familiar Sequences subtest scores on the *Clinical Evaluation of Language Fundamentals-Fourth Edition* (CELF-4) between the two groups of students. The independent-sample *t* tests evaluated the differences between the means of the two independent-groups MLD and Low Math/NLD for all the subtest variables from the WJ III ACH and CELF-4 subtests in order to determine if the mean for one group differed significantly from the mean value of the other group for all variables. The *Informal Mathematics Assessment*, utilizing the student clinical interview and an error analysis, was used to identify processes and strategies used by students to solve mathematical word problems and the types of mathematical word problem errors that they made (Babbitt, 1990; Jitendra & Kameenui, 1996; Muoneke, 2001).

Chapter IV

Results

This study explored the underlying factors associated with solving mathematical word problems. The performances of students with mathematics difficulties who were likely to have a learning disability (MLD) were compared to the performances of students with mathematics difficulties who were unlikely to have a learning disability (Low Math/NLD) in the areas of mathematics automaticity, calculation, applied problems, linguistics, and reading. This study also examined student problem-solving error types displayed by MLD and Low Math/NLD groups during word problem solving. This chapter described the findings including the (a) research questions and hypotheses, (b) results of data analyses for the research questions, and (c) summary of findings.

Research Questions and Hypotheses

The following research questions were addressed:

(1) What are the underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student mathematics and reading achievement from the *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests?

(2) Are there statistically significant differences in mathematics automaticity as shown by Math Fluency scores measured by the WJ III ACH between (a) students with MLD who received a score $\leq$25th percentile in Calculation, Math

Fluency, or Applied Problems on the WJ III ACH who were likely to have a learning disability according to the *Learning Disabilities Diagnostic Inventory* (LDDI) and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(3) Are there statistically significant differences in Calculation scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(4) Are there statistically significant differences in Applied Problems scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(5) Are there statistically significant linguistic differences in cognitive areas measured by the subtest scores of the CELF-4 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied

Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(6) Are there statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

(7) Are there differences in mathematical problem-solving error type percentages as measured by the categories of computation, operation, translation, and no attempt on the *Informal Mathematics Assessment* between (a)students with MLD who received a score ≤25^{th} percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25^{th} percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were not likely to have a learning disability according to the LDDI?

From these questions the following null hypotheses were posited:

(1) There are no statistically significant underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student mathematics and reading achievement from *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests.

(2) There are no statistically significant differences in mathematics automaticity as shown by Math Fluency scores measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(3) There are no statistically significant differences in Calculation scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(4) There are no statistically significant differences in Applied Problems scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(5) There are no statistically significant linguistic differences in cognitive areas measured by the subtest scores of the CELF-4 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(6) There are no statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math

Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

(7) There are no differences in mathematical problem-solving error type percentages as measured by the categories of computation, operation, translation, and no attempt on the *Informal Mathematics Assessment* between (a) students with MLD who received a score $\leq 25^{th}$ percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score $\leq 25^{th}$ percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were not likely to have a learning disability according to the LDDI.

Findings of Data Analyses

Dependent Variables

The dependent variables were the Receptive Language and Working Memory subtest scores from the CELF-4, which was used for assessing language processing, the Math Fluency subtest score from WJ III ACH used to assess mathematics automaticity, the Calculation subtest score from the WJ III ACH used to assess mathematics computation, and the Applied Problems subtest score from the WJ III ACH used to assess mathematical problem-solving skills. The Letter-Word Identification subtest score and the Reading Fluency subtest score from WJ III ACH were used to assess reading.

Independent Variables

The independent variables were (a) achievement level in mathematics and (b) the indication of a learning disability. The two levels of the independent variable for the indication of a learning disability assessed by teacher ratings on the *Learning Disabilities Diagnostic Inventory* (LDDI) were: (a) likely to have a learning disability and (b) unlikely to have a learning disability. The two levels of the independent variable for mathematics achievement were (a) low achievement in mathematics as determined by a subtest score less than or equal to the 25^{th} percentile on the Math Fluency, Calculation, or Applied Problems subtest scores from the *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and (b) achievement in mathematics greater than the 25^{th} percentile as determined by the Math Fluency, Calculation, and Applied Problems subtest scores from the *Woodcock-Johnson III Tests of Achievement* (WJ III ACH).

The findings of the data analyses for each research question and hypothesis were reported as follows:

What are the underlying factors associated with solving mathematical word problems?

Research question #1.

(1) What are the underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student mathematics and reading achievement from the *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests?

Null Hypotheses

(1) There are no statistically significant underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student mathematics and reading achievement from *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests.

Factor analysis was the statistical procedure used to identify the underlying dimensions known as factors associated with solving mathematical word problems. Subtests from the WJ III ACH and the CELF-4 were selected based on the likelihood that their measures reflected the different factors such as cognitive processing, language processing, visual spatial processing, long term memory, working memory, and processing speed in solving mathematical word problems. Exploratory factor analysis (EFA) was used (Fabrigar, Wegener, MacCallum, & Strahan, 1999) to search for underlying factors in the measured variables and to identify commanalities through the use of correlations between the students' subtest scores from the WJ III ACH mathematics and reading subtests and the CELF-4 linguistic subtests and mathematical word problems. The common factors were variables that were influenced by one or more of the same common factors and thus were correlated (Fabrigar, Webener,MacCallum, & Strahan, 1999). Because factor analysis uses an interdependence technique, it was employed to identify the interrelationships existing among all the variables simultaneously.

The common R-Type factor analysis, placing cases in rows and variables in columns, was used to examine dependent variables and group them according to underlying structures or factors. Principal Components analysis (PCA) was the factor extraction method used in SPSS as shown in Table 4.1. PCA is the most common form of factor analysis. While using PCA, the assumption was made that the measured variables were linearly related to the factors. After finding the linear variable combination with the maximum amount of variance, PCA removed the variance and then sought a second linear combination and removed that variance, and so on (Garson, 2006). Therefore, first extracted factor by PCA accounted for the largest amount of the variability among the measured variables, the second extracted factor the next most variability, and so on. The variability of a factor is called an eigenvalue (Green, Salkind, & Akey, 2000), and a factor's eigenvalue measured the variance in all of that factor's variables.

Table 4.1 Factor Extraction and Explanation of Variance

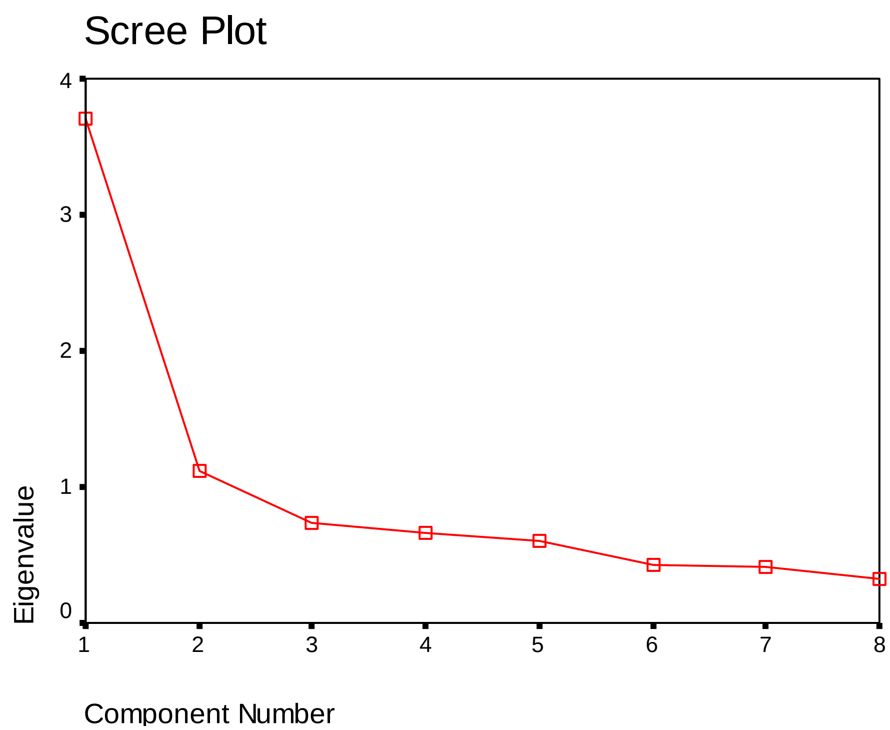
Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.710	46.370	46.370	3.710	46.370	46.370
2	1.119	13.989	60.359	1.119	13.989	60.359
3	.734	9.181	69.549			
4	.662	8.278	77.818			
5	.604	7.551	85.369			
6	.433	5.418	90.787			
7	.416	5.202	95.988			
8	.321	4.012	100.000			

Extraction Method Principal Component Analysis

Two criteria were used to determine the number of factors to rotate. The first criterion (Kaiser criterion) was to retain each factor that had an eigenvalue greater than one (Pedhazur & Schmelkin, 1991). Factors with a low eigenvalues explained little of the variance and might be ignored and did not need to be retained (Garson, 2006). Only two factors had eigenvalues greater than one (3.710 and 1.119). The second criterion was to examine the plot of the eigenvalues and retain each factor with an eigenvalue in the steep slope or sharp descent part of the plot before the eigenvalues started to level off. The Cattell scree test, as shown in Figure 4.1, plotted the components as the X axis and their corresponding eigenvalues as the Y axis. The first component with the largest amount of the variability was placed farthest to the left on the graph. The later components, placed to the right on the graph, showed the eigenvalues dropping. The rule for the Cattell scree test is to keep all components in the sharp descent of the slope and to discard all components after the slope has leveled off.

Figure 4.1 Cattell Scree Plot for Eigenvalues Values



After examining the scree plot, it was determined that two factors were in the steep slope or sharp descent part of the plot and should be rotated. Between the first two factors the slope was quite steep but after the second factor the slope decreased and began to level off. To aid in the interpretation of factors, a varimax rotation (orthogonal rotation) was used. Table 4.2 shows the results of the rotation.

Table 4.2. Varimax Rotation of Eigenvalues and Explanation of Total Variance.

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
Factor	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.710	46.370	46.370	2.793	34.910	34.910	2.294	28.674	28.674
2	1.119	13.989	60.359	1.138	14.221	49.131	1.637	20.457	49.131
3	.734	9.181	69.540						
4	.662	8.278	77.818						
5	.604	7.551	85.369						
6	.433	5.418	90.787						
7	.416	5.202	95.988						
8	.321	4.012	100.000						

Extraction Method: Maximum Likelihood.

Using the varimax rotation had the effect of differentiating the original variables by the extracted factor and made it easier to identify a variable with the factors (Garson, 2006). The two rotated factors are shown in Table 4.3. Those measures loading most heavily on Factor 1 were Word Problems (.696), Applied Problems (.661), Calculation (.653), Working Memory (.618), and Receptive Language (.550). Letter-Word Identification loaded almost equally on Factor 1 (.410) and Factor 2 (.431). Measures loading heaviest on Factor 2 were Reading Fluency (.921) and Math Fluency (.584).

Table 4.3 Two Rotated Factors with Loadings on Matrix

Rotated Factor Matrix

Subtest	Factor	
	1	2
Word Problems	.696	.147
Applied Problems WJ III ACH	.661	.239
Math Fluency WJ III ACH	.198	.584
Calculation WJ III ACH	.653	.252
Letter-Word Identification WJ III ACH	.410	.431
Reading Fluency WJ III ACH	.231	.921
Receptive Language CELF-4	.550	.284
Working Memory CELF-4	.618	.198

Extraction Method: Maximum Likelihood.
Rotation Method: Varimax with Kaiser Normalization.
Rotation converged in 3 iterations.

To identify the two factors the descriptions of the subtests were reviewed. The mathematical word problems selected for Word Problems from the *Informal Mathematics Assessment* were used to assess the student's mathematical problem-solving abilities. The problem was read to the student and the student also had a text copy of the problem. The student then solved the problem and the student's answer was analyzed for correctness and types of errors (computational, operational, translation, and no attempt). Problem-solving abilities included listening comprehension, analyzation, reasoning, and understanding of number. The Applied Problems subtest from the WJ III ACH was designed to assess a student's ability to analyze, reason, and solve math word problems. Many of the problems included visual stimuli such as pictures of objects and questions were presented orally. After listening to the word problem, the student needed to recognize the required procedure and appropriate operation, ignore extraneous information, and perform simple calculations. The Math Fluency subtest from the WJ III ACH was a 3-minute timed test, which assessed the student's ability to solve quickly simple addition, subtraction, and multiplication facts. The test had a 3-minute time limit. The Calculation subtest from the WJ III ACH assessed the student's skill in performing mathematical computations. The Letter-Word Identification subtest from the WJ III ACH assessed the student's skill in identifying words in isolation. The student was not required to know any word's meaning. The Reading Fluency subtest from the WJ III ACH was a 3-minute timed test, which assessed the student's ability to quickly read simple sentences with easy vocabulary words and determine if the statements were true.

The Receptive Language was composed of two subtests, Concepts and Following Directions and Word Classes, from the CELF-4. Concepts and Following directions measured the student's ability to interpret, remember, and carry out spoken directions. Word Classes measured the student's ability to understand logical relationships in associated words. Working Memory was composed of the results of two subtests, Number Repetition and Familiar Sequences, from the CELF-4. Number Repetition measured the student's ability to repeat forward and backward random digit sequences. Familiar Sequences, which was timed, measured the student's ability to mentally manipulate and sequence auditory or verbal information. Table 4.4 shows the subtests from the WJ III ACH and the CELF-4 and the abilities that these subtests measure.

Table 4.4 Subtests and Measured Abilities

Subtest	Abilities Measured by Subtests						
	Processing Speed	Analyzing /Reasoning	Number Awareness	Processing Visual Information	Language Compre-hension	Auditory Memory	Retrieval from long fixed memory
Word Problems		X	X		X		
Applied Problems		X	X	X	X		
Math Fluency	X			X			X
Math Calculation		X	X	X			
Letter-Word Identifi-cation				X			X
Reading Fluency	X	X		X	X		X
Receptive Language		X		X	X	X	
Working Memory	X	X			X	X	

Reading Fluency (.921), Math Fluency (.584), and Letter-Word Identification (.431) loaded heaviest onto Factor 2. The chart showed that there were two categories that they had in common: processing visual information and retrieval from long fixed memory. Retrieval from long fixed memory was the one category that these three subtests had in common in which there were no other subtests that also measured skills in that category. Processing visual information was eliminated for Factor 2 because Applied Problems (.239), Math Calculation (.252), and Receptive Language (.284), which also required student skills in processing visual information, had loadings of less than .3 on Factor 2. Therefore, retrieval from long fixed memory was identified as Factor 2.

To identify Factor 1, the categories that Word Problems (.696), Applied Problems (.661), Math Calculation (.653), Receptive Language (.550), and Working Memory (.618) had in common were identified. Analyzing/Reasoning was the only category that all of those subtests had in common, however, all those subtests except Math Calculation also measured Language Comprehension. Therefore, Factor 1 should be a factor such as abstract thinking that includes both Analyzing/Reasoning and Language Comprehension.

Factor 1, the largest underlying factor associated with solving mathematical word problems, was identified as Abstract Thinking and Factor 2 was identified as Retrieval from Long Fixed Memory. Therefore, the null hypothesis #1 was rejected. There were underlying factors associated with solving mathematical word problems as identified by correlations among variables measuring student mathematics and reading achievement from *Woodcock-Johnson III Tests of Achievement* (WJ III ACH) and receptive language from the *Clinical Evaluation of Language Functions-4th Edition* (CELF-4) subtests, and the factors were Abstract Thinking and Retrieval from Long Fixed Memory.

Are there statistically significant differences in mathematics automaticity between (a) students with MLD and (b) students with Low Math/NLD?

Research question #2.

(2) Are there statistically significant differences in mathematics automaticity as shown by Math Fluency scores measured by the WJ III ACH between (a) students with MLD who received a score $\leq$25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH who were likely to have a

learning disability according to the *Learning Disabilities Diagnostic Inventory* (LDDI) and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

Null hypothesis.

(2) There are no statistically significant differences in mathematics automaticity as shown by Math Fluency scores measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

Historically, the group of students whose math achievement scores are below the 30th percentile have included students who have learning disabilities and students who are developmentally delayed in the area of mathematics (Fuchs & Fuchs, 1998; Geary, 1993; Jordan & Hanich, 2000). Both groups of students are often placed in the same classroom and receive the same instruction. The students with learning disabilities may be receiving full inclusion and minimal special education support with the general education classroom teacher responsible for math instruction. Therefore, it is important to determine if there are differences between these groups such that instruction can be adjusted to meet the needs of both groups. Both groups have difficulties in math, but it is important to determine if both groups have difficulties in the same areas of mathematics. Students with learning disabilities may need differentiated instruction which addresses their specific needs. To determine if there were differences within the group of students whose mathematics achievement scores were $\leq 25^{th}$ percentile in the areas of Math Fluency, Calculation, or Applied Problems (which are areas of mathematics related to word problem solving), the group was sorted into two smaller groups. Students were assigned to the two groups based upon their scores for the dependent variables Calculation, Math Fluency, Applied Problems, and their LDDI profile. Students who received a score $\leq 25^{th}$ percentile on Calculation, Math Fluency, or Applied Problems and a LDDI profile indicating they were likely to have a learning disability were assigned to the group of students MLD. Students who received a score $\leq 25^{th}$ percentile on Calculation, Math Fluency, or Applied Problems and a LDDI profile indicating they were unlikely to have learning disabilities were assigned to the group of students Low Math/NLD. An independent *t*-test was conducted on the dependent variables, scores on the WJ III ACH and CELF-4 subtests, to analyze statistical differences between the two smaller groups of students: (a) students with mathematics difficulties who were likely to have a learning disability (MLD) and (b) students with mathematics difficulties who were unlikely to have a learning disability (Low Math/NLD). The total mean percentage score for each subtest for each group was obtained by adding all the test scores for each subtest for group members and then determining the average. The Math Fluency mean score for students with MLD in Group 1 (M =32.50, SD = 26.30) was higher than the mean score for students with Low Math/NLD in Group 2 (*M* =14.56, *SD* =12.08). The results are shown in Table 4.5.

Table 4.5 Means for Math Scores for Students with MLD and Students with Low Math/NLD

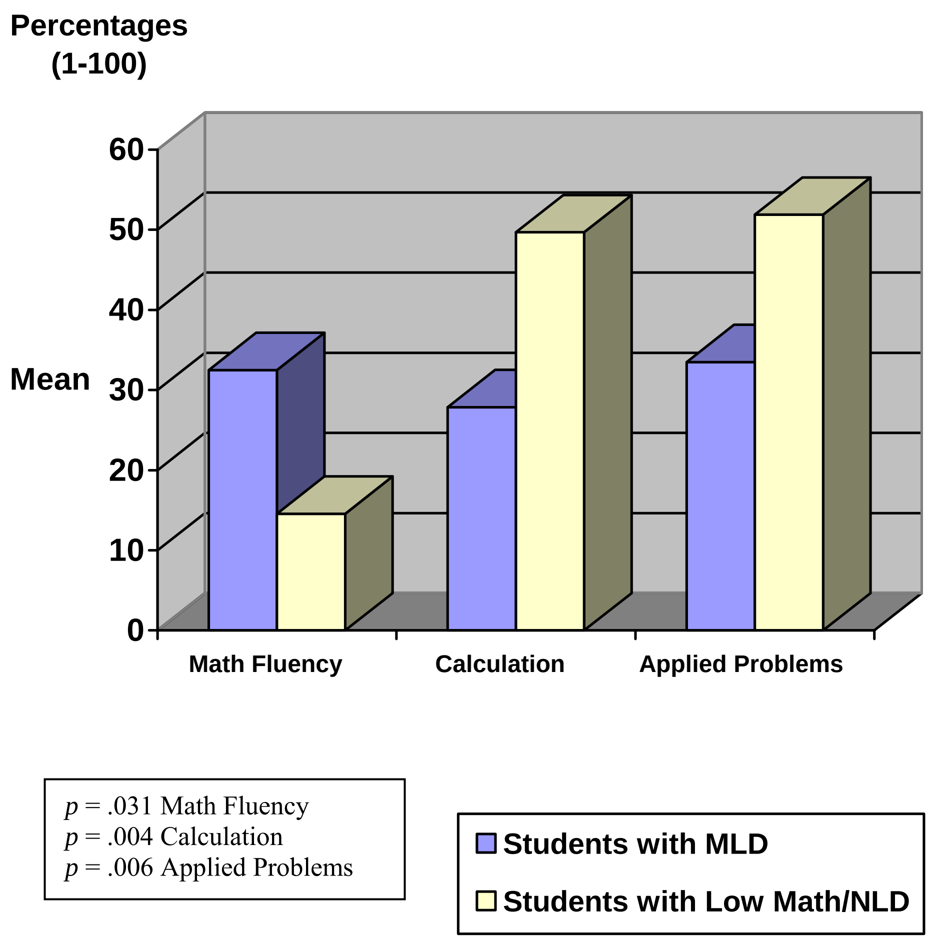
Math Scores for Students with MLD and Students with Low Math/NLD

Subtest	Group	*N*	*M*	*SD*	*SE*
Math Fluency	1-MLD	14	32.50	26.30	7.03
Math Fluency	2-Low Math/NLD	16	14.56	12.08	3.02
Calculation	1-MLD	14	27.86	16.92	4.52
Calculation	2-Low Math/NLD	16	49.69	21.42	5.36
Applied Problems	1-MLD	14	33.50	17.10	4.57
Applied Problems	2-Low Math/NLD	16	51.88	16.49	4.12

Note. WJ III ACH mean scores are expressed in percentages and have a mean of 50.
CELF-4 scaled scores have a range of 1-19 with a mean of 10.
Group 1 = Students with MLD
Group 2 = Students with Low Math/NLD

The Math Fluency mean scores for students with MLD in Group 1 and students with Low Math/NLD in Group 2 are graphically displayed in Figure 4.2.

Figure 4.2 Bar Graph of Math Scores for Students with MLD and Students with Low Math/NLD



The Independent Samples test was significant, *t* (17.723) =2.345, *p* =.031. Students with MLD in Group 1 (*M* =32.50, SD = 26.30) scored significantly higher than Group 2 students with Low Math/NLD (*M* = 14.56, *SD* =12.08) did. The results are shown in Table 4.6. Therefore, the null hypothesis #2 was rejected. There was a significant difference in mathematics automaticity favoring Group 1 as shown by Math Fluency scores between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, and Applied Problems on the WJ III ACH and were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and were unlikely to have a learning disability according to the LDDI. The effect size for the *d* index was .9 which was large (Cohen, 1988).

Table 4.6 Math Independent Samples Test Results for Students with MLD and Students with Low Math/NLD

Independent Samples Test for Students with MLD and Students with Low Math/NLD

Subtest	*t*	*df*	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Math Fluency	2.345	17.723	**.031***	17.94	7.65
Calculation	-3.114	27.743	**.004***	-21.83	7.01
Applied Problems	-2.985	27.168	**.006***	-18.38	6.16

Note. Equal variances not assumed.
*$p < .05$

Are there statistically significant differences in Calculation scores between (a) students with MLD and (b) students with Low Math/NLD?

Research question #3.

(3) Are there statistically significant differences in Calculation scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the

LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

Null hypothesis.

(3) There are no statistically significant differences in Calculation scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

The Calculation mean score for students with Low Math/NLD in Group 2 (*M* = 49.69, SD =21.42) was higher than the mean score for students with MLD in Group 1 (*M* = 27.86, *SD* =16.92). The results are shown in Table 4.5.

The Calculation mean scores for students with MLD in Group 1 and students with Low Math/NLD in Group 2 are graphically displayed in Figure 4.2.

The Independent Samples test was significant, t (27.743) = −3.114, p = .004. Students with Low Math/NLD in Group 2 (*M* =49.69, SD =21.42) scored significantly higher than Group 1 students with MLD (*M* = 27.86, *SD* =16.92) did. The results are shown in Table 4.6. Therefore, the null hypothesis #3 was rejected. There was a significant difference in Calculation scores favoring Group 2 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied

Problems on the WJ III ACH and were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, and Applied Problems on the WJ III ACH and were unlikely to have a learning disability according to the LDDI. The effect size for the *d* index was 1.1 which was large (Cohen, 1988).

Are there statistically significant differences in Applied Problems scores between (a) students with MLD and (b) students with Low Math/NLD?

Research question #4.

(4) Are there statistically significant differences in Applied Problems scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

Null hypothesis.

(4) There are no statistically significant differences in Applied Problems scores as measured by the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th

percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

The Applied Problems mean score for students with Low Math/NLD in Group 2 (*M* = 51.88, *SD* =16.49) was higher than the mean score for students with MD in Group 1 (*M* = 33.50, *SD* =17.10). The results are shown in Table 4.5.

The Applied Problems mean scores for students with MLD in Group 1 and students with Low Math/NLD in Group 2 are graphically displayed in Figure 4.2.

The Independent Samples test was significant, t (27.168) = −2.985, p = .006. Students with Low Math/NLD in Group 2 (*M* =51.88, *SD* =16.49) scored significantly higher than Group 1 students with MD (*M* = 33.50, *SD* =17.10) did. The results are shown in Table 4.6. Therefore, the null hypothesis #4 was rejected. There was a significant difference in Applied Problems scores in favor of Group 2 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, and Applied Problems on the WJ III ACH and were unlikely to have a learning disability according to the LDDI. The effect size for the *d* index was 1.1, which was large according to Cohen (1988).

Are there statistically significant linguistic differences in cognitive areas as shown by CELF-4 subtest scores between (a) students with MLD and (b) students with Low Math/NLD?

Research question #5.

(5) Are there statistically significant linguistic differences in cognitive areas measured by the subtest scores of the CELF-4 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

Null hypothesis.

(5) There are no statistically significant linguistic differences in cognitive areas measured by the subtest scores of the CELF-4 between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

An independent *t*-test was conducted on the dependent variables, scores on the CELF-4 subtests, to analyze statistical differences between two groups (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, and Applied Problems on the WJ III ACH and were unlikely to have a learning disability according to the LDDI. The Receptive Language and Working Memory scores, used in factorial analysis, were replaced for purposes of finer discrimination with scores from their component subtests Concepts & Following Directions and Word Classes for Receptive Language and Number Recognition and Familiar Sequences for Working Memory. The total mean percentage score for each subtest for each group was obtained by adding all the test scores for each subtest for group members and then determining the average. The results, as shown in Table 4.7, indicated that students with MLD in Group 1had a lower mean score for all CELF-4 subtests than all mean scores for students with Low Math/NLD in Group 2 did.

Table 4.7 Means for CELF-4 Scores for Students with MLD and Students with Low Math/NLD

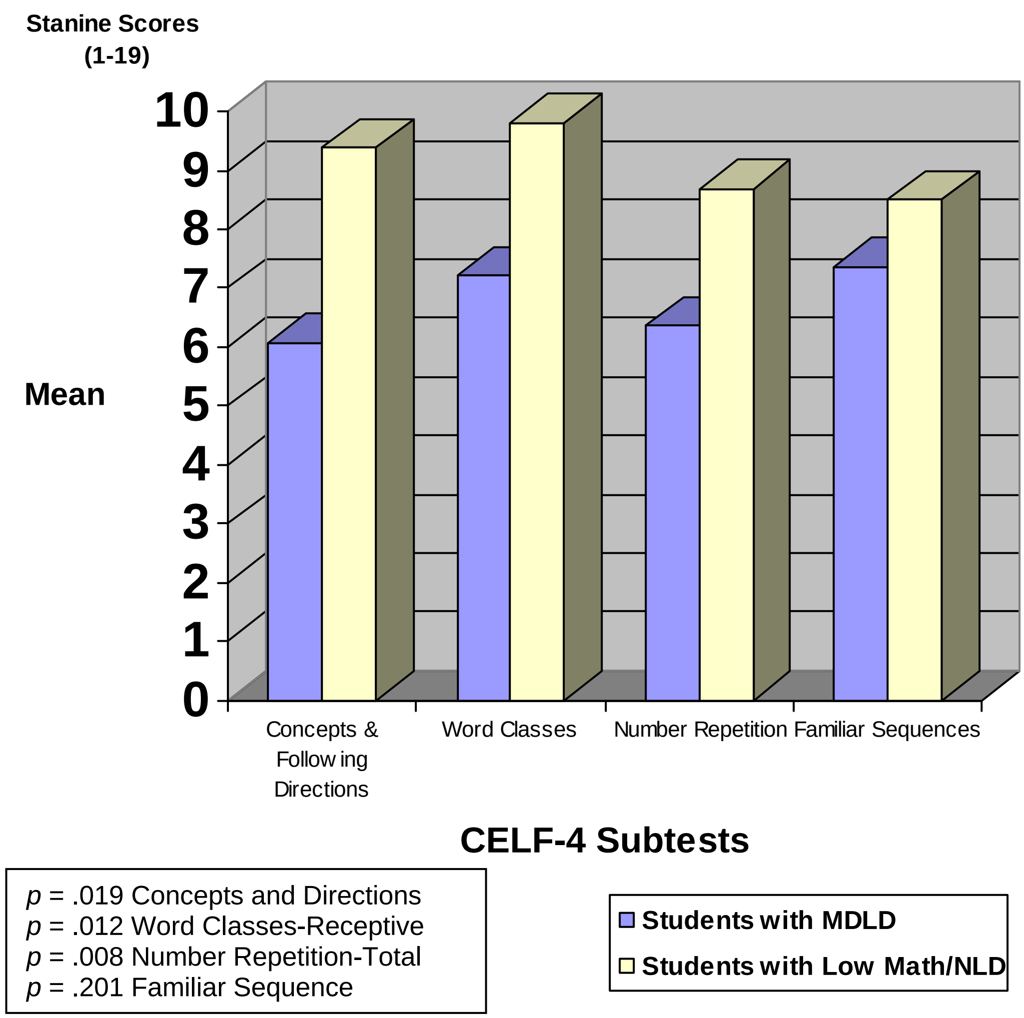
Group for Students with MLD and Students with Low Math/NLD

CELF-4 Subtest	Group	*N*	*M*	*SD*	*SE*
Concepts & Following Directions	1-MLD	14	6.07	4.12	1.10
	2-Low Math/NLD	16	9.38	2.80	.70
Word Classes-Receptive	1-MLD	14	7.21	2.75	.74
	2-Low Math/NLD	16	9.81	2.51	.63
Number Repetition-Total	1-MLD	14	6.36	2.06	.55
	2-Low Math/NLD	16	8.69	2.39	.60
Familiar Sequences	1-MLD	14	7.36	2.37	.63
	2-Low Math/NLD	16	8.50	2.39	.60

Note. CELF-4 scaled scores have a range of 1-19 with a mean of 10.
Group 1 = Students with MLD
Group 2 = Students with Low Math/NLD

The CELF-4 mean scores for students with MLD in Group 1 and students with Low Math/NLD in Group 2 are graphically displayed in Figure 4.3.

Figure 4.3 Bar Graph of CELF-4 Means of Students with MLD and Students with Low Math/NLD



The Independent Samples test was significant for the Concepts & Following Directions subtest, $t(17.933) = -2.530$, $p = .002$. Students with Low Math/NLD in Group 2 (M =10.47, SD = 1.88) scored significantly higher than Group 1 students with MLD (M = 6.07, SD = 4.12) did. The results are shown in Table 4.8. The effect size for the *d* index was .9, which was large (Cohen, 1988). The Independent Samples test was significant for Word Classes-Receptive, $t\ (26.184) = -2.689$, $p = .025$. Students with Low Math/NLD in Group 2 (M = 9.53, SD = 2.47) scored significantly higher than Group 1 students with MLD (M = 7.21, SD = 2.75) did. The effect size for the *d* index was 1.0, which was large (Cohen, 1988). The results are shown in Table 4.8. The Independent Samples test was significant for Number Repetition-Total, $t\ (26.996) = -3.057$, $p = .005$. Students with Low Math/NLD in Group 2 (M = 8.80, SD = 2.24) scored significantly higher than Group 1 students with MLD (M = 6.36, SD = 2.06) did. The effect size for the *d* index was 1.0, which was large (Cohen, 1988). The results are shown in Table 4.8. The Independent Samples test was not significant for Familiar Sequence subtest, $t\ (26.464) = -2.863$, $p = .008$. Students with Low Math/NLD in Group 2 (M = 9.80, SD = 2.21) did not score significantly higher than Group 1 students with MLD (M = 7.36, SD = 2.37) did. The results for all the CELF-4 subtests are shown in Table 4.8. The null hypothesis #5 was rejected for Concepts & Following Directions, Word Classes-Receptive, and Number Repetition-Total but not for Familiar Sequence. There are statistically significant linguistic differences in language areas measured by the Concepts & Following Directions, Word Classes-Receptive, and Number Repetition subtest scores on the CELF-4 between (a) students with MLD who received a score

≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, and Applied Problems on the WJ III ACH and were unlikely to have a learning disability according to the LDDI.

Table 4.8 CELF-4 Independent Samples Test Results

for Students with MLD and Students with Low Math/NLD

Independent Samples Test for Students with MLD and Students with Low Math/NLD

CELF-4 Subtest	*t*	*df*	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Concepts & Following Directions	-2.530	22.453	**.019***	-3.30	1.31
Word Classes Receptive	-2.689	26.601	**.012***	-2.60	.97
Number Repetition Total	-2.870	27.998	**.008***	-2.33	.81
Familiar Sequences	-1.310	27.538	.201	-1.14	.87

Note. Equal variances not assumed.
*$p < .05$

Are there statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD and (b) students with Low Math/NLD?

Research question #6.

(6) Are there statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in

Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI?

Null Hypotheses

(6) There are no statistically significant reading differences in areas measured by the Letter-Word Identification and Reading Fluency subtest scores of the WJ III ACH between (a) students with MLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score ≤25th percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were unlikely to have a learning disability according to the LDDI.

Mathematical word problems are directly related to the use of language. In order to solve a mathematical word problem, all of the language processes associated with mathematics in addition to the language processes associated with reading is needed to understand and solve the mathematics problem. Students with reading difficulties find that their reading problems interfere with their ability to solve word problems (Smith, 1994). Because of the role that reading plays in solving mathematical word problems, reading subtests were included as part of the assessment. Letter-Word Identification and Reading Fluency were the two subtests from the WJ III ACH used to measure students' abilities in the area of reading. The Letter-Word Identification mean score for students with Low Math/NLD in Group 2 (*M* = 49.31, *SD* =24.68) was higher than the mean score for students with MLD in Group 1 (*M* =24.31, SD =20.10). The Reading Fluency mean score for students with Low Math/NLD in Group 2 (*M* =37.25, *SD* =28.49) was higher than the mean score for students with MLD in Group 1 (*M* =30.38, SD =21.99). The mean scores for reading are shown in Table 4.9.

Table 4.9 Means for WJ III ACH Reading Scores

for Students with MLD and Students with Low Math/NLD

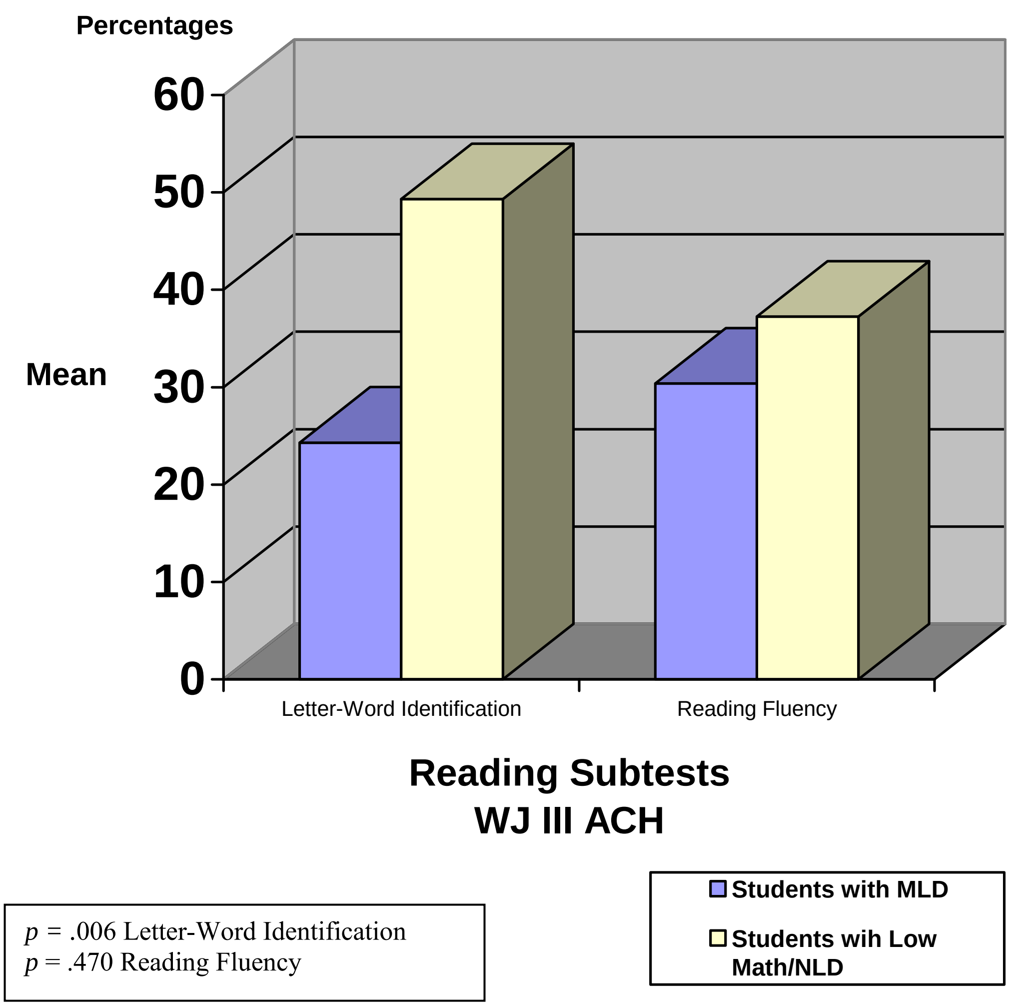
Group Statistics for Students with MLD and Students with Low Math/NLD

WJ III ACH Subtest	Group	*N*	*M*	*SD*	*SE*
Letter-Word Identification	1-MLD1	14	24.31	20.10	5.58
	2-Low Math/NLD2	16	49.31	24.68	6.17
Reading Fluency	1-MLD1	13	30.38	21.99	6.10
	2-Low Math/NLD2	16	37.25	28.49	7.12

Note. WJ III ACH mean scores are expressed in percentages and have a mean of 50.
CELF-4 scaled scores have a range of 1-19 with a mean of 10.
Group 1 = students with MLD
Group 2 = students with Low Math/NLD

The Letter-Word Identification and Reading Fluency mean scores for students with MLD in Group 1 and students with Low Math/NLD in Group 2 are graphically displayed in Figure 4.4.

Figure 4.4 Bar Graph of WJ III ACH Reading Subtests Means Scores of Students with MLD and Students with Low Math/NLD



p = .006 Letter-Word Identification
p = .470 Reading Fluency

For Letter-Word Identification the Independent Samples test was significant, *t* (26.997) =-3.007, *p* = .006. Students with Low Math/NLD in Group 2 (*M* =49.31, SD

=24.68) scored significantly higher than Group 1 students with MLD (*M* = 24.31, *SD* =20.10) did. The effect size d index was 1.1, which was large (Cohen, 1988). For Reading Fluency the Independent Samples test was not significant, *t* (26.950) =-.732, *p* = .470. Students with Low Math/NLD in Group 2 (*M* =37.25, SD =28.49) did not score significantly higher than Group 1 students with MLD (*M* = 30.38, *SD* =21.99) did. The results are shown in Table 4.10.

Table 4.10 Reading Independent Samples Test Results

for Students with MLD and Students with Low Math/NLD

Independent Samples Test for Students with MLD and Students with Low Math/NLD

Reading Subtests WJ III ACH	*t*	*df*	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Letter-Word Identification	-3.007	26.997	**.006***	-25.00	8.32
Reading Fluency	-.732	26.950	.470	-6.87	9.38

Note. Equal variances not assumed.
*p < .05

Are there differences in Mathematical problem-solving error type percentages on the Informal Mathematics Assessment between (a) students with MLD and (b) students with Low Math/NLD?

Research question #7.

(7) Are there differences in mathematical problem-solving error type percentages as measured by the categories of computation, operation, translation, and no attempt on the *Informal Mathematics Assessment* between (a)students with MLD who received a score $\leq 25^{th}$ percentile in Calculation, Math Fluency, or Applied

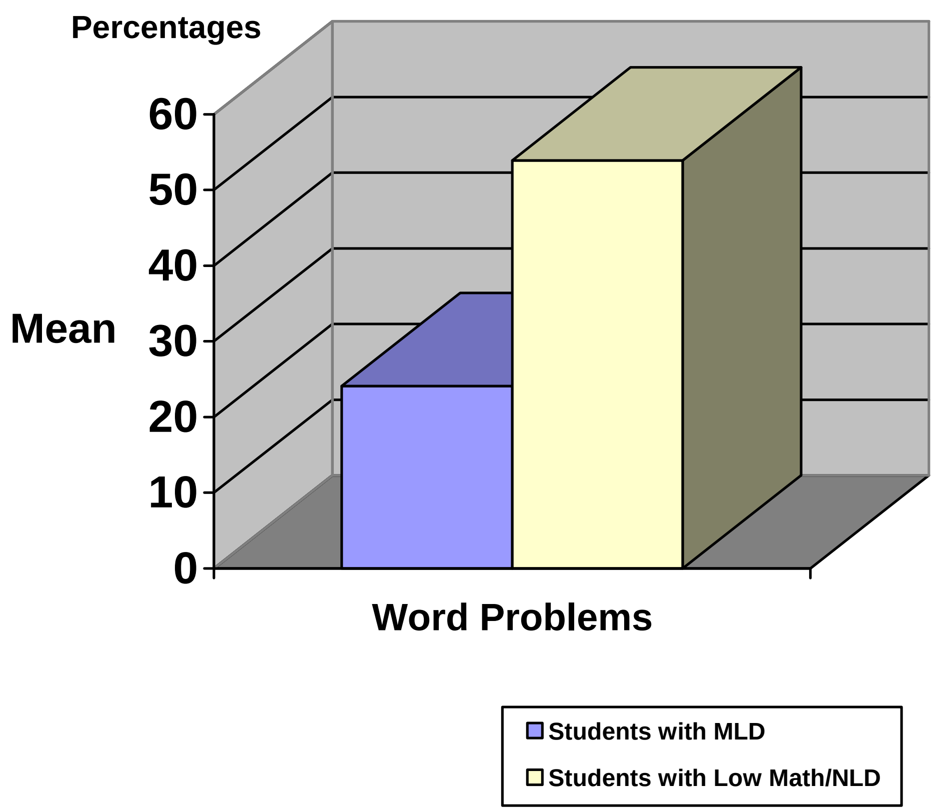
Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score $\leq 25^{th}$ percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were not likely to have a learning disability according to the LDDI?

Null Hypotheses

(7) There are no differences in mathematical problem-solving error type percentages as measured by the categories of computation, operation, translation, and no attempt on the *Informal Mathematics Assessment* between (a) students with MLD who received a score $\leq 25^{th}$ percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were likely to have a learning disability according to the LDDI and (b) students with Low Math/NLD who received a score $\leq 25^{th}$ percentile in Calculation, Math Fluency, or Applied Problems on the WJ III ACH and who were not likely to have a learning disability according to the LDDI.

The results for mathematics word problems that were solved by all of the students were analyzed. The percentage of problems correctly solved by students in each group was computed. The students with MLD correctly solved 24.1% of the problems. Students with Low Math/NLD correctly solved 53.91% of the problems. Results are shown in Figure 4.5.

Figure 4.5 Bar Graph of Percentage of Word Problems Correct of Students with MLD and Students with Low Math/NLD



An error analysis was also performed on the mathematical word problems. Each student's written work and verbal explanation were analyzed to determine the types of errors made by students in each of the groups. The percentage of error type demonstrated by students in each of the groups was computed and is shown in Table 4.11.

Table 4.11 Percentages of Error Types for Math Word Problems Attempted by Students with MLD and Students with Low Math/NLD

Error Categories	Types of Errors	Percentage of Error Type	
		MLD	Low Math/ NLD
Computation Errors	Non-Math Fact Errors	1.786	3.906
	Math Fact Errors	3.571	3.125
	Grouping Errors	1.786	.781
	Careless Errors	5.357	7.813
	Inverse Operation Chosen	1.786	4.688
	Simpler Operation Chosen	3.571	0
	Total Computation Errors	17.85	20.313
Translation Errors	Misinterprets Language in Problem	.892	2.344
	Irrelevant Rules or Inadequate Strategies	38.393	14.063
	Failure to Complete Problem	7.143	2.344
	Total Translation Errors	46.428	18.751
No Attempt	No Attempt Made	11.607	7.031
Correct	Correct	24.107	53.906

Error types were combined into categories (computation, translation, no attempt, and correct) and percentages were derived for each category (Babbitt, 1990; Jitendra & Kameenui, 1996; Radatz, 1979). The percentage of errors found in each category for students with MLD is shown in Figure 4.6. The percentage of errors found in each category for students with Low Math/NLD is shown in Figure 4.7. Students with Low Math/NLD had a greater percentage of problems correct than students with MLD did.

Figure 4.6 Math Word Problems Category Errors for Students with MLD

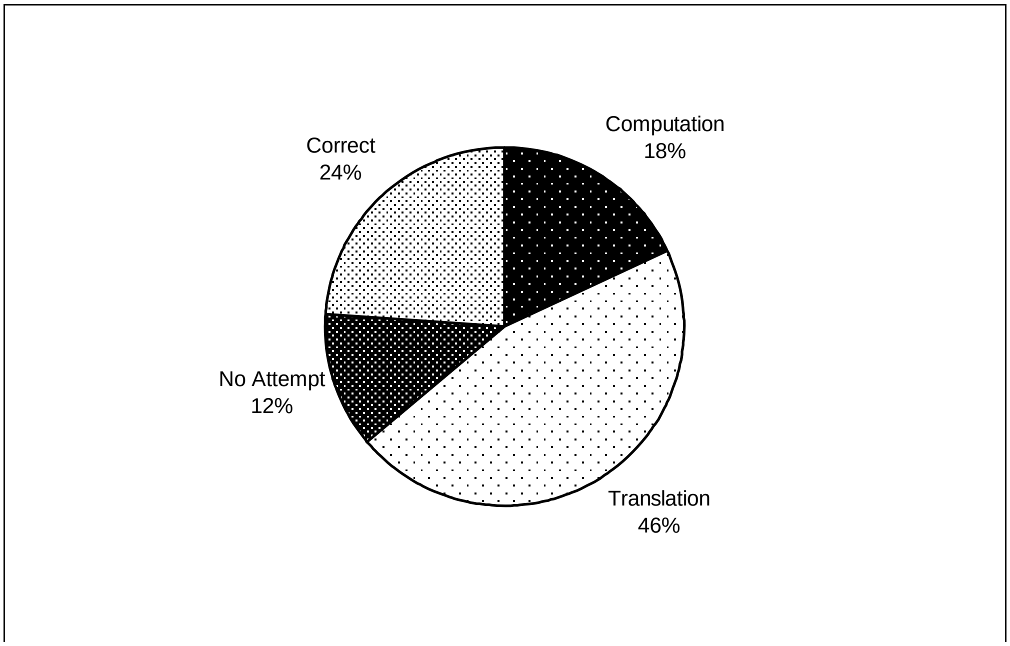
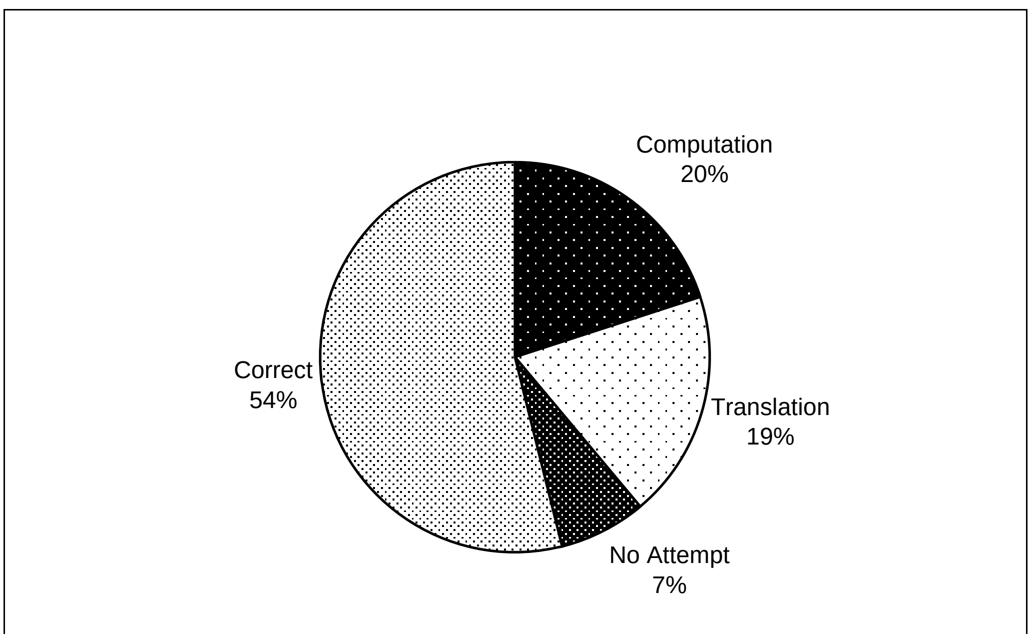


Figure 4.7 Math Word Problems Category Errors for Students with Low Math/NLD



Summary of Data

Using correlations among variables measuring student mathematics (Applied Problems, Math Fluency, and Calculation) and reading (Letter-Word Identification and Reading Fluency) achievement from the *Woodcock-Johnson III Tests of Achievement* and Receptive Language and Working Memory from the *Clinical Evaluation of Language Functions-4th Edition,* two underlying factors associated with solving mathematical word problems were identified. The factors were Abstract Thinking and Retrieval from Long Fixed Memory.

The results of this study showed that there were statistically significant differences in means between students with mathematics difficulties (subtest scores $\leq$ 25th percentile on Math Fluency, Calculation, or Applied Problems on the WJ III ACH) and likely to have a learning disability (MLD) and students with mathematics difficulties subtest scores ($\leq$ 25th percentile on Math Fluency, Calculation, or Applied Problems on the WJ III ACH) and unlikely to have a learning disability (Low Math/NLD) in the areas of automaticity (Math Fluency), which favored students with MLD, and Calculation and Applied Problems, which favored students with Low Math/NLD, as measured by WJ III ACH subtests scores.

In the area of Math Fluency, students with MLD (M =32.50, SD = 26.30) scored significantly higher than students with Low Math/NLD (M = 14.56, SD =12.08) did. The Independent Samples test was significant, t (17.723) =2.345, p =.031. The effect size for the d index was .9 which was large (Cohen, 1988).

In the area of Calculation, students with Low Math/NLD (*M* = 49.69, *SD* =21.42) scored significantly higher than students with MLD (*M* = 27.86, *SD* =16.92) did. The Independent Samples test was significant, *t* (27.743) = −3.114, *p* = .004. The effect size for the *d* index was 1.1 which was large (Cohen, 1988).

In the area of Applied Problems, students with Low Math/NLD (*M* =51.88, *SD* =16.49) scored significantly higher than students with MD (*M* = 33.50, *SD* =17.10) did. The Independent Samples test was significant, *t* (27.168) = −2.985, *p* = .006. The effect size for the *d* index was 1.1, which was large according to Cohen (1988).

There were also statistically significant linguistic differences in cognitive areas as shown by differences in means between students with mathematics difficulties (subtest scores ≤ 25th percentile on Math Fluency, Calculation, or Applied Problems on the WJ III ACH) and likely to have a learning disability (MLD) and students with mathematics difficulties subtest scores (≤ 25th percentile on Math Fluency, Calculation, or Applied Problems on the WJ III ACH) and unlikely to have a learning disability (Low Math/NLD) in the areas of Concepts & Following Directions, Word Classes-Receptive, and Number Repetition-Total as measured by CELF-4 subtest scores. Although students with Low Math/NLD scored higher than students with MLD on Familiar Sequences, the difference was not significant.

For the Concepts & Following Directions subtest, students with Low Math/NLD in Group 2 (*M* =10.47, SD = 1.88) scored significantly higher than Group 1 students with MLD (*M* = 6.07, *SD* = 4.12) did. The Independent Samples test was significant t(17.933) = −2.530, *p* = .002. The effect size for the *d* index was .9, which was large (Cohen, 1988).

For Word Classes-Receptive, students with Low Math/NLD in (*M* = 9.53, SD = 2.47) scored significantly higher than students with MLD (*M* = 7.21, *SD* = 2.75) did. The Independent Samples test was significant t (26.184) = −2.689, p = .025. The effect size for the *d* index was 1.0, which was large (Cohen, 1988).

For Number Repetition-Total, students with Low Math/NLD in (*M* = 8.80, SD = 2.24) scored significantly higher than students with MLD (*M* = 6.36, *SD* = 2.06) did. The Independent Samples test was significant t (26.996) = −3.057, p = .005. The effect size for the *d* index was 1.0, which was large (Cohen, 1988).

For Familiar Sequence subtest, students with Low Math/NLD in Group 2 (*M* = 9.80, SD = 2.21) did not score significantly higher than students with MLD (*M* = 7.36, *SD* = 2.37) did. The Independent Samples test was not significant t (26.464) = −2.863, p = .008.

There were statistically significant differences in means between students with mathematics difficulties (subtest scores ≤ 25^{th} percentile on Math Fluency, Calculation, or Applied Problems on the WJ III ACH) and likely to have a learning disability (MLD) and students with mathematics difficulties subtest scores (≤ 25^{th} percentile on Math Fluency, Calculation, or Applied Problems on the WJ III ACH) and unlikely to have a learning disability (Low Math/NLD) in the area of reading (Letter-Word Identification and Reading Fluency) as measured by WJ III ACH subtests scores.

The Letter-Word Identification mean score for students with Low Math/NLD in (*M* = 49.31, *SD* =24.68) was significantly higher than the mean score for students with

MLD (*M* =24.31, *SD* =20.10). The Independent Samples test was significant, *t* (26.997) =-3.007, *p* = .006. The effect size d index was 1.1, which was large (Cohen, 1988).

The Reading Fluency mean score for students with Low Math/NLD (*M* =37.25, *SD* =28.49) was not significantly higher than the mean score for students with MLD (*M* =30.38, *SD* =21.99). The Independent Samples test was not significant, *t* (26.950) =-.732, *p* = .470.

In an error analysis of the word problems, students with MLD had fewer problems correct than students with Low Math/NLD did. Students with Low Math/NLD made fewer translations and no attempt errors than students with MLD did. Although students with MLD made more overall errors, a smaller percentage of their errors were computational errors than the errors of students with Low Math/NLD. Students with Low Math/NLD had higher means on all achievement and linguistic subtests than students with MLD did with the exception of Math Fluency.

CHAPTER V

DISCUSSION

The purpose of this research was to determine the factors related to solving mathematical word problems and to examine the differences in characteristics between students with low achievement in mathematics who were likely to have a learning disability (MLD) and students with low achievement in mathematics who were unlikely to have a learning disability (Low Math/NLD). The study included examining the students' mathematics performance on the Math Fluency, Calculation, and Applied Problems subtests on the WJ III ACH; their reading performance on Letter-Word Identification and Reaching Fluency subtests on the WJ III ACH; and their linguistic performance on the Concepts & Following Directions, Word Classes, and Number Repetition subtests on the CELF-4; and their accuracy and the types of errors committed when solving mathematical word problems on the *Informal Mathematics Assessment* . Conclusions regarding students with low achievement in mathematics and with learning disabilities and students with low achievement in mathematics and without learning disabilities and the types of error patterns made during word problem solving can be drawn from the results.

Factors Related to Solving Mathematical Word Problems

Successful problem-solving requires adequate linguistic, computational, memory, and cognitive abilities (Engler, Culatta, & Horn, 1987; Parmar, 1992: Parmar, Cawley & Frazita, 1996; Rivera, 1997). The Informal Mathematics Assessment, which was based on solving actual mathematical word problems, involved understanding language, reasoning, and applying mathematics. The word problems were read to the student so that the student's answers would not be influenced by a student's lack of reading proficiency (Jitendra & Kameenui, 1996). In order to measure more accurately the student's actual mathematics problem-solving abilities rather than measure the student's reading or visual-spatial abilities (Fletcher, 1985; McLean & Hitch, 1999), the student was not required to read text or interpret diagrams. Students with Low Math/NLD had a higher percentage (53.9%) of correct answers to Word Problems than students with MLD (24.1%) even when the influence of reading was removed. Previous studies by Roark and Conway (1997) and Geary (1993) also found that students with low achievement in mathematics without LD had more answers correct when solving mathematics word problems than students with low achievement in mathematics with LD.

The ability to solve word problems appears strongly related to abstract thinking as shown by the fact that Word Problems loaded most heavily (.696) onto Abstract Thinking. Because comprehension, reasoning, and working memory (Swanson & Beebe-Frankenberger, 2004) are viewed as components of Abstract Thinking, it was not surprising that Working Memory also loaded heavily (.618) onto Abstract Thinking. Working memory conveys the idea of manipulating information and making that information available when conceptualization or abstract thinking is occurring in the process of problem solving. Swanson and Beebe-Frankenberger (2004) showed that at least 30% of the variance in problem-solving performance was accounted for by working memory. In this study, the category Working Memory was composed of two subtests from the CELF-4: Number Repetition Total and Familiar Sequences. Students with Low Math/NLD (*M* = 8.69, *SD* = 2.39) scored significantly higher than students with MLD (*M* = 6.36, *SD* = 2.06) on Number Repetition Total which was to be expected based on the work of Swanson and Beebe-Frankenberger (2004). However, there was no significant difference between the scores of the students with Low Math/NLD (*M* = 8.50, *SD* = 2.39) and students with MLD (*M* = 7.36, *SD* = 2.37) on Familiar Sequences or the scores of students with Low Math/NLD (*M* = 37.25, *SD* = 28.49) and students with MLD (*M* = 30.38, *SD* = 21.99) on Reading Fluency. Both tests were timed. The other subtest that was timed was Math Fluency. This timed subtest was the only subtest on which students with MLD (*M* = 32.50, *SD* = 26.30) scored significantly higher than students with Low Math/NLD (*M* = 14.56, *SD* = 12.08). The difference between the three timed subtests was that language played a greater role in Familiar Sequences and Reading Fluency than in Math Fluency (Woodcock & Mather, 2001).

The importance of language also appeared in Factor 1. With Factor 1 working memory (Swanson, Cochran, & Ewers, 1990) might have been a part of the factor but more of the factor was devoted to the conceptualization of ideas conveyed in language words and symbols or in abstract thinking when problem solving.

The word problems were read to the student so no reading was required (Jitendra & Kameenui, 1996) although text was provided if the student wished to look at it. Subtests from the WJ-III ACH and the CELF-4 were selected because their measures should have reflected the abilities required for successful problem solving. Applied problems also involved having the student listen as word problems were read aloud. In addition, Applied Problems required the student to look at and interpret pictures and diagrams to solve the mathematics problem (Woodcock & Mather, 2001). Applied Problems loading for Factors 1 and 2 were .661 and .239 respectively. Receptive Language and Working Memory loaded most heavily onto the first factor .550 and .618 respectively. All of these subtests required a considerable amount of conceptualization, analyzation, and reasoning on the part of the student to complete the tasks. It was therefore concluded that the Factor 1 was described as abstract thinking.

Reading Fluency loaded most heavily onto Factor 2 (.921) as did Math Fluency (.584). Initially one might surmise that the second factor was related to automaticity and speed because the student had unlimited time on all other subtests with the exception of Working Memory. However, because Working Memory, which also had a processing speed component, loaded so little (.198) onto Factor 2, it was unlikely that Factor 2 was related to processing speed. Reading Fluency, Letter-Word Identification, and Math Fluency all required the student to read either words or mathematical symbols (Woodcock & Mather, 2001). Therefore, Factor 2 was related to some extent to the language recognition of words and symbols. Reading Fluency required reading simple words in simple sentences. Math Fluency required reading basic mathematics facts and writing down the answer. If the student had the basic mathematics facts memorized, there was little reasoning required to reach a solution, and the response process should be similar to that of reading a sight word—a mere retrieval from long fixed memory (Fuchs, et al., 2006). Letter-Word Identification required reading words in isolation. Initially the task required the student to recognize and name sight words. As the words became more difficult, the student used phonics or other strategies to decode words that were not in the student's reading vocabulary. The task of decoding required more reasoning than the task of just identifying the known sight words. Because Letter-Word Identification required two major skills, it loaded similarly onto both Factors 1 and 2 (.410 and .431). Reading Fluency, Math Fluency, and Letter-Word Identification all required retrieval of information from long fixed memory (Swanson & Beebe-Frankenberger, 2004). However, when Letter-Word Identification began requiring the student to apply phonics skill to decode unknown words, the abilities of analyzation, reasoning, and abstract thinking were needed (Rourke, 1991). Word Problems and Working Memory did not require the student to look at anything in order to complete their task, and, therefore, visual recognition of words or symbols from long fixed memory was not required. The student only had to understand information provided auditorially. Therefore, there was a low loading of Word Problems and Working Memory respectively .147 and .198 onto the factor of retrieval from long fixed memory.

The factor of abstract thinking (.696) appeared to be more crucial to solving Word Problems than retrieval from long fixed memory (.147) especially because the problem was read to the student. Retrieval from long fixed memory and the recognition of words and symbols were more important in solving mathematical word problems when pictures and diagrams were presented to the student (Jitendra, 2002) than they were in Applied Problems (.239). When computational mathematics problems were written, the loading on retrieval from long fixed memory increased because students were required to work with mathematical symbols that had been previously memorized. Scores from Calculation (.252) and Math Fluency (.584) support this conclusion.

Factor 1 was identified by considering the categories that Word Problems (.696), Applied Problems (.661), Calculation (.653), Receptive Language (.550), and Working Memory (.618) had in common based on descriptions of the subtests in the manuals of the CELF-4 and the WJ III ACH. Those categories were Analyzing/Reasoning and Language Comprehension. Calculation, according to the descriptions of the subtest, was not designed to measure abilities in language comprehension, although an ability in language comprehension was actually required to complete successfully all subtests that contained written information in the form of words or mathematical symbols (Montague, 1992) as well as information supplied verbally. Because all of the subtests provided information in either written or verbal form, they measured to some degree a student's language comprehension abilities. Similarly, all the subtests measured to some extent the student's ability to analyze and reason. With Letter-Word Identification the student was expected to identify the words through recall from long-term memory, but the student was actually using analysis and reasoning to identify some of the words through phonics or association with words with similar spelling patterns. It was assumed in Math Fluency that the student would use recall from long fixed memory to identify mathematics facts, however, students do not always have mathematics facts memorized and may rely on other methods that require reasoning to figure out the answer. The result was that all the subtests measured to some extent the ability found under the Analyzing/Reasoning category. Therefore, Factor 1 needed to be a factor that included both

Analyzing/Reasoning and Language Comprehension. Abstract thinking appeared to fit this requirement and was identified as Factor 1.

There were other reasonable choices to consider for naming the two factors. Factor 2 retrieval from long fixed memory might have been identified as long-term memory or as automaticity. The choice of automaticity was rejected because automaticity encompasses speed and accuracy. For Factor 2 speed did not appear relevant because students had as much time as needed. This is not to say that automaticity does not play an important role in problem solving. Throughout the assessments, students displayed numerous examples of incorrect answers due to lack of automaticity of basic mathematics facts (Fuchs et al., 2006; Geary, Hoard, & Hamson, 1999).

Students with MLD and Students with Low Math/NLD

For this study the term mathematics difficulties referred to students who had at least one standardized mathematics achievement score (Applied Problems, Calculation, or Math Fluency) that fell at or below the 25th percentile. The students with and without learning disabilities were so designated based on their LDDI profile. On the *Woodcock-Johnson III Tests of Achievement* there was a significant statistical difference between the scores of the two groups on Applied Problems, Math Fluency, Calculation, and Letter-Word Identification. The group of students with low achievement in mathematics without learning disabilities (Low Math/NLD) had a higher mean than the group of students with low achievement in mathematics with learning disabilities (MLD) on all of the WJ III ACH subtests except Math Fluency. On Math Fluency students with MLD had a statistically significant difference and a higher mean than students with Low Math/NLD did. Reading Fluency was the one WJ III ACH subtest on which there was not a significant statistical difference between the scores of the two groups. Interestingly, it appears that the students with Low Math/NLD had greater difficulties in fluency than the students with MLD did. On Math Fluency 15 of 16 students with Low Math/NLD received a score $\leq 25^{th}$ percentile compared to 7 of 13 students with MLD. A possible explanation for the difference might be that students with MLD have spent more time practicing basic math facts than students with Low Math/NLD. This possibility is supported by a study measuring regression of computation skills during summer break. In the study, Allinder and Fuchs (1991) surmised that students with LD did not regress significantly during the summer because they tended to stay on a particular skill until mastery during the school year. Because the subtest was timed, neither group had time to check for careless errors. On the Error Analysis, students with Low Math/NLD had a higher percentage of total computational errors than students with MLD did (15.625 and 12.500), students with Low Math/NLD had more careless errors than students with MLD did (7.813 and 5.357), and students with Low Math/NLD had more non-math fact errors than students with MDLD did (3.906 and 1.786). Computational errors are more likely to be the result of excessive speed and lack of accuracy than the other categories. Translation errors reflect errors in abstract thinking (Jitendra, Kameenui, 1996). Students with MLD had a greater percentage of total translation errors than students with Low Math/NLD did (46.428 and 18.751). The greater number of translation and no attempt errors made by students with MLD supports previous research by Woodward, Monroe, and Baxter (2001) which found that even with intervention students with MLD would continue to say, "I don't know" or attempt to solve the word problem using irrelevant information. Overall, students with Low Math/NLD had a greater percentage of answers correct than students with MLD did (53.906 and 24.107). When solving mathematical word problems, students with Low Math/NLD had more correct answers and more computational errors than students with MLD did. Students with MLD had fewer correct answers and more translation errors than students with Low Math/NLD did.

It appears that based upon their performance in solving mathematical word problems students with MLD form a group with distinct characteristics from students with Low Math/NLD. Students with Low Math/NLD have an overall higher mathematical performance and tend to have fluency problems. Students with Low Math/NLD would be good candidates for intensive instruction in basic math facts memorization and the development of self-regulatory skills so that they could monitor their own behavior and careless errors. Students with Low Math/NLD appear to be students that with treatment could be expected to perform eventually at grade level (Geary, 1993). Students with MLD have deficits in areas of analyzation, reasoning, and abstract thinking. These areas are much more difficult to remediate. In addition to their performance in problem solving, differences between the two groups in abstract thinking were also demonstrated by their scores on the *Clinical Evaluation of Language Fundamentals-Fourth Edition* (CELF-4). There was a statistically significant difference between the two groups' subtest scores on Concepts & Following Directions, Word

Classes, and Number Repetition. These three subtests are all related to language. Examining the means for the two groups showed that the means for all of the CELF-4 subtests fell within average range for students with Low Math/NLD while none of the means fell within average range for students with MLD. Students with learning disabilities have processing deficits in the area of language (IDEA, 2004). Low scores on the CELF-4 subtests would be anticipated for students with learning disabilities. Because students with MLD have low language scores they demonstrate an area of weakness in language which is indicative of students with LD (Jitendra, 2002).The students with Low Math/NLD did not demonstrate the weakness in language often seen with students with LD. The difficulties that the students with MLD show in the areas of language and abstract thinking could be due to processing disorders.

Limitations

Limitations to this study are noted. The battery of tests including the *Informal Mathematics Assessment* that was given to 85% of the students was administered by one certified diagnostician who was also the primary researcher for the study. However, this limitation was addressed by audio recording all students' responses for the *Informal Mathematics Assessment*. A copy of 20% of the participants' audio recordings and their responses on paper were provided to a second researcher who was also a doctoral student in education. Interrater reliability was 95%. The advantage of having one person examine almost all the students was in the consistency it provided. Differences in personalities and styles of establishing rapport with students did not need to be considered. With the audio recording the second examiner was able to verify interpretation of student responses.

Future Research

A future study could include a comparison of native English and native Spanish speakers to determine if the same results are found regardless of native language when examining the relationship of language to solving mathematical word problems. When this study began in the schools, teachers, administrators, and parents were disappointed that students whose native language was not English could not be included in the study. They were very anxious that their Spanish-speaking students be included in any follow up study that included non-native English speakers. There is concern amongst public school staff that these students experience difficulties in school and yet there seems to be a paucity of valuable assessment information available that can be used to assist teachers with providing appropriate instruction to these students. This study could be replicated with a Spanish speaking population to determine whether the results extend across languages or are relegated to just an English speaking population. At the time that the study began, the *Woodcock-Johnson III Tests of Achievement* was available in English and Spanish but the *Clinical Evaluation of Language Fundamentals-Fourth Edition* (CELF-4) was not. The CELF-4 is now available in Spanish so a follow-up study could easily be conducted in Spanish.

Math Fluency was the one subtest with a significant difference between students with MLD and students with Low Math/NLD. Geary (1993, 2003) favoring students with Low Math/NLC. A follow up study should include an actual reading comprehension assessment. Based on the results of the reading comprehension assessment by a student with MLD, the student should be assigned to (a) a group of students with MLD whose reading comprehension score $\leq$ 25th percentile or (b) students with MLD whose reading comprehension score is > 25th percentile. This assignment would allow the two groups to be compared similar to the previous studies of Rourke (1989; 1991; 1998) and Jordan and Hanich (2000) who explored co-morbid disabilities for students with LD with weaknesses in mathematics and reading. Those studies selected students as having LD based on achievement scores whereas this future study would identify students as having LD based on the results of the LDDI or a similar measure.

During the clinical interviews of the *Informal Mathematics Assessment* some students, upon additional probing, indicated that they did not know why they were following a certain procedure but only that it was what they thought the teacher wanted. This tendency by the students to do what they thought the teacher wanted them to do promoted the misapplication of learned rules when they were presented with novel problems in the assessment situation. Other students said, “I don’t know” so that they could move on to the next problem. In the area of teacher education, a future study could incorporate the training of teachers in the use of the interview and error analysis. The training procedure would be similar to that of the instruction of miscue analysis for reading. Teachers’ behavioral changes would be measured as would student progress with before and after interviews and error analysis.

Implications of the Results

This study has similarities to previous studies (Fuchs, et al., 2006; Kavale et al., 1994; Ysseldyke et al., 1982) that have looked for quantitative differences between students identified with mathematics learning disabilities and those who had low achievement in mathematics without learning disabilities. Difficulties the previous studies encountered were that the groups of students labeled LD were not necessarily comparable (Kavale et al., 1994; Ysseldyke et al., 1982). In previous studies students were placed in LD groups based either on identification by the local school district or by a low mathematics achievement score (Morris et al., 1994). This meant that when groups labeled LD were compared between studies the students within the groups were not derived from the same sample population of students or with the same criterion reference point. Some of the sample was composed of students who were merely low achieving and others who met local school district criteria as learning disabled. Because the local school district criteria for learning disabilities varies between school districts as well as between states, the samples of students identified as LD by school districts vary as well (Mercer, King-Sears, & Mercer, 1990; Zigmond, 1993) and may not be comparable. The *Learning Disabilities Diagnostic Inventory* (LDDI) provides an opportunity for researchers to place students into groups using a method other than just low achievement scores (Hammill & Bryant, 1998). The group of students with LD will have been selected for having indications of mathematics learning disabilities and compared with a group of students, who are without indications of mathematics learning disabilities but who have low achievement in mathematics. Because these students are categorized based upon the characteristics of MLD that they display, it seems reasonable to anticipate that these groups would show a statistically significant difference in mathematics scores for some of the variables. If the LDDI can potentially identify students as LD, it might be possible to move from an assessment that includes formalized testing with IQ tests and standardized achievement tests to an informal assessment that includes the LDDI, student portfolio, teacher ratings, and criterion reference tests such as those given by states. The interest in replacing the IQ-discrepancy model with the response to intervention (RTI) model has once again made identification of students with learning disabilities an area of concern (Fletcher, Coulter, Reschly, & Vaughn, 2004; Kavale, Holdnack, & Mostert, 2006). RTI has three tiers of instruction for students (Vaughn & Fuchs, 2003). RTI is predicated on providing early interventions to all students who are not successful with the unmodified instruction provided in the general education classroom (Fletcher et al., 2004). Becoming eligible for special education services through using the RTI model requires time as the student proves his/her nonresponsive to the instruction provided at the Tier 1 and 2 levels. If the LDDI could be incorporated as a component of school-wide screening processes currently being put into place as a means of identifying students who are at-risk of failure (Fletcher et al., 2004), students could be potentially considered as likely to have a learning disability based on the combination of their low achievement, the results of the LDD, and other assessments.

The LDDI has proven to be an effective instrument in identifying students with processing disorders indicative of learning disabilities. Because the LDDI examined areas of reasoning, listening, reading, writing, and speaking as well as mathematics, it appeared to be sensitive to the difficulties with abstract reasoning and language that the students with low achievement with learning disabilities displayed**.** Use of the LDDI may be an effective measure in preventing under and over identification of students with MLD coupled with other measures such as progress monitoring data to reflect student response to intervention.

Under identification denies services and assistance to students with MLD. Without the additional resources and instruction designed to meet their needs, students with MLD are prevented from experiencing school success. Students with MLD may not experience school success if they are provided with intervention that is responsive to their needs (Lyon et al., 2001). Under identification of students with MLD is made likely because of current eligibility practices. Current eligibility practices do not require the presence of an intrinsic processing disorder in either the discrepancy formula or RTI models in order to determine that a student has a learning disability and is eligible for special education services (Fletcher et al., 2004, Frankenberger & Fronzaglio, 1991). By not using the possession of an intrinsic processing disorder as a means of identification, it is likely that students with an intrinsic processing disorder will not always be identified as having a learning disability, and due to their lack of identification, these students with MLD will go without special services. Due to misidentification, students with MLD may be seen as merely low achieving and placed in remedial mathematics programs whose instruction is aimed at students with low achievement without LD. Because these programs do not provide the type of unique and individualized instruction required by students with MLD, students with MLD will be unable to obtain the achievement levels acquired by students with MLA for whom the remedial mathematics programs were designed (Geary, 1993; Fuchs & Fuchs, 1998). The progress and success in school of these students with MLD will be limited without the needed appropriate instruction.

Over identification presents the opposite problem. Over identification allocates funds to students who do not have MLD but may only have MLA. These students with MLA may only be in need of remedial instruction that can be provided through general education. Students with MLA do not need the specific and different instruction individualized to meet the unique needs of students with LD and provided through special education (Fuchs & Fuchs, 1998; Kavale, Fuchs, & Scruggs, 1994). Students with MLA do not need to receive special education services, which are almost twice the cost of an education provided through general education (Parrish, 1995).

With the use of the LDDI which uses teacher ratings that we know have a high reliability of accuracy (Gresham, MacMillan, & Bocian, 1997), the inclusion of a processing deficit is included into the mix of components needed for identification of a learning disability. RTI relies heavily on achievement levels and student response to appropriate instruction whereas the discrepancy formula examines achievement and IQ. Neither of these models takes into consideration whether the student possesses a processing deficit which is thought to be indicative of a learning disability (Ginsburg, 1997).

RTI is based upon providing scientifically based interventions expected to improve the skills of the student who is low achieving (IDEA, 2004). The intervention most often provided for students with low achievement in mathematics is in the area of basic mathematics facts or automaticity. The results of this study support previous research indicating that automaticity or mathematics fluency is an area of weakness for students with MLD (Geary, 2004). The students with MLD in this study obtained a mean percentage of 32.5 on the Math Fluency subtest of the WJ III ACH. Students with Low Math/NLD received an even lower mean percentage of 14.56 on the Math Fluency subtest of the WJ III ACH. Previous research (Fuchs, 2005; Geary, 1993) indicates that with intensive intervention in mathematics fluency and computation students with Low Math/NLD will show improvement in their mathematics skills while students with MLD will demonstrate little or no improvement. This lack of responsiveness to the intervention is the method that RTI uses to identify students with MLD whose lack of improvement is assumed to be due to their processing deficits (Swanson & Beebe-Frankenberger, 2004). Because both students with MLD and students with Low Math/NLD need to improve their mathematics fluency, this skill area might be a reasonable initial choice for an area for intervention.

However, a student’s improvement in mathematics fluency does not guarantee an improvement in the ability to solve mathematics word problems. With the acquiring of mathematics fluency, it is expected that students with Low Math/NLD should improve in their ability to solve mathematics word problems because more working memory is available that can be used for problem solving (Swanson & Beebe-Frankenberger, 2004). Unlike students with MLD, students with Low Math/NLD follow a typical developmental pattern (Geary, 1993). Even if students with MLD should experience some improvement in mathematics fluency, it is hypothesized that they will continue to demonstrate difficulties in mathematics word problem solving due to their processing deficits which affect conceptual understanding, sequential processing, language processing (Miller & Mercer, 1997; Rourke & Conway, 1997), working memory, and cognitive processing (Geary 1993; Swanson, 1993) especially if intervention is not provided in word problem solving.

This study demonstrated the difference in language abilities between students with MLD and students with Low Math/NLD as shown by their CELF-4 subtest scores. All of the CELF-4 subtest scores of the students with Low Math/NLD fell within average range (a stanine score of 8-12 is average) whereas all of the CELF-4 subtest scores for the students with MLD were below average. Because mathematics word problem solving is so interconnected to language, the performance of the students with MLD when solving mathematics word problems is greatly impacted as shown by a mean percentage score of only 24.17 correct answers on the *Informal Mathematics Assessment* compared to students with Low Math/NLD who answered over half the problems correctly (mean percentage score of 53.906.) The vocabulary of mathematics is often difficult for students with MLD as shown by the percentage of translation errors they made when solving mathematics word problems (46.428). Mathematics vocabulary requires students to understand the meaning of words as well as mathematical symbols (Wiig & Semel, 1984) especially for mathematics word problems. Successful problem solving requires more than just having the student translate words into mathematical symbols. Students must recognize the pattern and type of procedure required to solve the mathematical word problem. With intensive and carefully structured instruction, students with MLD have learned to solve four basic types of one-step arithmetic problems (Jitendra, 2002; Jitendra & Kameenui, 1996).

Multi-step mathematical word problem-solving is much more complex than one-step arithmetic problems and provides the opportunity for the misapplication of irrelevant rules or inadequate strategies. On the *Informal Mathematics Assessment,* students with MLD had a percentage of error of 38.393 due to the application of irrelevant rules or inadequate strategies compared to students with Low Math/NLD with 14.063. Students with MLD tended to select procedural rules with which they were familiar when solving word problems even when the rules did not apply to a particular problem. Students with MLD treated each problem that was slightly different from the pattern with which they were familiar as a novel situation. They were unable to generalize the procedures that they already knew to what they perceived to be a novel situation.

Because previous studies have concentrated on studying mathematics calculation or mathematics operations rather than word problems, this study will add to the small but growing volume of studies on word problem solving (Jitendra & Xin, 1997; Montague, 1992). Past studies, which have focused on mathematical word problems, were usually intervention studies and measured the number of correct answers. This study focused on the problem solving behaviors which students with and without MLD utilized when faced with solving a mathematical word problem. This information was gathered not just through observation but also through student interviews, which is a type of assessment infrequently used (Woodward & Baxter, 1997) even though it is quite effective.

This study also provides implications for the classroom teacher. During interviews students explained their reasoning behind the steps they took in solving the problems. Sometimes their series of errors just happened to generate a correct answer for a specific problem. When they generalized their error-ridden procedure that happened to produce a one-time correct answer to a new problem, they, of course, were mystified as to why their answer to the new problem was incorrect. Looking only at their written work, it would be near impossible for their teacher to recreate the rationale behind their process without the help of their thoughts (Woodward & Baxter, 1997). Instead of teachers only marking answers either correct or incorrect, by using the interview process and error analysis, the classroom teacher could be trained to analyze and provide remediation in much the same fashion as teachers already use miscue analysis for reading (Woodward & Baxter, 1997).

In this study the relationship between language and solving mathematics word problems was seen to be tightly woven as shown by the scores on the CELF-4 and the *Informal Mathematics Assessment*. The poor performance of students with MLD on the *Informal Mathematics Assessment* (percentage of problems correct 24.107) supports the concerns of educators (Woodward & Baxter, 1997; Woodward & Montague, 2002) regarding the adequacy of traditional instructional methods for students with MLD. Teachers who have been most successful at having their students understand the new concepts present the vocabulary explicitly with semantic maps or graphic depictions (Bryant, 2005). Content is methodically sequenced (Carnine, 1998; Swanson, 2001). Because of the complexity of the content and procedures especially at the middle school level for mathematics word problem solving, instruction requires careful scaffolding (Jitendra, DiPiPi, & Perron-Jones, 2002; Montague, 1997, 1998). Mathematics skills are broken down into their component parts and also taught with explicit instruction within small groups after ascertaining that the students possess the prerequisite abilities. For those students who do not have the prerequisite skills, additional instruction is provided by the teacher. This supplemental instruction is provided by general education teachers within the general education classroom under Tier 2 in the RTI model (Bryant, 2005). Some tasks may require instructional adaptations (Van Luit, 2000) such as the delivery of instruction (e.g. procedural and conceptual strategies should be taught with explicit instruction), instructional content (e.g., more instruction on requisite skills such as mathematics fluency), instructional activities (e.g. verbalizations of cognitive strategies), and instructional materials (representations of concepts presented visually), and monitoring of the student's progress (Bryant, 2005).

This study will benefit the field of special education because it provides more information for answering the question: Is there a statistically significant quantitative difference between students who have low achievement in mathematics without indications of learning disabilities and those students who have low achievement in mathematics with indications of mathematics learning disabilities as recognized by teachers.

Summary

This study focuses on differences in solving mathematical word problems between students with low achievement in mathematics with learning disabilities (MLD) and students with low achievement in mathematics without learning disabilities (Low Math/NLD) by comparing their performances in mathematics, language, reading, and problem solving. This information was gathered through WJ III ACH and CELF-4 subtests and through interview and error analysis of their solving mathematical word problems provided by the *Informal Mathematics Assessment*.

This research study examined the characteristics of students with MLD and students with Low Math/NLD and the processes and strategies they used to solve mathematical word problems and their use of language. This study sought to discover a way in which students with MLD could be distinguished from students with MLA specifically in the area of mathematics word problems as they relate to language.

The results showed that there are significant differences between students with MLD and students with Low Math/NLD in favor of students with Low Math/NLD. When solving mathematical word problems, students with Low Math/NLD had more correct answers, more computational errors, and fewer translation errors than students with MLD did. Compared to students with MLD, students with Low Math/NLD have more mathematics fluency problems but a higher overall mathematical performance.

Students with MLD have problems with translating language into symbols and operations. Students with MLD have conceptual difficulties in the areas of analyzation, reasoning, and abstract thinking. Their difficulties in the areas of language and abstract thinking could be due to processing disorders (Swanson, 1993).

Appendix A: Consent from Parents

IRB# 2004090032

Informed Consent to Participate in Research

The University of Texas at Austin

Your child is invited to participate in a study that examines how children solve math word problems. My name is Paula Hartman, and I am a doctoral student in the School of Education at The University of Texas at Austin. This study is a continuation of my examination on how children learn arithmetic and especially math word problems. I am asking for permission to include your child in this study because your child's teacher has volunteered to participate in this study. This study is being conducted in the 3rd, 4th, 5th, and 6th grades in Round Rock I.S.D. schools. I have asked your child's teacher to send permission forms home with some of the students in the teacher's class. I expect to have at least 200 participants in the study.

If you allow your child to participate, your child may be selected to be assessed by myself or another researcher, in one to three sessions. Not all children will receive all assessments, and your agreement to allow your child to participate doesn't guarantee that your child will be included in the study.

Please read the information below and ask questions about anything you don't understand before deciding whether or not to allow your child to take part. I am available to answer all of your questions or you may ask your questions of your child's teacher. Your child's participation is entirely voluntary and you can refuse to allow your child to participate at any time without penalty or loss of benefits to which you or your child are otherwise entitled.

Title of Research Study:

Student Approaches for Solving Math Word Problems

Principal Investigator(s) (include faculty sponsor), UT affiliation, and Telephone Number(s):

The Principal Investigator for this study is Paula Hartman who is a graduate student in the School of Education at the University of Texas. You may contact her if you have questions about the study or if you later on decide that you no longer wish your child to participate in the study. Her faculty advisor and the sponsor for this study is Diane Bryant who is the Associate Dean for Teacher Education and Student Affairs.

Paula Hartman 475-6571 (office phone) or 219-6852 (home phone)
Diane Bryant 471-3223 (office phone)

Funding source:

This study receives no outside funding. The University of Texas researchers are volunteering their time.

What is the purpose of this study?

The purpose of this study is to examine differences in the ways that children solve math word problems and use language. Your child's teacher can help us understand how children solve math word problems by providing us with information about your child. With the information we gather on your children's skill levels in reading and math; their performance on formal tests taken this year or in previous years; and their characteristics on the *Learning Disabilities Diagnostic Inventory*, we hope to gain a better understanding of what characteristics influence the way in which children attempt to solve math word problems. We will share what we learn with educators. We hope that this new information will help teachers improve their instruction and result in children learning even more in school each year.

What will be done if your child takes part in this research study?

Your child's teacher will be able to share information about your child with the researchers. The researchers will assess your child in the areas of math, reading and language. If you choose to allow your child to participate in this study you are giving your permission for your child's classroom teacher:

- to complete a *Learning Disabilities Diagnostic Inventory* on your child
and give these forms to the researchers
- to give your child's report card grades to the researchers.

You are giving permission for Round Rock I.S.D. to provide the researchers with your child's scores on the following tests:

- Texas Assessment of Knowledge and Skills (TAKS)

- Cognitive Abilities Test
- Otis Lennon School Ability Test
- Stanford Achievement Test
- Wechsler Tests
- Woodcock-Johnson III
- Iowa Basic Skills Test

You are giving permission for your child to participate, in one to three sessions at your child's school in a quiet, well-lit room that is free of distractions, with a researcher who is trained to give diagnostic assessments.

- During the first session which should take 15-25 minutes, your child will be assessed using the Broad Math and Reading subtests from the Woodcock-Johnson III Tests of Achievement.
- During the second session, your child will be assessed using Verbal Comprehension, Concept Formation, and Visual Matching from the Woodcock-Johnson III Tests of Cognitive Ability, which will take 15 minutes, if no intelligence composite score is available from your child's school records. Your child will also be assessed with the *Clinical Evaluation of Language Functions-4th Edition* which will take 30 minutes unless scores from the assessment are already available.
- During the third session your child will be assessed using the *Informal Mathematics Assessment* which will take 10-20 minutes.

All of the sessions with your child will be recorded on audio tape. After the audiotape has been transcribed, it will be destroyed.

What are the possible discomforts and risks?

There are no known physical risks to your child if you allow your child to participate in this study. The only known risk to your child that we are aware of is the possibility that your child's information might not remain confidential. To reduce this risk we will keep the information about your child and the audio tapes in a locked office, and only the study's researchers will be allowed to see information that identifies your child. If you wish to discuss the information above or any other risks your child may experience, you may call the Principal Investigator listed on the front page of this form.

What are the possible benefits to you or to others?

Your child's results on the L.D.D.I. will be shared with your child's teacher. This information will help your child's teacher select appropriate instruction for your child. The language and math assessments will give your child's teacher information on the strategies that your child uses to solve math word problems. The benefit to the student is instruction that is possibly more appropriate for the student and more likely to meet the student's needs whereas the risk is a very small chance that the student's confidential information may be seen by someone who is not authorized to see the information.

If you choose to take part in this study, will it cost you anything?

It will not cost you or your child anything to participate in this study.

Will you receive compensation for your participation in this study?

You and your child will not receive any kind of compensation if your child volunteers to participate in this study.

What if you are injured because of the study?

There is no physical risk to your child, if your child participates in this study.

If you do not want to take part in this study, what other options are available to you?

Participation in this study is entirely voluntary. You are free to refuse to allow your child to be in the study, and your refusal will not affect your current or future relationships with The University of Texas at Austin or Round Rock I.S.D.

How can you withdraw from this research study and who should I call if I have questions?

You can withdraw your child from this study at any time. If you have any questions or **if you wish to stop your child's participation in this research study for any reason, you should contact:** Paula Hartman at (512) 475-6571 (office phone) or (512) 219-6852 (home phone)**. You are free to withdraw your consent and stop participation in this**

research study at any time without penalty or loss of benefits for which you or your child may be entitled. Throughout the study, the researchers will notify you of new information that may become available and that might affect your decision for your child to remain in the study.

In addition, if you have questions about your child's rights as a research participant, please contact Clarke A. Burnham, Ph.D., Chair, The University of Texas at Austin Institutional Review Board for the Protection of Human Subjects, 512/232-4383.

How will your privacy and the confidentiality of your research records be protected?

You and your child's privacy and confidentiality will be protected by keeping all research records in a locked office. Only the researchers will have access to information identifying you and your child. **Any information that is obtained in connection with this study and that can be identified with your child will remain confidential and will be disclosed only with your permission. Your child's name and your name will not appear in any written or verbal report of this research project.**

Authorized persons from The University of Texas at Austin and the Institutional Review Board have the legal right to review your research records and will protect the confidentiality of those records to the extent permitted by law. If the research project is sponsored then the sponsor also have the legal right to review your research records. Otherwise, your research records will not be released without your consent unless required by law or a court order.

If the results of this research are published or presented at scientific meetings, your identity will not be disclosed.

Will the researchers benefit from your participation in this study

The researchers will not benefit from your child's participation in this study beyond publishing or presenting the results of the study.

You may keep the copy of this consent form.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. If you later decide that you wish to withdraw your permission for your child to participate in the study, simply tell me. You may discontinue your child's participation at any time.

You have been informed about this study's purpose, procedures, possible benefits and risks, and you have received a copy of this Form. You have been given the opportunity to ask questions before you sign, and you have been told that you can ask other questions at any time. You voluntarily agree for your child to participate in this study. By signing this form, you are not waiving any of your legal rights or your child's legal rights.

______________________________	______________________________
Printed Name of Child	Printed Name of Child's Teacher

____

Signature of Parent(s) or Legal Guardian Date

____

Signature of Investigator Date

With your permission, Information from the Informal Mathematics Assessment and the scores from the Woodcock-Johnson III, the Clinical Evaluation of Language Functions-4th Edition, and the Learning Disabilities Diagnostic Inventory will be shared with your child's teacher and the teacher's supervisor or principal. Your signature below indicates that you are agreeing to have the scores obtained in this study shared with Round Rock I.S.D. staff. (Please check the appropriate box.)

☐ __I consent to have information from this study shared with Round Rock I.S.D. staff.

☐ __I **do not** consent to have information from this study shared with Round Rock I.S.D. staff.

_________________________________ ___________
Signature of Parent(s) or Legal Guardian Date

_________________________________ **3rd 4th 5th 6th**

Name of School (Circle Child's Grade)

As a representative of this study, I have explained the purpose, the procedures, the benefits, and the risks that are involved in this research study:

____ _____

Signature and printed name of person obtaining consent Date

____ _____

Signature of Principal Investigator Date

Please sign and return this page to your child's classroom teacher.

Paula Hartman 475-6571 (office phone) or 219-6852 (home phone)
School of Education, The University of Texas-Austin

Appendix B: Student Assent

B. Assent form for child between the ages of 7 and 12.

ASSENT FORM

Solving Math Word Problems

I agree to be in a study about solving math word problems and the way children learn. This study was explained to my parents/guardians and they said that I could be in it. The only people who will know about what I say and do in the study will be the people in charge of the study and my parents and teachers.

For this study my ability and skills test scores and my grades on my report card will be collected and compared to information my teacher provides about me on a learning inventory. I give permission for my teacher to share this information with the researchers.

I will also participate in three testing sessions in my school in a quiet, well-lit room that is free of distractions, with a researcher who is trained to give tests. During the first session which should take 15-25 minutes, I will take some tests about math and reading. During the second session, I might take some ability tests that will take 15 minutes. I will also take a test about language, listening, and following directions that will take 30-

40 minutes. During the third session I will solve some math word problems that will take 10-20 minutes. I will be tape recorded for all three sessions.

Writing my name on this page means that the page was read to me and that I agree to be in the study. I know what will happen to me. If I decide to quit the study, all I have to do is tell the person in charge.

____ ____________________

Child's Signature Date

____ ____________________

Signature of Researcher Date

Appendix C: Word Problem Error Analysis

Error Categories	Error Types	Code	Descriptions
Computation Errors	Defective Algorithm	DA	Student uses correct operation but fails due to errors other than number facts.
	Inadequate Facts	IF	Student uses correct operation and strategy but applies inaccurate basic math facts.
	Incomplete Algorithm	IA	Student fails to complete operation.
	Grouping Error	GE	Student makes errors when regrouping is involved.
	Careless Error	CE	Student makes errors due to carelessness.
Operational Errors	Wrong Operation	WO	Student uses incorrect operation to solve problem.
	Inverse Operation	IO	Student subtracts instead of adding or multiplying or vice versa.
	Simpler Operation	SO	Student chooses a simpler operation than that required by the problem.
	Random Choice	RC	Student response is unrelated to the given problem.
	Immature strategies	IS	Student chooses operation based upon key words, size of numbers.
Extraneous Information Errors	EI	Student uses extraneous information to solve problem.	
No Attempt Error	No Attempt Error	NA	Student fails to attempt the problems.

Source: Muoneke 2001; Jitendra & Kameenui, 1996

Appendix D: Word Problems

Gail had 43 music CDs in her collection. Then she bought 11 more CDs at a garage sale. How many CDs does Gail have now?
(Jitendra & Griffin, 1998)

Some kids are playing a jumping game on the sidewalk. You can only jump on one leg, and its Michael's turn. Michael gets 6 points for every time he jumps in the cement square, and he loses 4 points every time he jumps on a crack between the squares. Michael jumped on 7 squares and 3 cracks. How many points does Michael get?
(Woodward, Monroe, & Baxter, 2001)

A teacher grades 10 of her students' tests every half hour. It takes her one and one half hours to grade all of her students' tests. How many students are in her class?
(TIMMS S4)

Jill needs to earn $45.00 for a class trip. She earns $2.00 each day on Mondays, Tuesdays, and Wednesdays, and $3.00 each day on Thursdays, Fridays, and Saturdays. She does not work on Sundays. How many weeks will it take her to earn $45.00?
(NAEP Questions #7)

A thin wire 20 centimeters long is formed into a rectangle. If the width of this rectangle is 4 centimeters, what is its length?
(TIMMS K7)

5th graders Kara and Rani both have lemonade stands. Kara sells her lemonade at 5 cents a glass and Rani sells hers at 7 cents a glass. Kara sold 17 glasses of lemonade today and Rani sold 14 glasses. Both of the girls also sold 10 cookies for 5 cents each at their stands. Who made the most money? How much did she make?
(Muoneke)

A store sells shirts for $13.50 each. On Saturday, it sold 93 shirts. This was 26 more than it had sold on Friday. How much did the store charge for all the shirts sold on both days?
(Montague, M., 1993)

Mark was offered a job downtown that would give him a raise of $78 a month over his current salary, but his commuting costs would be $2 a day higher. If he works 22 days a month, what would be his net monthly increase in pay?
(Muoneke)

Appendix E: Teacher Consent

IRB# 2004090032

Informed Consent to Participate in Research

The University of Texas at Austin

You are being asked to participate in a research study. This form provides you with information about the study. The Principal Investigator (Paula Hartman) or her representative will also describe this study to you and answer all of your questions. Please read the information below and ask questions about anything you don't understand before deciding whether or not to take part. Your participation is entirely voluntary and you can refuse to participate without penalty or loss of benefits to which you are otherwise entitled.

Title of Research Study:

Student Approaches for Solving Math Word Problems

Principal Investigator(s) (include faculty sponsor), UT affiliation, and Telephone Number(s):

The Principal Investigator for this study is Paula Hartman, a graduate student in the School of Education at the University of Texas. The sponsor for this study is Diane Bryant who is the Associate Dean for Teacher Education and Student Affairs.
Paula Hartman 475-6571 (office phone) or 219-6852 (home phone)
Diane Bryant 471-3223 (office phone)

Funding source:

This study receives no outside funding. The University of Texas researchers are volunteering their time.

What is the purpose of this study?

All teachers of the 11,000 students in the 3rd, 4th, 5th, and 6th grade in Round Rock I.S.D. whose primary language is English are invited to participate in this study. The purpose of this study is to study differences in the ways that children in the 3rd, 4th, 5th, and 6th grade solve math word problems and use language. (This study will focus on students whose primary language is English. A later study may look at students whose first language is Spanish.) You can help us understand how children solve math word problems by providing us with information about your students and their intrinsic processing characteristics as shown on the Learning Disabilities Diagnostic Inventory. With the information we gather on your students' skill levels in reading and math; their performance on formal tests taken this year or in previous years; and their characteristics on the Learning Disabilities Diagnostic Inventory, we hope to gain a better understanding of what characteristics influence the way in which children attempt to solve math word problems. We will share what we learn with educators. We hope that this new information will assist teachers in making instructional decisions.

What will be done if you take part in this research study?

If you choose to participate in this study you are agreeing to:
to complete a Learning Disabilities Diagnostic Inventory (LDDI) on student participants and to give these completed forms to the researchers. (You will **not** be asked to complete a LDDI on every student participating in the study. (Most teachers will only complete an LDDI on one or two students. Completing an LDDI takes 10-20 minutes of time for each student and can be completed at the teacher's convenience.)

What are the possible discomforts and risks?

There are no known physical risks to you if you choose to participate in this study. The only known risk to you that we are aware of is the possibility that your information might not remain confidential. To reduce this risk we will keep the information you provide to the researchers in a locked office, and only the study's researchers will be allowed to see information that identifies you. If you wish to discuss the information above or any other risks you may experience, you may call the Principal Investigator listed on the front page of this form.

What are the possible benefits to you or to others?

Individual student's results on the L.D.D.I. will be shared with teachers. This will provide teachers with additional information when selecting appropriate instruction to meet an individual student's needs. If there are students who are failing the TAKS, then with this information the district will be able to design a more accurate method of early identification of students at-risk for failing the TAKS for purposes of early intervention and prevention of failure. Accurate early identification should assist the district in reducing the numbers of students who are not promoted and will eventually reduce the district's dropout rate since a high proportion of students who are retained never graduate from high school. The results of the study may improve rates of graduation, reduce dropouts, and provide appropriate instruction to students thereby increasing the numbers of students who are not retained.

If you choose to take part in this study, will it cost you anything?

It will not cost you anything to participate in this study.

Will you receive compensation for your participation in this study?

You will not receive any kind of compensation if you choose to participate in this study.

What if you are injured because of the study?

There is no physical risk to you if you participate in this study.

If you do not want to take part in this study, what other options are available to you?

Participation in this study is entirely voluntary. You are free to refuse to be in the study, and your refusal will not influence current or future relationships with The University of Texas at Austin and Round Rock I.S.D..

How can you withdraw from this research study and who should I call if I have questions?

If you wish to stop your participation in this research study for any reason, you should contact: Paula Hartman at (512) 475-6571 (office phone) or (512) 219-6852 (home phone)**. You are free to withdraw your consent and stop participation in this**

research study at any time without penalty or loss of benefits for which you may be entitled. Throughout the study, the researchers will notify you of new information that may become available and that might affect your decision to remain in the study.

In addition, if you have questions about your rights as a research participant, please contact Clarke A. Burnham, Ph.D., Chair, The University of Texas at Austin Institutional Review Board for the Protection of Human Subjects, 512/232-4383.

How will your privacy and the confidentiality of your research records be protected?

Authorized persons from The University of Texas at Austin and the Institutional Review Board have the legal right to review your research records and will protect the confidentiality of those records to the extent permitted by law. If the research project is sponsored then the sponsor also have the legal right to review your research records. Otherwise, your research records will not be released without your consent unless required by law or a court order.

If the results of this research are published or presented at scientific meetings, your identity will not be disclosed.

Will the researchers benefit from your participation in this *study*?

The researchers will not benefit from your participation in this study beyond publishing or presenting the results of the study.

Signatures:

As a representative of this study, I have explained the purpose, the procedures, the benefits, and the risks that are involved in this research study:

____ ____________________

Signature and printed name of person obtaining consent **Date**

You have been informed about this study's purpose, procedures, possible benefits and risks, and you have received a copy of this Form. You have been given the opportunity to ask questions before you sign, and you have been told that you can ask other questions at any time. You voluntarily agree to participate in this study. By signing this form, you are not waiving any of your legal rights.

____

Printed Name of Subject **Date**

____

Signature of Subject **Date**

____

Signature of Principal Investigator **Date**

____

Printed Name of School **Grade**

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Vita

Paula Ann Hartman was born in Panama City, Florida in 1953, the daughter of Mary and Paul Lane. After completing her work at Jefferson High School, Lafayette, Indiana, in 1971, she entered Indiana University at Bloomington. She received the degree of Bachelor of Science in August 1975. Following her graduation, she taught first grade in the Lafayette School District in Indiana. After marrying Daniel Hartman, she attended Purdue University in West Lafayette, Indiana before returning to Indiana University to complete her Masters Degree in Special Education in August of 1977. Upon graduation she was employed as a high school special education teacher in Owen Valley School District (1977-1981) in Spencer, Indiana. After the birth of her three sons, the family moved to Portland, Oregon where she was employed as a special education/consulting teacher (1984-1987) and as a diagnostician (1987-2000) in Portland Public Schools in Oregon. She attended graduate school at Portland State University (1985-1987) and at Lewis and Clark College (1989). In 2000, the family relocated to Texas where she took a position as a secondary special education teacher in the Lago Vista Independent School District (1990-1991). For the next few years she worked as a freelance editor for an educational publishing company (1991-1994) and also taught writing classes at Austin Community College (1993). From (1994-1999) she was employed as a diagnostician by the Round Rock Independent School District. In 1999 she was accepted into the doctoral program in the Special Education Department at The University of Texas at Austin.

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