Calculation of Slot Losses in High Frequency Electrical Machines

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Abstract--As new magnetic materials as well as applications emerge in electromechanical conversion systems, one finds that electrical machines are increasingly moving away from the mainstream power frequency to much higher frequencies. This shift is mostly being driven by the fact that for the same power a higher speed machine is smaller in size. Operating at higher frequencies has its advantages on size however one must ensure that this advantage is not offset by larger losses in the core as well as in the conductors carrying the high frequency currents. This paper discusses a method of calculating the slot losses in conductors. There are two aspects to this loss calculation, (a) most machines will have parallel conductors in a single coil, one must know the current distribution in these individual parallel paths and (b) the current in each conductor is not uniformly distributed through the cross-section but is skewed due to proximity and skin effects. Both these considerations must be accounted for when determining the dynamic loss factors.

Index Terms--losses, high frequency, machines, conductors

I. NOMENCLATURE

\( \vec{H} \) = magnetic field intensity (A-turns/m)

\( \hat{H}_{nb} \) = complex field intensity on the bottom of the nth conductor

\( \hat{H}_{nt} \) = complex field intensity on the top of the nth conductor

\( I \) = current (A)

\( \phi \) = phase angle of current (rads or degrees)

\( \omega \) = radian frequency of current in conductor (rad/s)

\( \delta \) = skin depth (m)

\( \mu_0 \) = permeability of free space

\( \sigma \) = electrical conductivity of conductor (S/m)

\( J_2 \) = complex current density in z direction (A/m²)

II. INTRODUCTION

Fig. 1 illustrates a typical coil with arrangement of conductors in a slot. The two slots show the location of the conductors from a single coil. It is to be noted that each turn in the coil is made of four parallel paths. This structure is important in high frequency machines especially as otherwise the conductor losses would be excessive as shall be shown shortly. The leakage flux produced by the conductors in the slot is predominantly transverse to the slot as shown in Fig. 2. Also shown in the figure is a plot of the flux density in the slot as a function of height from the bottom of the slot. The change in slope at the middle of the slot is due to the fact that the conductors in the top half have in general a current of a different phase compared to the bottom.

Fig. 1. Typical coil structure slot layout in electrical machines

Fig. 2. Direction and distribution of the leakage flux in a slot

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Calculation of ac losses in the laminated conductor bundle must be done in two steps: the first step is determining how the current distributes in the laminants and secondly determining the eddy current losses given the distribution of currents in the individual laminants.

III. TECHNICAL DISCUSSION

A. Determining the Distribution of Current in the Individual Laminants

Since at this stage the focus is on the current flow, a coupled circuit approach will be utilized. Which means that to determine the current distribution, we must know the slot inductances of the various laminants and their couplings with other laminants and phases.

The transposition that is inherent in a two layer coil by the twisting at the end turns is adequate to approximately equalize the currents in the laminants. However, the manner in which the phase at the bottom on the left hand slot of Fig. 1 couples to the top phase laminants is different. From Fig. 1 it can be seen that the coupling of the bottom phase with the laminant number 1 is consistently less than the coupling with laminant number 4. This coupling, of course, steadily increases from laminant 1 to 4. This by itself would not be a problem if there were to be a compensatory effect in the other slot. However, the layout in the other (right hand slot Fig. 1) is quite different, in that the upper phase links equally with all the laminants.

Fig. 3 shows the equivalent circuit used to perform this current distribution analysis. It shows the laminants as individual coils with resistance and self and mutual inductances with other laminants. Also shown in each individual path is a voltage source which corresponds to the voltage induced in these paths by the bottom phase in the slot. The inductance matrix only includes the leakage values.

The analysis is carried out in ac steady state mode. It is assumed that the total current in laminants is some value say \( I_\text{bottom} \) and the other phase in the slot is carrying a current \( I_\Phi \). Then based on the circuit of Fig. 3, for ‘n’ laminants in general we can write n-1 loop equations and the nth equation enforces the total current constraint i.e.

\[
\sum_{l=1}^{n-1} I_l = I_\text{bottom} - I_\Phi
\]

Here \( I_l \) is the current in each laminant.

Typical results for the current and the phase in each laminant are as shown in Figs. 4a and 4b respectively. This analysis was performed for a slot that was about 12.7 mm wide and about 50.8 mm deep. The slot layout was as shown in Fig. 1 with three turns per coil and four laminants per turn. Each of the 24 laminants were 1.14 mm thick. It was assumed that the laminant current summed to 10 A. It was assumed that the bottom phase was excited with the same total current amplitude. As can be seen from Fig. 4a the current at low frequencies starts of equally distributed at 2.5 A and then is significantly divergent in amplitude and phase at higher frequencies. In the limiting case at very high frequencies the distribution will attain a constant phase and amplitude and will no longer be a function of frequency.
Fig. 4b. Phase of the current in the 4 laminants

The reason for this non-uniform distribution can best be explained by considering Fig. 5. This is a schematic of the laminants, which are connected together at the end of the coils. The conduction current produces a magnetic field as shown in the figure. At low frequencies the induced current is not significant and the conduction current is uniformly distributed. As the frequency increases there is a tendency for the laminants to expel the flux linked between them. For the example in Fig. 5, this results in an induced current which adds to the conduction current in the top laminant and subtracts from the current in the bottom laminant. This skews the current towards the top laminant.

Fig. 5. Explanation for the non-uniform current distribution (longitudinal view of the conductor)

The important aspect to note here is that even though in the slot there appears to be no path for the induced current this path is established through the terminations of the coils.

B. Determining the Current Density Distribution in the Individual Laminants

Having determined the current between the filaments the next step is to determine how this current distributes within each laminant. The same mechanism that causes a non-uniform distribution of current also causes a non-uniform current density distribution within each filament. This will of course lead to additional losses.

To determine the current density distribution in each laminant we adopt the field approach. The fact that the field is predominantly transverse to the slot as shown in Fig. 2 simplifies the problem to that of a one dimensional ac diffusion problem. We do have to make use of the current distribution between the laminants to solve this problem. Say we know the current in each laminant in the slot and it is given by \( I_i \angle \Phi_i \), here \( I \) and \( \Phi \) are the amplitude and phase and the subscript ‘i’ refers to the location as shown in Fig. 1. Then we know that the boundary condition for the magnetic field intensity on either side of the \( n^{th} \) laminant as:

\[
\hat{H}_{nb} = \frac{\sum_{i=1}^{n-1} I_i \angle \Phi_i}{w_s} \quad \text{at the lower boundary}
\]

\[
\hat{H}_{nt} = \frac{\sum_{i=1}^{n} I_i \angle \Phi_i}{w_s} \quad \text{at the upper boundary}
\]

Here the subscripts ‘\( nb \)’ denote the ‘\( n^{th} \)’ conductor bottom and ‘\( nt \)’ denotes the same for the top of the conductor. The hat on the variable indicates that it is a phasor. Also ‘\( w_s \)’ is the slot width.

The problem that must now be solved for each laminant is shown in Fig. 6.

Fig. 6. Laminant placed in the reference frame with boundary conditions

The applicable partial differential equation [1] in the region of the laminant is

\[
\nabla^2 \mathbf{H} = \mu_0 \sigma \frac{\partial \mathbf{H}}{\partial t}
\]

(1)

Here \( \mu_0 \) is the permeability of free space and \( \sigma \) is the conductivity of laminant.
Since we are considering the ac steady state conditions and also because the problem is one-dimensional this partial diffusion equation simplifies to an ordinary differential equation as shown in (2).

\[
\frac{d^2 \hat{H}_x}{dy^2} = j\omega\mu_0\sigma \hat{H}_x
\]  

(2)

It is implicit here that solution is sought for the phasor \(\hat{H}_x\) and that the total solution for \(H_x\) as shown in (2).

\[
H_x = \text{Re}[\hat{H}_x e^{j\omega t}]
\]

(3)

The solution to equation (2) is given by (4)

\[
\hat{H}_x = A e^{\frac{y}{\delta}} + \hat{B} e^{-\frac{y}{\delta}}
\]

(4)

Here the constants \(A\) and \(\hat{B}\) must be determined with the help of the boundary conditions. The constant \(\delta\) is the ac skin depth and is given by the equation (5).

\[
\delta = \frac{2}{\sqrt{\omega\mu_0\sigma}}
\]

(5)

The following simultaneous equations are obtained when solving for the constants

\[
\begin{bmatrix}
\frac{1}{e^{\frac{y}{\delta}}} & 1 \\
e^{\frac{y}{\delta}} & e^{\frac{y}{\delta}}
\end{bmatrix}
\begin{bmatrix}
A \\
\hat{B}
\end{bmatrix}
= \begin{bmatrix}
H_{nb} e^{j\Phi_{nb}} \\
H_{nt} e^{j\Phi_{nt}}
\end{bmatrix}
\]

Solving these simultaneous equations we get the solutions

\[
A = H_{nt} e^{j\Phi_{nt}} - e^{\frac{d}{\delta}} H_{nb} e^{j\Phi_{nb}}
\]

\[
\frac{2\sinh\left(\frac{d}{\delta} (1+j)\right)}{2\sinh\left(\frac{d}{\delta} (1+j)\right)}
\]

\[
\hat{B} = -H_{nt} e^{j\Phi_{nt}} + e^{\frac{d}{\delta}} H_{nb} e^{j\Phi_{nb}}
\]

\[
\frac{2\sinh\left(\frac{d}{\delta} (1+j)\right)}{2\sinh\left(\frac{d}{\delta} (1+j)\right)}
\]

Thus all the constants of the phasor solution (4) are now known. The next step is to determine the current density in the conductor given the relationship (6).

\[
J = \nabla \times H
\]

(6)

this simplifies to

\[
j_z = -\frac{\partial \hat{H}_x}{\partial y}
\]

and one gets the expression for the current density

\[
j_z = -\frac{1}{\delta \sinh\left(\frac{d}{\delta} (1+j)\right)} \begin{bmatrix}
H_{nt} e^{\delta (1+j)} \cosh\left(\frac{y}{\delta} (1+j)\right) \\
H_{nt} e^{\delta (1+j)} \cosh\left(\frac{y}{\delta} (1+j)\right)
\end{bmatrix}
\]

(7)

Knowing the current density one can then obtain the losses from the relation

\[
\int \int \int \frac{\| \mathbf{J} \|}{2\sigma} dV
\]

Here the ‘*’ implies conjugate and the integration is performed over the volume of the laminant conductor.

C. Verification of the expression for the current density

It is worthwhile at this stage to check the validity of expression (7). We can do this by testing this expression under certain limiting cases where we can obtain simpler expressions. In the limiting case when the frequency is very low we know that the current density in the conductor is uniform and can be obtained by simply taking the total current in the conductor and dividing it by the cross-sectional area of the conductor. For low frequencies the skin depth is very large and the arguments of the hyperbolic functions tend to zero, making use of the limits as \(x\) approaches zero, \(\cosh(x)=1\) and \(\sinh(x)=x\) expression (7) collapses to

\[
j_z = \frac{\hat{H}_m - \hat{H}_{nb}}{d}
\]

(8)

From the expression for the boundary conditions above one can see that

\[
\hat{H}_m - \hat{H}_{nb} = \frac{I_n e^{\Phi_n}}{\omega s}
\]

and therefore in the limiting case the current density is obtained as

\[
j_z = -\frac{I_n e^{\Phi_n}}{\omega s d}
\]

(9)

Which is the expected result for low frequencies.

Now we can consider the other extreme in frequency i.e. high frequencies. In this case the skin depth \(\delta\) is very small and the argument for the hyperbolic functions becomes very large. In this case for large \(x\), \(\sinh(x)=\cosh(x)=e^x\). And the expression for the current density becomes.

\[
j_z = \frac{1}{\delta} \begin{bmatrix}
\hat{H}_m e^{\frac{y}{\delta} (1+j)} \\
\hat{H}_{nb} e^{\frac{y}{\delta} (1+j)}
\end{bmatrix}
\]

(10)

This expression represents two diffusion processes one on each of the boundaries of the laminant with the current density decaying rapidly within two to three skin depths. Fig. 7 shows the condition in the laminant at high frequencies and is typically the case under inductive limited conditions. The behavior on each boundary is as it would be on a semi-infinite plate [2]. The other check is that if we integrate over the thickness of the conductor the current in the plate equals \(I_n e^{\Phi_n}\).
1) Example Calculation

Having established the validity of the expressions the described procedure was applied to a specific example. This geometry analyzed is shown in Fig. 8 along with relevant dimensions in inches. The analysis was carried out at 250 Hz. The current distribution in the different laminants at 250 Hz is shown in Table 1. Laminant #1 is the bottommost one in the slot. The results in the form of an ac/dc resistance ratio at 250 Hz is shown in Fig. 9 for the different laminants.

<table>
<thead>
<tr>
<th>Laminant Number</th>
<th>Amplitude (A)</th>
<th>Phase(°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>4.024</td>
<td>16.972</td>
</tr>
<tr>
<td>#2</td>
<td>3.336</td>
<td>-2.537</td>
</tr>
<tr>
<td>#3</td>
<td>3.0</td>
<td>-20.022</td>
</tr>
</tbody>
</table>

D. FEM Verification

To make sure that the expressions for the losses as obtained from the closed form analysis are correct, the geometry shown in Fig. 8 with the current distribution of Table 1 was used in a finite element model. The agreement obtained between the two models i.e. fem and closed form was very good, the percent error in the loss calculation at 250 Hz is shown in Fig. 10.
IV. CONCLUSIONS

A procedure has been presented that to determine the slot losses in high frequency machines. There are two effects that must be accounted for, the first one is the distribution of current between the parallel paths in the coil and then finding out the current distribution within each laminant. The combined effect gives the slot losses that accounts for both proximity and skin effects. It can be seen from the example chosen that there is, even with thin laminants, a significant slot loss enhancement as can be seen in the ratio of ac/dc resistance in the conductor close to air gap.

V. REFERENCES


VI. BIOGRAPHY

Dr. Pratap received his B.S. at the University of Bombay, India on 1979. He received his MSE in 1982 and Ph.D. in 1996, both at The University of Texas at Austin.

Dr. Pratap's research specialty is in the area of rotating electrical machines, computational electromagnetics, and transients in pulsed electrical equipment. As Chief Scientist at The University of Texas at Austin Center for Electromechanics, he has developed expertise in the EM design, analysis, simulation and experimental performance assessment of pulsed power and high average power systems.

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