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THE EVOLUTION OF GALAXIES. II. CHEMICAL EVOLUTION COEFFICIENTS

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ABSTRACT

We present a detailed analysis of how stellar evolution determines the coefficients in theories of the chemical evolution of galaxies. With these coefficients, it is possible to make comparisons with observed abundances; these comparisons are insensitive to the history of the total stellar birthrate. From a detailed examination of numerical stellar evolutionary sequences we evaluate these coefficients. The agreement with observation is excellent.

Subject headings: abundances, stellar — galaxies — nucleosynthesis — stellar evolution

I. INTRODUCTION

In order to understand the evolution of galaxies we must understand the evolution of stars up to and through all stages of mass ejection. In this paper we develop a formalism which is applicable to perhaps all published models of galactic evolution. We will evaluate the coefficients in this formalism using recent stellar-evolution theory. Although a number of suggestive results are now available, the final state of stars is still sufficiently uncertain that we will emphasize primarily qualitative features and systematic behavior. It is extremely gratifying that prescriptions based on stellar-evolution models yield excellent *quantitative* agreement between observations and elementary models of Galactic history. There no longer exists a need to adjust the stellar-evolution picture to fit Galactic models.

The formalism described below is one which we have used in studies of galactic evolution (cf. Arnett 1971*a*, *b*; Talbot 1971; Talbot and Arnett 1971*a* [TA71]; Talbot and Arnett 1973; Arnett and Talbot 1974; Talbot 1973). An early version (I) used an amalgam of stellar-evolution theory of some three or so years ago to evaluate the necessary coefficients. This version was used in some of the older papers listed above, has been referred to by Quirk and Tinsley (1973) and Tinsley (1972), and was widely distributed in preprint form. At present we prefer a revised, more up-to-date version (II). When stellar-evolution and nucleosynthesis calculations currently in progress are completed, further revision and refinement should be possible. We present both versions for two reasons: first, to illustrate the surprisingly small changes which stem from the progress in stellar-evolution calculations; second, to precisely document the version I prescription, since the published references to it would be interpreted as this paper.

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In § II we describe our prescription for the composition of the ejecta of stars of different mass m and different initial composition.

Section III briefly discusses some aspects of chemical evolution in galaxies, especially in the solar neighborhood of our Galaxy. Emphasis is placed on those topics which are relatively independent of the history of the stellar birthrate. The results are summarized in § IV.

II. PRESCRIPTION FOR THE EJECTION OF NUCLEOSYNTHESIS PRODUCTS FROM STARS

a) *Formalism*

In TA71 we discussed in detail analytic and numerical solutions to a highly simplified astrophysical description of the end products of stellar evolution. The intent was to illustrate the high precision attainable with analytic solutions for Galactic evolution. Here we discuss the more realistic description of actual stellar-evolution end products.

The description of the evolution of the chemical composition of the gas in a galaxy involves a prescription for the fraction of a star of mass m which is ejected back into the interstellar medium in the form of chemical or isotopic species i, R_{mi}. In general, R_{mi} is a function of the initial chemical composition of the star. If we represent the abundance of species j in the interstellar gas by X_j, then provided the star-formation process does not select certain species over others, we may write

$$R_{mi} = \sum_j Q_{mij} X_j \,, \tag{1}$$

where the summation includes all species. The matrix Q_{mij} specifies the fraction of the mass of the star initially in the form of species j which is eventually ejected as species i. The *production matrix for a star* Q_{mij} is a function of m. In general the ij matrix element may depend upon the initial composition of the star, though most of the influence of composition upon R_{mi} is exhibited explicitly by the linear dependence upon X_j.

If one considers a generation of stars with an initial mass function (IMF) Ψ_m (normalized so that the integral over all stellar masses is unity), then the *production matrix for the generation* is

$$q_{ij} = \int_0^\infty \Psi_m Q_{mij} dm \,. \tag{2}$$

Some related quantities for which it is convenient to have separate symbols are:

$$f = \sum_{k,l} q_{k,l} X_l$$

$$= \sum_k q_{kl} \text{ (independent of } l\text{) if } Q_{mij} \text{ is independent of composition}\,, \tag{3a}$$

$$p_i = \Bigl(\sum_{j \neq i} q_{ij} X_j\Bigr) / (1 - f)\,, \tag{3b}$$

$$u_i = q_{ii}\,, \tag{3c}$$

and

$$\Lambda_i = (f - u_i)/(1 - f)\,. \tag{3d}$$

The quantity f is the fraction of mass ejected from a generation of stars; u_i is the fraction of species i which is ejected unchanged; and p_i is the *yield* of species i.

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A *primary* nucleosynthesis product is defined to be one with ^{1}H and/or ^{4}He as its progenitor—e.g., ^{12}C. If i is a primary heavy species, $q_{i1} = q_{i4}$ (we will use atomic weight to denote species whenever there is no chance of ambiguity). Since $X_1 + X_4 \simeq 1$ in essentially all cases of interest, it is convenient to use

$$p_i = q_{i1}/(1 - f) . \tag{3e}$$

The role of these coefficients easily appears in the very elementary model which assumes instantaneous recycling of the gas through stars (TA71), a time-invariant IMF, no mass inflow, and a well-mixed interstellar gas. In that case (cf. Searle and Sargent 1972, TA71, and Talbot and Arnett 1973)

$$d\mathscr{M}_g/dt = -\mathscr{B} + f\mathscr{B} , \tag{4a}$$

$$d\mathscr{M}_{gi}/dt = -\mathscr{B}X_i + \mathscr{B}\sum_j q_{ij}X_j , \tag{4b}$$

so

$$dX_i/dy = p_i - \Lambda_i X_i , \tag{4c}$$

where $\mathscr{M}_g$ is the mass of gas, $y = \ln[\mathscr{M}_g(0)/\mathscr{M}_g(t)]$, $\mathscr{B}$ is the rate of consumption of gas by star formation, and $\mathscr{M}_{gi}$ is the mass of gas in the form of species i. In this limiting model our yield p_i is the same as the more general quantity Searle and Sargent (1972) define as y. Their y includes time-delayed effects as older generations of star eject matter; consequently, it depends upon the history of the stellar birthrate. Our yield is defined for a single generation.

In more complicated models the more general quantities q_{ij} appear prominently. In comparing abundance ratios, it is the ratio of matrix elements of q_{ij} which is important, not their absolute size. *This latter statement remains true to a high degree of precision even when one lifts the restrictions on $\mathscr{M}_g/\mathscr{M}$, completeness of mixing, time invariance of the IMF, and mass inflow.*

b) Production Matrix for a Star (Q_{mij})

In this discussion we neglect the dependence of Q_{mij} upon the initial composition of stars. All mass loss is regarded as taking place at the end of the star's life. These simplifications probably do not cause serious error in most situations. The elements of Q_{mij} are specified in terms of the following quantities defined for a star of mass m: d is the mass fraction which remains as a stellar remnant upon the death of the star; q_c is the mass fraction within which material has been processed through helium burning into carbon/oxygen or to heavier nuclei; q_4 is the mass fraction within which material has undergone H $\rightarrow$ ^{4}He processing; q_3 is the mass fraction within which any original ^{3}He is converted to ^{4}He or heavier species; w_N is the mass fraction external to q_4 in which initial ^{12}C and/or ^{16}O is converted into ^{14}N; w_3 is the mass fraction ejected in the form of newly created ^{3}He; $w_c = q_c - d$ is the mass fraction which has been processed through helium burning and ejected; and χ_i is the mass fraction of w_c which is ejected in the form of the individual heavy species i.

Table 1 lists the elemental and isotopic species which we consider at this time, together with the adopted symbol for subscripts and the solar-system abundances we adopt for comparisons in § III.

Table 2 gives the prescription for the components of Q_{mij} which are not identically zero. For this investigation we have arbitrarily set $\chi_{CO}/\chi_h = 3$ which will automatically reproduce (approximately) the solar-system abundance ratio. This is the only parameter in Q_{mij} which has been chosen on the basis of observed abundances. Preliminary

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TABLE 1

NUCLEAR SPECIES CONSIDERED

Element(s)	Symbol	Solar Abundance Adopted from Cameron (1973)
Hydrogen	1	0.770
Deuterium	2	*3.85×10^{-5}
Helium 3	3	*1.60×10^{-5}
Helium 4	4	0.214
Carbon 12 and Oxygen 16	CO	1.17×10^{-2}
Nitrogen 14	N	1.23×10^{-3}
Heavier Species:		
Neutron rich	nr	4.2×10^{-4}
Remainder	h	3.8×10^{-3}

* Reeves *et al.* (1973) estimate $X_2 = (3.5 \pm 1.5) \times 10^{-5}$ and $X_3 = (2 \pm 1) \times 10^{-5}$.

stellar-evolution results seem to support this choice. All other parameters are based upon stellar models. Eventually the set of χ_i will be available from stellar-evolution and explosive-nucleosynthesis calculations.

For completeness we note that for cosmochronology calculations (e.g., Talbot 1973) we assume that r-process nuclei are produced in proportion to the heavy-element species h. The proper calculation of u_i for radioactive r-process species involves allowing for their decay while they are being stored in stars.

i) *Prescription of Q_{mij} (Version I)*

This prescription was based on our amalgamation of the work of Iben (1965; 1966*a*, *b*, *c*; 1967*a*, *b*), Paczyński (1970), Schwarzschild and Härm (1958), Deinzer and Salpeter (1964), and Talbot and Arnett (1971*b*). Specifically, we imposed the current view (Arnett and Clayton 1970; Arnett 1971*a*, 1973) that the dominant factor in the production of elements beyond ^{12}C is explosive nucleosynthesis in massive stars $m \gtrsim m_2 \simeq 8$ to 9 $M_\odot$ and the suggestion by Paczyński (1970) that stars in mass interval $m \lesssim m_1 \simeq 4$–5 $M_\odot$ lose only the material *above* the hydrogen-burning shell. We rather arbitrarily set to zero the primary production of ^{4}He and heavier species by stars for $m_1 \leq m \leq m_2$; that some enrichment of the ejecta might occur cannot be disputed at this time.

For stars more massive than m_2 we assumed that the mass fraction which has undergone complete hydrogen burning has had its entire initial CO content converted to N. In those zones which have been processed through helium burning, we assumed that the original CO and N become neutron-rich (nr) material (such as ^{18}O, ^{22}Ne, ^{23}Na, ^{25,26}Mg, and ^{27}Al [see Arnett and Clayton 1970; Couch and Arnett 1972]). The prescription for w_N was designed to approximate Iben's results. The prescription was based on an assumption that CO consists of 33 percent ^{12}C by mass. The specification of w_3 was composed from the results of Iben (1967*b*), with the equilibrium ^{3}He abundances decreased by a factor of $5^{1/2}$ to correct for the revised ^{3}He(^{3}He, $2p$)^{4}He cross-section (Winkler and Dwarakanath 1967). This specification assumed that all of the ^{3}He produced in Iben's models is eventually ejected from the star. We have no assurance that this does in fact happen.

We have assumed that deuterium (D) is not produced and ejected by any star, and that all D which forms the initial composition of a star of less than 2 $M_\odot$ is converted into heavier species. As an approximation for stars of greater than 4 $M_\odot$, we assume that all D outside q_3 is converted into ^{3}He. Inside q_3, ^{3}He achieves an equilibrium value independent of the initial D abundance, and the equilibrium abundance of D

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TABLE 2

DEFINITION OF THE NONZERO COMPONENTS OF THE Q_{mij} MATRIX

	PROGENITOR j							
PRODUCT i	1	2	3	4	CO	N	nr	h
1	$1 - q_4 - w_3$*							
2		w_2						
3	w_3	$1 - q_3 - w_2$	$1 - q_3$					
4	$q_4 - q_c$*	$q_3 - q_c$	$q_3 - q_c$	$1 - q_c$				
CO	$\chi_{CO} w_c$	$\chi_{CO} w_c$	$\chi_{CO} w_c$	$\chi_{CO} w_c$	$1 - q_4 - w_N$			
N					$w_N + q_4 - q_c$	$1 - q_c$		
nr					w_c	w_c	1 − d	
h	$\chi_h w_c$	$\chi_h w_c$	$\chi_h w_c$					1 − d

* See table 3 for alternate specification in the P mass interval.

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TABLE 3

PRESCRIPTION FOR PARAMETERS IN THE Q_{mij} MATRIX. VERSION I

Mass Interval Designation	Mass Interval	Specification of d and q's
W	$m \le m_1$	$d = \min(m_{wd}/m, 1)$ $q_c = q_4 = d$
1	$m_1 < m \le m_2$	$d = m_n/m$ $q_c = q_4 = d$
2	$m_2 < m \le m_p$	$d = m_n/m$ $q_c = 0.6f + (1 - f)m_n/m_2$ $q_4 = 0.2 + 0.45f$ where $f = \log(m/m_2)/\log(m_p/m_2)$
P*	$m > m_p$	$d = m_n/m$ $q_c = 10/m$ $q_4 = 0.74 - 0.09(m_p/m)^2$

For Full Mass Range

$q_3 = \max(d, 0.60)$

$w_N = 0.33 \max(0, 0.37 + 0.16 \log m - q_4)$

$w_3 = 4.7 \times 10^{-4} m^{-2} A,$

where $A = 0$	if $d \ge 0.7$,
$= 50(0.7 - d)^2$	if $0.6 \le d < 0.7$,
$= 1.0 - 50(d - 0.5)^2$	if $0.5 \le d < 0.6$,
$= 1.0$	if $d < 0.5$.
$w_2 = 0$	if $m \le 2$,
$= (1 - q_3)(m - 2)/2$	if $2 < m \le 4$,
$= 1 - q_3$	if $m > 4$.

* For this mass interval we assume that pulsational mass ejection causes helium to be ejected before the normal course of $H \rightarrow {}^4He$ reaches completion, according to the general principles discussed by Talbot and Arnett (1971*b*). Consequently, we modify the specification in table 2 to

$$Q_{1,1} = 1 - q_4 - w_3 + 0.75(q_4 - q_c) \quad \text{and} \quad Q_{4,1} = 0.25(q_4 - q_c)\,.$$

is effectively zero. A smooth interpolation is made for stars of mass between 2 and 4 $M_\odot$. This prescription is based upon the results of Bodenheimer (1966).

The prescription we developed from these considerations is given in table 3. The following values of the mass parameters were employed: $m_{wd} = 0.7\ M_\odot$ is the white-dwarf mass; $m_n = 1.4\ M_\odot$ is the neutron-star mass; $m_1 = 5\ M_\odot$ is the boundary between masses leaving white dwarfs and those leaving neutron stars; $m_2 = 9\ M_\odot$ is the boundary between masses which eject CO and heavier species and those which do not; and $m_p = 60\ M_\odot$ is the upper mass limit for pulsational stability in the main-sequence phase. If one wishes, "black hole" may be substituted for "neutron star."

ii) *Current Prescription of Q_{mij} (Version II)*

Instead of having the discontinuity in the stellar remnant mass at $m = m_1$, we now assume

$$d = m_r/m\,, \tag{5}$$

where

$$m_r = m \quad \text{if} \quad m \le 0.7\ M_\odot\,,$$

$$= 1.4\ M_\odot \quad \text{if} \quad m \ge 4\ M_\odot\,. \tag{6}$$

and m_r varies linearly with m in between.

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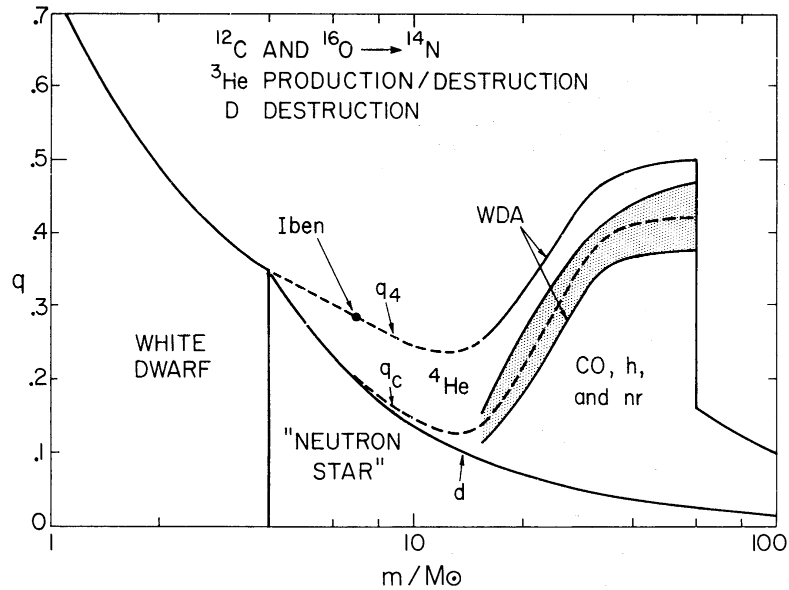

FIG. 1.—Stellar evolution parameters in the prescription for the production matrix Q_{mij}, Version II. The mass fraction d, q_4, and q_c are shown as functions of stellar mass m. Stars below 4 $M_\odot$ are assumed to leave a white dwarf remnant. All stars above 4 $M_\odot$ are assumed to leave a 1.4$M_\odot$ remnant which may be identified with neutron stars (or black holes). The values of the parameters are adopted using stellar models of Iben, Paczyński, and Arnett.

For the mass range 4 $M_\odot \leq m \leq 60$ $M_\odot$, the parameters q_c and q_4 are no longer approximated by analytic formula; interpolation between tabulated points is used. Figure 1 illustrates the adopted values based in part upon (*a*) calculations by Arnett (1972*a*, *b*), (*b*) the 7 $M_\odot$ model of Iben (1972), and (*c*) some interpolation for those aspects not explicitly given in the above.

Although details differ between Versions I and II, these produce only minor variations in the parameters which govern the chemical history of the Galaxy. Consequently, this suggests that the parameters which will result from current stellar-evolution calculations will not appreciably alter the discussion which follows.

In Figure 1 we show: (1) (*solid curves*) d from equations (5) and (6) and q_4 and q_c ($m > 15$ $M_\odot$) from Arnett (1972*a*, *b*); (2) (*shaded region*) the region in which shell-helium-burning products are convected; and (3) (*dotted curves*) the adopted values for q_4 and q_c. The placement of the q_4 curve has been strongly influenced by Iben's (1972) 7 $M_\odot$ model.

Our original prescription for the processing of CO into ^{14}N outside the hydrogen-burning shell was based on Iben's models which had not traveled up the giant branch for the second time. Recently Iben (1972) has published the envelope composition of a later 7 $M_\odot$ model which, upon reignition of the hydrogen shell, did additional processing of the envelope CNO species. Specifically, in that model the production of ^{14}N from ^{12}C and ^{16}O was about 1.6 times what our Version I prescription used. Owing to the uncertainty about the extent to which this single 7 $M_\odot$ model is representative of other masses and the uncertainty in the effects of subsequent evolution, it is not clear how one should prescribe $Q_{m,\mathrm{N,CO}}$.

We have employed two prescriptions. The first (denoted A) consists of simply adjusting the Version I prescription to match Iben's (1972) model at 7 $M_\odot$. The second (denoted B) is designed to provide an upper limit to $Q_{m,\mathrm{N,CO}}$; it consists of assuming that *all* CO exterior to q_4 is converted to N, in stars of all mass. There certainly exists

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no line of argument that this is true, but it does provide an upper limit which, with case A, should bracket the true situation. The two cases are:

$$\text{A:} \qquad w_{\rm N} = 0.33 \max\,[0.0, 0.56 + 0.16 \log m - q_4]\,;$$

$$\text{B:} \qquad w_{\rm N} = 1 - q_4\,.$$

It should be noted that we discuss here *only secondary* production of ^{14}N; that is, the production of ^{14}N via the CNO cycle from the *initial* ^{12}C and ^{16}O abundance in the star. We have not postulated the primary production of ^{14}N via CNO processing of ^{12}C or ^{16}O which is *produced* in the star itself. Should double-shell flashing occur in the manner supposed by Cameron and Fowler (1971), it may be necessary to introduce such primary production.

For case A we have not altered the prescriptions for the production and depletion of ^{3}He or the depletion of D. It is apparent, however, that the deep envelope convection in Iben's model which processes CO to ^{14}N will also deplete ^{3}He and D. Consequently, in the case B treatment of ^{14}N we assume that all initial D and ^{3}He are converted into heavier species. It is possible, for example, that some ^{3}He will be converted into ^{7}Li in a manner similar to that discussed by Cameron and Fowler (1971).

By these considerations we regard our case A prescription for ^{3}He production to be either (*a*) an upper limit to the true ^{3}He production or (*b*) the production for ^{3}He plus ^{7}Li. By the same considerations, the case A prescription for the depletion of D is most likely a lower limit to the actual depletion.

For mass above m_p we now adopt $q_4 = 0.50$; otherwise we use the prescription given in table 3. We now employ $m_1 = 4\,M_\odot$.

Although our prescription for Q_{mij} will be modified as stellar-evolution theory for late stages improves, we feel that the primary uncertainties are qualitatively understood. The most uncertain mass range is probably the interval $m = 4$–$15\,M_\odot$. This mass interval is a controversial one today for the pulsar, supernova, and explosive-nucleosynthesis theorists. Our discussions above suggest that it also may be the most important mass interval for understanding the production of ^{14}N and ^{3}He as well as the production of ^{7}Li and the *s*-process *a la* Cameron and Fowler (1971).

Currently we assume that this mass interval is not a significant source of primary heavy species as a whole. The adopted small mass interval between d and q_c for $m = 4$–$9\,M_\odot$ contributes only a few percent or less to the total when integrated over plausible initial mass functions. That it might be a significant source of some low-abundance species (for example, the *r*-process) cannot be disputed at this time.

c) The Production Matrix for a Generation of Stars (q_{ij})

i) *Choice of Initial Mass Function*

The computation of q_{ij} (eq. [2]) involves the initial mass function (IMF). The current solar-neighborhood IMF is the only one known with substantial reliance, though certainly not the only one of interest. This local IMF above about 3 $M_\odot$ may be found from the luminosity function of intrinsically bright stars. We adopt the luminosity function of McCusky (1966), stellar lifetimes from Iben (1967*c*) and Stothers (1966), and the M_v–m relation for luminosity class V stars in the list of Harris, Strand, and Worley (1963). We find that the IMF from $m \simeq 3$–$50\,M_\odot$ is well represented by a power law $m^{-\mu}$ with μ around 1.3–1.8. The uncertainty lies primarily in the M_v–m relation for bright stars. (For reference, the Salpeter [1955] IMF has $\mu = 1.35$.)

Below about 1 $M_\odot$ the observed luminosity function yields only the IMF integrated over all past time. *If* the IMF has been constant, this data may be used to ascertain the IMF. There is, however, the problem that it is difficult to obtain the faint end of the

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luminosity function. Consequently, it is not known what fraction of the IMF is made up of very-low-mass stars. Further, we feel that there is (1) no indication that the IMF has been constant in time, and (2) no compelling evidence that it has varied. For one alternative to Schmidt's (1963) conclusion that the IMF has varied, see Talbot and Arnett (1973) which discusses a model in which the star formation rate is enhanced by the presence of heavy elements in the interstellar gas.

Together with the luminosity function of McCusky (1966)—which is based in part on that of van Rhijn—we have incorporated the data on low-mass stars from Weistrop (1972). We will emphasize results for an IMF midway between her cases van Rhijn I and van Rhijn III. We will assume that the IMF has been constant in time. *Most of the results in this paper are expressed in such a way that they are independent of these uncertainties in the IMF, including being independent of time variations of the low-mass end of the IMF.*

The IMF between about 1 $M_\odot$ and 3 $M_\odot$ must be considered within the context of a specific model for the history of the solar neighborhood (e.g., Salpeter 1955, 1959; Schmidt 1959, 1963). For the purposes of the survey results discussed here, we will only consider the case where the power law $m^{-\mu}$ continues down to $m = 1\ M_\odot$. Specific models of the evolution of the solar neighborhood may require a radically different shape for the IMF in this interval. Our experience has indicated, however, that very satisfactory models may be made with this simple power-law form in this interval (e.g., Talbot and Arnett 1973; Arnett and Talbot 1973). In addition, it agrees reasonably well with the observed luminosity function of young clusters.

Because essentially all of the nonzero features of the Q_{mij} prescriptions are found at $m > 1\ M_\odot$, the ratios of all of the q_{ij} elements to one another are independent of the IMF below 1 $M_\odot$. Consequently, the q_{ij} depend upon the IMF via only the power μ and the constant ζ in the parametrization

$$\Psi_m = \zeta(\mu - 1)m^{-\mu} \qquad \text{for} \qquad m \geq 1\,. \tag{7}$$

The constant ζ is that fraction of the mass in the IMF which consists of stars of $m > 1\ M_\odot$. The lower portion of the IMF affects the chemical evolution of a galaxy through ζ only.

From the data presented by Weistrop (1972) we estimate that ζ(McCusky, without Weistrop's data) ~ 0.35, ζ(van Rhijn III) ~ 0.22, and ζ(van Rhijn I) ~ 0.27.

In the following we will present results assuming $\zeta = 0.25$. The reader with insight into the low end of the luminosity function may easily modify the results according to his ideas on ζ. Furthermore, the reader with insight on how the low end of the IMF may have varied with time or space will find that an excellent approximation will be achieved by assuming that q_{ij} is time dependent according to

$$q_{ij}(t) = [\zeta(t)/0.25]q_{ij} \quad \text{(this paper)}\,. \tag{8}$$

If in deriving the IMF there exists a misinterpretation of the numbers of high-mass stars compared with intermediate or low-mass stars, the principal effect would be to change q_{ij} through changing ζ.

ii) *Results*

Figure 2 shows the product $\Psi_m Q_{mij}$ (for Version IIA) as a function of m for certain species; $\zeta = 0.25$ and $\mu = 1.55$ were employed in this illustration. Note that the production of ^{3}He and N occur predominantly at about 1.6 $M_\odot$ and 2.5 $M_\odot$, respectively. The peak of the mass ejection (denoted by f) occurs at about 1.15 $M_\odot$.

Table 4 lists $q_{\mathrm{CO},1}$, $q_{\mathrm{N,CO}}$, and $q_{3,1}$ for Versions I and II (A and B) and for $\mu = 1.3$ and 1.8. (If the reader is interested in any q_{ij} for some other value of μ, it is generally

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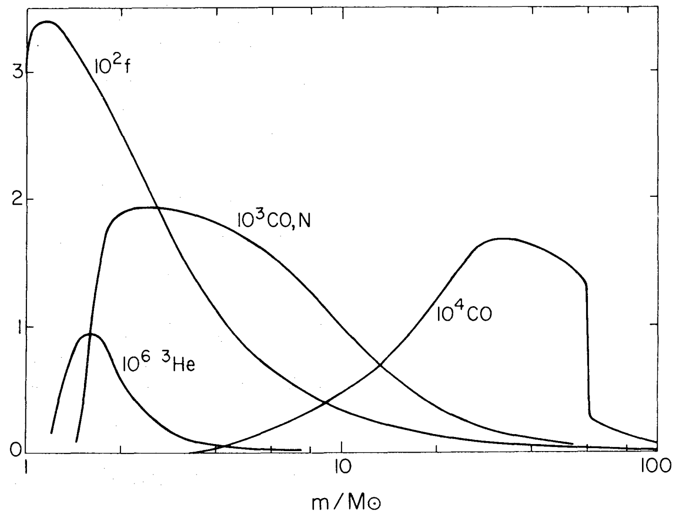

FIG. 2.—Relative contributions of stars of varying mass m to the primary production of CO and ^{3}He and the secondary production of ^{14}N. The ordinate is $\Psi_m Q_{mij}$, the production matrix (Version IIA) weighted by the initial mass function for the case $\zeta = 0.25$, $\mu = 1.55$. The curve labeled f is $\Psi_m R_m$, the total ejected mass fraction weighted by the initial mass function.

a good approximation to assume that $\log q_{ij}$ is a linear function of μ.) Version IIA probably represents the best approximation, and $\mu = 1.3$ and 1.8 probably bracket the expected range for that parameter. There is an uncertainty of at least 20 percent or so in all of the numbers owing to the uncertainty in ζ.

Versions I and II are based in large part on standard hydrostatic evolutionary calculations of single nonrotating stars. Their differences are a rough measure of the uncertainty in that theory. There are many other uncertainties, involving multiplicity, rotation, and/or hydrodynamics for example.

Table 5 shows the complete q_{ij} matrix for Version IIA with $\mu = 1.55$, the midpoint of the range. Some additional quantities of interest are

$$f = 0.17\,, \qquad p_{\rm CO} = 8.5 \times 10^{-3}$$

$$q_{\rm N,CO}/q_{\rm nr,CO} = 4.3\,, \qquad q_{\rm N,CO}/q_{\rm CO,1} = 5.7\,,$$

and

$$p_3 = 10^{-5}[1.02(X_1/0.77) + 0.055(X_2/10^{-5})]\,.$$

For our current prescription, $p_h = p_{\rm CO}/3$ by definition. More detailed stellar-evolution calculations will produce the yields p_i for a large number of primary species.

TABLE 4

SOME ELEMENTS OF THE q_{ij} MATRIX FOR THE TWO VERSIONS OF Q_{mij} AND DIFFERENT μ

	$10^2 q_{\rm CO,1}$		$10^2 q_{\rm N,CO}$		$10^5 q_{3,1}$	
VERSION	1.3 μ	1.8 μ	1.3 μ	1.8 μ	1.3 μ	1.8 μ
I.	1.36	0.65	6.53	1.77	1.1	2.2
IIA.	0.95	0.42	7.4	2.6	0.7	1.3
IIB.	0.95	0.42	18.4	14.9	0.0	0.0

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TABLE 5

THE q_{ij} MATRIX FOR VERSION IIA WITH $\zeta = 0.25$ AND $\mu = 1.55$

	PROGENITOR j							
PRODUCT i	1	2	3	4	CO	N	nr	h
1..........	0.15	...	...	...	...	...	...	...
2..........	...	5.6(−2)	...	...	...	...	...	...
3..........	1.1(−5)	4.7(−2)	0.10	...	...	...	...	...
4..........	1.0(−2)	5.8(−2)	5.8(−2)	0.16	...	...	...	...
CO........	7.1(−3)	7.1(−3)	7.1(−3)	7.1(−3)	0.12	...	...	...
N.........	...	...	...	...	4.0(−2)	0.16	...	...
nr.........	...	...	...	...	9.4(−3)	9.4(−3)	0.17	...
h..........	2.4(−3)	2.4(−3)	2.4(−3)	2.4(−3)	...	...	...	0.17

NOTE.—$X.X\,(Y)$ denotes $X.X \times 10^y$.

If the relative production of nuclides i and j depend upon stellar mass, then comparisons of observed and theoretical p_i/p_j will place constraints on μ independent of ζ (see Arnett 1971b for an example of this).

When considering secondary-to-primary ratios such as $q_{\mathrm{N,CO}}/q_{\mathrm{CO,1}}$, the result is also independent of ζ; however, the uncertainty due to that in μ and in the version of Q_{mij} is large. Again, improved stellar-evolution calculations will eliminate the lattter uncertainty, allowing observed abundances to place strong constraints on the upper portion of the IMF.

The comparison of the observed and theoretical values of a single species (as opposed to ratios of pairs of species) involves not only μ but also the uncertain quantities ζ and ln $[\mathscr{M}/\mathscr{M}_g]$.

III. MODEL-INSENSITIVE RESULTS

a) *Abundance Ratios*

Provided the initial abundance $X_i(0)$ is zero and X_i remains very small, the production of any species i in the instantaneous recycling approximation is given by the following special case of equation (4c):

$$dX_i/dy = p_i\,, \tag{9}$$

where $y = \ln\,[\mathscr{M}(t)/\mathscr{M}_g(t)]$. Equation (9) does not explicitly contain the total stellar birthrate. That birthrate just governs the relationship between y and time t. If the variation in the IMF is due only to a variation in ζ, then by adopting a reference primary species z we may write

$$dX_i/dX_z = p_i/p_z\,, \tag{10}$$

where from equation (8) it is clear that the right-hand side of equation (10) is independent of ζ. It is then straightforward to solve for the following: If i is a primary species, then

$$X_i/X_z = p_i/p_z = q_{i1}/q_{z1}\,; \tag{11a}$$

and if k is a secondary species with z as its progenitor, then

$$2X_k/X_z^2 = q_{k,z}/q_{z,1}\,. \tag{11b}$$

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We have performed many numerical experiments with a variety of stellar-birthrate prescriptions and IMF's with variable ζ. These verify that equations (11a) and (11b) provide excellent approximations for $\mathscr{M}_g/\mathscr{M} \gtrsim 0.05$. They even supply reasonably good approximations in cases in which $\mathscr{M}_g/\mathscr{M} < 0.05$, there is mass infall into the system, or there is incomplete mixing in the interstellar gas. By "reasonably good approximations" we mean typical errors of the order of 5 percent and less than 30–50 percent in the worst cases, always far smaller than the uncertainties in the q_{ij}'s.

We may now test our prescriptions for q_{ij} by comparing them with the solar-system abundances. This test is independent of whether ζ in the IMF has varied with time and independent of the history of the total stellar birthrate—subject to the following qualifications: (*a*) If the interstellar gas has been incompletely mixed, all of our relations apply only to mean abundances. If the Sun possesses abundances which deviate statistically from the mean, accurate comparisons require models of the extent with which abundances should correlate with one another. (*b*) If there has been infall of primordial material, then our calculations in this paper are inadequate for those species (other than ^{1}H) which have appreciable primordial abundances. Large-scale radial gas flow in the Galactic disk has a similar effect, especially if metal-enriched gas is brought into the region under consideration.

i) *Heavy Primary Species*

In this paper we make no comparisons between primary species heavier than ^{4}He—this has been amply done throughout the recent investigations of explosive nucleosynthesis (cf. Arnett and Clayton 1970 for a review). We forced the production matrix to give an approximately correct ratio for X_{CO}/X_h. This was done in order to make a crucial comparison between the secondary product ^{14}N and the primary species ^{12}C and ^{16}O.

ii) CNO

For the comparison of ^{14}N with ^{12}C and ^{16}O we have the solar-system value

$$(2X_{\mathrm{N}}/X_{\mathrm{CO}}^2)_\odot = 18\,,$$

whereas our predicted ratio for $\mu = 1.55$ is

$$q_{\mathrm{N,CO}}/q_{\mathrm{CO,1}} = 5.7 \qquad \text{and} \qquad 23$$

for Versions IIA and IIB, respectively. Values for other cases are given in table 6. For a proper appreciation of these numbers we mention: (1) the observed number is uncertain by at least 30 percent; and (2) the theoretical number varies over the range 2.7–35.

We see that there is an underproduction of ^{14}N unless we invoke Version IIB. Considering the great uncertainty in the eventual evolution of the stellar models upon which Version IIA ^{14}N production is based, we feel that the agreement between theory and observations is adequate. In particular, we see no *need* to invoke the *assumed* primary production of ^{14}N as was done by Truran and Cameron (1971); this statement does not preclude the possibility that future stellar models or observations may require such a process.

Recently Paczyński (1973) has discussed carbon depletion in the envelopes of main-sequence stars. Incorporating his results into a modification of our case A, we find

$$q_{\mathrm{N,CO}}/q_{\mathrm{CO,1}} = 7.7\,, \qquad 7.5\,, \qquad \text{and} \qquad 10.3$$

for $\mu = 1.3$, 1.55, 1.8, respectively. These are intermediate between case A and the complete-mixing case B, but are not quite as large as observations appear to require.

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TABLE 6

A Survey of Some Important Chemical Evolution Parameters: $\zeta = 0.25$

A.

μ	$10^2 p_{CO}$ Version I	$10^2 p_{CO}$ Version II	$q_{N,CO}/q_{CO,1}$ Version I	$q_{N,CO}/q_{CO,1}$ Version IIA	$q_{N,CO}/q_{CO,1}$ Version IIB
1.30	1.70	1.19	4.8	7.7	19
1.55	1.28	0.85	2.9	5.7	23
1.80	0.79	0.50	2.7	6.1	35

B.

μ	Versions IA and IIA Λ_2	Versions IA and IIA Λ_3	Versions IA and IIA $10^5 p_3'$	Versions IB and IIB $\Lambda_2 = \Lambda_3$
1.30	0.152	0.117	5.75	0.244
1.55	0.138	0.0815	12.5	0.205
1.80	0.131	0.0504	19.8	0.182

iii) *Neuton-rich* (*nr*) *Heavy Species*

Our prescription for the group labeled nr was one which we knew to be deficient. We prescribed it as a secondary process (actually there is also a small tertiary contribution). The neutron enrichment is supplied by primordial ^{14}N or the ^{14}N product of CNO processing of primordial ^{12}C and ^{16}O. There is another source of neutron enrichment during hydrostatic carbon burning (Arnett and Truran 1969; Arnett 1972*c*). This is a primary process for producing nr species. Current stellar-evolution calculations suggest that this primary process is comparable to the secondary process.

Our secondary prescription is such that

$$q_{nr,CO}/q_{CO,1} = 1.33$$

for all versions and μ. The solar-system ratio,

$$(2X_{nr}/X_{CO}^2)_\odot = 6.1 ,$$

is larger. These figures are consistent with there being a primary source of X_{nr} which is the same order of magnitude as our secondary. It appears that stellar-evolution calculations will satisfactorily resolve this problem.

iv) ^4He

The ratio $q_{4,1}/q_{CO,1} = 1.4$ in table 5 is typical of this ratio for other values of μ. This suggests that stellar nucleosynthesis may vary X_4 by a few percent ($\Delta X_4 \lesssim 0.03$) provided the IMF is constant (see Talbot and Arnett 1971*b*).

v) D *and* ^3He

The production and/or destruction of ^3He and D must be handled more carefully than equation (9). We assume that D is only destroyed, never produced, by stars; the appropriate version of equation (4c) is

$$dX_2/dy = -\Lambda_2 X_2 . \qquad (12)$$

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The solution may be expressed as a function of the reference primary species z:

$$\frac{dX_2}{dX_z} = -\frac{\Lambda_2}{p_z} X_2 \tag{13}$$

or

$$X_2 = X_2(0) \exp(-\Lambda_2 X_z/p_z) . \tag{14}$$

We allow for both production and destruction of ^{3}He. The production occurs in stars of about 1.3–2 $M_\odot$; consequently the sources have sufficiently long lives that the instantaneous recycling approximation is only marginally adequate. The solution of equation (4c) for ^{3}He may be written in terms of D:

$$X_3 + X_2 - p_3' = [X_3(0) + X_2(0) - p_3'] \exp(-\Lambda_3 X_z/p_z) , \tag{15}$$

where

$$p_3' = q_{31} X_1/(f - u_3) . \tag{16}$$

Table 6 lists Λ_3, Λ_2, and p_3' (with $X_1 = 0.77$) for case A (for Versions I and II these quantities are identical) and for case B where $\Lambda_2 = \Lambda_3$ and $p_3' \equiv 0$.

First consider case B. Upon adopting $X_{\rm CO}$ from table 1 and $p_{\rm CO}$ from table 6, we find

$$X_2/X_2(0) = X_3/X_3(0) = 0.57 , \quad 0.75 , \quad 0.84 \tag{17}$$

for $\mu = 1.3$, 1.55, 1.8, respectively.

If we were to assume a constant IMF with $\mu = 1.55 \pm 0.25$ and employ $y = \ln[\mathscr{M}/\mathscr{M}_g] = 2.3 \pm 0.7$, then we would find

$$X_2/X_2(0) = X_3/X_3(0) = 0.63 \pm 0.12 . \tag{18}$$

The low value of 0.51 corresponds to the assumptions adopted by Truran and Cameron (1971) and agrees with the value they state. We regard the larger values as more likely, however.

For case A we find

$$X_2/X_2(0) = 0.70 , \quad 0.83 , \quad 0.88 \tag{19}$$

and

$$\begin{aligned} X_3 &= 0.76 X_3(0) + 0.060 X_2(0) + 1.38 \times 10^{-5} \\ &= 0.89 X_3(0) + 0.067 X_2(0) + 1.32 \times 10^{-5} \\ &= 0.95 X_3(0) + 0.073 X_2(0) + 0.95 \times 10^{-5} \end{aligned} \tag{20}$$

for $\mu = 1.3$, 1.55, and 1.8, respectively.

Reeves *et al.* (1973) discuss the primordial solar abundance of X_2 and X_3; their estimates are

$$X_2 = 3.5 \pm 1.5 \times 10^{-5} , \qquad X_3 = 2 \pm 1 \times 10^{-5} . \tag{21}$$

The value of $X_2(0)$ is consequently

$$\begin{aligned} X_2(0) &\simeq 4.2 \pm 1.3 \times 10^{-5} \text{ for case A} , \\ X_2(0) &\simeq 4.7 \pm 1.9 \times 10^{-5} \text{ for case B} . \end{aligned} \tag{22}$$

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These cannot be distinguished between, and they suggest a low-density universe with $\rho_0/\theta^3 \simeq 5.5 \times 10^{-31}$ g cm^{-3}, adopting the models of Wagoner, Fowler, and Hoyle (1967) ($\theta = T_0/3°$ K; ρ_0 and T_0 are the present mean density and temperature of the Universe).

For case B,

$$X_3(0) \simeq 2.7 \pm 1.4 \times 10^{-5}\,,$$

which is consistent with the $X_3(0) \simeq 3.0 \times 10^{-5}$ which the above low-density universe would predict. We conclude that case B is consistent with a big-bang production of all D and all ^{3}He.

The various uncertainties are sufficiently large that one cannot make a clear-cut statement about case A. A large range of plausible initial $X_3(0)$ and $X_2(0)$ results in an overproduction of ^{3}He. For example, in the low-density universe which just matches the D abundance, the value of $X_3(0) \simeq 3.0 \times 10^{-5}$ and equation (20) yields $X_3 \simeq 4.3 \times 10^{-5}$ if $\mu = 1.55$. This is greater than the observed value, and any lower-density universe would increase this discrepancy. Universes with slightly higher densities would satisfy ^{3}He better, but would produce too little D. There are a large class of solutions between cases A and B which are acceptable.

These results are consistent with the argument that the requirements on ^{14}N are such that some sort of envelope mixing is required in intermediate and high-mass stars. If all of the ^{3}He produced in the envelopes of Iben's models were to be converted to ^{7}Li in the manner suggested by Cameron (1955) and Cameron and Fowler (1971), then ^{7}Li would be greatly overproduced in the Galaxy. (It could produce X_7 as large as about 5×10^{-6}, whereas the Cameron [1973] value for X_7 is about 10^{-9}. The abundance in the interstellar gas is unknown, but it is not uncommon for T Tauri stars to have Li abundances 3–30 times solar.) This suggests that the Cameron (1955) ^{7}Be transport mechanism works with an efficiency no greater than about 1 percent—or in only 1 percent of the stars which produce ^{3}He.

It is perhaps interesting to note that if the low-density big-bang calculations are not relevant and the initial abundances of D and ^{3}He were zero, then the case A stellar production of ^{3}He is just about the required amount. A Galactic source of D would be necessary, and a reconsideration of the ^{14}N problem would be required.

Note.—Except for the comparison with the result of Truran and Cameron (1971) at equation (18), this entire discussion of D and ^{3}He is independent of uncertainties in the lower portion of the IMF [valid for arbitrary $\zeta(t)$ in eq. (8)] and independent of the choice of $\mathscr{M}_g/\mathscr{M}$. It of course does depend upon our adopted values for the solar abundances (table 1) and our prescription for Q_{mij}.

b) *Abundance of* CO

In the preceding discussion we found satisfactory agreement between predictions based upon our Q_{mij} prescriptions and observations for various abundance ratios of heavy elements and the abundances of D and ^{3}He. This was accomplished in a fashion independent of the low portion of the IMF (ζ) and the history of the stellar birthrate in the solar neighborhood; the comparison involved only stellar-evolution predictions and the shape of the upper portion of the IMF (parametrized by μ).

In order to compare for a single species the observed and predicted absolute abundance, the magnitude of the yield p (proportional to ζ) is involved. Also, the history of star formation enters because one needs the value of $y = \ln(\mathscr{M}/\mathscr{M}_g)$ at *solar-system formation.* The latter is only approximately known in the solar neighborhood at the *present epoch* ($\mathscr{M}_g/\mathscr{M}$ is about 0.05 to 0.20 depending upon one's choice of Galactic mass distribution model and corrections for gas other than H I). The value of $\mathscr{M}_g$ is frequently taken to be an exponentially decreasing function of time, and in this fashion one extrapolates backward in time for the age of the Sun, 4.7×10^9 years.

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We will compare the solar-system abundance with theoretical values of X_{CO} as a function of (1) version of $Q_{m;j}$, (2) IMF (ζ and μ), and (3) $y = \ln(\mathscr{M}/\mathscr{M}_g)$. These comparisons *are* model dependent; however, our experience has been that the simple solution to equation (9),

$$X_{CO} = p_{CO} y \tag{24}$$

allows comparisons to be made with the introduction of an error less than a factor of two. From the solar system abundance we find the following required values for p_{CO}:

$\mathscr{M}_g/\mathscr{M}$.............	0.05	0.10	0.20	0.30
$(10^2 p_{CO})_{obs}$.........	0.39	0.51	0.73	0.98 .

To these values one should compare the theoretical values of p_{CO} in table 6.

For no combination of $\mathscr{M}_g/\mathscr{M}$, μ, and version is there a discrepancy of more than a factor of 4.3 between the observed and theoretical abundance for CO. For the most probable values of the parameters the agreement is excellent, especially considering the uncertainties in the stellar models and the simplicity of this galactic-evolution model. Making a closer comparison between $(p_{CO})_{obs}$ and $(p_{CO})_{th}$ requires more sophisticated models which resolve the problem of the paucity of metal-poor stars (e.g., Schmidt 1963). This involves a discussion of models which *do* depend upon the precise details of star formation. We discuss those separately (Talbot and Arnett 1973; Arnett and Talbot 1974). Those solutions do not alter the satisfactory agreement between theoretical chemical evolutionary coefficients and observed abundances which was discussed above—except to improve the agreement for the CO abundance.

IV. SUMMARY

We have presented a prescription for the quantities required to compute the chemical evolution of galaxies. We have expressed comparisons with the solar-system abundances in such a manner as to be independent of (1) variations in the lower portion of the initial mass function, (2) the history of the stellar birthrate in the solar neighborhood, and (3) inhomogeneous mixing of the interstellar gas *provided* the Sun is a good sample of the mean abundances.

In this paper there has been no attempt (or need) to deal with any prescribed time history of the solar neighborhood. Consequently, for example, nucleo-cosmochronologies were not discussed.

The agreement between observations and theory is well within the many uncertainties. The primary uncertainty is the degree of envelope mixing in late, intermediate-mass stars. (No mixing and complete mixing bracket the observational requirements of ^{14}N; this may be an argument that such mixing does occur.) This uncertainty concerns only the abundances of the secondary species ^{14}N and the easily destroyed species D and ^{3}He; this uncertainty has essentially no effect on primary species such as CO.

Furthermore, it should be clear that for any evolved star-gas system there are broad characteristics of abundances which are independent of the history of the stellar birthrate in the system. All abundances vary in a manner which may be parametrized by, e.g., X_{CO}. *Any* two systems with the same X_{CO} will have very nearly the same set of abundances; varying the slope of the upper part of the luminosity function may produce minor variations in the abundance ratios.

As noted by Searle and Sargent (1972) and Searle (1972), the absolute level of abundances depends upon $\ln [\mathscr{M}/\mathscr{M}_g]$. The mean value of the ratio $\mathscr{M}_g/\mathscr{M}$ varies by less than a factor of 20 for the spiral- and irregular-type galaxies reviewed by Roberts (1969). Consequently we would anticipate variations in primary-species abundances of less than a factor of 3. The measurements of abundances in our Galaxy are uncertain

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by a factor of this order; other galaxies, more so. Order-of-magnitude improvements in observational techniques may be required to reach the level of being able to use observed abundances to constrain parameters in models of the history of galactic nucleosynthesis.

We have also examined aspects of galactic evolutionary models which *do* depend upon the precise details of star formation (for example, G-dwarf metallicities and cosmochronologies); detailed discussion is presented separately (Talbot and Arnett 1973; Arnett and Talbot 1974).

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