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Three Essays on Industrial Organization

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Three Essays on Industrial Organization

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Three Essays on Industrial Organization

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This thesis consists of three chapters. Chapter 1 explores the impact of compatibility regulation on the technological transition in industries with indirect network externalities. This chapter contrasts the scenario where firms make their own compatibility decisions with the scenario where compatibility is mandatory and shows that firms are better off in the first scenario and worse off in the second scenario. In some cases, technological transition takes place if there is no regulation, but may not take place if the compatibility regulation is in place. Beside regulation, the technological transition in these industries may be held back by either the coordination problem or the compensation problem. The analysis culminates by showing conditions in which these problems can be eliminated. Chapter 2 explores the code-release decision of profit-maximizing software firms. Equilibrium results show that

firms will not release code if the complementarity coefficient is either too low or too high. If the open source community can produce high quality open source software, then both firms may adopt the open source approach. If the open source community is moderately efficient and the complementarity coefficient falls in a middle range, then the decision to adopt open source approach depends on the efficiency gap between the two firms. Chapter 3 explores the impact of keyword auction on online retailers' pricing strategies. The incumbent has positioning advantage on the search engine but the new entrant can bid for the sponsored advertisement place and neutralize such advantage. In equilibrium, preemptive advertisement exists. In one scenario, there is a pure equilibrium in which the incumbent charges a higher price and outbids the entrant in the sponsored ads auction. In the other scenario, there is a unique mixed equilibrium in which the incumbent can only partially deter the entrant from moving up.

Contents

Chapter 1

Network Externalities, Minimal Compatibility, Coordination and Innovation

1.1 Introduction

1.1.1 Innovation in Television and Video Game Industries

Transition to color TV in 1950s In the U.S., the transition from black-and-white to color television took place during the 1950s and the early 1960s. The first color program which conformed to RCA/NTSC standard was aired by NBC Television Network in mid 1950s. NBC was virtually the sole color broadcaster until 1965-66 when CBS and ABC decided to join the bandwagon.

The RCA/NTSC system allows color images to be transmitted and received on either monochrome or color receivers. Therefore, consumers with monochrome receivers could watch RCA/NTSC programs on their old machines, but only in black-and-white. Similarly, owners of color TV sets could watch black-and-white TV programs aired by stations using the old technology. The RCA/NTSC system have been in place for the last 40 years and is still widely used throughout the U.S. although the transition to the new HD technology has been initiated more than a decade ago.

Transition to HDTV in 1990s The industry is now in the transition to the next generation of television broadcasting. The new standard, called the Grand Alliance System, was recommended by the Advisory Committee on Advanced Television Service (ACATS) and was adopted by the Federal Communications Commission (FCC) in 1995. The first HDTV program was aired nationwide in October 1998. However, by the end of 2005, only a handful number of HDTV channels have been available to consumers and only in major cities such as New York, Washington DC and Los Angeles. Most households in the US still use Analog TV sets which cannot receive digital signals.

The new digital system is not compatible with the existing analog system. However, the incompatibility issue may be solved by "digital-to-analog" converter box which analog TV owners can purchase with low price. This will allow an analog TV set to receive HDTV signals, but only at SD quality. Moreover, the US government also granted the industry 6 MHz of additional spectrum for free, enabling TV broadcasters to broadcast both analog signals and digital signals to consumers. The industry does not have to give back this additional spectrum until 85 percent of television households are capable of receiving digital signals.

As a consequence, during the present transition period, analog TV owners can still watch HDTV programs on their machines. Similarly, owners of HDTV sets can watch analog broadcasts since almost all HDTV sets have the built-in capacity to receive analog signals.

Video Game Industry (VGI) between 1970s-2000s The first video game console was launched by Magnavox in 1972, under the name Odyssey. Since then, the console producers and game developers have introduced 6 different generations of video game systems.

The generation beginning with Odyssey lasted from 1972–1977, and featured Odysseyand Atari's and PONG. The second generation was from 1977-1982, generally marked as the "Golden Age" of the game industry, and featured the Atari 2600, Odyssey^2 and Intellivision. The industry suffered from a serious crisis in 1982-1985 which was marked as the industry's "Dark Age". Sales decreased worldwide and many firms went bankrupt. During the peak of the golden age, the sale of the game industry was more than \$3 billion a year in America alone; in 1985, at the end of this dark age, video game sales reached only \$100 million worldwide.

The forth generation of video game systems was driven by two technological innovations: lower-cost memory chips and higher-power 8-bit microprocessors. It lasted for four years (1985–1989) and featured the Nintendo Entertainment System (NES), Sega Master System and Nintendo GameBoy. The Nitendo GameBoy alone had 100 million units shipped worldwide in various configurations.

The fifth generation (1989–1995) featured 16-bit processors, more detailed graphics, and more imaginative games. This generation was dominated by Nintendo and Sega. The sixth generation (1995–1998) featured high-powered microprocessors and dedicated graphics processors that enabled extremely realistic graphics and game play. Dominating systems during this period includes the Sega Saturn, Sony Playstation and Nintendo 64. The Sony Playstation alone had 50 million units sold worldwide.

The seventh generation (1998–2006) features dominating systems Sony Playstation 2, Microsoft Xbox and Nintendo GameCube. The Xbox alone had 11 millions unit sold worldwide in just two years 2001-2003. VGI is now moving to the seventh generation with the introduction of Xbox 360 from Microsoft, PlayStation 3 from Sony and Nintendo Revolution from Nintendo.

During this whole period, whenever a new generation of consoles is introduced, game developers followed immediately and developed new games to run on the new console. In most cases, the new games were not compatible with older consoles. Even though consumers had an installed console base, they often abandoned the old consoles and acquired a new ones. Such a dynamic process does not exist in television industry.

1.1.2 Similarities and Differences

The two industries share some prominent features of industry with network externalities:

Network Effects Both TVI and VGI are examples of industries with indirect network externalities (also called market-mediated network externalities). In these industries, the utility that a consumer derives from hardware (TV receiver, game console) depends on the availability and diversity of complementary content (software, TV programs). A hardware without any complementary content is useless for consumers. Likewise, content can not be "consumed" unless users have compatible hardware. As content for a given type of hardware become more available, that hardware becomes more attractive to users.

Due to network effects, both industries face the same coordination problem in innovation, generally called a "chicken-and-egg" problem: consumers do not purchase new hardware if they can not find enough content for the new hardware, but they do not buy the new content without the new hardware. Therefore, without efficient coordination, no single content producer wants to be the first mover and create markets for other content producers.

Installed Bases Both TVI and VGI have to deal with an installed base problem in the transition to a new technology. Making a product compatible with the installed base means the firm can secure a market for its product. However, if consumers can find program/software/games which work with their installed base, then the installed base plus these program/software/games serves as a reservation payoff, which lowers consumers' incentives to switch to the new technology.

Hardware's Minimal Compatibility in both TVI and VGI Hardware firms in both TVI and VGI often make new generation hardware backwardly compatible with older generation contents. For example, most Xbox games can be played on Xbox 360; and most HDTV sets have the capacity to receive and decode analog signals. The compatibility of new hardware with older contents is minimal: the quality of the whole system is defined by the component with lower quality.

Beside these similarities, TVI and VGI have some major differences:

Content's Minimal Compatibility in TVI As discussed above, owners of old TV sets can view "new" contents on their machines. This is because the new content is made minimally compatible with the older machines. The compatibility of the new content with the older machines takes the form of minimal compatibility. For example, a owner of a monochrome receiver can only watch a movie in black-and-white quality when the movie may actually be broadcasted in color.

Content's Incompatibility in VGI Games written for the newer generation console can not run on older generation consoles. For example, Call of Duty 2, a game written for the Xbox 360, will not run on the Xbox. Users who really want to play new games have to obtain a new console.

Therefore, the main difference between TVI and VGI is that new content is minimally compatible with old hardware in TVI, but completely incompatible in VGI. While broadcasters in TVI have to comply with government regulations to make their new content compatible with older machines, game developers have a compatibility choice: they can decide whether their new generation games can be played on older generation consoles or not. History shows that they have choosen to make new games incompatible with older consoles.

Intuitively, this seems to be a irrational decision. When most consumers have the installed base (such as the normal Xbox) and few have new generation consoles (such as the Xbox 360), making the game compatible only with the Xbox 360 appears to be risky, if not irrational. However, most, if not all, game developers choose to make their games compatible with either one generation or the other, but not both.

1.1.3 Paper's Objectives and Organization

As we discussed in the case studies above, innovation in VGI has proceeded faster than in the TVI. It took the TVI at least 15 years to switch from black-and-white to color broadcasts, and it may take more than 15 years to switch from standard definition (SD) to high definition (HD). The video game industry, however, has experienced 7 generations of game consoles within the last 30 years.

Can the above difference in innovation paths be explained by the compatibility choice? Specifically, if software producers and broadcasters can choose to make their new products incompatible with the installed base, then should we expect a more dynamic pattern of innovation? What is the rationale of regulation on compatibility? These are questions which we believe have both theoretical and practical importance. Section 2 below presents a benchmark model and compares the possibility of innovation in both the *laissez-faire scenario* where firms can make their own compatibility decision and the *regulated scenario* where firms have to make their products minimally compatible with the installed base.

When we try to answer these questions, we came up with even more questions which we believe have equal importance. We find that innovation in such industries is held back not only by the compatibility regulation, but also by two factors which we identify as the "*compensation problem*" and the "*coordination problem*". Firms face the *compensation problem* when it is collectively better off for them to switch to the new technology. However, due to the asymmetry in profitability, some firms find it unprofitable to switch, rendering the whole industry trapped in the less advanced technology. Thus, innovation in such situation may occur only if firms can make side payments from one to the others. Firms face the *coordination problem* when it is strictly better off for all of them to switch to the new technology, but due to the fact that innovation takes place only as a coordinated effort, firms find it strictly worse off to switch if everyone else do not switch. As a consequence, innovation can not take place and the industry is de-facto standardized in the less advanced technology.

How can a decentralized industry escape from the trap of coordination failure? What kind of mechanism a decentralized industry could use to overcome the compensation failure? Section 3 provides a brief discussion about the "cheap talk" approach to coordination problem. Section 4 presents an restricted side-contracting framework which can essentially solve the compensation problem and offers explicit conditions for the elimination of inefficiency. Section 5 offers a brief conclusion.

1.1.4 Related Literature

There are two major strands of literature related to our paper.

The first strand directly related to our paper is on network effects. Previous literature on network effects emphasize the strategic behavior of hardware firms (Church and Gandal (1993), Chou and Shy (1990)) or software firms (Church and Gandal (1992), Desruelle, Gaudet and Richelle (1996)). Some assume one firm producing both hardware and software and focus on the firms' choices of compatibility or incompatibility (Matutes and Regibeau (1988), Marinoso (2001)). Michihiro and Rob (1998) use stochastically evolutionary approach to study what they called "the bandwagon games". Their paper focuses only the demand side, however.

Both Church and Gandal (1992) and Desruelle, Gaudet and Richelle (1996) assume monopolistic competition in software market: each firm produces only one software product with increasing return to scale, and the market is assumed to have free entry. Church and Gandal (1992) further assume the incompatibility between the two systems and they identify two kinds of equilibria: de-facto standardization (only one system exists), and a coexistence (each system share half of the market). Desruelle, Gaudet and Richelle (1996) explore also the case when the two systems are compatible and find that only a symmetric equilibrium exists. They find that the equilibria might be monopolistic or duopolistic, symmetric or asymmetric, depending on the size of the fixed costs.

The main differences between our approach and the existing literature are in the way we characterize innovation and the way we characterize regulation on compatibility. In the existing literature, the authors often look at the innovation problem as a standardization issue: which technology shall be de-facto standardized given different choices of the firms and which equilibrium outcome is socially efficient?. In our paper, we characterize innovation as an problem which may require coordination and side-contracting between firms. Moreover, the existing literature disregards the regulation issue, which plays a significant role in industries such as TVI.

The second strand directly related to our paper is on side contracting. Coase (1959) and Coase (1960) state that if property rights are well defined, and bargaining is costless, then rational agents faced with externalities should contract to come to an efficient outcome. Jackson and Wilkie (2005) characterize the outcomes of games when players may make binding offers of strategy-contingent side payments before the game is played. They show that the side payments do not always lead to efficient outcomes, despite complete information and costless contracting. However, the authors show that when the number of players is greater than two, then inefficiency can not be eliminated by their side-contracting structure.

This paper offers an alternative restricted side-contracting framework based on restrictions imposed by contract law. I show that inefficiency can in fact be eliminated by restricted side-contracting. Under some conditions, efficiency is the unique equilibrium result of the game.

1.2 Benchmark Model

1.2.1 Model Specification

Technology

In this benchmark model, we assume that there are N proprietary content products offered by N firms, $N \geq 2$. Firms can either offer their products in low quality (A) or high quality (B) and incur fixed cost e_k and marginal cost c_k ($k = A, B$), $e_H > e_S$ and $c_B > c_A$.

There are two non-proprietary hardware technologies H (high quality) and L (low quality). The marginal cost of hardware is s_H and s_L for types H and L; $s_H > s_L$. Hardware production is assummed to involve no fixed cost. Since hardware technologies are non-proprietary, each is offered to consumers at marginal costs.

A system is a combination of hardware and content. A *low quality system* is comprised of low quality hardware and low quality content. A *high quality system* is comprised of high quality hardware and high quality content. If a system is comprised of both low quality and high quality components, then the cross-standard compatibility issue arises. In industries such as VGI and TVI, the compatibility between components from different standards is not perfect. This leads to the concept of "minimal compatibility".

Components from different standards are said to have minimal compatibility if the quality derived from consumption of a content is defined by the quality of the component which has lower standard.

We assume that hardware H is minimally compatible with low quality contents. Thus, a consumer can alway view contents of type A on hardware H, but the viewing experience derived from these quality contents is no different from viewing experience the consumer could derive from viewing them on hardware L. If firms choose to make their high quality contents compatible with the low quality hardware L, they have to pay an additional fixed cost $e_c \geq 0$.

In following sections, the phases "high quality standard" and "new/more advanced technology" are used exchangably to refer to B and H; the phases "low quality standard" and "old/less advanced technology" are used exchangably to refer to A and L.

Preferences

We assume that hardware only facilitates the consumption of contents and therefore provides no stand-alone benefit. Each consumer purchases at most one unit of hardware. The number of consumers in the market is normalized to 1. All consumers are assumed to have installed base hardware L.

To represent consumer preferences over contents, we adopt CES utility function[1]. Let t_i be the type of content $i : t_i = A, B$. Let x_{it_i} be the amount of content i consumed by a consumer. Let $U\left(\{x_{it_i}\}_{i=1}^{N}, h\right)$ be the consumer's utility function, h is the best hardware owned by the consumer, $h \in \{H, L\}$.

We will prove below that if a content i is offered in high quality, then all contents with ranking above i will be provided in high quality. Thus, in this section, it's enough to characterize the utility in the case that the first M contents are offered in high quality ($M \leq N$) and the last $(N - M)$ contents are offered in low quality.

Therefore, the explicit form of $U\left(\{x_{it_i}\}_{i=1}^{N}, h\right)$ is characterized as:

- If the first M contents are offered in high quality, and the last $N - M$ contents are offered in low quality ($M \leq N$), and if all high quality contents are minimally compatible with hardware L, then:

$$U(\{x_{it_i}\}_{i=1}^{N}, L) = \sum_{i=1}^{M} (x_{iB})^{\beta} + \sum_{i=M+1}^{N} (x_{iA})^{\beta}, \tag{1.2.1}$$

$$U(\{x_{it_i}\}_{i=1}^{N}, H) = \sum_{i=1}^{M} (a_{iB}x_{iB})^{\beta} + \sum_{i=M+1}^{N} (x_{iA})^{\beta}. \tag{1.2.2}$$

- If the first M contents are offered in high quality, and the last $N - M$ contents are offered in low quality ($M \leq N$), and if all high quality contents are not compatible with hardware L, then the consumer's utility derived from

[1] The CES utility function is used widely in modelling variety. For example, see Dixit and Stiglitz [1977], Church and Gandal [1989], Church and Gandal [1992], Chou and Shy [1990].

consumption of N contents are:

$$U(\{x_{it_i}\}_{i=1}^{N}, L) = \sum_{i=M+1}^{N} (x_{iA})^{\beta}, \tag{1.2.3}$$

$$U(\{x_{it_i}\}_{i=1}^{N}, H) = \sum_{i=1}^{M} (a_{iB}x_{iB})^{\beta} + \sum_{i=M+1}^{N} (x_{iA})^{\beta}. \tag{1.2.4}$$

The parameter $\beta < 1$ characterizes the concavity of the utility function. $\{a_{iB}\}_{i=1}^{M}$, $a_i > 1, \forall i \in \{1, ..., M\}$ is the vector utility coefficients characterizing the superiority of high quality standard. We assume that given everything else equal, consumers have strictly different rankings over different high quality contents: $a_i \neq a_j, \forall i \neq j,\ i, j \in \{1, ..., M\}$. Moreover, without loss of generality, assume that contents with higher ranking is more desirable to consumers: $a_i > a_{i+1}$, $\forall i \in \{1, ..., M\}$.

It is worth noting that the utility fuction specified above is additively seperable. Thus, without budget constraint, the consumer's optimal consumption of a content i is not affected by the properties (prices, qualities, compatibilities) of other contents.

The assumption that $a_i > a_{i+1}$, $\forall i \in \{1, ..., M\}$ is not artificial. In the television industry, for example, it is widely recognized that movies are most beneficial to consumers in HD. Sports programs are runner-ups while educational and news are far behind. Independent and local programs are likely to have lowest ranking of all. In a survey conducted by eMarketer inc. in 2003, consumers' responses regarding their interests in viewing broadcast contents in HD strongly support this assumption: 63% households showed interest in HD movie channels, 49% showed interest in HD sport channels, 23% in concerts, 21% in drama, 19% in sitcoms, 18% in news, and 11% in soaps.

Game structure and Equilibrium Concept

Consider a one shot "innovation" game with three stages. In the first stage, content providers decide either to provide low quality content product (type A) or high quality content product (type B) and incur development cost e_A or e_B ($e_A < e_B$). If a firm chooses to offer a content in high quality, it also decides whether the new content is minimally compatible with the old hardware or not at all. The decisions made by firms in the first period become public knowledge immediately after the termination of stage 1.

In the second stage, consumers enter hardware market and make purchase decisions, knowing the quality and compatibility choices that content providers have made in the first stage. Since hardware is offered at marginal cost, the prices for hardware H and L are s_H and s_L respectively.

In the last stage, firms choose prices simultaneously. Consumers purchase some amount of each content product.

We adopt SPE as the equilibrium concept. In the equilibrium analysis below, we first solve for optimal prices and quantities in the last stage, taking types of hardware and contents as given. Then, the consumers' choice of hardware in the second stage is identified given the optimal prices and consumption. Finally, we consider the choices of content firms regarding quality and compatibility of their products.

1.2.2 Equilibrium Analysis

Optimal Consumption

In the last stage, consumers choose the optimal consumption based on the prices of contents (p_i, $i = 1, ..., N$), the type of hardware they possess, $h \in \{L, H\}$, and the compatibility feature of each content. Since the utility function has the additive separability structure, the consumption decision can be divided into two cases:

Only hardware L available: If content i is offered in low quality, or if content i is offered in high quality and minimally compatible with the old hardware, then the consumption decision regarding content i is:

$$\max_{x_i}\{(x_{it_i})^{\beta} - p_{it_i}x_{it_i}\}. \tag{1.2.5}$$

The corresponding solution is $x^*_{it_i} = (\frac{p_{it_i}}{\beta})^{\frac{1}{\beta-1}}$. If content i is offered in high quality, but not compatible with the hardware L, then the optimal consumption of i is zero.

Hardware H is available: If content i is offered in low quality, then since H is minimally compatible with A, the consumption decision is similar to the above problem and the solution is $x^*_{iA} = (\frac{p_{iA}}{\beta})^{\frac{1}{\beta-1}}$. If content i is offered in high quality instead, then the consumption decision is characterized by:

$$\max_{x_{iB}}\{(a_i x_{iB})^{\beta} - p_{iB}x_{iB}\}, \tag{1.2.6}$$

which have unique solution $x^*_{iB} = \frac{1}{a_i}(\frac{p_{iB}}{a_i\beta})^{\frac{1}{\beta-1}}$.

Since $a_i > a_j > 1$ for $i < j$ by assumption, it is straightforward from the solution above that if both contents have high quality and are priced equally ($p_{iB} = p_{jB}$), then a consumer with high quality hardware will consume content i more than content j ($x^*_{iB} > x^*_{jB}$).

Optimal Pricing

Suppose in the last stage, there is α measure of consumers possessing high quality hardware, and λ measure of consumers possessing only low quality hardware. Since consumers are homogeneous, we have $\alpha \in \{0, 1\}$ and $\lambda + \alpha = 1$. Let $\widetilde{\Pi}_i\,(p_{it_i}, \alpha, \lambda, t_i, k_i)$ be firm i's profit in the last period; t_i is the quality of product i, $t_i \in \{A, B\}$ and $k_i \in \{0, 1\}$ is a dummy variable. $k_i = 0$ if i is not compatible with hardware L and $k_i = 1$ if i is minimally compatible with L.

The optimal pricing problem for firm i is characterized as:

$$\max_{p_i}\{\widetilde{\Pi}_i\left(p_{it_i},\alpha,\lambda,t_i,k_i\right)\}. \tag{1.2.7}$$

The solution for problem (1.2.7) is $p_{iA}^{*}=\frac{c_A}{\beta}$ and $p_{iB}^{*}=\frac{c_B}{\beta}$. The algebraic details for this solution can be found in the Appendix A. From the solution, it's clear that the firm i's optimal price $p_{it_i}^{*}$ does not depend the number of firms on the market (N) and the utility coefficient a_i.

Substitute $p_{it_i}^{*}$ into $\widetilde{\Pi}_i\left(p_{it_i},\alpha,\lambda,t_i,k_i\right)$ and let $\Pi_i\left(\alpha,\lambda,q_i,k_i\right)$ be firm i's profit in the first period: $\Pi_i\left(\alpha,\lambda,t_i,k_i\right)=\widetilde{\Pi}_i\left(p_{it_i}^{*},\alpha,\lambda,q_i,k_i\right)-e_{t_i}$ and let

$$g\left(c\right)=\left(1-\beta\right)\beta^{\frac{1+\beta}{1-\beta}}c^{\frac{\beta}{\beta-1}}, \tag{1.2.8}$$

then firm i's profit in the first period can be characterized generally as:

$$\Pi_i\left(\alpha,\lambda,t_i,k_i\right)=g\left(c_{t_i}\right)\left[\alpha\left(a_{it_i}\right)^{\frac{\beta}{1-\beta}}+\lambda k_i\right]-\left[e_{t_i}+k_ie_c\right].[2] \tag{1.2.9}$$

From (1.2.9), if a firm i offers its content in low quality, then the firm's profit does not depend on the market determinant α and λ: $\Pi_i\left(\alpha,\lambda,A,1\right)=g(c_A)-e_A$. Moreover, if consumers does not purchase hardware H, then firm i's profit is strictly lower if it provides high quality content: $\Pi_i\left(0,1,B,k_i\right)=k_ig\left(c_B\right)-\left[e_{t_i}+k_ie_c\right]<\Pi_i\left(0,1,A,1\right)$.

Since $\beta<1$, $g\left(c\right)$ is decreasing with respect to c. $\Pi_i\left(\alpha,\lambda,t_i,k_i\right)$ is increasing

[2] The explicit form of $\Pi_i\left(\alpha,\lambda,t_i,k_i\right)$ is:

$$\begin{aligned}
\Pi_i\left(\alpha,\lambda,A,1\right) &= \left(\alpha+\lambda\right)\left[\left(1-\beta\right)\beta^{\frac{1+\beta}{1-\beta}}\left(c_A\right)^{\frac{\beta}{\beta-1}}\right]-e_A,\\
\Pi_i\left(\alpha,\lambda,B,1\right) &= \left[\left(1-\beta\right)\beta^{\frac{1+\beta}{1-\beta}}c_B^{\frac{\beta}{\beta-1}}\right]\left[\alpha(a_i)^{\frac{\beta}{1-\beta}}+\lambda\right]-\left(e_B+e_c\right),\\
\Pi_i\left(\alpha,\lambda,B,0\right) &= \alpha(a_i)^{\frac{\beta}{1-\beta}}\left[\left(1-\beta\right)\beta^{\frac{1+\beta}{1-\beta}}c_B^{\frac{\beta}{\beta-1}}\right]-e_B.
\end{aligned}$$

with respect to a_i and decreasing with respect to c_{t_i}. Thus, if two products are both offered in high quality, have the same compatibility property k and the market determinant is such that $\alpha = 1$, then the higher ranking firm has strictly higher profit than the lower ranking firm: $\Pi_i(1, 0, B, k) > \Pi_j(1, 0, B, k)$, $\forall i < j$. In other words, if the industry successfully switches to the high quality standard, then firms with higher rankings get more than firms with lower rankings. Thus, high ranking firms always have stronger incentive to innovate.

The following two lemmas offer some simple but useful observations:

Lemma 1. *In equilibrium, if content i is offered in high quality, then all contents $j \leq i$ are also offered in high quality.*

Proof. By definition, $\alpha \in \{0, 1\}$ is the measure of consumers with high quality hardware. If $\alpha = 0$, firm i is strictly worse off if it provides high quality content.

Because content i is offered in high quality, it must be that $\alpha = 1$, and:

$$\Pi_i(1, 0, B, k_i) \geq \Pi_i(1, 0, A, 1). \tag{1.2.10}$$

Moreover, we have $a_{i-1} > a_i$ by assumption. Therefore:

$$\Pi_{i-1}(1, 0, B, k_i) > \Pi_i(1, 0, B, k_i), \tag{1.2.11}$$

The strict inequality sign comes from the fact that $\alpha = 1$.

Finally,

$$\Pi_{i-1}(\alpha, \lambda, A, 1) = \Pi_i(\alpha, \lambda, A, 1) = g(c_A) - e_A. \tag{1.2.12}$$

Combining (1.2.10), (1.2.11), and (1.2.12), we have $\Pi_{i-1}(1, 0, B, k_i) > \Pi_{i-1}(1, 0, A, 1)$. In other words, providing the content in high quality is the strictly dominant strategy for firm $i - 1$. $\square$

Based on the result established in lemma 1, we assume that the first M firms provide their products in high quality and the remaining $N-M$ firms provide their products in low quality, $0 \leq M \leq N$.

Lemma 2. *In equilibrium, if firm i offers its content in high quality, then firm i also make its content incompatible with L.*

The intuition behind (and the proof of) Lemma 2 is straightforward: since i is offered in high quality, it must be that consumers purchase H in the second period, or $\alpha = 1$. But if a firm i anticipates that all consumers will purchase hardware H, then making content i compatible with hardware L is strictly unnecessary.

Lemma 2 explains why in a laissez-faire environment, such as in the VGI, where innovating firms can choose between minimal compatibility and incompatibility, they choose incompatibility.

Hardware Choices

In the second stage, consumers enter the hardware market and make purchase decisions. Since they are all installed-base owners, they have no interest in hardware L and only decide whether to obtain hardware H or not.

Lemma 1 establishes that if content i is offered in high quality, all contents with an index less than i are also offered in high quality. As discussed above, let M be the number of high quality contents, then the first M contents have high quality and the last $N-M$ content products have low quality. We also know from Lemma 2 that the high quality contents are incompatible with the low quality hardware L.

If a consumer does not purchase H, her net surplus is equal to the consumption surplus she derives in the last period. In the last period, she can only consume the last $N-M$ contents due to the incompatibility between the first M contents and the hardware L.

$$W(M,L)=\sum_{i=M+1}^{N}\left[(x_{iA}^{*})^{\beta}-p_{iA}^{*}x_{iA}^{*}\right]=\frac{1}{\beta}(N-M)\,g(c_A)\,. \tag{1.2.13}$$

If a consumer purchases H instead, then her net surplus is equal to the consumption surplus she derives in the last period minus the cost of hardware H:

$$\begin{aligned} W(M,H) &= \sum_{i=1}^{M}\left[(a_i x_{iB}^{*})^{\beta}-p_{iB}^{*}x_{iB}^{*}\right]+\sum_{i=M+1}^{N}\left[(x_{iA}^{*})^{\beta}-p_{iA}^{*}x_{iA}^{*}\right] && (1.2.14)\\ &= \frac{1}{\beta}g(c_B)\sum_{i=1}^{M}(a_i)^{\frac{\beta}{1-\beta}}+\frac{1}{\beta}(N-M)\,g(c_A)-s_H. && (1.2.15)\end{aligned}$$

Comparing $W(M,L)$ and $W(M,H)$, a consumer will purchase H iff:

$$\frac{1}{\beta}g(c_B)\sum_{i=1}^{M}(a_i)^{\frac{\beta}{1-\beta}}\geq s_H. \tag{1.2.16}$$

Let $w(c_B,M)=\frac{1}{\beta}g(c_B)\sum_{i=1}^{M}(a_i)^{\frac{\beta}{1-\beta}}$. Then $w(c_B,M)$ reflects the additional benefit a consumer derives from the new hardware and the left hand side of (1.2.16) reflects the cost for the new hardware. Consumers purchase H iff the additional benefit exceeds the cost and not purchase it otherwise.

By definition, $w(c_B,M)$ is increasing with respect to M and decreasing with respect to c_B. Define

$$M_t=\arg\min_{M}\left\{w(c_B,M)-s_H:\text{s.t. }\ w(c_B,M)\geq s_H\right\}. \tag{1.2.17}$$

Then M_t is the threshold value for industry innovation: consumers will not adopt the high quality standard (purchase H) if the number of innovating firms (M) is less than the threshold value: $M<M_t$. $M=M_t$ is the minimum number of innovating firms needed to induce installed base owners to discard their existing hardware and

purchase a new one.

Before we proceed to the next section, in order to avoid trivial cases, we make the following assumptions:

Assumption 1: $g\left(c_B\right)\sum_{i=1}^{N}(a_i)^{\frac{\beta}{1-\beta}} > \beta s_H$.

Assumption 2: $g\left(c_A\right) - e_A \geq 0$.

Assumption 1 guarantees that if all firms offer high quality products, then consumers will adopt the new technology (purchase H). If assumption 1 is violated, then innovation can never happen and the industry is de-facto standardized at low quality and we have a trivial case. Assumption 2 guarantees that given the absence of a higher standard, firms still find it profitable to operate on the content market. Thus, the industry can survive without the presence of the new standard.

Quality Choices

In the first period, firms simultaneously determine their product's quality, taking into account that if the first M_t firms do not unanimously offer high quality products, then consumers will not purchase new hardware and there will be no market for high quality products.

Let $\triangle\Pi_i$ be the net profit firm i gets from offering content i in high quality instead of in low quality, given all consumers purchase H :

$$\triangle\Pi_i = \Pi_i\left(1,0,B,0\right) - \Pi_i\left(1,0,A,1\right) = \left[g\left(c_B\right)\left(a_i\right)^{\frac{\beta}{1-\beta}} - e_B\right] - \left[g\left(c_A\right) - e_A\right]. \quad (1.2.18)$$

Firm i will not offer its content in high quality unless $\triangle\Pi_i \geq 0$. Let M_p be the lowest ranking firm which offers high quality content given $\alpha = 1$:

$$M_p = \arg\min_i \left\{\triangle\Pi_i : \triangle\Pi_i \geq 0\right\}. \quad (1.2.19)$$

Proposition 1. *If $M_p < M_t$, then the innovation game has an unique SPE. On the* *equilibrium path,* $t_i = A$*,* $h = L$*,* $p_i = \frac{c_A}{\beta}$ *and* $x_i = (\frac{p_{iA}}{\beta})^{\frac{1}{\beta-1}}, \forall i$*.*

The intuition (and the proof) for proposition 1 is straightforward. Since the number of firms willing to adopt the new standard given the availability of market for high quality products ($\alpha = 1$) is M_p, and M_p is less than the threshold value required to induce consumers to adopt the new standard, no innovation can take place. Therefore, in equilibrium, firms only provide low quality contents and consumers do not purchase new hardware.

According to Proposition 1, industry innovation might not occur even if it is collectively profitable for the first M_t firms to do so. To see this, assume that all consumers have hardware H. Suppose that $\sum_{i=1}^{M_p} \triangle\Pi_i + \sum_{i=M_p+1}^{M_t} \triangle\Pi_i > 0$. If the first M_p firms could compensate/make transfer to the remaining $M_t - M_p$ firms so that all of the first M_t firms find it profitable to provide high quality products to consumers, then they might be able to collectively induce consumers to adopt the new technology. We identify this particular problem as “compensation problem”:

The Compensation Problem: In industries where products exhibits network effects and consumers have installed base, innovation can only occur if the number of firms willing to innovate reaches some critical threshold value. In such industries, firms face compensation problem when it is collectively better off for them to switch to the new technology, however, due to the asymmetry in profitability, some firms find it unprofitable to switch, rendering the whole industry trapped in the less advanced technology.

Thus, innovation is not Pareto-improving when the compensation problem exists, but it is a socially optimal. The internalization/compensation of losses incurred by firm $j \in \{M_p + 1, ..., M_t\}$ could be done via merger and acquisition. However, other mechanisms might work as well. The analysis of the Coasian contracting approach to this problem is of particular interest and shall be discussed in detail later in the paper.

Proposition 2. *If $M_p \geq M_t$, then the innovation game has two pure SPEs. On the equilibrium path of the first SPE, $t_i = A$, $h = L$, $p_i = \frac{c_A}{\beta}$ and $x_i = (\frac{p_{iA}}{\beta})^{\frac{1}{\beta-1}}, \forall i$; and on the equilibrium path of the second SPE, $t_i = B, \forall i \leq M_p$ and $t_i = A, \forall i > M_p$, $h = H$, $p_i = \frac{c_{t_i}}{\beta}$ and $x_i = (\frac{p_{iA}}{\beta})^{\frac{1}{\beta-1}}, \forall i \leq M_p$ and $x_i = \frac{1}{a_i}(\frac{p_{iB}}{a_i\beta})^{\frac{1}{\beta-1}}, \forall i > M_p$.*

Again, the intuition (and the proof) for proposition 2 is straightforward. Note that firms $j \in \{M_p+1, ..., N\}$ always offer low quality products. Therefore, the game reduces to the coordination game between the first M_p firms. Consider a firm $i \in \{1, ..., M_p\}$. If all other firms $j \neq i$, $j \in \{1, ..., M_p\}$ choose B, then i's best response is also B. If all other firm $j \neq i$, $j \in \{1, ..., M_p\}$ choose A, then i's best response is also A.

The first SPE characterized in proposition 2 exhibits a coordination failure: all firms offer low quality products and consumers do not purchase high quality hardware. The second SPE exhibits the coordinated equilibrium: the first M_p firms offer high quality products, the last $N - M_p$ firms offer low quality products, and consumers adopt the new technology (purchase H).

We identify the situation in the first equilibrium characterized by proposition 2 as "coordination problem":

The Coordination Problem: In industries where products exhibits network effects and consumers have installed base, innovation can only occur if the number of firms willing to innovate reaches some critical threshold value. In such industries, firms face a coordination problem when it is strictly better off for all of them to switch to the new technology, but due to the fact that innovation takes place only as a coordinated effort, firms find it strictly worse off to switch if everyone else do not switch. As a consequence, innovation can not take place and the industry is de-facto standardized in the less advanced technology.

Thus, innovation is both Pareto-improving and socially optimal when the compensation problem exists. When multiple equilibria exists, the equilibrium selection problem arises. We shall discuss this issue more detail in the discussion section below.

1.2.3 Regulation on Compatibility

As we already discussed in the introduction section, the nature of digital technology allows broadcasters to provide programs incompatible with the analog TV sets to households. However, the government mandates that every carriers should transmit both analog and digital signals during the transition period. This regulation makes broadcasters' products have the minimal compatibility feature discussed above. Also as noted, the government subsidies firms by providing them free additional spectrum, enabling them to transmit both type of signals at the same time.

In this section, we will uncover the scenario in which firms are obliged to make their high quality products compatible with the low quality hardware. We also assume that firms are compensated for the cost of compatibility ($e_c = 0$).

Lower Interest in New Technology:

In the second period, consumers make hardware choice. Let r stand for minimal compatibility regulation. Consumers purchase H iff $W(M, H) \geq W_r(M, L)$. $W_r(M, L)$ is the payoff from a system comprised of hardware L, the first M high quality (minimally compatible) content products and last $(N - M)$ low quality content products. Since B is minimally compatible with L, $W_r(M, L)$ is characterized as:

$$W_r(M, L) = \frac{M}{\beta} g(c_B) + \frac{(N - M)}{\beta} g(c_A). \tag{1.2.20}$$

Therefore, $W(M, H) \geq W_r(M, L)$ is equivalent to:

$$\frac{1}{\beta} g(c_B) \sum_{i=1}^{M} \left[(a_i)^{\frac{\beta}{1-\beta}} - 1 \right] \geq s_H. \tag{1.2.21}$$

Let $\widetilde{w}\left(c_B, M\right) = \frac{1}{\beta} g\left(c_B\right) \sum_{i=1}^{M} \left[\left(a_i\right)^{\frac{\beta}{1-\beta}} - 1\right]$ and define M_t' as:

$$M_t' = \arg\min_{M} \left\{\widetilde{w}\left(c_B, M\right) - s_H : \text{s.t. } \widetilde{w}\left(c_B, M\right) \geq s_H\right\}. \tag{1.2.22}$$

Then it is immediate that $\widetilde{w}\left(c_B, M\right) < w\left(c_B, M\right)$ and $M_t' > M_t$. The threshold number of innovating firms required in the regulation scenario is strictly higher than the threshold number in the laissez-faire scenario. Consequently, the condition for industry innovation is harder in regulation scenario than in laissez-faire scenario. We characterize this observation formally in the following proposition:

Proposition 3. *If $M_t \leq M_p < M_t'$, then industry innovation is possible under laissez-faire but impossible under regulation*

The proof of Proposition 3 is straightforward. Since $M_p \geq M_t$, Proposition 2 establishes that there exists a SPE in which industry innovation occurs under laissez-faire scenario. However, since $M_p < M_t'$, the number of firms willing to offer high quality products is smaller than the number needed to induce consumers to adopt the high quality standard. Thus, innovation does not occur in the regulation scenario. However, this does not necessarily mean that consumers are worse off under regulation.

Regulation and Consumer Protection

Consider the situation characterized by proposition 3. Under laissez-faire scenario, consumers adopt the high quality standard, but not under compatibility regulation. Intuitively, when contents are minimally compatible with old hardware, the installed base becomes more valuable to consumers and increase their "reservation" utilities. Since under regulation, consumers do not adopt the new technology even if M_p firms offering high quality products, $W_r\left(M_p, L\right) > W\left(M_p, H\right)$. Since $W\left(M_p, H\right)$ is also the consumer's surplus under laissez-faire, consumers are strictly better off under compatibility regulation than under laissez-faire. We characterize this observation formally in the following proposition:

Proposition 4. *If $M_t \leq M_p < M_t'$, then regulation on compatibility makes consumers strictly better off and content providers strictly worse off.*

For $M_t \leq M_p < M_t'$, in laissez-faire environment, consumer are “forced” to adopt the new technology because firms does not offer compatible contents. In that case, regulation works as a mechanism to protect consumers from “unwanted” innovation, but it comes with the loss of profit from the firms' side. Whether laissez-faire leads to higher total social surplus depends on whether the profit gain from firms' side can equate the loss in consumers' surplus.

It is worth noting that under compatibility regulation, we still have the coordination problem and the compensation problem. Regarding the coordination problem, if $M_p \geq M_t'$, then the game has two SPEs, one exhibits the coordination failure and one exhibits successful coordination. Regarding the compensation problem, if $M_p < M_t'$, then there is still the case in which $\sum_{i=1}^{M_p} \triangle\Pi_i + \sum_{i=M_p+1}^{M_t'} \triangle\Pi_i > 0$: the first M_t' firms are collectively better of to switch to the new technology, but due to the asymmetry in profitability, some firms find it strictly worse off to switch to new standard. As a consequence, innovation does not occur.

We establishes these observations as a corollary:

Corollary 1. *Under regulation on compatibility, the compensation problem exists iff $M_p < M_t'$ and*

$$\sum_{i=1}^{M_p} \triangle\Pi_i + \sum_{i=M_p+1}^{M_t'} \triangle\Pi_i > 0, \tag{1.2.23}$$

the coordination problem exists iff $M_p \geq M_t'$ and the equilibrium play involves no innovation.

It is also worth noting that if $M_p \geq M_t'$, then in the coordinated equilibrium, all consumers adopt the new technology regardless of the presence of the compatibility regulation. In such case, the regulation is strictly redundant.

1.3 Discussion

Rationale of Regulation on Compatibility

The results established in proposition 3 and 4 state that regulation on compatibility is not justifiable on the technology standpoint. It does not facilitate the transition. Rather, it complicates the transition and make it harder for firms to switch to the more advanced technology. In a laissez-faire scenario, innovating firms make their advanced contents incompatible with low quality hardware. By doing so, they lower the reservation benefits of the consumers and provide them with a stronger incentive to adopt the new standard. What have been going on in the video game industry for the last three decades seems to fit with this scenario. Life span of each technological generation is short and consumers frequently upgrade their consoles whenever a new version comes out, knowing that game developers will never bother writing games for outdated consoles.

However, the US government might have other things to worry about beside the technological innovation. If the regulation is dropped, as it was first expected to be so at the end of 2006, the 21 million American households (approximately 20 percent of total number of households) who receive only over-the-air-broadcasts will have their TV sets go dark on Jan. 1, 2007, unless they purchase sets with DTV tuners or subscribe to cable or satellite television services. Since this 20% households belongs to the poorest section of the population who might not be able to resolve the problem themselves, the government have a good reason to protect their access to television. As one reporter puts it “digital television is a civil right”.

In fact, as the Congress recently set a firm date for termination of analog signals, it also passed a law that would provide an initial $990 million, and as much as $1.5 billion, to help Americans buy converter boxes that would keep their old, analog televisions working when the digital transition is completed.

Notwithstanding the retarding effect of regulation on innovation, there exit ways in which firms could overcome the compensation and coordination problems, at least in principle. The remainder of this paper will provide discussion about cheap talk and Coasian contracting.

Cheap Talk and Equilibrium Selection

Proposition 2 and corollary 1 establish that coordination problems exist in both the regulation scenario and the laissez-faire scenario. How firms can avoid the "bad" equilibrium and successfully coordinate on the efficient equilibrium is a topic extensively discussed in the literature regarding equilibrium selection. One approach is to impose Nash refinements (payoff dominance vs. risk dominance, stable vs. non-stable). Other directions involves communication among players. Most relevant to our discussion is the "cheap talk" approach (Farrell (1987), Park (2002) among others).

Farrell studies an entry game between two identical firms. Only one firm can be profitably accommodated by the market. If both firms enter, they will both receive strictly negative payoff. If both firms choose to stay out, then they will both receive zero payoff. Thus, the game has two pure strategy Nash equilibrium and one mixed strategy equilibrium.

Farrell assumes that before players actually play the game, they have one round of communication. Communication consists of each firm saying "In" or "Out". Then they play the game with payoffs unchanged by the first round of communication. The communication in the first round has the nature of a cheap talk because it is assumed to be costless and it does not directly affect either player's payoff.

Farrell proceeds by assuming that if the signals in the communication round are "In" for the first firm and "Out" for the second firm, then the first firm will enter with probability one. If both firms send the same signal, they will play the symmetric mixed equilibrium in the entry game.

Based on this structure, Farell shows that the probability of coordination failure is reduced by cheap talk. The assumption regarding interpretation of signals plays a crucial role in Farell's results. The rationale behind it is explained by the author: "Because there are no payoff links between the two periods, this is an assumption. It seems a reasonable one: once an equilibrium of the original entry game become focal through being "agreed on," it will be followed."

If we utilize Farell's framework and add one more round of communication between firms before the actual innovation game is played, then the first round is served as a "practice" round: each firm can costlessly announce its preference (innovate or not innovate) and learn about the others' preferences. If all firms reveal their true preferences, then innovation is "agreed on" and shall be played in the actual game. Given this structure, no firm want to conceal its true preferenc. Consequently, coordination becomes the unique equilibrium of the extended game. In other words, cheap talk can eliminate the coordination problem which arises in our innovation game.

Communication and Coordination between Broadcasters

The race for a standard HD system started at the end of 1980s and continued until the Grand Alliance was formed in May 24th, 1993, including proponents of the four competing systems: GI, Zenith, AT&T, MIT, Thompson, Sarnoff and Philip. The system proposed by the Alliance was finally accepted as the new standard for US broadcast industry in 1995. The "innovation" game between broadcasters started after the new standard was announced.

There is strong evidence that communication between broadcasting firms was extensive. They have an active organization called the National Association of Broadcasters (NAB), which has strong political influence. In 1987, they jointly petitioned the FCC to investigate for potential of advanced TV technology. After the Grand Alliance came up with a new television system, the broadcaster industry again collectively lobbied the government to give them additional spectrum. In letters, speeches and testimony before congressional committees, broadcasters espoused the virtues of HDTV. The message was clear: they would use the digital spectrum to offer high-definition television. An executive of the NAB said that TV stations "will use this spectrum for HDTV, pure and simple."

The government supported the broadcast industry's arguments about the importance of making the transition to digital. But not everyone agreed that they needed a second full 6 MHz channel, or that they should get the additional spectrum for free. Opponents called it a spectrum "giveaway" and proposed giving broadcasters only the amount of spectrum necessary to transmit a single standard-definition digital signal while making them pay for the additional spectrum.

Broadcasters vehemently oppose the idea that they should have to pay for additional spectrum. Their arguments was switching to the new system is costly, and if they have to pay for additional spectrum, then might be unable to provide HD services to consumers. As a consequence, consumers would be deprived of one of the great benefits of digital technology – high-definition television. The government finally supported the broadcast industry on that matter and the Congress passed the Telecommunications Act of 1996, giving each broadcasting station an additional spectrum of 6 MHz free of charge.[3]

With such extensive communication and collective action, one would not have suspected that broadcasters would fall in the trap of coordination failure. However,

[3] Senate Commerce Committee Chairman John McCain called the broadcaster give-away "one of the great scams in American history"

the transition to HDTV has been much slower than expected. In 1997, FCC set a target date of 2006 for the cessation of analog broadcast service. The Congress then passed the Balanced Budget Act of 1997 confirming December 31, 2006 as the transition completion date. However, the cessation date is not universally mandatory, since it allows broadcasters to file for an extension. Consequently, the broadcasters have delayed the transition and the 2006 deadline was never taken seriously. Facing the possibility that this transition might take forever unless stronger regulation was in place, the Congress has passed a new legislation in February 8th, 2006, which set February 17, 2009 as the final deadline for termination of all analog broadcast services.

What has been happening since 1997? Why are some broadcasting firms are committed to the HD, and others not? Extensive communication between firms leaves no room for misinterpretation of each other's preference. Since the benefit of HDTV is universally agreed upon broadcasters and consumers, our only other explanation for the excessive delay is the compensation problem. When industry innovation is held back by the problem of ownership and the compensation effect, is there any way a decentralized industry can do except to consolidate? In the section below we will discuss the implication of Coase's theorem within the framework of limited side-contracting.

1.4 The Compensation Problem and Side Contracting

In this section, we explore the possibility of an decentralized solution to the compensation problem. Our central question is: when it is collectively efficient for firms to adopt the more advanced technology, can firms overcome the compensation problem to achieve efficiency via negotiation/contract design? Our approach to the problem is inspired by what generally called "the Coase Theorem". As Jackson and Wilkie (2005) put it:

> "The simple but powerful idea put forth by Coase (1960) says that if property rights are well defined, and bargaining is costless, then rational agents faced with externalities should contract to come to an efficient outcome. Roughly speaking, with fully symmetric information and no transactions costs, agents should be able to come to an agreement that supports an efficient strategy profile as an equilibrium point of the game with side payments."

Examples often used to demonstrate Coase's idea are the "tragedy of the common" games: countries fishing in the same international water can design "side contracts" in the form of international fishing agreements; industrial countries around the world can design international pollution agreements which controls the amount of national emissions. In such agreements, some promises of strategy-contingent payments (side payments) are often included. The side payments, when implemented, can help promote efficiency by changing the incentives of the involved parties so that they see more fully the total impact or value that their actions generate.

1.4.1 Enforceable Contracts

Game theoretic modeling of side payments has been championed by Jackson and Wilkie (2005). Their approach is to view a game as being embedded in a larger game where in a first stage players may engage in side contracting that can effectively rewrite pay-off functions to depends on strategies subsequently chosen. The players then play the eventual altered game in the second stage. The goal is to see whether or not inefficient equlibria (and/or "efficient" strategy profiles) survive the tranformation of the original games

Under JW's structure, each player selects a binding *transfer function* which maps from the set of pure strategy profiles to the set of positive real numbers. In other words, players can contract with others on each and every single profile.

They cannot punish other players with negative transfers, but they can effectively commit to pay other players contingent on their own actions. For example, under JW's structure, a player can effectively design a contract which states that if he does not play strategy X_i, then he will pay every other players a positive amount t_i.

The JW structure leads to a somewhat strange result: inefficiency can be eliminated in games with two players, but can not be eliminated in any game with more than two players. For example, by utilizing side contracts, two players in a tragedy of the commons game can be certain that inefficient fishing will never be played in an equilibrium, but three players can not escape inefficiency.

The reason behind such a result is subtle: in the JW structure, each player can commit to play any particular strategy. If there are only two players, then player 1 can always "undo" player 2's commitment by proposing a counter transfer. But if there are three players, for example, then player 1 can not undo player 2's commitment because player 2's commitment involves not only player 1, but also player 3.

Besides leading to the somewhat strange result mentioned earlier, the JW side payment structure is not generally enforceable/applicable under contract law. For example, the common law system does not enforce the self-commitment characterized in JW. Legal theorists define contracts as *promises* that the law will enforce. The law provides remedies if a promise is breached and recognizes the performance of a promise as a duty. To be legally binding as a contract, a promise must be exchanged for adequate *consideration*. Adequate consideration is a benefit or detriment which a party receives which reasonably and fairly induces them to make the promise.

Promises that are purely gifts are not considered enforceable because the personal satisfaction the grantor of the promise may receive from the act of giving is normally not considered adequate consideration. A typical expression of nonenforcement of gratuitous promises follows:

> The underlying principle of consideration would seem to be negative, - a denial that ordinarily there is sufficient reason why gratuitous promises should be enforced. From a nude pact no obligation arises. The courts have not felt impelled to extend a remedy to one who seeks to get something for nothing. English law accordingly will not usually enforce a promise unless it is given for value, or the promise for value, i.e., something which the law must assume to be of some value to the promisor and which the parties make the subject of bargain or exchange.[4]

The self-commitment in JW is a promise without adequate consideration. When a player make a promise which states that "if I do not play strategy X_i, then I will pay every other players a positive amount t_i", then such promise is not used to exchange to any consideration other than the hope that it might affect other players' action. Hence, it is not enforeable under contract law.

In the following analysis, we characterize only enforceable promises. We assume that players can design binding contracts with side payments contingent only on the actions to be taken by other players. Self-commitment is not allowed, Surprisingly, under some conditions, this leads to elimination of inefficient equilibrium when the number of players is greater or equal than three.

1.4.2 Definitions

Because our goal is to explore the possibility of a decentralized solution to the compensation problem, we restrict ourselves to the case in which compensation problem arises. For simplicity, we collapse the innovation game described above

[4]Ballantine, Mutuality and Consideration, supra note 50, at 121.

to only the first stage where firms decide which technology they pursue. We also normalize the profit associated with old technology to zero. Since the game only involves the first M_t firms (or M_t' firms in the regulation scenario), we simply neglect the remaining firms[5]. Moreover, since we want to contrast our results with the possible results in the JW structure, we only discuss the case where the number of involving firms is greater or equal to 3.

Our innovation game can be describe as: $N \geq 3$ players play a one shot, simultaneous move game. Player i's pure strategy space is $X_i = \{A, B\}$, with $X = \times_i X_i$. Let $\triangle(X_i)$ denote the set of mixed strategies for player i , let $\triangle = \times_i \triangle(X_i)$ and let $\triangle_{-i} = \times_{j \neq i} \triangle(X_j)$. Denote generic elements of X_i, $X, \triangle(X_i)$, $\triangle$ and $\triangle_{-i}$ by x_i, x, μ_i, μ, μ_{-i}, respectively.

In the innovation game, the payoff $v_i : X \to R$ is characterized by the following assumptions:

- If a player plays A, he always gets zero payoff regardless of actions committed by other players[6].
- If player i plays B and there exists a player $j \neq i$ who plays A, then i's payoff is $-c$, with $c > 0$.
- If everyone plays B, then player i gets payoff $\pi_i > 0$.

Assume $\pi_i > \pi_{i+1}$ and $\exists M < N$ such that $\pi_M \geq 0$ and $\pi_{M+1} < 0$.

We are now ready to characterize the extended innovation game with side contracting.

[5]Consumers will upgrade hardware only if the first M_t firms (or M_t' firms in the regulation scenario) switch to the new technology. We already show in the above section that if firm i does not provide high quality content product, then all firms indexed higher than i ($j > i$) will not offer high quality products. Therefore, the strategic roles of the remainng $N - M_t$ firms (or $N - M_t'$ firms in the regulation scenario) can be neglected.

[6]This comes from our normalization of profit associated with old technology to zero. We established in the above section that if a firm does not innovate, it always receive a fixed amout of profit regradless of actions taken by other firms. This result come from the fact that high quality hardware is minimally compatible with low quality content products.

Innovation Game with Side Contracting

A set $N = \{1, ..., n\}$, $n \geq 3$ of players interact in two stages. At stage 1, players simultaneously announce binding transfer functions that reward other players based on their actions. At stage 2, players play the above innovation game with payoffs modified by the transfers announced in the first period.

The first-stage transfer functions The transfer functions that player i announces in the first period are given the vector of functions $t_i = (t_{i1}, ..., t_{in})$, where $t_{ij} : X_j \to R+$ represents the promises to player j as a function of the action that is played by j in the second-period game. For example if x_j is played by j in the second period, then i transfers $t_{ij}(x_j)$ to player j.

Let $t = (t_1, ..., t_n)$. Also, denote by t_i^0 the degenerate transfers such that $t_{ij}^0(x_j) = 0$, $\forall j \in N$, $\forall x_j \in X_j$, and let $t^0 = (t_1^0, ..., t_n^0)$.

The characterization of the transfer function in this setting is more restricted than the characterization in JW. JW's setting allows agents to contract based on every strategy profile $t_{ij} : X \to R+$. This setting only allows agents to contract on the action that other agents undertake in the second period game, $t_{ij} : X_j \to R+$.

The modified payoffs Assuming transferable utility, the pay-off to player i given a profile of transfer functions t and a play x in the second-period game is then

$$U_i(x, t) = v_i(x) + \sum_{j \neq i} \left(t_{ji}\left(x_i\right) - t_{ij}\left(x_j\right)\right). \tag{1.4.1}$$

So, given a profile of transfer functions t and a mixed strategy μ played in the second-period game, the expected utility to player i is

$$EU_i(\mu, t) = \sum_x \mu(x) \left[v_i(x) + \sum_{j \neq i} \left(t_{ji}\left(x_i\right) - t_{ij}\left(x_j\right)\right)\right]. \tag{1.4.2}$$

Let $NE(t)$ denote the set of (pure and mixed) Nash equilibria of the second-stage game where payoffs are modified according to (1.4.1) and (1.4.2). This is the set of Nash equilibria taking a profile of transfer functions t as given, and only varying the strategies in the second-period game. Let $NE(t^0)$ denote the set of (pure and mixed) Nash equilibria of the game without transfer.

Supportable strategies and payoffs A pure strategy profile $x \in X$ of the second-stage game together with a vector of payoffs $u \in R^n$ such that $\sum_i u_i = \sum_i v_i(x)$ is *supportable* if there exists a subgame perfect equilibrium of the two stage game where some t is played in the first stage and x is played in the second stage (on the equilibrium path), and $U_i(x, t) = u_i$.

If $x \in NE(t^0)$, we say that x survives if $(x, v(x))$ is supportable by some t in the extended game.

The idea behind supportability is two folds:

- Given a strategy profile x, for example, x is the most efficient profile of the game, supportability asks the question whether x can be a part of a SPE of the extended game with side contracting. If the answer is yes, then we know that there is a possibility that x will be played as long as players can design binding transfers before they actually play the game.

- Given an equilibrium x', for example, x' is an inefficient equilibrium of the game, supportability asks the question whether x' can be eliminated from the equilibrium path of every SPE of the extended game with side contracting. If the answer is yes, then we know that players will never play the bad equilibrium x' if they can design binding transfers before they actually play the game.

Only when the efficient profile and payoff (x, u) is supportable, and all other (x', u') such that $x' \neq x$ are not supportable, then we know that players will achieve efficiency with probability 1.

1.4.3 An Example with 3 Players

Denote the strategy profile $\{x : x_i = A, \forall i\}$ by x_A and $\{x : x_i = B, \forall i\}$ by x_B. In order to demonstrate the main idea of side contracting, and how the JW setting and our setting works, we first consider an innovation game with only three players. The following is the normal form of the game:

The payoff matrix if player 3 plays A:

$$\begin{array}{ccc} & & \text{player 2} \\ & & \begin{array}{cc} A & B \end{array} \\ \text{player 1} & \begin{array}{c} A \\ B \end{array} & \begin{array}{cc} (0,0,0) & (0,-c,0) \\ (-c,0,0) & (-c,-c.0) \end{array} \end{array} \tag{1.4.3}$$

The payoff matrix if player 3 plays B:

$$\begin{array}{ccc} & & \text{player 2} \\ & & \begin{array}{cc} A & B \end{array} \\ \text{player 1} & \begin{array}{c} A \\ B \end{array} & \begin{array}{cc} (0,0,-c) & (0,-c,-c) \\ (-c,0,-c) & (\pi_1,\pi_2,\pi_3) \end{array} \end{array} \tag{1.4.4a}$$

Assume that $\pi_1 > \pi_2 > 0$ and $\pi_3 < 0$ but that $\pi_1 + \pi_2 + \pi_3 > 0$. In other words, total surplus received by 1 and 2 exceeds the loss incurred by 3. The unique equilibrium of the innovation game is $x_A = \{x : x_i = A, \forall i\}$ whereas the efficient strategy profile is $x_B = \{x : x_i = B, \forall i\}$. Without side contracts, efficiency is not achievable. Moreover, cheap talk does not help to achieve efficiency because it does not alter players' payoffs. Since with the unchanged payoff matrix, A is the strictly dominant strategy for player 3, pre-game communication will not make him prefers strategy B. Thus, to achieve efficiency, we need to rely on side contracting.

Jackson and Wilkie's Setting:

In the JW setting, since x_A is an equilibrium of the original game with more than 2 players, x_A always survives in an extended game with side contracting. In other words, even if efficiency is achievable via side-contracting, players might still end up with the inefficient outcome.

To see that, consider a transfer scheme such as the following:

$$t_{ij} = \begin{cases} 2\pi_1 & \text{if } x_i = B \\ 0 & \text{otherwise} \end{cases}, \forall i,\, \forall j \neq i. \tag{1.4.5}$$

According to the above transfer, each player commits to pay other players $t = 2\pi_1$ if he plays B and pay nothing if he plays A. With such commitment, it is immediate that x_A is the unique equilibrium of the second stage[7]. The only question left is whether a player, for example, 1, wants to commit to such a transfer scheme. The answer is "yes", because selecting other transfer function only makes sense if 1 expect the second stage's play is something else instead of x_A. To get such a result, 1 has to use his deviated transfer (t') to manipulate other players. In the innovation game, the only sensible alternative is x_B for the second stage. In order to make player 2 play B, his deviated transfer should satisfies:

$$0 + 2\pi_1 + t'_{12}\left(x'\right) \leq \pi_2 + 2\pi_1 + t'_{12}\left(x_B\right) - 2\left(2\pi_1\right). \tag{1.4.6}$$

Where $x' = \{B, A, B\}$, t'_{12} is the transfer scheme which 1 considers deviating to. The left hand side is the payoff player 2 would get from playing A (zero payoff, plus transfer from player 3 and 1), the right hand side is the payoff player 2 would get from playing B (π_2 plus transfer from player 3 and 1 minus his transfer to the other two players since player 2 still keeps his transfer commitment).

[7]Suppose all 3 players play t_{ij} in the first stage. In the second stage, given the two other players play A, player i will also play A because the payoff he receives from playing A is π_i and the payoff he receives from playing B is $-c - 2\left(2\pi_1\right) < \pi_i$.

(1.4.6) implies that

$$t'_{12}(x_B) \geq 4\pi_1 - \pi_2 > 3\pi_1. \tag{1.4.7}$$

A similar incentive problem applies to player 3. Therefore, $t'_{13}(x_B) \geq 4\pi_1 - \pi_3 > 3\pi_1$. As a consequence, the payoff for player 1 at x_B is less than:

$$\pi_1 + 2\pi_1 + 2\pi_1 - 3\pi_1 - 3\pi_1 = -\pi_1 < 0. \tag{1.4.8}$$

Thus, deviating to t'_{12} is not profitable to 1. Similarly, 2 and 3 do not find it profitable to deviate to another transfer scheme either. In other words, the inefficient equilibrium x_A always survives in JW's setting.

Our Restricted Setting:

In our restricted setting, players can not make "self commitment". Therefore, the transfer from i to j can only condition on j's action taken in second stage. To simplify notation, denote $t_{ij}(x_j = B)$ by $t_{ij}(B)$ and $t_{ij}(x_j = A)$ by $t_{ij}(A)$. We will show that given this setting, x_A will not survive if π_1 large enough. To show that x_A will not survive, assume the opposite that $(x_A, v(x_A))$ is supportable by some transfer t. We first establish that if x_A survives, then the transfer t which support the $(x_A, v(x_A))$ should satisfy $t_{ij}(A) = 0$. This result is immediate since $v_i = 0$ is the secure payoff for player i (he can not get lower than that) and x_A is a equilibrium of the original game. Hence, player i has no reason to transfer any positive amount to other players to convince them to play x_A. Any positive t_i would simply be a gratuitous transfer.

With $t_{ij}(A) = 0$, the transfer in the first stage can be characterized as:

$$t_{ij}(x_j) = \begin{cases} 0 & x_j = A \\ t_{ij}(B) & x_j = B \end{cases}, \forall i. \tag{1.4.9}$$

Beside $t_{ij}(A) = 0$, there is nother condition which $t_{ij}(x_j)$ should satisfy: the transfer $t_{ij}(B)$ should be such that no player wants to deviate by selecting a null transfer in the first stage and plays B in the second stage. By playing (1.4.9) in the first stage and A in the second stage, player i gains 0. If player i plays null transfer t_i^0 in the first stage and B in the second stage, he gets $-c + \sum_{j \neq i} t_{ji}(B)$. Therefore, the tranfer $t_{ij}(x_j)$ should satisfy:

$$0 \geq -c + \sum_{j \neq i} t_{ji}(B), \forall i. \tag{1.4.10}$$

Assuming that 2 and 3 do not deviate, consider player 1. If 1 does not deviate from t_1, then 1 receives payoff $v_1 = 0$. If 1 deviates and selects the transfer function t_1' such that $t_{1i}'(A) = 0$ and:

$$t_{1i}'(B) = c + t_{i1}(B) + t_{ij}(B) - t_{ji}(B), \; j \neq i, \; j \neq 1, \tag{1.4.11}$$

then player 2 and 3 will find it profitable to play B. It is because if i $(i = 2, 3)$ plays B, he gets at least:

$$-c + t_{1i}'(B) + t_{ji}(B) - t_{i1}(B) - t_{ij}(B) = 0 \tag{1.4.12}$$

Given such t_1' and x_B is played in the second stage, player 1 gets:

$$\pi_1 - 2c - \sum_{i=2}^{3} \left[t_{i1}(B) + t_{ij}(B) - t_{ji}(B) \right] + \sum_{i=2}^{3} t_{i1}(B), \; j \neq i, j \neq 1, \tag{1.4.13}$$

which is equal to:

$$\pi_1 - 2c - \sum_{i=2}^{3} \left[t_{ij}(B) - t_{ji}(B) \right] = \pi_1 - 2c. \tag{1.4.14}$$

In sumary, if $(x_A, v(x_A))$ is supportable at all, then the vector of transfer t

which support $(x_A, v(x_A))$ should satisfy (1.4.9) and (1.4.10). However, if $\pi_1 > 2c$, then player 1 always find it strictly better off to deviate. Player 1 can always choose a transfer:

$$t'_{1i}(x_i) = \begin{cases} c + t_{i1}(B) + t_{ij}(B) - t_{ji}(B) & \text{if } x_i = B \\ 0 & \text{o.w.} \end{cases}, \tag{1.4.15}$$

with $i \neq 1$, $j \neq 1$, $j \neq i$. With tranfer $t'_{1i}(x_i)$, x_B becomes the only equilibrium of the second stage game. Player 2 and 3 each get zero payoff, and player 1 gets $(\pi_1 - 2c) > 0$.

In other words, given $(\pi_1 - 2c) > 0$, the inefficient equilibrium x_A can not survive.

1.4.4 Efficiency with N Players

We now come back to the general innovation game with (restricted) side contracting. Before establishing the general efficiency results, we have a few simple but useful observations:

Lemma 3. *No adoption $\{x : x_i = A, \forall i\}$ is the unique equilibrium for the original innovation game*

The result established by the above lemma comes straight from our analysis of the benchmark model.

Lemma 4. *If $\sum_{i=1}^{N} \pi_i \leq 0$, then $(x_B, u(x_B))$, $\forall u$ such that $\sum_{i=1}^{N} u_i(x_B) = \sum_{i=1}^{N} \pi_i$, is not supportable in the extended game with side contracting.*

Proof. By contradiction: Suppose players choose a vector of transfer functions $t = \{t_1, ..., t_N\}$ in the first period. In the subsequent period, if x_B is an equilibrium, it must be that:

$$\pi_i + \sum_{j \neq i} t_{ji}(B) - \sum_{j \neq i} t_{ij}(B) \geq 0. \tag{1.4.16}$$

The above condition applies for all $i = 1, ..., N$. Aggregateing across all i leads to:

$$\sum_{i=1}^{N}[\pi_i + \sum_{j\neq i} t_{ji}(B) - \sum_{j\neq i} t_{ij}(B)] \geq 0, \tag{1.4.17}$$

or $\sum_{i=1}^{N} \pi_i \geq 0$, contradicting with the assumption that $\sum_{i=1}^{N} \pi_i \leq 0$. □

A more interesting question are (1) whether $(x_B, u(x_B))$ is supportable for some $u(x_B)$ such that $\sum_{i=1}^{N} u_i(x_B) = \sum_{i=1}^{N} \pi_i$ if $\sum_{i=1}^{N} \pi_i \geq 0$; and (2) which condition can guarantee that x_A does not survive. Proposition 5 below establishes that efficiency is achievable if $\sum_{i=1}^{N} \pi_i \geq 0$ and proposition 6 establishes that if we also have $\pi_1 > (N-1)c$, then efficiency is the only equilibrium outcome of the extended innovation game with side contracting.

Proposition 5. *If $\sum_{i=1}^{M-1} \pi_i + \sum_{i=M}^{N} \pi_i \geq 0$, then there exists an $u(x_B)$, $\sum_{i=1}^{N} \pi_i = \sum_{i=1}^{N} u_i(x_B)$, such that $(x_B, u(x_B))$ is supportable.*

Proof. The main idea of the proof is to construct a vector of transfer functions that can support x_B.

Essentially, since the last $(N-M)$ players are strictly worse off if they plays B, the first M players have to share the burden of compensating them. Suppose that each player $i \in \{1, ..., M\}$ contribute:

$$\left(-\frac{\sum_{l=M+1}^{N} \pi_l}{\sum_{l=1}^{M} \pi_l}\right)\pi_i \leq \pi_i{}^8. \tag{1.4.18}$$

So that the total sum of compensation would be

$$\sum_{i=1}^{M}\left(-\frac{\sum_{l=M+1}^{N} \pi_l}{\sum_{l=1}^{M} \pi_l}\right)\pi_i = -\sum_{l=M+1}^{N} \pi_l. \tag{1.4.19}$$

[8] The inequality comes from the assumption that $\sum_{l=1}^{N} \pi_l > 0$. Thus, $\sum_{l=1}^{M} \pi_l \geq -\sum_{l=M+1}^{N} \pi_l$.

Let i be a player in $\{1, ..., M\}$ and j be a player in $\{M+1, ..., N\}$, define the transfer from i to j by:

$$t_{ij}(x_j) = \begin{cases} -\frac{\pi_i \pi_j}{\sum_{l=1}^{M} \pi_l} & \text{if } x_j = B \\ 0 & \text{o.w.} \end{cases}. \tag{1.4.20}$$

The total transfer player j receives from the first M players is

$$\sum_{i=1}^{M} \left(-\frac{\pi_i \pi_j}{\sum_{l=1}^{M} \pi_l} \right) = -\pi_j. \tag{1.4.21}$$

For $i, k \in \{1, ..., M\}$, define $t_{ik} = 0$. For $j, h \in \{M+1, ..., N\}$, define $t_{jh} = 0$. For $k \in \{1, ..., M\}$ and $h \in \{M+1, ..., N\}$, define $t_{hk} = 0$.

With the vector of transfers t defined above, player i's modified payoff at x_B is:

$$u_i(x_B) = \begin{cases} \left(1 + \frac{\sum_{l=M+1}^{N} \pi_l}{\sum_{l=1}^{M} \pi_l}\right) \pi_i \geq 0 & \text{if } i \in \{1, ..., M\} \\ 0 & \text{if } i \in \{M+1, ..., N\} \end{cases} \tag{1.4.22}$$

Thus, x_B becomes an equilibrium of the second period game.

Also, with the above transfer structure, no player wishes to deviate by selecting a different transfer function: the last $N-M$ players have no incentive because they do not make any transfer to any other player. The first $M-1$ players have no incentive to deviate since any increase in their transfer is unnecessary, any reduction in their transfer means coordination failure and they immediately get 0 payoff. □

Proposition 6. *If $\sum_{i=1}^{M-1} \pi_i + \sum_{i=M}^{N} \pi_i \geq 0$ and $\pi_1 \geq (N-1)c$, then $(x_B, u(x_B))$, with $u(x_B)$ defined in (1.4.22) is the unique supportable strategy profile and payoffs.*

Proof. In order to prove the proposition, we have to show that any other strategy profile $x \neq x_B$ can not be played on equilibrium path of any SPE. Showing that x_A does not survive given $\pi_1 \geq (N-1)c$ is a straightforward extension of the 3 players game characterized in section (4.3) and is presented in Appendix A.

Our remaining task is to show that any strategy profile $\widehat{x}$ such that $\widehat{x} \neq x_A$ and $\widehat{x} \neq x_B$ can not be supported. Let $n_A(\widehat{x})$ and $n_B(\widehat{x})$ be the sets of players playing A and B respectively according to strategy profile $\widehat{x}$. $n_A(\widehat{x})$ and $n_B(\widehat{x})$ are both non-empty.

Suppose by contradiction that $\widehat{x}$ is supported by a vector of transfer t. Then since the total payoff of every players at $\widehat{x}$ is $-cn_B(\widehat{x}) < 0$, at least one player should receive a strictly negative payoff. Let h be that player. In the first stage, player h can always deviate by selecting the null transfer t_h^0 and plays A in the second stage. By doing so, he can secure his payoff equals to zero. Therefore, no transfer t could support $\widehat{x}$.

As a result, $(x_B, u(x_B))$, with $u(x_B)$ defined in (1.4.22) is the unique supportable strategy profile and payoffs. $\square$

1.5 Conclusion

In explaining the excessive delay in the television industry's transition to HD, this paper characterizes an industry with network effects, minimal compatibility and installed base. I contrast the scenario where firms can make the compatibility decision with the scenario where the government requires firms to make their products compatible with the installed base. I find that if firms can make their own compatibility decisions, they will make their products incompatible with the installed base, providing stronger incentives for consumers to switch to the new system. Regulation significantly slows down innovation, since it creates stronger incentives for consumers to utilize the installed base. I also show that the excessive delay may come from completely different sources. Consumers adopt the new technology and purchase new hardware only if there is a significant number of "new" products. Thus, if the number of firms switching to new technology is lower than a threshold level, the innovating firms will incur losses because the market for "new products" is not successfully created.

Consequently, innovation can be held back by either a "coordination problem" or a "compensation problem". The coordination problem exists when firms fail to switch to the new technological standard even if the innovation benefits all. It arises under when firms face multiple equilibria and no mechanism exists which can eliminate the inefficient equilibrium. The compensation problem exists if firms fail to switch to the new technological standard although it is collectively optimal for all firms, but switching to the new technology makes some firms strictly better off while making others strictly worse off. Thus, the worse-off firms do not switch and the innovation is not realized.

To solve the compensation problem, we characterize a restricted side-contracting framework based on restrictions imposed by contract law. I show that inefficiency can in fact be eliminated by restricted side-contracting. Under some conditions, efficiency is the unique equilibrium result of the game. In term of legal enforceability, this framework fits more than the one proposed by Jackson and Wilkie. Moreover, when the number of players greater than 2, the restricted side-contracting framework also leads to the possibility of elimination of inefficient equilibria, which is not possible under Jackson and Wilkie's framework.

Chapter 2

Software Industry: Quality Competition and Code Release

"We felt we could still be true to the open-source roots and at the same time go to a business model that was proven." - Larry Augustin, cofounder of SourceForge.Net, a website hosting more than 100 thousands open source projects.

"*Let me be clear - Microsoft has no beef with open source.*" - Craig Mundie, chief research and strategy officer at Microsoft.

2.1 Introduction

Open source software (OSS) such as Linux operating system or the Apache web server is software of which the source code is published and free for everyone to inspect and modify. OSS is generally available for download free of charge or with a nominal price. Although open source software has been an integral part of the computer industry since its birth, the economics of open source did not get the attention of the economic research community until just recently. The initial interest of the literature on OSS has been to explain the motivations underlying the activity of programmers who allocate their time to open source projects without a direct monetary reward. The participation of a programmer in the open source community has been found to be similar to the involvement of an academic professor in the research community. Thus, programmers' motivations are commonly explained in light of private information and signaling models (Lerner and Tirole (2002), Hertel (2003), Bessen (2002) among others). The dynamics of a community containing such programmers is investigated in the context of social network models and bandwagon games.(Bitzer and Schroder (2006)).

While the incentive of open source developers has been widely studied, little has been done in explaining the engagement of profit-maximizing firms. In the last ten years, there has been a growing number of firms releasing their source code into the open source community (OSC), including big names such as Oracle, IBM[1], HP, Compaq, Sun Microsystem and Netscape. Lerner and Tirole (2002a) and Schmidt and Schnitzer (2002) suggested in their discussion papers that profit-maximizing firms may want to release their software to the community to sell more complementary goods, for example, hardware or other software programs. In this paper we hold this reasoning to a careful scrutiny, and find that the issue is surprisingly subtle.

When a firm releases software, the straightforward benefit from such an approach is to reduce production costs . Consider for example the Linux operating system. A cost analysis conducted in 2002[2] showed that Debian GNU/Linux version 2.2 contained over fifty-five million source lines of code, and the study estimated that it would have cost 1.9 billion dollars (year 2000 U.S. dollars) to develop by conventional means. Profit-maximizing firms taking the open source approach such as RetHat can rely on the OSC in developing the Linux kernel and other supporting software to avoid this development cost. However, as this approach reduces firms' production costs, it also attracts entry. When a source code is released and contributed by the community, it is freely available for any entrant to pick up and commercialize. For instance, Sun Microsystem released StarOffice in June 2000 and a few years later, the open source version of StarOffice became available (under the name OpenOffice.Org, or OOo). Several commercial firms around the world picked up this OSS, customized it, and sold their customized versions as proprietary. Among these versions are Workplace by IBM, MultiMedia Office by Platasoft, KaiOffice by KaiSource and WPS Office by KingSoft.

Free entry makes code-release a bad strategy since it quickly brings down

[1] IBM has recently become one of the most notable examples of commercial entities taking open source approach. Since 2001, the company has released a large number of source code projects to the community, including Jikes Research Virtual Machine (RVM) in October 2001, Java-based Cloudscape Database in August 2004, Object Rexx in November 2004, Unstructured Information Management Architecture (UIMA) in January 2006, and Autonomic Software in June 2006.

[2] http://people.debian.org/~jgb/debian-counting/counting-potatoes/

the firm's profit. Even in the case of no entry, the firm has to compete with the OSS which it helped create. After Netscape released Mozilla in March 1998, the non-profit Mozilla foundation was established and introduced Mozilla Firefox – an increasingly popular internet browser. Meanwhile, the proprietary Netscape Navigator, which is based on the open source Mozilla, did not succeed and has been almost forgotten.

Promoting sales of products or services which are complementary to the OSS is another reason to release a source code. However, it is not always the case. Suppose the software market has two competing products. By releasing its product to the OSC, a firm normally expects the source code to be developed by the OSC and will subsequently become more competitive than the rival (proprietary) software product. As the open source software earns a larger marketshare, the involved firm will generate more sales of the complementary products or services. However, the OSC may turn out to be poorly managed and cannot contribute much to the open code. In such cases, releasing a software may increase the competitiveness of the rival firm. On the other hand, if the potential profit from selling complementary goods is too high, neither firms may want to release their code to the OSC because the OSC does not act on behalf of any profit-maximizing firm and may provide an OSS with quality lower than the code-released-firm wanted. Worst of all, the released source code may not be picked up and developed at all because the rival firm also releases their code and the OSC may find the code of the rival software more attractive.

In a nutshell, code-release strategy effectively reduces production costs but it also invites entry. Firms may want to release their software to promote sales of complementary goods. Nevertheless, their decisions to release code depend on a complex combination of the OSC's competency, the complementarity between the software of interest and the firm's related commercial products, and the firms' production efficiency. In this respect, finding the right conditions in which a profit-maximizing firm may adopt the open source approach is an important question, both theoretically and practically. This paper develops a theoretical model in which firms choose a production approach (either the open source or the proprietary approach), quality levels, and pricing strategies in a Bertrand competition framework for differentiated products. Each firm benefits from sale of its software in two ways: the direct profit from the software sale, and the indirect profit from complementary sales, which is increasing with respect to the marketshare of the firm's software.

Our analysis is related to the models of quality competition developed by Shaked and Sutton (1982), Ronnen (1991), Motta (1993), and Lehmann-Grube (1997). These papers analyzed two stage Bertrand models in which duopoly firms choose quality levels in the first stage and engage in price competition in the second stage. Our paper extends their models to incorporate the firms' decision regarding production methods and the complementarity effect of the software of interest with other related goods.

The code-release by commercial firms has also been analyzed in other settings. In Baake and Wichmann (2004)'s, a software firm can release some modules of their source code to reduce production costs. The authors recognize the problem of free entry but they assume that there is at most one entrant and the code-released firm can invest more on the remaining proprietary modules of their software to create an effective entry barrier. Hawkins (2004) presents a model in which firms benefit from increasing sales of complementary products and releasing source code reduces their production costs. The author focused more on the comparative statics of the given parameters rather than characterizing a full competition model with quality setting and pricing dimensions. Finally, Mustonen (2005) presents a model in which a monopolist decides whether to support the OSC in order to create compatibility between the firm software and the OSS. Compatibility increases consumer's evaluation of both software products as well as the competition betwen the two products as they become more alike.

The next section provides formal characterization of the model. Equilibrium analysis is provided in section 3. Section 4 offers some concluding remarks.

2.2 The Model

There are two software firms in the market, firm 1 and firm 2, possessing two sets of incomplete source code which can be used to produce two software products with similar functionalities. At the first stage of the game, the firms choose either to adopt the proprietary approach (PA) or the open source approach (OA). If a firm chooses the OA and releases its source code, then the open source community (OSC) may develop the source code in the second stage. If a firm chooses the PA, then it has to develop the source code by itself in the second stage. For notational convenience, we address a firm taking the the proprietary approach as "PA firm" and a firm taking the open source approach as "OA firm".

In the second stage, PA firms simultaneously choose respective quality levels for their products. We denote the quality of software i by s_i $(i = 1, 2)$. The production of quality s_i involves development cost $F_i(s_i)$ which is independent of output and convex in the chosen quality, $F' \geq 0$, $F'' > 0$. Without loss of generality, we assume that the source code of software 1 is more cost efficient than the source code of software 2. Thus, if both firms choose the PA in the first stage and identical quality s in the second stage then $F_1(s) \leq F_2(s)$. To ensure that the game always has an interior and bounded solution with both firms being active in the market, we further assume that $F_k(0) = 0$, $F_k'(0) = 0$, $\lim_{s \to \infty} F_k'(s) = \infty$. Throughout the paper, unless stated otherwise, we also assume that $F_k(2s) < s$ $(k = 1, 2)$ for sufficiently small s.

If the first stage results in one PA firm and one OA firm, then the OSC will develop an open source software (OSS) based on the released code. If both firms chose the open source approach, then the OSC will pick up only the code of product 1 because it is more cost efficient. We assume the OSC has capacity limit and if the OSC picks up a code, it will develop an OSS with quality $\overline{s}$, which is common knowledge.

In the last stage, the PA firms engage in simultaneous price competition. Since software can be multiplied easily, we assume the PA firms incur zero variable cost in this stage. If there is at least one firm releasing source code in the first stage, then the OSS of quality $\overline{s}$ will be available free of charge to consumers in this stage.

A firm benefits from the software sale in two ways: a direct profit from the sale of the software itself and an indirect profit from selling complementary goods. To keep the analysis tractable, we assume that firm i's indirect profit is a linear function of software i's marketshare. Particularly, let q_i be the market share for product i. Firm i's profit from the sales of complementary goods or services is assumed to be nq_i where n is the "complementarity coefficient". The indirect profit nq_i does not depends on whether the software i is an OSS or a proprietary software.

For the demand side, there are N consumers with unit demand. Consumers differ in taste parameter θ, and they get a net payoff from buying a product of quality s at price p:

$$U(s,p) = \theta s - p, \tag{2.2.1}$$

The consumer taste parameter θ is uniformly distributed on $[0, \theta^0]$. If neither of the firms decide to release their codes in the first stage, then in the last stage, only two proprietary products are available and a consumer of type θ will buy from at least one of the firm if $U(s_i, p_i) \geq 0$ for some i and will purchase from the firm that offers him the best price/quality combination. If at least one firm releases their code in the first stage, then consumers can download the OSS free of charge in the last stage. In this case, a consumer only purchases the proprietary software (PS) if the net payoff from buying the PS exceeds $\theta\overline{s}$. Without further loss of generality, the total number of consumers, N, as well as θ^0 are normalized to unity. Similar demand structure was utilized by Tirole (1988), Ronnen (1991), Choi and Shin (1992), and Lehmann-Grube (1997).

2.3 Equilibrium Analysis

In the first period, each firm chooses between the PA and the OA. As a result, three corresponding scenarios can emerge in the following stages: if both firms choose the PA in the first stage, then two proprietary software will be developed in the second stage and the two firms will engage in price competition in the last stage. We address this subgame as the "PS/PS" subgame. If only one firm chooses the PA approach in the first stage, then in the second stage, the PA firm will develop their proprietary software and compete with the OSS in the last stage. We address this subgame as the "PS/OSS" subgame. Finally, if both firms choose the OA in the first stage, then only software 1 will be developed by the OSC in the second stage and there will be no competition in the last stage. In this case, firm 2 receives zero profit, every consumer downloads the OSS ($q_{OSS} = 1$), and firm 1 (the incumbent) makes a profit n.

2.3.1 The PS/OSS subgame

Suppose firm i adopts the PA and firm j adopts the OA in the first stage. In the last stage, the OSS with quality $\overline{s}$ is available for free download. Firm i has positive marketshare only if it chooses quality $s_i \geq \overline{s}$ in the second stage. Suppose $s_i \geq \overline{s}$. In the last stage, the demand for firm i is $q_i = 1 - p_i / (s_i - \overline{s})$, [3] and the revenue

[3] The indifferent consumer is θ such that $\theta s_i - p_i = \theta\overline{s}$. Thus, $\theta = \frac{\theta\overline{s}+p_i}{s_i}$. The marketshare of firm i is $q_i = 1 - \frac{\theta\overline{s}+p_i}{s_i}$ and the marketshare of the OSS is $\frac{\theta\overline{s}+p_i}{s_i}$.

function for firm i is

$$R_i(s_i, \overline{s}) = (1 - \frac{p_i}{s_i - \overline{s}})(p_i + n).$$

Maximizing $R_i(s_i, \overline{s})$ with respect to p_i yields the solution $p_i = (s_i - \overline{s} - n)/2$ if $s_i \geq n + \overline{s}$ and $p_i = 0$ otherwise. In other words, if the quality of the proprietary software is low, the PA firm will make it freely available for download in order to get maximum marketshare. If the quality is higher, then the PA firm may want to raise the price above zero and start receiving revenue from the software market. Since the quality s_i is sufficiently high, when the firm raises their price above zero, the gain from increasing revenue in the software market outweighs the reduction in the firm's marketshare (and thus, reduction in the complementary profit.)

In the case $p_i = 0$ and $s_i = \overline{s}$, we assume the break-even rule that consumers will choose the free proprietary software instead of the OSS. With this rule, the revenue function for firm i is

$$R_i(s_i, \overline{s}) = \begin{cases} \frac{(s_i - \overline{s} + n)^2}{4(s_i - \overline{s})} & \text{if } s_i > \overline{s} + n \\ n & \text{if } s_i \leq \overline{s} + n. \end{cases}$$

Firm i's profit is

$$\pi_i = R_i(s_i, \overline{s}) - F_i(s_i) = \begin{cases} \frac{(s_i - \overline{s} + n)^2}{4(s_i - \overline{s})} - F_i(s_i) & \text{if } s_i \geq \overline{s} + n \\ n - F(s_i) & \text{if } s_i \leq \overline{s} + n. \end{cases} \quad (2.3.1)$$

If the firm chooses quality level $s_i \leq \overline{s} + n$ and if $F_i(\overline{s}) > n$, the PA firm can not profitably compete against the OSS and the optimal quality level is $s_i = 0$. Suppose that $F_i(\overline{s}) \leq n$. Choosing a quality level $s_i \in (\overline{s}, \overline{s} + n]$ is not an optimal strategy for the PA firm since $p_i = 0$ and firm i's profit is decreasing with respect to $s_i \in [\overline{s}, \overline{s} + n]$. If the firm chooses the quality level $s_i = \overline{s}$, then the corresponding profit is $\pi_i = n - F(\overline{s}) \geq 0$.

Nevertheless, if the firm chooses the quality level $s_i > \overline{s} + n$, then s_i must satisfy the following first order condition:

$$\frac{2\left(s_i-\overline{s}+n\right)\left(s_i-\overline{s}\right)-\left(s_i-\overline{s}+n\right)^2}{4\left(s_i-\overline{s}\right)^2}-F_i^{'}\left(s_i\right)=0,$$

or simply

$$F_i^{'}\left(s_i\right)=\frac{1}{4}-\frac{n^2}{4\left(s_i-\overline{s}\right)^2}. \tag{2.3.2}$$

Let s_i^0 be a solution for (2.3.2). Since $F_i'\left(s_i\right)$ is an increasing function, $\lim_{s\to\infty}F'\left(s\right)=\infty$ and $F_i^{'}\left(s_i^0\right)\leq\frac{1}{4},\forall n$; the optimal quality s_i^0 as a function of n is bounded from above by an S such that $F_i^{'}\left(S\right)=0.25$. Thus, given $\overline{s}$ fixed, the inequality $s_i^0>\overline{s}+n$ can not hold for n sufficiently large. Mathematically, there exists an $\overline{n}$ such that the local optimal quality s_i^0 ceases to be the global optimal quality if $n\geq\overline{n}$. In other words, for all $n\geq\overline{n}$, it is globally optimal for the PA firm to choose $s_i=\overline{s}$ and $p_i=0$. This observation is important for the later analysis and is summarized in the following proposition:

Proposition 7. *Given a constant $\overline{s}$, there exists an $\overline{n}$ such that for all $n\geq\overline{n}$, the PS/OSS subgame has an unique Nash equilibrium in which the PA firm chooses quality $s_i=\overline{s}$ and price $p_i=0$.*

Proof. Choose $\overline{n}$ such that $F_i\left(\overline{s}\right)\leq\overline{n}$ and $\overline{s}+\overline{n}>S$, $i=1,2$. By $F_i\left(\overline{s}\right)\leq\overline{n}$, providing a software with nonzero quality is profitable for the PA firm. By $\overline{s}+\overline{n}>S$, and $s_i^0<S$, the condition $s_i^0>\overline{s}+\overline{n}$ does not hold. Thus, the unique optimal quality for the PA firm is $s_i=\overline{s}$ and the corresponding price is $p_i=0$. □

By the break-even rule, if $n\geq\overline{n}$, the proprietary software is available to consumers for free and the marketshare of the OSS is zero. The corresponding profit for the OA firm is zero. If $n<\overline{n}$ and $s_i^0>\overline{s}+n$, the residual marketshare for the OSS is positive and is characterized by:

$$q_{OSS} = \frac{p_i}{s_i - \overline{s}} = \frac{(s_i - \overline{s} - n)}{2(s_i - \overline{s})} = \frac{1}{2} - \frac{n}{2(s_i - \overline{s})},$$

and the profit for the OA firm is also positive and is characterized by:

$$\pi_{j,OSS} = \left[\frac{1}{2} - \frac{n}{2(s_i - \overline{s})}\right] n. \tag{2.3.3}$$

Now, suppose that n is fixed, since the optimal quality s_i^0 as a function of n is bounded from above by an S such that $F_i^{'}(S) = 0.25$, the inequality $s_i^0 > \overline{s} + n$ can not hold for $\overline{s}$ sufficiently large. Moreover, as $\overline{s}$ increases, the inequality $F_i(\overline{s}) \leq n$ eventually will not hold. In other words, when the open source community becomes sufficiently effective in producing high quality open source software, the PA firm can not profitably compete against the OSC and the firm's optimal strategy is setting zero quality.

Corollary 2. *Given n fixed, there exists an $\overline{S}$ such that for every $\overline{s} \geq \overline{S}$, the PS/OSS has a unique equilibrium in which the PA firm chooses zero quality.*

Proof. Let $\overline{S}$ be defined such that $F_i\left(\overline{S}\right) > n$ and $F_i^{'}\left(\overline{S}\right) > \frac{1}{4}$, $i = 1, 2$. Since $\lim_{s\to\infty} F_i(s) = \lim_{s\to\infty} F_i^{'}(s) = \infty$, such $\overline{S}$ exists. Now since $F_i\left(\overline{S}\right) > n$, any $s_i \in (0, \overline{s} + n]$ is strictly dominated by $s_i = 0$. Also since $F_i^{'}\left(\overline{S}\right) > \frac{1}{4}$, any $s_i \in (\overline{s} + n, \infty)$ is strictly dominated by $s_i = \overline{s} + n$. In other words, $s_i = 0$ in equilibrium. □

2.3.2 The PS/PS Subgame

Consider a subgame in which both firms choose the PA in the first stage. This is a two stage quality competition game and is equivalent to the game characterized in Ronnen (1991) and Lehmann-Grube (1997), except that in their model, $F_1(s) = F_2(s)$ and $n = 0$. In this subgame, two firms simultaneously choose quality levels and after observing the rival's quality, firms simultaneously choose price levels. Note first that choosing identical quality $s_i = s_j = s$ in the second stage will lead to a standard Bertrand competition in the third stage and both firms receive zero revenue from software sale. Thus, in the standard setting developed by Ronnen and Lehmann-Grube, quality pooling is not an equilibrium strategy. In this model, firms receive both direct profit from software sale and indirect profit as well. However, quality pooling is not an equilibrium strategy either. To see that, suppose by contradiction that both firms offer the same quality s and choose price zero. Each firm has 50% marketshare and receives a profit of $\pi_k(s,s) = \frac{1}{2}n - F_k(s)$, $k = 1, 2$. Given the rival firm plays s, firm k receives $\pi_k(s,s) = \frac{1}{2}n - F_k(s)$ if not deviating from s. However, by increasing the software's quality to $s + \varepsilon$, with ε is arbitrarily small, and play $p_k = 0$, firm k receives:

$$\pi_k(s+\varepsilon, s) = n - F_k(s+\varepsilon).$$

Since $F_k(s)$ is continuous, one can easily select an ε such that $F_k(s_k+\varepsilon) < F_k(s) + \frac{1}{2}n$. With such ε, the deviated profit is strictly greater than the quality-pooling profit. As a result, quality-pooling is not an equilibrium in the PS/PS subgame.

Since quality-pooling is not an equilibrium strategy, we denote the high quality firm by i and the low quality firm by j $(s_i > s_j)$. In the last stage, a consumer of type θ purchases from firm j if $\theta s_j - p_j \geq 0$ and $\theta s_j - p_j \geq \theta s_i - p_i$. Similarly, a consumer of type θ purchases from firm i if $\theta s_i - p_i \geq 0$ and $\theta s_i - p_i \geq \theta s_j - p_j$. The last-stage demand functions for firm i and j are (see Figure 1):

$$q_i = 1 - \frac{p_i - p_j}{s_i - s_j} \text{ and } q_j = \frac{p_i - p_j}{s_i - s_j} - \frac{p_j}{s_j}. \tag{2.3.4}$$

In the last stage, firms maximize revenue $R = pq + nq$, taking s_i and s_j as

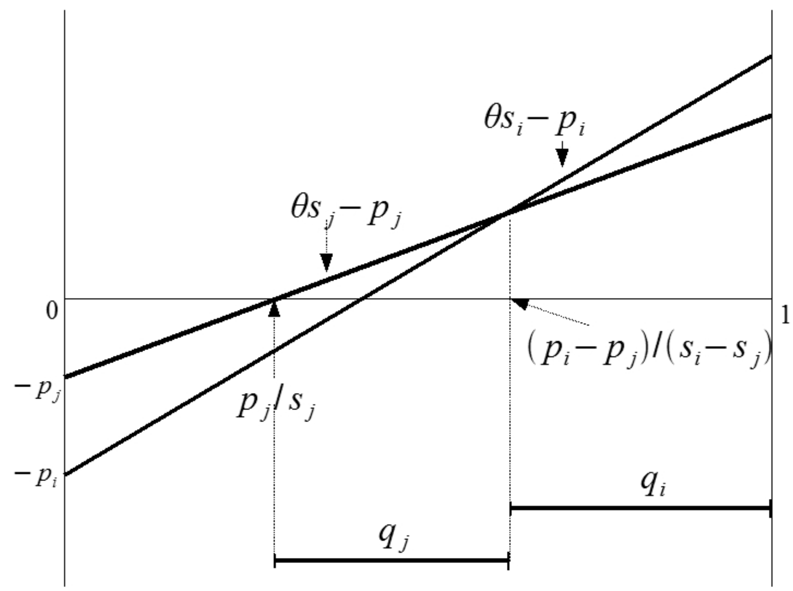

Figure 2.1: Demands for firm i and j. A consumer of type $\theta = \frac{p_i - p_j}{s_i - s_j}$ is indifferent between purchasing from i or j. A consumer of type $\theta = \frac{p_j}{s_j}$ is indifferent between purchasing from j or not purchasing anything.

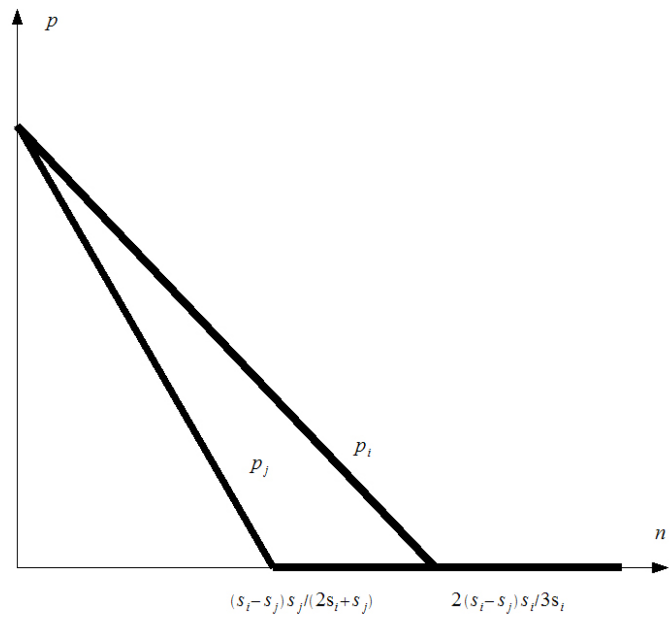

Figure 2.2: Prices p_i, p_j as functions of complementarity coefficient n

given. The corresponding optimal prices for firm i and j are

$$p_i = \frac{2(s_i - s_j)s_i - 3ns_i}{(4s_i - s_j)}, \tag{2.3.5}$$

$$p_j = \frac{(s_i - s_j)s_j - n(2s_i + s_j)}{(4s_i - s_j)}. \tag{2.3.6}$$

From (2.3.5) and (2.3.6), the high quality firm's price (p_i) and the low quality firm's price (p_j) are both decreasing with respect to the complementarity coefficient n. The low quality firm chooses zero pricing if $n \geq \frac{(s_i-s_j)s_j}{(2s_i+s_j)}$. The high quality firm choose zero pricing strategy if $n \geq \frac{2(s_i-s_j)}{3}$. Since $\frac{2(s_i-s_j)}{3} > \frac{(s_i-s_j)s_j}{(2s_i+s_j)}$, there exists a range of n in which the high quality firm i still charges a strictly positive price and the low quality firm offers the software free of charge (see figure 2).

The formal characterization of prices is:

$$p_i = \begin{cases} \frac{2(s_i - s_j)s_i - 3ns_i}{(4s_i - s_j)} & \text{if } n \geq \frac{2(s_i - s_j)}{3} \\ 0 & \text{otherwise.} \end{cases}$$

$$p_j = \begin{cases} \frac{(s_i - s_j)s_j - n(2s_i + s_j)}{(4s_i - s_j)} & \text{if } n \geq \frac{(s_i - s_j)s_j}{(2s_i + s_j)} \\ 0 & \text{otherwise.} \end{cases}$$

Given the above price functions, the corresponding revenues are

$$R_i\left(s_i, s_j\right) = \begin{cases} \frac{1}{(4s_i - s_j)^2}\left(s_i - s_j\right)\left(2s_i + n\right)^2 & \text{if } n \leq \frac{(s_i - s_j)s_j}{(2s_i + s_j)} \\ \frac{(2s_i + n)}{(4s_i - s_j)^2}\left[\left(s_i - s_j\right)\left(2s_i - s_j\right) + 3ns_i\right] & \text{otherwise} \\ n & \text{if } n \geq \frac{2(s_i - s_j)}{3}. \end{cases}$$

and

$$R_j\left(s_i, s_j\right) = \begin{cases} \frac{s_i(s_i - s_j)(s_j + 2n)^2}{s_j(4s_i - s_j)^2} & \text{if } n \leq \frac{(s_i - s_j)s_j}{(2s_i + s_j)} \\ \frac{2(s_i - s_j)s_i - 3ns_i}{(s_i - s_j)(4s_i - s_j)}n & \text{otherwise} \\ 0 & \text{if } n \geq \frac{2(s_i - s_j)}{3}. \end{cases}$$

In the quality setting stage, firms maximize $\pi = R - F(s)$ with respect to s. The game either has equilibrium in pure strategies or in mixed strategies or both. The necessary first order conditions for a Nash equilibrium in pure strategies are

$$\frac{\partial R_i\left(s_i, s_j\right)}{\partial s_i} = F_i'\left(s_i\right) \tag{2.3.7}$$

$$\frac{\partial R_j\left(s_i, s_j\right)}{\partial s_j} = F_j'\left(s_j\right), \tag{2.3.8}$$

Let $\left\{s_i^*, s_j^*\right\}$ be the solution to the above system of equations. $\left\{s_i^*, s_j^*\right\}$ is a equilibrium of the subgame if and only if the low quality firm (j) and the high quality firm (i) have no desire to trade places by respectively setting quality higher than

s_i^* or lower than s_j^*. As noted by Lehmann-Grube, these conditions do not always hold for all functional forms of $F_i(s)$ and $F_j(s)$. The PS/PS subgame may not have equilibrium in pure strategies. Nevertheless, it can be shown that PA firms always receive strictly positive profit in this PS/PS subgame. This result is essential for the analysis in the next section and is stated formally in the following proposition:

Proposition 8. *If both firms adopt the proprietary approach in the first stage, then both firms will receive strictly positive profits in the corresponding PS/PS subgame.*

The detail of the proof can be found in the appendix. Intuitively, the proof proceeds by showing that given firm j plays $\overline{G}_j(s)$, with $\overline{G}_j(s)$ being either a pure strategy or a mixed strategy, firm i can always find a strategy $G_i(s)$ such that

$$\pi_i\left(G_i(s), \overline{G}_j(s)\right) > 0. \tag{2.3.9}$$

Let $\overline{G}_i(s)$ be the firm i's best response given firm j's plays $\overline{G}_j(s)$, the following inequality must be true

$$\pi_i\left(\overline{G}_i(s), \overline{G}_j(s)\right) \geq \pi_i\left(G_i(s), \overline{G}_j(s)\right) > 0$$

In other words, firm i always receive strictly positive profits regardless of firm j's action. Hence, both firms receive strictly positive profits in the PS/PS subgame.

By proposition 2, let's fix an equilibrium for the PS/PS subgame and denote $\pi_{1,PS/PS}$ and $\pi_{2,PS/PS}$ the profits for firm 1 and firm 2. Both $\pi_{1,PS/PS}$ and $\pi_{2,PS/PS}$ depend on n but not on $\overline{s}$. In the next section, we will provide comparisons of the PS/PS profits and the PS/OSS profits.

2.3.3 The Open Source Approach vs. The Proprietary Approach

In the first stage, the firms choose to adopt either the proprietary approach (PA) or the open source approach (OA) with knowledge of the equilibria in the corresponding subgames. In the subgame where both firms release their source codes, firm 1 has total profit of n and firm 2 has zero profit. Let $\pi_{1,PS/PS}$ and $\pi_{2,PS/PS}$ be the equilibrium profits in the PS/PS subgame. Let $\pi_{1,PS/OSS}$ and $\pi_{2,OSS}$ be the equilibrium profits in the PS/OSS subgame in which firm 2 releases the source code. Let $\pi_{1,OSS}$ and $\pi_{2,PS}$ be the equilibrium profits in the PS/OSS subgame in which firm 1 releases the source code. Firms' decisions in the first period can be summarized in the following normal form game:

			Firm 2
		PA	OA
Firm 1	PA	$\pi_{1,PS/PS}, \pi_{2,PS/PS}$	$\pi_{1,PS/OSS}, \pi_{2,OSS}$
	OA	$\pi_{1,OSS}, \pi_{2,PS/OSS}$	$n, 0$

Without explicit functional forms of $F_1(s)$ and $F_2(s)$, it is almost impossible to compare the values in the above payoff matrix. However, some important observations can be derived without further loss of generalization. The first observation is that by proposition 1, for all $n \geq \overline{n}$, the PS/OSS subgame has an unique equilibrium in which the profit for the OA firm is zero. In this case, neither of the two firms want to release their source codes. Consequentially, $\{$PA, PA $\}$ is the unique equilibrium outcome in the first stage for all $n \geq \overline{n}$. In addition releasing source code is not a optimal strategy either if $n = 0$ since the profit for the OA firm is zero ($\pi_{i,OSS} = 0$, $i = 1, 2$) and $\pi_{i,PS/PS} > 0$, $i = 1, 2$ by proposition 2. Thus, $\{$PA, PA $\}$ is also the unique equilibrium outcome in the first stage in case of zero complementarity coefficient. The following proposition extends this observation:

Proposition 9. *Given a constant $\overline{s}$, there exists an $\underline{n}$ such that for all $n \leq \underline{n}$ and for all $n \geq \overline{n}$, there exists an equilibrium in which both firms adopt the proprietary approach.*

Proof. Fix a constant $\overline{s}$ which is non-zero and finite. Proposition 1 directly implies that given $n \geq \overline{n}$, choosing the OA approach is a dominated strategy. Suppose $n < \overline{n}$, we will show that there exists an $\underline{n}$ such that for all $n \leq \underline{n}$, choosing the OA is also a dominated strategy.

From the analysis of the PS/PS subgame above, we have $\pi_{1,PS/PS} > 0$ and $\pi_{2,PS/PS} > 0$. It is sufficient to show that as n goes to zero, the profits $\pi_{i,OSS}$ goes to zero, $i = 1, 2$. From the analysis of the PS/OSS subgame, we have:

$$\pi_{i,OSS} = \begin{cases} \{\frac{1}{2} - \frac{n}{2[s_i(n)-\overline{s}]}\}n & \text{if } s_i(n) > \overline{s} + n \\ 0 & \text{otherwise} \end{cases}, i = 1, 2.$$

$s_i\left(n\right)$ is the solution to the first order condition stated in (2.3.2):

$$F_i^{'}\left(s_i\right) = \frac{1}{4} - \frac{n^2}{4\left(s_i - \overline{s}\right)^2}.$$

If the condition $s_i(n) > \overline{s} + n$ does not hold, the profit function $\pi_{i,OSS}$ takes zero value and the OS approach is not optimal for firm i. If the condition $s_i(n) > \overline{s} + n$ holds, then $s_i\left(n\right)$ is bounded from above by $\overline{S}$ with $\overline{S}$ is defined by $F_k\left(\overline{S}\right) = \frac{1}{4}$, $k = 1, 2$, and below by $\overline{s} + n$. Thus,

$$n \leq s_i\left(n\right) - \overline{s} \leq \overline{S} - \overline{s},$$

and

$$\lim_{n \to 0} \{\frac{1}{2} - \frac{n}{2\left[s_i\left(n\right) - \overline{s}\right]}\}n = 0$$

Hence, there exists an $\underline{n}$ such that $\forall n \leq \underline{n}$, $\pi_{1,PS/PS} \geq \pi_{1,OSS}$ and $\pi_{2,PS/PS} \geq \pi_{1,OSS}$. To sum up, we have estalished that if $n \leq \overline{n}$ or if $n \geq \overline{n}$, there exists an equilibrium where both firms adopt the proprietary approach. $\square$

Intuitively, code release makes sense only if a firm can derive some indirect monetary reward, which in our model is the complementarity profit. Given the complementarity coefficient is negligible or zero, the indirect reward does not exist and neither firms have the incentive to release codes. On the other hand, if the indirect reward is high, firms want to make sure they have good marketshare. The analysis in the PS/OSS subgame revealed that if a PA firm competes with the OSC in this high indirect reward context, the PA firm will provide their product free of charge and will have maximum marketshare while the OA firm receives nothing. To avoid this situation, neither firm 1 or firm 2 wants to release their source code.

The second observation is that the competency of the open source community also affects the firms' choice between the open source approach and the proprietary approach. If the OSC is ineffective in developing codes, for example, $\overline{s} = 0$, neither firms would want to release their source code regardless of the complementarity coefficient n. If the OSC is sufficiently effective in producing the open source software, then firms may want to choose the open source approach when the complementarity coefficient is reasonably high.

Proposition 10. *If there exists an n such that*

$$\min\left\{\pi_{1,PS/PS}, \pi_{2,PS/PS}\right\} < n$$

then there exists $\overline{S}$ such that for all $\overline{s} \geq \overline{S}$, the game has an equilibrium in which both firms adopt the open source approach.

Proof. Fix an n such that

$$\min\left\{\pi_{1,PS/PS}, \pi_{2,PS/PS}\right\} < n.$$

Now, consider $\overline{S}$ defined in corollary 1. For all $\overline{s} \geq \overline{S}$, suppose firm 1 chooses the open source approach in the first stage. Firm 2 can either choose the proprietary approach or the open source approach.

- If firm 2 chooses the OA, then its source code is abandoned and the firm receives zero complementary profit.

- If firm 2 chooses the PA, then since $\overline{s} \geq \overline{S}$, firm 2 can not profitably compete against the OSS and its optimal strategy is to be inactive ($s^0_{2,} = 0$).

Similarly, if firm 1 chooses the proprietary approach and firm 2 chooses the open source approach, then for $\overline{s} \geq \overline{S}$, firm 1 can not profitably compete against the OSS and their optimal strategy is to be inactive ($s^0_1 = 0$). The game is summarized in the following normal form:

			Firm 2
		PA	OA
Firm 1	PA	$\pi_{1,PS/PS}, \pi_{2,PS/PS}$	$0, n$
	OA	$n, 0$	$n, 0$

Since $\min\left\{\pi_{1,PS/PS}, \pi_{2,PS/PS}\right\} < n$, at least one firm wants to deviate from playing PA. If $\pi_{1,PS/PS} < n$ then playing OA is the dominating strategy for firm 1 and {OA,OA} is an equilibrium of the game. If $\pi_{1,PS/PS} \geq n$ then $\pi_{2,PS/PS} < n$ and if firm 1 chooses PA, firm 2's best response is playing OA and the corresponding payoff for firm 1 is zero. Hence {OA,OA} is the equilibrium of the game. Therefore, given the above n and $\overline{s} \geq \overline{S}$, both firms adopt the open source approach in equilibrium.

□

By proposition 4, the more efficient firm may prefer the proprietary approach given the less efficient firm adopting the same approach ($\pi_{1,PS/PS} \geq n$). However, since the reward from switching to the open source approach is more appealing for the less efficient firm, this firm prefers to abandon the proprietary approach ($\pi_{2,PS/PS} < n$). This, in turn, prompts the more efficient firm to adopt the open source approach. This effect resembles the bandwagon effect in a dynamic setting and it is reasonable to conjecture that such bandwagon effect can be established in a dynamic model given relevant ranges of the parameters.

The third observation is that the efficiency gap between the two firms also has a significant impact on the outcome in the first stage. Suppose the constants $n, \overline{s}$ are such that the game does not fall into either one of the two scenarios characterized in proposition 3 and proposition 4. Suppose also the function $F_1(s)$ is unchanged and consider the function $F_2(s, m)$ with the assumptions that $\frac{\partial F_2}{\partial m} > 0$, $m \geq 1$, $F_2(s, 1) \geq F_1(s)$, and $\lim_{m \to 0} F(s, m) = \infty$, in addition to the original assumptions stated in the model specification. With this structure, firm 2 becomes less efficient as m increases. Assume that with $m = 1$, the subgame PS/PS is on equilibrium path and consider the case $m > 1$. Proposition 2 establishes that $\pi_{2,PS/PS} > 0$ no matter how large m is (as long as the assumption $F_2(2s, m) < s$ $(k = 1, 2)$ for small s still holds). Even if this assumption does not hold, firm 2 still receives strictly positive profit in the PS/PS subgame unless firm 1 charges zero price. It is because given any $m > 0$, firm 2 can choose a quality $\xi(m) > 0$ so small that the corresponding price $p_2 = 0$ and the production cost $F_2(\xi(m), m)$ is negligible since $F_2(0, m) = 0$ and $\lim_{s \to 0} F_2'(s, m) = 0$ by assumption. With quality $\xi(m) > 0$, $p_2 = 0$, and $F_2(\xi(m), m)$ negligible, firm 2 gains strictly positive marketshare which is bounded away from zero if $p_1 > 0$. Thus, if $p_1 > 0$, then $\pi_{2,PS/PS} > 0$ regardless of firm 2's efficiency. In the limit, the first order condition specifying firm 1's optimization given m goes to infinity is

$$F_1(s_1^p) = \frac{1}{4} - \frac{3n^2}{16(s_1^p)^2}. \tag{2.3.10}$$

The corresponding profit for firm 2 is

$$\pi_{2,PS/PS} = \frac{p_1}{s_1} n = \frac{1}{2} n - \frac{3n^2}{4s_1}. \tag{2.3.11}$$

If firm 2 takes the OA, its earns a profit

$$\pi_{2,OSS} = \frac{1}{2}n - \frac{n^2}{2\left(s_1^0 - \overline{s}\right)},$$

where s_1^0 solves

$$F_1^{'}\left(s_1\right) = \frac{1}{4} - \frac{n^2}{4\left(s_1 - \overline{s}\right)^2}. \tag{2.3.12}$$

Firm 2 will select the OA if $\pi_{2,OSS} \geq \pi_{2,PS/PS}$, which is equivalent to

$$2s_1^p < 3\left(s_1^0 - \overline{s}\right).$$

Note that $\pi_{2,PS/PS} > 0$ only if $p_1 > 0$ which is possible only if $n < \frac{2}{3}s_1^p$. If $s_1^p \leq \frac{3n}{2}$, firm 2 will receive zero payoff if it chooses the proprietary approach since firm 1 will offer a higher quality software with zero price. Together with the conclusion that firm 2 will select the OA if $2s_1^p < 3\left(s_1^0 - \overline{s}\right)$, we have the following proposition:

Proposition 11. *Suppose $\pi_{2,OSS} > 0$, $\pi_{1,PS/OSS} > n$ and consider the case when firm 2's production costs increase infinitely. If $s_1^p \leq \frac{3n}{2}$ then firm 2 chooses the OA while firm 1 chooses PA in equilibrium. If $s_1^p > \frac{3n}{2}$ and $2s_1^p < 3\left(s_1^0 - \overline{s}\right)$, then firm 2 also chooses the OA while firm 1 chooses PA in equilibrium.*

By proposition 5, when the efficiency gap between the two firms are large, the less efficient firm may not be able to profitably compete against the more efficient firm and may want to drop the proprietary approach in favor of the open source approach. The efficiency of the open source community, however, does not pose a strong threat to the more efficient firm. The more efficient firm can still use the proprietary approach, producing high quality software and making a profit. This is intuitively consistent with the actual competition between Sun Microsystem vs. Microsoft in the market for office suite. StarOffice did not get any significant marketshare until its source code was released in July 2000. Soon after the introduction of OpenOffice.Org version 2.0 by the open source community, the new OSS has gained significant marketshare against Microsoft Office and Sun has been able to get some profit from selling a commercial version of OpenOffice.Org. However, Microsoft Office is still the dominating software in this market and Microsoft has no reason to release their software's source code, at least in the foreseeable future.

2.4 Conclusion

Open source approach is a relatively new production method which is only available in the digital age. By using this business strategy, a profit-maximizing firm can substantially cut its production costs related to source code development. However, after releasing a software's source code, the firm no longer own the code. The open source community will generally develop an open source software based on the released code and make it available to consumers free of charge. The firm, however, can provide complementary services (RedHat, Sun Microsystem) or promote sales of related products (IBM, HP). Aside from rivals' strategies, a firm's decision to release their software's source code ultimately depends on (1) how effective the open source community is in producing a high quality open source software; (2) the complementarity coefficient linking the software's marketshare with the profitability of other businesses conducted by the firm; and (3) the efficiency gaps among the involved firms in the software market. This paper develops a theoretical model in which firms choose production approaches, quality levels and pricing strategies in a Bertrand competition framework for differentiated products. The main findings can be summarized as follows:

- If releasing code has little complementarity effect on the profitability of other businesses undertaken by the firms, then the proprietary approach is more desirable. Moreover, if the effect is too strong, then firms will not risk abandoning their source codes and will also choose the proprietary approach.

- If the complementarity effect is neither too strong nor too weak, then profit-maximizing firms may consider adopting the open source approach. If the open source community is competent at producing high quality open source software, then every involved firms may adopt the open source approach. In this case, a firm's motivation to adopt the open source approach may either be driven by the possibility of higher profit or by fear that the rival firm may release the code and it is not profitable to compete against a free, high quality open source software.

- If the complementarity effect and the OSC's competency are neither too strong nor too weak, then the decision to adopt the open source approach depends on the efficiency gap between the two firms. Facing a large efficiency gap, the less efficient firm may find it more profitable to release their source code than to compete head-on with the more efficient firm. Meanwhile, the potential competition from a free but low quality OSS is not significant to make the more efficient firm abandoning the proprietary approach.

Chapter 3

Organic Search, Sponsored Advertisement and Quality of Search Engines

3.1 Introduction

Search engine advertising (keyword advertising/sponsored ads) was first introduced by GoTo (later renamed as Overture Services) in 1998. Yahoo! acquired Overture in 2003 and re-branding it to Yahoo! Search Marketing in 2005. Google adopted the business model and modified it to incorporate click feedback in 2002. Since its birth, the search engine advertising has become a phenomenally successful business model with the combined revenue of the two industry leaders Yahoo! and Google exceeding \$11 billion in 2005. The simple auction mechanisms utilized by Google and Yahoo have captured great interest from auction theorists. Edelman et al. (2005), Varian (2006) and Liu et al. (2006) among others offer different theoretical models in explaining the efficiency of these mechanisms.

The general approach utilized by the existing literature on keyword auctions is to analyze a game of assigning N agents to M slots, $N < M$ (Varian (2006)). Each agent is assumed to have a fixed valuation of each slot. The equilibrium outcome is generally such that the bidder with highest valuation get the first slot, the bidder with the second highest valuation gets the second slot, and so on. Little attention has been paid in understanding the relationship between this auction mechanism and retailer's pricing strategies in the market.

The Internet has been widely recognized as a platform which may allow consumers to reduce search cost and narrow down price dispersion. However, the Internet can not completely remove search cost. Even with the most sophisticated search technology, shopping for an item online is still a challenging task. In fact, the underlying reason for an online retailer to bid for a sponsored link is to attract consumers who may otherwise not find his online store. Unlike the traditional sequential search pattern, a search on search engine (SE) always returns the results as a list with retailers being ranked in fixed order. A firm on the top of the list is more accessible to consumers than a firm on the lower part of the list. Because it is quite common that search engines return hundreds of thousands of links for each query, it is unlikely that consumers can find retailers in the lower part of the list. Being on the top of the lists means that consumers can find the "incumbents" easily and these retailers can use this advantage to charge more. Retailers with low ranking have no choice but to reduce prices in order to attract consumers with low search cost. Arbatskaya (2006) analyzed a model in which consumers search in a fixed order and showed that firms with better position charge more in equilibrium. Her findings can be applied to the search engine environment where each online retailer is assigned to a rank which he can not change.

With the introduction of keyword advertisement, however, firms can effectively change their ranks quickly by having their names displayed in the sponsored section. Since new entrants often find themselves buried under a large number of links, improving rank is essential for survival. The keyword auction provides a good mechanism for the new entrants to quickly move up on the list and make themselves more accessible to consumers. Meanwhile, established and well-known firms often have their links placed on the very top of the list. However, it is common that these firms also advertise on the sponsored section. Their incentive to advertise comes partly from trying to preempt new entrants from moving up.

While the ordered search model and the keyword auction models introduced above provide insights into the nature of retail markets on search engines, an incorporation of these two models will provide a deeper understanding of the pricing and bidding strategies faced by online retailers. To the literature on search, such incorporation can answer the question of whether price dispersion still exists and in what form. To the literature on keyword auctions, such incorporation endogenizes the bidders' valuations and provides insights into the relationship between pricing and bidding. On the practical side, such incorporation provides strategic guideline to incumbents and entrants alike in the retail market on search engines.

Our paper characterizes a duopoly model with sequential search and a sponsored ads auction. In a homogeneous goods market, there are two retailers, an "incumbent," who possesses a better virtual location, and an entrant. Both retailers announce their prices in the first stage, bid for the right to display their links in the sponsored section in the second stage and face Bertrand competition in the last stage. Consumers are different in terms of searching efficiency. We show that in some cases, the game exhibits an equilibrium in pure strategies in which the incumbent charges a higher price, becomes the winner in the auction and successfully preempts the entrant from moving up on the list. The entrant charges a lower price and becomes the provider of the residual market for those with low search costs. We also show that in some circumstances, the game has an unique equilibrium in mixed strategies in which both firms randomize the price they want to charge in the first stage.

Our model characterizations and analytical findings are consistent with recent reports by various empirical works. For example, in modeling consumer's behavior, we take into account that consumers are different in terms of search cost. Haring (2004) conducted a survey which shows that users click-through to just a few links of the results pages and that nearly 70% of the users do not examine more than the first two pages of results, corresponding to the first 20 hits. Machill et al. (2003) shows that 81% of the users evaluated only the first page of results, further 13% the first and second pages, implying that only 6% considered more than the first 20 entries. Our finding about price dispersion is consistent with reports by Baylis and Perloff (2002), Smith and Brynolfsson (2001), and Ellision and Ellision (2001). The pure equilibrium characterized in our paper is consistent with the empirical evidence provided by Baylis and Perloff (2002). These authors studied the online market for cameras and found also that firms' price-rank ordering is consistent over time. The authors argued that such a rigid pattern contradicts the mixed equilibrium found in the theoretical literature of sequential search. Haring (2004) studied the online market for contact lenses and found that firms with higher organic ranks tend to charge higher prices than firms with lower organic ranks. The author suggests that online retailers with a superior search engine rank are able to exploit this prominent position through price mark-ups. This possibility is also suggested by Smith et al. (2000). Preemptive advertising found in our model is consistent with empirical findings by Animesh et al. (2005). They found that in markets for "search goods"[1], retailers with higher ranks bid more aggressive than retailers with lower ranks. These authors did not offer any explanation of why such behavior exists in the retail market on search engine.

Our paper relates to two different sets of literature. the first set is on consumer search, information gatekeepers and price dispersion. The other one is on keyword auctions. The literature on consumer search is vast. However, directly related to our paper is the ordered search model by Arbatskaya (2006). Arbatskaya studies a search model where consumers search with a fixed order. She shows that the equilibrium prices and profits decline in the order of search. Our paper also consider retailers listed in a fixed order, but in our model, retailers can bid for the sponsored ads slot and upset the existing order. Baye and Morgan (2001) and Baye and Morgan (2002) consider the role of information gatekeepers on price dispersion and show that the gatekeeper's profits are maximized in an equilibrium where the product market exhibits price dispersion. The information gatekeepers characterized in their papers are shopbots which are different from generic search engines characterized in our paper. We also benefited from the discussion in Edelman et al. (2005), Varian (2006) and Liu et al. (2006) about the nature of keyword auctions.

[1]The authors used the SEC framework to catergorize products. According to the SEC framework, attributes of goods can be analyzed in terms of three properties – search, experience, and credence. Search goods have characteristics that are identifiable through inspection and prior to purchase. Our model considers a homogenous goods which consumers have public knowledge of its value. Thus, the type of products considered in our model matches perfectly with the definition of search good in Animesh et al.

Section 2 of this paper provides the formal characterization of the model and reports main findings. Section 3 provides a short summary and discussion about directions for future research. All the technical proofs are presented in the Appendix.

3.2 The Model

3.2.1 Description of Search

Consider a duopoly model with two online retailers each selling the same homogeneous non-durable goods and facing no capacity constraints. The marginal cost is constant and the same for all retailers. Without loss of generality, we assume that the marginal cost is zero. There is a continuum of consumers normalized to 1. Consumers have unit demands and have an identical valuation, v, of the good, v being the maximum price a consumer would ever be willing to pay for the good.

In order to make purchase decision, consumers need to locate the retailers by using a monopolist search engine. To conduct a search, consumers first enters a query, and then the search engine will then generate two reference lists. One is the *sponsored* list, on which retailers pay for position. The other is the *organic* list, which is automatically generated by the search engine's algorithm[2]. We assume that consumers are equally efficient in using the search engine. Particularly, we assume that all consumers can obtain these reference lists with zero cost.

For each set of keywords, there is a unique organic list which is the same to every consumers. Each retailer has only one link shown in the organic list. The size of the organic list is L ($2 \leq L < \infty$) and the list contains both *relevant* and *irrelevant* links. A link is relevant if it belongs to one of the two retailers and is irrelevant otherwise. Without further loss of generality, we assume one retailer is an established incumbent and with the link placed on the top of the organic list.

[2]The appendix contains a visual demonstration of the organic list and a sponsored list.

The other retailer is a new entrant with the link placed in the bottom of the organic list. Define the quality of the search engine as the proportion of relevant links in the organic list. A search engine has highest quality if the organic list contains only relevant links ($L = 2$). The search engine firm can increase or decrease the value of L at will to maximize its revenue.

The sponsored list is assumed to contain only one link and is placed on the top of the organic list. In the second stage, the link is sold to retailers in a second price sealed auction. The highest bidder gets the sponsored slot but pays the seller the value of the second highest bid.

3.2.2 The Timeline of the Game

The game has three stages. In the first stage, retailers simultaneously choose retail prices. Prices announced in the first stage can not be changed in the second stage. In the second stage, retailers bid against each other for the sponsored slot. Each bidder specifies how much he would want to pay the SEF for each time a consumer clicks his link. If there is a draw, then the sponsored slot is randomly assigned to one of the two bidders. In the last stage, consumers conduct searches make purchase decision. Consumers have unit demands and have full information about prices charged by the retailers[3]. However, the consumers can not access the retailer's websites directly and have to locate them using the search engine. The equilibrium concept referred to in the following analysis is subgame perfect equilibrium (SPE).

[3]The model is somewhat similar to the following spartial model: Consider two gas stations positioning on the same highway comming through a town. Towners who need gas can either go to the first station, which is closer to the town, or to the other one, which is further away from the town. The prices charged by the two gas stations are public knowledge. A real estate company has just built a new gas station which is even closer to the town than either of the existing stations. The gas retailers in the two existing stations can bid against each other for the right to rent the newly erected facility but they can not change the announced price.

3.2.3 Consumers' Search and Demand Functions

In the last stage consumers base on the results returned by the search engine and choose links they want to visit. We assume that consumers know the size of the result list and the exact locations of the two relevant links. Consumers are differentiated in terms of search cost. For a consumer of type θ, going through a list of k links costs him a search cost of θk, θ is uniformly distributed on the support $\left[0, \overline{\theta}\right]$, $\overline{\theta} < 1$. Thus, the density of θ is $\frac{1}{\overline{\theta}}$.

As we assumed above, the incumbent occupies the top position in the organic list and the entrant occupies the bottom position of the organic list. Without the auction, the search cost for a consumer of type θ is zero if she wants to visit the incumbent and $\theta\left(L-1\right)$ if she wants to visit the entrant. We assume that $v > 2\overline{\theta}$: if the entrant's link is placed third from top-down, then given the entrant charges zero price, even the consumer with highest search cost can still find it worthwhile to purchase from the second retailer.

With the presence of the auction, however, her search costs depend on which bidder won the auction in the second stage. If the incumbent won the auction, his position is still on the top of the result list and consumers can visit him zero search cost. If the entrant won the auction, then the entrant occupies the highest position and the incumbent is pushed down to the second highest position. Thus, consumer θ's search cost is θ if she wants to visit the incumbent and zero if the wants to visit the entrant.

Let the prices charged by the retailers be p_1 and p_2 where p_1 is the price of the incumbent. The demands for the incumbent and the entrant in the last stage depend on which bidder won the auction in the second stage. There are two scenarios:

The incumbent won the auction:

A consumer of type θ receives a net payoff $v-p_1$ if she purchases from the incumbent and $v-p_2-\theta L$ if she purchases from the entrant. Thus, a consumer θ is indifferent between purchasing from the incumbent and from the entrant iff $v-p_2-\theta L = v-p_1$, or $\theta = \frac{p_1-p_2}{\bar{\theta}L}$. The third-stage demand functions for the incumbent and the entrant are (figure 1):

$$q_1(p_1,p_2) = \begin{cases} 1 & \text{if } p_1 \leq \min\{p_2, v\} \\ 1-\frac{p_1-p_2}{\bar{\theta}L} & \text{if } p_2 \leq p_1 \leq \min\left\{\bar{\theta}L+p_2, v\right\} \\ 0 & \text{otherwise} \end{cases} \tag{3.2.1}$$

and

$$q_2(p_1,p_2) = \begin{cases} 1 & \text{if } p_2 < \min\left\{v, p_1-\bar{\theta}(L-1)\right\} \\ \frac{p_1-p_2}{\bar{\theta}L} & \text{if } p_1-\bar{\theta}(L-1) \leq p_2 \leq \min\{v, p_1\} \\ 0 & \text{otherwise} \end{cases} \tag{3.2.2}$$

respectively.

The entrant won the auction:

Since the entrant won the auction, his link is placed on the top of the organic list and the incumbent's link is placed second from top-down. A consumer of type θ receives a net payoff $v-p_2$ if she purchases from the entrant and $v-p_1-\theta$ if she purchases from the incumbent. As a consequence, if the incumbent's price is higher, then no consumer would want to visit his shop. The second-stage demand functions for the incumbent and the entrant are

$$q_1(p_1,p_2) = \begin{cases} \frac{1}{\theta}(p_2-p_1) & \text{if } p_1 \leq \min\{p_2, v-\theta\} \\ 0 & \text{otherwise} \end{cases} \tag{3.2.3}$$

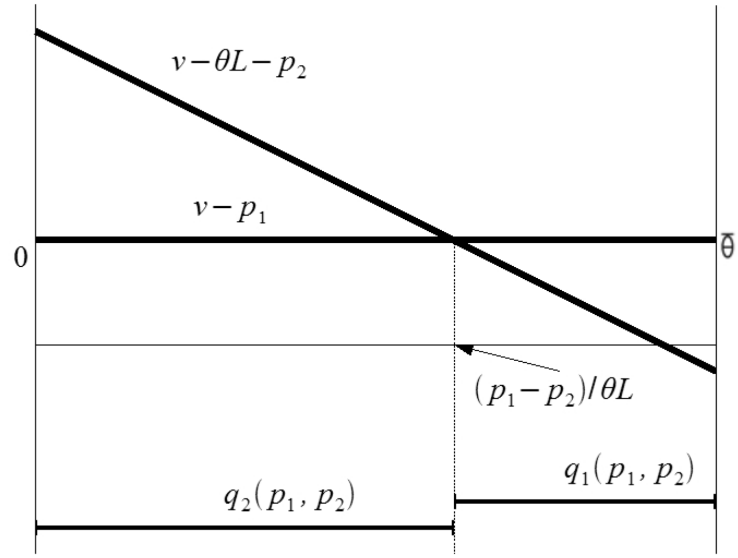

Figure 3.1: The incumbent won the auction in the second stage and serves the segment of consumers with high search cost. The entrant serves the segment of consumers with low search cost.

and

$$q_2(p_1, p_2) = \begin{cases} 1 & \text{if } p_2 \leq \min\{p_1, v\} \\ 1 - \frac{1}{\theta}(p_2 - p_1) & \text{if } p_1 < p_2 \leq v \\ 0 & \text{otherwise} \end{cases} \tag{3.2.4}$$

respectively.

3.2.4 Equilibrium Bids

In the second stage, the two retailers bid against each other for one slot of sponsored link. Each bidder submits a bid which specifies how much money he would want to pay the SEF for one click made by the consumers (b_i). The marginal revenue for each additional consumer purchase for bidder i is p_i. Since neither of the bidders can change the price announced in the first stage, bidder i's equilibrium bid is not higher than p_i ($b_i \leq p_i$). To derive bidders' valuations and equilibrium bids, consider two scenarios:

The incumbent charges a higher price ($p_1 > p_2$):

We have established in the above section that if $p_1 > p_2 + \overline{\theta} L$ or if $p_1 > v$, then the incumbent would have zero demand in the last stage. Since setting such a high price is not optimal for the incumbent, suppose that $p_2 < p_1 \leq \min\left\{\overline{\theta}\left(L-1\right) + p_2, v\right\}$. Before we derive the equilibrium bids, it is worthwhile to note that factor to determine bidders' valuations of the sponsored link is the fact that $p_1 > p_2$.

- If the incumbent loses the auction, the entrant's link become more accessible and since the incumbent's price is higher, no consumer would want to visit the incumbent. As a consequence, the incumbent payoff is zero if he loses the auction. In the case he wins, he gets p_1 from each unit sold to a consumer. Thus, the incumbent's maximum bid for each consumer click is p_1.

- Determining the entrant's valuation for each consumer click is more challenging. If the entrant wins the auction, every consumer will purchase from him. Thus, his revenue is p_2. If he loses the auction, he can still sell to the consumers with low search cost and get a net payoff

$$\pi_2\left(w_1\right) = \left(p_1 - p_2\right) p_2 / \overline{\theta} L.^4 \tag{3.2.5}$$

  When the entrant wins, he can not subtract the number of consumers who would purchase from him even if he lose the auction,[5] out of the total number

[4] In this formula, the notation w_1 in $\pi_2\left(w_1\right)$ stands for the fact that retailer 1 (the incumbent) winning the auction. We will use the similar notation w_2 later in the case the winner is the entrant.

[5] For example, the entrant receives 1000 visits if he does not win the auction and 2000 visits if he wins the auction. The problem here is that if he wins the auction, all of the 2000 consumers will visit him via the link displayed in the sponsored section. The search engine firm charges the entrant for EACH consumer visit via the link displayed in the sponsored section. Thus, the entrant has to pay the SEF for the total of 2000 visits. He can not subtract 1000 visits–which he would receive even if he does not win–out of the total 2000 visits.

of consumers visiting his link. Thus, although he receives only an extra number of $1-(p_1-p_2)/\overline{\theta}L$ visits in the case of winning, he has to pay the auctioneer for all 1 visits. Thus, his valuation of each click on the sponsored link is his total *extra* payoff gained via the sponsored link divided by total visits in the case of winning:

$$\overline{b}_2 = p_2[1-\frac{p_1-p_2}{\overline{\theta}L}]. \tag{3.2.6}$$

Since the winner pays the seller the second highest bid, neither of the bidder would want to bid lower than his reservation. As a consequence, the equilibrium bids are such that if $p_2 < p_1$, $b_2 = \overline{b}_2$ and $b_1 = p_1$.

The entrant charges a higher price ($p_2 \geq p_1$):

Since no retailer would want to charge higher than v in equilibrium, we consider only the case $p_1 < p_2 \leq v$. If the entrant charges higher than $p_1+\overline{\theta}$, then even if he won the auction, his sale would still be zero. Therefore, suppose $p_1 \leq p_2 \leq \min\left\{p_1+\overline{\theta}; v\right\}$. If the entrant loses the auction, he will receive no consumer visit and zero payoff. If he wins the auction, he gets p_2 for each consumer click. Thus, his valuation for each click is p_2.

On the other hand, if the incumbent wins the auction, he serves the whole market and get a total revenue of p_1. If he loses the auction, his payoff is $\pi_1^2(w_2) = (p_2-p_1)\,p_1/\overline{\theta}$. By the same argument applied above in the derivation of entrant's equilibrium bid in the case $p_1 > p_2$, we can derive the incumbent's equilibrium bid in this case as:

$$\overline{b}_1 = p_1[1-\frac{(p_2-p_1)}{\overline{\theta}}]. \tag{3.2.7}$$

Since the winner pays the seller the second highest bid, neither of the bidder would want to bid lower than his reservation. As a consequence, the equilibrium bids are such that if $p_2 > p_1$, $b_1 = \overline{b}_1$ and $b_2 = p_2$.

From both scenarios above, its not difficult to see that the winner is always the retailer who charges higher price. We have the following lemma:

Proposition 12. *The equilibrium bid function for the incumbent is* $b_1 = p_1$ *if* $p_1 > p_2$ *and* $b_1 = \bar{b}_1$ *if* $p_1 < p_2$*. The equilibrium bid function for the entrant is* $b_2 = p_2$ *if* $p_2 > p_1$ *and* $b_2 = \bar{b}_2$ *if* $p_2 < p_1$*. The retailer announcing a higher price in the first stage wins the auction in the second stage.*

Proof. The above discussion established that if $p_i > p_{-i}$, then $b_i = p_i$ and $b_{-i} = \bar{b}_{-i}$. Thus, the first part of the proposition follows naturally. For the second part, we need to show that if $p_i > p_{-i}$, then $b_i > b_{-i}$. In other words, we need to show that $p_i > \bar{b}_{-i}$.

First, suppose that $p_2 \geq p_1$. According to (3.2.7), $p_2 > \bar{b}_1$ is equivalent to:

$$p_1[1 - \frac{(p_2 - p_1)}{\bar{\theta}}] < p_2,$$

which can be rearranged as

$$1 - \frac{(p_2 - p_1)}{\bar{\theta}} < \frac{p_2}{p_1}. \tag{3.2.8}$$

Since $p_2 > p_1$, the right hand side is greater than one and the left hand side is smaller than one. Thus, (3.2.8) is correct. In other words, $p_2 > \bar{b}_1$. Since the the entrant has higher valuation, and since both bidders bid his true valuation, the entrant wins the auction and pays the seller the price which is equal to $\bar{b}_1$.

The proof for the case $p_1 > p_2$ can be shown in the same way. $\square$

3.2.5 Equilibrium Pricing

In the first stage, the retailers simultaneously choose prices, taking into account that announced prices can not be changed in the following stages and whoever charges higher price will win the auction in the second stage. To solve for equilibrium prices and profits, we first establish that the subgame perfect equilibrium exhibits price dispersion in pure strategies. The result is similar to the price dispersion in pure strategies found by Arbatskaya (2006). However, in Arbatskaya (2006), retailers have fixed positions (for example, in a oriental bazaar) and can not change their relative position via position auction. Thus, it is quite intuitive in her model that firms with more advantageous position charge higher prices. In our model, the retailers interact in a three-stage game and the entrant has an option to reverse the existing order by charging a higher price in the first stage and winning the position auction in the second stage. Thus, it is not obvious in the context of our model that the entrant always charge lower price.

Lemma 5. *(existence of price dispersion) In equilibrium, $p_1^* \neq p_2^*$.*

Proof. Suppose otherwise that the incumbent and the entrant charge the same price, $p_1^* = p_2^* = p \leq v$. The retailers have equal chance of winning in the second stage and the equilibrium bids are such that $b_1 = b_2 = p$. Each retailer receive zero profit.

- If $p < v$, the entrant is strictly better of to deviate. To see that suppose the entrant charges $p_2 = p + \varepsilon$, $0 < \varepsilon < v - p$. Since $p_2 > p_1^*$ and by proposition 1, the entrant wins the auction and receives a net payoff

$$[1 - \frac{1}{\overline{\theta}}(p_2 - p_1^*)]\,(p_2 - b_1) = (1 - \frac{\varepsilon}{\overline{\theta}})\varepsilon. \tag{3.2.9}$$

  Meanwhile, if the entrant does not deviate, he receives zero payoff. Since $\overline{\theta}$, p, and v are all fixed, define $\widehat{\varepsilon}$ by

$$0 < \widehat{\varepsilon} < \min\left\{\overline{\theta}, v - p\right\}$$

  Since $\overline{\theta} > 0$ and $v - p > 0$, such $\widehat{\varepsilon}$ exists. Given $\widehat{\varepsilon}$, if the entrant deviates and plays $p_2 = p + \widehat{\varepsilon}$, his deviated payoff is strictly positive.

- If $p = v$, the incumbent is strictly better off to deviate. To see that suppose the incumbent charges $p_1 = p - \varepsilon$, $\varepsilon > 0$. Since $p_1 < p_2$, the incumbent loses the auction and does not have to pay the auctioneer. His deviated payoff is

$$[\frac{1}{\overline{\theta}} (p_2 - p_1^*)]p_1 = \frac{\varepsilon}{\overline{\theta}} (p - \varepsilon)$$

Since $\overline{\theta}$, p, and v are all fixed, define $\widetilde{\varepsilon}$ by

$$0 < \widetilde{\varepsilon} < \min \{p, v - p\}$$

Since $p > 0$ and $v - p > 0$, such $\widetilde{\varepsilon}$ exists. Given $\widetilde{\varepsilon}$, if the entrant deviates and plays $p_2 = p - \widetilde{\varepsilon}$, his deviated payoff is strictly positive.

In other words, at least one retailer always find it profitable to deviate from $p_1^* = p_2^*$. □

Proposition 1 and Lemma 2 imply that if a pure strategy equilibrium exists, then there are only two possible scenarios. The first scenario involves the incumbent charges a higher price and wins the auction. The second scenario involves the entrant charges a higher price and wins the auction in the second stage.

Scenario 1: The entrant charges a higher price ($p_2^* > p_1^*$)

Note first that it is not optimal for either retailer to raise the price over v. Note also that it is not optimal for the entrant to raise price higher than $\overline{\theta} + p_1$ because doing so would lead to zero demand even if the entrant wins the auction. Consider the case where $p_1 < p_2 \leq \max \left\{p_1 + \overline{\theta}; v\right\}$. By Lemma 1, the entrant wins the auction and pays the SEF $\overline{b}_1$. In the third stage, the entrant receives $1 - (p_2 - p_1) / \overline{\theta}$ visits and his total profit is

$$\pi_2(p_1,p_2,w_2) = \left(p_2 - \bar{b}_1\right)\left[1 - \frac{p_2 - p_1}{\overline{\theta}}\right], \tag{3.2.10}$$

where w_2 stands for the fact that the winner is the entrant. (3.2.10) is equivalent to

$$\pi_2(p_1,p_2,w_2) = \left[p_2 - p_1 + \frac{(p_2 - p_1)\,p_1}{\overline{\theta}}\right]\left[1 - \frac{p_2 - p_1}{\overline{\theta}}\right]. \tag{3.2.11}$$

Meanwhile, the incumbent loses the auction, his link is pushed down to the second place and attracts only consumers with lower search cost. The incumbent's total profit is

$$\pi_1(p_1,p_2,w_2) = \frac{1}{\overline{\theta}}(p_2 - p_1)\,p_1. \tag{3.2.12}$$

Maximizing (3.2.11) with respect to p_2 yields the unique solution $p_2 = \frac{1}{2}\left(\overline{\theta} + 2p_1\right)$. Maximizing (3.2.12) with respect to p_1 yields the unique solution $p_1 = \frac{1}{2}p_2$. Substituting p_1 into p_2 leads to $p_1(w_2) = \frac{1}{2}\overline{\theta}$ and $p_2(w_2) = \overline{\theta}$. The corresponding profits for the incumbent and for the entrant are $\pi_1(p_1(w_2), p_2(w_2)) = \frac{1}{4}\overline{\theta}$ and $\pi_2(p_1(w_2), p_2(w_2)) = \frac{3}{4}\overline{\theta}$. respectively. Since $2\overline{\theta} \leq v$ by assumption, our conditions that $p_1 < v$ and $p_2 \leq v$ is met. Note that the prices and corresponding profits do not depend on the value of L. This result will have a crucial role later on in determining the equilibrium of the game.

Scenario 2: The incumbent charges a higher price $(p_1^* > p_2^*)$

Again, note that it is not optimal for the either of the retailer to raise the price over v. Also, it is not optimal for the incumbent to raise the price over $p_2 + \overline{\theta}L$ because doing so would lead to zero demand even if he wins the auction. Thus, consider the case where $p_2 < p_1 \leq \min\left\{p_2 + \overline{\theta}L, v\right\}$. By lemma 1, the incumbent wins the auction and pays the SEF the price which is equal to $\bar{b}_2$. Let us disregard the corner solutions for now. The entrant's total profit is

$$\pi_2(p_1, p_2, w_1) = \frac{p_1 - p_2}{\overline{\theta}L} p_2, \tag{3.2.13}$$

where w_1 stands for the fact that the winner is the incumbent. The incumbent's total profit is

$$\pi_1(p_1, p_2, w_1) = \left(p_1 - \overline{b}_2\right)\left[1 - \frac{p_1 - p_2}{\overline{\theta}L}\right], \tag{3.2.14}$$

which is equivalent to

$$\pi_1(p_1, p_2, w_1) = [p_1 - p_2 + \frac{(p_1 - p_2)\, p_2}{\overline{\theta}L}](1 - \frac{p_1 - p_2}{\overline{\theta}L}). \tag{3.2.15}$$

Maximizing (3.2.13) with respect to p_2 yields the unique solution: $p_2 = \frac{1}{2}p_1$. Maximizing (3.2.15) with respect to p_1 also yields the unique solution:

$$p_1 = p_2 + \frac{\overline{\theta}L}{2}. \tag{3.2.16}$$

Substituting $p_2 = \frac{1}{2}p_1$ into (3.2.16) leads to $p_1(w_1) = \overline{\theta}L$ and $p_2(w_1) = \frac{1}{2}\overline{\theta}L$. The corresponding profits are $\pi_1(p_1, p_2, w_1) = \frac{3\overline{\theta}L}{8}$ and $\pi_2(p_1, p_2, w_1) = \frac{\overline{\theta}L}{4}$. Since $\overline{\theta}L < \frac{3}{2}\overline{\theta}L$, the condition $p_1 \leq p_2 + \overline{\theta}L$ is satisfied. Let $\overline{L} = v/\overline{\theta}$. For all $L > \overline{L}$, $p_1(w_1) > v$ and for all $L < \overline{L}$, $p_1(w_1) < v$. The complete characterization of the optimal prices in scenario 2 takes the following form:

$$p_1(w_1) = \begin{cases} \overline{\theta}L & \text{if } L \leq \overline{L} \\ v & \text{if } L > \overline{L} \end{cases} \tag{3.2.17}$$

$$p_2(w_1) = \begin{cases} \frac{1}{2}\overline{\theta}L & \text{if } L \leq \overline{L} \\ \frac{1}{2}v & \text{if } L > \overline{L} \end{cases} \tag{3.2.18}$$

The analysis of the two scenarios above provides two candidates for the SPE. In order to establish that a pair of prices $\{p_1(w_i), p_2(w_i)\}$ is an equilibrium, we need to show that $p_1(w_i)$ is the global best response for the incumbent given the entrant charges $p_2(w_i)$ and $p_2(w_i)$ is the global best response for the entrant given the incumbent charges $p_1(w_i)$. In other words, we need to check that the high-price retailer does not want to leapfrog backwards and become low-price retailer. Similarly, the low-price retailer does not want to leapfrog and become the high-price retailer.

For example, the local optimal prices in scenario 1 above is $p_1(w_2) = \frac{1}{2}\overline{\theta}$ and $p_2(w_2) = \overline{\theta}$. This local optimal price is the equilibrium price only if given the entrant charges $\overline{\theta}$, the incumbent does not want to charge higher than $\overline{\theta}$; and given the incumbent charges $\frac{1}{2}\overline{\theta}$, the entrant does not want to charge lower than $\frac{1}{2}\overline{\theta}$.

We can establish one set of conditions in which we can rank p_1^* and p_2^*.

Proposition 13. *If $L \leq 6$, $\overline{\theta}L + \frac{1}{2}\overline{\theta} > v \geq \overline{\theta}L$ and*

$$7\left(v - 6\overline{\theta}\right)\left[1 - \frac{v - 6\overline{\theta}}{\overline{\theta}}\right] < \frac{3}{2}\overline{\theta},$$

then the game has a Nash equilibrium in pure strategies in which $p_1 = \overline{\theta}L$, $p_2 = \frac{1}{2}\overline{\theta}L$. The game does not have any Nash equilibria in pure strategies, otherwise.

Proof. See Appendix. □

Intuitively, the proof shows that the pair of prices $\left(\frac{1}{2}\overline{\theta}, \overline{\theta}\right)$ is too low and the incumbent always find it better off to deviate by charging a price higher than $\overline{\theta}$. Thus, there does not exist a pure equilibrium such that $p_2 > p_1$. The existence of a pure equilibrium in which $p_1 > p_2$ depends on the value of L, $\overline{\theta}$ and v. The discussion in section (2.5.2) revealed that the entrant charges $p_2(w_1) = \frac{1}{2}\overline{\theta}L$, increasing with respect to L. Thus, $\overline{b}_2$ is also increasing with respect to L. Since the incumbent is the winner, he has to pay the auctioneer an amount equal to $\overline{b}_2$ for each consumer visit. When L becomes sufficiently large, it is unprofitable for the incumbent to be the winner and he will have the incentive to bid lower than the entrant.

On the other hand, since $p_1(w_1) = \overline{\theta}L \leq v$, the entrant can raise the price over $\overline{\theta}L$. However, the entrant can not raise their price over v. Thus, if $\overline{\theta}L$ is sufficiently close to v, deviation is not worthwhile for the entrant. If $v - \overline{\theta}L$ is sufficiently large, deviation becomes profitable.

Proposition 3 has a direct implication about preemptive advertising. In the pure equilibrium characterized in proposition 3, the sponsored slot has no intrinsic value to the incumbent since he already on the top of the organic list and consumers can access his link easily. However, the incentive of the incumbent in winning the auction is to deter the entrant from moving up on the list. If the entrant won the auction, the incumbent would lose his positioning advantage. In equilibrium, this incentive happens to be strong enough to induce the incumbent to outbid the entrant and win the auction. This result is consistent with the empirical findings by Animesh et al. (2005) in markets for "search goods."

If the conditions in proposition 3 are not met, then the game has a unique mixed equilibrium. Let $F_1(p_1)$ and $F_2(p_2)$ be the cumulative distribution of prices for the incumbent and the entrant. Let $[\underline{p}_1, \overline{p}_1]$ be the support of F_1 and $[\underline{p}_2, \overline{p}_2]$ be the support of F_2. It is easy to verify that $\underline{p}_1 = \underline{p}_2 = \underline{p}$ and $\overline{p}_1 = \overline{p}_2 = \overline{p}$. To see that, suppose for example that $\underline{p}_1 > \underline{p}_2$. The entrant can not be indifferent between $p_2 \in [\underline{p}_2, \underline{p}_1]$ since $\forall p_2 \in [\underline{p}_2, \underline{p}_1]$, the incumbent's price is strictly higher than the entrant's price and the profit function for the entrant is characterized (3.2.13), which does not have a constant value on a non empty interval $[\underline{p}_2, \underline{p}_1]$.

Also, in the range $[\underline{p}, \overline{p}]$, $F_i(p)$ has no mass point, since otherwise the other seller could decrease (increase) its price around the mass point by an arbitrarily small amount and experience a discontinuous shift in profit. The support $[\underline{p}_2, \overline{p}_2]$ is also a interval with no gaps[6].

[6] To see that, note first that if there is a gap in the support of F_1, then the exact same gap should appear in the support of F_2 since otherwise the entrant will not be indifferent between two prices p_2 and $p_2^{'}$ when p_2 belongs to the gap of F_1 and $p_2^{'}$ is not. Given a gap in $[\underline{p}, \overline{p}]$, the chance of winning the auction is constant when a firm plays price levels at the lower end of the gap, in

Given the two functions $F_1(p_1)$ and $F_2(p_2)$ having a common support which is an interval $[\underline{p}, \overline{p}]$ and there is no mass points, the expected profit for the entrant is

$$\begin{aligned} \pi_2(F_1, F_2) &= \int_{p_2}^{\overline{p}} \frac{(p_1 - p_2) p_2}{\overline{\theta} L} f_1(p_1) \, dp_1 + \\ & \int_{\underline{p}}^{p_2} (p_2 - p_1)(1 + \frac{p_1}{\overline{\theta}})[1 - \frac{p_2 - p_1}{\overline{\theta}}] f_1(p_1) \, dp_1, \end{aligned} \tag{3.2.19}$$

and the expected profit for the incumbent is

$$\begin{aligned} \pi_1(F_1, F_2) &= \int_{p_1}^{\overline{p}} \frac{(p_2 - p_1) p_1}{\overline{\theta}} f_2(p_2) \, dp_2 + \\ & \int_{\underline{p}}^{p_1} (p_1 - p_2)(1 + \frac{p_2}{\overline{\theta} L})[1 - \frac{p_1 - p_2}{\overline{\theta} L}] f_2(p_2) \, dp_2. \end{aligned} \tag{3.2.20}$$

In order to find F_1 and F_2, we first examine the support $[\underline{p}, \overline{p}]$ of these two functions. If the incumbent plays $p_1 = \underline{p}$, then his payoff is

$$\pi_1(\underline{p}, F_2) = \int_{\underline{p}}^{\overline{p}} \frac{(p_2 - \underline{p}) \underline{p}}{\overline{\theta}} f_2(p_2) \, dp_2 = \frac{1}{\overline{\theta}} \underline{p} \left[E(p_2) - \underline{p} \right].$$

If the incumbent plays $p_1 = \overline{p}$, then his payoff is

$$\pi_1(\overline{p}, F_2) = \int_{\underline{p}}^{\overline{p}} (\overline{p} - p_2)(1 + \frac{p_2}{\overline{\theta} L})[1 - \frac{\overline{p} - p_2}{\overline{\theta} L}] f_2(p_2) \, dp_2$$

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the gap, and at the higher end of the gap. With a constant probability of wining, and constant expected values of the other firm's price, the profit function of retailer i has an inverted U-shape and the retailer i can not be indifferent between these three price levels.

Let $h_1(p)$ and $g_1(p)$ be defined by

$$\begin{aligned} h_1(p) &= \frac{1}{\overline{\theta}} p\left[E(p_2) - p\right], \\ g_1(p) &= \int_{\underline{p}}^{p} (p - p_2)\left(1 + \frac{p_2}{\overline{\theta} L}\right)\left[1 - \frac{p - p_2}{\overline{\theta} L}\right] f_2(p_2)\, dp_2. \end{aligned}$$

Since $h_1(p)$ is maximized at $p = \frac{1}{2}E_2(p_2)$ and $h_1(p)$ has inverted U-shape, F_1 is the optimal price scheme only if $\underline{p} \leq \frac{1}{2}E_2(p_2)$. Moreover, $g_1(p)$ is maximized at:

$$p = \frac{\overline{\theta}^2 L^2 + 3\left(\overline{\theta} L\right) E_2(p_2) + 2E_2\left(p_2^2\right)}{2\left(\overline{\theta} L + E(p_2)\right)},$$

and $g_1(p)$ has inverted U-shape, F_1 is the optimal price scheme for the incumbent only if

$$\overline{p} \geq \frac{\overline{\theta}^2 L^2 + 3\left(\overline{\theta} L\right) E_2(p_2) + 2E_2\left(p_2^2\right)}{2\left(\overline{\theta} L + E(p_2)\right)}. \tag{3.2.21}$$

Similarly, if retailer 2 plays $p_2 = \underline{p}$, his payoff is

$$\pi_2\left(\underline{p}, F_1\right) = \int_{\underline{p}}^{\overline{p}} \frac{\left(p_1 - \underline{p}\right)\underline{p}}{\overline{\theta} L} f_1(p_1)\, dp_1 = \frac{1}{\overline{\theta} L}\underline{p}\left[E(p_1) - \underline{p}\right],$$

and if he plays $p_2 = \overline{p}$, his payoff is

$$\pi_2(\overline{p}, F_1) = \int_{\underline{p}}^{\overline{p}} (\overline{p} - p_1)\left(1 + \frac{p_1}{\overline{\theta}}\right)\left[1 - \frac{\overline{p} - p_1}{\overline{\theta}}\right] f_1(p_1)\, dp_1.$$

By the same method , F_2 is the optimal price scheme only if $\underline{p} \leq \frac{1}{2}E_1(p_1)$ and

$$\overline{p} \geq \frac{\overline{\theta}^2 + 3\overline{\theta} E_1(p_1) + 2E_1\left(p_1^2\right)}{2\left[\overline{\theta} + E_1(p_1)\right]} \tag{3.2.22}$$

This discussion can be summarized in the following lemma:

Lemma 6. *In equilibrium, the upper bound and the lower bound of the support* $[\underline{p},\overline{p}]$ *are such that:*

$$\begin{aligned}
\underline{p} &\leq \min\left\{\frac{1}{2}E_1(p_1), \frac{1}{2}E_2(p_2)\right\} \\
\overline{p} &\geq \max\left\{\frac{\overline{\theta}^2 + 3\overline{\theta}E_1(p_1) + 2E_1\left(p_1^2\right)}{2\left[\overline{\theta} + E_1(p_1)\right]}, \frac{\overline{\theta}^2 L^2 + 3\left(\overline{\theta}L\right)E_2(p_2) + 2E_2\left(p_2^2\right)}{2\left(\overline{\theta}L + E(p_2)\right)}\right\}
\end{aligned}$$

Since F_1 and F_2 are equilibrium pricing strategies for the incumbent and the entrant, any price $p \in \left(\underline{p},\overline{p}\right)$ should satisfy the following conditions

$$\begin{aligned}
\pi_1(p_1, F_2(p)) &= \pi_1(p_1^{'}, F_2(p)),\ \forall p_1, p_1^{'} \in [\underline{p},\overline{p}], \\
\pi_2(F_1(p), p_2) &= \pi_2(F_1(p), p_2^{'}),\ \forall p_2, p_2^{'} \in [\underline{p},\overline{p}].
\end{aligned}$$

which essentially saying that $\pi_1(p_1, F_2(p))$ is a constant for all $p_1 \in [\underline{p},\overline{p}]$ and $\pi_2(F_1(p), p_2)$ is also a constant for all $p_2 \in [\underline{p},\overline{p}]$. In other words, the following first order condition (3.2.23) must hold for all $p_1 \in [\underline{p},\overline{p}]$ and condition (3.2.24) must hold for all $p_2 \in [\underline{p},\overline{p}]$.

$$\int_{p_1}^{\overline{p}} \frac{p_2 - 2p_1}{\overline{\theta}} f_2(p_2)\,dp_2 + \int_{\underline{p}}^{p_1} [1 - \frac{2(p_1 - p_2)}{\overline{\theta}L}](1 + \frac{p_2}{\overline{\theta}L}) f_2(p_2)\,dp_2 = 0 \tag{3.2.23}$$

$$\int_{p_2}^{\overline{p}} \frac{p_1 - 2p_2}{\overline{\theta}L} f_1(p_1)\,dp_1 + \int_{\underline{p}}^{p_2} [1 - \frac{2(p_2 - p_1)}{\overline{\theta}}](1 + \frac{p_1}{\overline{\theta}}) f_1(p_1)\,dp_1 = 0 \tag{3.2.24}$$

Solving for $f_1(p)$ and $f_2(p)$, we have:

$$f_1(p) = A[e]^{\frac{2L}{\overline{\theta}(L+1)}p}\left[\overline{\theta}^2 L + (L+1)\overline{\theta}p\right]^{1+\frac{2}{(L+1)^2}}, \tag{3.2.25}$$

$$f_2(p) = B[e]^{\frac{2}{\overline{\theta}L(L+1)}p}\left[\overline{\theta}L(L+1)p + \left(\overline{\theta}L\right)^2\right]^{-\frac{3L^2+2L+1}{(L+1)^2}}. \tag{3.2.26}$$

A and B are scalars such that $\int\limits_{\underline{p}}^{\overline{p}} f_1(p)\,dp = 1$ and $\int\limits_{\underline{p}}^{\overline{p}} f_2(p)\,dp = 1$.

In the event that the conditions stated in proposition 3 are not met, then both retailers play mixed strategies in equilibrium. The two cumulative distribution functions F_1 and F_2 have to satisfy conditions (3.2.23) and (3.2.24), while the common support $\left[\underline{p}, \overline{p}\right]$ satisfies the two conditions stated in lemma 1. We have the following proposition:

Proposition 14. *If the conditions stated in proposition* 3 *are not met, then the game has unique mixed equilibrium* $F_1(p_1), F_2(p_2)$ *characterized by (3.2.26) and (3.2.25) on the support* $[\underline{p}, \overline{p}]$*, which satisfies the conditions stated in lemma* 1.

3.2.6 The Auctioneer's Revenue

The auctioneer's revenue come from selling the sponsored advertisement slot. The key control variable for the SEF is the fact that retailers' pricing strategies and corresponding bids depend on the value of L. The auctioneer can increase/decrease the value of L to maximize their revenue from the auction.

In the case in which the game has a pure equilibrium, proposition 3 states that there is at most one L that makes the retailers choosing pure strategies. In this case, the auctioneer's revenue is

$$R_1(p_1^*, p_2^*) = \overline{\theta}L(1 - \frac{\overline{\theta}L - \frac{1}{2}\overline{\theta}L}{\overline{\theta}L})^2 = \frac{1}{4}\overline{\theta}L. \tag{3.2.27}$$

If the auctioneer increases/decreases the value of L, then the two retailers will play mixed strategies. In the case of mixed equilibrium, given a pair of prices (p_1, p_2), if $p_1 > p_2$, the auctioneer receives

$$R_1(p_1, p_2) = p_2(1 - \frac{p_1 - p_2}{\overline{\theta}L})^2,$$

and if $p_2 > p_1$, the auctioneer receives:

$$R_2\left(p_1, p_2\right) = p_1(1 - \frac{p_2 - p_1}{\bar{\theta}})^2.$$

Thus, given the incumbent and the entrant playing F_1 and F_2, the expected revenue of the auctioneer is

$$ER = \int_{\underline{p}}^{\overline{p}} \left[\int_{\underline{p}}^{p_1} R_1\left(p_1, p_2\right) f_2\left(p_2\right) dp_2 + \int_{p_1}^{\overline{p}} R_2\left(p_1, p_2\right) f_2\left(p_2\right) dp_2 \right] f_1\left(p_1\right) dp_1 \quad (3.2.28)$$

Since F_1, $F_2, \overline{p}$ and $\underline{p}$ are functions of L, the revenue in (3.2.28) is also a function of L - the size of the organic list. It is still unclear to us how ER changes when L increases. If ER is not maximized at $L = 2$, then the auctioneer can increase his revenue by introducing irrelevant links into the organic list. The introduction of irrelevant links increases the consumers' search cost and reduce social welfare. In the case of more than one search engine provider, and if the qualities of different search engines are comparable and there is no switching cost between the search engines, then a higher quality search engine may increase social welfare. However, it is still an open question for us regarding how the consumers' surplus would change with respect to L. If an reduction in L leads to an increase in prices an increase in consumer's expected cost, then consumers may prefer a search engine with lower quality. Otherwise, they would prefer the search engine with highest quality.

The current development in the search engine market is consistent with this prediction. The search engine market is currently dominated by Google and Yahoo!. Google holds approximately 50% market share and Yahoo holds 22% market share. Up until recently, most technical reviews suggest that for ordinary users, Google's search engine and Yahoo!'s search engine are not significantly different in terms of quality. However, Yahoo! is now introducing the Beta version of Yahoo! Mindset, a search engine platform which can customize the organic list according to the searcher's preferences. On Yahoo! Mindset, searchers can specify if they want the organic list to be more commercial or more informational. Thus, a commercial searcher can effectively instruct the search engine to eliminate all irrelevant non-commercial links in the organic list[7].

3.3 Discussion

Consumers use search engine to locate online retailers. Established retailers often have their links displayed first on the organic list while new entrants have their links buried under a large number of irrelevant links. Both incumbents and entrants have incentive to advertise on the sponsored section. For the incumbents, the incentive is to preempt rival firms from improving their ranks. For the entrants, the incentive is to improve their ranks and become more competitive. This paper characterizes the equilibrium of a game between two online retailers facing a predetermined order of virtual locations and having a chance to upset that order in an auction for sponsored location. Our main conclusions can be summarized as follows:

- In the keyword auction, bidders' valuations of the sponsored link depends on the price they announced in the first stage. If they announce a higher price, they must win the auction since they will not have positive sale if they lose. In equilibrium, the retailer charging a higher price bids more and wins the auction.

- Price dispersion exists either in terms of a pure equilibrium in which the incumbent charges a higher price and the entrant charge a lower price, or in terms of a mixed equilibrium in which both firms randomize their prices.

[7]Yahoo! Mindset is not a shopbot since it does not display prices and does not allow consumers to rank the links in the organic list with respect to prices.

- Preemptive advertisement can be found in the context of the pure equilibrium, in the form that the incumbent wins the auction and effectively deter the entrant from changing ranks. In the mixed equilibrium, there only exists partial preemption: the incumbent does not completely block the entrant.

Although we consider only a duopoly model, our main conclusions do not change if the number of retailers is greater than two. Similarly, our qualitative results are robust to the variation of the number of sponsored advertisement slots, as long as this number is smaller than the number of online retailers in the market.

There are a few limitations in our model. Full information is a strong assumption. We assume that consumers know how many retailers on the market, their exact locations on the organic list and their announced prices, which may be not be a realistic assumption in many online trading environment. For example, consumers looking for an item online may obtain an organic list containing millions of links. They may not know how many online retailers are selling this item and where these retailers are located. Whether our conclusions are still valid in such scenario is an interesting question for future research.

Another important feature needs to be pointed out is the static nature of our model. A dynamic setting would allow for periodical changes in the ranking of positions. For example, as the entrant obtains richer sale history and becomes more popular, the position of his link will move up on the organic list and may eventually replace the position of the incumbent's link. Studying equilibrium prices and bids in such dynamic environment to shed light on the long term pattern of position ranking is another interesting question for future research.

Appendix A

Proofs for Chapter 1

A.1 Optimal Pricing

Consider the optimal pricing problem for firm i:

$$\max_{p_i}\{\widetilde{\Pi}_i\,(p_{it_i}, \alpha, \lambda, t_i, k_i)\}. \tag{A.1.1}$$

The explicit forms of $\widetilde{\Pi}_i\,(p_{it_i}, \alpha, \lambda, t_i, k_i)$ can be found in three cases:

- If $t_i = A$, then $k_i = 1$ since low quality contents are always compatible with hardware L. Consumers with hardware H and consumers with hardware L purchase the same amout of content i: $x^*_{iA} = (\frac{p_{iA}}{\beta})^{\frac{1}{\beta-1}}$. Thus, the profit maximization becomes:

$$\max_{p_i}\{(\alpha+\lambda)\,[(\frac{p_{iA}}{\beta})^{\frac{1}{\beta-1}}\,(p_{iA}-c_A)]\}, \tag{A.1.2}$$

  which has unique solution: $p^*_{iA} = \frac{c_A}{\beta}$.

- If $t_i = B$ and $k_i = 1$, then consumers with hardware L purchase $x^*_{iB} = (\frac{p_{iB}}{\beta})^{\frac{1}{\beta-1}}$ and consumers with hardware H purchase $x^*_{iB} = \frac{1}{a_i}(\frac{p_{iB}}{a_i\beta})^{\frac{1}{\beta-1}}$. Thus, firm i's pricing problem becomes:

$$\max_{p_i}\{\alpha\frac{1}{a_i}(\frac{p_{iB}}{a_i\beta})^{\frac{1}{\beta-1}}(p_{iB}-c_B)+\lambda(\frac{p_{iB}}{\beta})^{\frac{1}{\beta-1}}(p_{iB}-c_B)\}, \tag{A.1.3}$$

which also has unique solution: $p^*_{iB}=\frac{c_B}{\beta}$.

- If $t_i=B$ and $k_i=0$, then then consumers with hardware L purchase $x^*_{iB}=0$ and consumers with hardware H purchase $x^*_{iB}=\frac{1}{a_i}(\frac{p_{iB}}{a_i\beta})^{\frac{1}{\beta-1}}$. Thus, firm i's pricing problem is:

$$\max_{p_i}\{\alpha\frac{1}{a_i}(\frac{p_{iB}}{a_i\beta})^{\frac{1}{\beta-1}}(p_{iB}-c_B)\}, \tag{A.1.4}$$

and the solution is the same as the solution in the problem (A.1.3): $p^*_{iB}=\frac{c_B}{\beta}$.

A.2 Supplemental Proof of Proposition 6

To show that x_A will not survive, assume the opposite that $(x_A, v(x_A))$ is supportable by some transfer t. We first establish that if x_A survives, then the transfer t which support the $(x_A, v(x_A))$ should satisfy $t_{ij}(A)=0$. This result is immediate since $v_i=0$ is the secure payoff for player i (he can not get lower than that) and x_A is a equilibrium of the original game. Hence, player i has no reason to transfer any positive amount to other players to convince them to play x_A. Any positive t_i would simply be a gratuitous transfer.

With $t_{ij}(A)=0$, the transfer in the first stage can be characterized as:

$$t_{ij}(x_j)=\begin{cases} 0 & x_j=A \\ t_{ij}(B) & x_j=B \end{cases}, \forall i. \tag{A.2.1}$$

Beside $t_{ij}(A)=0$, there is nother condition which $t_{ij}(x_j)$ should satisfy: the transfer $t_{ij}(B)$ should be such that no player wants to deviate by selecting a null transfer in the first stage and plays B in the second stage. By playing (A.2.1) in the first stage and A in the second stage, player i gains 0. If player i plays null transfer t_i^0 in the first stage and B in the second stage, he gets $-c+\sum_{j\neq i} t_{ji}(B)$. Therefore, the tranfer $t_{ij}(x_j)$ should satisfy:

$$0 \geq -c + \sum_{j\neq i} t_{ji}(B), \forall i. \tag{A.2.2}$$

Assuming that other players do not deviate, consider player 1. If 1 does not deviate from t_1, then 1 receives payoff $v_1 = 0$. If 1 deviates and selects the transfer function t_1' such that $t_{1i}^{'}(A) = 0$ and:

$$t_{1i}'(B) = c + t_{i1}(B) + \sum_{j\neq 1, j\neq i} [t_{ij}(B) - t_{ji}(B)], \tag{A.2.3}$$

then B is the best response for all other players ($i = 2...N$). It is because if i plays B, he gets at least:

$$-c + t_{1i}'(B) + t_{ji}(B) - t_{i1}(B) - t_{ij}(B) = 0 \tag{A.2.4}$$

Given such t_1' and x_B is played in the second stage, player 1 gets:

$$\pi_1 - (N-1)c - \sum_{i=2}^{N} \left\{ t_{i1}(B) + \sum_{j\neq 1, j\neq i} [t_{ij}(B) - t_{ji}(B)] \right\} + \sum_{i=2}^{N} t_{i1}(B), \tag{A.2.5}$$

which is equal to:

$$\pi_1 - (N-1)c - \sum_{i=2}^{N} \sum_{j\neq 1, j\neq i} [t_{ij}(B) - t_{ji}(B)] = \pi_1 - (N-1)c. \tag{A.2.6}$$

In summary, if $(x_A, v(x_A))$ is supportable at all, then the vector of transfer

t which support $(x_A, v(x_A))$ should satisfy (A.2.1) and (A.2.2). However, if $\pi_1 > (N-1)c$, then player 1 always find it strictly better off to deviate. Player 1 can always choose a transfer:

$$t'_{1i}(x_i) = \begin{cases} ccc + t_{i1}(B) + \sum_{j\neq 1, j\neq i} [t_{ij}(B) - t_{ji}(B)] & \text{if } x_i = B \\ 0 & \text{otherwise} \end{cases}, \qquad \text{(A.2.7)}$$

with $i \neq 1$, $j \neq 1$, $j \neq i$. With tranfer $t'_{1i}(x_i)$, x_B becomes the only equilibrium of the second stage game. Player $i = 2, ..., N$, each get zero payoff, and player 1 gets $\pi_1 - (N-1)c > 0$.

In other words, given $\pi_1 - (N-1)c > 0$, the inefficient equilibrium x_A can not survive.

Appendix B

Proofs for Chapter 2

B.1 Proof of Proposition 8

We will prove that if both firms adopt the proprietary approach in the first stage, then both firms receive strictly positive profits in the corresponding PS/PS subgame. We will show that firm 2 can always sustain strictly positive profit regardless of firm 1's strategies. The same approach can be used to derive the conclusion for firm 1 and will not be repeated. In the following proof, we will first consider the case in which firm 1 utilizes a pure strategy s_1. In the second part of the proof, we will consider the case in which firm 1 utilizes a mixed strategy $G_1(s)$.

Fix a *small* n such that $F(2n) < n$. Suppose firm 1 plays s_1 and $s_1 > \frac{3}{2}n$. If firm 2 chooses $s_2 < s_1$, then firm 2's profit is

$$R_2(s_1, s_2) = \begin{cases} \frac{\left[s_1(s_1-s_2)(s_2+2n)^2\right]}{s_2(4s_1-s_2)^2} & \text{if } n \leq \frac{(s_1-s_2)s_2}{2s_1+s_2} \\ \frac{2(s_1-s_2)s_1-3ns_1}{(s_1-s_2)(4s_1-s_2)}n & \text{otherwise} \\ 0 & \text{if } n \geq \frac{2}{3}(s_1-s_2). \end{cases}$$

Since $s_1 > s_2$, there exists $s_2 > 0$ such that $(s_1 - s_2) \geq \frac{3}{2}n$. As s_2 goes to zero, the condition $n \leq \frac{(s_1-s_2)s_2}{2s_1+s_2}$ eventually will not hold and $\lim_{s_2 \to 0} \pi_2(s_1, s_2) =$

$\frac{n}{4s_1}\left[2s_1-3n\right]>0.$

Suppose now that firm 1 plays s_1 and $s_1 \leq \frac{3}{2}n$. If firm 2 chooses $s_2 = s_1+\varepsilon$, $0<\varepsilon<\frac{1}{2}n$ then firm 2's profit is

$$R_2\left(s_2,s_1\right)=\begin{cases} \frac{1}{\left(4s_2-s_1\right)^2}\left(s_2-s_1\right)\left(2s_2+n\right)^2 & \text{if } n\leq\frac{\left(s_1-s_2\right)s_2}{2s_1+s_2} \\ \frac{2s_2+n}{\left(4s_2-s_1\right)^2}\left[\left(s_2-s_1\right)\left(2s_2-s_1\right)+3ns_2\right] & \text{otherwise} \\ n & \text{if } n\geq\frac{2}{3}\left(s_1-s_2\right). \end{cases}$$

Since $s_2-s_1=\varepsilon<\frac{1}{2}n$, $R_2\left(s_2,s_1\right)=n$. Firm 2's profit is $\pi\left(s_1,s_2\right)\geq n-F_2\left(\frac{3}{2}n+\varepsilon\right)>0$.

Fix a *small* n such that $F\left(2n\right)<n$. For the case in which firm 1 plays a mixed strategy $G_1\left(s\right)$ on a support $[\underline{q},\overline{q}]$, in the spirit of Varian (1980), $G_1\left(s\right)$ can not have any mass point and the support has no gap. If $\overline{q}\leq\frac{3}{2}n$, firm 2 can always set $s_2=\frac{3}{2}n+\varepsilon$, $\varepsilon<\frac{1}{2}n$ and receives a profit greater or equal to $n-F\left(\frac{3}{2}n+\varepsilon\right)>0$. In the following discussion, assume that $\overline{q}>\frac{3}{2}n$.

- Suppose that $\underline{q}=0$ and firm 2 plays $s_2=\varepsilon>0$ such that $\varepsilon<\frac{3}{2}n$. On the range $s_1\in[0,\varepsilon)$, firm 2 is the high quality firm but its price is zero due to $s_2=\varepsilon<\frac{3}{2}n$ (recall the discussion in section 3.2 about firms' pricing). On the range $s_1\in[\varepsilon,\overline{q}]$, firm 2 is the low quality firm. As ε goes to zero, firm 2's profit is

$$\begin{aligned}\lim_{\varepsilon\to0}\pi_2\left(G_1,\varepsilon\right) &= \lim_{\varepsilon\to0}\int_{\frac{3}{2}n+\varepsilon}^{\overline{q}}\left[\frac{2\left(s_1-s_2\right)s_1-3ns_1}{\left(s_1-s_2\right)\left(4s_1-s_2\right)}n\right]dG_1\left(s_1\right), \\ &= \int_{\frac{3}{2}n}^{\overline{q}}\frac{n}{4s_1}\left[2s_1-3n\right]dG_1\left(s_1\right)>0.\end{aligned}$$

- Finally, suppose that $\underline{q}>0$. let firm 2 plays $\varepsilon<\underline{q}$. Firm 2 is the low quality firm. On the range of price that firm 1 chooses zero pricing, firm 2 does not make any profit. On the range of price that firm 1's price is strictly positive, then firm 2's revenue is strictly positive. As ε goes to zero, the limit of firm 2's profit is

$$\begin{aligned}
\lim_{\varepsilon \to 0} \pi_2 \left(G_1, \varepsilon\right) &= \lim_{\varepsilon \to 0} \int\limits_{\max\left\{\underline{q}, \frac{3}{2}n+\varepsilon\right\}}^{\overline{q}} [\frac{2\left(s_1 - s_2\right)s_1 - 3ns_1}{\left(s_1 - s_2\right)\left(4s_1 - s_2\right)}n] dG_1\left(s_1\right), \\
&= \int\limits_{\max\left\{\underline{q}, \frac{3}{2}n\right\}}^{\overline{q}} \frac{n}{4s_1}\left[2s_1 - 3n\right] dG_1\left(s_1\right) > 0.
\end{aligned}$$

Thus, firm 2 can always guarantee strictly positive profit despite firm 1's strategies. Likewise, firm 1 can always guarantee strictly positive profit regardless of firm 2's strategies. In other words, both firms receive strictly positive profit in the PS/PS subgame.

Appendix C

Proofs for Chapter 3

C.1 Appendix C1: Proof of proposition 13

We will consider two cases $p_2 > p_1$ and $p_1 > p_2$. We first show that there is no pure strategy equilibrium such that $p_2 > p_1$ and then show that pure equilibrium $p_1 > p_2$ exists if the condition presented in the proposition is satisfied.

C.1.1 Case 1: $p_2 > p_1$.

The discussion in (2.5.1) established that $p_1(w_2) = \frac{1}{2}\overline{\theta}$ and $p_2(w_2) = \overline{\theta}$. The corresponding profits for the incumbent and for the entrant are $\pi_1(p_1(w_2), p_2(w_2)) = \frac{1}{4}\overline{\theta}$ and $\pi_2(p_1(w_2), p_2(w_2)) = \frac{3}{4}\overline{\theta}$. respectively. Suppose that the entrant holds his price at $p_2(w_2) = \overline{\theta}$. The price $p_1(w_2) = \frac{1}{2}\overline{\theta}$ is the incumbent's equilibrium price only if the incumbent does not find it profitable to deviate by selecting a price higher than $\overline{\theta}$. If the incumbent charges a price higher than $\overline{\theta}$, he gets

$$\pi_1(p_1, p_2(w_2), w_1) = [p_1 - \overline{\theta} + \frac{(p_1 - \overline{\theta})\overline{\theta}}{\overline{\theta}L}][1 - \frac{p_1 - \overline{\theta}}{\overline{\theta}L}]. \tag{C.1.1}$$

Maximizing (C.1.1) with respect to p_1 yields the unique solution: $p_1^{'} = \overline{\theta} + \frac{\overline{\theta}L}{2}$,Taking into account the condition that $p_1 \leq v$, the optimal deviated price is

$$p_1^{'}(L) = \begin{cases} \overline{\theta} + \frac{\overline{\theta}L}{2} & \text{if } \overline{\theta} + \frac{\overline{\theta}L}{2} \leq v \\ v & \text{otherwise} \end{cases}. \tag{C.1.2}$$

The corresponding deviated profit is

$$\pi_1(p_1, p_2(w_2), w_1) = \begin{cases} \frac{\overline{\theta}(L+1)}{4} & \text{if } \overline{\theta} + \frac{\overline{\theta}L}{2} \leq v \\ \left(v - \overline{\theta}\right)[1 + \frac{1}{L}][1 - \frac{v - \overline{\theta}}{\overline{\theta}L}] & \text{otherwise} \end{cases} \tag{C.1.3}$$

Since $\overline{\theta}(L+1)/4 > \overline{\theta}/4$, the incumbent is strictly better off to deviate if if $\overline{\theta} + \frac{\overline{\theta}L}{2} \leq v$. If $\overline{\theta} + \frac{\overline{\theta}L}{2} > v$ instead, then since the function

$$f(x) = (1 + \frac{1}{L})x(1 - \frac{x}{\overline{\theta}L}) \tag{C.1.4}$$

is increasing with respect to x in the range $x < \overline{\theta}L/2$, also since $\frac{\overline{\theta}L}{2} > v - \overline{\theta} > \overline{\theta}$, the following statement must be true:

$$f\left(v - \overline{\theta}\right) > f\left(\overline{\theta}\right) = \overline{\theta}(1 + \frac{1}{L})(1 - \frac{1}{L}). \tag{C.1.5}$$

The left hand side of (C.1.5) is strictly larger than $\frac{1}{4}\overline{\theta}$ since $L \geq 2$. Thus, the incumbent is always strictly better off to deviate.

C.1.2 Case 2 $p_1 > p_2$.

Since $p_1 > p_2$, the discussion in (2.5.2) established that $p_1(w_1) = \overline{\theta}L$ and $p_2(w_1) = \frac{1}{2}\overline{\theta}L$. The corresponding profits are $\pi_1(p_1, p_2, w_1) = \frac{3\overline{\theta}L}{8}$ and $\pi_2(p_1, p_2, w_1) = \frac{\overline{\theta}L}{4}$. If the incumbent deviates and chooses price $p_1^{'} < \frac{1}{2}\overline{\theta}L$, then his optimal deviated price is

$$p_1^{'} = \frac{1}{2} p_2 \left(w_1 \right) = \frac{1}{4} \overline{\theta} L \tag{C.1.6}$$

and the corresponding profit is

$$\pi_1 \left(p_1^{'}, p_2, w_2 \right) = \frac{1}{16} \overline{\theta} L^2. \tag{C.1.7}$$

The maximum deviated profit is strictly higher than $\pi_1 \left(p_1, p_2, w_1 \right)$ iff $L > 6$.

Now, consider the incentive of the entrant. If the entrant deviates by charging a price higher than $\overline{\theta} L$ then, his optimal deviated price is

$$p_2^{'} = \begin{cases} \overline{\theta} L + \frac{1}{2} \overline{\theta} & \text{if } \overline{\theta} L + \frac{1}{2} \overline{\theta} \leq v \\ v & \text{otherwise.} \end{cases} \tag{C.1.8}$$

and the corresponding profit is

$$\pi_2 \left(p_1, p_2^{'}, w_2 \right) = \begin{cases} \frac{1}{4} \left(\overline{\theta} + \overline{\theta} L \right) & \text{if } \overline{\theta} L + \frac{1}{2} \overline{\theta} \leq v \\ \left(v - \overline{\theta} L \right) \left(1 + L \right) \left[1 - \frac{v - \overline{\theta} L}{\overline{\theta}} \right] & \text{otherwise.} \end{cases} \tag{C.1.9}$$

Since $\frac{1}{4} \left(\overline{\theta} + \overline{\theta} L \right) > \frac{1}{4} \overline{\theta} L$, $\pi_2 \left(p_1, p_2^{'}, w_2 \right) > \pi_2 \left(p_1, p_2, w_2 \right)$ if $\overline{\theta} L + \frac{1}{2} \overline{\theta} \leq v$.

If $\overline{\theta} L + \frac{1}{2} \overline{\theta} > v$, let $y = v - \overline{\theta} L$ and consider the function $f \left(y \right) = \left(1 + L \right) y \left[1 - \frac{y}{\overline{\theta}} \right]$. $f \left(y \right)$ is strictly increasing on the range $0 \leq x \leq \frac{1}{2} \overline{\theta}$. If $x = 0$, then $f \left(y \right) = \pi(p_1, p_2^{'}, w_2) = 0$, deviation is strictly worse off. If $x = \frac{1}{2} \overline{\theta}$, then $f \left(y \right) = \frac{1}{4} \left(\overline{\theta} + \overline{\theta} L \right) > \pi_2 \left(p_1, p_2, w_2 \right)$, deviation is strictly better off. Thus, given v and $\overline{\theta}$, there exists an $\overline{L}$ such that deviation is strictly worse off if $L > \overline{L}$ and weakly better off if $L \leq \overline{L}$.

Since the entrant is strictly better off to deviate if $L > 6$ and the incumbent is strictly better off to deviate if $L \leq \overline{L}$. In other words, there exists a Nash equilibrium in pure strategy if $L \leq 6$, $\overline{\theta} L + \frac{1}{2} \overline{\theta} > v \geq \overline{\theta} L$ and

$$7 \left(v - 6 \overline{\theta} \right) \left[1 - \frac{v - 6 \overline{\theta}}{\overline{\theta}} \right] < \frac{3}{2} \overline{\theta}. \tag{C.1.10}$$

C.2 Appendix C2: Derivation of Conditions (3.2.21) and (3.2.22).

C.2.1 Condition (3.2.21):

Note that:

$$g_1(p) = \int_{\underline{p}}^{p} (p - p_2)(1 + \frac{p_2}{\overline{\theta} L})[1 - \frac{p - p_2}{\overline{\theta} L}] f_2(p_2)\, dp_2. \tag{C.2.1}$$

Maximizing $g_1(p)$ leads to the following condition:

$$g_1^{'}(p) = \int_{\underline{p}}^{p} \left[1 - \frac{2(p - p_2)}{\overline{\theta} L}\right](1 + \frac{p_2}{\overline{\theta} L}) f_2(p_2)\, dp_2 = 0, \tag{C.2.2}$$

which can be rearranged as

$$\begin{aligned} 0 \quad = \quad & \int_{\underline{p}}^{p} f_2(p_2)\, dp_2 + \frac{1}{\overline{\theta} L}\int_{\underline{p}}^{p} p_2 f_2(p_2)\, dp_2 - \frac{2p}{\overline{\theta} L}\int_{\underline{p}}^{p} f_2(p_2)\, dp_2 + \frac{2}{\overline{\theta} L}\int_{\underline{p}}^{p} p_2 f_2(p_2)\, dp_2 \\ & - \frac{2p}{\overline{\theta}^2 L^2}\int_{\underline{p}}^{p} p_2 f_2(p_2)\, dp_2 + \frac{2}{\overline{\theta}^2 L^2}\int_{\underline{p}}^{p} p_2^2 f_2(p_2)\, dp_2. \end{aligned} \tag{C.2.3}$$

or,

$$\left[1 - \frac{2p}{\overline{\theta} L}\right]\int_{\underline{p}}^{p} f_2(p_2)\, dp_2 + \left[\frac{3}{\overline{\theta} L} - \frac{2p}{\overline{\theta}^2 L^2}\right]\int_{\underline{p}}^{p} p_2 f_2(p_2)\, dp_2 + \frac{2}{\overline{\theta}^2 L^2}\int_{\underline{p}}^{p} p_2^2 f_2(p_2)\, dp_2 = 0. \tag{C.2.4}$$

If $p > \overline{p}$, then $\int_{\underline{p}}^{p} f_2(p_2)\, dp_2 = 1$, $\int_{\underline{p}}^{p} p_2 f_2(p_2)\, dp_2 = E_2(p_2)$, and $\int_{\underline{p}}^{p} p_2^2 f_2(p_2)\, dp_2 = E_2\left(p_2^2\right)$. The above equation can be rewritten as:

$$1 - \frac{2p}{\overline{\theta} L} + \left[\frac{3}{\overline{\theta} L} - \frac{2p}{\overline{\theta}^2 L^2}\right] E_2(p_2) + \frac{2}{\overline{\theta}^2 L^2} E_2\left(p_2^2\right) = 0, \tag{C.2.5}$$

or

$$p=\frac{\overline{\theta}^2L^2+3\left(\overline{\theta}L\right)E_2\left(p_2\right)+2E_2\left(p_2^2\right)}{2\left[\overline{\theta}L+E_2\left(p_2\right)\right]}. \tag{C.2.6}$$

If $\overline{p}$ is strictly smaller than the price in (C.2.6), then playing $\overline{p}$ is a strictly dominated strategy and that would contradict with the assumption that $\overline{p}$ is the upper bound of the equilibrium strategy F_1. Thus, the necessary condition for F_1 to be the equilibrium mixed strategy for the incumbent is

$$\overline{p}\geq\frac{\overline{\theta}^2L^2+3\left(\overline{\theta}L\right)E_2\left(p_2\right)+2E_2\left(p_2^2\right)}{2\left[\overline{\theta}L+E_2\left(p_2\right)\right]}. \tag{C.2.7}$$

C.2.2 Condition (3.2.22):

Note that

$$g_2\left(p\right)=\int\limits_{\underline{p}}^{p}\left(p-p_1\right)\left(1+\frac{p_1}{\overline{\theta}}\right)\left[1-\frac{p-p_1}{\overline{\theta}}\right]f_1\left(p_1\right)dp_1. \tag{C.2.8}$$

Maximizing $g_2\left(p\right)$ leads to the following condition:

$$g_2^{'}\left(p\right)=\int\limits_{\underline{p}}^{p}\left[1-\frac{2}{\overline{\theta}}\left(p-p_1\right)\right]\left(1+\frac{p_1}{\overline{\theta}}\right)f_1\left(p_1\right)dp_1=0, \tag{C.2.9}$$

which can be rearranged as

$$\frac{2p}{\overline{\theta}}\int\limits_{\underline{p}}^{p}\left(1+\frac{p_1}{\overline{\theta}}\right)f_1\left(p_1\right)dp_1=\int\limits_{\underline{p}}^{p}\left(1+\frac{p_1}{\overline{\theta}}\right)f_1\left(p_1\right)dp_1+\frac{2}{\overline{\theta}}\int\limits_{\underline{p}}^{p}\left(1+\frac{p_1}{\overline{\theta}}\right)p_1f_1\left(p_1\right)dp_1. \tag{C.2.10}$$

If $p>\overline{p}$, then the above equation is similar to

$$2[\overline{\theta}+E_1\left(p_1\right)]p=\left[\overline{\theta}^2+3\overline{\theta}E_1\left(p_1\right)+2E_1\left(p_1^2\right)\right], \tag{C.2.11}$$

or,

$$p_1 = \frac{\overline{\theta}^2 + 3\overline{\theta}E_1(p_1) + 2E_1\left(p_1^2\right)}{2\left[\overline{\theta} + E_1(p_1)\right]}. \tag{C.2.12}$$

If $\overline{p}$ is strictly smaller than the price in (C.2.12), then playing $\overline{p}$ is a strictly dominated strategy and that would contradict with the assumption that $\overline{p}$ is the upper bound of the equilibrium strategy F_2. Thus, the necessary condition for F_2 to be the equilibrium mixed strategy for the entrant is

$$\overline{p} \geq \frac{\overline{\theta}^2 + 3\overline{\theta}E_1(p_1) + 2E_1\left(p_1^2\right)}{2\left[\overline{\theta} + E_1(p_1)\right]}. \tag{C.2.13}$$

C.3 Appendix C3: Demonstration of Organic List and Sponsored List

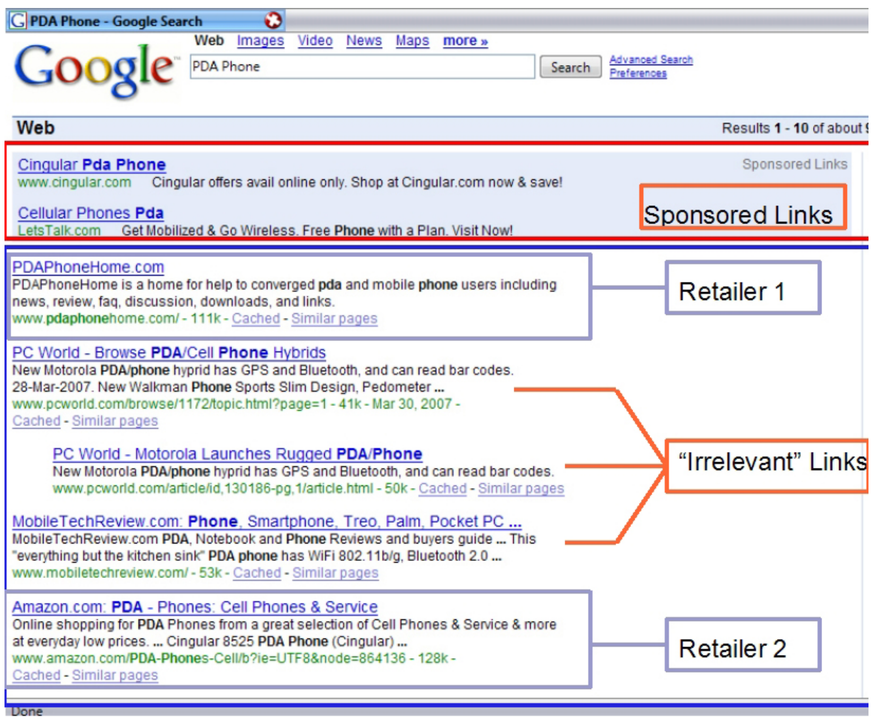

Figure C.1: For each querry entered, the search engine generates a sponsored list and an organic list. The sponsored list is placed on the top of the organic list. The organic lists may contain millions of links. The sponsored list generally does not contain more than 4 links

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