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Actuator Gear Train Design and Material Selection Method for Collaborative Robots

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Actuator Gear Train Design and Material Selection Method for Collaborative Robots

by

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Abstract

Actuator Gear Train Design and Material Selection Method for Collaborative Robots

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Collaborative robots are industrial robots especially designed for interaction with humans and their work environments. Traditional actuators usually have high stiffness, stable structure, long life, and high repeatability, but are quite heavy, expensive, energy consuming, and suffer from slow response. For collaborative robots, a new kind of actuator gear train with relatively high performance and accuracy, lightweight, low cost, and quick response to the environment, is needed.

In this research, a complete design method for collaborative robot actuator gear trains has been developed, including an actuator structure study, gear train type choice, gear train parameter optimization and standardization, gear train performance evaluation, and material selection.

Different form traditional gear train design method, which only determines gear teeth numbers according to reduction ratio requirements, the optimization design developed in this research seeks to achieve the best gear train performance based on the

optimum assignment of all input parameters. An exhaustive method is applied determine the constraint relationships between the input parameters and the design requirements. Several assumptions are made for the input parameters to simplify the design process process and improve computational efficiency. The parameters are assumed to be continuous instead of discrete at first. After design optimization, the diametral pitch, number of teeth and face width are standardized.

For original robot actuator gear train design, the material is usually chosen from metal alternatives. However, the gear train can be made of various materials, including steel, cast irons, nonferrous alloys, and even plastics. Materials with lower density and lower cost, among other properties, can significantly improve gear train performance once basic strength and fatigue limit requirements are met. Material selection based on the material properties directly can confuse the designer, so a material selection process based the final gear train performance (torque capacity, weight, torque density, inertia, and responsiveness) to support meaningful and effective selection. The ELECTRE III multiple criteria decision analysis method is used to compare material alternatives to facilitate material selection.

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Chapter 1: Introduction

From small sweeping robots at home, industrial collaborative robots in workshop, to large high payload lift robotic arms, robots have become more and more professional, and indispensable in people's lives.

The advantages of industrial robots, like high degree of accuracy, indefatigable continuous working ability, and lower cost than human labor, make them the best choice in the workshop for repetitive works. And people have started to give more delicate and complicated work to robots to accomplish. Collaborative robots are developed for interactions with humans and environments. Collaborative robots could liberate humans from labor, increase working efficiency, and even improve product quality. [1]

The actuators in the market now are usually standard parts, and can be used for different kinds of robots. They usually have high stiffness, stable structure, long life time, high repeatability, but are heavy, expensive, and energy consuming, with slow responsiveness. For collaborative robots, a new kind of actuator, particularly with a lightweight gear train, low price, and with quick responsiveness to the environment, is needed.

In this thesis, a comprehensive design method for the collaborative robot actuator gear train is described, including gear train type selection, parameter optimization and standardization, gear train performance evaluation, and material selection.

1.1 Purpose & Motivation

Industrial robots have been more and more widely used and have become important tools in industry. According to the International Federation of Robotics (IFR), by the end of 2013, the volume of sales of industrial robots was between 1,332,000 and 1,600,000. [2] And total sales were about US$ 25.5 billion, including software and

peripherals, for year 2011 only. [3] The quickly increasing robot market requires improved robots with higher performance and better services.

The collaborative robot is a new type of robot that is undergoing rapid development. Compared with traditional industrial robots, collaborative robots have are lightweight (similar to the human arm), highly responsive to operators and the environment to ensure safety, and low in cost in terms of energy consumption.

As a result, the structure of collaborative robots designed specifically for these unique requirements. A new type of actuator that features a gear train with relatively high accuracy, light weight, low price, and quick responsiveness to environment, is needed.

1.2 Problems and Solutions

Collaborative robot actuators have distinctive features compared to traditional robot actuators. Particularly for gear train design, the problems designers encounter and possible solutions approaches are discussed in this section from the perspective of three aspects.

1.2.1 Gear Train Type and Structure

Compared with human arms, the overall mass of lightweight robots is still higher, yet the payload is typically low. [4] For this reason, increasing the torque capacity while reducing the weight of are important design goals.

A good understanding of industrial robots and actuator structure helps to quantify design requirements, and contributes to structural design and system optimization. In Chapter 3, the performance of commonly used gear trains is presented and compared to support the selection of one that is appropriate for collaborative robot actuators.

1.2.2 Parameter Optimization and Standardization

Given a gear train type and robot actuator requirements, design optimization is the next step. In traditional gear design, the designer may follow an exhaustive approach based on gear teeth numbers. [5] Several problems exist in this method, such as non-standard parameter values, causing the design to have less than optimal best performance, while consuming excessive design time.

The design method developed in Chapter 5 assumes all the parameters are continuous instead of discrete at first. The design method assumes that certain gear train parameters are input parameters. These input parameters are selected based on a detailed study of gear train structure and limitations. Design optimization is the process of finding the input parameter values that achieve the best torque capacity performance. Finally, a standardization process is applied to adjust the continuous parameter values to industry standard values to ensure availability of components and to reduce cost.

The parameters that are most influential in determining gear train performance are the diametral pitch, number of teeth, face width, and gear diameter. After clarifying mechanical and geometry limitations and the input and output relationships between these parameters, we can determine which individual parameters must be assign values.

To achieve the highest performance, an exhaustive search method is first applied as a benchmark. To simplify computation in the design process, constraints between input parameters and design requirements are identified. This reduces the number of parameter values that must be evaluated simultaneously. When this reduced set of parameters is used to simulate gear train performance, accuracy has been demonstrated higher than 90.7% compared to the exhaustive search benchmark.

Parameters that are standardized are diametral pitch, number of gear teeth and face width. The diametral pitch and face width have a standard series to choose from.

Since diametral pitch has the strictest limitations, it is assigned first. The method also includes provisions for allowing clearance for installation.

1.2.3 Gear Train Performance and Material Selection

Almost all traditional robot actuator gear train design are fabricated from steel. Actually, a gear train can be made of various materials in addition to steel, such as cast irons, nonferrous alloys, and even plastics. Steel is most commonly used because of its high strength, relatively low cost and the mature technology in manufacture. [6] But plastic also has very good properties, which make it fit for some special applications in collaborative robot design. Materials and manufacture technology are discussed in Chapter 4.

Selection of material based only on material properties is generally not effective because it is difficult to understand what the properties actually mean for a particular gear train application. [7] For example, we know that increased surface hardness is desirable, but we may not know exactly which hardness level is need for a particular situation. For instance, for a given application, is it better to choose a material with higher surface hardness but with lower core strength? So gear train performance calculation is quite important, as it can help the designer make a better choice. Calculation of gear train performance based on different materials connects the gear train design process and material selection.

The material selection method developed in Chapter 6 compares gear train performance with design requirements, in addition to direct comparison of the properties of different candidate materials. The ELECTRE III multiple criteria decision analysis tool is introduced to facilitate comparison of different materials based on these multiple criteria.

1.3 CONTRIBUTIONS

To develop a comprehensive design method for collaborative robot actuator gear trains, knowledge of actuator structure, gear train types, material properties, design standards, and selection methods are all needed. The following tasks have been completed to develop the proposed design method:

1. Study robot actuator structure, and clarify the requirements and limitations of collaborative robots.
2. Discuss and compare different types of gear trains, and determine the most appropriate gear train type for collaborative robots.
3. Study available gear materials, including both metals and plastics, and develop an understanding of gear manufacture and post treatment technology.
4. Develop a comprehensive gear train design method.
5. Discuss the structure and design method for three types of star compound gear train, the selected gear train structure, and clarify the parameter optimization design method.
6. Provide a way to standardize gear train parameters.
7. Calculate gear train performance based on different materials to support material selection for gear trains.
8. Develop a method for multiple criteria material selection.

Chapter 2: Industrial Robots and Actuator Structure

2.1 Industrial Robots and increased market

The ISO 8373 standard defines an industrial robot as an "automatically controlled, reprogrammable, multipurpose manipulator with three or more axes, which can be either fixed in place or mobile for use in industrial automation applications". [8] The discussion of industrial robots for this thesis is focused on robotic arms.

Industrial robots are already widely used in assembly, packaging, welding, and other repetitive work, and have become important tools in industry. According to the International Federation of Robotics (IFR), by the end of 2013, the volume of sales of industrial robots was between 1,332,000 and 1,600,000. [2] Total sales for the year 2011 were about US $25.5 billion, including software and peripherals. [3] The remainder of this chapter surveys industrial robots by category, identifying features that impact actuator design.

2.2 Collaborative Robots

Early robotic arms were just imitations of human arms. By introducing artificial intelligence, robots have become smart bionic devices that are more frequently used in sophisticated applications, medical robotics, rehabilitation and therapy. [9] This progress has motivated designers to develop new robots that can work in cooperation with humans. Collaborative robots are industrial robots designed for interaction with humans and environments, and require a "lightweight design with high load-to-weight ratio and high motion velocity". [10]

This chapter focuses on existing industrial collaborative robots and the structure of actuators for such robots, analyzing the resulting requirements for actuators, especially gear train design.

2.2.1 Traditional Lightweight Robot Arms

Collaborative robots use lightweight robot-arms due to the desire to make robot arms similar to human arms. The main improvements for robots in the current market focus on "decreasing the weight of the arm, increasing the payload up to human physical capabilities, achieving desired compliance of joints, and decreasing energy consumption". [10] Table 2-1 below lists characteristics of some commercially available lightweight robot arms. Figure 2-1 shows several examples of these arms.

LWR type	DOF	Range (mm)	Weight (kg)	Payload (kg)	Repeatability(mm)	Tip speed (m/s)	Compliance
Barret	7	1000	25	4	+/- 0.10	3	N
KR Agilus 6 R700	6	706	50	6	+/- 0.03	2	N
LWA Powerball	6	700	12.5	6	+/- 0.06		N
LWA PA10	7	930	35	10	+/- 0.10	1.55	N
SIA5F	7	559	30	5	+/- 0.06		N
VS-6577G-B	6	854	36	7	+/- 0.03	8.2	N
LBR iiwa 7R800	7	800	22.3	7	+/- 0.10		Y
UR5	6	850	18.4	5	+/- 0.10	1	Y

Table 2-1：Lightweight Robot Arms parameter [10]

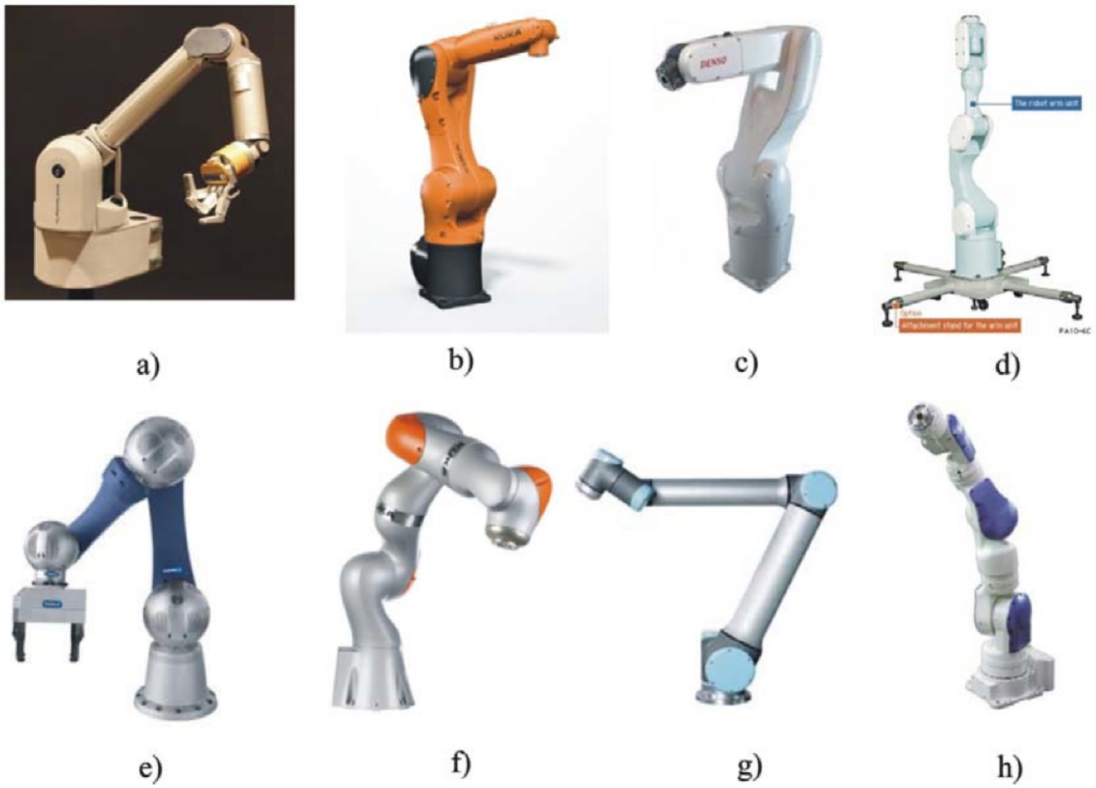

Figure 2-1： Example of lightweight industrial arms [10]

Traditional lightweight robots (LWR) are “mainly intended for industrial applications with demand of high accuracy and repeatability” [10]. Traditional lightweight robots usually have high stiffness and relatively high weight, which makes them quite dangerous to operators, and protected working areas are needed to isolate them from operators. Several quite popular and well known industrial robots in the market are: KUKA KR Agilus 6 R700 [11], Shunk LWA Powerball [12], Mitsubishi LWA PA10 [13], Yaskawa-Motoman SIA5F [14], Denso VS-6577G-B [15], and Barett robotic arm [16].

2.2.2 LWR with Compliance

The second category, still a topic of ongoing worldwide research, is the LWR with compliance with its actuators. One example is the KUKA LBR iiwa [17] lightweight robotic arm, which makes use of torque sensors in actuators to detect collisions and external forces. The compliance in the joints makes the robot safe to work with operators, so they can achieve quite difficult and complex functions. The only problem for this robot is the high cost for the devices and system design. They are still not widely used because of very high costs.

2.2.3 Technical Requirements

To be acceptable by the market, certain technical requirements for collaborative robots must be met. For example, the overall mass of lightweight robots is still higher than a human's arm, while the payload is too low. And, as already mentioned, the cost of collaborative robots is too high compared to traditional lightweight robot arms.

Seeking to achieve these goals, we assume the following criteria must be satisfied:

- Maximize the payload.
- Minimize the mass of the robot arm.
- Minimize energy consumption.
- Minimize cost.
- Provide data acquisition system with numerous sensors.
- Provide target joint compliance.
- Maximize precision (repeatability).
- Durable construction resistant to mechanical impacts, temperature extremes, and chemical and moisture influence.

Finally, based on the analysis above, the central issues of collaborative robot gear train design are as follows:

- Choose a stable, effective, and easy to assemble the structure to ensure system stability and low cost.
- Develop a method to maximize gear train torque capacity to increase the robot payload level.
- Optimize gear train performance, with respect to such metrics as torque density and responsiveness, to provide the gear train with quick responsiveness to the outer environment.
- Reduce the gear train weight to make the robot lighter, and therefore reduce energy consumption and impact force if the robot contacts humans. In this study, we reduce gear train weight by selecting an appropriate material with lower density.
- Reduce the cost, including raw material cost, manufacture cost, and assembly cost, which means choosing a simple structure and a material that costs less and has better machinability.

2.3 Actuator Structure

For better gear train design, a good understanding of robot actuator structure is necessary. In general, an electro-mechanical actuator is an integrated unit consisting of a motor, gear train, bearings, shafts, brakes, a gear housing, seals, and lubricants. And for some actuators, sensors, clutches, and other accessories are also included. Figure 2-2 shows the mechatronic joint design of the DLR-LWR-III.

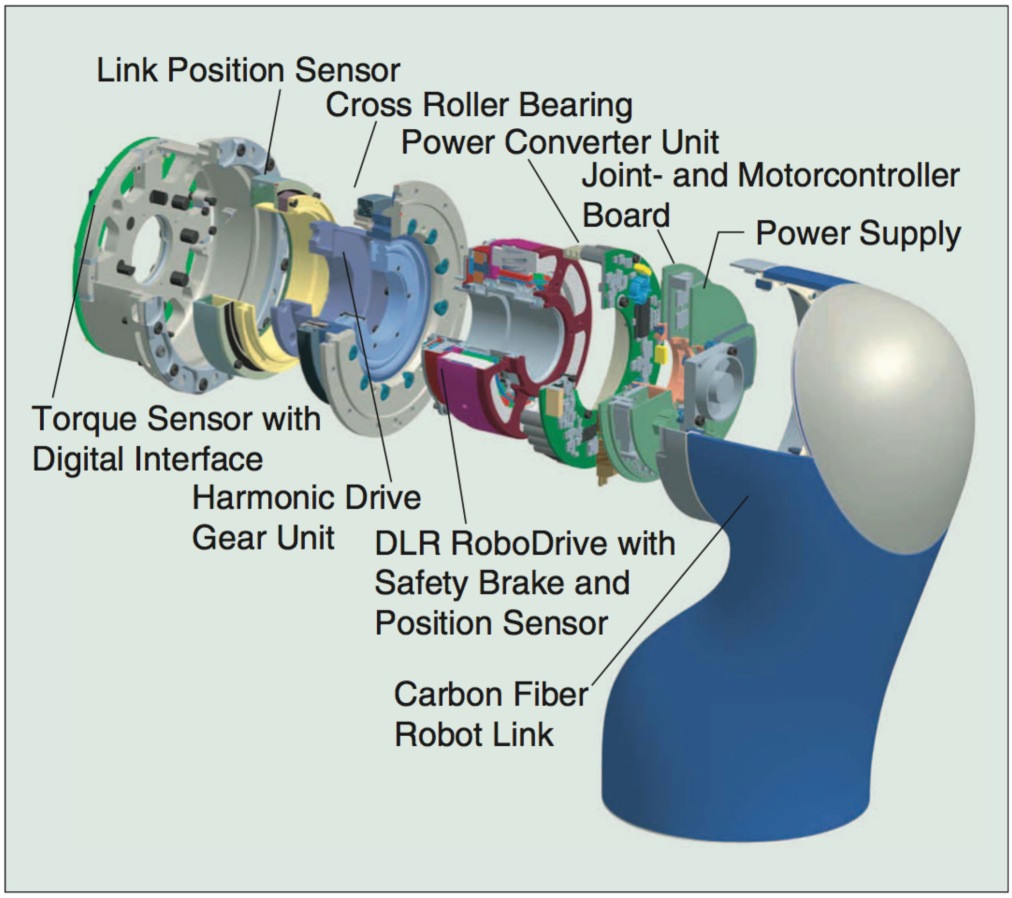

Figure 2-2: The mechatronic joint design of the DLR-LWR-III, including actuation, electronics, and sensing. [4]

2.3.1 Motors

The motors we discuss in this thesis are electro-mechanical actuators (EMA), which have some advantages compared to traditional hydraulic or pneumatic actuators. [18] For collaborative robots, motors with high torque at moderate speed, low energy loss, and fast dynamic response are preferred. Performance metrics and design parameters, such as rated voltage, current, power, torque and speed, must be evaluated to determine the basic requirements for gear train design.

2.3.2 Gear Train

The gear train is a system of machine elements that transmits the power and motion from the motor to the final output. The gear train can also amplify the torque and reduce the speed to meet the requirements of the load. A gear train allows a designer to modify the power, speed, and torque for specific applications. An added benefit is that the gear train isolates the motor from shocks to the output of the actuator. [19] Gear train design is quite critical for the actuator performance, and this subsystem will be carefully and completely discussed in Chapter 3.

2.3.3 Bearings

Bearings are also critical components of mechanical systems in the actuator. Bearings connect stationary parts and moving parts while allowing rotation between them. In actuator designs in this thesis, bearings are mainly used to support star gears and reduce rotational friction.

2.3.3.1 Bearing Types

Various different types of bearings have been developed to allow rotational motion in mechanical structures. Commonly used bearings in actuators include journal bearings, ball bearings and roller bearings.

Journal bearings are formed by fitting a shaft into a hole that is large enough to hold the shaft (See Figure 2-3). A journal bearing makes use of pressurized oil to separate the contacting surfaces of the shaft and the hole. The oil film reduces friction and wear, thereby improving the bearing's performance. Journal bearings have a simple structure and are comparatively small, but friction in journal bearings is still relatively higher than in roller bearings. Journal bearings are appropriate for situations where the space is limited, mechanical requirements are not strict, and a source of pressurized oil can be provided.

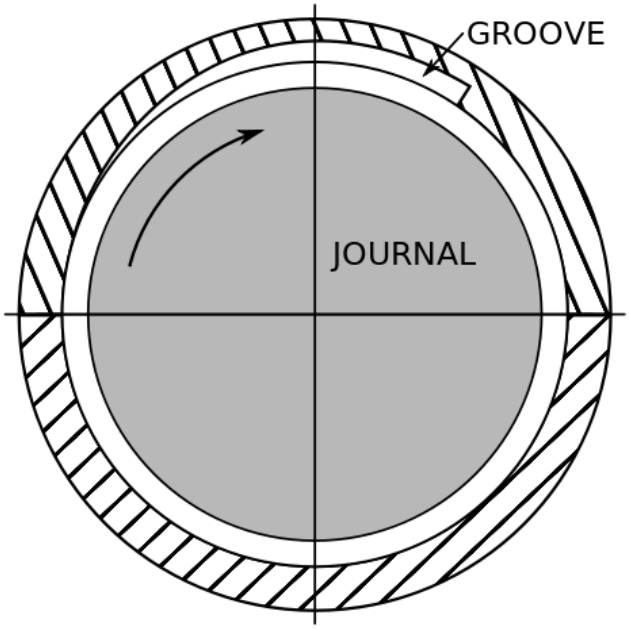

Figure 2-3: Journal Bearing [20]

Ball bearings take use of rolling balls to support the radial force between the stationary part and the moving part. Ball bearings perform excellently, with very low losses to friction. However, a ball bearing is relatively large in the radial direction (See Figure 2-4).

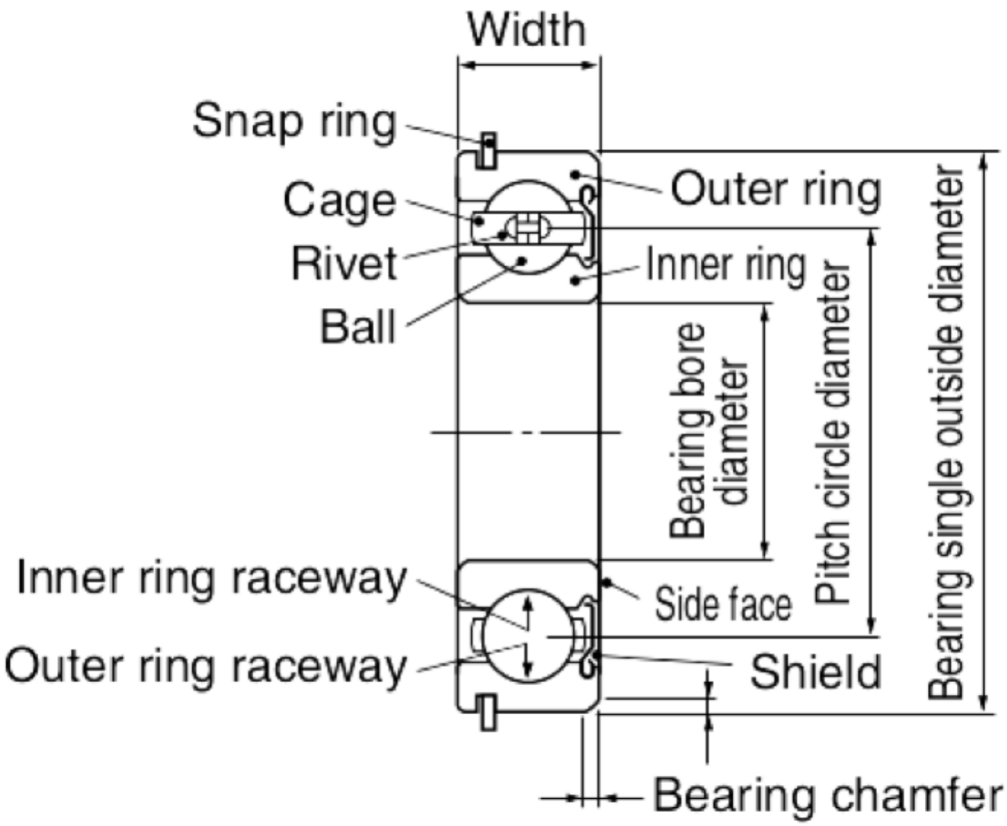

Figure 2-4: Ball Bearing [21]

Roller bearings take use of cylindrical rolling elements. Since a cylinder contacts a surface with a line instead of a point (as in ball bearings), roller bearings typically have higher load capacity than ball bearings of the same size. Tapered bearings are roller bearings designed to withstand axial forces.

2.3.3.2 Bearing Selection

Choosing the right bearing type for a gear train is quite important. In general, for actuators with strict diameter and space constraints, there is no choice and only journal bearings can be used. The advantages of the journal bearing are the small dimension and

simple structure. Its disadvantage is the relatively higher friction. The choice of lubricant, discussed below, will influence the performance of the journal bearing significantly.

If space constraints allow, ball bearings or roller bearings should be selected over journal bearings. Ball bearings are used for light loads, and roller bearing can bear much heavier loads. For robots working in with the presence of shock and vibration, a tapered bearing may be chosen to resist axial forces.

2.3.4 Shafts

Shafts are indispensable components in motion and power transmission. The number of the shafts is determined by the number of individual gear train elements.

When transmitting power, because the material at the outer radius of the shaft bears most of the torque, shafts can be manufactured as a tubes to reduce weight. To connect a shaft to a motor, a keyway is needed. The design of the shaft should fully consider the impact and stress concentrations that result from the keyway.

When gears are connected by roller bearings, an interference fit is used, so the diameter of the shaft should be a little bit larger than the inner diameter of the bearing. In contrast, when gears are connected by journal bearings, a clearance fit is used, so the diameter of shaft should be a little bit smaller the journal.

2.3.5 Brakes

A brake is a mechanical device to slow, stop and inhibit relative motion between different components. In most cases, a brake makes use of friction to convert the kinetic energy to heat.

In general, for robot actuators, brakes have three functions. First, they lock robot axes after use. Second, they quickly stop an axis to ensure safety in emergencies, which is extremely important for collaborative robots. And third, brakes can protect the motor and

mechanical components from failing under heavy loads or impacts beyond their design limits. The most common types of brakes are disc brakes and drum brakes. A disc brake uses pairs of calipers to squeeze a disc connected to the rotational parts. [22] A drum brake uses a set of non-rotating shoes or pads to press against the rotating part. [22] Accessibility for replacement should be considered in structural design when the robot is designed for long life or when the brake is used frequently.

2.3.6 Clutches

For robot actuators, clutches quickly connect and disconnect the power between the motor and the mechanical elements. Generally, the clutch is an optional part for traditional LWR, because the brake can stop the motion. But for collaborative robot actuators with compliance, the study of clutches is meaningful. [23] And to meet the requirements of collaborative robots, clutches should be lightweight, small, and durable, with quick response. Commonly used clutches are friction clutches and dog clutches.

The friction clutch transmits movement and power by the friction between surfaces. Similar to the disc brake, the friction is caused by pressure between pairs of plates. In general, more plates for a multi-plate clutch means higher torque capacity. [24] Multiple plates also reduce wear on the plate surfaces, and therefore increase lifetime (See Figure 2-5). When the power should be cut off, pressure applied on the plates is reduced, and relative motion between different plates is allowed.

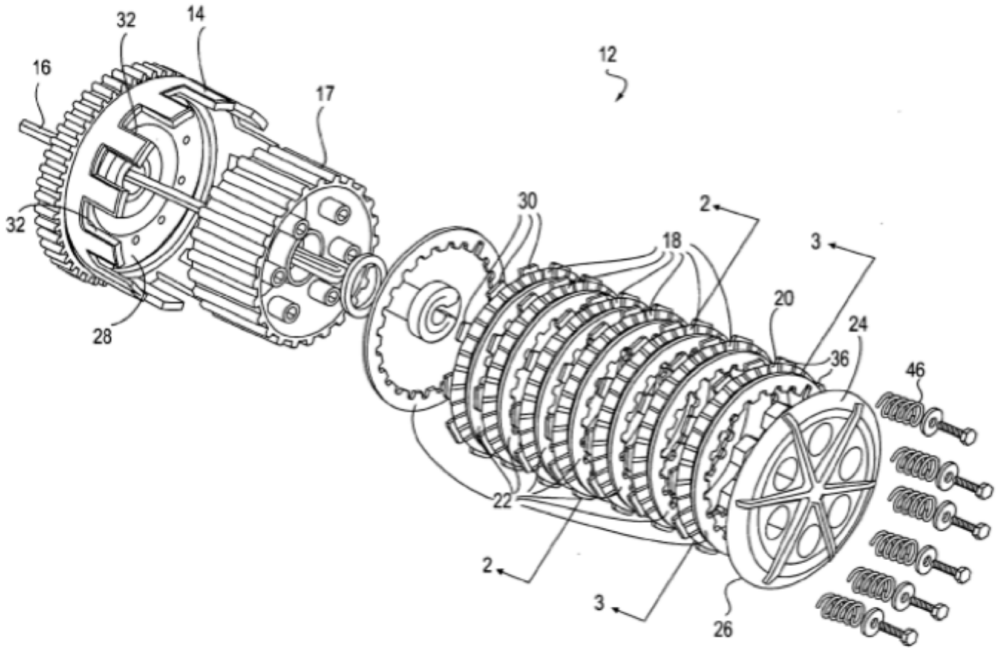

Figure 2-5: Multi-Plate Clutch [25]

The dog clutch couples rotating shafts by interference instead of friction, as shown in Figure 2-6. Compared with friction clutches, dog clutches cause two parts to rotate at the same speed, and allow no slip. Dog clutches require high teeth strength, and can carry exceptional loads. However, when the dog clutch teeth engage, shock results if the speed of the two components is different.

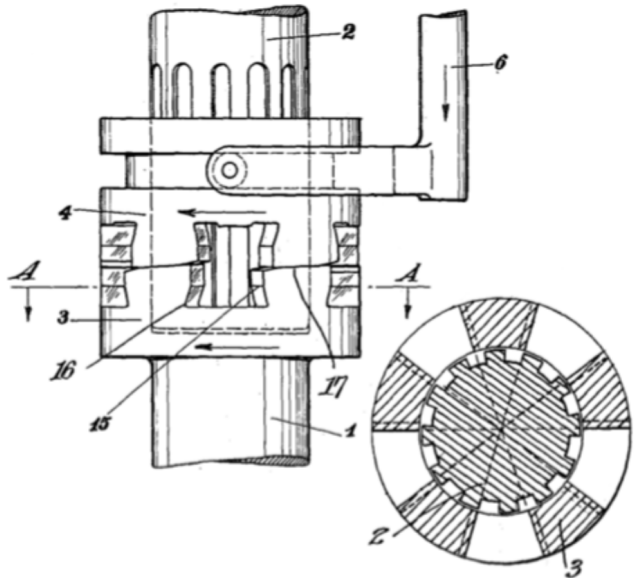

Figure 2-6: Dog Clutch [26]

2.3.7 Cooling and Lubricant

For robot actuators, lubricants are mainly used for bearings and the gear train. They perform three critical tasks for mechanical machines: friction reduction, gear protection and heat transfer. [27]

For bearings, lubricants can significantly reduce the friction between the surfaces of stationary parts and rotational parts, and therefore increase the energy efficiency, which is one of the objectives of this research. Lubricants can reduce the likelihood of several types of gear failure, such as surface pitting and wear. A good lubricant can improve system stability and increase gear train lifetime. Liquid lubricants, like oil, can also effectively act as coolants, which becomes important when the performance of components in electro-mechanical actuators degrades as temperature increases. Grease is an appropriate lubricant for light load applications. [28] The importance of lubricants will be discussed in more detail in Chapter 4.

2.3.8 Seals

For actuators, seals prevent the lubricant from escaping while preventing debris from entering. [29]

Gear train performance can be influenced significantly by lubrication condition, and, when oil lubricant is used, seals are indispensable. For gear trains that use grease as a lubricant, seals may be unnecessary.

Debris in the lubricant can cause severe wear on gear teethfaces and result in catastrophic failure. Even though changing the oil regularly removes debris, using seals is the most effective way to prevent debris from entering. Sometimes seals are integrated into the bearings, so there is no need to design them. [19]

2.3.9 Sensors and Actuator Intelligence

Actuators have become more and more intelligent through the includes of embedded sensors. Ideally, the actuator should be aware of its own state and the outer environment. [30] Table 2-2 lists commonly used sensors in actuators.

Sensor Domain	Primary Choice	Secondary Choice
Position	Absolute Encoder	Resolver
Velocity	Incremental Encoder	Tacho-generator
Acceleration	Ferraris' Principle	Force-balance
Torque	Piezo-electric	Surface Acoustic Wave
Temperature	Thermistor	Resistance Temp Detector
Noise	Condenser Microph.	Piezo-electric Microph.
Vibration	Piezo-electric	Piezo-resistive accel.
Magnetic Field	Hall Effect	Closed Loop Hall Effect
Current	Magneto-resistive	Closed Loop Hall Effect
Voltage	Closed Loop Hall Effect	Closed Loop Hall Effect

Table 2-2: Sensors for intelligent actuators [19]

A position sensor is critical to the operation of the motor, which requires position information for its control method. In general, the position sensor is integrated inside the motor. Additional position sensors can be used to monitor the angular position of the robot axes to provide the controller with more information. Examples of position sensors include potentiometers, resolvers, and different kinds of encoders. [31]

Velocity and acceleration sensors can provide more information for the motor and the system. With more detailed information about the movement of the axes, new control strategies can be developed to improve the performance of the robot, and new functions can be realized.

Torque sensors are helpful in monitoring the torque status, external forces and torques, and load condition of the robot axes. Torque sensors are currently used in collaborative robots like the KUKA LBR iiwa. [17] Even though the torque output of the motor can be estimated based on voltage and current, the torque at the end point can be

quite different from the calculated torque due to backlash and other interference in the gear train. The torque sensor has been proven to be an effective tool to monitor the working status for collaborative robots.

The vibration sensor is used for detecting operational quality for traditional robots, but, for collaborative robots, it becomes a new tool to ensure safety and protect humans working with robots. Collisions between a robotic arm and humans or other obstacles can be easily sensed by a vibration sensor and transmitted to the controller to deal with. The disadvantage of this control method is that a vibration sensor is more easily affected by environmental noise. Accelerometers are often used as vibration sensors.

2.4 Chapter Summary

In this chapter, industrial robots, particularly collaborative robots, are introduced. The structure of the actuator and important elements in actuator design are discussed to assist gear train selection. Through this study of collaborative robot characteristics, we expect our final gear train to have relatively high performance, to be lightweight and low in cost, and to meet the strength requirements of the actuator.

Chapter 3: Gear Trains

Gear trains are widely used in industrial equipment as a general element of transmitting power and motion in machinery. Important functions of the gear train are relocating a force or torque and amplifying the torque and while reducing the speed to meet the requirements of the load. And for this reason, gear trains work well with motors, and perform an irreplaceable role in actuator design. A gear train allows a designer to adjust the power, speed, and torque available for the intended application.

Gear trains also have disadvantages. Compared with a motor only, adding a gear train to the system causes complexity, backlash, noise, and some power losses. These disadvantages can be minimized by good design. This chapter introduces gear design information that is useful in the preliminary design of electromechanical actuators.

3.1 Gear Basics

A gear is the fundamental element of a gear train. Gear technology is relatively mature, and the gear has proven to be a reliable, efficient and versatile method of power transfer that provides much design flexibility for engineers.

3.1.1 Basic Gear Nomenclature

A brief summary of relevant gear terminology is important to understanding the research that is presented here. Gear standards have been developed by both the American Gear Manufacturers Association (AGMA) and the International Organization for Standardization (ISO). For this research, the AGMA standards were largely adopted. The symbols used in this thesis to represent important gear parameters are listed in Table 3-1.

AGMA Symbol	Unit	Definition
N_P	#	Number of Teeth on Pinion
N_g	#	Number of Teeth on Gear
D	in	Diameter
P_d	1/in	Diametral Pitch
g	#	Reduction Ratio
F	in	Face Width
ϕ	Degree	Pressure Angle
φ	Degree	Helix Angle

Table 3-1: Basic Gear Geometric Parameters

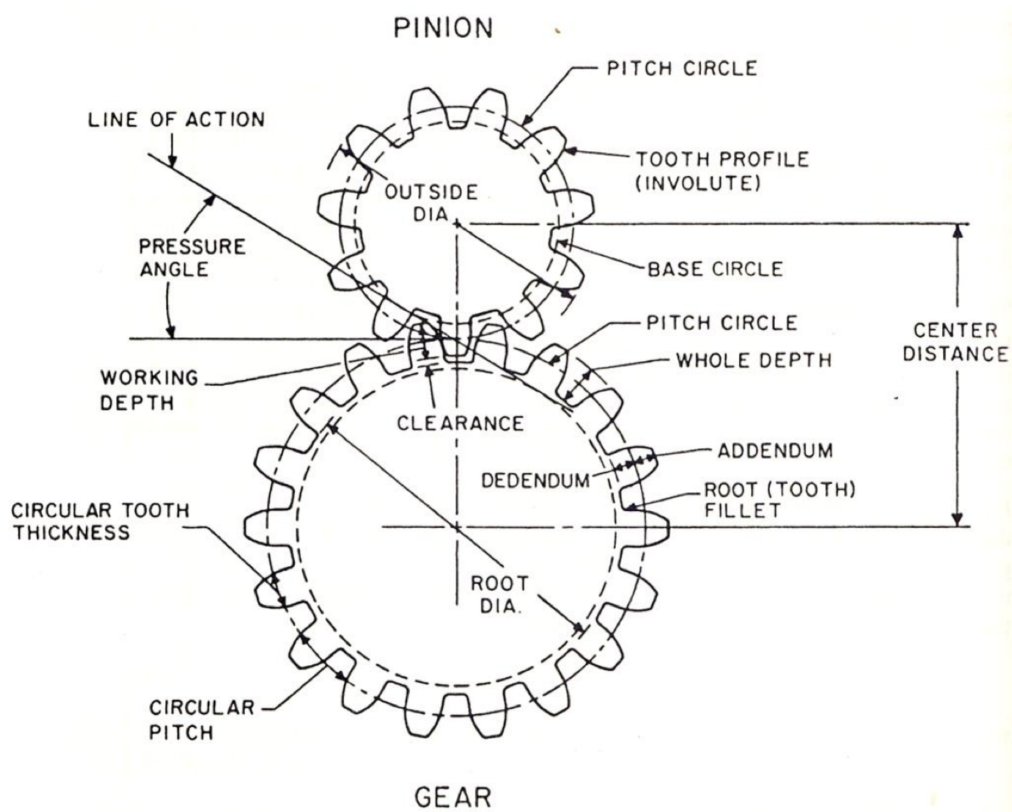

Figure 3-1: Basic Gear Nomenclature [32]

- Pinion and gear

  The gear with fewer gear teeth and the smaller diameter of two meshed gears is called the pinion (see Figure 3-1). The larger one is referred to as the gear. For the designs in this study, the pinion is always the driving element and the gear is the driven element.

- Pitch diameter

  The pitch diameter is the diameter of the pitch circle, which is an imaginary circle that is the working circle when the gears are replaced by disks for the same motion transmission. [33]

- Diametral pitch P_d (1/in)

  The tooth shape on meshed gears is always the same. The diametrical pitch is the measure of the tooth size in the AGMA standard. The diametral pitch is the ratio of the number of teeth on the gear to the pitch diameter (in units of inches):

$$P_d = \frac{N}{D}$$

- Module m (mm)

  The module is the measure of tooth size in the ISO standard. The module is the ratio of the pitch diameter (in units of millimeters) to the number of teeth:

$$m = \frac{D}{N}$$

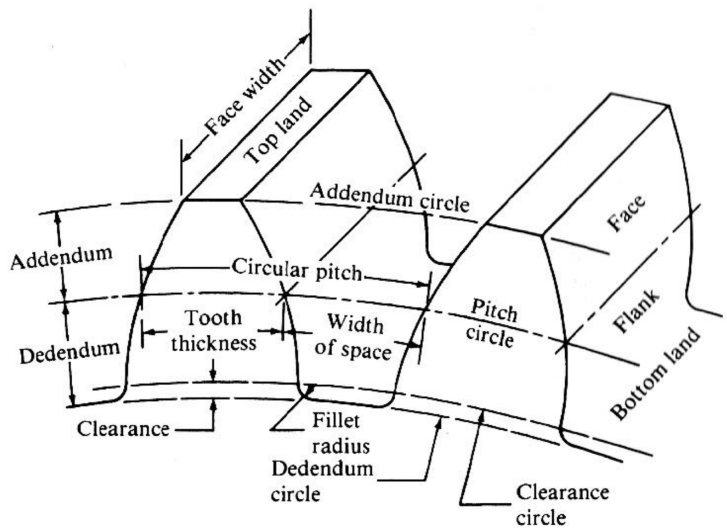

Figure 3-2: Basic Gear Teeth Nomenclature [34]

- Line of action

  The line of action is the line through the pitch point and tangential to the base circle. In general, the force between two meshed gears is transmitted along the line of action. The line of action is a very important geometric feature for mechanical analysis of gears.

- Pressure angle ϕ

  The pressure angle is the angle between the line of action and the common tangent of the pitch circles of two meshed gears.

- Pitch point

  The pitch point is the point of tangency of two pitch circles. [35]

3.1.2 Gear Types

3.1.2.1 External Gear and Internal Gear

An external gear is a gear with the teeth on the outer surface. Conversely, an internal gear has its teeth on the inner surface. In actuator gear train design, use of an internal gear as the last gear of a star compound gear train provides a greater reduction ratio than an external gear.

3.1.2.2 Spur gears

Spur gears are the most commonly used gears in industry because of their relatively simple structure, which makes the design and manufacturing easier, and low cost. Spur gears have teeth parallel to the axis of rotation, and therefore can only support loads perpendicular to the axis of rotation, not in the axial direction. The pitch-line velocities of spur gears are limited to 4000 fpm to avoid vibrations and noise. [36]

3.1.2.3 Helical gears

Helical gears have their teeth cut at an angle to the axis of rotation. For this reason, helical gears can resist forces applied in the axial direction. Therefore, a gear train composed of helical gears tends to be more stable, and works very well in an environment with higher vibration. Another advantage of helical gears compared to spur gears is that the teeth of helical gears come into contact gradually in the meshing process, and therefore work smoothly.

3.1.3 Tooth Profile

For angular motion and force to be smoothly transmitted throughout the meshing cycle, the mesh must also provide a constant angular velocity throughout the meshing action. The involute curve of a circle meets these requirements. As shown in Figure 3-3,

the involute profile is produced by “unwrapping a string wound around the base circle of the gear”. [35]

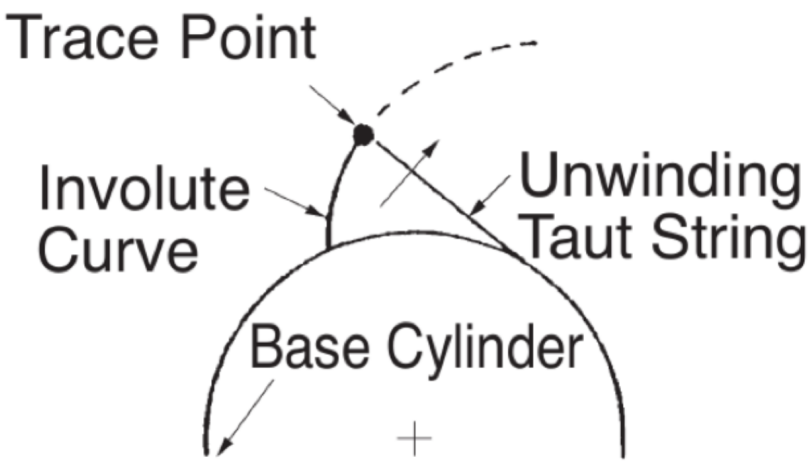

Figure 3-3: Generation of involute profile [37]

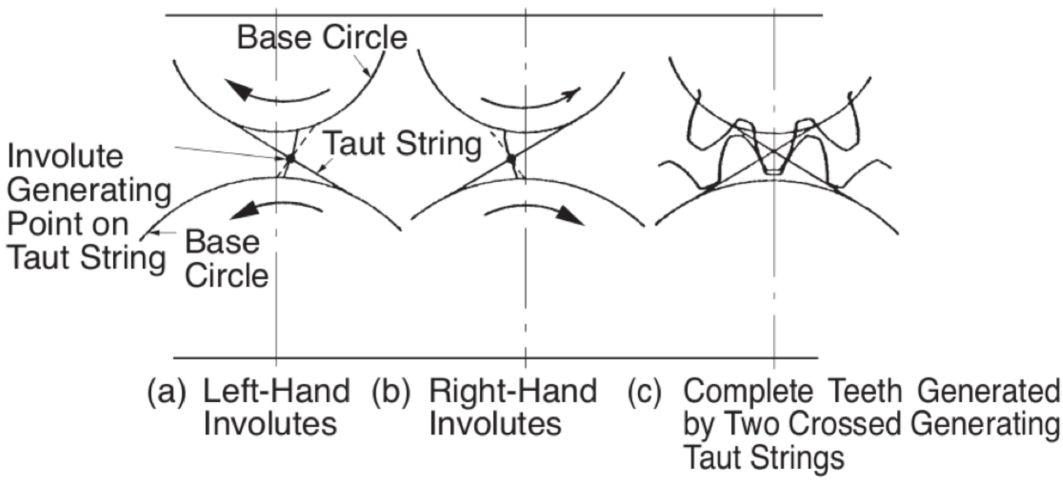

Figure 3-4: Mesh of Involutes [37]

Figure 3-4 shows that the pitch point of two meshed gears with involute tooth profiles is always at the center of the mesh, which allows the gear train to run smoothly. For this study, the involute gear tooth profile was selected and assumed for the design process developed.

3.1.4 Diametral Pitch Choice

The diametral pitch is used to describe the size of the gear teeth. As the diametral pitch is the ratio of the number of teeth to the pitch diameter, a smaller diametral pitch means fewer teeth on one gear, and therefore the teeth are larger and stronger. However, the module, the parameter to measure tooth size in ISO standard, is opposite. Since the module is the ratio of the diameter to the number of teeth, a larger module means larger and stronger teeth.

The relation between the module and the diametral pitch is:

$$P_d = \frac{25.4}{m}$$

where the diametral pitch is in units of 1/in, and the module is in units of mm.

Figure 3-5: Gear Tooth Size as a Function of Diametral Pitch [38]

Figure 3-5 illustrates the relative gear sizes with different diametral pitches. A gear tooth with a larger diametral pitch can transmit larger forces and torques. In an actuator gear train, the gear closer to the output shaft needs to bear larger torque, so it is better to choose a gear with a smaller diametral pitch. This phenomenon is discussed in a later chapter where the design process is illustrated.

The actual diametral pitch and module cannot be chosen freely without limitations. To reduce manufacturing cost, both AGMA and ISO have provided a standards to assist both manufacturers and designers. These organizations have established a series of modules and diametral pitches commonly used in industry (see Table 3-2). A gear chosen from these series can be easily purchased at low price. Gear

sizes that are not in a series must be custom fabricated and may be very expensive. A search of McMaster-Carr Supply Company (City, State, USA) reveals that gears with diametral pitches between 6 and 64 are easy to find, and it is better to choose gears with diametral pitches in this range. [39]

Diametral Pitches in General Use	
Coarse pitch	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine pitch	20, 24, 32, 40, 48, 64, 96, 120, 150, 200
Modules in General Use	
Preferred	1,1.25,1.5,2,2.5,3,4,5,6,8,10,12,16,20,25,32,40,50
Next Choice	1.125,1.375,1.75,2.25,2.75,3.5,4.5,5.5,7,9,11,14,18,22,28,36,45

Table 3-2: Commonly Used Diametral Pitches and Modules [40]

3.1.5 Pressure Angle Choice

The pressure angle also determines the gear tooth shape. Because the line of action goes through the pitch point and is tangential to the base circle, a smaller pressure angle means a larger base circle. With the same diametral pitch, a gear with a smaller pressure angle usually has a larger tooth thickness (see Figure 3-6). An increase in pressure angle also decreases the tangential force, but increases the radial force. Gears and cutting tools with pressure angles of 20°are most commonly used in industry; thus, a pressure angle of 20° is chosen for the design process developed in this research.

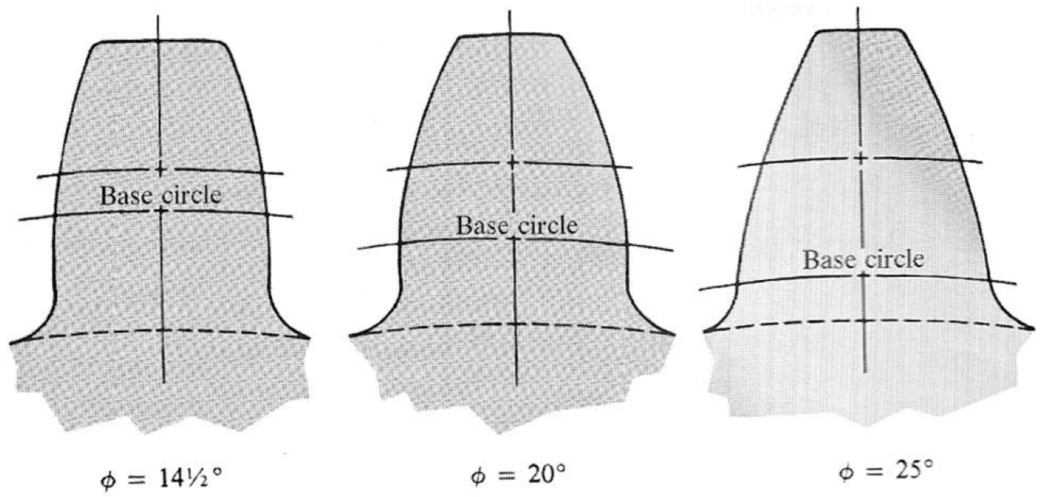

Figure 3-6: Involute Tooth Form for varying Pressure angles [34]

The AGMA has also provided standards for other gear teeth features, such as the addendum, the dedendum, whole depth, and clearance, based on the diametral pitch and pressure angle (see Table 3-3). Therefore, once P_d and $\emptyset$ are chosen, most of the other parameters about a gear are determined.

	Coarse Pitch $P_d < 20$	Fine Pitch $P_d \geq 20$
Pressure Angle	20° or 25°	20°
Addendum	$1.000/P_d$	$1.000/P_d$
Deddendum	$1.250/P_d$	$(1.200/P_d)$+0.002(min)
Whole Depth	$2.250/P_d$	$(2.200/P_d)$+0.002(min)
Working Depth	$2.000/P_d$	$2.000/P_d$
Clearance (basic)	$0.250/P_d$	$(0.200/P_d)$+0.002(min)
Clearance (shaved or ground teeth)	$0.350/P_d$	$(0.350/P_d)$+0.002(min)
Circular Tooth Thickness	$1.571/P_d$	$1.571/P_d$

Table 3-3: Standard Proportions – AGMA Full-Depth Gear Teeth [36]

3.1.6 Number of Gear Teeth

The reduction ratio of a gear train is usually calculated as the ratio of the number of teeth of the gear to that of the pinion. Thus, the number of gear teeth is the most important parameter to choose when a specific reduction ratio is required.

The number of gear teeth is limited by the manufacturing method. Phenomena such as undercutting, which can reduce the bending torque capacity of the gear, should be avoided. A minimum number of teeth should be used to avoid interference:

$$N_{min} = \frac{2}{sin^2\emptyset}$$

The minimum number of gear teeth is determined by the pressure angle, and decreases when a larger pressure angle is used. As discussed earlier, a pressure angle of 20° is chosen for the design examples in this thesis. Thus, the minimum number of gear teeth used in the design process is 18 (see Table 3-4). Even though there are no strict limitations for the maximum number of gear teeth, it is better to choose gears with less than 135 teeth to avoid difficulties in gear manufacture. [41] In this thesis, the number of gear teeth is chosen from 18 to 135. To avoid having too many teeth on a gear, the reduction ratio for one reduction stage is no larger than 6.

Tooth Form	Minimum number of teeth
14.5^{o} , involute, full-depth	32
20° , involute, full-depth	18
25° , involute, full-depth	12

Table 3-4: Recommended minimum number of pinion teeth [34]

3.2 GEAR TRAIN

Different combinations of meshed gears form a surprising number of different kinds of gear trains, which have different properties and advantages to meet the design requirements.

3.2.1 Simple Gear Pairs

Simple gear pairs make use of the simplest type of gears, each fixed on one shaft separately, to transmit torque and power. Generally, with simple gear pairs, each gear needs one shaft, and all shafts are parallel to each other, as shown in Figure 3-7.

Simple gear pairs can be combined using several gears with the same diametral pitch, and the gears between the driver gear and the driven gear are called idler gears. A characteristic of simple gear pairs is that the idler gears do not affect the gear ratio. The gear ratio is determined only by the number of teeth of the first driver gear and the last driven gear, as shown in the equation below:

$$r = \frac{N_B}{N_A}\frac{N_C}{N_B}\frac{N_D}{N_C} = \frac{N_D}{N_A}$$

Thus, simple gear pair trains usually cannot achieve a very high reduction ratio, and their main objective is to relocate the torque and power.

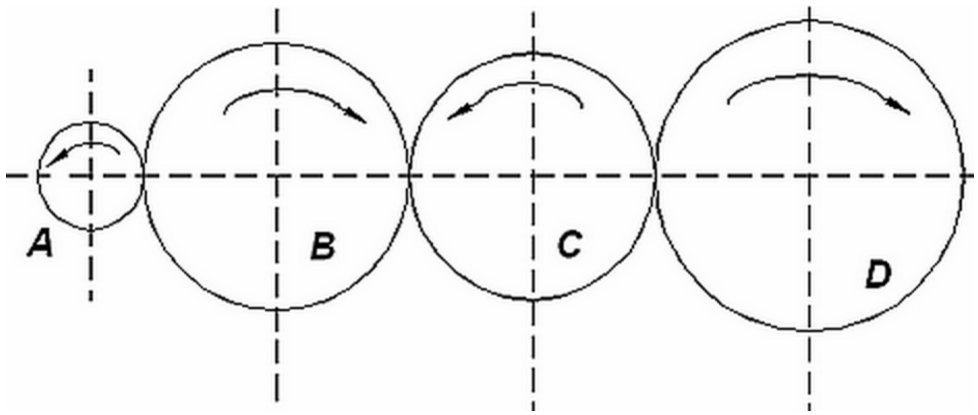

Figure 3-7: Simple Gear Pairs [42]

Another characteristic of simple gear pair trains is the rotation direction changes as each successive gear is added. The driven gear has the same rotation direction as the driver gear with an odd number of idler gears, and the opposite direction with an even number of idlers.

3.2.2 Split Path Gearing

A split path gear train is actually not one kind of gear train, but a gear combination method. In a split path gear train, several gears of the same size are connected to the driver gear and finally rejoin back to the driven gear, as shown in Figure 3-8.

The objective of a split path gear train is to improve the total load capacity by sharing a heavy load with multiple gears. Additionally, a symmetric structure also contributes to system stability, which is quite obvious on planetary gear trains. Split path gear sets can be applied in other types of gear trains to share heavy loads.

In a split path gear train, the shaft locations are fixed, as with a simple gear pair train. Only one degree of freedom exists, although torque may be divided between several shafts. [40] Split path gear trains require high manufacturing and installation

accuracy. When the loads are distributed unevenly among multiple gears, the one with the largest load may fail. Additionally, any asymmetry can cause excessive vibration.

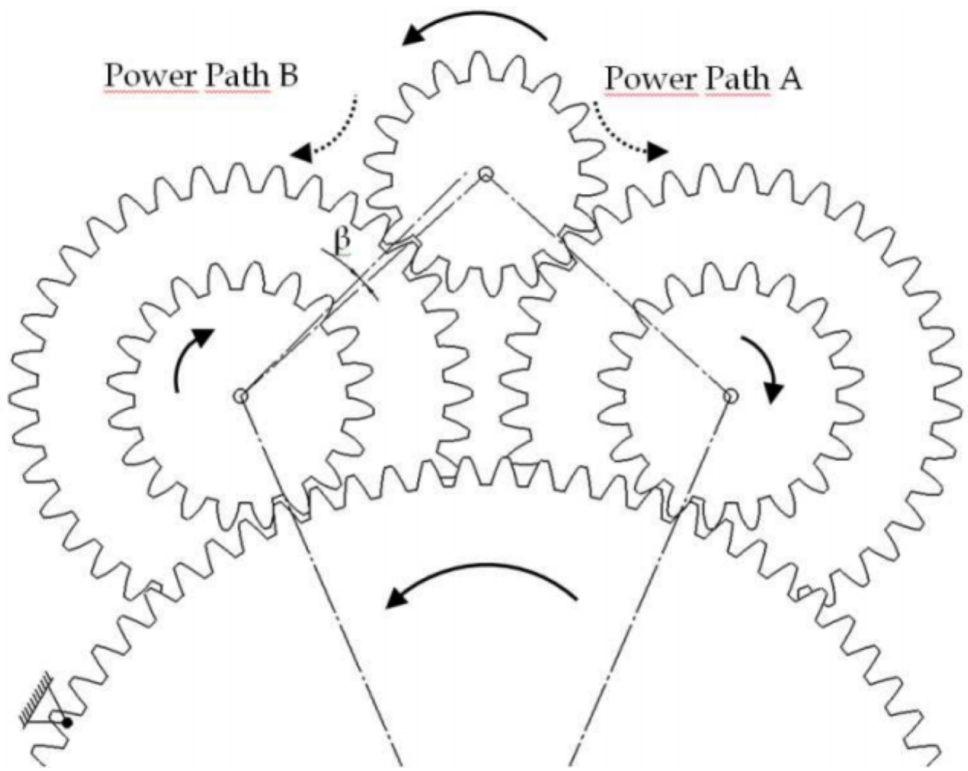

Figure 3-8: Split Path Gearing [43]

3.2.3 Compound Gearing

A compound gear is actually a combination of two simple gears fixed on one shaft (see gears No.2 and No.3 in Figure 3-9). In most cases, the diameters of the two simple gears are different. While these two simple gears have the same angular velocity, they will have different pitch line velocities.

Each of the two simple gears will then be meshed with another gear. For example, if a large simple gear is connected to the pinion, the larger diameter compared to the driver gear results in a reduction ratio. When the small gear is connected to the driven

gear, the smaller diameter compared to the driven gear causes another reduction ratio. The total reduction ratio is the product of these two. Compound gearing performs well in speed reduction and requires less space compared with other gear train types, and therefore is widely used in actuator gear trains.

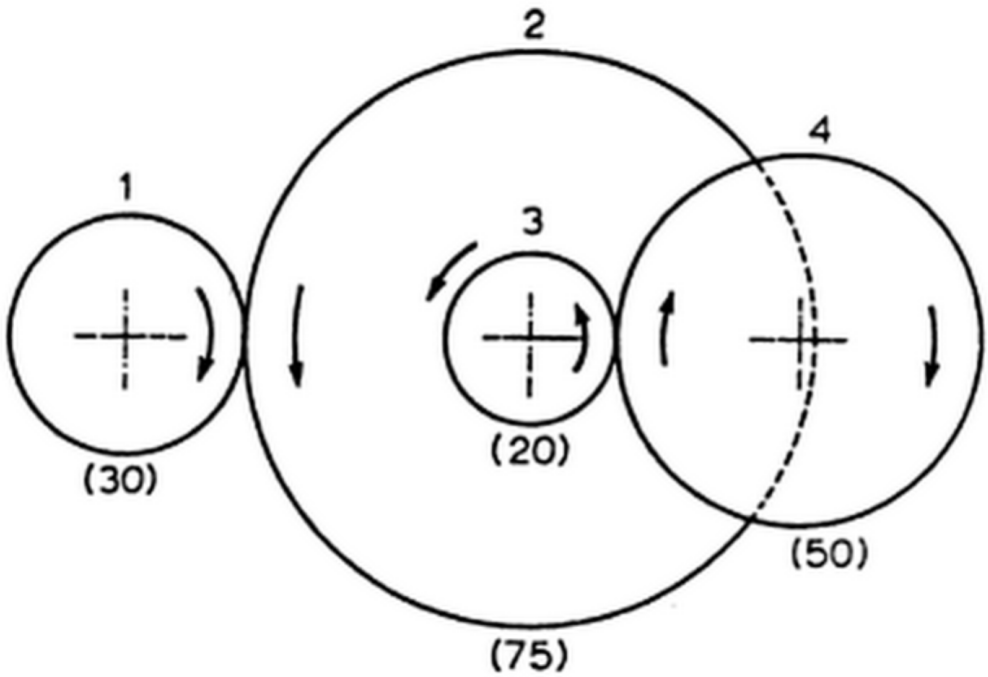

Figure 3-9: Compound Gearing [44]

3.2.4 Epicyclic Gear Sets

Epicyclic gear sets are composed of a sun gear, an internal ring gear and several planet gears in the split path gearing pattern. The sun gear and the internal ring gear are concentric, and the planet gears are symmetric to the common axis of the sun gear and the ring gear. The planet gears mesh with both the sun gear and the ring gear, and therefore, have the same rotation speed and revolution speed. To increase stability, a

carrier is always used to link these multiple planet gears. Each planet gear can rotate freely around the axis provided by the carrier.

Epicyclic gear sets have two degrees of freedom, which means one more constraint is needed to determine the state of the gear set. The constraint is determined by choosing one gear to be fixed. There are three different options, each resulting in a gear set with a specific type of behavior.

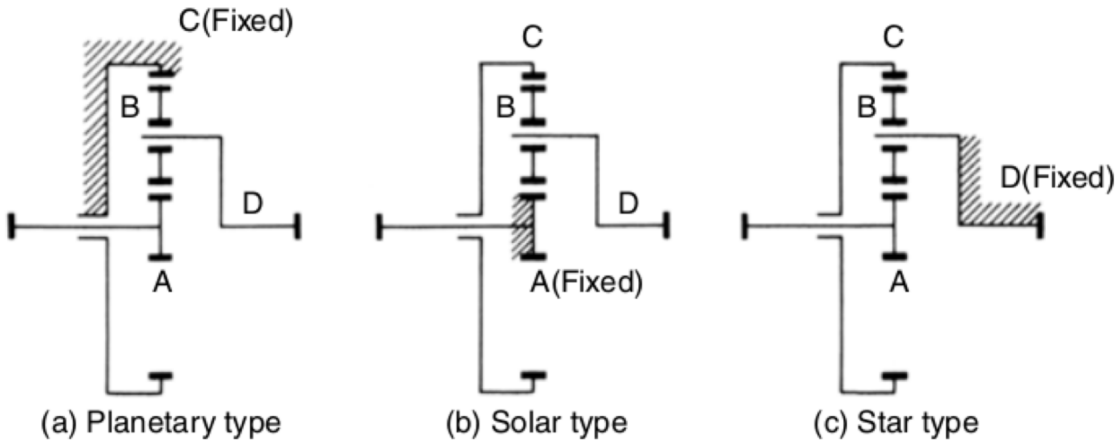

Figure 3-10: Basic Planetary Transmission Types [45]

As shown in Figure 3-10, when the ring gear is fixed, the gear set is called a planetary gear set. When the sun gear is fixed, the gear set is known as a solar gear set. And when the carrier of the planet gears is fixed, the gear set is called a star gear set.

3.2.5 Hypocycloidal Gearing

Hypocyclic gear sets are a special type of compound epicyclic gear train. However, the hypocyclic gear sets have no sun gear, and the input is an eccentric shaft. A wobble gear is created by fixing two external gears together. There are also two internal

ring gears. The first ring gear is fixed, and the second one is the output. An example of hypocycloidal gearing is shown in Figure 3-11.

For analysis purposes, a hypocycloidal gear set can be divided into two parts, the left part and the right part. Because the shaft is mounted to the wobble gear eccentrically, it acts as a carrier. Both parts thus work similar to an epicyclic gear set.

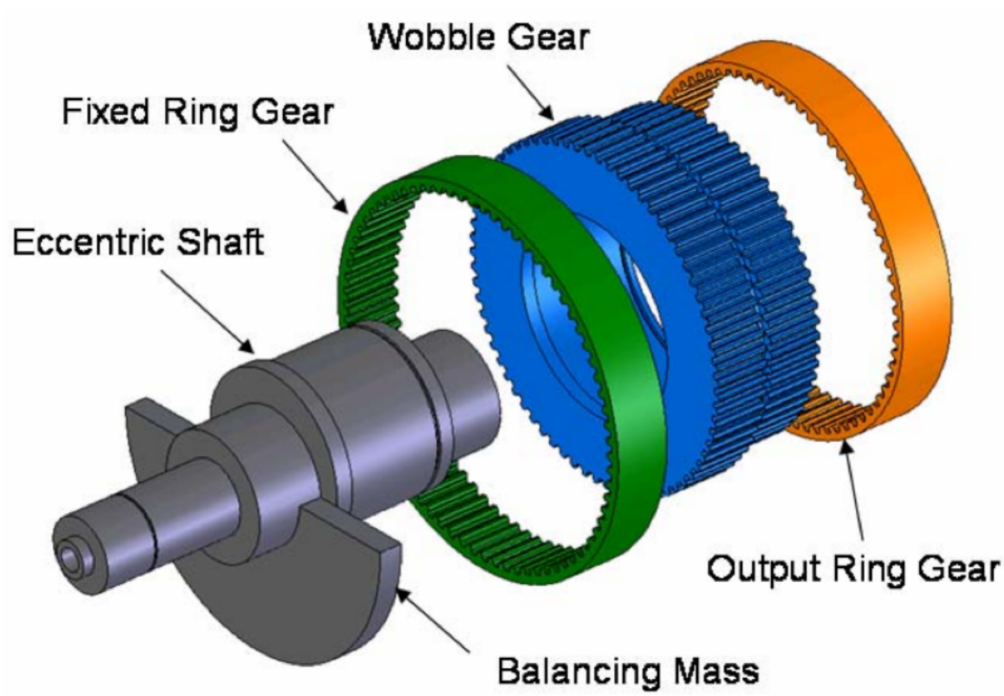

Figure 3-11: Hypocyclic Gear Train [46]

Due to their use of the differential gearing principle, hypocyclic gear sets are capable of extremely high gear ratios. The velocity ratio of a hypocyclic gear set is:

$$g = \frac{N_{w1}N_{r2}}{N_{w2}N_{r1} - N_{w1}N_{r2}} = \frac{d_{w1}d_{r2}}{d_{w2}d_{r1} - d_{w1}d_{r2}}$$

where N_{w_1} is the first wobble gear, N_{w_2} the second wobble gear, N_{r_1} the first ring gear, and N_{r_2} the second ring gear. It can readily be seen that if the product of N_{w_2} and N_{r_1} approaches the product of N_{w_1} and N_{r_2}, the velocity ratio approaches infinity. This is practically limited by the requirement of an integer number of teeth and standardized tooth sizes.

A hypocyclic gear train can achieve an extremely high gear reduction ratio. But there are still several difficulties to overcome. Since the shaft is eccentric, the gear train actually operates in an unbalance state, and additional counterbalancing is needed. Additionally, hypocyclic gear sets are very sensitive to manufacturing accuracy, and small errors result in large vibrations and resulting operational difficulties.

3.2.6 Star Compound Gear Train

The Star Compound Gear Train (SCGT) is already widely used for its outstanding characteristics. The SCGT can have a relatively high reduction gear ratio, and can transfer large power and torque. It also has a simple structure and is very small. A SCGT is simple in structure, and is easy to manufacture and assemble. In a SCGT, all the axes of the meshing gears are parallel to each other and are in fixed rigid structures. All the gears used for a SCGT are spur gears or helical gears, with no intersecting-axes gears like bevel gears, worm gears, and hypoid gears.

To achieve exact reduction ratio and dimension requirements, three types of SCGT have been developed for different situations.

1. One-Stage Star Compound Gear Train (1-Stage SCGT)
2. Pancake-Type Two-Stage Star Compound Gear Train (P-Type 2-Stage SCGT)
3. Coffee Can-Type Two-Stage Star Compound Gear Train (C-Type 2-Stage SCGT)

The 1-Stage SCGT is a two-grade reducing mechanism, and the P-Type and C-Type 2-Stage SCGT are three-grade and four-grade reducing mechanisms, respectively.

3.2.7 Gear Train Summary

A full and comprehensive discussion of the commonly used gear trains is quite helpful to final selection for the design of a robotic actuator. For collaborative robots, the reduction ratio, structure compactness, system stability, manufacture and installation complexity, and cost should all be considered.

Simple gear pair sets work for situations when the power should be relocated from the original position. The final reduction gear ratio is determined only by the number of teeth of the driver gear and the driven gear, and therefore is equal to a gear train with only one grade of reduction. Also, the volume needed for the simple gear pair trains is relatively large.

Split path gearing is a method to use multiple gears to share a heavy load, and can be used on other kinds of gear trains to improve the system torque capacity and system stability. For split path gears, manufacturing and installation accuracy are quite important.

A compound gear can provide a large total reduction ratio. For larger gear meshes with a smaller gear in the earlier stage, and with the small gear meshing with a larger gear in next stage, two reduction grades are created by only one compound gear.

An epicyclic gear train uses split path gearing to share the torque for the planet gears. By choosing different fixed gears, epicyclic gear trains are quite flexible. With the carrier gear fixed, the epicyclic gear train is called a star gear train and has better stability than two other alternatives.

A hypocyclic gear train can achieve a very high reduction ratio. However, several technical problems must be solved and the cost must be reduced before this gear train type becomes a leading product in the market. Hypocyclic gear sets are very sensitive to manufacture and installation accuracy.

Among all the gear train types discussed above, the star compound gear train is selected as the best gear train design to fulfill the requirements of robot arm actuators. For different reduction ratios and geometry requirements, three types of SCGT, 1-Stage SCGT, 2-Stage P-Type SCGT, and 2-Stage C-Type SCGT are designed separately, and their properties are also discussed to illustrate their separate advantages and the preferred design for each type.

3.3 Gear Train Features

For collaborate robots, some features of gear trains require additional attention. To ensure safety when robots work together with operators, the working conditions of the robots should be monitored and controlled closely. However, errors and delays caused by certain gear train features may reduce the performance of the robotic arm. Two such features worth noting are backlash and lost motion.

3.3.1 Backlash

Backlash is the phenomenon caused by the width of tooth space being larger than the width of the tooth (see Figure 3-12). Although zero-backlash gears are necessary in some applications such as precision positioning mechanisms, in most applications, some backlash is recommended to account for manufacturing tolerances, temperature effects, and tooth lubrication. Table 3-5 shows the recommended backlash corresponding to certain diametral pitches and center distances. [34]

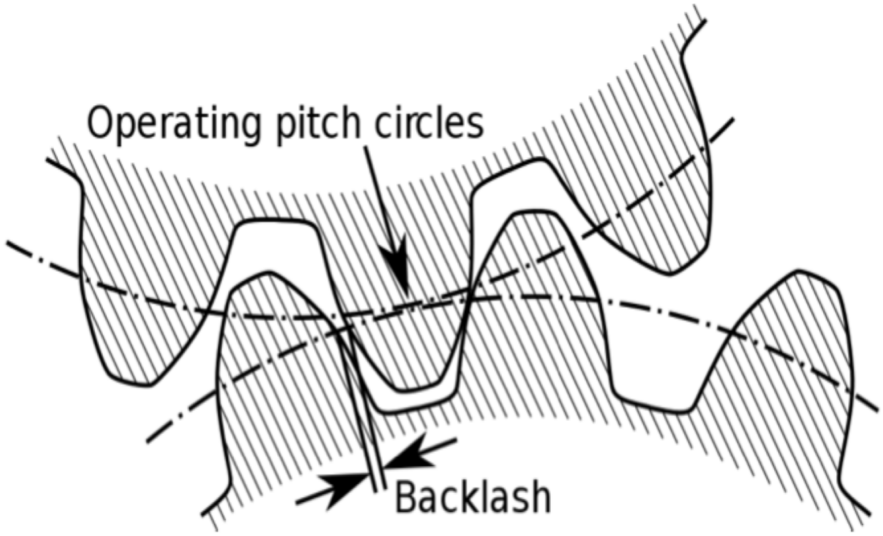

Figure 3-12: Backlash [34]

Diametral Pitch	Circular Pitch	Center Distance, C (in)				
		2	4	8	16	32
18	0.17	0.005	0.006			
12	0.26	0.006	0.007	0.009		
8	0.39	0.007	0.008	0.010	0.014	
5	0.63		0.010	0.012	0.016	
3	1.05		0.014	0.016	0.020	0.028
2	1.57			0.021	0.025	0.033
1.25	2.51				0.034	0.042

Table 3-5: Recommended minimum backlash (in) for coarse pitch gears [34]

3.3.2 Lost Motion

In addition to backlash, there exists some degree of lost motion in gear trains. As the load switches directions, the load will shift from one face of the tooth to the other. The deformation that takes place during this transition is known as lost motion. Lost motion must be considered in highly loaded precision gear trains, as the precision is reduced as backlash and lost motion increases. Figure 3-13 illustrates backlash and lost motion in the load direction switch process.

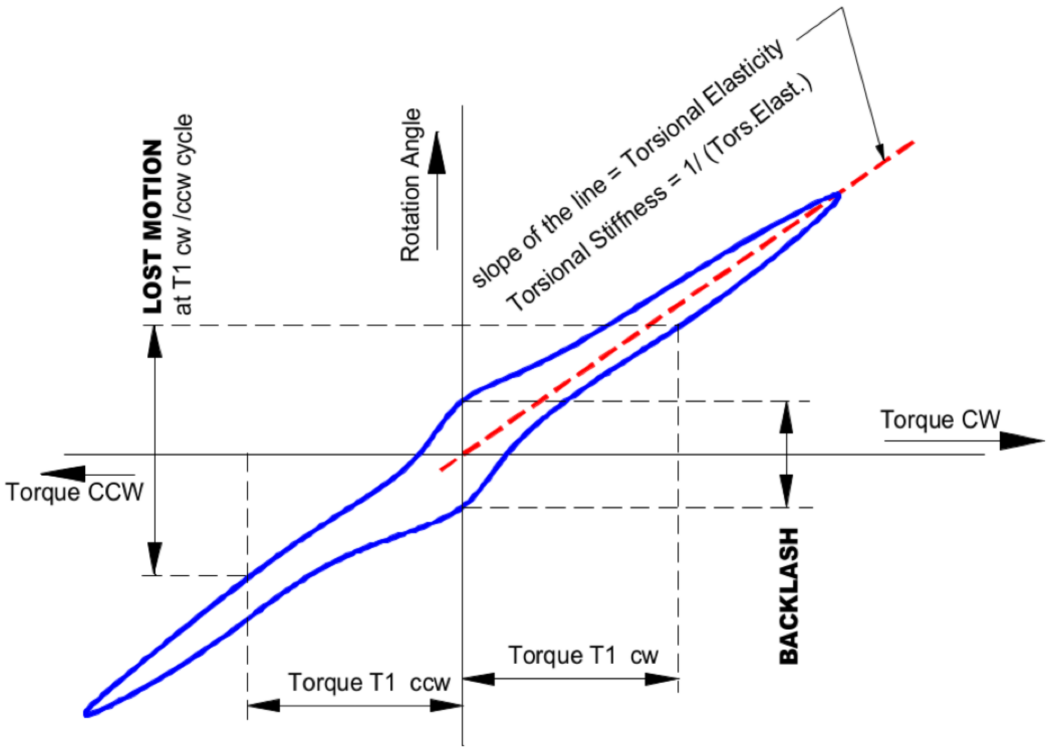

Figure 3-13: Backlash & Lost Motion [47]

Backlash and lost motion phenomena can be ignored in situations where the required accuracy is not high. For collaborative robots, however, these factors must be considered. The backlash of a gear train may delay the response time for a torque sensor,

and may cause safety problems. Lost motion can reduce the position control accuracy of robotic arms. Therefore, these factors must be accounted for in the design of actuator gear trains for precision robots.

3.4 Gear Failure

Gear failure is closely related to system stability and actuator life. The life of a gear train should be considered during the design process. In general, life is measured as the number of cycles. Consideration of gear failure types and ways to avoid failure increases reliability in gear train performance. AGMA has classified 36 modes of gear failure under the broad categories of wear, scuffing, plastic deformation, contact fatigue, cracking, fracture, and bending fatigue, as shown in Table 3-6.

Category			
Contact Fatigue	Wear	Plastic Deformation	Scuffing
Pitting	Adhesion	Indentation	
Micro-pitting	Abrasion	Cold Flow	
Subcase Fatigue	Polishing	Hot Flow	
	Corrosion	Rolling	
	Fretting Corrosion	Tooth Hammer	
	Scaling	Rippling	
	Cavitation	Ridging	
	Erosion	Burr	
	Electrical Discharge	Root Fillet Yielding	
	Rippling	Tip-to-root Interference	
Bending Fatigue	Cracking	Fracture	
Low-cycle Fatigue	Hardening Cracks	Brittle Fracture	
High-cycle Fatigue	Grinding Cracks	Ductile Fracture	
	Rim and Web Cracks	Tooth Shear	
	Case/Core Separation	Fracture After Plastic Deformation	
	Fatigue Cracks		

Table 3-6: Gear Failure Nomenclature Recommended by the American Gear Manufacturers Association (AGMA) [48]

From the gear failure table shown above, many gear failure types have been identified. In our study, several of these have been included for consideration in the design process.

Bending fatigue is a major cause of gear failure. As the inner structure of the gear teeth changes due to cyclic bending loads, the strength of the gear teeth decreases, and resistance to impact is reduced. The bending fatigue limit is mainly determined by material strength, core hardness, and ultimate tensile stress. Two methods are employed to avoid bending failure. First, heat treatments or other treatments after gear formation

can significantly increase the resistance to fatigue. For example, a through-hardened gear performs much better than one made of the original material. Second, an appropriate design for the gear train ensures a significant factor of safety in its torque capacity.

Pitting fatigue influences wear, and nearly all fatigue occurs at the gear tooth surface. This type of failure depends on the material surface hardness, the lubrication condition, working impact, and vibration. The pitting fatigue limit of the material represents its ability to resist pitting failure. The main method employed to avoid pitting fatigue from the material perspective is to harden the material surface properly. Additionally, external factors can be improved, such as the working environment, by reducing vibrations and impurities in the lubricant.

3.5 Chapter Summary

This chapter first introduced basic knowledge of gears. The terminology and formulas used in gear train design have been discussed. Several different gear train types were introduced and compared, and the star compound gear train was selected the most appropriate configuration for actuators in collaborative bots. Gear train features, particularly backlash and lost motion, which are important considerations for collaborative robots, and gear failure modes were discussed. In next chapter, gear materials and manufacturing methods are discussed.

Chapter 4: Material Characteristics

A gear train can be made of various materials, such as steel, cast iron, nonferrous alloys, and even plastics. Steel is most commonly used because of its high strength, relatively low cost and the mature manufacturing technology. Plastic gears also have their own advantages, such as low weight, low cost, dirt tolerance, and no additional lubrication needed. To realize objectives of lightweight and low cost design, materials that have low density, good machinability and are inexpensive will be favored, assuming the mechanical requirements are met. Multiple criteria must be considered in material selection to make a comprehensive comparison.

In general, material selection for gears is based mainly on four perspectives: mechanical properties, manufacture and processing, cost, and other properties. The mechanical properties include strength, fatigue resistance, and surface hardness, among others. Manufacture and processing considerations include machinability, maturity of manufacturing technology, and property improvement post heat treatment. Other properties here refer to the material's special ability in a specific environment. For example, stainless steel is appropriate for machines used for processing food, particularly when the gear is exposed to food. Plastic gears work well without lubrication.

This chapter consists of a brief introduction and comparison of material alternatives for gears. Manufacturing technologies and material treatment methods are also discussed.

4.1 Metallic Materials

Metallic gear materials can be divided into ferrous and nonferrous alloys. The most commonly used ferrous alloys are surface hardened and through hardened carbon alloy steels. Other ferrous alloys used for gears are cast irons, cast steels, powder

metallurgy (P/M) irons and steels, stainless steels, and tool steels. [6] Commonly used nonferrous alloys are copper-base, aluminum-base, and zinc-base alloys.

4.1.1 Steel

Steels are the most commonly used materials for transmissions subjected to heavy loads, "because of their high strength-to-weight ratio and relatively low cost, they are the most widely used gear materials for heavy duty, power transmission applications". [6] Most steel gears are made of carbon and low-alloy steels. Carburized gears achieve very high performance after carburization. The material used in the design example is carburized steel.

4.1.2 Cast Irons

Cast iron actually is a family of metallic materials, including gray iron, ductile cast iron, and P/M irons. Cast iron gears have "good resistance to wear", but the bending strength capacity is usually about one third that of steel. [6]

Gray cast iron should not be used under situations with shock loads because it has poor resistance to impact, and ductile iron has relatively good strength and hardness after heat treatment. [6]

4.1.3 Nonferrous Alloys

Copper-base, aluminum-base, and zinc-base alloys account for much of the nonferrous gear materials used. [6] Tin bronze is copper-tin that is deoxidized with phosphorus, which has a rough surface and good corrosion resistance. [6] Aluminum bronzes are light in weight and attain improved mechanical properties through heat treatments, and "as the strength of aluminum bronzes increases, their ductility decreases". [6]

4.2 Plastic Materials

Properly used plastic gears can replace steel in some specific situations. And they also have some very good properties, such light weight, low cost, and operation without lubrication. [49]

Plastic gears were not considered for high power, high speed applications until recently. They are now widely accepted due to the development of new materials with better properties, improved molding technology, and cost reduction. [6]

4.2.1 Advantages and Disadvantages

Among the characteristics responsible for the growth in plastic gear usage, the following properties are the most significant: [6]

- Low cost (due to injection molding)
- Fast manufacturing speed
- Complex shapes possible
- No finishing operations needed
- Lower density (light weight and inertia)
- Ability to dampen impact
- Ability to operate with minimal or no lubrication
- Low coefficient of friction
- Smooth, quiet operation

There are also limitations of plastic gears relative to metal gears, including:

- Lower torque capacity
- Poor resistance to high temperatures
- Lower accuracy
- High initial mold costs (expensive when quantity is low)
- Easily affected by harsh chemicals

4.2.2 Plastic material categories

4.2.2.1 Nylons

Nylons are among the most commonly used plastic materials today. Commonly used nylons are Nylon 6, 6/6, and 6/12. Nylon parts are accepted by the market for their "outstanding toughness and wear resistance, low coefficient of friction, and excellent electrical properties and chemical resistance". [6]

4.2.2.2 Acetal

Acetal is a very stable plastic material known for its "low water absorption rate", relatively high resistance to fatigue and low coefficient of friction. [6] Acetal can be used in relatively severe environments, especially with poor lubrication and exposure to harsh chemicals.

4.3 Manufacturing Methods and Post Treatments

For the irreplaceable function and large scale use of gears in industry, quite a lot of research and development have been performed to improve their manufacturing and post treatment technology. Among the manufacturing methods, material removal methods are the most mature technologies for metal gears, and injection molding is widely used for plastic gears.

4.3.1 Metal Removal Processes

Metal removal processes are the oldest and most mature technologies for metal gear manufacture. Figure 4-1 indicates that metal removal processes for gear manufacture can be divided into four stages: gear cutting, soft finishing, hardening, and hard finishing. For gears without post treatment, the last two processes can be eliminated.

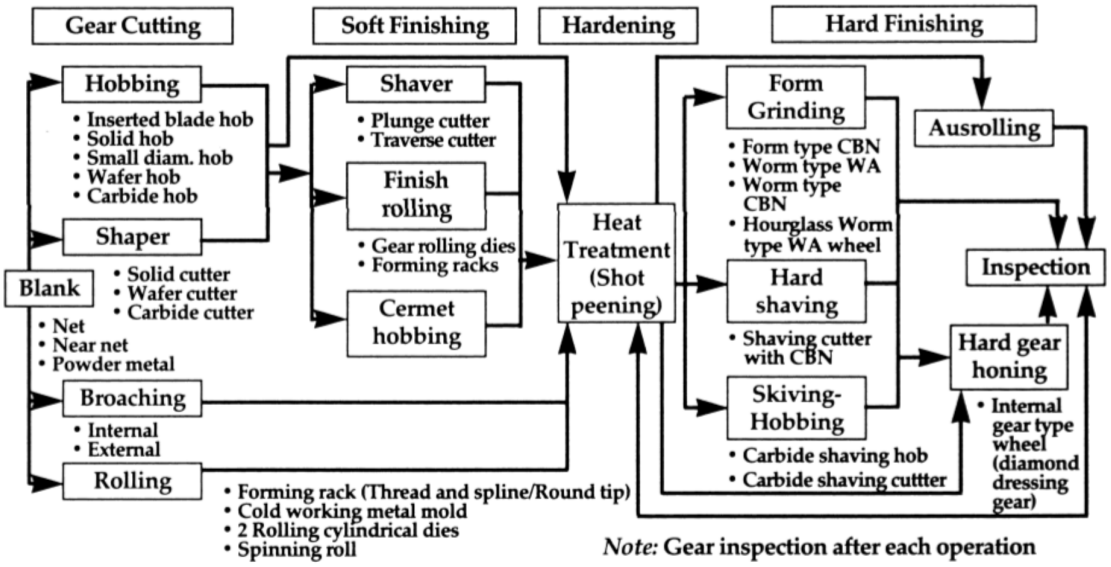

Figure 4-1: Examples of various gear manufacturing processes [6]

4.3.1.1 Gear Cutting

Commonly used gear cutting methods are hobbing and shaping. Hobbing makes use of a tool that resembles a worm gear with helical teeth. Hobbing can fabricate gears with almost any number of gear teeth [6], and therefore guarantee the manufacturability of gears with gear teeth numbers specified by the design method in Chapter 5. The shaping method makes use of a shaper to cut the gear through the face width in a manner similar to a pinion meshing with a gear. [6]

4.3.1.2 Gear Finishing

There are many different finishing methods for gear manufacture. Only grinding and shaving are considered here. Grinding is a process that shapes the surface with a rotating abrasive wheel. This process is used to smooth the tooth surface or “remove heat treatment distortion from large gears”. [6] Shaving is another finishing operation to

"shave off small amounts of metal as the gear and cutter are meshed at an angle to one another". [6]

4.3.2 Injection Molding and Power Metallurgy Processing

Injection molding is a method to form softened plastic into a desired shape "with a relatively cool cavity under pressure" and is "widely used for high volume production of thermoplastic resin gears". [6] Gears fabricated through injection molding achieve quality levels ranging from 6 to 8, as described in next chapter. [6]

Power metallurgy (P/M) processing forms a powder in desired shape after heating it to "an elevated temperature below the melting point". [6] P/M can be used to fabricate metal gears, such as cast iron gears, but is not appropriate for plastic gears, and it is only applied in high volume production.

4.3.3 Post Treatments

Post treatments are also quite important for gears, as they greatly improve gear properties. There are various heat treatment processes designed for gears. Surface hardening produces a hard surface layer on the gear and improves the surface quality. With surface hardening, failures caused by surface wear can be avoided. The through hardening method also increases the strength of the teeth, but a hardness gradient is developed because "the outside of a gear is cooled faster than the inside" in the treatment process. [6] In general, the surface hardening method is more common. Carburizing, carbonitriding, and nitriding are other hardening methods.

4.4 LUBRICATION

Gear performance is largely influenced by the lubrication conditions. Poor lubrication increases the risk of gear failure. The lubricant between two surfaces decreases pressure concentration problems while protecting the gear surface.

4.4.1 Lubrication Related Failures

Pitting is a common failure for gear teeth related to the lubrication conditions, and results from extremely high contact stresses. Pitting usually does not cause failure directly but may result in crack or damage to the tooth structure, increasing the likelihood of bending failure.

Another common lubrication related gear failure is wear, including adhesion and abrasion. Adhesion happens "when the oxide layers on the gear teeth surfaces in contact are disrupted and bare metal is exposed." [6] Abrasion is usually caused by hard particles and impurities. All lubrication related gear failures can be significantly reduced by using appropriate lubricants.

4.4.2 Lubricant Selection

The choice of lubricant depends on the gearing types, operating speed, load level, and temperature, among other factors. [6] Most gears are lubricated by oil or grease.

4.4.2.1 Oil

Oil is the most widely used lubricant because it has good lubricating and cooling properties, and can be used for both gears and bearings. Another advantage for oil is that the impurities are removed when the oil is changed. The disadvantage of oil is that seals must be used to prevent oil leakage and escape, and periodic maintenance is also needed.

4.4.2.2 Grease

Grease is a kind of semisolid lubricant especially fit for low speed, low load applications. Grease is easy to apply and the sealing requirements are less stringent because the grease is highly viscous. However, the performance of grease is not as good as oil in other respects. First, lubrication is not as effective, because the grease may not cover the whole contact surface of the meshed gear and may be squeezed out under high

pressure. Second, debris and contamination is hard to remove from the grease and may cause wear on the gear surface. Finally, grease is not as good a coolant as oil.

4.5 SUMMARY

In this chapter, both metal and plastic gear materials have been introduced. Steel is the most widely used gear material because of its high strength and mature manufacturing technology. Plastic has its own advantages, such as relatively low cost, good manufacturability, and lower density. Plastic has limitations compared to metal gears, including lower strength and lower dimensional accuracy.

Manufacturing and post treatment technology for gears, including metal gear cutting, injection molding, and hardening treatments have been introduced. Lubrication technology to improve the gear train performance is also discussed.

The next chapter discusses gear train design standards, the gear train design process developed in this research, and gear train performance.

Chapter 5: Gear Train Design

5.1 American Gear Manufacturers Association (AGMA) Standard

The design of gear trains should follow a standard procedure to meet the demands of strength, dimensional accuracy, life, and application requirements. In this study, the basic design and analysis of gear trains follows the standard ANSI/AGMA 2001-D04 *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*. [50]

More than twenty parameters, including gear geometry, material properties, gear manufacturing quality, and life (number of cycles under mesh pressure) are involved in the design process. [36] All of these parameters may affect the gear train performance, load capacity, and expected life. The standards published by AGMA help in balancing these parameters to maximize performance. [40]

5.1.1 Nomenclature

A clear and comprehensive description of the terms used in the AGMA gear train design standard is provided in Table 5-1. Several of these parameters are discussed in more detail in this section.

Nomenclature			
Symbol	Description	Symbol	Description
σ_{ab}	Bending fatigue strength	σ_{ac}	Surface fatigue strength
S_{at}	Allowable bending stress (lb/in^2)	S_{ac}	Allowable contact stress (lb/in^2)
W_t	Tangential tooth load	f^t	Tangential force transmitted
K_O	Overload factor	K_v	Dynamic factor
K_S	Size factor	K_T	Temperature factor
K_R	Reliability factor	K_B	Rim thickness factor
K_m	Load distribution factor		
S_F	Safety factor - bending	S_H	Safety factor - pitting
J	Geometry factor - bending	I	Geometry factor - pitting
Y_N	Bending stress cycle factor	Z_N	Contact stress cycle factor
C_p	Elastic coefficient ($\sqrt{lbf/_{in^2}}$)	F	Face width (in)

Table 5-1: Parameters Used in Gear Train Design

5.1.1.1 K_0 Overload Factor

The overload factor K_0 is a factor for situations when the momentary load can be larger than that during normal operation. It can be regarded as a safety factor to resist shock impact. The overload factor should be larger when shock loads on input and output is stronger. [51] The designer should predict the degree of shocks before the design process, and assign the overload factor. Table 5-2 shows values for this factor in different situations.

Impact from Prime Mover	Impact from Load Side of Machine		
	Uniform Impact Load	Medium Impact Load	Heavy Impact Load
Uniform Load (Motor, Turbine, Hydraulic Motor)	1.0	1.25	1.75
Light Impact Load (Multicylinder Engine)	1.25	1.5	2.0
Medium Impact Load (Single Cylinder Engine)	1.5	1.75	2.25

Table 5-2: Overload Factor Look-up Table [36]

5.1.1.2 K_v Dynamic Factor

The dynamic factor is affected by the working pitch line velocity, and the quality number of the gear. The pitch line velocity is the product of the rotation velocity and the gear diameter, which can only be determined after the gear dimensions are chosen. And as the gear diameter may be adjusted in the design process, the dynamic factor should be updated when the diameter changes. The quality number is determined by the material and manufacture accuracy. The quality number is assigned as 11 for steel gears in general cases, and ranges from 6 to 8 for plastic. [6]

Figure 5-1 below shows the trend of dynamic factor with different quality numbers and pitch line velocities.

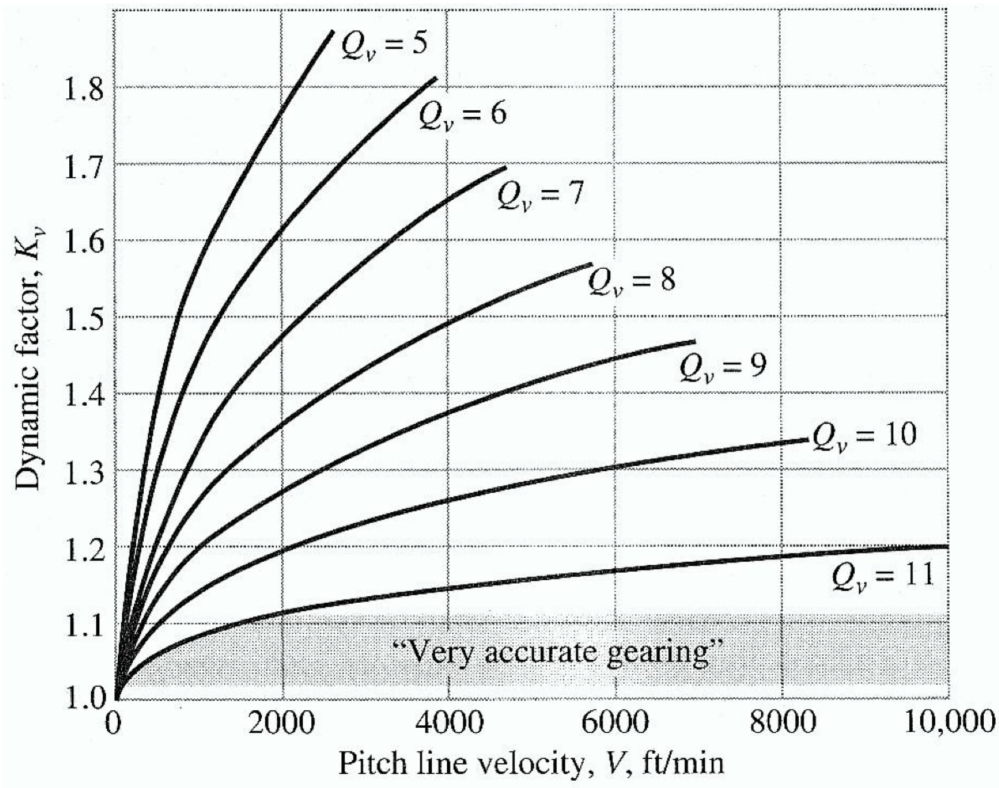

Figure 5-1: Dynamic Factor K_v [36]

5.1.1.3 K_s Size Factor

The size factor accounts for material strength differences when the dimensions of the gear teeth are large. [52] The AGMA suggests that, the size factor K_s can be taken to be 1.00 for most gears if no more information is provided.

5.1.1.4 K_T Temperature Factor

Higher temperature sometimes affects material properties significantly, softening materials and accelerating wear; therefore, a temperature factor is included. [36] Generally, this factor is set to 1 if the gear train working temperature is below 250° C. If the temperature is expected to be higher than this, the temperature factor is calculated as:

$$K_T = (460.0 + T)/620.0$$

5.1.1.5 K_R Reliability Factor

The reliability factor is determined by reliability requirements (expressed as a percentage). A higher reliability requires a more conservative design, and a higher reliability factor is recommended. Suggested reliability factors are given in Table 5-3.

Reliability (%)	K_R
90	0.85
99	1.0
99.9	1.25
99.99	1.5

Table 5-3: Reliability Factor for Specific Reliability [36]

5.1.1.6 K_B Rim Thickness Factor

The rim thickness factor is a factor for the geometry of the gear. A thin rim weakens the gear. The rim thickness factor can be set at 1 if the following requirement is met:

$$\frac{t_R}{h_t} \geq 1.2$$

where t_R is the rim thickness, and h_t is the gear teeth height. See Figure 5-2 for illustrations of these parameters.

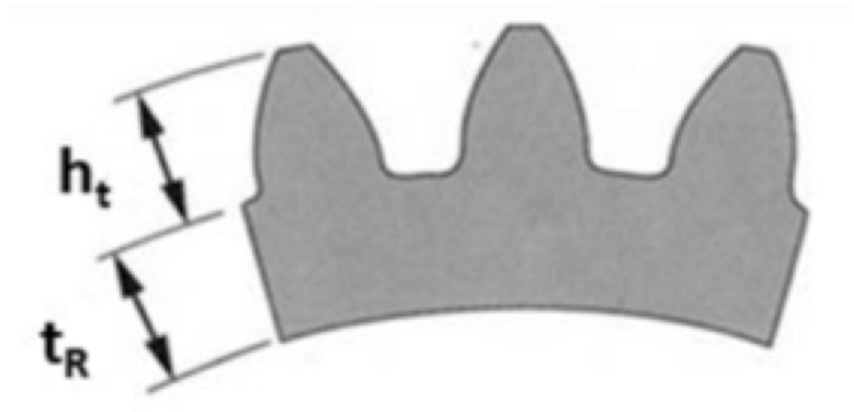

Figure 5-2: Rim thickness and gear depth [36]

5.1.1.7 K_m Load Distribution Factor

The load distribution factor K_m accounts for uneven distribution of load across the face width caused by shaft misalignments. So face width and mounting accuracy determine the load distribution factor. [5]

Table 5-4 shows typical values for the load distribution factor based on mounting conditions and gear face width. It is important to note that, in the gear width design process, an initial face width is chosen, and the face width changes with each iteration, so the load distribution factor should be updated before the stress is recalculated each time.

Support Properties and Gear Quality	Face Width, in			
	0 to 2	6	9	>16
Accurate mountings, small bearing clearances, minimum deflections, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, more bearing clearance, less accurate gears, contact across full face	1.6	1.7	1.8	2.2
Combinations of mounting properties and gearing precision that produce less than full face contact	2.2 or higher			

Table 5-4: Load Distribution Factor K_m Look-up Table [40]

5.1.1.8 Geometry Factors J (Bending Strength) and I (Surface Strength)

Geometry Factors *J* and *I* account for the extent to which load sharing occurs with different gear shapes and sizes. [5] AGMA provides look-up tables to find the values for *J* and *I*, as shown in Table 5-5. To simplify and automate the design process, curve fitting is used to create a surface model of this data, as shown in Figure 5-3. [41]

***I* AND *J* FACTORS FOR:**[1]

14.5	DEG. PRESSURE ANGLE	2.157	WHOLE DEPTH FACTOR
25.0	DEG. HELIX ANGLE	0.024	TOOTH THINNING FOR BACKLASH
0.157	TOOL EDGE RADIUS		LOADED AT TIP

50 PERCENT LONG ADDENDUM PINION ($x_1 = 0.50$)

50 PERCENT SHORT ADDENDUM GEAR ($x_2 = -0.50$)

GEAR		PINION TEETH																	
		12		14		17		21		26		35		55		135	
TEETH		P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	I																
	J	U	U														
14	I			0.057													
	J	U	U	0.38	0.19												
17	I			0.086		0.073											
	J	U	U	0.39	0.22	0.40	0.23										
21	I			0.117		0.102		0.087									
	J	U	U	0.39	0.24	0.40	0.25	0.41	0.26								
26	I			0.148		0.131		0.114		0.099							
	J	U	U	0.39	0.27	0.40	0.28	0.42	0.29	0.43	0.29						
35	I			0.188		0.170		0.151		0.134		0.112					
	J	U	U	0.40	0.29	0.41	0.30	0.42	0.31	0.43	0.32	0.45	0.34				
55	I			0.240		0.222		0.203		0.185		0.161		0.127			
	J	U	U	0.40	0.32	0.41	0.33	0.42	0.34	0.44	0.35	0.45	0.37	0.47	0.39		
135	I			0.311		0.295		0.279		0.263		0.242		0.210		0.144	
	J	U	U	0.41	0.36	0.42	0.37	0.43	0.38	0.44	0.39	0.46	0.41	0.48	0.43	0.51	0.47

1 The letter "U" indicates a gear tooth combination which produces an undercut tooth form in one or both components and should be avoided. See Section 7 and Fig 7–1.

Table 5-5: Geometry Factor Look-up Table [53]

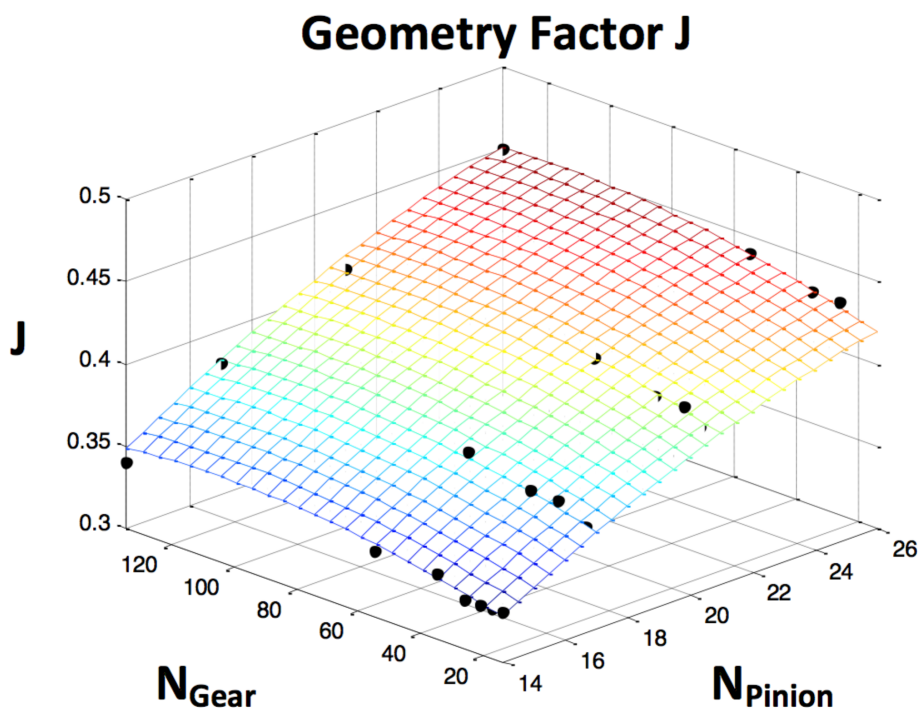

Figure 5-3: Curve fitting for Geometry Factor [41]

5.1.1.9 Safety Factor S_F and S_H

Safety factors account unpredicted situations. The values for these factors depend on application situations, designer experience and system requirements. For the purposes of this research, safety factors are set to 1.3. For more conservative designs, these factors can be set as high as 1.7 . [5]

5.1.1.10 Stress Cycle Factors Y_N and Z_N

Life is an important performance criterion for gear trains. Based on experimental results that are summarized in Figures 5-4 and 5-5, when the number of load cycles is around 10^7, these factors can be chosen to be 1. As the figures show, the hardness of the material also affects these factors.

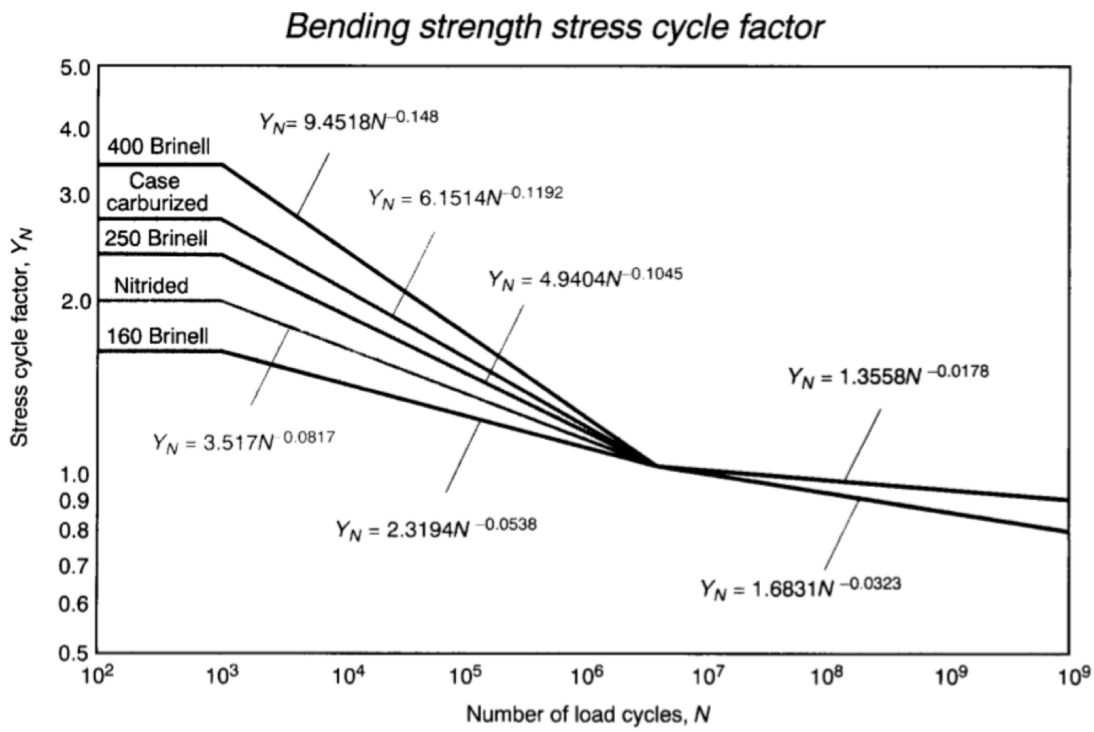

Figure 5-4: Bending Stress Cycle Factor [51]

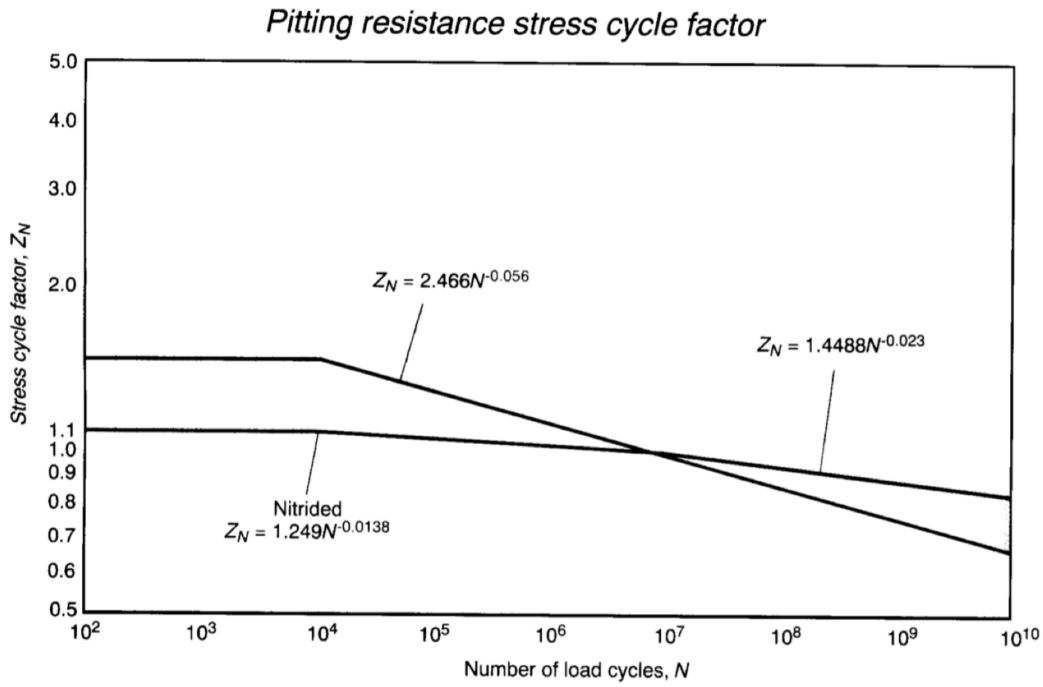

Figure 5-5: Pitting Stress Cycle Factor [51]

5.1.1.11 Allowable Stress Values

Allowable bending and surface stresses are important parameters for gear material used in the design process. The bending and surface strengths for carburized and case-hardened steel are shown in Table 5-6. Other materials are discussed in next chapter.

	Grade 1	Grade 2	Grade 3
Max. allowable bending stress (ksi)	55	70	75
Max. allowable surface stress (ksi)	180	225	275

Table 5-6: Allowable Stress Values for Various Steel Quality Grades [36]

AGMA has categorized steels into three grades, and Grade 2 is selected in this study. So the stress values used in this chapter are:

Allowable bending stress - 70 ksi

Allowable contact stress - 225 ksi

5.1.1.12 C_p Elastic Coefficient

The elastic coefficient accounts for the different performance of different materials with respect to rolling and sliding. For example, earlier pitting and failures may occur when materials adhere to each other. The elastic coefficient is chosen to be 2300 for a pinion and gear that are both made of steel [5]

5.1.1.13 AGMA Factors and Values for Design

Based on the discussion of the factors in AGMA standard, the values adopted in this research are shown in Table 5-7. In the table, "undefined" means the value of this factor cannot be determined before the design process starts. Several factors must be updated as the design process progresses.

Symbol	Value	Symbol	Value	Symbol	Value
K_0	1				
K_v	Undefined	Pitch line velocity	Undefined	Input rotate speed	6000 r/min
				Diameter	Undefined
		Q_v	11		
K_A	1.5	Impact from Machine Load	Medium		
		Impact from Mover Load	Medium		
K_s	1				
K_T	1	Temperature	200 F		
K_R	1	Reliability	99%		
K_B	1				
K_m	Undefined				
K_l	1				
J & I	Undefined	Pinion Teeth Number	Undefined		
		Gear Teeth Number	Undefined		
S_F & S_H	1.3				
Y_N & Z_N	0.9	Number of Load Cycles	10^7		
		Brinell Hardness	600 Bhn		
S_{at}	70 ksi				
S_{ac}	225 ksi				
C_p	2300				

Table 5-7: Values of Factors for Gear Train Design

5.1.2 Face Width

In general, the torque capacity of a gear increases as the gear face width increases. However, misalignment issues can occur for larger face widths. Therefore, there should be a maximum allowable face width for a given gear mesh. In general, the maximum face width is recommended not to exceed twice pinion pitch diameter. [54]

To calculate a desirable face width, AGMA has provided an iterative method to satisfy both bending and surface stress constraints on all meshes. However, to start the design process, an initial face width value must be chosen. A new parameter, F_{rule}, is introduced to assist the designer in this choice. F_{rule} represents the relationship between the gear diameter and the face width. In our calculation, F_{rule} is chosen to be 1 at first, and is adjusted in the iterative process. [5]

$$F = D \times F_{rule}$$

5.2 Design Method

A complete gear train design method is developed in this section, and it is applied to the design of three types of SCGTs. Gear train design, especially for gear trains with high reduction ratios and multiple stages, is quite complicated because the values of many parameters need to be determined. What we would like is a simple and quick method which results in a gear train with excellent performance.

In traditional gear train selection [5], the number of gear teeth is set first according to the gear reduction ratio requirement. Then, other parameters like diametral pitch are calculated based on the number of teeth and the gear train dimension requirements. Finally, several designs are compared to find the one with the best torque performance.

Several problems exist with this method. First, the values of parameters such as diametral pitch calculated through these methods are not always in the standard series for selection, which means they are only for reference and not actual design results. Before this approach can be used to direct real design and manufacture, a process to standardize all the parameters is required. Second, exhaustive method is time consuming, particularly when a large reduction ratio and multiple stages are required. And typically, with no basis

in theory, the process will miss the best design choice. A better approach is to find the design with the best performance based on continuous parameters, and then the final design can be chosen by comparing the performance of several feasible designs with standard parameter values.

5.2.1 Design Procedure

The general idea of the design method in this study is to initially assume all the parameters in design are continuous instead of discrete. Then, after a comprehensive study of the gear train structure and performance limits, several parameters are selected as the input parameters. Next, parameter values are selected which achieve the best performance. Finally, standard parameter values are chosen and one set of values is selected according to the actual design situation. With this design method, other gear train performance metrics, like weight, torque density, inertia, and responsiveness, are also considered and used to facilitate the final material selection.

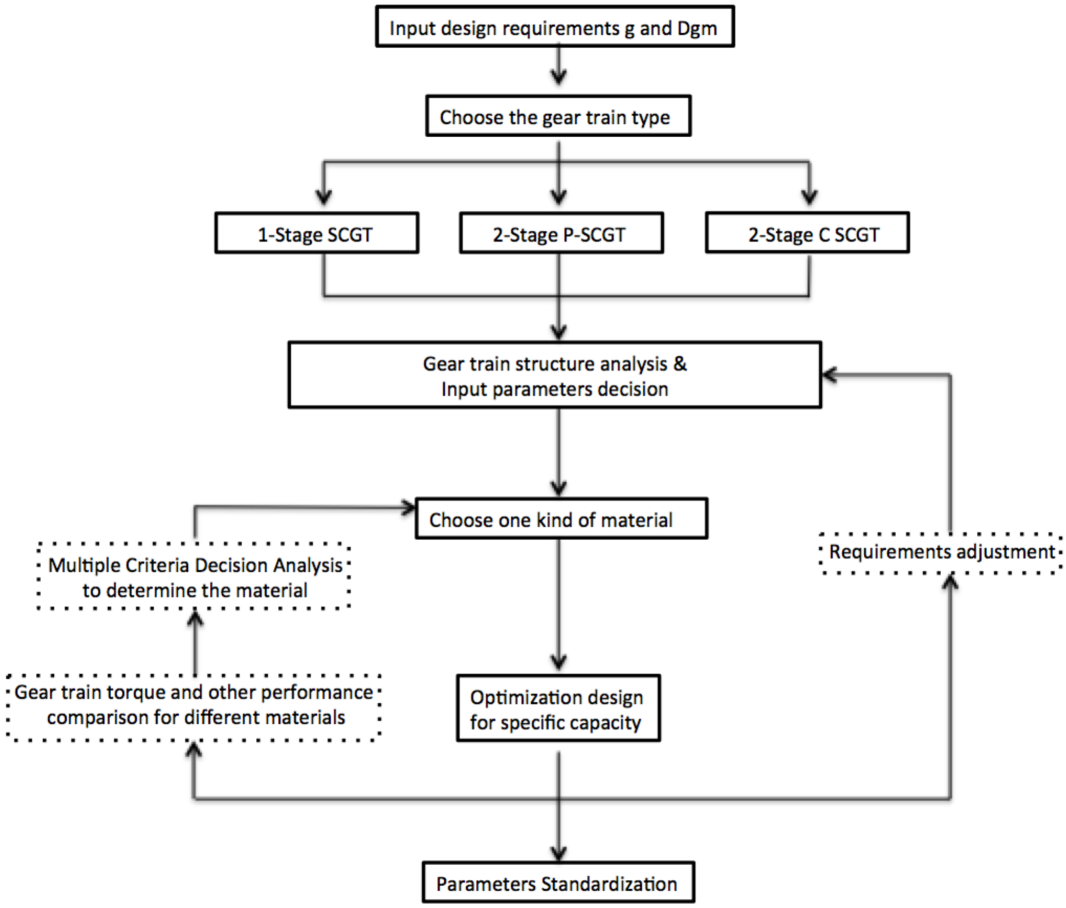

Figure 5-6: Design Method Procedure

The complete design procedure is shown in Figure 5-6. To begin, a gear train type is selected first, according to the given desired reduction ratio, g, and the gear train mesh diameter, D_{gm}.

The 1-Stage SCGT is usually designed for reduction ratios no larger than 27, which will be explained below. For larger reduction ratios, the 2-Stage P-Type or C-Type SCGT can satisfy collaborative robot requirements. The P-Type is mainly appropriate for situations with a constraint on the actuator length, while the C-Type is appropriate for situations with a constraint on the diameter.

After selecting the right gear train type, the next step is to conduct a comprehensive study of the gear train structure and performance limits, and determine the degree of freedom in the assignment of parameter values. Then the parameter values that achieve best performance on specific input pairs (g and D_{gm}) are calculated.

The steps in the dotted boxes, "requirement adjustment", and "material comparison and selection processes", are optional and depend on the requirements. The "requirement adjustment" step is taken when the requirements are not strict and some adjustment to improve the system performance is allowable. For example, when the torque capacity obtained for the given reduction ratio and mesh diameter largely exceeds expectations, a smaller diameter can be chosen to make the product more compact and lighter. Material selection is more fully discussed in the next chapter.

5.2.2 Fatigue Limit Calculation and Face Width Decision

The AGMA methodology checks both the bending stress and contract stress to ensure the safety of the gear train. The two fundamental equations are:

$$S_t = f^t K_0 K_v K_S \frac{P_d}{F} \frac{K_m K_B}{J}$$

$$S_c = C_P \sqrt{f^t K_O K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I}}$$

According to AGMA, bending stress and contact stress should satisfy the following equations:

$$S_t \leq \frac{S_{at}}{S_F} \frac{Y_N}{K_T K_R}$$

$$S_c \leq \frac{S_{ac}}{S_H}\frac{Z_N}{K_T}\frac{C_H}{K_R}$$

For one compound gear, the power transmitted from the large gear to the small gear should be the same, which is the product of the torque and the angular velocities. And since the large star and small star gears have the same angular velocity, the torque on them is the same:

$$\left(\frac{D_{LS}}{2}\right) \times {f_1}^t = \left(\frac{D_{SS}}{2}\right) \times {f_2}^t$$

where ${f_1}^t$ is the tangential force on the large star gear, and ${f_2}^t$ is the tangential force on the small star gear.

Define a new parameter here, amplification factor, r_A, as the ratio of the pitch diameter of the large gear to that of the small gear:

$$r_A = \frac{D_{LS}}{D_{SS}}$$

$${f_2}^t = \frac{D_{LS}}{D_{SS}} \times {f_1}^t = r_A \times {f_1}^t$$

An iterative process should be applied to ensure all gear widths meet the strength requirements. To begin with, as noted previously in the face width discussion, the initial F_{rule} is chosen as 1. The iterative face width calculation process requires ensuring that $S_t \leq S_{at}$ and $S_C \leq S_{ac}$ for all the gear meshes.

Consider the 1-stage SCGT as an example:

$${f_1}^t = \frac{S_{at1}\, F_1 J_1}{K_0 K_{v1} K_S K_{m1} K_B P_{d1}}$$

The corresponding load on the gear teeth in the second mesh is:

$$f_2{}^t = r_A \times f_1{}^t$$

The bending stress in mesh 2 corresponding to this load is then:

$$S_{t2} = f_2{}^t K_0 K_{v2} K_S \frac{P_{d2}}{F_2} \frac{K_{m2} K_B}{J_2}$$

Here we make sure the bending strength is high enough for the second mesh when the first gear mesh is loaded at its highest level. If the bending stress is larger than the material bending strength, the tangential force should be updated:

$$Updated\ f_1{}^t = \frac{S_{at2}}{S_{t2}} \times f_1{}^t\ if\ S_{t2} > S_{at2}$$

The same process should be followed for pitting fatigue:

$$S_{c1} = C_P \sqrt{f_1{}^t K_O K_{v1} K_s \frac{K_{m1}}{d_{p1} F_1} \frac{C_f}{I_1}}$$

$$Updated\ f_1{}^t = (\frac{S_{ac1}}{S_{c1}})^2 \times f_1{}^t\ if\ S_{c1} > S_{ac2}$$

$$S_{c2} = C_P \sqrt{f_2{}^t K_O K_{v2} K_s \frac{K_{m2}}{d_{p2} F_2} \frac{C_f}{I_2}}$$

$$Updated\ f_1{}^t = (\frac{S_{ac2}}{S_{c2}})^2 \times f_1{}^t\ if\ S_{c2} > S_{ac2}$$

With the new ${f_1}^t$, we compute the new S_{t1}, S_{t2}, S_{c1}, S_{c2}, and face width:

$$F_{t1} = \frac{S_{t1}}{S_{at1}} \times F_1$$

$$F_{t2} = \frac{S_{t2}}{S_{at2}} \times F_2$$

$$F_{c1} = \frac{S_{c1}}{S_{ac1}} \times F_1$$

$$F_{c2} = \frac{S_{c2}}{S_{ac2}} \times F_2$$

$$F_1 = Greater\ of\ F_{t1}\ and\ F_{c1}$$

$$F_2 = Greater\ of\ F_{t2}\ and\ F_{c2}$$

5.3 Design for One-Stage Star Compound Gear Train

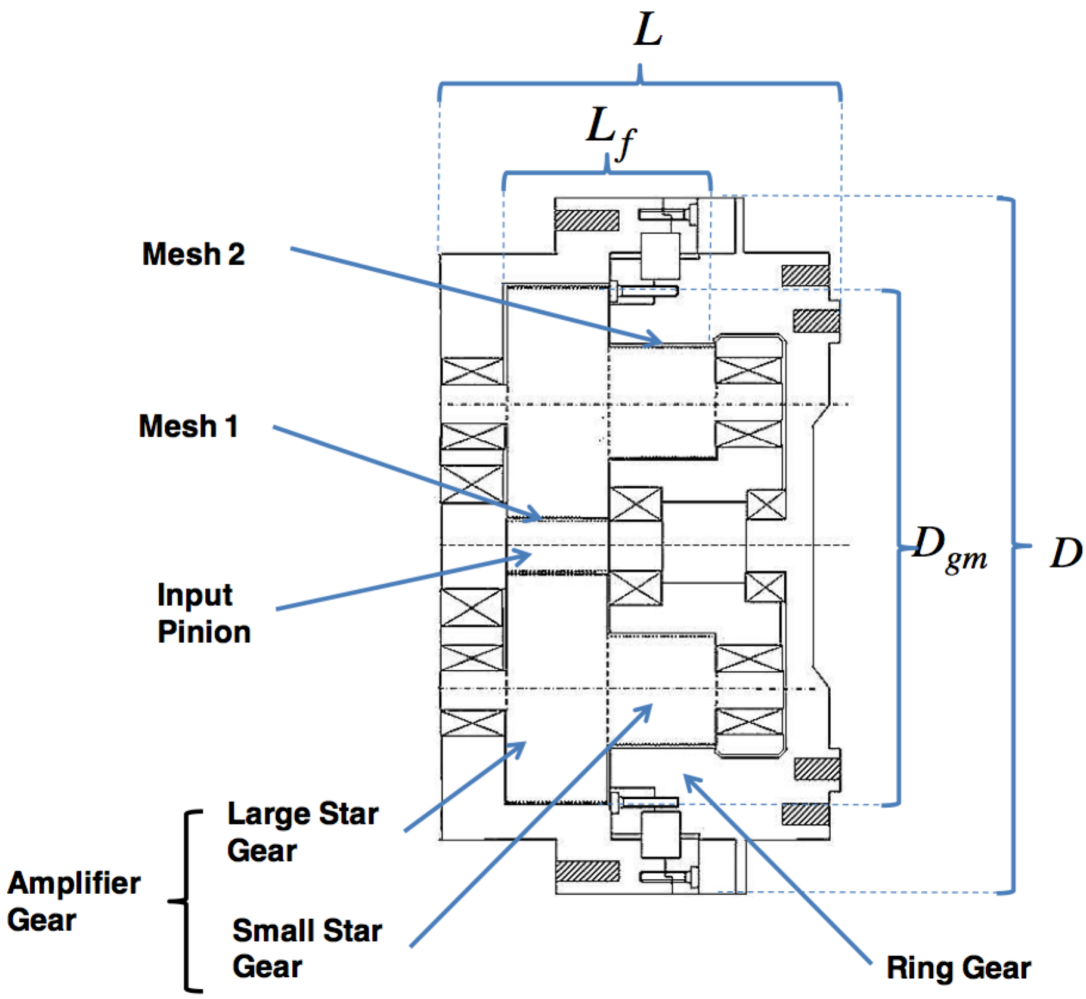

Figure 5-7: 1-Stage Star Compound Gear Train [5]

Figure 5-7 shows the architecture of a 1-Stage Star Compound Gear Train. Table 5-8 lists the design parameters for a 1-stage SCGT. To complete a design, values must be assigned for all these parameters.

Nomenclature			
Symbol		Description	
Mesh 1		Mesh between Input Pinion and First Large Star gears	
Mesh 2		Mesh between First Small Star gears and the Second Large gear	
g	Whole gear reduction ratio	D_{gm}	Gear train mesh diameter
g_1	Gear ratio of Mesh 1	g_2	Gear ratio of Mesh 2
N_P	Number of teeth of input pinion	N_{LS}	Number of teeth of the large star gear
N_{SS}	Number of teeth of the small star pinion	N_R	Number of teeth of ring gear
P_{d1}	Diametral pitch of gears in Mesh 1	P_{d2}	Diametral pitch of gears in Mesh 2
D_P	Diameter of the input pinion	D_{LS}	Diameter of the large star gear
D_{SS}	Diameter of the small star gear	D_R	Diameter of the Ring gear
F_1	Face width of gears in Mesh 1	F_2	Face width of gears in Mesh 2

Table 5-8: Terminology for One-Stage Star Compound Gear Train

5.3.1 1-Stage SCGT Structure Analysis

The gear train mesh diameter D_{gm} and gear ratio g are the primary input parameter pairs:

$$g = g_1 g_2 = \frac{\prod N_{Driven\ Gears}}{\prod N_{Driving\ Gears}} = \frac{N_{LS}}{N_P}\frac{N_R}{N_{SS}}$$

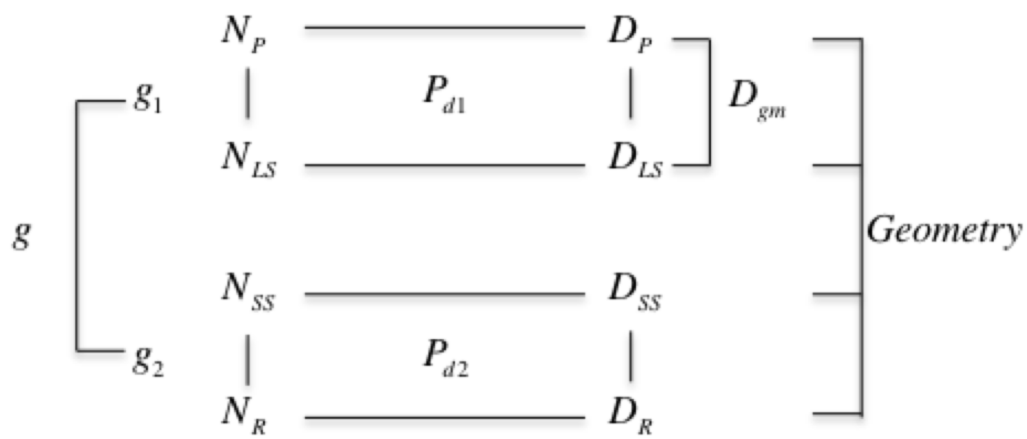

Figure 5-8: 1-Stage SCGT Parameters Relationship

For a gear train design, relationships between all the parameters should be clarified so that we know how many degrees of freedom exist for the parameter design. The relationships are depicted graphically in Figure 5-8. For a 1-Stage SCGT, everything will be determined if the following parameters are assigned values:

$$N_P, N_{LS}, N_{SS}, N_R, D_P, D_{LS}, D_{SS}, D_R$$

However, these parameters are not independent, because of constraints in the gear structure:

$$N_i = P_{dj} * D_i$$

$$N_{gj} = g_j * N_{pj}$$

$$D_{gj} = g_j * D_{pj}$$

where *j* represents the index of the mesh, *j* = 1, 2, and *i* represents the index of the gear, *i* = 1, 2, 3, 4. So, for a SCGT with *j* mesh contacts, there are 3*j* unknown parameters that determine the whole system, and for the 1-Stage SCGT, there are six unknown

parameters. Additionally, we have design inputs (g, D_{gm}) and geometry constraints which reduce the parameters we need to consider in the optimization process:

$$g = g_1 * g_2$$

$$D_{gm} = D_P + 2 * D_{LS}$$

$$D_R = D_P + D_{LS} + D_{SS}$$

So ultimately, three unknown parameters must be considered.

5.3.2 Optimization of the Design

Torque capacity is one of the most important properties in actuator design, and is therefore the first criterion considered in the design process. A 3-D graph is used to show the relationship between the torque capacity, the reduction ratio and the gear train mesh diameter. The reduction ratio, g, ranges from 8 to 36, and the gear train mesh diameter ranges from 1 to 12 in. The core content for the design optimization is to study how different input pairs affect the maximum torque.

The first step of the optimization is to determine which three parameters to choose among all eight parameters. The first two parameters are the number of teeth of the pinion in each mesh (N_P, N_{SS}). As discussed in Chapter 3, the minimum number of teeth to avoid interference that is also readily available is 18, and the maximum number is 25, when considering the gear should not be too large. And the last parameter selected is the reduction ratio of the first reduction stage, g_1.

Intuitively, the number of teeth of the pinion should be the minimum value because, from the formula $N = P_d * D$, we know fewer teeth usually means a smaller diameter, which will drive the system to a larger torque capacity. The reduction ratio g_1 is actually hard to determine, so let $g_1 = g_2 = \sqrt{g}$. To find the best choice for these parameters, an exhaustive method was applied to all the parameter combinations, and the

highest performance with each input pair has been selected. The corresponding input parameters have been obtained. In this exhaustive method, the reduction ratio ranges from 3 to 6, and the number of gear teeth ranges from 18 to 25, as discussed in Chapter 3.

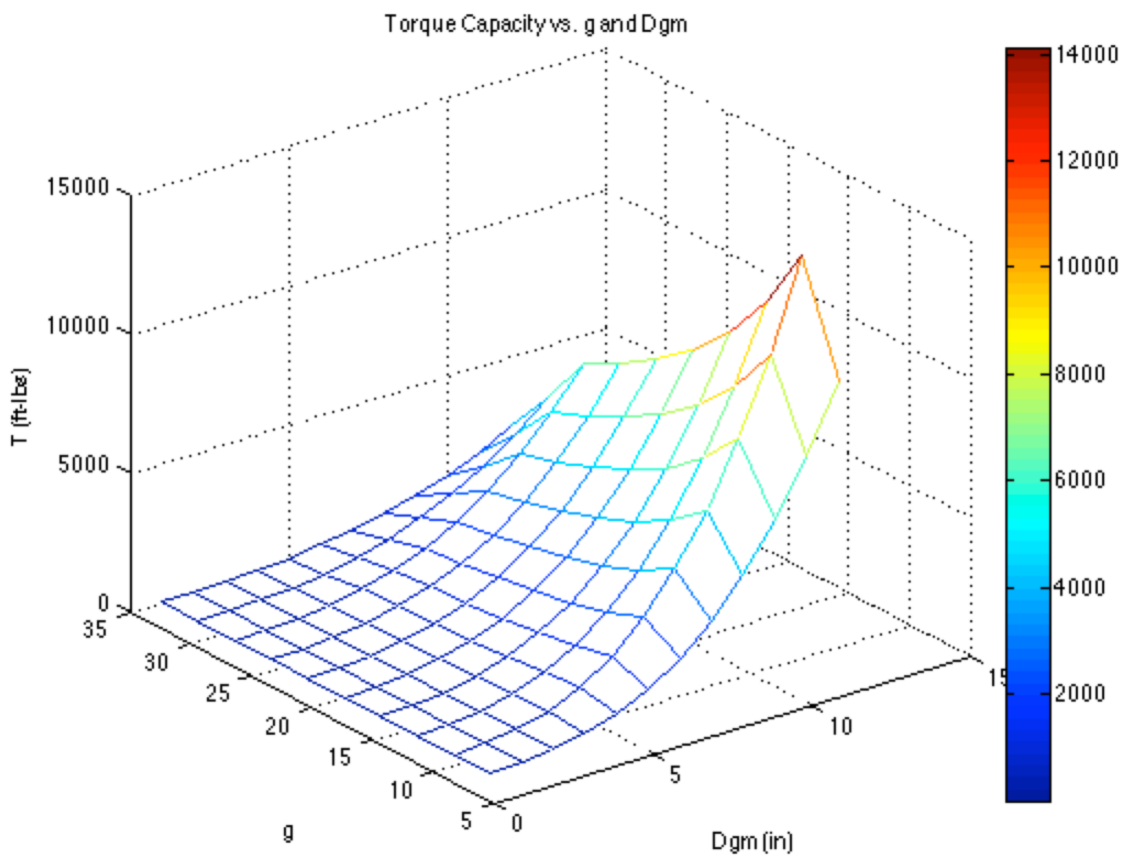

Figure 5-9: Largest Torque Capacity for g and D_{gm} Input

Figure 5-9 indicates the 1-Stage SCGT torque capacity versus reduction ratio g, and gear train mesh diameter D_{gm}. From this graph, we can clearly see that the torque capacity increases with increasing gear train mesh diameter and decreasing reduction ratio.

However, the calculation process for this exhaustive method is very complicated and time-consuming. So we continue to study how to assign these parameters to achieve high torque capacity.

The graph indicates that the best input parameter choices seem to be closely related to the total reduction ratio, but have no relationship with the diameter at all. Through a curve fitting process, the following guidelines were obtained:

$$N_P = \begin{cases} 18, & g < 29 \\ 25, & g \geq 29 \end{cases}$$

$$N_{SS} = 18$$

$$g_1 = \begin{cases} 3, & g < 10.84 \\ 0.1718 * g + 1.1376, & 10.84 \leq g \leq 28.30 \\ 6, & g > 28.30 \end{cases}$$

Even though these guidelines are calculated through the data obtained from the torque capacity performance, the correctness of these assumptions should be demonstrated. The torque capacity was calculated based on these three assumptions, and the results are shown in Figure 5-10.

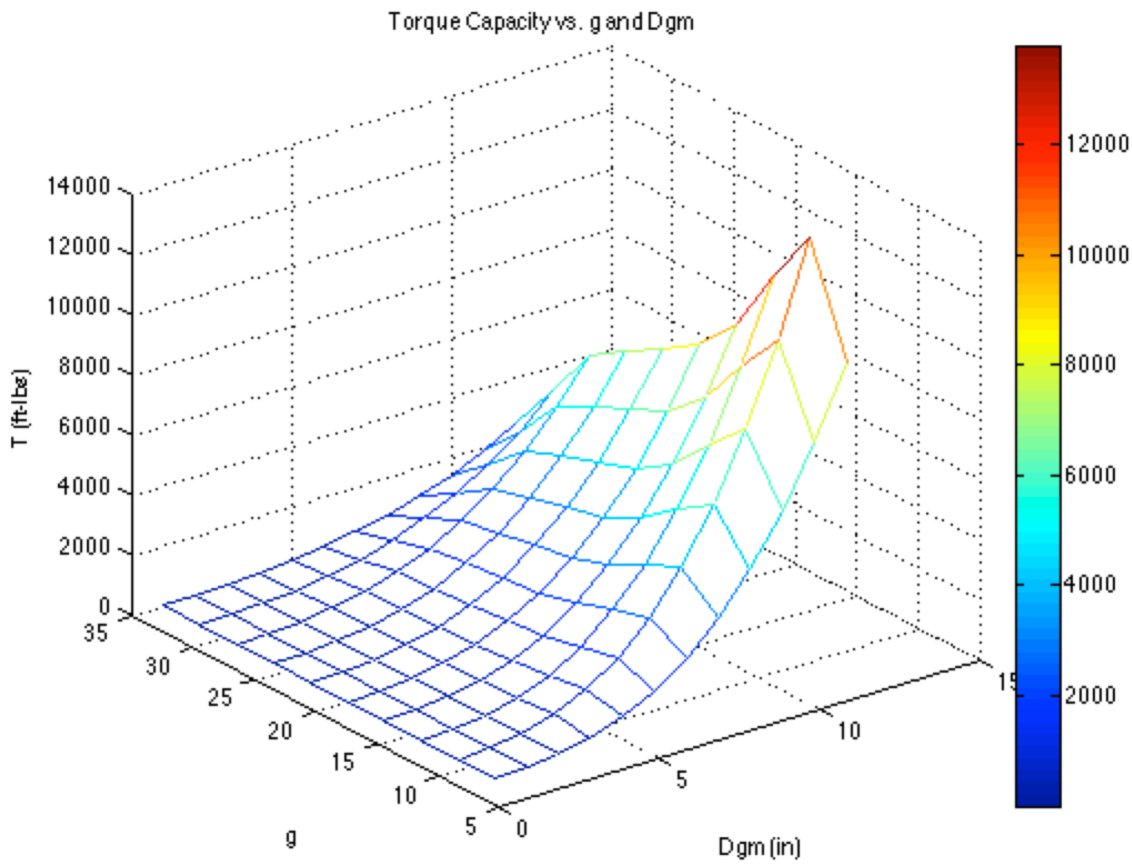

Figure 5-10: Torque Capacity Based on the Parameters Assumptions

Compared with the largest torque capacity on all the samples, the torque obtained based on the assumptions ranges from 91.47% to 1. The largest errors occur at the edge of the graph. Therefore, the assumptions made above work quite well, and can greatly simplify the calculation.

5.3.3 Result Discussion

Based on the assumed parameters, two obvious limits were found with respect to the reduction ratio. When the total reduction ratio is below 11, or above 28, the choices of the number of teeth of the pinion and the reduction ratio in the first stage have both reached their limits. This explains why the graph is not smooth when the total reduction

ratio is near 11 or 28. Figure 5-11 shows that the torque drops for points within the area between reduction ratios of 11 and 28. Therefore, based on the parameter assumptions discussed above, the best reduction ratio range for a 1-Stage SCGT is between 11 and 28.

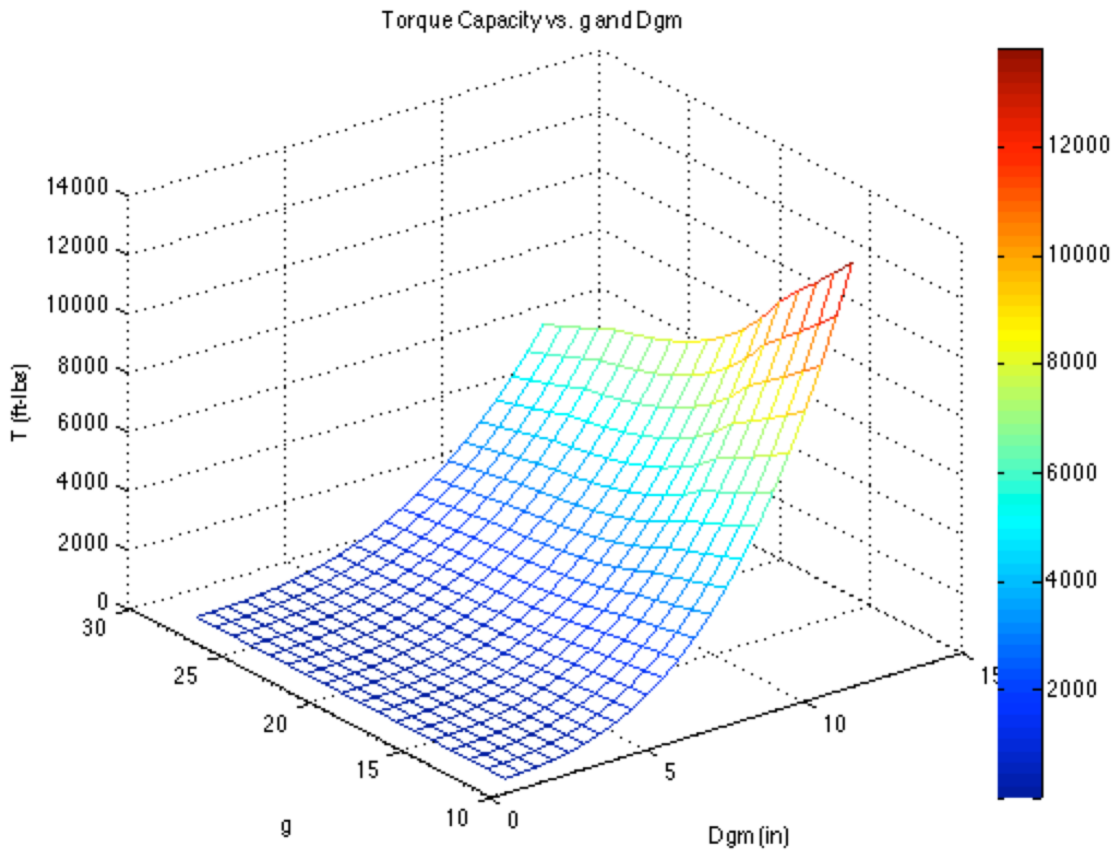

Figure 5-11: Torque Capacity in Best Selection Field

5.4 P-TYPE 2 STAGE SCGT

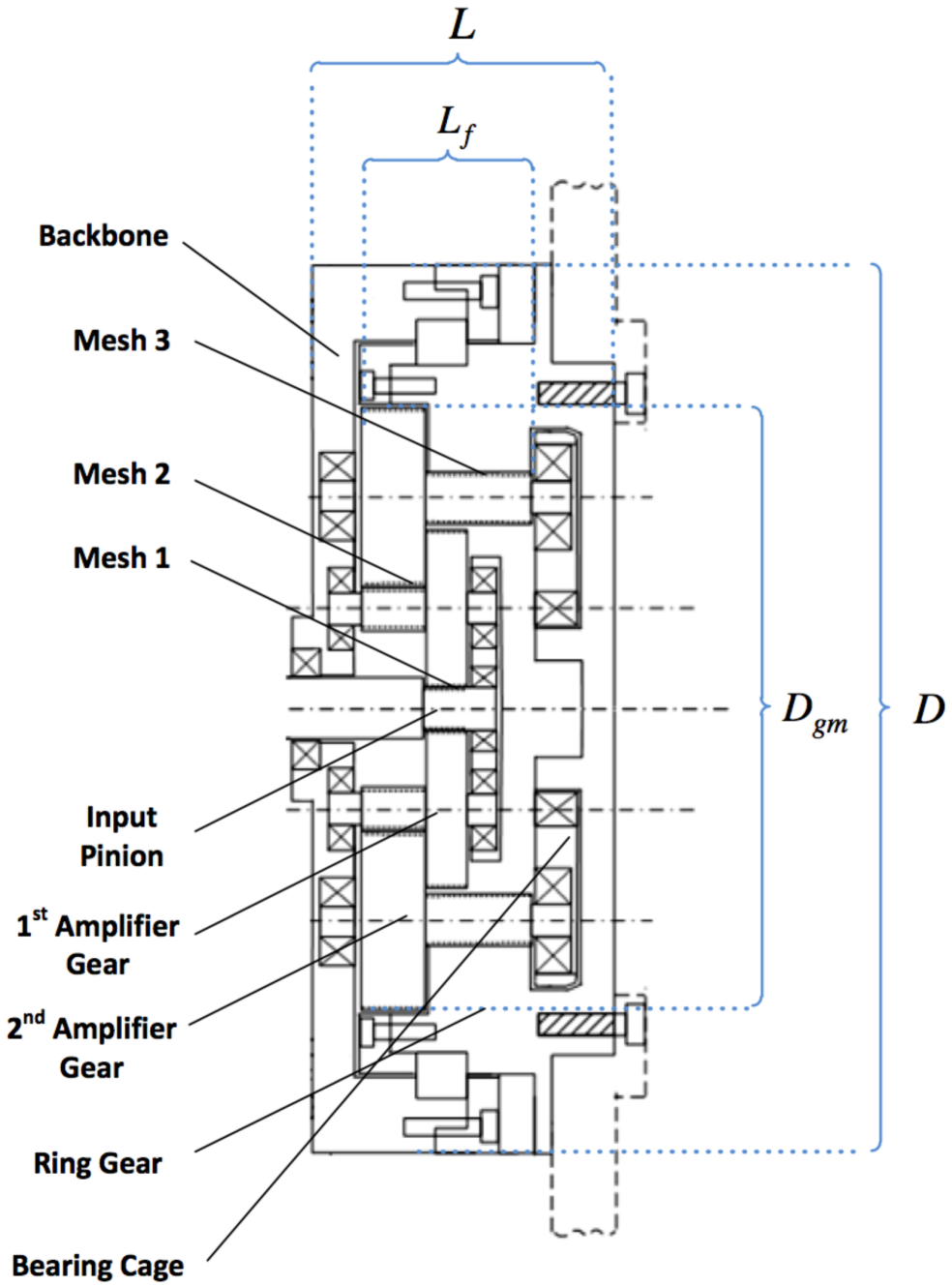

Figure 5-12: 2-Stage P-Type SCGT Structure [5]

Figure 5-12 shows a representative layout of a typical Pancake-Type Two-Stage SCGT, and the nomenclature is defined in Table 5-9. The reduction ratio is:

$$g = g_1 g_2 g_3 = \frac{\prod N_{Driven\ Gears}}{\prod N_{Driving\ Gears}} = \frac{N_{LS1}}{N_P}\frac{N_{LS2}}{N_{SS1}}\frac{N_R}{N_{SS2}}$$

From the figure, we see that two more gears are added to the diameter of the whole gear train, which increases the reduction ratio and the gear train mesh diameter Thus, the total length is almost the same as the length of the 1-Stage SCGT. That is why, when there are diameter constraints for gear trains, the P-Type SCGT configuration should not be selected. Table 5-9 lists the design parameters for a 2-stage P-Type SCGT.

Nomenclature			
Symbol		Description	
Mesh1		Mesh between input pinion and the first large star gears	
Mesh2		Mesh between the first small star gears and the second large gears	
Mesh3		Mesh between the second Small star gears and the ring gear	
g	Whole gear reduction ratio	g_1	Gear reduction ratio of Mesh 1
g_2	Gear reduction ratio of Mesh 2	g_3	Gear reduction ratio of Mesh 3
N_P	Number of teeth of input pinion	N_{LS1}	Number of teeth of the first large star gear
N_{SS1}	Number of teeth of the first small star pinion	N_{LS2}	Number of teeth of the second large star gear
N_{SS2}	Number of teeth of the second small star pinion	N_R	Number of teeth of the ring gear
P_{d1}	Diametral pitch of gears in Mesh 1	P_{d2}	Diametral pitch of gears in Mesh 2
P_{d3}	Diametral pitch of gears in Mesh 3		
D_P	Diameter of the input pinion	D_{LS1}	Diameter of the first large star gear
D_{SS1}	Diameter of the first small star gear	D_{LS2}	Diameter of the second large star gear
D_{SS2}	Diameter of the second small star gear	D_R	Diameter of the ring gear
D_{gm}	Gear train mesh diameter		
F_1	Face width of gears in Mesh 1	F_2	Face width of gears in Mesh 2
F_3	Face width of gears in Mesh 3		

Table 5-9: Nomenclature for 2-Stage P-Type SCGT

5.4.1 2-Stage P-Type SCGT Structure Analysis

Similar to the 1-Stage SCGT, this section discusses the relationships between the parameters and the number of independent parameters.

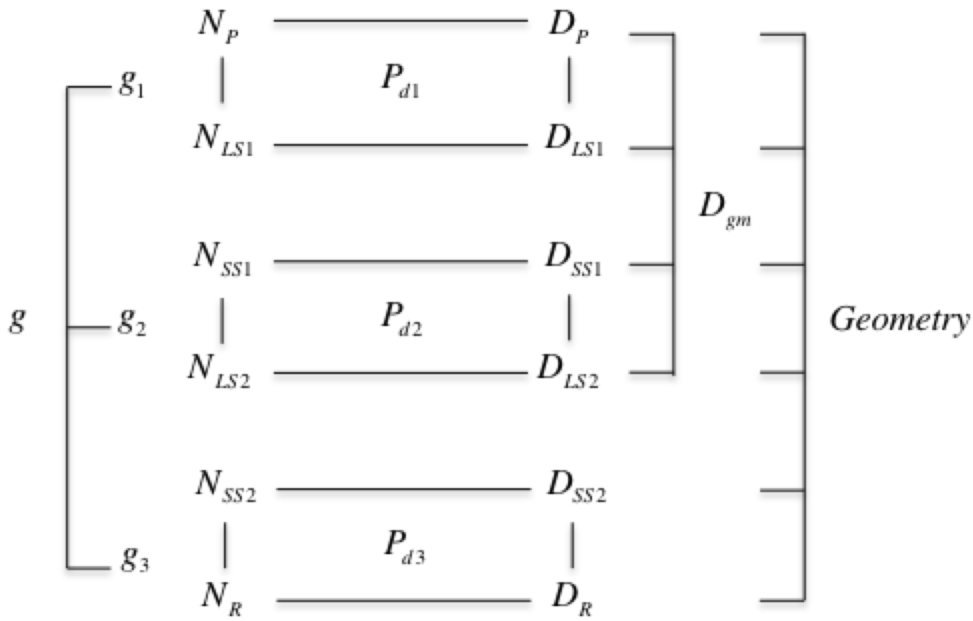

Figure 5-13: 2-Stage P-Type SCGT Parameters Relationship

As shown in Figure 5-13, for a 2-Stage P-Type SCGT, there are 12 parameters for the whole gear train:

$$N_P, N_{LS1}, N_{SS1}, N_{LS2}, N_{SS2}, N_R,\ D_P, D_{LS1}, D_{SS1}, D_{LS2}, D_{SS2}, D_R$$

With three mesh contacts, nine parameters determine the state of the P-Type SCGT fully. We also have three other design inputs and geometry constraints:

$$g = g_1 * g_2 * g_3$$

$$D_{gm} = D_g + D_{LS1} + D_{SS1} + 2 * D_{LS2}$$

$$D_R = D_g + D_{LS1} + D_{SS1} + D_{LS2} + D_{SS2}$$

Thus, six independent input parameters can be selected to optimize the design.

5.4.2 Optimization of the Design

For a 2-Stage P-Type SCGT, values for six input parameters must be assigned. This is a large design space, and it will be difficult for the designer to choose values that lead to a successful design. The six input parameters are:

$$g_1, g_2, N_P, N_{SS1}, N_{SS2}, P_{d1}$$

To assist the designer, similar to the 1-Stage SCGT, an exhaustive search was conducted with respect to all the parameters. The number of teeth for pinions ranged from 18 to 25, the reduction ratio ranged from 3 to 6, and the diamentral pitch ranged from 6 to 150. The diamentral pitch was limited to 150 because, even though a pitch of 200 is standard, it is difficult to find gears with this pitch in the market. [39]

The largest torques for all the input pairs are shown in Figure 5-14.

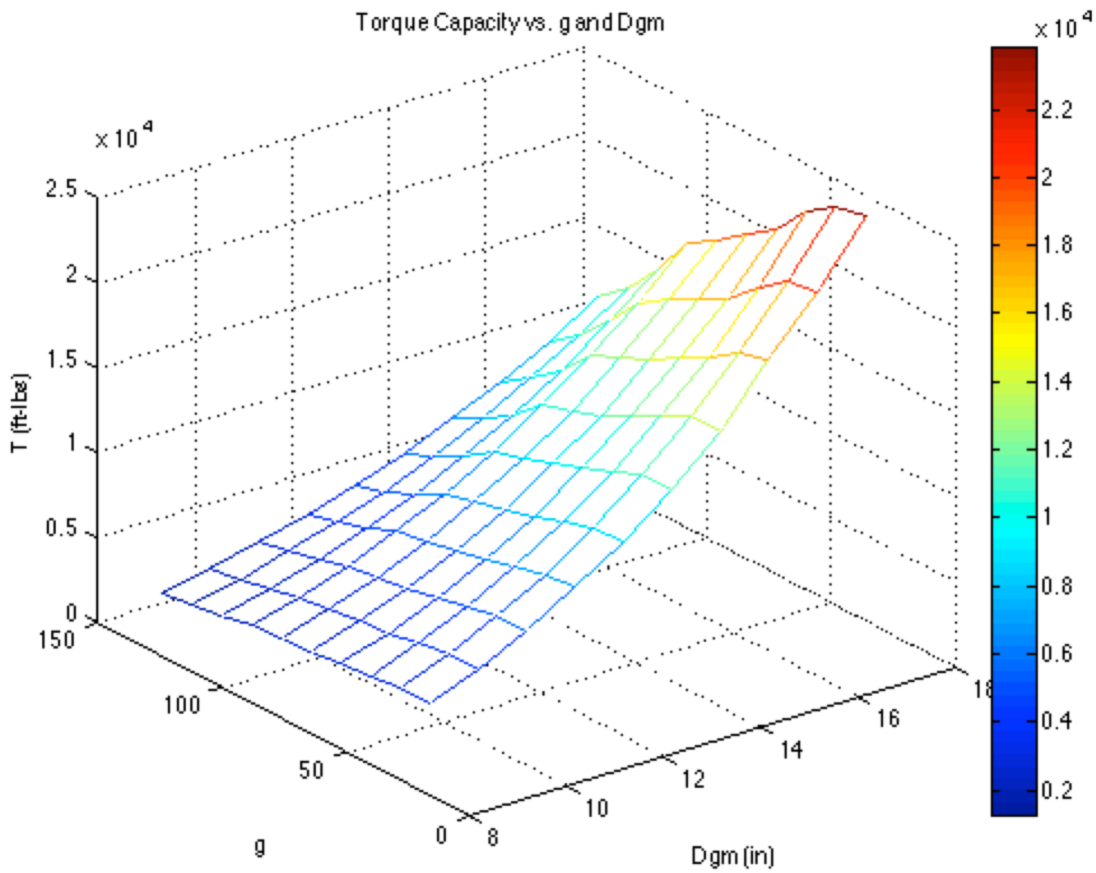

Figure 5-14: Best Torque Capacity Performance on Input Parameters

Similar to the 1-Stage SCGT, guidelines for the selected parameters were identified. Through calculation and curve fitting, the following guidelines were obtained:

$$N_{ss1} = 18$$

$$N_{ss2} = 18$$

$$g_1 = \begin{cases} 0.1046 * g + 0.6879, & g < 50.78 \\ 6 & g \geq 50.78 \end{cases}$$

$$g_2 = \begin{cases} 3, & g \leq 51.72 \\ 0.0574 * g + 0.0315, & 51.72 < g < 103.98 \\ 6, & g \geq 103.98 \end{cases}$$

$$P_{d1} = 0.5097 * g - 4.1505 * D + 68.1001$$

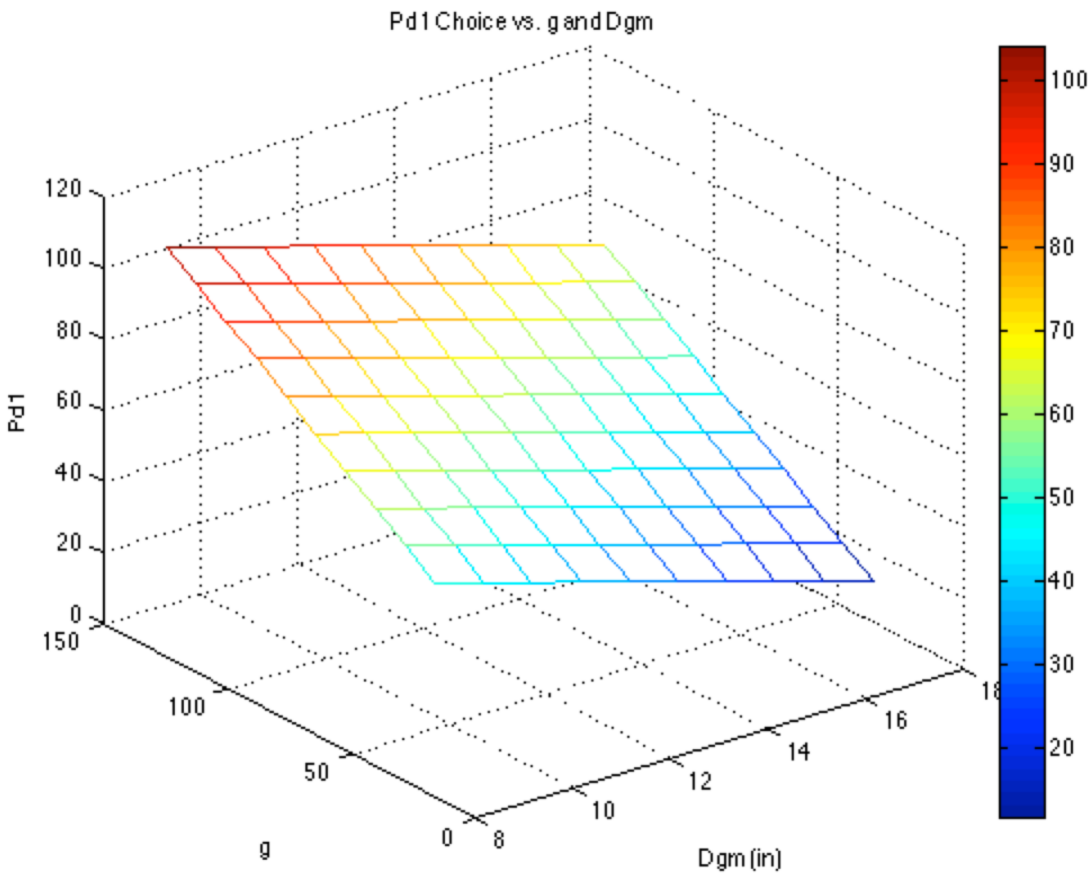

Figure 5-15: Pd1 Choice for Input Parameter Pairs

Based on the calculations, the assignment of the number of teeth for the pinion in the first mesh is totally random, and no guidelines could be identified. However, for the second and third meshes, the largest torque is achieved when the number of teeth on each pinion is equal to the minimum, 18.

The selection of the reduction ratio for each stage is only related to the total reduction ratio, and has no relevance to the gear train mesh diameter. There are also two obvious limits for the reduction ratio. When the total reduction ratio is below 51 or above

103, the reduction ratio for each is below 6, which meets the constraint set at the beginning of the process.

The diametral pitch for the first mesh is related to both the total reduction ratio and the gear train mesh diameter, as shown in Figure 5-15. The pitch increases with increasing reduction ratio and decreasing gear train mesh diameter.

To demonstrate the correctness of the assumptions for the input parameters, the torque was calculated based on these assumptions (see Figure 5-16). For N_P, the largest torque can be identified by comparing all the choices, but the resulting value is much lower than the result obtained from the exhaustive search.

In comparison to the torque capacity values obtained from the exhaustive search, the torques obtained based on the above assumptions ranges from 90.6% to 1 on all the samples selected. Thus, the assumptions are quite good. When a 2-Stage P-Type SCGT is designed, only one parameter N_P is undetermined, which saves the designer quite a lot of time.

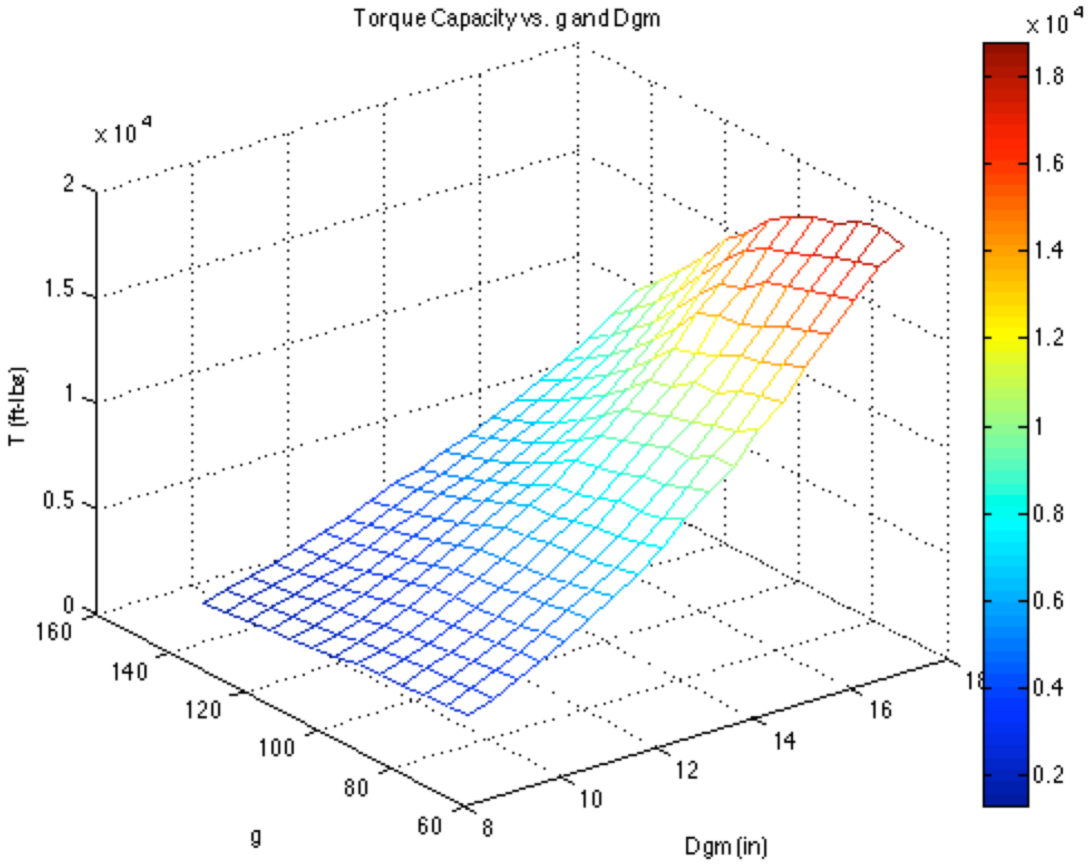

Figure 5-16: Torque Capacity Based on Input Assumption

5.5 2-STAGE C-TYPE SCGT

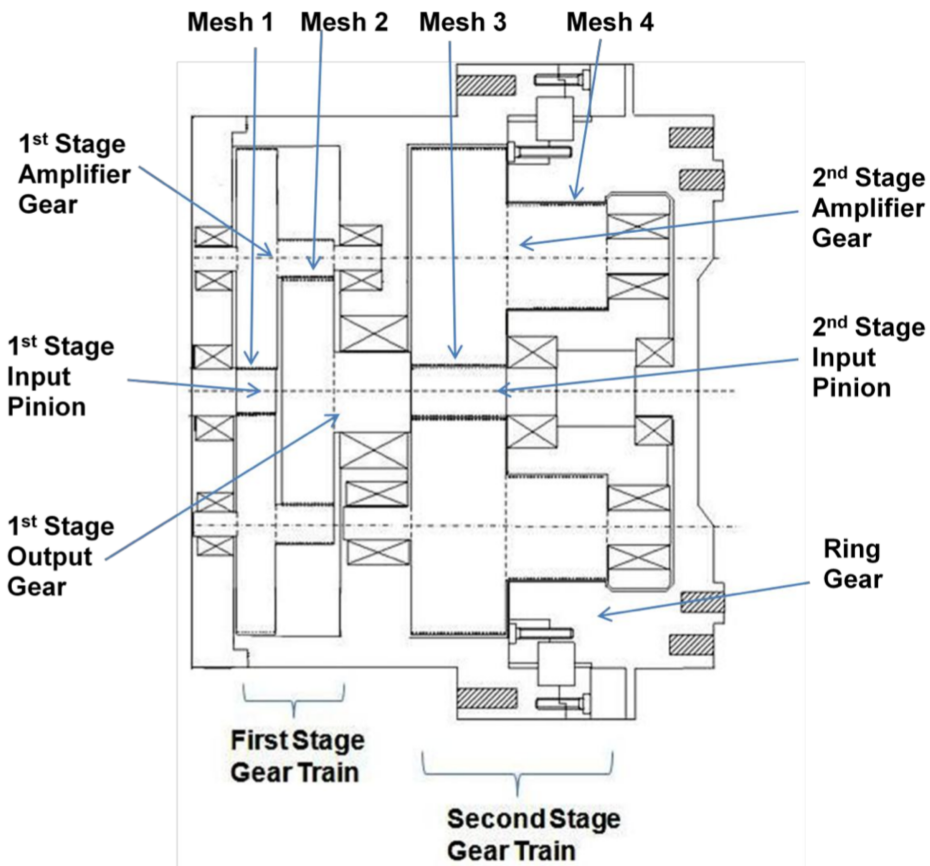

Figure 5-17: 2-Stage C-Type SCGT Structure [5]

Figure 5-17 shows a Coffee-Can Type Two-Stage SCGT structure. Actually, the right side of the C-Type 2-Stage SCGT is the same as the 1-Stage SCGT. The left side is a little different, because an external gear is used instead of an internal ring gear. The overall gear reduction is given in the equation below, and the nomenclature for this gear train type is given in Table 5-10.

$$g = g_1 g_2 g_3 g_4 = \frac{\prod N_{Driven\ Gears}}{\prod N_{Driving\ Gears}} = \frac{N_{LS1}}{N_P}\frac{N_{LS2}}{N_{SS1}}\frac{N_{LS3}}{N_{SS2}}\frac{N_R}{N_{SS3}}$$

Nomenclature			
Symbol		Description	
Mesh1		Mesh between input pinion and first large star gears	
Mesh2		Mesh between first small star gears and the second large gear	
Mesh3		Mesh between second small star gears and the third large gear	
Mesh4		Mesh between Third Small Star gears and the Ring gear	
g	Whole gear reduction ratio	D_{gm}	Gear train mesh diameter
g_1	Reduction ratio of Mesh 1	g_2	Reduction ratio of Mesh 2
g_3	Reduction ratio of Mesh 3	g_4	Reduction ratio of Mesh 4
N_P	Number of teeth of input pinion	N_{LS1}	Number of teeth of the first large star gear
N_{SS1}	Number of teeth of the first small star pinion	N_{LS2}	Number of teeth of the second large star gear
N_{SS2}	Number of teeth of the second small star pinion	N_{L32}	Number of teeth of the third large star gear
N_{SS3}	Number of teeth of the third small star pinion	N_R	Number of teeth of ring gear
P_{d1}	Diametral pitch of gears in Mesh 1	P_{d2}	Diametral pitch of gears in Mesh 2
P_{d3}	Diametral pitch of gears in Mesh 3	P_{d4}	Diametral pitch of gears in Mesh 4
D_P	Diameter of the input pinion	D_{LS1}	Diameter of the first large star gear
D_{SS1}	Diameter of the first small star gear	D_{LS2}	Diameter of the second large star gear
D_{SS2}	Diameter of the second small star gear	D_{LS3}	Diameter of the third large star gear
D_{SS3}	Diameter of the third small star gear	D_R	Diameter of the ring gear
F_1	Face width of gears in Mesh 1	F_2	Face width of gears in Mesh 2
F_3	Face width of gears in Mesh 3	F_4	Face width of gears in Mesh 4

Table 5-10: Nomenclature for 2-Stage C-Type SCGT

5.5.1 2-Stage C-Type SCGT Structure Analysis

For a gear train with four mesh contacts, there is a total of 16 parameters for the gear train design:

$$N_P, N_{LS1}, N_{SS1}, N_{LS2}, N_{SS2}, N_{LS3}, N_{SS3}, N_R, D_P, D_{LS1}, D_{SS1}, D_{LS2}, D_{SS2}, D_{LS3}, D_{SS3}, D_R$$

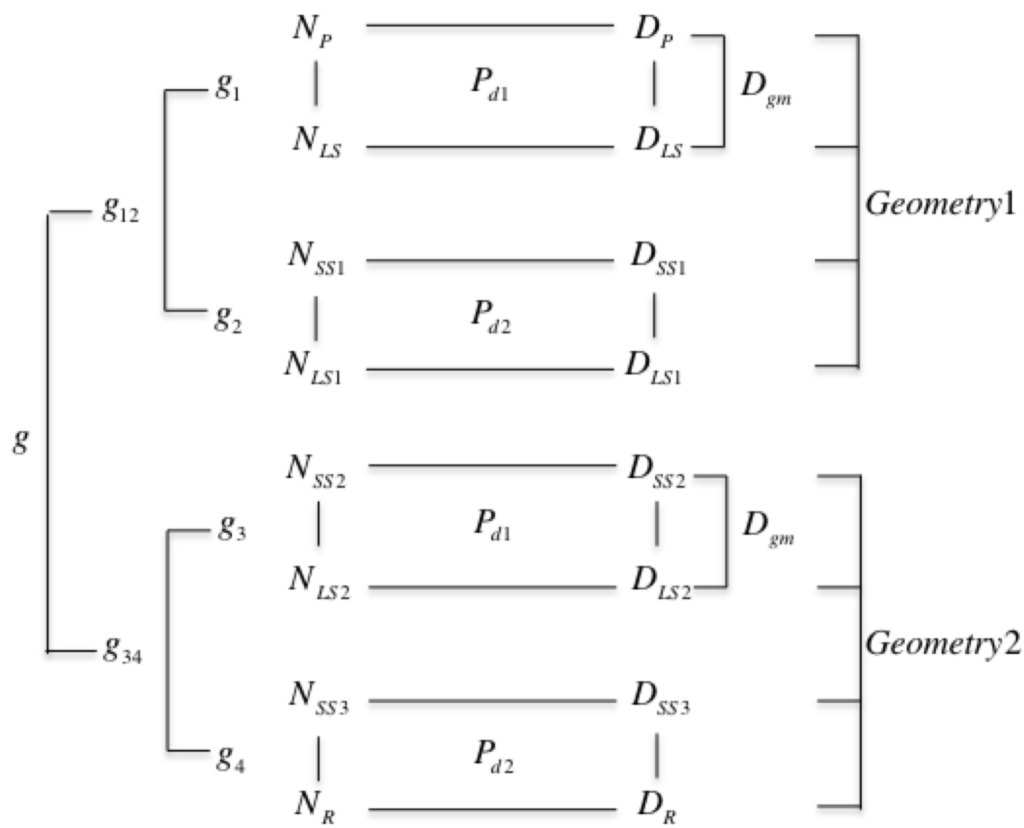

Figure 5-18: C Type SCGT Parameters Relationship

The relationships among the parameters for this type of gear train are depicted in Figure 5-18. For a gear train with 4 mesh contacts, 12 individual parameters must be assigned values, and five other design and geometry constraints are listed below:

$$g = g_1 * g_2 * g_3 * g_4$$

$$D_{gm} = D_P + D_{LS} \times 2$$

$$D_{gm} = D_{SS2} + 2 * D_{LS3}$$

$$D_P + D_{LS1} = D_{LS2} + D_{SS1}$$

$$D_R = D_{SS2} + D_{LS3} + D_{SS3}$$

Note that the gear train mesh diameter of the left stage is assumed to be the same as that of the right stage. In the standardization process, this assumption is changed to constrain the diameter of the left stage to be no bigger than the right stage. Because the right stage is mainly used to bear the large output torque, it deserves priority in the design process. After considering the above constraints, we are left with seven unknown parameters to be selected.

5.5.2 Optimization of the Design

Based on the similarities between the right stage of the 2-Stage C-Type SCGT and the 1-Stage SCGT, the simulation can be divided into two parts, the first two mesh contacts, and the last two.

The first two gear meshes have nearly the same size as the last two, but the load torque is much lower, which means almost no torque calculation is needed for the left two meshes. We focus on the reduction ratio. In a manner similar to the 1-Stage SCGT, we set $N_P = N_{LS1} = 18$, $D_{gm} = 6$, and use P_{d1} and P_{d2} as inputs, with the reduction ratio the only remaining unknown parameter.

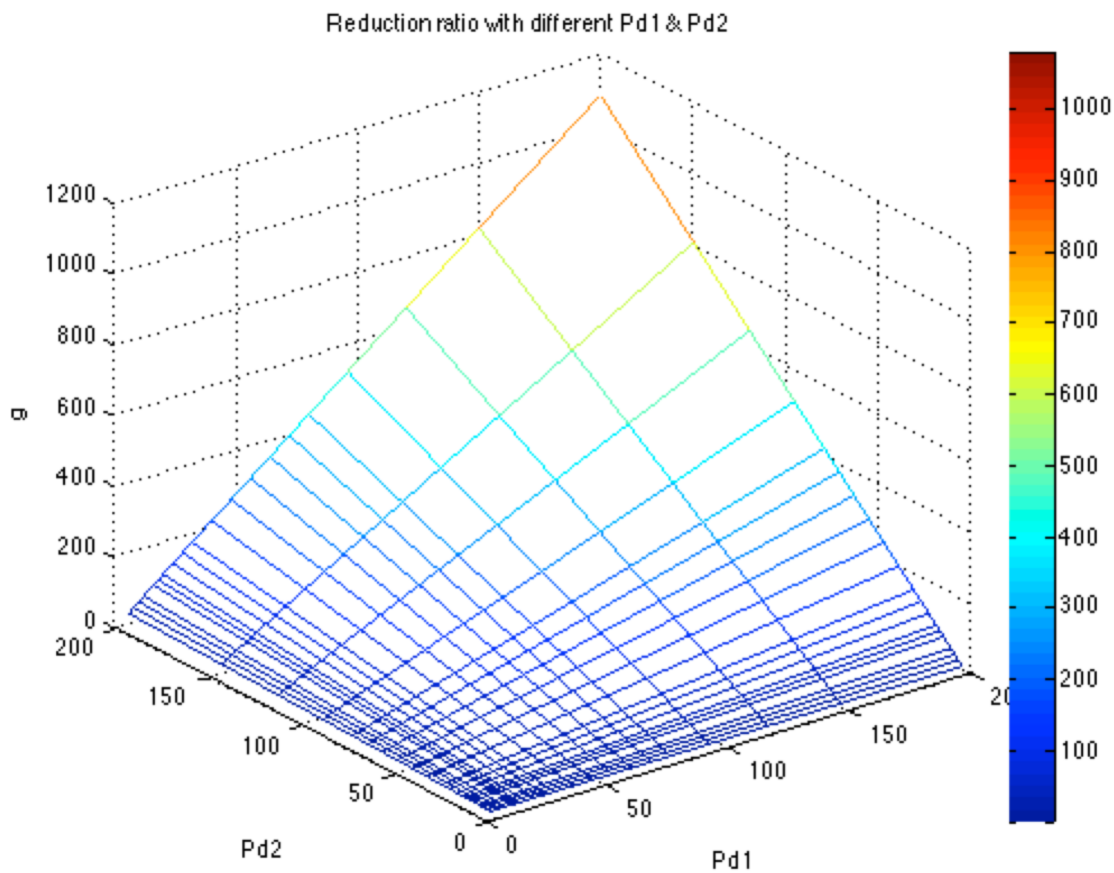

Figure 5-19: Reduction ratio vs. Pd1 & Pd2

Figure 5-19 illustrates that nearly all reduction ratios can be obtained with appropriate combinations of P_{d1} and P_{d2}. This means the left part of this gear train only operates as a gear reduction multiplier, and its torque performance is not important.

The design process for the right stage of the 2-Stage C-Type SCGT is the same as that for a 1-Stage SCGT with $g' = {}^{g}/_{g_1 g_2}$. Based on the discussion of the 1-Stage SCGT, we know the 1-Stage SCGT performs best when the reduction ratio is about 11.3; therefore, we choose $g_3 \times g_4 = 11.3$ and $g_1 \times g_2 = g/11.3$.

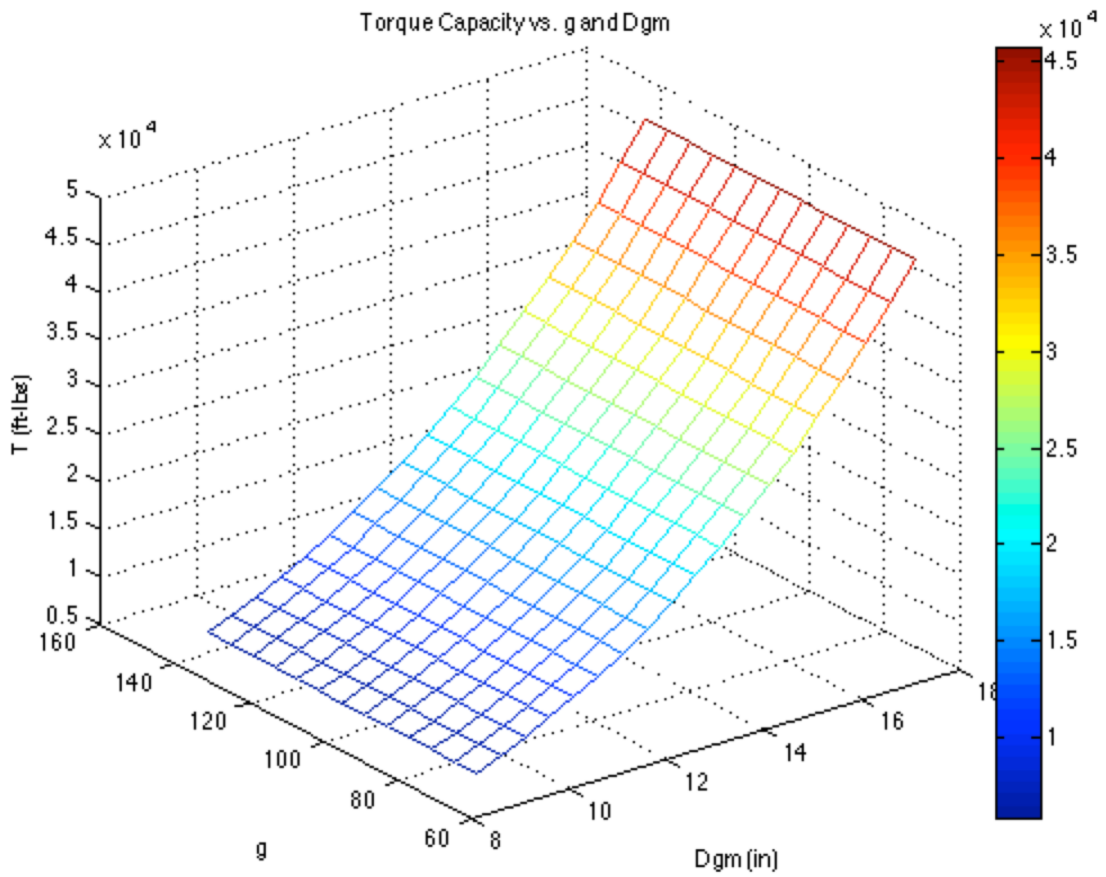

Figure 5-20: Torque Capacity of 2-Stage C-Type SCGT

Since the reduction ratio requirement can always be achieved by the first two mesh contacts, the *g* requirement does not impact the final torque capacity. So the torque capacity for 2-Stage C-Type SCGT is not affected by the reduction ratio, as shown in Figure 5-20.

5.5.3 Comparison of 2-Stage P-Type SCGT and C-Type SCGT

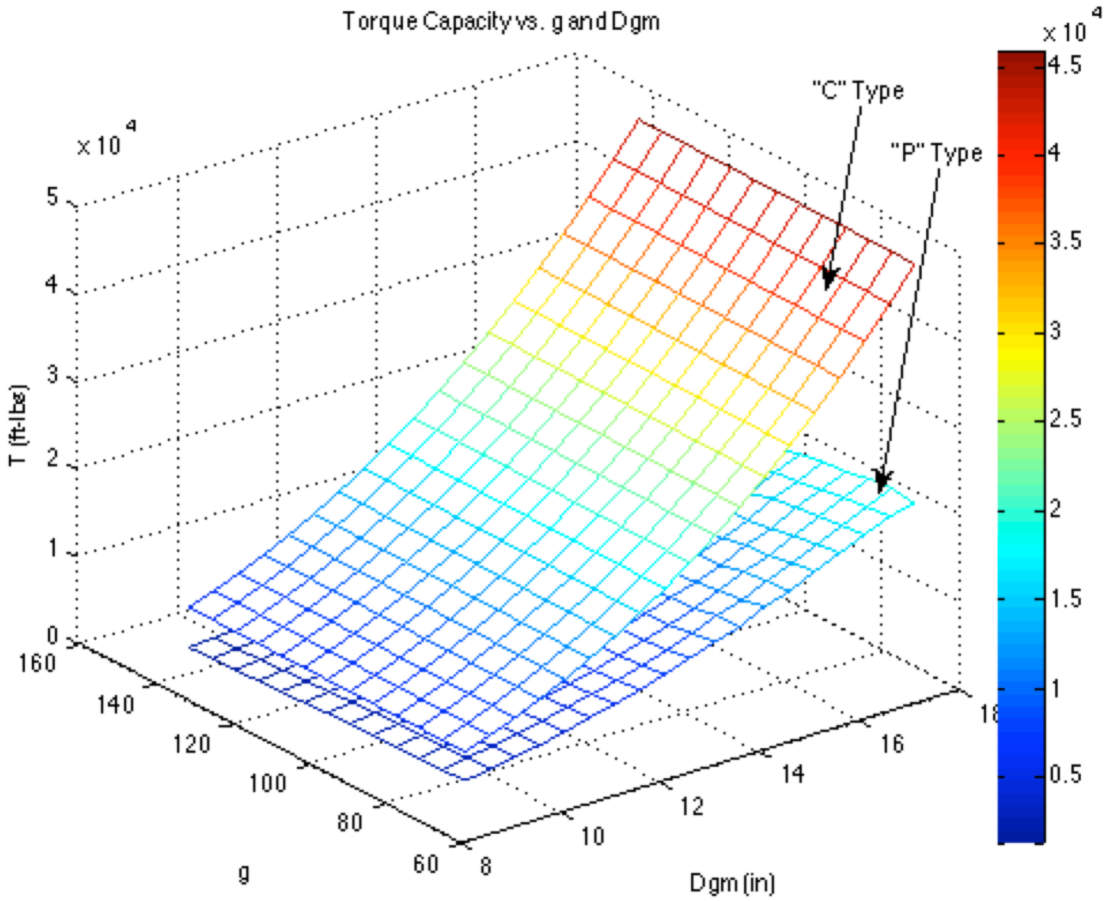

Figure 5-21: C and P Type Torque Comparison with Same g & D_{gm}

Figure 5-21 compares torque carrying capacity of the P-Type SCGT and the C-Type SCGT, and shows there is an obvious advantage for the C-Type in torque capacity. However, the P-Type also has advantages, and cannot be replaced by the C-Type for all situations. The biggest issue with the C-Type is its length, which is more than twice that of the P-Type.

For a P-Type SCGT, the total length of the gear train is:

$$L_P = F_2 + F_3$$

For the C-Type SCGT, the total length of the gear train is:

$$L_C = F_1 + F_2 + F_3 + F_4 + Clearance$$

To have a better understanding of these two gear trains, a comparison of their lengths was conducted. The results are shown in Figure 5-22.

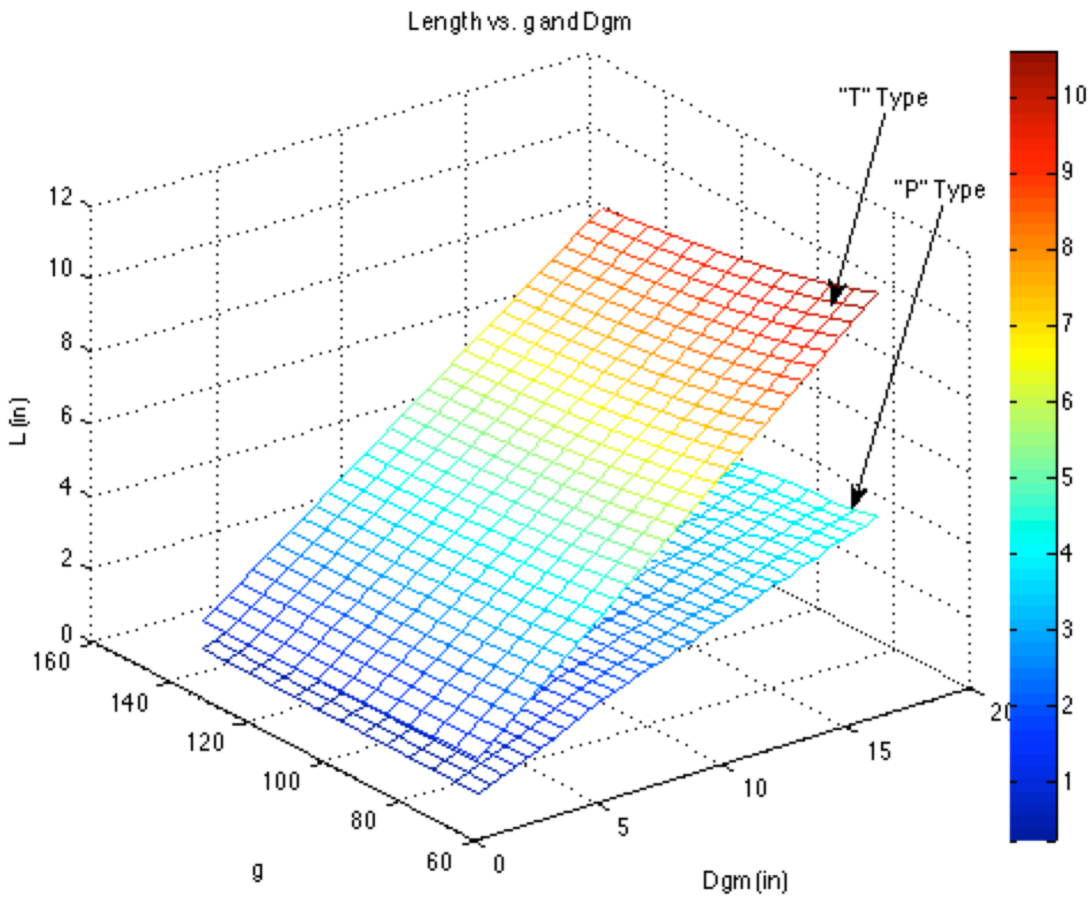

Figure 5-22: C and P Type Length Comparison

From this graph, we clearly see the advantage of the P-Type SCGT. As we have mentioned, the actual length of the C Type gear train is the sum of the length of four gears plus the clearance between mesh 2 and mesh 3 required for the bearing. The total

length of the C Type is almost three times that of the P Type. This is another criterion for designers to consider in choosing an appropriate gear train.

5.6 Gear Train Performance Evaluation

The performance of a gear train can be evaluated in many other ways besides the torque capacity. For example, a collaborative robot requires an actuator with light weight and quick responsiveness. Therefore, a set of performance criteria should be evaluated and compared to assist the design process. A full evaluation of the gear train also facilitates material selection, as will be seen in next chapter.

Here we choose the following four performance criteria for evaluation:

1. Weight (W)
2. Effective Inertia (I)
3. Torque Density (T_d)
4. Responsiveness (R)

As we have discussed previously, in gear train type selection, when the reduction ratio exceeds 27, we choose either a P-Type or C-Type two-stage SCGT if there are no dimension requirements. Comparing the performance of these two kinds of gear train with respect to these other performance criteria provides the designer with more detailed information with which to make a decision.

5.6.1 Weight

For a 1-Stage SCGT, the total weight is the sum of the pinion, the second gear (a combination of the large star gear and the small star gear), and the ring gear. The weight is proportional to the density of the material. The detailed calculations are shown below:

$$V_P = \frac{\pi (D_P)^2}{4} F_1$$

$$V_g = \frac{\pi(D_{LS})^2}{4} F_1 + \frac{\pi(D_{SS})^2}{4} F_2$$

$$V_R = \frac{\pi[(D_{gm})^2 - (D_R)^2]}{4} F_2$$

$$W = \rho_P V_P + N_g \rho_g V_g + \rho_R V_R$$

where N_g is the number of star gears, with $N_g = 3$ for this design.

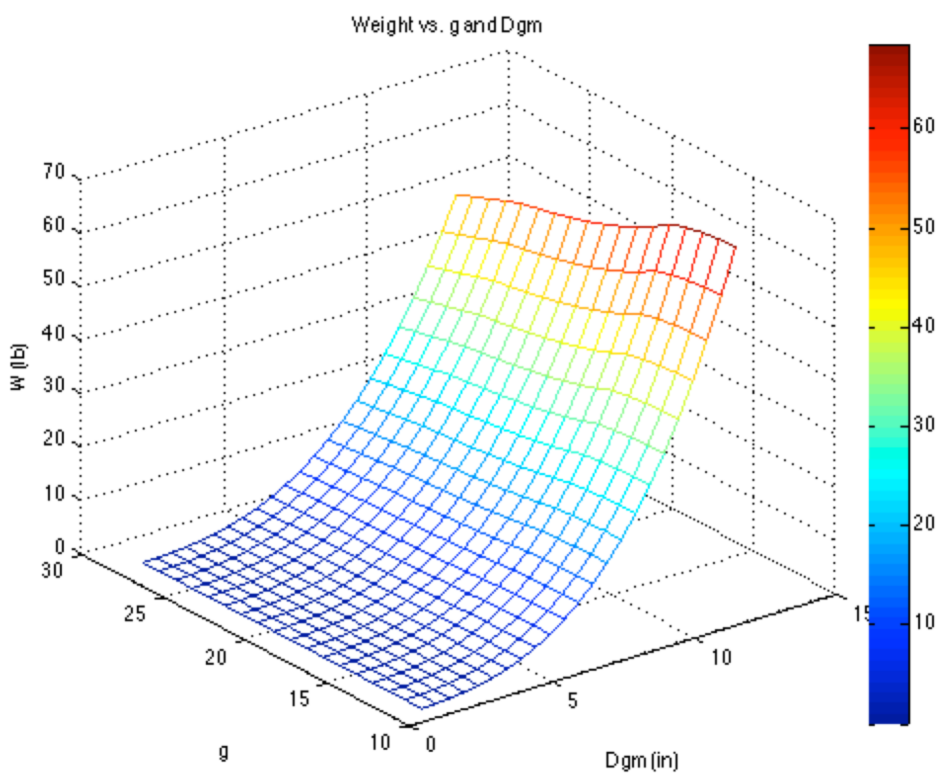

Figure 5-23: Weight vs. g and D_{gm} for 1-Stage SCGT

As shown in Figure 5-23, the weight of the gear train grows as the gear train mesh diameter increases, while decreasing slightly for higher reduction ratios. The weight of a gear train is proportional to the material density, gear train length, and the square of the diameter. The reason for the reduction is that increasing *g* decreases the total length.

For a 2-Stage P-Type SCGT, the weight is calculated as follows:

$$V_P = \frac{\pi (D_P)^2}{4} F_1$$

$$V_{g1} = \frac{\pi (D_{LS1})^2}{4} F_1 + \frac{\pi (D_{SS1})^2}{4} F_2$$

$$V_{g2} = \frac{\pi (D_{LS2})^2}{4} F_2 + \frac{\pi (D_{SS2})^2}{4} F_3$$

$$V_R = \frac{\pi [(D_{gm})^2 - (D_R)^2]}{4} F_3$$

$$W = \rho_P V_P + N_g \rho_{g1} V_{g1} + N_g \rho_{g2} V_{g2} + \rho_R V_R$$

For a 2-stage C-Type SCGT, the weight is calculated as follows:

$$V_P = \frac{\pi (D_P)^2}{4} F_1$$

$$V_{g1} = \frac{\pi (D_{LS1})^2}{4} F_1 + \frac{\pi (D_{SS1})^2}{4} F_2$$

$$V_{g2} = \frac{\pi (D_{LS2})^2}{4} F_2 + \frac{\pi (D_{SS2})^2}{4} F_3$$

$$V_{g3} = \frac{\pi(D_{LS3})^2}{4} F_3 + \frac{\pi(D_{SS3})^2}{4} F_4$$

$$V_R = \frac{\pi[(D_{gm})^2 - (D_R)^2]}{4} F_4$$

$$W = \rho_P V_P + N_g \rho_{g1} V_{g1} + \rho_{g2} V_{g2} + N_g \rho_{g3} V_{g3} + \rho_R V_R$$

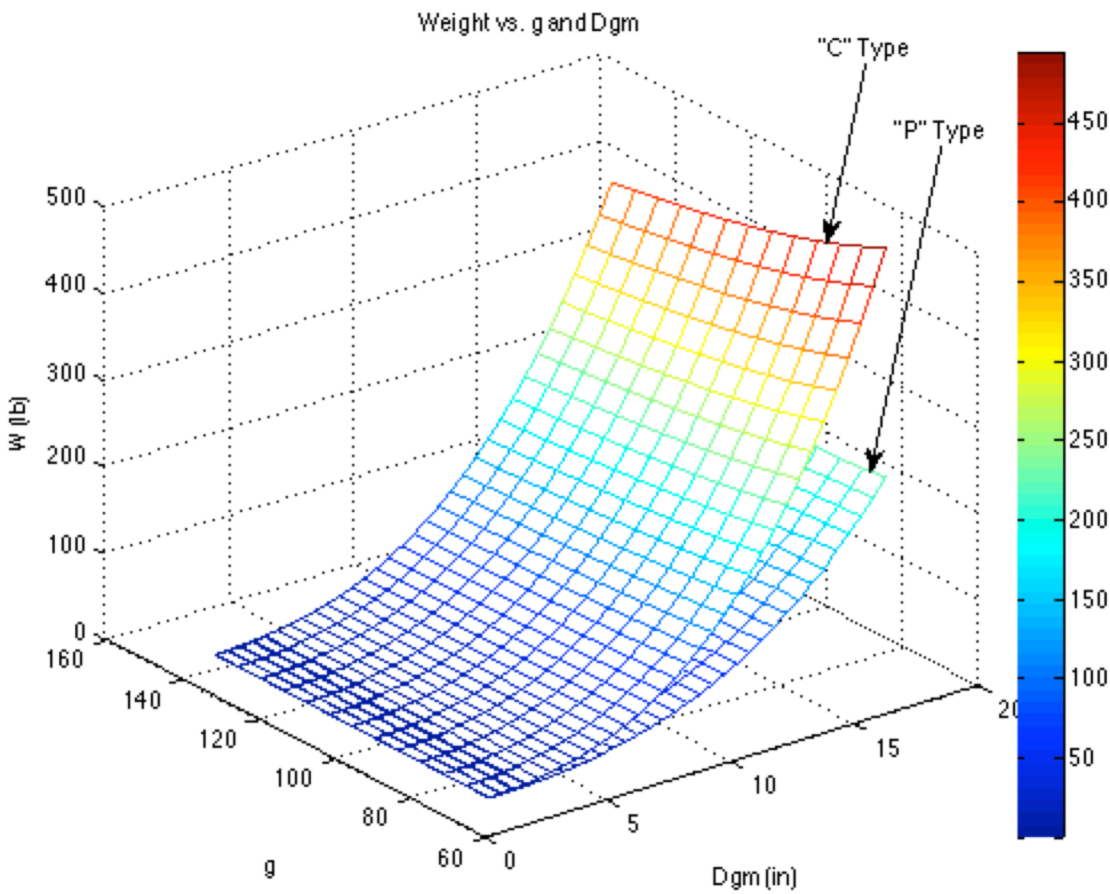

Figure 5-24: Comparison of Weight for C and P Type of SCGT

As shown in Figure 5-24, the total weight of the C-Type is much larger than the weight of the P-Type. Their relationship is quite similar to performance with respect to the total length.

5.6.2 Torque Density

Torque density is the ratio of the torque to the weight:

$$T_D = \frac{T}{W}$$

\A higher torque density means more torque capacity can be provided with the same weight. This is a very important criterion, especially when a gear train with low weight and high torque capacity is needed.

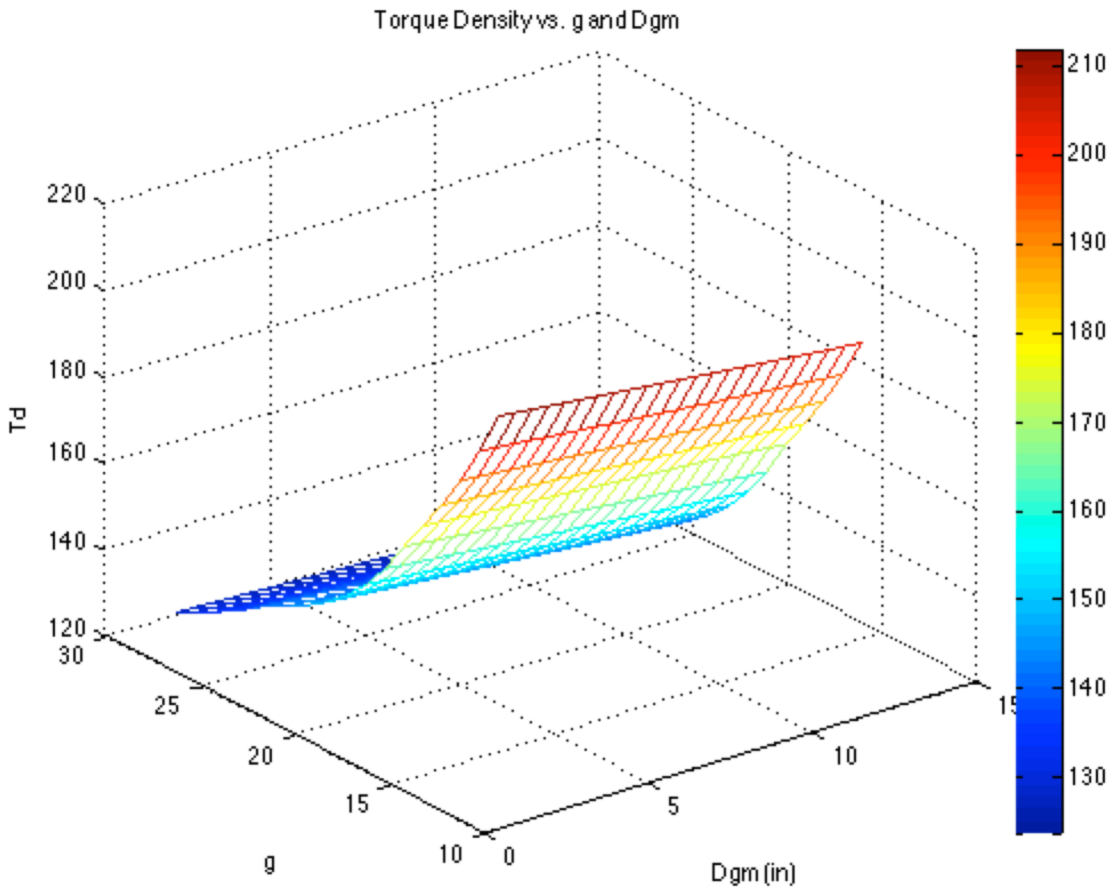

Figure 5-25: Torque Density for 1-Stage SCGT

As shown in Figure 5-25, the torque capacity for the 1-State SCGT decreases as the reduction ratio increases, while staying nearly constant as the gear train mesh diameter varies.

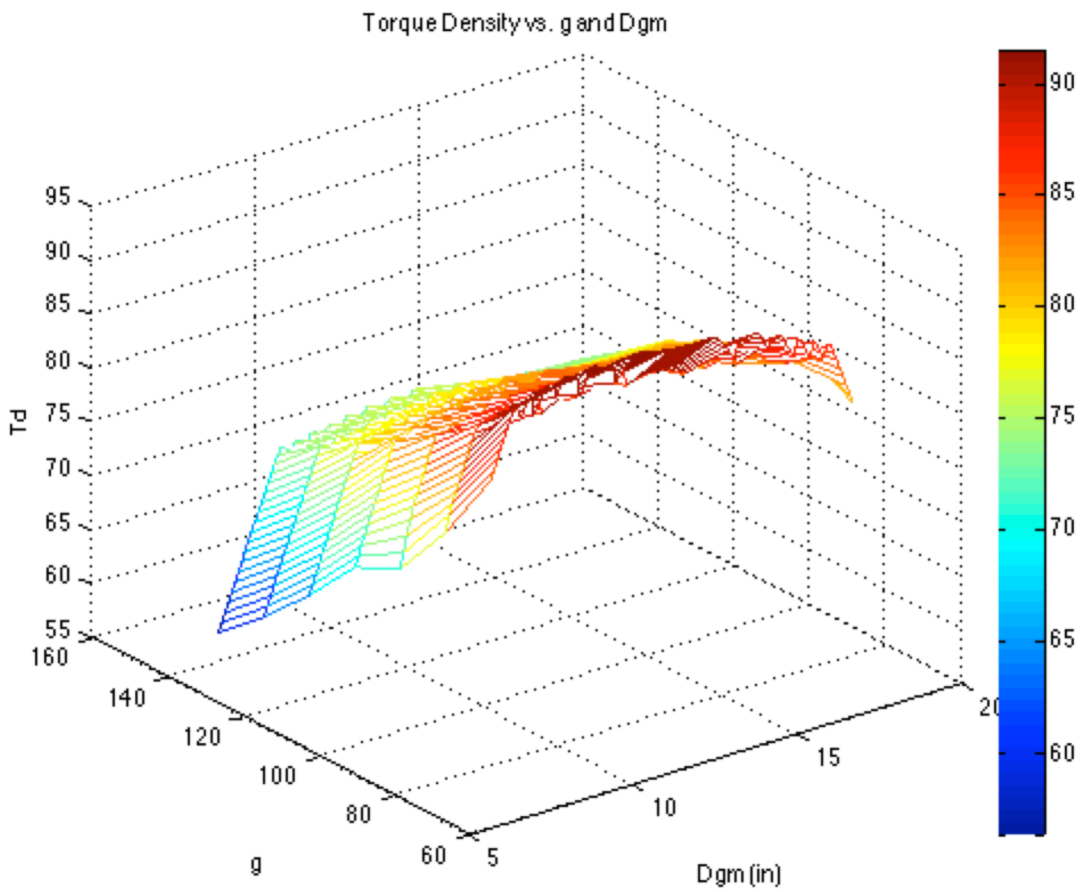

Figure 5-26: Torque Density of 2-Stage P-Type SCGT

Because the 2-Stage C-Type SCGT is actually a combination of two 1-Stage gear trains, there is no added value in evaluating its torque density. Figure 5-26 above shows the torque density relationship of a P-Type SCGT. The torque density of P-Type SCGT cannot be compared to a 1-Stage SCGT, because they have quite different reduction ratios. And even though the torque density reduces with increasing reduction ratio, the slope of the P-Type SCGT performance surface is much steeper than that of a 1-Stage SCGT.

5.6.3 Inertia

Inertia is defined as the resistance of a gear train to rotational motion. In general, a smaller inertia is expected to result in a gear train that responds quickly under a specific applied torque.

The inertia of a hollow cylinder is calculated using following equation:

$$I = \rho L \frac{\pi \left({d_o}^4 - {d_i}^4 \right)}{32}$$

where, d_o is the outer diameter (diametral diameter) of the gear and d_i is the inner diameter of the hollow space for the bearing. Here we use a factor K_O defined as:

$$d_i = K_O \, d_o$$

The inertia equation then becomes:

$$I = K \rho L \frac{\pi \left({d_o}^4 \right)}{32}$$

where $K = 1 - {K_0}^4$, and in this study, $K_0 = 1/2$, and $K = 31/32$.

The total inertia for the whole gear train is the inertia of the pinion, star gear, and the ring gear. It is worth noting that the center of the star gear is not coincident with the center of the pinion, and therefore the total inertia increased by the product of the weight of the star gears and the square of the distance between two axes.

For a 1-Stage SCGT, the inertia is calculated as follows:

$$I_P = K\,\rho_P\,F_1\,\frac{\pi\,\left(D_P{}^4\right)}{32}$$

$$I_g = K\,\rho_g\,F_1\,\frac{\pi\,\left(D_{LS}{}^4\right)}{32} + K\,\rho_g\,F_2\,\frac{\pi\,\left(D_{SS}{}^4\right)}{32}$$

$$I_R = \rho_R\,F_2\,\frac{\pi\,\left(D_{gm}{}^4 - D_R{}^4\right)}{32}$$

$$I = I_P + N_g\,[\,I_g + W_g * (D_P + D_{LS})^2/4] + I_R$$

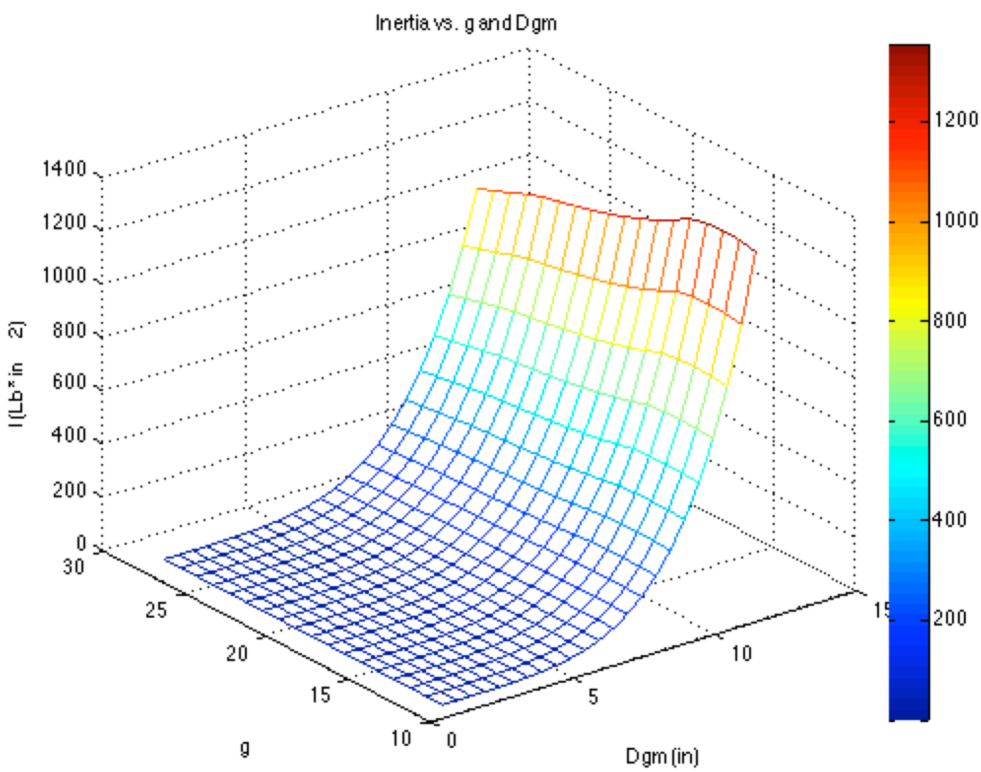

Figure 5-27: Inertia vs. g and D_{gm} for 1-Stage SCGT

As shown in Figure 5-27, the inertia of 1-Stage SCGT positively correlates with the gear train mesh diameter and negatively correlates with the reduction ratio. Based on the calculation process, the inertia is proportional to the fifth power of the gear train mesh diameter. The decrease in inertia with reduction ratio is also caused by the decreasing gear train length.

For a P Type SCGT, the inertia is calculated as follows:

$$I_P = K\,\rho_P\,F_1\,\frac{\pi\,\left({D_P}^4\right)}{32}$$

$$I_{g1} = K\,\rho_{g1}\,F_1\,\frac{\pi\,\left({D_{LS1}}^4\right)}{32} + K\,\rho_{g1}\,F_2\,\frac{\pi\,\left({D_{SS1}}^4\right)}{32}$$

$$I_{g2} = K\,\rho_{g2}\,F_2\,\frac{\pi\,\left({D_{LS2}}^4\right)}{32} + K\,\rho_{g2}\,F_3\,\frac{\pi\,\left({D_{SS2}}^4\right)}{32}$$

$$I_R = K\rho_R\,F_3\,\frac{\pi\,\left({D_{gm}}^4 - {D_R}^4\right)}{32}$$

$$I = I_P + N_g\,[\,I_{g1} + W_{g1} * (D_P + D_{LS1})^2/4] + N_g\,[\,I_{g2} + W_{g2} * (D_P + D_{LS1} + D_{SS1} + D_{LS2})^2/4] + I_R$$

For a C Type SCGT, the inertia is calculated as follows:

$$I_P = K\,\rho_P\,F_1\,\frac{\pi\,\left({D_P}^4\right)}{32}$$

$$I_{g1} = K\,\rho_{g1}\,F_1\,\frac{\pi\,\left({D_{LS1}}^4\right)}{32} + K\,\rho_{g1}\,F_2\,\frac{\pi\,\left({D_{SS1}}^4\right)}{32}$$

$$I_{g2} = K\,\rho_{g2}\,F_2\,\frac{\pi\,\left({D_{LS2}}^4\right)}{32} + K\,\rho_{g2}\,F_3\,\frac{\pi\,\left({D_{SS2}}^4\right)}{32}$$

$$I_{g3} = K\,\rho_{g3}\,F_3\,\frac{\pi\,\left({D_{LS3}}^4\right)}{32} + K\,\rho_{g3}\,F_4\,\frac{\pi\,\left({D_{SS3}}^4\right)}{32}$$

$$I_R = \rho_R\,F_4\,\frac{\pi\,\left({D_{gm}}^4 - {D_R}^4\right)}{32}$$

$$I = I_P + N_g\,[\,I_{g1} + W_{g1} * (D_P + D_{LS1})^2 + [\,I_{g2}] +$$

$$+ N_g\,[\,I_{g3} + W_{g3} * (D_{SS2} + D_{LS3})^2/4] + I_R$$

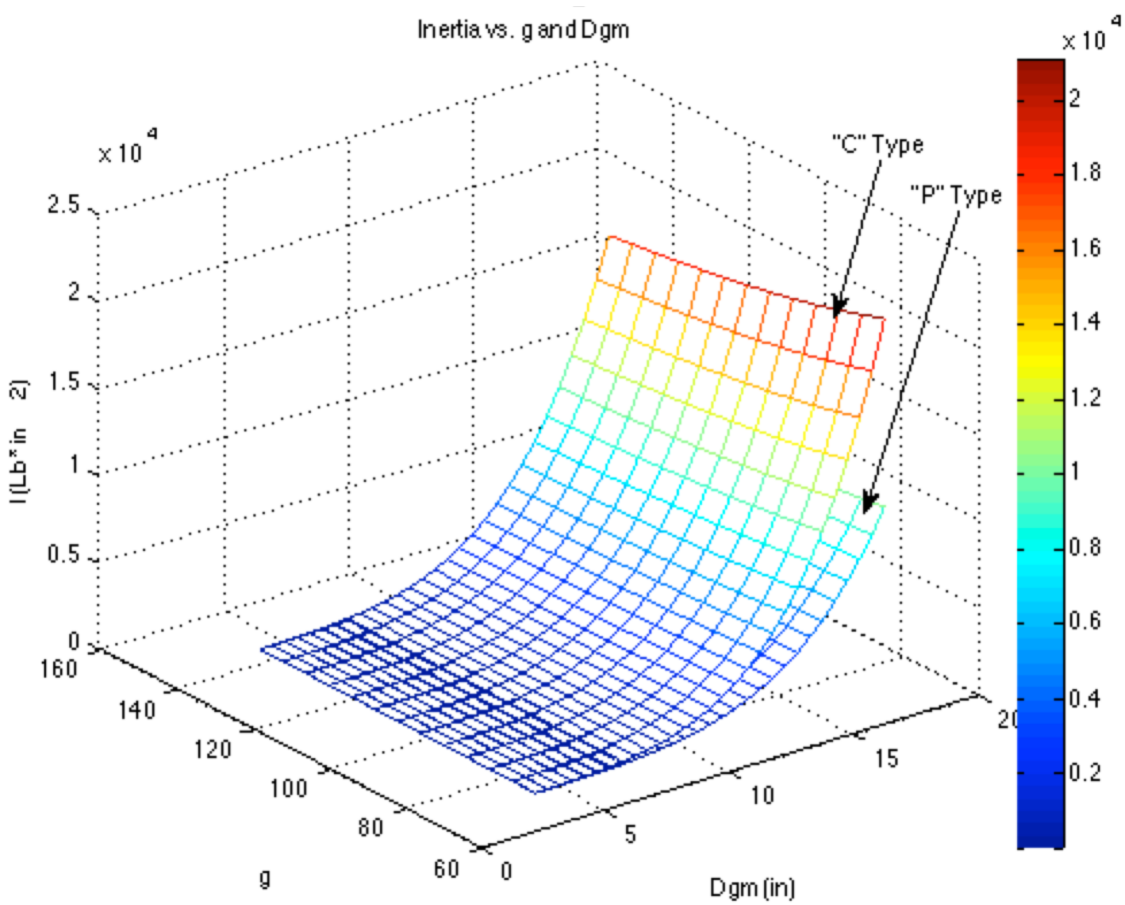

Figure 5-28: Comparison of Inertia for C and P Type of SCGT

As shown in Figure 5-28, the inertia of the C Type is bigger than the P Type under the same reduction ratio and gear train mesh diameter. Besides the difference in structure, the length of the gear train is the main reason for this difference.

5.6.4 Responsiveness

The responsiveness of a gear train is the ratio of the torque to the inertia, which reflects the ability for the velocity to change.

$$R = \frac{T}{I}$$

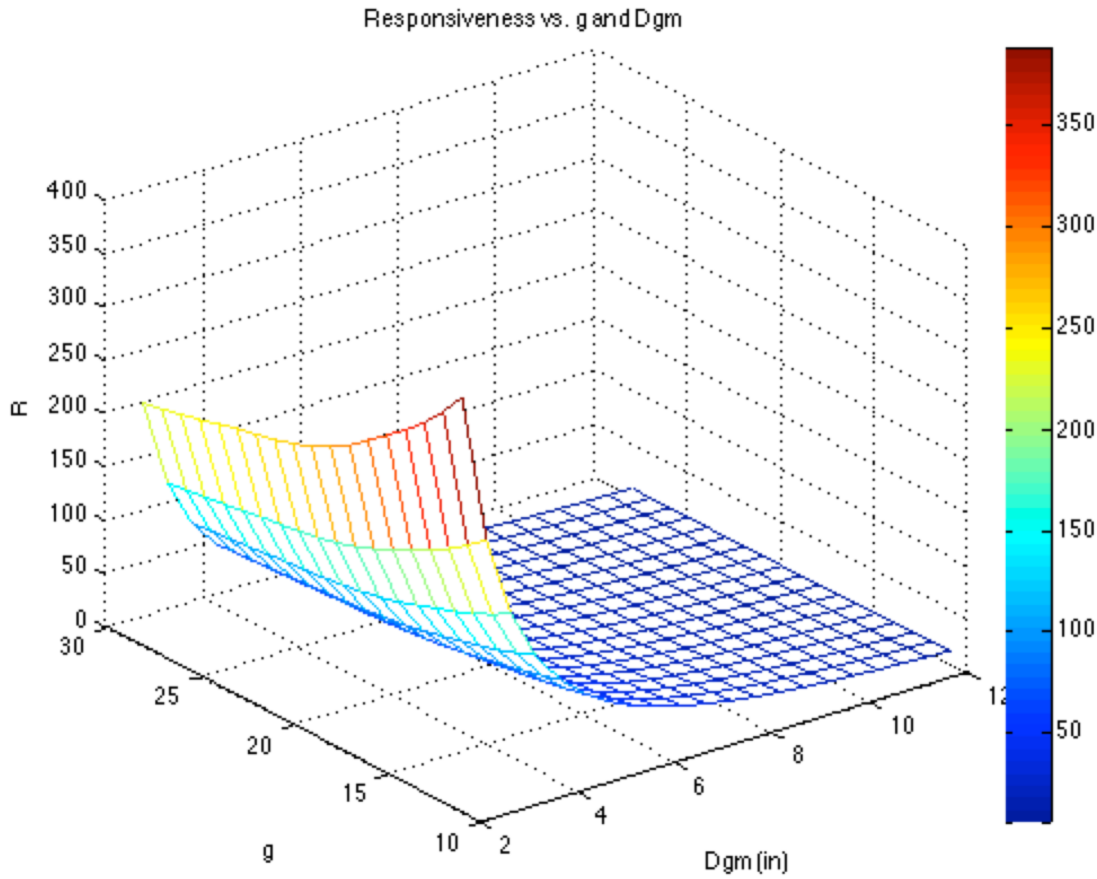

Figure 5-29: Responsiveness vs. g and D_{gm} for 1-Stage SCGT

Figure 5-29 illustrates the relationship of responsiveness with overall gear reduction ratio and the overall gear train diameter for a 1-Stage SCGT. As the figure shows, the responsiveness decreases as the reduction ratio and gear train mesh diameter increase. One guideline inspired by this graph is that, when a relatively large responsiveness is needed and one of the two input pairs is fixed, the responsiveness can be increased by decreasing the mesh diameter.

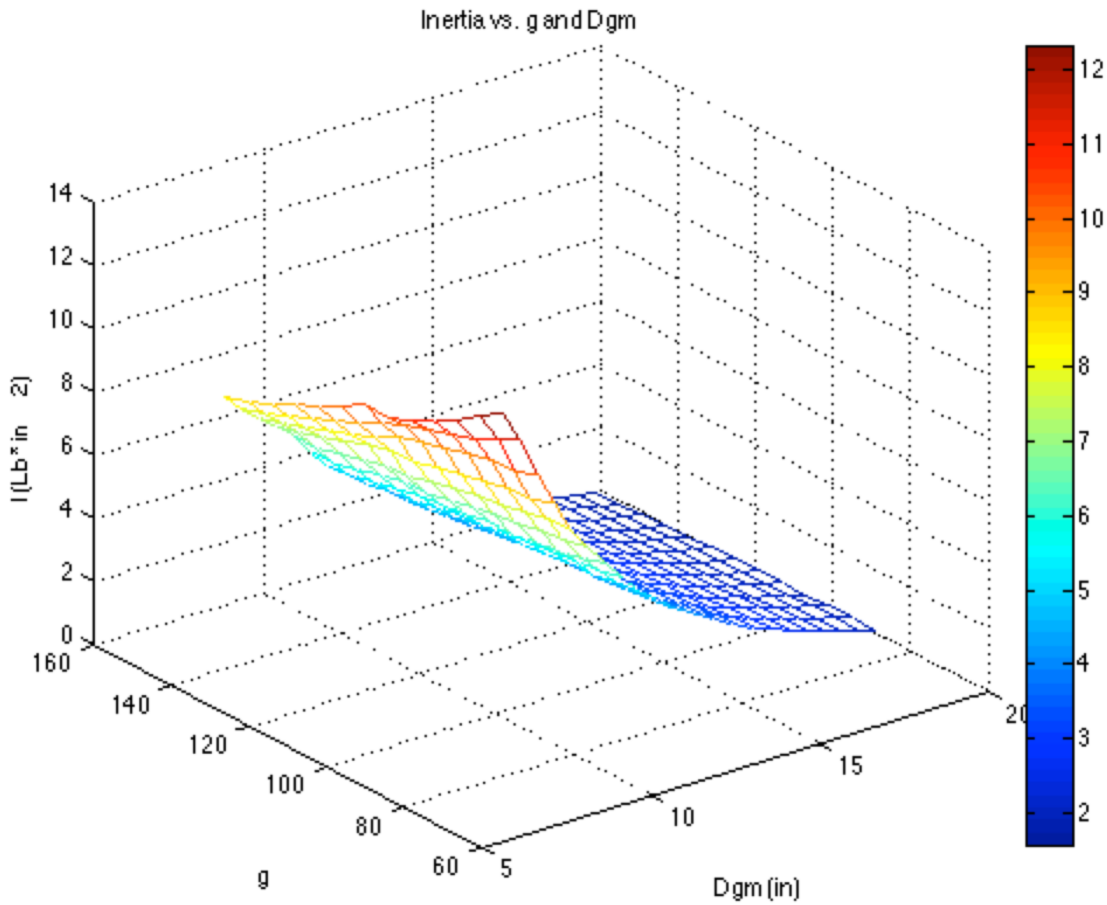

Figure 5-30: Responsiveness vs. g & D_{gm} for 2-Stage P-Type SCGT

As shown in Figure 5-30, the responsiveness of a P-Type 2-Stage SCGT has the same trend as a 1-Stage SCGT relative to the reduction ratio and diameter. The responsiveness of a P-Type SCGT looks much smaller than that of a 1-Stage SCGT, but actually they have totally different reduction ratios and cannot be directly compared.

5.7 Standardization of Parameters

The design method developed in this chapter is based on finding the largest torque capacity (or other performance criterion) assuming the input parameter pairs of reduction ratio and diametral pitch. As demonstrated previously, the proposed method is quite

appropriate for the three selected types of gear trains. The assumptions made for the 1-Stage SCGT and 2-Stage P-Type SCGT work quite well.

However, for a real gear train design, continuous parameter values may result in strange parameter values for gears, perhaps impossible to fabricate. In this section, a standardization process is demonstrated that operates on the results from the optimization design. After standardization, the design is feasible using widely adopted gear series.

5.7.1 Standard Gear Parameters

As discussed previously, for a gear, three parameters should be standardized:

P_d – Diametral Pitch

N – Number of gear teeth

F – Face width

The design process ends when these three parameters are determined. However, each of these three parameters has limitations.

Among these three parameters, the diametral pitch is the hardest one to be standardized, for it must be selected from a specific series. The number of teeth should be an integer. And, in general, even though only gears with specific numbers of teeth are commonly available, this parameter is more flexible than the diametral pitch. The common diametrical pitches and modules are shown in Table 5-11.

Diametral Pitches in General Use	
Coarse pitch	2, 2 1/4, 2 1/2, 3, 4, 6, 8, 10, 12, 16
Fine pitch	20, 24, 32, 40, 48, 64, 96, 120, 150, 200
Modules in General Use	
Preferred	1,1.25,1.5,2,2.5,3,4,5,6,8,10,12,16,20,25,32,40,50
Next Choice	1.125,1.375,1.75,2.25,2.75,3.5,4.5,5.5,7,9,11,14,18,22,28,36,45

Table 5-11: Commonly Used Diametral Pitches and Modules [55]

Based on gear manufacturers' information, gears with diametral pitches of 18, 24, 28, and 56 can also be found. [39] In general, the series for diametral pitch selection used in this research is as follows:

[6, 8, 10, 12, 16, 18, 20, 24, 28, 32, 40, 48, 56, 64, 72, 80, 96, 120, 150, 200]

Next, the face width also has a series, and the series used in this study is:

[1/16, 1/8, 3/16, 1/4, 5/16, 3/8, 7/16, 1/2, 9/16, 5/8, 11/16, 3/4, 13/16, 7/8, 15/16, 1, 1.125, 1.25, 1.375, 1.5, 1.625, 1.75, 1.875, 2, 2.25, 2.5, 2.75, 3, 3.25, 3.5, 3.75, 4]

The standardization of the face width is relatively more flexible, because it is the last parameter to be chosen and does not interfere with the standardization process of the other two parameters.

5.7.2 Standardization Method

The central idea of this standardization method is to use the parameters calculated through the design and optimization process, and then compare the performance of the points which are close to the selected peak and find standard gears that are close to the continuous values found from optimization.

Similar to design optimization, the independent parameters of the new system should be analyzed first. This time, the total reduction ratio, and gear train mesh diameter

are no longer given as inputs, which means the individual parameters of the whole system have larger values than the values obtained from optimization design.

The first parameter set is the diametral pitch, because it is very limited and deserves high priority. Since $P_d = {}^{N}/_{D}$, a little change in the diametral pitch will make a big different in the number of teeth, so multiple choices should be evaluated to make sure the new parameters are close to the optimal point. The method suggested here is to start with the closest standard diametral pitches that bracket the pitch determined from the analysis presented above. Therefore, for the 1-Stage SCGT, four choices are evaluated; for 2-Stage P-Type SCGT, eight choices are evaluated, and for 2-Stage C-Type SCGT, 16 choices are evaluated for the final selection.

Next, the other individual parameters are assigned values according to the design requirements. For example, if the design requires a strict reduction ratio, or the number of gear teeth is fixed, these values can be chosen as the input parameters, and other parameters without such constraints can be calculated. In our examples, we choose to assign several gear teeth numbers first, and the value assigned is just the number we get in the optimization design. Last, the face width is chosen, completing the standardization process. If needed, other gear train performance metrics are also determined for the final selection.

The final selection is the one that fulfills the requirements best, and has relatively good performance with respect to the important criteria. During final selection, the following criteria can be used to assist designers in making a decision:

1. Check if there are strict design requirements. For example, if D_{gm} is strictly limited, a value for D_{gm} that exceeds the original design should not be selected.
2. Check that all the gear teeth numbers are easy to find.

3. Compare the performance of the rest of the choices, and choose the one with the best performance. In our examples, the torque capacity has been compared.

Another two points worth considering in the gear teeth number assignments are:

1. The number of gear teeth should not be an integral multiple of that of the pinion in the same mesh, to avoid having the same teeth contact each other in all rotations.
2. The teeth number should be adjusted to leave enough clearance for assembly. This point is carefully discussed in our examples.

5.7.3 Example: 1-Stage SCGT Gear Parameter Optimization

This example is a gear train design for a 1-Stage SCGT with input values $g = 18$ and $D_{gm} = 6$. With one geometry constraint for 1-Stage SCGT, and with values assigned to the reduction ratio, g, and gear train mesh diameter, D_{gm}, there are five input parameters remaining to be assigned. Because P_{d1} and P_{d2} will be determined by our selection method, the other three input parameters in our example are N_p, N_{ss}, and N_{ls}.

Values for P_{d1} and P_{d2} should be chosen from the standard series close to the result obtained from the optimization design. For the resulting values, $P_{d1} = 28.56$, $P_{d2} = 17.51$; therefore, four combinations of 32/28, and 18/16 are evaluated. N_p, N_{ss}, and N_{ls} are rounded to the nearest integers.

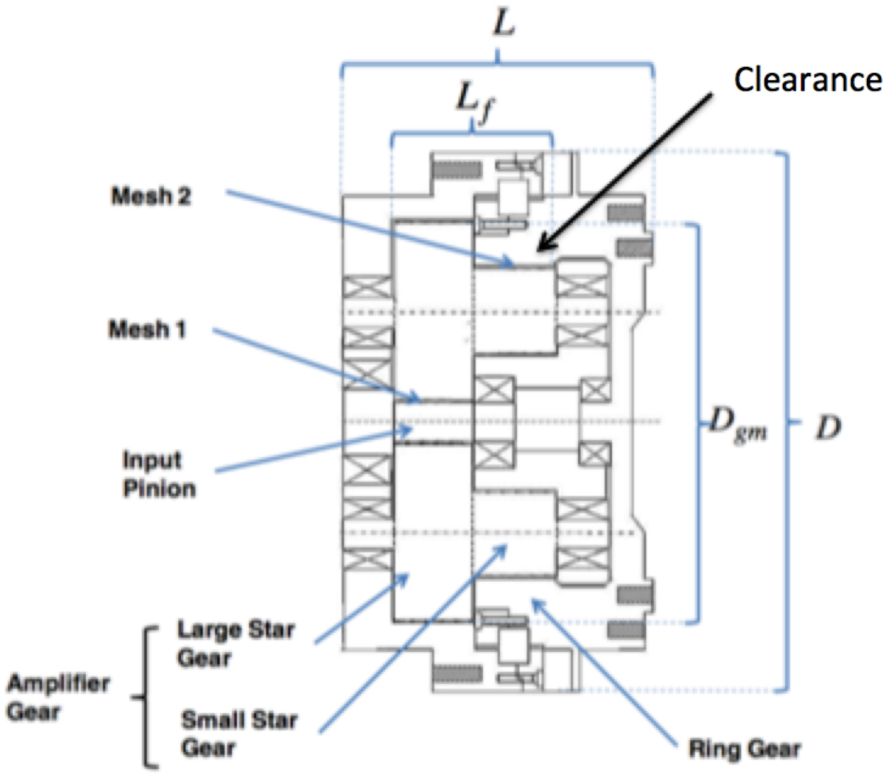

Figure 5-31: Clearance for 1-Stage SCGT [5]

One point worth noticing is that, for a real gear train, clearance should be provided for assembly. (See Figure 5-31) Clearance is nominally provided between the star gears and the ring gear here, but actually it is shared by all the mesh contacts. So the geometry constraint is adjusted as:

$$D_R \geq D_P + D_{LS} + D_{SS}$$

For this example, the following calculations are used:

$$D_R = D_P + D_{LS} + D_{SS},$$

$$N_R = D_R \times P_{d2}$$

Update N_R: $N_R = ceil\,(N_R) + n$

Update D_R: $D_R = N_R / P_{d3}$

where, *ceil* is a function that returns the next integer larger than its argument, and n is a flexible non-negative integer to adjust the clearance. In this example, n is chosen to be 0 or 1. The choices for 1-Stage SCGT parameters are shown in Table 5-12.

	Pd1	Pd2	g1	g2	g	Dgm	Clearance
Original	28.38	17.66	4.23	4.26	18.00	6	0
1	32	18	4.22	4.00	16.89	5.31	0.063
2	28	18	4.22	4.39	18.53	6.07	0.032
3	32	16	4.22	3.67	15.48	5.31	0.063
4	28	16	4.22	4.00	16.89	6.07	0.018
	Np	Nls	Nss	Nr	F1	F2	T
Original	18	76.14	18	76.60	0.62	1.02	1169.17
1	18	76	18	71	0.625	0.8125	899.79
2	18	76	18	79	0.625	1.125	1174.98
3	18	76	18	66	0.625	0.6875	824.81
4	18	76	18	72	0.6875	0.9375	1290.48

Table 5-12: Standard Parameter Choices for 1-Stage SCGT

In the final selection process from these four choices, we assume there is a strict constraint for the gear train mesh diameter, D_{gm}, and any diameter that exceeds the original design is eliminated. So design 2 and design 4 do not meet the requirements. Next, we compare design 1 and design 3. For design 1, N_r is 71, which is not a commonly used number of teeth. If we adjust N_r to 72, it will be exactly four times larger than N_{ss}, which violates the guideline that the number of teeth on the gear should not be an integral multiple of that on the pinion in the same mesh. Based on this consideration, design 3 is chosen.

5.7.4 Example: 2-Stage P-Type SCGT Gear Parameters Optimization

This example is the design of a 2-Stage P-Type SCGT with input g = 96 and $D_{gm} = 8$. For a 2-Stage P-Type SCGT, because there is only one geometric constraint, eight parameters must be assigned values.

Through the optimization design, P_{d1} = 83.83, P_{d2} = 35.29, and P_{d3} = 6.57; therefore, eight combinations of 96/80, 40/32, 8/6 are listed in Table 5-13.

With no other design requirements, N_p, N_{ss1}, N_{ss2}, N_{ls1}, and N_{ls2} are selected as the other five input parameters. The value for each of these parameters is chosen to be the closest integer. It is worth noting that, with N_{ls1} = 132, and N_p = 22, N_{ls2} is six times larger than N_p. To avoid integral multiples of gear teeth in meshed pairs, N_{ls2} should be set to 130.

There is also one clearance value to set, and the geometric constraint is changed to:

$$D_R \geq D_P + D_{LS1} + D_{SS1} + D_{LS2} + D_{SS2}$$

The same process is performed to determine D_R, to leave enough clearance for assembly:

$$D_R = D_P + D_{LS1} + D_{SS1} + D_{LS2} + D_{SS2}$$

$$N_R = D_R \times P_{d3}$$

$$\text{Update } N_R\text{: } N_R = ceil\ (N_R) + n$$

$$\text{Update } D_R\text{: } D_R = N_R / P_{d3}$$

	Pd1	Pd2	Pd3	g1	g2	g3	g0	Dgm	Clearance	T
0	83.83	35.29	6.57	6	5.54	2.89	96.00	8	0	3365.0
1	96	40	8	5.91	5.56	3.06	101.85	7.05	0.07	1764.8
2	80	40	8	5.91	5.56	3.22	107.41	7.38	0.13	1550.9
3	96	32	8	5.91	5.56	3.39	112.96	8.42	0.08	3561.2
4	96	40	6	5.91	5.56	2.56	85.19	7.05	0.11	1968.0
5	80	32	8	5.91	5.56	3.50	116.67	8.74	0.01	3500.8
6	80	40	6	5.91	5.56	2.67	88.89	7.38	0.13	1711.3
7	96	32	6	5.91	5.56	2.78	92.59	8.42	0.04	3892.1
8	80	32	6	5.91	5.56	2.89	96.30	8.74	0.05	3852.8
	Np	Nls1	Nss1	Nls2	Nss2	Nr	F1	F2	F3	
0	22	132	18	99.75	18	51.97	0.19	0.51	1.13	
1	22	130	18	100	18	55	0.1875	0.5	1	
2	22	130	18	100	18	58	0.1875	0.5	0.9375	
3	22	130	18	100	18	61	0.25	0.5	1.125	
4	22	130	18	100	18	46	0.1875	0.5	1.125	
5	22	130	18	100	18	63	0.25	0.5625	1.125	
6	22	130	18	100	18	48	0.1875	0.5	1.125	
7	22	130	18	100	18	50	0.25	0.5	1.25	
8	22	130	18	100	18	52	0.25	0.5625	1.25	

Table 5-13: Standardization Parameters for 2-Stage P-Type SCGT

Of the eight designs under final consideration, the torque performance drops significantly for the designs with D_{gm} smaller than original design. This illustrates why it is not a good idea to enforce the strict diameter constraint on D_{gm} at first. In designs 3, 5, 7, and 8, D_{gm} only exceeds the original design slightly, but the torque performance is really good.

If we assume the strict constraint for D_{gm} is 9, we can accept any of the designs. After comparing g, N_r, and T, we choose design 7 as the final choice, with a reduction ratio of 92.59, number of teeth on the ring gear of 50, and torque capacity of 3892.1 ft-lb.

5.7.5 2-Example: Stage C-Type SCGT Gear Parameters Optimization

This example is a gear train design for a 2-Stage C-Type SCGT with inputs g = 200 and $D_{gm} = 6$. For a 2-Stage C-Type SCGT, 12 parameters are initially unknown. The following three geometric relationships reduce the number to nine:

$$D_{gm} = D_P + 2 \times D_{LS1}$$

$$D_P + D_{LS1} - D_{SS1} = D_{LS2}$$

$$D_{SS2} + D_{LS3} + D_{SS3} = D_R$$

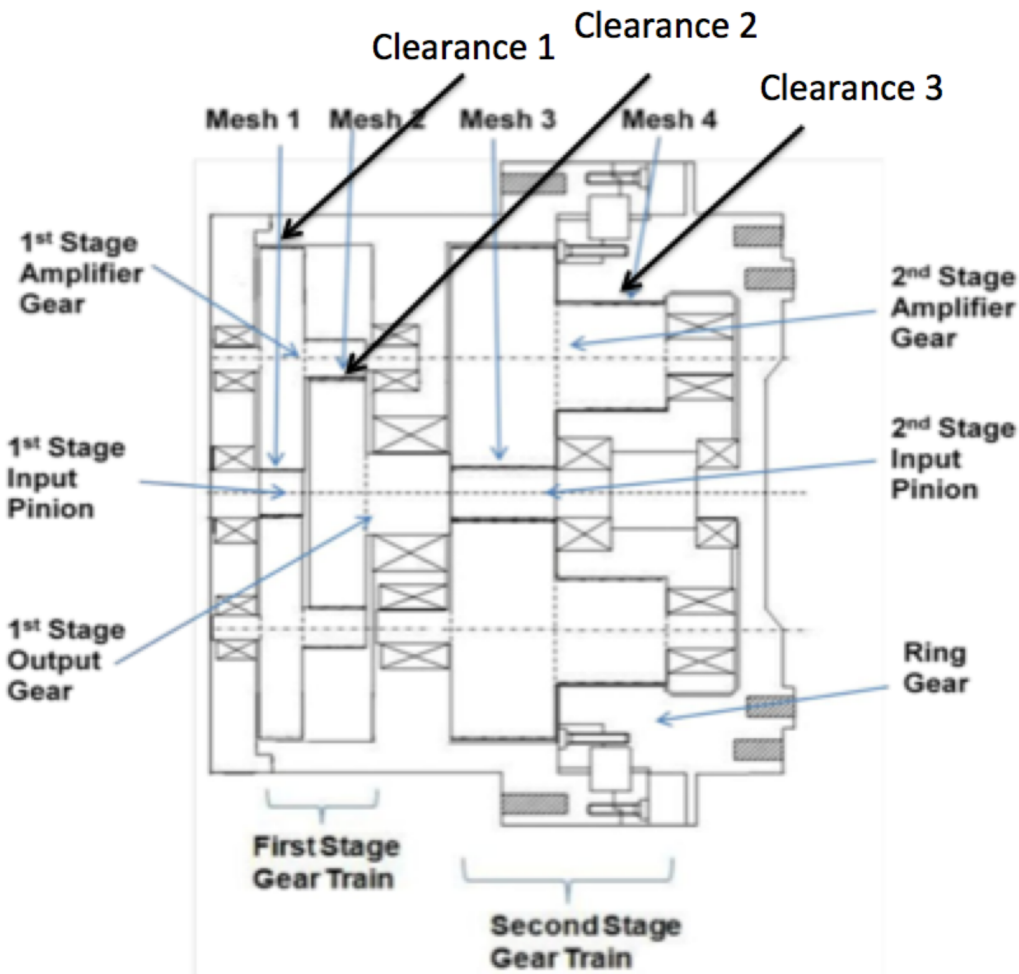

Figure 5-32: Clearance for 2-Stage C-Type SCGT [5]

Additionally, three clearances are needed to allow assembly (see Figure 5-32):

$$Clearance\ 1 = D_{gm} - (D_P + 2 \times D_{LS1})$$

$$Clearance\ 2 = D_P + D_{LS1} - D_{SS1} - D_{LS2}$$

$$Clearance\ 3 = D_R - (D_{SS2} + D_{LS3} + D_{SS3})$$

From the optimization design, the diametral pitches are $P_{d1} = 28.24$, $P_{d2} = 28.24$, $P_{d3} = 21.47$, and $P_{d4} = 14.06$. Therefore, 16 combinations of 32/28, 32/28, 24/20, 16/12 are listed in Tables 5-14 through 5-16.

For the other five parameters, we choose N_p, N_{ss1}, N_{ss2}, N_{ss3}, and N_{ls3} as inputs, and the values assigned are all integers close to the results calculated during optimization. All standard parameter values are shown in the following tables.

	Pd1	Pd2	Pd3	Pd4	g1	g2	g3	g4	g0
0	28.24	28.24	21.47	14.06	4.21	4.21	3.08	3.67	200.00
1	32	32	24	16	4.22	4.17	3.06	3.72	200.09
2	28	32	24	16	3.61	4.17	3.06	3.72	171.13
3	32	28	24	16	4.22	3.50	3.06	3.72	168.07
4	32	32	20	16	5.17	5.11	3.06	4.28	345.17
5	32	32	24	12	4.22	4.17	3.06	3.06	164.25
6	32	32	20	12	5.17	5.11	3.06	3.44	277.93
7	32	28	24	12	4.22	3.50	3.06	3.06	137.97
8	32	28	20	16	5.17	4.33	3.06	4.28	292.64
9	28	32	24	12	3.61	4.17	3.06	3.06	140.48
10	28	32	20	16	4.44	5.17	3.06	4.28	300.15
11	28	28	24	16	3.61	3.56	3.06	3.72	146.03
12	32	28	20	12	5.17	4.33	3.06	3.44	235.64
13	28	32	20	12	4.44	5.17	3.06	3.44	241.68
14	28	28	24	12	3.61	3.56	3.06	3.06	119.88
15	28	28	20	16	4.44	4.39	3.06	4.28	254.96
16	28	28	20	12	4.44	4.39	3.06	3.44	205.30

Table 5-14: Standardization Parameters for 2-Stage C-Type SCGT

	Np	Nls1	Nss1	Nls2	Nss2	Nls3	Nss4	Nr	T
0	18	75.73	18	75.73	18	55.42	18	66.06	1993.3
1	18	76	18	75	18	55	18	67	1393.2
2	18	65	18	75	18	55	18	67	1393.2
3	18	76	18	63	18	55	18	67	1393.2
4	18	93	18	92	18	55	18	77	1239.9
5	18	76	18	75	18	55	18	55	1038.9
6	18	93	18	92	18	55	18	62	2076.4
7	18	76	18	63	18	55	18	55	1038.9
8	18	93	18	78	18	55	18	77	1239.9
9	18	65	18	75	18	55	18	55	1038.9
10	18	80	18	93	18	55	18	77	1239.9
11	18	65	18	64	18	55	18	67	1393.2
12	18	93	18	78	18	55	18	62	2076.4
13	18	80	18	93	18	55	18	62	2076.4
14	18	65	18	64	18	55	18	55	1038.9
15	18	80	18	79	18	55	18	77	1239.9
16	18	80	18	79	18	55	18	62	2076.4

Table 5-15: Standardization Parameters for 2-Stage C-Type SCGT (continued)

	F1	F2	F3	F4	C1	C2	C3	Dgm
0	0.22	0.64	0.84	1.10	0	0	0	6
1	0.25	0.5625	0.75	1.125	0.021	0.031	0.021	5.33
2	0.25	0.5625	0.75	1.125	0.048	0.058	0.021	5.33
3	0.25	0.6875	0.75	1.125	0.021	0.045	0.021	5.33
4	0.1875	0.5625	0.6875	1.125	0.025	0.031	0.037	6.40
5	0.25	0.5625	0.75	0.8125	0.021	0.031	0.042	5.33
6	0.1875	0.5625	0.9375	1	0.025	0.031	0.017	6.40
7	0.25	0.6875	0.75	0.8125	0.021	0.045	0.042	5.33
8	0.25	0.6875	0.6875	1.125	0.025	0.040	0.037	6.40
9	0.25	0.5625	0.75	0.8125	0.048	0.058	0.042	5.33
10	0.1875	0.5625	0.6875	1.125	0.043	0.031	0.037	6.40
11	0.3125	0.6875	0.75	1.125	0.048	0.036	0.021	5.33
12	0.25	0.6875	0.9375	1	0.025	0.040	0.017	6.40
13	0.1875	0.5625	0.9375	1	0.043	0.031	0.017	6.40
14	0.3125	0.6875	0.75	0.8125	0.048	0.036	0.042	5.33
15	0.25	0.6875	0.6875	1.125	0.043	0.036	0.037	6.40
16	0.25	0.6875	0.9375	1	0.043	0.036	0.017	6.40

Table 5-16: Standardization Parameters for 2-Stage C-Type SCGT (continued)

The tables show that design16 is the best choice, if the diameter is not strictly constrained. For design 16, the total reduction ratio is 205.30, and the torque capacity is 2076.4 ft-lb.

5.8 CHAPTER SUMMARY

In this chapter, a complete gear train design method was developed and illustrated with examples of the design of three types of SCGTs. First, the gear train design standard

from AGMA was introduced to guide the gear train design process and the face width calculation. Then, all steps in the design method were described. Three types of star compound gear trains were designed separately based on their own characteristics after a detailed study of their structure.

To simplify the process of optimizing gear designs, an exhaustive search was used to determine the gear train parameters that the performance is most sensitive to, and guidelines were developed for input values for these parameters. The designs that result from using these guidelines were shown to be very close to the results obtained from the exhaustive search.

Various gear train performance metrics, including the torque capacity, weight, torque density, inertia, and responsiveness, were computed using the design method. These data are used in the next chapter for the material selection.

In the final step, all parameters values through the optimization design were converted to standard values.

In next chapter, the material selection process is explained completely.

Chapter 6: Material Selection

The selection of materials is critical for collaborative robot actuator gear train design, because lower density materials can improve gear train performance with respect to such metrics as weight, torque density, and responsiveness. Also, lower cost and better machinability can reduce cost and enhance competitiveness in the market.

In this chapter gear train material selection is conducted based on two different kinds of criteria. In the first case, where information about the gear train is not yet known, material selection is based directly on material properties such as bending fatigue limit and ultimate tensile strength. In the second case, the requirements of the gear train are known, and the design method developed in Chapter 5 is applied to calculate the gear train performance, such as torque capacity, weight, inertia, torque density and responsiveness. In this case, material selection is based on these performance metrics instead of material properties.

In the selection process based on gear train performance metrics, these values are compared with the design requirements and with each other. In the comparison process with each other, when there are multiple material alternatives and criteria, the selection process becomes very complicated. For this situation, a Multiple Criteria Decision Analysis (MCDA) method, ELECTRI III is introduced to assist the final selection.

6.1 MCDA: ELECTRE III and Selection Based on Material Properties

Multiple Criteria Decision Analysis is a statistical method concerned with decision-making assistance facing multiple criteria and difference alternatives. In the decision-making process, there may be some conflicting criteria worth evaluating together and carefully. For example, in gear train material selection, in most cases, high strength always comes with high density and price. So, a method to balance these

properties is quite necessary. In most cases, the final solution for multiple criteria decision analysis is not unique, but depends on the particular situation and the decision maker's preferences. [56] The optimal solution based on economics may be quite different than that based on performance. In this study, the ELECTRE III method is introduced to help the designer compare material alternatives based on multiple criteria. This method provides the decision maker the freedom to define the threshold and weight for each decision criterion.

ELECTRE was first introduced by Bernard Roy and SEMA Consultancy Company in the 1960s. Based on its convenience and adaptability, several variants of methods have been developed for specific situations. [57] The ELECTRE method has been applied in business [58], design [59], and the energy field [60] among others.The family of methods includes ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE TRI [57]. Among them, ELECTRE III is mainly used to rank all alternatives with several different thresholds and weights on different criteria.

6.1.1 ELECTRE III Method

The ELECTRE III method is different from traditional criteria selection in three aspects. First, a concept of "as least as good as" is introduced, and the extent of preference is also expressed quantitatively, which gives the designer finer distinction among the criteria to make a choice. Second, several kinds of thresholds make the evaluation process more targeted and comprehensive. And finally, comparison is conducted between every pair of alternatives, and the results are listed in a matrix to allow the decision maker to make a comprehensive selection. The process of ELECTRE III method is detailed introduced by Buchanan [61], and the introduction in this chapter is based on his work.

To understand the ELECTRE III method better, two concepts, “preferred” and “indifference”, are introduced first. In selection, “preferred” means a clear preference of one alternative, and “indifference” means the performance of two alternatives on one specific criterion is too close, and should not be a standard for the designer to make a choice, and other criteria should be considered.

To help the reader to understand, only two alternatives, a and b, are used for the following example, and $g(a)$ and $g(b)$ are their performances based on criterion g. The preference standard in traditional comparison relationship is as follows: a is preferred to b if $g(a) > g(b)$. But in the ELECTRE III method, an indifference threshold q is introduced, such that, when $g(a) > g(b) + q$, then a is preferred to b.

The introduction of the indifference threshold, q, is quite reasonable. Sometimes, a small difference for a specific criterion will not impact the final performance, and therefore should be ignored in criteria evaluation. And for some selection methods, the method of indicating one alternative is better than another for a small difference in one criterion may hide the importance of large differences in other criteria.

The second threshold introduced is the preference threshold, p. The preference threshold is larger than the indifference threshold q. And once the difference is larger than the preference threshold, $g(a) - g(b) > p$, then a will be preferred for that criterion. If the difference is between q and p, $q < g(a) - g(b) \leq p$, the preference of a is not that strong. [61]

With the factors p and q, a relationship named concordance has been defined, which means we can define a standard for which alternative a is at least as good as b. However, a different kind of relationship called discordance should also be considered. And another veto threshold, v_j, is used. Once the difference is larger than the veto threshold, the alternative with worse performance here can no longer be “at least as good

as" the one with better performance, even if its performance is better in all other criteria. The veto threshold is a very strict threshold. It should only be used when one criterion is extremely important.

The choice of appropriate thresholds is not easy in most realistic decision-making situations. They can be determined only after deep consideration of the design requirements, the actual situation and even comparison between the performance data. But these thresholds provide more freedom and flexibility for the designer to rank the material alternatives accurately to meet real situations according to the design requirements.

6.1.2 Material Characteristics

To explain how ELECTRE III method works to help the designer to compare material alternatives on multiple criteria, properties of several different materials are collected and listed in Table 6-1. The materials we choose here are composed of both metals like cast iron, carburized steel, and aluminum, and plastics such as nylon and acetal resins. And the properties include hardness (surface and core), fatigue limit (surface and bending), strength (UTS and yield), density, cost and thermal conductivity. The cost property is the price per cubic foot of the raw material from the online industrial and commercial facilities suppliers. [39] To rank the material alternatives accurately, data about their machinability and maintain are quite important. However, there seems to be no commonly used evaluation standards for these two properties, and they are not used in this work.

Materials	Physical properties								
	Hardness		Fatigue limit		Strength				
	Surface (Bhn)	Core (Bhn)	Surface (MPa)	Bending (MPa)	UTS (MPa)	Yield (MPa)	Density (kg/m3)	Cost	Thermal conductivity W(M/K)
Cast iron (M1)	200	200	330	100	380	275.74	7300	38.26	47
Ductile iron (M2)	220	220	460	360	880	551.49	7100	25.31	75
Cast alloy steel (M3)	270	270	630	435	845	620.42	7700	19.36	50
Through hardened steel (M4)	270	270	670	540	1190	710	7850	22.15	44.5
Surface hardened steel (M5)	542	226	1160	680	1850	710	7850	24.55	44.5
Carburized steel (M6)	647	297	1500	920	2300	979	7850	43.33	52
Nylon 6 (M7)	112	112	250	102	230	210	1570	19.24	0.299
Polyphenylene Sulfide (M8)	120	120	155	86	143	139	1400	10.44	0.53
Acetal resins (M9)	90	90	205	97	190	230	1160	8.58	0.17
Titanium (M10)	334	334	1450	510	950	880	4600	126.28	22
Aluminum (M11)	150	150	607	386	550	490	2700	6.91	153

Table 6-1: Material Properties [6][7][39][62][63][64][65]

It is important to note that the properties of one kind of material can be quite different if the type or treatment technology is different. For example, for different types of ductile iron, the Brinell hardness ranges from 156 Bhn to 302 Bhn, tensile strength ranges from 415 Mpa to 830 Mpa, and yield strength ranges from 275 Mpa to 620 Mpa. [6] The data listed in Table 6-1 only reflect the properties of one kind of material, but this serves the purposes of discussion for this material selection method. In a real design situation, accurate data should be collected according to the actual situation.

6.1.3 ELECTRE III Method Process and Material Selection

The first step of the ELECTRE III method is to develop the measurement of the concordance, which means the extent to which one alternative is “at least as good as” each of the others:

$$c_j(a,b) = \begin{cases} 1, & if\ g_j(a) - g_j(b) \geq -q_j \\ 0, & if\ g_j(a) - g_j(b) \leq -p_j \\ \frac{p_j + g_j(a) - g_j(b)}{p_j - q_j}, & otherwise \end{cases} \quad [60]$$

For a comprehensive comparison of two alternatives, the concordance of all criteria should be combined:

$$C(a,b) = \frac{1}{k}\sum_{j=1}^{r} k_j c_j(a,b),\ where\ k = \sum_{j=1}^{r} k_j \quad [60]$$

where k_j is the weight for criterion j.

Table 6-2 lists the indifference thresholds, preference thresholds and the weights for the material properties. Since the designer still does not have information about the gear train or the final product requirements, the thresholds can only be determined through comparison of the properties between the different materials. The indifference threshold is chosen as a value a little larger than the smallest difference between materials, and the preference threshold is chosen as two or three times the value of the indifference threshold. Because the material properties seem to be quite abstract to designers, it is hard to select a targeted threshold; this problem will be discussed later in selection based on gear train performances.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
Indifference threshold (q)	50	50	200	150	300	200	1000	5	5
Preference threshold (p)	100	100	400	300	600	400	3000	10	10
Weights	1	1	1	1	1	1	1	1	1

Table 6-2: Indifference and Preference Thresholds and Weights

Comparison has been performed between every two material alternatives, and the results are listed in Table 6-3. The values in the matrix are the measurements of concordance, which means the extent to which the material corresponding to the row is "at least as good as" that corresponding to the column.

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
M1	1.00	0.58	0.49	0.39	0.33	0.34	0.78	0.78	0.78	0.25	0.52
M2	1.00	1.00	0.98	0.97	0.56	0.38	0.87	0.78	0.78	0.51	0.67
M3	1.00	0.89	1.00	0.98	0.60	0.47	0.89	0.80	0.78	0.68	0.67
M4	1.00	0.89	1.00	1.00	0.67	0.47	0.89	0.78	0.78	0.72	0.67
M5	1.00	0.89	1.00	1.00	1.00	0.56	0.88	0.78	0.78	0.73	0.67
M6	1.00	0.78	0.89	0.89	0.89	1.00	0.78	0.78	0.78	0.89	0.67
M7	0.72	0.39	0.23	0.22	0.22	0.22	1.00	0.92	0.89	0.22	0.54
M8	0.76	0.29	0.22	0.22	0.22	0.22	1.00	1.00	1.00	0.22	0.54
M9	0.67	0.37	0.23	0.22	0.22	0.22	1.00	1.00	1.00	0.22	0.57
M10	0.78	0.78	0.78	0.78	0.54	0.44	0.78	0.78	0.78	1.00	0.73
M11	1.00	0.90	0.78	0.65	0.49	0.33	0.99	0.98	0.97	0.52	1.00

Table 6-3: Concordance Matrix

The second step is to calculate the discordance, and the third threshold called the veto threshold is defined. The discordance for each criterion j, $d_j(a,b)$ is calculated as:

$$d_j(a,b) = \begin{cases} 0, & if\ g_j(a) - g_j(b) \geq -p_j \\ 1, & if\ g_j(a) - g_j(b) \leq -v_j \\ \frac{-p_j + g_j(b) - g_j(a)}{v_j - p_j}, & otherwise \end{cases} \quad [60]$$

Since the influence of the on the final performance of the gear train is unknown, it is irrational to exclude some materials just because of one criterion. Therefore, the veto thresholds shown in Table 6-4 are all larger than the difference of material properties.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
Veto threshold	1000	1000	2000	2000	3000	2000	10000	200	50

Table 6-4: Veto Thresholds

The next step is to combine the result of concordance and discordance to produce a credibility matrix. [60] The credibility for each pair of alternatives is defined as:

$$S(a,b) = \begin{cases} C(a,b) \\ C(a,b) \prod_{j \in J(a,b)} \frac{1-d_j(a,b)}{1-C(a,b)}, \ where\ d_j(a,b) > C(a,b) \end{cases} \quad [60]$$

The credibility value is the same as the concordance if the discordance does not exceed the concordance. And once discordance exceeds concordance, the concordance value should be modified. One special situation is that, if the difference between two alternatives on one criterion is larger than the veto threshold, the discordance value is 1 and the credibility is 0 according to the formula. The credibility matrix of selection based on material properties is listed in table 6-5.

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
M1	1.00	0.58	0.49	0.39	0.32	0.14	0.78	0.78	0.78	0.18	0.00
M2	1.00	1.00	0.98	0.97	0.56	0.37	0.87	0.78	0.78	0.51	0.00
M3	1.00	0.89	1.00	0.98	0.60	0.47	0.89	0.80	0.78	0.68	0.00
M4	1.00	0.89	1.00	1.00	0.67	0.47	0.89	0.78	0.78	0.72	0.00
M5	1.00	0.89	1.00	1.00	1.00	0.56	0.88	0.78	0.78	0.73	0.00
M6	1.00	0.78	0.89	0.89	0.89	1.00	0.78	0.78	0.78	0.89	0.00
M7	0.21	0.00	0.00	0.04	0.02	0.00	1.00	0.92	0.89	0.12	0.00
M8	0.27	0.00	0.00	0.04	0.01	0.00	1.00	1.00	1.00	0.10	0.00
M9	0.16	0.00	0.00	0.04	0.01	0.00	1.00	1.00	1.00	0.11	0.00
M10	0.78	0.00	0.78	0.78	0.54	0.40	0.78	0.78	0.78	1.00	0.00
M11	1.00	0.90	0.78	0.65	0.49	0.20	0.99	0.98	0.97	0.52	1.00

Table 6-5: Credibility Matrix

The credibility matrix clearly presents the comparison relationship between all the alternatives. It can be used as a reference to rank the alternatives. The following step simplifies the comparison process by modifying the value to 1 or 0 with a defined standard. Define $\lambda = max_{a,b \in A} S(a,b)$, and $s(\lambda)$ as the tolerance (0.15 here), then the credibility which is bigger than $\lambda - s(\lambda)$ is set to 1, and otherwise, it is set to 0. [60] The final matrix T is shown in Table 6-6.

$$T(a,b) = \begin{cases} 1, & if\ S(a,b) > \lambda - s(\lambda) \\ 0, & otherwise \end{cases}$$

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
M1	1	0	0	0	0	0	0	0	0	0	0
M2	1	1	1	1	0	0	1	0	0	0	0
M3	1	1	1	1	0	0	1	0	0	0	0
M4	1	1	1	1	0	0	1	0	0	0	0
M5	1	1	1	1	1	0	1	0	0	0	0
M6	1	0	1	1	1	1	0	0	0	1	0
M7	0	0	0	0	0	0	1	1	1	0	0
M8	0	0	0	0	0	0	1	1	1	0	0
M9	0	0	0	0	0	0	1	1	1	0	0
M10	0	0	0	0	0	0	0	0	0	1	0
M11	1	1	0	0	0	0	1	1	1	0	1

Table 6-6: Matrix *T*

In the matrix *T*, a "1" means the material corresponding to that row is "at least as good as" the material in the corresponding column. The qualification of the material is then defined as the difference between the sum of the row and the sum of the column, thereby creating a list of alternative qualifications (see Table 6-7). In general, the alternative with a larger qualification number means it has a higher priority.

M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
−6	0	0	0	4	5	−5	−1	−1	−1	5

Table 6-7: Material qualification

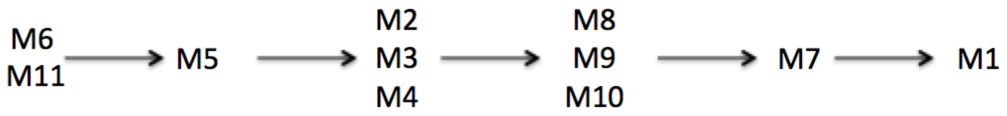

Figure 6-1: Priority of All Material

From the final ranking, we find that M6 (Carburized Steel), and M11 (Aluminum) are the highest priorities (see Figure 6-1). M5 (Surface hardened steel) has the second highest priority.

The reason why we get this ranking is that, among all nine criteria, six criteria relate to material strength, fatigue limit, and hardness. So M6, M11, and M5 all perform well, as their values for strength, hardness and fatigue limit are very high. The materials M1 and M7 each have relatively low, and perform poorly with respect to density compared to M8, M9, and M10. For these reasons they have the lowest priorities. Even though some materials are clearly preferred, we rank all material alternatives instead of providing only one choice to allow the designer flexibility to make the final choice after deeper and comprehensive consideration.

The ELECTRE III method based on the material properties directly assists designers, but there are still several drawbacks to the method. First, lower thresholds for the some criteria, especially strength and fatigue limit, should be established based on constraints of the particular design proejct. For example, a material with high performance for most criteria should still be eliminated if its strength cannot meet the specific design requirements. Second, it can be difficult to assign values for thresholds and weights, and some material properties may seem too abstract to designers. For

example, they may not understand how a parameter like ultimate tensile strength affects the final performance of the gear train. On the other hand, a threshold based only on comparison with other materials is really not necessarily effective. The design method described in Chapter 5 builds a direct connection between the material properties to the design requirements. However, instead of setting thresholds on material properties, setting thresholds on the gear train performance is much easier and potentially more meaningful.

6.2 Material Properties for Gear Train Design

In this section, to illustrate the use of the ELECTRE III method based on gear train performance, five performance criteria are selected: torque capacity, weight, torque density, inertia, and responsiveness. All data are calculated for the design of a 1-Stage SCGT with reduction ratio, g, of 18, and gear train mesh diameter, D_{gm} of 6. A comparison of gear train performances of three kinds of materials, carburized steel, aluminum, and nylon 6, are shown in Figures 6-1, 6-2, 6-2, 6-4, 6-5, to provide the designer an intuitive illustration of their relative performance.

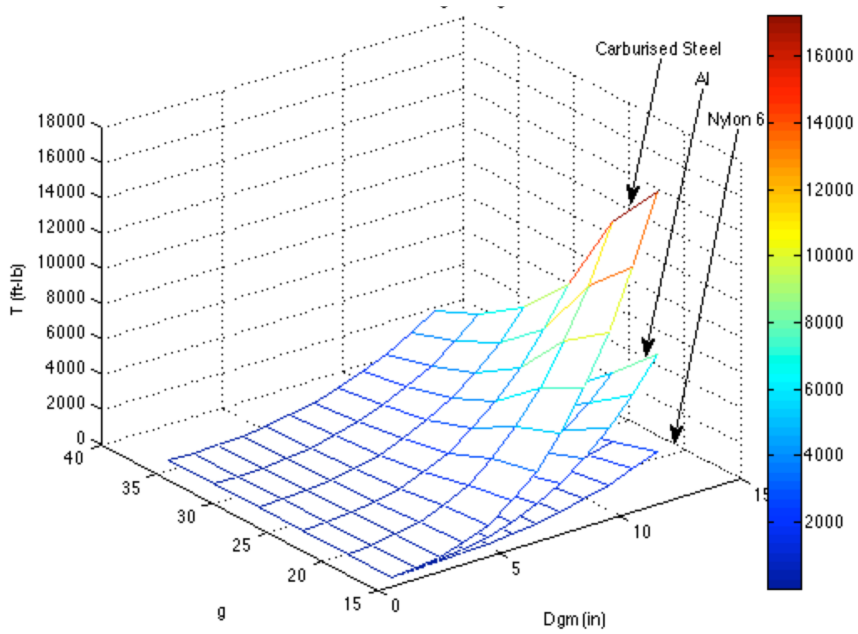

Figure 6-2: Torque Comparison for Different Materials

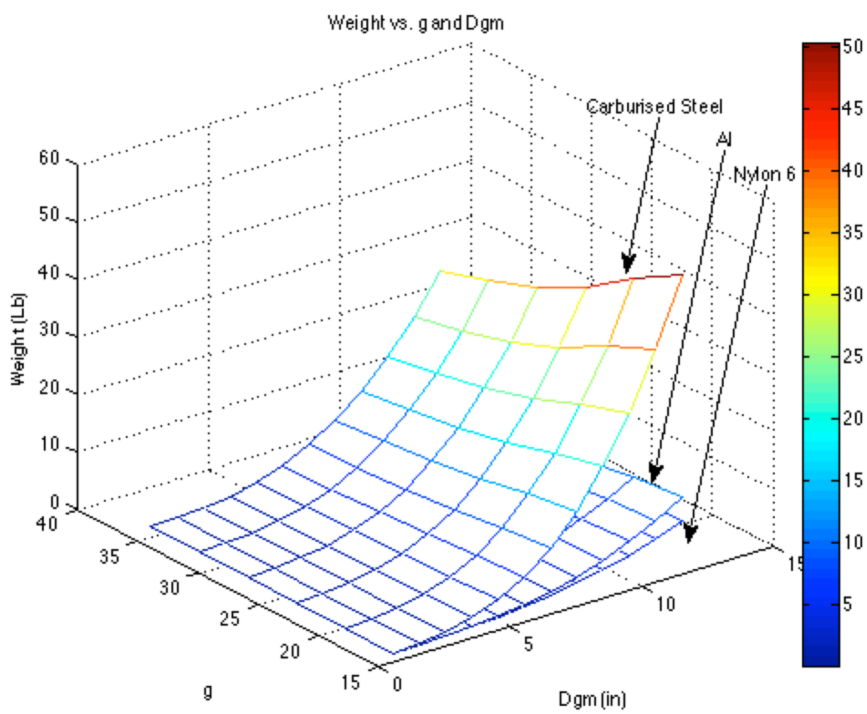

Figure 6-3: Weight Comparison for Different Materials

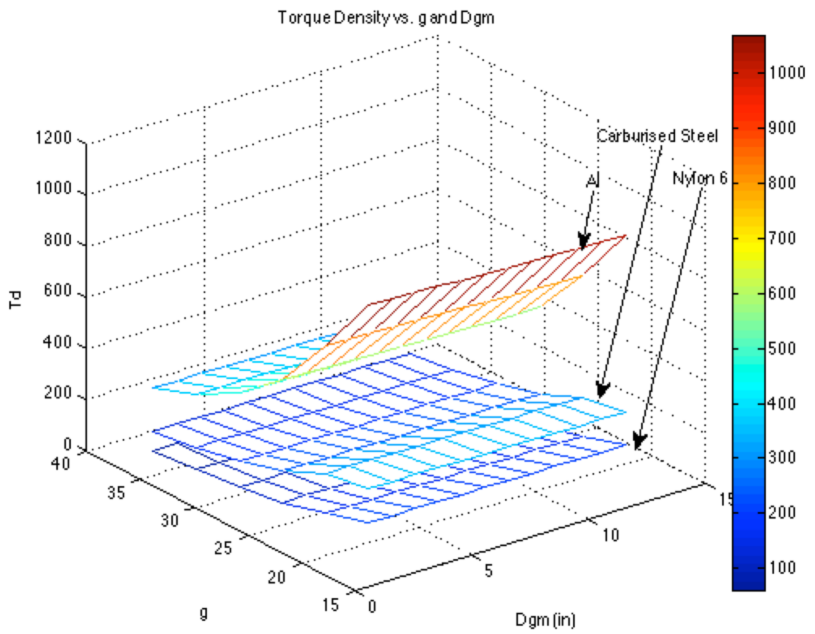

Figure 6-4: Torque Density Comparison for Different Materials

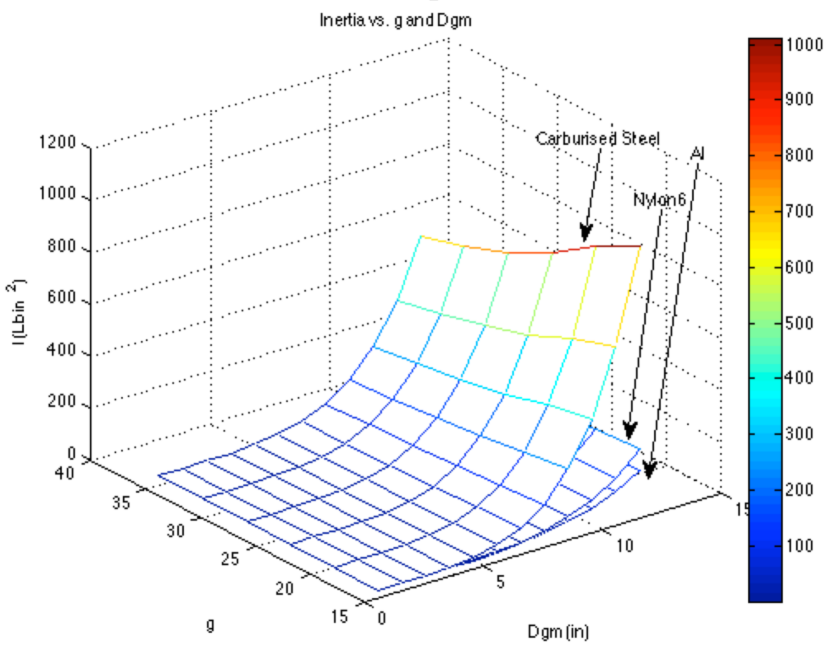

Figure 6-5: Inertia Comparison for Different Materials

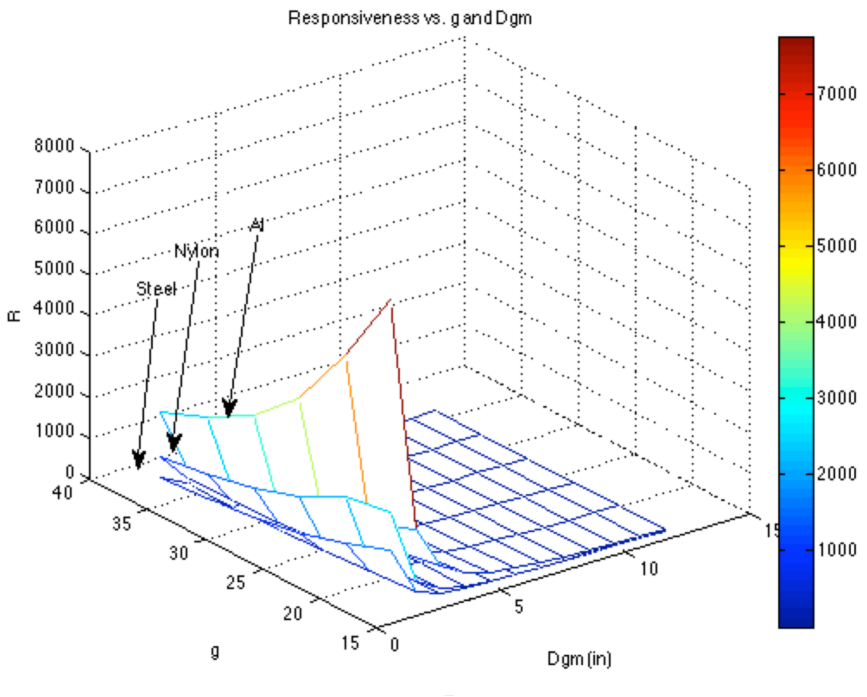

Figure 6-6: Responsiveness Comparison for Different Materials

Among all figures, the carburized steel gear train performs best in torque capacity, but worst in weight, inertia, and responsiveness, because steel has large fatigue limit, strength and hardness, while the density is very high compared with the other two kinds of materials. This result indicates that the steel is actually not always the best choice for a gear train, especially when weight, inertia, and responsiveness are strictly limited.

Aluminum performs best in torque density, inertia, and responsiveness, because it has relatively good strength properties and is lightweight. Nylon does not perform quite as well in comparison based on these five criteria, but it also has other good properties, which will be mentioned in the following selection process.

6.3 MATERIAL SELECTION BASED ON GEAR TRAIN PERFORMANCE REQUIREMENTS

Material selection based on gear train performance, rather than material characteristics alone, is significantly different, because the designer has a much better intuitive understanding of the meaning of these criteria, so that basic performance requirements can be addressed directly. For this example, the material selection criteria are torque capacity, weight, torque density, inertia, responsiveness, and cost and thermal conductivity as shown in Table 6-8. Gear train performance metrics are determined through the design method described in the previous chapter.

Material	Gear Train Performance						
	Torque (ft-lb)	Weight (lb)	Torque Density (ft-lb/lb)	Inertia (lb*in^2)	Responsiveness (ft-lb/ft*in^2)	Cost (Dollar/in^3)	Thermal Conductivity W(M/K)
Cast iron	247.83	7.15	34.66	36.19	6.85	38.26	47
Ductile iron	892.17	6.95	128.28	35.20	25.34	25.31	75
Cast Alloy iron	1078.04	7.54	142.93	38.18	28.24	19.36	50
Through hardened steel	1338.26	7.69	174.04	38.92	34.38	22.15	44.5
Surface hardned steel	1685.22	7.69	219.16	38.92	43.30	24.55	44.5
Carburised steel	2216.25	7.71	287.51	38.99	56.84	43.33	52
Nylon 6	252.78	1.54	164.37	7.78	32.47	19.24	0.299
Polyphenylene Sulfide	213.13	1.37	155.42	6.94	30.70	10.44	0.53
Acetal resins	240.39	1.14	211.56	5.75	41.80	8.58	0.17
Titanium	1263.91	4.51	280.50	22.81	55.42	126.28	22
Aluminum	956.61	2.64	361.70	13.39	71.46	6.91	153

Table 6-8: Gear Train Performance Criteria [38] [62]

6.3.1 Gear Train Performance Requirements

In the selection process based on gear train performances, the performance metrics should be compared with both the design threshold and with each other. Comparing gear train performance with the design threshold is a quite important step that eliminates materials that cannot meet requirements. For example, when a gear train

requires a specific torque capacity to ensure system stability, eliminating materials that cannot achieve this threshold not only ensures the selection quality but also saves time due to comparing too many material in the next step. This step can be very hard to conduct when there are only material properties.

For example, consider three constraints for a particular gear train:

1. Torque capacity no smaller than 220 ft-lb.
2. Torque density no smaller than 100 ft-lb/lb.
3. Price for unit raw material no more than 50 US$/in^3.

Based on these three design requirements, three material alternatives, cast iron, PPS and titanium are eliminated immediately. The remaining materials are compared in Table 6-9.

#	Material
M1	Ductile Iron
M2	Cast Alloy Iron
M3	Through Hardened Steel
M4	Surface Hardened Steel
M5	Carburized Steel
M6	Nylon 6
M7	Acetal Resins
M8	Aluminum

Table 6-9: Materials for Comparison

6.3.2 Thresholds and Weights

Using gear train performance instead of the properties of the materials makes the selection process meaningful, and the assignment of thresholds and weights much easier. This section summarizes the rationale for the threshold settings.

The veto threshold is a very strict threshold. When the difference of performance between two alternatives is larger than the veto threshold, the one with the lower value has no opportunity to be selected, even if it performs better than its competitors on all other criteria. Among the seven criteria in this example, the veto threshold can only be used for cost when the actuator gear train material is being selected. Since all the other criteria have no veto threshold requirements, the veto threshold can be set to a value bigger than the largest difference between all alternatives.

The indifference threshold and preference threshold should be assigned according to the value of all alternatives, and the actual situation. In general, the indifference threshold determines the sensitivity of the criterion. The preference criteria and weights determine which criteria receive more attention. For this study, one set of thresholds and weights are assigned as shown in Table 6-10. And the corresponding concordance matrix is shown in Table 6-11.

	C1	C2	C3	C4	C5	C6	C7
q	50	1	50	5	10	5	5
p	200	2	100	20	30	20	20
v	2000	10	300	35	50	30	150
w	1	1	1	1	1	1	1

Table 6-10: New Threshold and Weight Setting

	M1	M2	M3	M4	M5	M6	M7	M8
M1	1.00	0.86	0.86	0.68	0.57	0.70	0.46	0.14
M2	0.86	1.00	0.86	0.75	0.58	0.71	0.58	0.21
M3	0.86	1.00	1.00	0.86	0.60	0.71	0.63	0.19
M4	0.86	0.99	1.00	1.00	0.76	0.71	0.61	0.18
M5	0.73	0.86	0.86	0.87	1.00	0.57	0.57	0.33
M6	0.71	0.71	0.71	0.69	0.47	1.00	0.95	0.36
M7	0.71	0.71	0.71	0.71	0.60	1.00	1.00	0.43
M8	1.00	0.93	0.86	0.86	0.86	0.98	0.90	1.00

Table 6-11: Concordance Matrix

The concordance matrix reflects the relative preference between each pair of alternatives, and is determined by the indifference threshold and the preference threshold. The value in each cell reflects the extent to which the material in the corresponding row is “as least as good as” the material in the corresponding column. In this matrix, we find the numbers in the eighth row are all relatively large, and the numbers in the eighth column are relatively small (except for the eighth cell itself). This means M8 (Aluminum) is at least as good as all other materials.

	M1	M2	M3	M4	M5	M6	M7	M8
M1	1.00	0.86	0.86	0.68	0.50	0.70	0.30	0.01
M2	0.86	1.00	0.86	0.75	0.58	0.71	0.24	0.01
M3	0.86	1.00	1.00	0.86	0.60	0.64	0.21	0.02
M4	0.86	0.99	1.00	1.00	0.76	0.63	0.19	0.03
M5	0.73	0.86	0.86	0.87	1.00	0.34	0.00	0.00
M6	0.71	0.71	0.71	0.69	0.02	1.00	0.95	0.00
M7	0.71	0.71	0.71	0.71	0.02	1.00	1.00	0.00
M8	1.00	0.93	0.86	0.86	0.86	0.98	0.90	1.00

Table 6-12: Credibility Matrix

The credibility matrix is shown in Table 6-12. The difference between the credibility matrix and the concordance matrix is that the credibility matrix accounts for the veto threshold.

In the next step, set $\lambda_s = 0.1$. Here, this means that all entries in the S matrix larger than 0.9 are accepted, and will be replaced with 1, while any value below this threshold will be replaced with 0. (See Table 6-13)

	M1	M2	M3	M4	M5	M6	M7	M8
M1	1	0	0	0	0	0	0	0
M2	0	1	0	0	0	0	0	0
M3	0	1	1	0	0	0	0	0
M4	0	1	1	1	0	0	0	0
M5	0	0	0	0	1	0	0	0
M6	0	0	0	0	0	1	1	0
M7	0	0	0	0	0	1	1	0
M8	1	1	0	0	0	1	1	1

Table 6-13: Matrix T

The final selection order is shown in Figure 6-7 below.

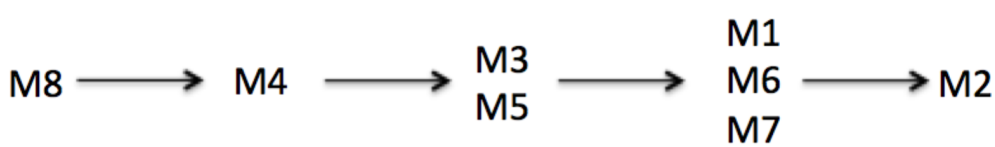

Figure 6-7 Priority of Selected Materials

6.4 Chapter Summary

In this chapter, the ELECTRE III method for multiple criteria decision analysis method is proposed for comparison and selection of materials for gear train applications. Using the design method from Chapter 5 and material properties to calculate gear train performance plays an important role in gear train material selection. Material selection for a gear train should be based on comparison to both the design requirements and comparison of material properties to each other. First, the basic performance limitation should be met to ensure system safety and stability. Second, a comparison of all

alternatives helps the designer to find the one alternative that best fits the design situation.

Chapter 7: Summary and Conclusions

7.1 Summary of Present Research

The objective of this research is to develop a comprehensive collaborative robot actuator gear train design method, including gear train type choice, gear train structure study, optimization and standardization of parameters, gear train performance evaluation, and material selection.

In Chapter 2, the actuator structure and components are described thoroughly. Based on the requirements of collaborative robots, the gear train is expected to be lightweight and low cost, and exhibit high performance.

Chapter 3 contains a detailed study of gear trains. From the introduction of different gear train types, a good understanding of gear design is useful for the process. The star compound gear train is preferred to transmit power and motion. The star compound gear train is based on star gear sets, compound gears, and the split path gear idea. The star compound gear train has the advantages of high reduction ratio, compact structure, and ease to manufacture and installation.

In Chapter 4, various different gear materials, including metals and plastic are described to inform the design. Manufacture, post-treatment technology and lubrication are all discussed.

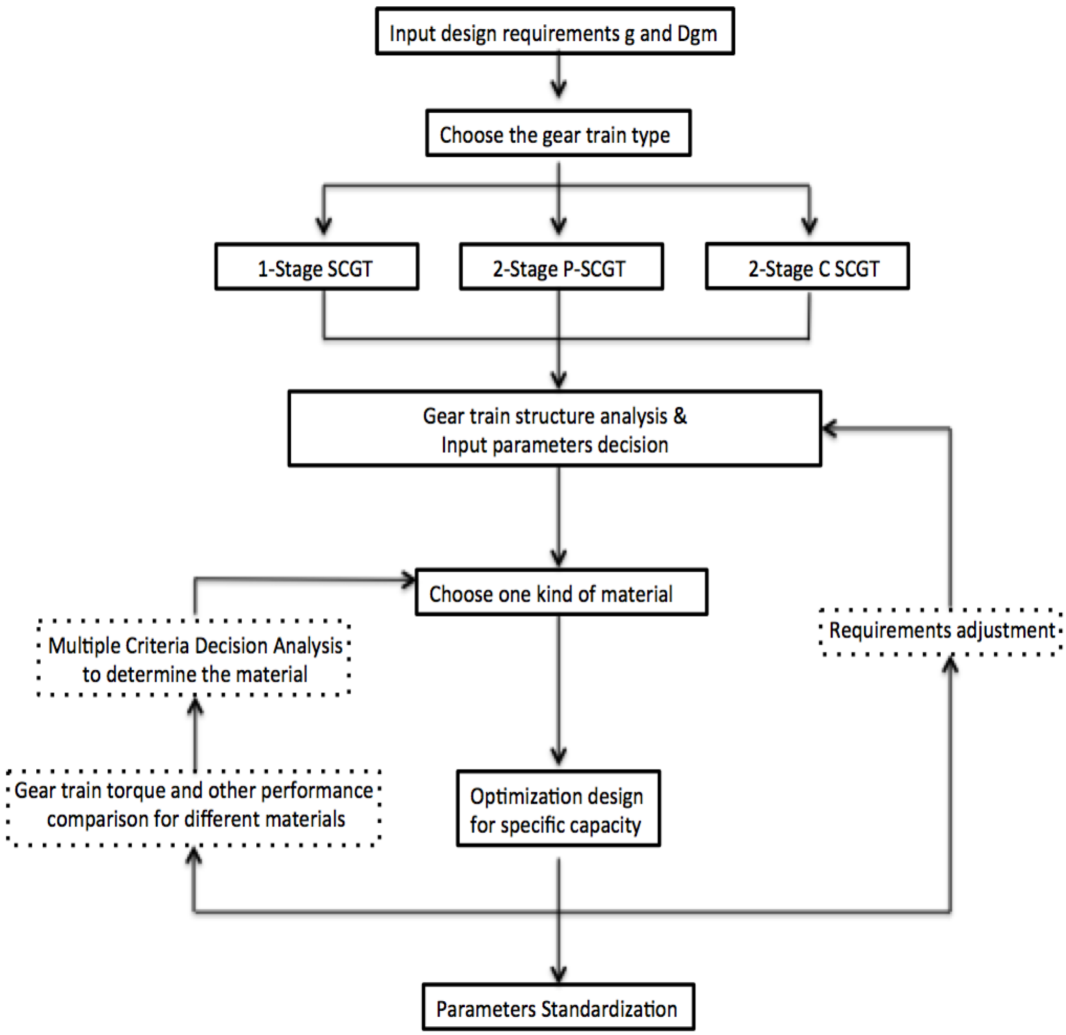

Figure 7-1: Completive Gear Train Design Process

Chapter 5 presents on the design process developed in this research (see Figure 7-2). The AGMA standard process is used to calculate the gear face width. A thorough analysis of the gear train structure is conducted to identify the individual parameters necessary to completely specific the gear train. For a 1 stage SCGT, three parameters are needed, and for a 2 stage P-Type SCGT and C-Type SCGT, the numbers of individual parameters are six and seven, respectively.

From the results of simulation,s several assumptions can be used to simplify gear train design calculations. For example, the best choice of diametral pitch of the first mesh for P-Type SCGT has a linear relationship with both reduction ratio and gear train mesh diameter (see Figure 7-3). The simplified calculations based on the assumptions have an accuracy of more than 90.7% for all the points chosen.

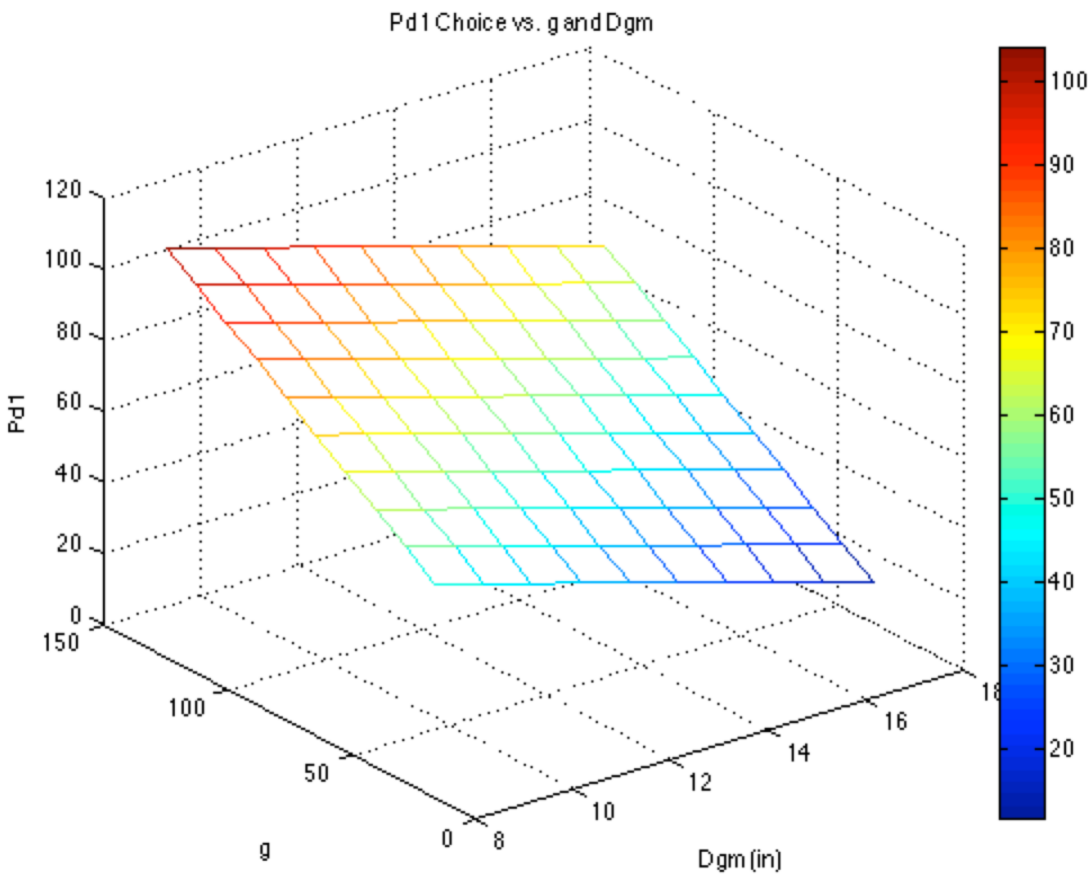

Figure 7-2: Dp1 Choice on Input Parameters Pairs

A standardization method is applied to modify the parameter values calculated by the optimization design process to be industry standard values. The standardization method starts with the diametral pitch. For gear trains with 2 meshes, four choices are

provided to the designer for final selection (see Table 7-1). For gear train with three and four meshes, the number of choices is eight, and 16, respectively.

Note that enough clearance should be left to ensure assemblability of the gear train. Also, the number of teeth of the gear should not be integral multiples of that of the pinion in the same mesh.

	Pd1	Pd2	g1	g2	g	Dgm	Clearance
Original	28.38	17.66	4.23	4.26	18.00	6	0
1	32	18	4.22	4.00	16.89	5.31	0.063
2	28	18	4.22	4.39	18.53	6.07	0.032
3	32	16	4.22	3.67	15.48	5.31	0.063
4	28	16	4.22	4.00	16.89	6.07	0.018
	Np	Nls	Nss	Nr	F1	F2	T
Original	18	76.14	18	76.60	0.62	1.02	1169.17
1	18	76	18	71	0.625	0.8125	899.79
2	18	76	18	79	0.625	1.125	1174.98
3	18	76	18	66	0.625	0.6875	824.81
4	18	76	18	72	0.6875	0.9375	1290.48

Table 7-1: Standard Parameter Choices for 1 Stage SCGT

The standard for the final selection is as follows:

1. Check strict design requirements. For example, if the *Dgm* is strictly limited, then solutions for which *Dgm* exceeds the requirement should be eliminated.

2. Check all gear teeth numbers for availability. The design is good if gears with the desired teeth numbers are easy to find.

3. Compare the performance of the remaining choices, and choose the one with the best performance. In our examples, the torque capacity is compared.

Next, we start the material selection process. Selection of material based on material properties directly is not a good idea, because in most cases, designers do not understand what the properties actually mean for gear train performance.

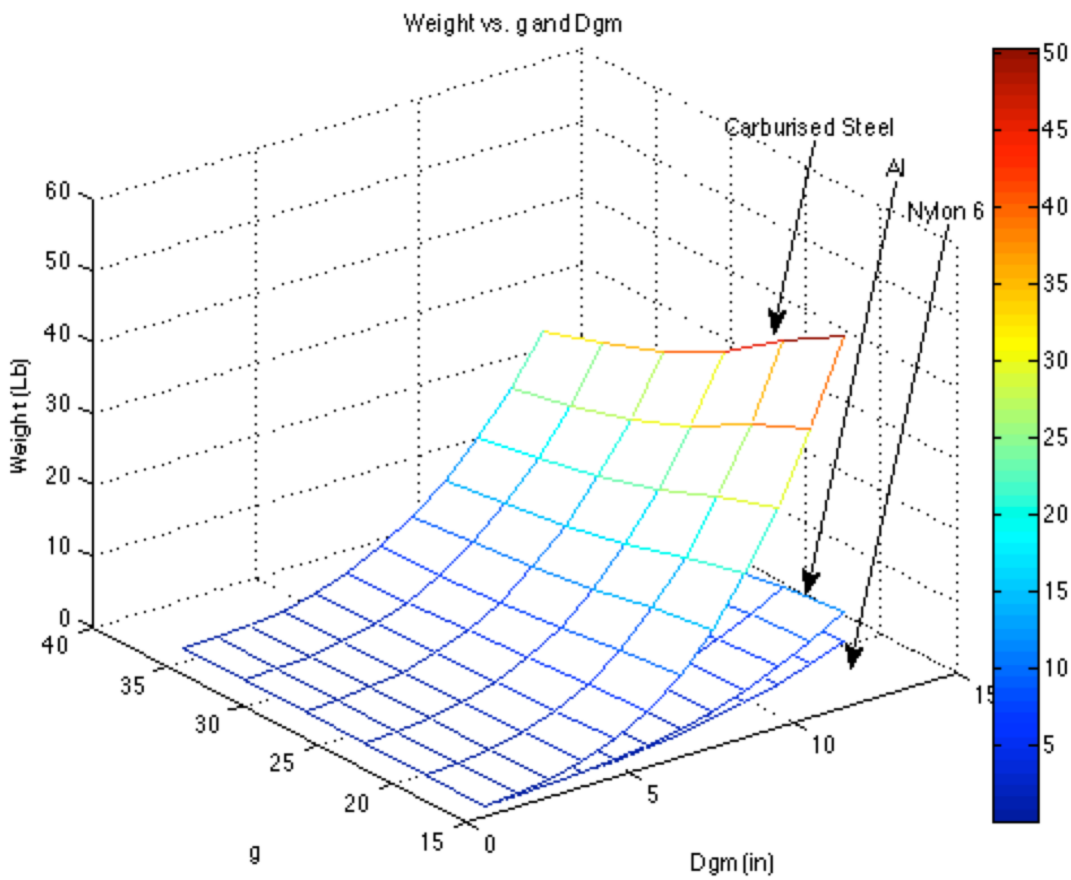

Figure 7-3: Performance comparison

At the end of gear train design, the performance metrics of gear trains of different materials, including the torque capacity, weight, torque density, inertia, and responsiveness, are calculated (see Figure 7-4). The material selection process is based on the comparison of the gear train performance with design requirements and with each other. First, the performance of the gear train should meet the required target value. Second, comparison between different materials on multiple criteria should be performed when several existing alternatives all meet the target value. The ELECTRI III method, a multiple criteria decision analysis tool, is adopted to solve this problem.

In the ELECTRI III method, three thresholds are introduced: the indifference threshold, q, preference threshold, p, and the veto threshold, v. The final selection is based on the T matrix (see Table 7-2). In each T matrix cell, the number 1 means the material in the row is at least as good as the material in the column. The qualification number for each material is obtained by subtracting the sum of the column fromthe sum of the row. The rank is determined by this qualification number.

	M1	M2	M3	M4	M5	M6	M7	M8
M1	1	0	0	0	0	0	0	0
M2	0	1	0	0	0	0	0	0
M3	0	1	1	0	0	0	0	0
M4	0	1	1	1	0	0	0	0
M5	0	0	0	0	1	0	0	0
M6	0	0	0	0	0	1	1	0
M7	0	0	0	0	0	1	1	0
M8	1	1	0	0	0	1	1	1

Table 7-2: T Matrix for Material Selection

7.2 Conclusion

The star compound gear train performs quite well for the requirements of the collaborative robots. It has a compact structure, enough torque capacity, and can fulfill the reduction ratio requirements.

Design optimization improves the gear train performance. However, the number of parameter values that must be evaluated requires intensive computation and increases design time. The simplifying assumptions developed for the design method result in calculations that are 90.7% accurate compared to exhaustive calculation, which is quite good.

The standardization method successfully standardizes the gear train parameters. In the examples, the standard design has quite similar torque capacity and dimensions compared to the optimized design. Additionally, clearance for installation can also be provided when determining the number of gear teeth.

The calculation of gear train performance with different materials successfully connects the gear train design with material selection. The material selection method based on the ELETRE III method chooses the preferered material based on comparison with design requirements and relative comparisons of the performance metrics of different materials.

7.3 Recommended Future Work

This actuator gear train design method can successfully help designers complete the design of a gear train with high performance, standard parameters and appropriate material. However, there are several areas in which the method can be improved.

First, only spur gears are used in this design process. Except for the cost factor, helical gears have quite good properties, such as high torque capacity, stability, and shock

resistance. Thus, adding consideration of helical gears will broaden the design space for gear trains.

Second, a more detailed actuator structure study is need. In actual design, some aspects of the actuator should be designed simultaneously with the gear train. For example, the bearing type is determined by the diameter of the gear and can make a difference to the transmitted force. Such details can help the designer build a more accurate model.

Third, more criteria should be used in the material selection process, such as machinability and routine maintenance, which are quite important for the material selection. The reason for their absence is that not enough information and standards are available for these criteria. A method to consider these aspects should be developed.

Fourth, in the parameter standardization process, a method to control the clearance for installation could be combined with evaluation of gear train backlash and lost motion control for collaborative robots.

And finally, the standardization method can be more detailed. In this reserach, to simplify the final selection, only the diametral pitch is discussed in detail. From the final result, some parameters that satisfy our need appear make this simplification reasonable. If more parameters can be considered, a choice with even better performance could be found.

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