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Uncertainty in Proved Reserves Estimation by Decline Curve Analysis

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Abstract

Uncertainty in Proved Reserves Estimation by Decline Curve Analysis

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Proved reserves estimation is a crucial process since it impacts aspects of the petroleum business. By definition of the Society of Petroleum Engineers, the proved reserves must be estimated by reliable methods that must have a chance of at least a 90 percent probability (P90) that the actual quantities recovered will equal or exceed the estimates.

Decline curve analysis, DCA, is a commonly used method; which a trend is fitted to a production history and extrapolated to an economic limit for the reserves estimation. The trend is the "best estimate" line that represents the well performance, which corresponds to the 50th percentile value (P50). This practice, therefore, conflicts with the proved reserves definition.

An exponential decline model is used as a base case because it forms a straight line in a rate-cum coordinate scale. Two straight line fitting methods, i.e. ordinary least square and error-in-variables are compared. The least square method works better in that the result is consistent with the Gauss-Markov theorem.

In compliance with the definition, the proved reserves can be estimated by determining the 90th percentile value of the descending order data from the variance. A conventional estimation using a principal of confidence intervals is first introduced to quantify the spread, a difference between P50 and P90, from the variability of a cumulative production.

Because of the spread overestimation of the conventional method, the analytical formula is derived for estimating the variance of the cumulative production. The formula is from an integration of production of rate over a period of time and an error model. The variance estimations agree with Monte Carlo simulation (MCS) results. The variance is then used further to quantify the spread with the assumption that the ultimate cumulative production is normally distributed. Hyperbolic and harmonic models are also studied. The spread discrepancy between the analytics and the MCS is acceptable. However, the results depend on the accuracy of the decline model and error used. If the decline curve changes during the estimation period the estimated spread will be inaccurate.

In sensitivity analysis, the trend of the spread is similar to how uncertainty changes as the parameter changes. For instance, the spread reduces if uncertainty reduces with the changing parameter, and vice versa.

The field application of the analytical solution is consistent to the assumed model. The spread depends on how much uncertainty in the data is; the higher uncertainty we assume in the data, the higher spread.

Table of Contents

List of Tables

List of Figures

Chapter 1: Introduction

There is a finite volume of hydrocarbons that can be produced from any given well, reservoir or field. It follows that at some point in the productive life, the production rate will decline, ultimately reaching the economic limit, which is when hydrocarbon production will terminate because producing rate or net revenue are reduced to the level that the current income equals to the cost of producing it. This is generally referred to as the "ultimate recovery" and is equal to the reserves at the beginning before production commences. As hydrocarbons are produced, reserves will decline as cumulative production increases, but the sum of cumulative production and reserves always equal to the estimated ultimate recovery at that time. The ultimate recovery remains an estimate and the actual figures will not be known for certain until the last well is plugged and abandoned at some date in the future.

Reliable reserve figures are crucial to the petroleum business throughout a property or field lifetime. At early stages, oil and gas recovery is required to solve specific engineering problems like the exploitation and development of a reservoir, or the construction of processing units, pipelines and refinery plants. At later stages after sufficient actual performance data have become available, reserves estimation remains essential for its use for accounting and financing purposes. Oil companies, in general, rely on the reserve figures as an integral part of profitability studies, financing, evaluating and trading of oil and gas properties. Therefore, the calculation of oil and gas reserves is the most critical and demanding aspect of any cash flow projection.

The petroleum engineer is called upon to furnish estimates of reserves or estimates of ultimate recovery of oil and gas from a particular producing unit. The practice of reservoir engineering is almost entirely devoted to assignments of this nature. There are abundant methods for determining reserves, which can be broadly classified in two groups; i.e. those based upon reservoir rock and fluid properties and those determined from reservoir production performance. Decline curve analysis, or DCA, is one of the deterministic methods to estimate the reserves from production data. Its accuracy depends on the amount of production history available.

<u>Decline Curve Analysis</u>

DCA is an empirical statistical method of analyzing production performance. When it is applied to performance data to predict future performance from that production unit, it is essentially using the reservoir as analogy. This procedure assumes that current well and reservoir conditions will remain unchanged in the future.

Records of production data are usually available by individual producing properties or leases. There are several ways of plotting these data depending upon the nature of the reservoir and the amount of hydrocarbon recovery to date. One basic technique is to plot producing rate against cumulative production on a coordinate grid for a constant decline case or on a semi-log grid for other cases. For example, in the case of exponential decline, the declining production trend from this plot is a straight line and future production can be forecast by extrapolating the straight line trend to the economic limit or to zero production.

There is an argument that the extrapolation of decline curves is an empirical, statistical method and not a true engineering approach. However, the techniques of DCA are widely for evaluating the potential of a producing property in a case in which information is sparse or the production could not support an extensive and expensive reservoir engineering project. The absence of sufficient log, core, and fluid analysis, static bottomhole pressure surveys, and well-test data precludes application of more advanced analysis techniques. The DCA also has the advantage of minimizing the time spent studying assets and are easily understood, it is, therefore, usually favorably used to evaluate potential of marginal properties; and it has proven to be a very satisfactory method of forecasting future production.

Reserves Classification

There are various reserves classifications by numerous organizations. However, the ones categorized by the Society of Petroleum Engineers, SPE, and the World Petroleum Congresses, WPC, have been commonly accepted as standards for reserves classification across the industry. In 1996 after years of synergy between the two organizations, they developed a common set of reserves definitions. The definitions are assigned beyond the reserves to less certainty resources, which are subcategorized to contingent and prospective resources as illustrated in Figure 1. The U.S. Securities and Exchange Commission, or SEC, has also adopted this set of definitions to its rule of oil and gas reporting. The proved reserves are required to report to the SEC, while other reserves are optional. For the resources, they are documented as a portfolio of a company.

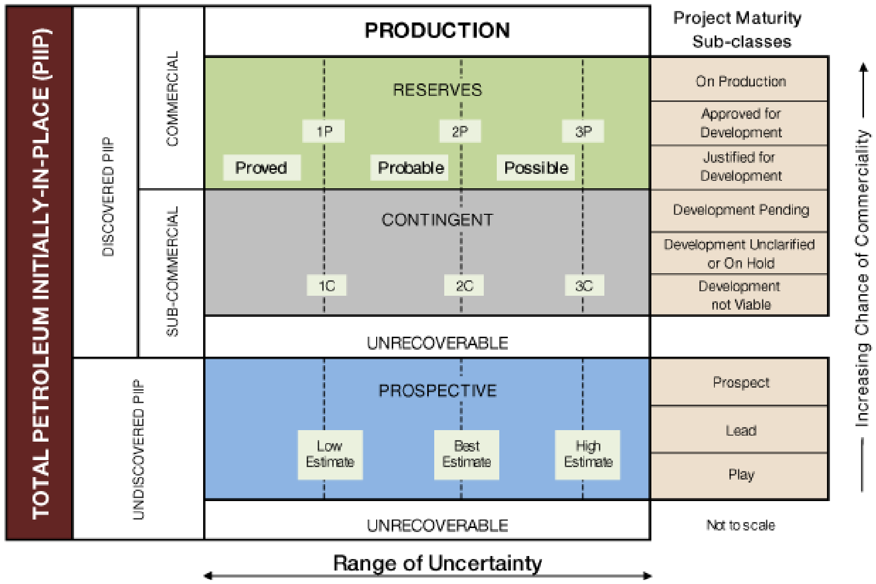

Figure 1 - Resources classification framework

(SPE 2011)

All reserve estimates involve some degree of uncertainty or probability distribution. The uncertainty of reserves can be classified into proved and unproved reserves. Unproved reserves are further subdivided into probable and possible reserves. Proved reserves, in general, have an existence probability of greater 90%; thus a symbol for proved reserves can be P90. Probable reserves are the quantities of recoverable hydrocarbons whose data are similar to those used for proved reserves, but lacking in certainty. Probable reserves, denoted by P50, are generally considered to be those that have a probability for overall reserves to be produced greater than 50%. Possible reserves, denoted by P10, utilize more uncertain data indicating possible reserves to be less likely than probable reserves to be commercially recoverable. Possible reserves must have a probability for overall reserves to be recovered at least 10%. Figure 2 depicts the reserves terminology associated with the uncertainty in term of percentiles.

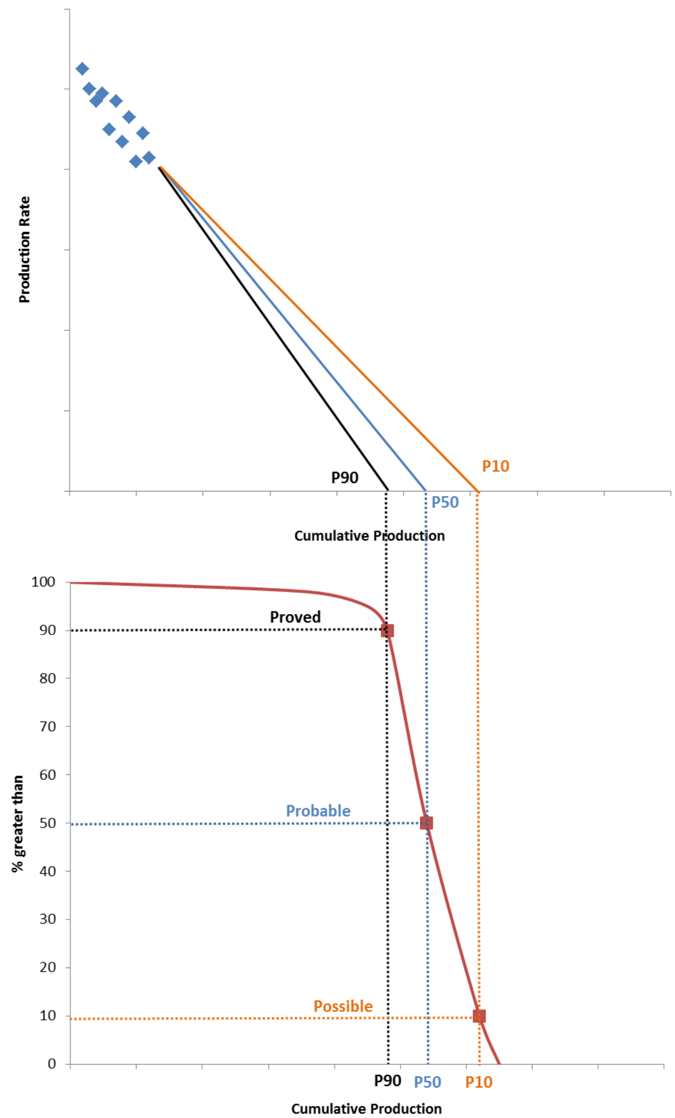

Figure 2 - SPE reserves terminology

The proved reserves are the highest valued category among the reserves classification. Because of high degree of confidence that the volume will be recovered, they have all commercial aspects addressed and considered as primary assets of Exploration and Production companies. This study focuses on the uncertainty in estimation of proved reserves. In accordance with the definition adopted by the SPE and WPC (reproduced entirely in Appendix A);

"Proved reserves are those quantities of petroleum which, by analysis of geological and engineering data, can be estimated with reasonable certainty to be commercially recoverable, from a given date forward, from known reservoirs and under current economic conditions, operating methods, and government regulations."

These figures of reserves can be derived from decline curves by projecting the producing rates to an assumed economic limit. Specifically for the exponential decline model, if the production decline has been plotted against cumulative oil production, the ultimate recovery can then be directly read from the economic limit on the X-axis (Figure 3). If the production decline is plotted against time, it is necessary to integrate the space under the curve in order to derive the remaining or ultimate recoveries (Figure 4).

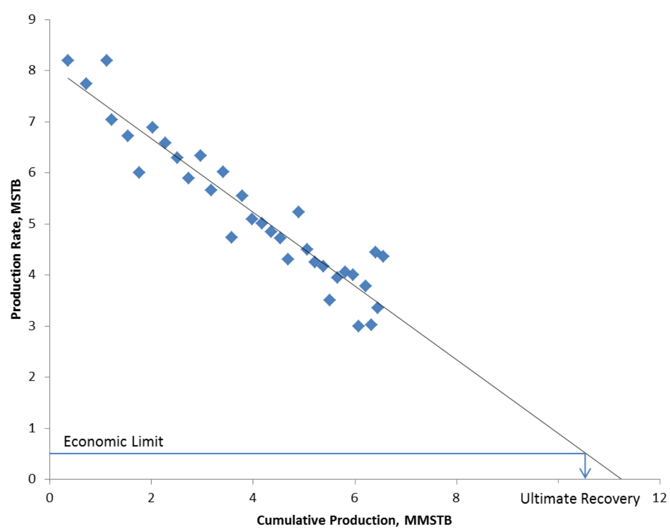

Figure 3 - Rate-cumulative DCA with projected ultimate recovery at the economic limit

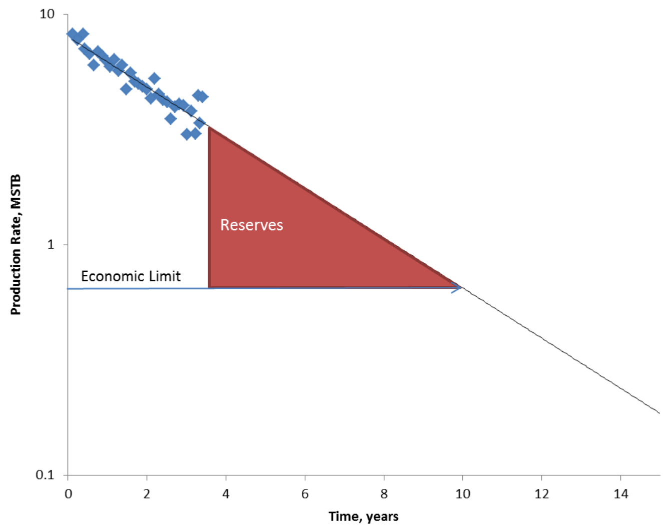

Figure 4 - Semi log of rate-time DCA with estimated reserves at the economic limit

In compliance with the definition, the methods used for estimating the reserves must have a high degree of confidence at the level of at least a 90 percent probability that the quantities actually recovered will equal or exceed the estimate. Decline curve extrapolations have been used extensively to estimate the proved reserves. In other words, the decline curve analysis method is generally accepted for its reasonable certainty in proved reserves estimation.

Curve fitting becomes a critical part of DCA as it impacts the extrapolations to ultimate recovery. Typically, actual data points will not follow a smooth decline so it will be necessary to draw the best line through the data points with least residuals between the line and data, ignoring outliers and atypical points caused by unrelated changes. The ultimate recovery from extrapolating this line to an economic limit is a “best estimate” of the recoverable volume, which represents the estimate considered to be the closest to the quantity that actually will be recovered from the reservoir. In other words, this line accounts the best chance that the data would follow this trend, it is sensible to call this line as the 50^{th} percentile line; meaning that there is a 50 percent chance that the actual ultimate recovery would fall below the estimate. However, this practice would fail to comply with the proved reserves definition that requires certain level of confidence, at least 90 percent.

This leads to the question that whether DCA should be continued practicing this way for proved reserves booking. To answer this question, a spread between the 50^{th} percentile and 90^{th} percentile lines must be determined. The objectives of this research are (i) to explore traditional knowledge for spread determination, (ii) to create a new analytical solution to quantify the spread if the traditional means fail, (iii) to analyze if the spread is substantial, (iv) to study sensitivity of the spread to other parameters such as numbers of measured data and economic limits, (v) to examine its application in actual field data, and (vi) to make recommendations to solve this issue and comply with the definition.

The remaining sections of this thesis are organized as follow. Chapter 2 is a literature review from previous research of reserves estimation and variance of intercepts estimation. Chapter 3 describes methods and discussions of this research. It shows a comparison between straight line fitting methods for exponential decline, the reasons why the least squares method is used for curve fitting, and the failure of traditional means to determine the spread. In addition, method of Monte Carlo simulation and analytical formulation and derivation are explained in this chapter. Chapter 4 contains results of analytical solution compared to Monte Carlo simulation, sensitivity analysis and a field application. The last chapter, 5, is the conclusions and recommendations of this research.

Chapter 2: Literature Review

The majority of the study involves the DCA technique, which is a reservoir engineering empirical technique that extrapolates trends in the production data from oil and gas wells. Typically, decline curve analysis is conducted on a plot of rate versus time or rate versus cumulative production. The most commonly used trending equations are those first documented by Arps (1945). Later on, the type-curve approach has been developed for analyzing extended producing histories. The fundamental theory of Arps equations is reviewed in this chapter.

The problem statement of this study is to determine the variability of the cumulative production so as to be able to estimate P90 reserves from the best fit trend which is more likely to be P50 reserves. The principle of the variance of intercepts, particularly the X-axis intercept, is anticipated to be the solution to the variability of the cumulative. The literature related to the estimation of the variance of the intercept is, then, studied in this chapter.

2.1 ARPS EMPIRICAL EQUATIONS AND TYPE-CURVE APPROACH

Arps (1945, 1956) applied a mathematical treatment to coalesce earlier works and concepts to unify the theory on the rate-time-cumulative production characteristics of production decline curves.

Arps defined the rate of change of the flow rate in terms of the decline rate D as.

$$D = -\frac{1}{q}\frac{dq}{dt} \qquad \text{(Eq. 1)}$$

He also defined the time-rate-change of the reciprocal of the decline rate in terms of b-component.

$$b = \frac{d(1/D)}{dt} \qquad \text{(Eq. 2)}$$

The b-component term should remain constant as the producing rate declines from an initial value to some later value. However, in many circumstances, changes in operating conditions cause these values to change. This variation therefore requires the well history to be divided into segments, with each segments representing a time of common exponent value.

Arps then developed equations representing exponential, hyperbolic, and harmonic production declines from the basic definitions of D and b. He defined

(i) b=0 to be the exponential case, for which the decline rate is constant,

(ii) 0<b<1 for the hyperbolic case, for which the decline rate is proportional to a fractional power b of the production rate, and

(iii) b=1 for the harmonic case, for which the decline rate is proportional to the production rate.

The exponential case, also called "constant decline", is the most commonly used because it is frequently observed when analyzing production data and it is also simple to use. The hyperbolic case is used in the case that an early decline is anticipated but a prolonged life is expected. Such a case cannot be adequately modeled by the exponential case because of the anticipated late life flattening of the rate. The harmonic case is a special case of the hyperbolic case where b=1, which is often found in horizontal wells, strong water drives and steam soak operations.

On the chart of Figure 5 are shown the trends of these three types of rate-time and rate-cumulative curves on regular coordinate paper, semi-log paper, and log-log paper.

Table 1 summarizes relationships between the decline rate, producing rate, time and cumulative production equations according to the classification of production decline curves.

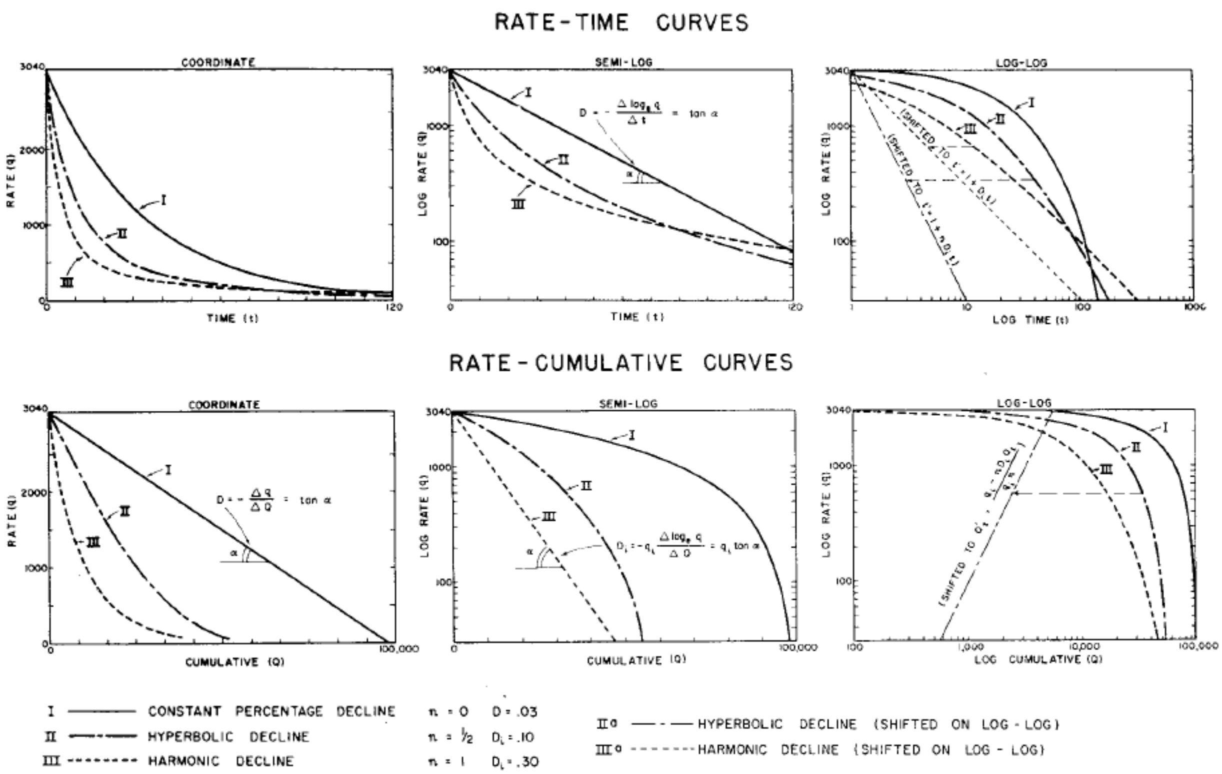

Figure 5 - Types of production decline curves on coordinate, semi-log, and log-log scales

(Arps, 1956)

	Decline Rate	Producing Rate, q	Elapsed Time, t	Cumulative Production, Q_t
Exponential	$\dfrac{\ln\left(\dfrac{q_i}{q_t}\right)}{t}$	$q_i e^{-D_i t}$	$\dfrac{\ln\left(\dfrac{q_i}{q_t}\right)}{D_i}$	$\dfrac{q_i - q_t}{D_i}$
Hyperbolic	$\dfrac{D_i}{D_t} = \left(\dfrac{q_i}{q_t}\right)^b$	$\dfrac{q_i}{(1 + bD_i t)^{\frac{1}{b}}}$	$\dfrac{\left(\dfrac{q_i}{q_t}\right)^b - 1}{bD_i}$	$\dfrac{q_i}{D_i(1-b)}\left[1 - \left(\dfrac{q_t}{q_i}\right)^{1-b}\right]$
Harmonic	$\dfrac{D_i}{D_t} = \dfrac{q_i}{q_t}$	$\dfrac{q_i}{1 + D_i t}$	$\dfrac{q_i - q_t}{D_i q_t}$	$\dfrac{q_i}{D_i}\ln\left(\dfrac{q_i}{q_t}\right)$

q_i - initial production rate $\quad$ D_i - initial decline rate

q_t - production rate at time t $\quad$ D_t - decline rate at time t

Table 1 - Summary of Arps Equations

(Poston and Poe, 2008)

When the decline curve does not fit one the three cases, it is more difficult to determine the appropriate values of b-component and the decline rate, D_i. However, Fetkovich (1980) developed a type curve to simplify that determination. He combined solutions to the diffusivity and Arps equations to provide a more general analysis method for covering a wide range of conditions. Figure 6 is a copy of the Fetkovich type curve for hyperbolic decline curves. By determining which of the type curves best match the production data, the correct value of b and D_i can be determined.

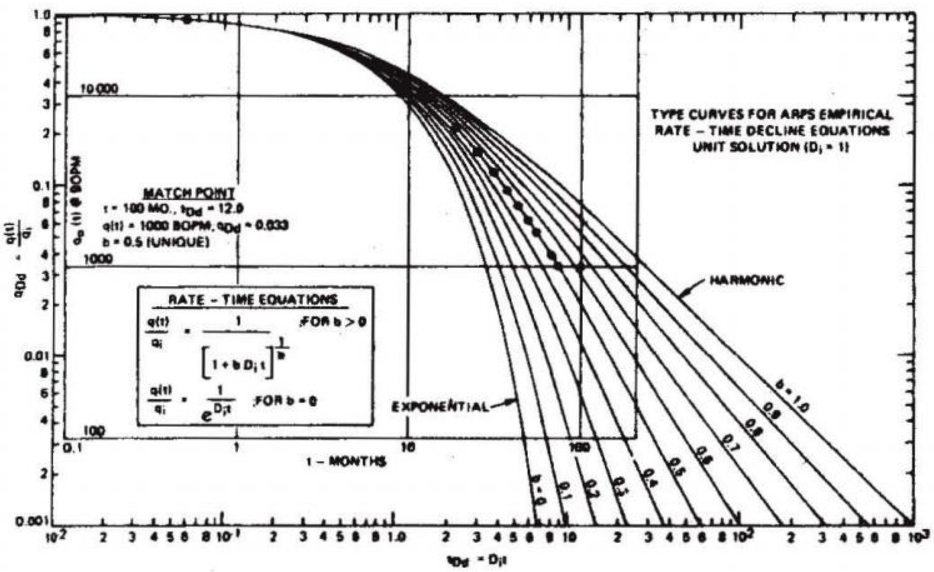

Figure 6 - Type-curve match of Arps hyperbolic decline example

(Fetkovich, 1980)

Apart from analyzing production data, the Fetkovich’s type-curve empirical equation has been adopted as a foundation for further development such as type curves for unconventional reservoir and for specific flow condition.

2.2 Estimation of Variance of Intercept

A "discrimination problem" provides a solution to estimate the unknown X-variables and bracket them in confidence intervals. As depicted in Figure 3, the ultimate recovery of an exponential decline model can be directly read from the chart at either the X-axis intercept or the economic limit. The discrimination problem may be applicable to this case which the variability of unknown X or the X-axis intercept is to be determined.

Let a regression model can be estimated by

$$\hat{\beta}_0 = Y_i - \hat{\beta}_1 X_i \qquad \text{(Eq. 3)}$$

where (X_i, Y_i) denote the measurements on each of n individuals

The regression model has ave

Miller (1981) applied a Bonferroni prediction interval and showed that limits of the (1-α) confidence region are given by

$$x = \bar{X} + \frac{(Y_i - \bar{Y})}{\hat{\beta}_1 \left[\frac{F_{1,n-2}^{\alpha/2} S^2}{\hat{\beta}_1^2 \sum_{i=1}^{n} (X_i - \bar{X})^2} - 1 \right]} \pm \frac{\sqrt{(Y_i - \bar{Y})^2 - \left[\frac{F_{1,n-2}^{\alpha/2} S^2}{\hat{\beta}_1^2 \sum_{i=1}^{n} (X_i - \bar{X})^2} - 1 \right] \left[F_{1,n-2}^{\alpha/2} S^2 (1 + \frac{1}{n}) - (Y_i - \bar{Y})^2 \right]}}{\hat{\beta}_1 \left[\frac{F_{1,n-2}^{\alpha/2} S^2}{\hat{\beta}_1^2 \sum_{i=1}^{n} (X_i - \bar{X})^2} - 1 \right]}$$

(Eq. 4)

where $F_{1,n-2}^{\alpha/2}$ is the critical value of F-distribution with significance level α and n-2 degrees of freedom

Jensen et al. (2000) estimated the same problem but replaced S^2 with the mean squared error, MS_ε, which is an approximation using the data. They also used a t-distribution instead. The limits, or error bounds, of X-variable are now given by

$$x = \overline{X} + \frac{(Y_i - \overline{Y})}{\hat{\beta}_1\left[\frac{\left(t_{n-2}^{\alpha/2}\right)^2 MS_\varepsilon}{\hat{\beta}_1^2 \sum_{i=1}^{n}(X_i - \overline{X})^2} - 1\right]} \pm \frac{\sqrt{(Y_i - \overline{Y})^2 - \left[\frac{\left(t_{n-2}^{\alpha/2}\right)^2 MS_\varepsilon}{\hat{\beta}_1^2 \sum_{i=1}^{n}(X_i - \overline{X})^2} - 1\right]\left[t^2 MS_\varepsilon(1+\frac{1}{n}) - (Y_i - \overline{Y})^2\right]}}{\hat{\beta}_1\left[\frac{\left(t_{n-2}^{\alpha/2}\right)^2 MS_\varepsilon}{\hat{\beta}_1^2 \sum_{i=1}^{n}(X_i - \overline{X})^2} - 1\right]}$$

(Eq. 5)

where $t_{n-2}^{\alpha/2}$ is the critical value of two-sided t-distribution with significance level α and n-2 degrees of freedom

However, these limits may not exist. For example, in the case of the X intercept, if the line slope could be zero at the required level of confidence, the X-axis intercept may be infinite. That is, if the ordinary least square regression line is parallel to the X-axis, i.e. $\hat{\beta}_1 = 0$, the intercept will be unbounded. Consequently, only meaningful confidence intervals can be obtained from (Eq. 5) if $\hat{\beta}_1$ is significantly nonzero.

Chapter 3: Method and Discussions

3.1 MONTE CARLO SIMULATION

"Monte Carlo simulation", or MCS, is a common technique for propagating uncertainties in the various aspects of a system to the predicted performance. It is a type of simulation that explicitly and quantitatively represents uncertainties. Monte Carlo simulation relies on the process of explicitly representing uncertainties by specifying inputs as probability distributions. If the inputs describing a system are uncertain, the prediction of future performance is inevitably uncertain.

In MCS, the entire system is simulated many times, i.e. over a hundred times. Each simulation is equally likely, referred to as a realization of the system. For each realization, all of the uncertain parameters are sampled (i.e., a single random value is selected from the specified distribution describing each parameter). The system is then simulated through time (given the particular set of input parameters) such that the performance of the system can be computed. This result is a large number of separate and independent results, each representing a possible result for the system. The results of the independent system realizations are assembled into probability distributions of possible outcomes. As a result, the outputs are not single values, but probability distributions. (GoldSim software introduction)

In this research, the uncertainty is production rate, which possibly comprises of errors from either rate fluctuations itself or measurement errors. Since the production rate is uncertain, the prediction of ultimate recovery is consequently uncertain; this is because cumulative production, which leads to ultimate recovery, is an integration of production rate over time.

In each realization, a single random value, or epsilon (ε) is selected from a uniform distribution representing the magnitude of errors that may occur in the system. The epsilon is then used to manipulate production rate to be erroneous rate. This process is so called "perturbating". After perturbations are added to the rate, the perturbed rates can produce hundreds of production profile; and distribution of ultimate recovery can be generated.

The next section contains general assumptions upon which the MCSs in this research are based, and then we follow with a detailed procedure of the Monte Carlo simulation.

3.1.1 General Assumptions

- Production rate, $\tilde{q}$, is the uncertain variable. It is a random, independent variable; the cumulative production, $\tilde{N}_p$, is the dependent variable.
- The true production profile can be forecasted with any DCA model. For most cases, we use the exponential decline model at one month time intervals. The initial rate, q_i , is 10,000 STB/d and decline rate, D_i , is 0.025/month. However, other models will also be used in the next section.
- The uncertainty associated with production rate measurement and/or rate fluctuation is arbitrary; the impact of varying uncertainties will be studied in the sensitivity analysis section. In most cases, we apply uncertainty at +/- 50% of certain values. Let epsilon, ε, be an uncertainty and uniformly distribute along U(-0.5,0.5).
- The economic limit is 50 STB/d.

- All the above variables are constant for the majority of the study. However, we will study their influences, i.e. the initial rate, the decline rate, the uncertainty, to the spread in the sensitivity analysis section.
- Volumetric estimates can be described by a log-normal distribution as the volumetric estimates consist mainly of multiplications of independent normally distributed parameters. Assuming the ultimate recovery is also based on the same distribution as the volumetric estimates, therefore, the probability distribution of the ultimate recovery is presumed to be log-normal.
- The system is simulated with 100 realizations.
- In compliance with the reserves notation, the data are arranged in decreasing order. The 90^{th} percentile value (P90) is the proved reserves, which have 90% certainty of the volume being recovered; while the 50^{th} (P50) and the 10^{th} (P10) percentile values are the probable and possible reserves, respectively.
- There is no cumulative production accounted before the prediction; therefore, the ultimate recovery is equal to the reserves.

3.1.2 Procedure of Monte Carlo Simulation

1. Generate production profile and cumulative production, referred as the true values, using exponential decline

$$q_t = q_i e^{-Dt} \quad \text{(Eq. 6)}$$

$$N_{p_t} = \frac{q_i - q_t}{D} \quad \text{(Eq. 7)}$$

2. Generate random numbers, ε, from a uniform distribution U(0,1)
3. Produce epsilons from multiplication of random numbers from 2. and arbitrary uncertainty

$$\varepsilon = (2\delta)[0,1] - \delta \quad \text{(Eq. 8)}$$

   where δ is the range of the uncertainty, which is ±0.5 in the base case

4. Generate the uncertain production histories, referred as perturbed rates, for a hundred realizations by applying epsilons to the true values, then determine the associated cumulative production by numerical integration.

   Imposing the condition that the expected value of a set of perturbed rates in each time step must be equal to true value of the rate at that particular time step

$$\tilde{q} = q_t(1+\varepsilon) = q_i e^{-D_i t}(1+\varepsilon) \quad \text{(Eq. 9)}$$

5. Calculate perturbed cumulative production corresponding to the set of perturbed rates

$$\tilde{N}_{pi} = \tilde{q}_i \Delta t + \tilde{N}_{pi-1} \quad \text{(Eq. 10)}$$

6. Fit a straight line to a set of perturb values of each realization (i.e. Cumulative production in x-axis and production rate in y-axis) by the fitting method, which will be presented in the next sections.
7. Calculate the variance of the perturbed cumulative production, Var($\tilde{N}_p$), at every time step and time interval and compare with estimates from the analytical formula, which will be presented in the next sections.
8. Determine the spread from the estimated variances (i.e. square of standard deviation, σ^2). As ultimate recovery is assumed normally distributed, the spread between its 50th and 90th percentiles, P50 and P90, can be calculated using critical value of the t-distribution and standard deviation.

   $$(P50 - P90) = t_{(0.1,\infty)} \times \sigma_{N_p} \qquad \text{(Eq. 11)}$$

   where $t_{(0.1,\infty)}$ is the critical value of two-sided t-distribution with significance level α=0.1 and infinite degrees of freedom

9. Determine the ultimate recovery of each realization from the best fit lines from step 5 that reaches the economic limit. Find the 90th percentile of ultimate recovery and compare with analytical estimate using the 50th percentile of ultimate recovery as a baseline and subtract with spread from step 8.

3.2 COMPARISON OF ORDINARY LEAST SQUARES AND MEASUREMENT ERROR MODEL

As known, the exponential decline curve, or constant percentage decline, forms a straight line on a semi-log rate-time plot and a coordinate rate-cum plot. Our study is, then, based mostly on the exponential decline curve of rate-cum on a regular coordinate scale as it could be analyzed in a straightforward manner; however, the study will be further extended to other decline models.

The relationship between the cumulative production and the production rate of the exponential decline is

$$q_t = -D_i N_p + q_i \qquad \text{(Eq. 12)}$$

The linear regression can then be determined from the rate-cum plot of the exponential cases. The slope of the linear regression is the decline rate and the intercept is the initial rate of the decline. Therefore, the straight line fitting method used in the study must be accurate and unbiased as much as possible because the curve fitting is the most critical part of the DCA technique. In this section, we study two fitting methods which are ordinary least square method and error-in-variables model. The best one will be used as the fitting method in our study.

When an independent variable consists of constant values at which corresponding measurements of the dependent variable are made (only dependent variable is assumed subject to error); it is well known that the ordinary least squares method, OLS, is the best unbiased linear estimator. However, in many circumstances, both the independent and dependent variables are subject to error and OLS may be inappropriate since its estimated slope is underestimated (Riggs et al. 1978) and biased toward zero (Fuller 1987). The measurement error model, thus, should be used to estimate the slope and intercept.

For our model, the production rate is the independent variable and the cumulative production is the dependent variable since the cumulative production is an integration of rate over a time period. This seems consistent to the case where OLS is applicable. However, it may be argued that both the independent and dependent variable are subject to errors because once there are errors in production rates, the errors also accumulate in the cumulative production. Therefore, the estimators of the measurement error model may provide better estimates.

Essential formulas for line fitting of OLS and measurement error model methods are as follow.

3.2.1 Ordinary Least Squares Method

Let a regression model have a relationship of the form

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \qquad \text{(Eq. 13)}$$

where i = 1,2,…,n and n is the number of data points, (x_i, y_i) denote the true values, β_0 and β_1 are true intercept and true slope, respectively; and ε_i are errors.

The estimators of OLS for slope and Y-axis intercept are given by

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - \overline{X})^2} \qquad \text{(Eq. 14)}$$

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X} \qquad \text{(Eq. 15)}$$

where (X_i, Y_i) denote the measurements on each of n individuals

The estimator of $\hat{X}_i$ can be determined by manipulating (Eq. 3)

$$\hat{X}_i = \frac{\hat{Y}_i - \hat{\beta}_0}{\hat{\beta}_1} \qquad \text{(Eq. 16)}$$

Or the X-axis intercept can then be estimated by $-\hat{\beta}_0 / \hat{\beta}_1$.

3.2.2 Measurement Error Model *(Classical Errors-in-Variables Model)*

The regression model of (Eq. 13) can be written in the form of the measurement error model as

$$(Y_t, X_t) = (y_t, x_t) + (e_t, u_t) \qquad \text{(Eq. 17)}$$

where (Y_t, X_t) is observed, y_t is the true value of the dependent variable, x_t is the true value of the independent variable, and (e_t, u_t) is the vector of measurement errors.

In the case that the ratio of measurement variances, (Eq. 18), is known, the model becomes a classical errors-in-variables model, EIV.

$$\delta = \frac{\sigma_e^2}{\sigma_u^2} \qquad \text{(Eq. 18)}$$

The estimator of EIV for slope is given by

$$\hat{\beta}_1 = \frac{S_{YY} - \delta S_{XX} + [(S_{YY} - \delta S_{XX})^2 + 4\delta S_{XY}^2]^{1/2}}{2S_{XY}} \qquad \text{(Eq. 19)}$$

where S_{YY}, S_{XY}, and S_{XX} are the sample variance of y (Eq. 20), covariance (Eq. 21), and variance of x (Eq. 22), respectively.

$$S_{YY} = \frac{\sum_{t=1}^{n}(Y_t - \bar{Y})^2}{n-1} \qquad \text{(Eq. 20)}$$

$$S_{XY} = \frac{\sum_{t=1}^{n}(X_t - \bar{X})(Y_t - \bar{Y})}{n-1} \qquad \text{(Eq. 21)}$$

$$S_{XX} = \frac{\sum_{t=1}^{n}(X_t - \bar{X})^2}{n-1} \qquad \text{(Eq. 22)}$$

The estimator of the EIV for intercept is similar to OLS, as shown in (Eq. 15).

3.2.3 Methods testing

True values are generated from an exponential decline model with q_i of 10,000 STB/d and D_i of 0.025 /month. The production history of the true values is depicted in Figure 7. Extrapolation to the X-axis intercept is the ultimate recovery, which is 12.02 MMSTB.

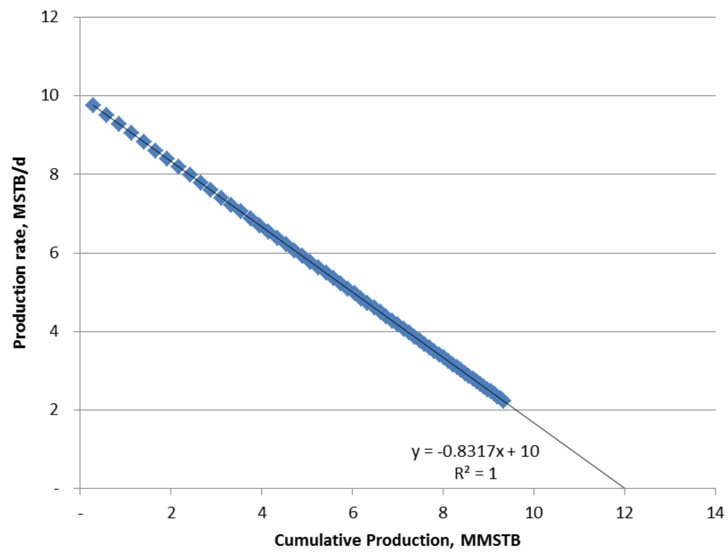

Figure 7 - Exponential decline of true values used in methods testing

The testing starts by following step 1 to 5 of the MCS. In step 5, the two methods are used to fit the perturbed data. EIV is fitted with perfect information of the ratio of errors variances. The ultimate recovery or the X-axis intercept can, then, be determined from estimated slope and intercept from each method. The results from 100 realizations are plotted as cumulative distribution function, CDF, as shown in Figure 8. The $90^{th}/50^{th}/10^{th}$ percentile values of each method can then be calculated from the CDF and tabulated in Table 2.

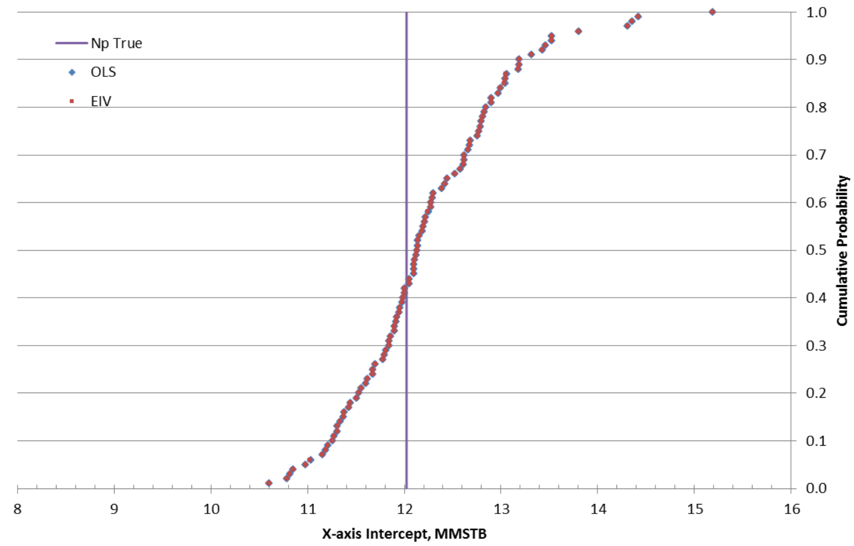

Figure 8 - Cumulative distribution function of the X-axis intercepts from 100 realizations

x-axis Intercept, MMSTB	**P90**	**P50**	**P10**	**σ**	**Error of P50 from true value**
True value		12.02			
OLS	11.28	12.14	13.20	0.84	0.94%
EIV	11.28	12.14	13.20	0.84	0.93%

Table 2 - Results of methods testing compared to the true value

It is obvious that the two methods provide the same estimates. This is because of the substantial difference between the magnitudes of cumulative production, X, and production rate, Y. This makes the values of $\hat{\beta}$ calculated by the EIV approach the values given by the OLS. The unknown ratio of error variances is, for this case, not a problem as the OLS still provide good estimates for a straight line fitting. The applicability of OLS means that no error in X is assumed, all errors are subject to only Y. The OLS is, therefore, selected as the fitting method for further study in the next sections.

Since the estimators by OLS have proved to be accurate for our problem, the data must be consistent with the Gauss-Markov theorem. The theorem says that the ordinary least squares coefficient estimators are the best of all linear unbiased estimators (BLUE) of β_0 and β_1, where "best" means "minimum variance".

GAUSS-MARKOV THEOREM

Sen et al. (1990) claims that the least squares is a good (BLUE) estimator if the following conditions are met.

$$E(\varepsilon_i) = 0 \qquad \text{(Eq. 23)}$$

This implies that the expectation $E(y)$ of y_i actually is $\beta_0 + \beta_1 E(x)$ for all i.

$$E(\varepsilon_i \varepsilon_j) = 0 \qquad \text{(Eq. 24)}$$

This condition is required for the samples to be uncorrelated for all $i \neq j$.

$$Var(\varepsilon_i) = E(\varepsilon_i - E(\varepsilon_i))^2 = E(\varepsilon_i^2) = \sigma^2 \qquad \text{(Eq. 25)}$$

This condition is imposed to prevent heteroscedasticity, which is when some data have different variabilities or variances from others.

These conditions can be obviously found from one of the field data. Figure 9 is the field data from Yates oil field in West Texas. The straight line can be fitted to the production history; the straight line, therefore, represents the true decline of the reservoir.

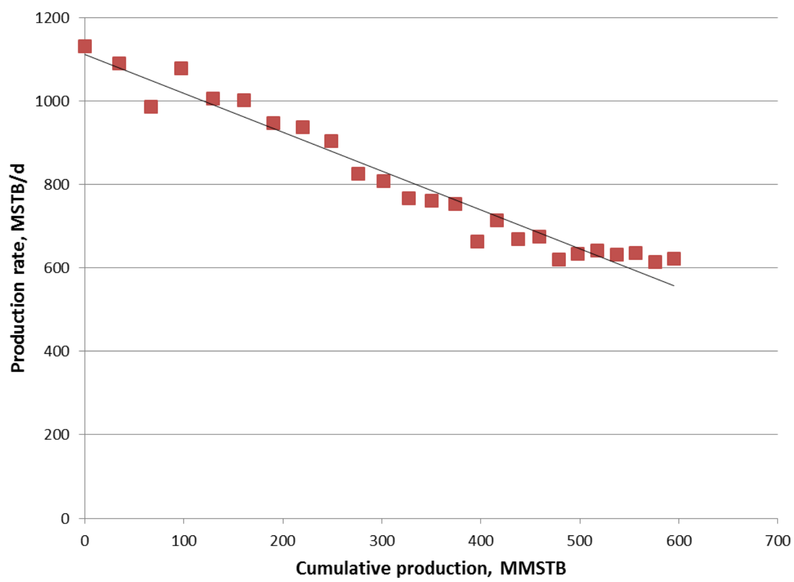

Figure 9 - Production history from Yates oil field during 2000-2002

The errors associated with the true values (the straight line) can then be calculated from residuals, (Eq. 26), and shown in the terms of relative error, (Eq. 27).

$$residual = q - \hat{q} \qquad \text{(Eq. 26)}$$

where q is an observed rate and $\hat{q}$ is the predicted rate

$$\%error = \frac{q - \hat{q}}{\hat{q}} \qquad \text{(Eq. 27)}$$

Table 3 shows the production history of the Yates oil field as well as the calculated residuals and the relative errors. The average of the relative errors is 1.41%, which is nearly zero. This result supports our claim that the errors have zero expected value.

Days	Rate (MSTB/d)	Cum (MMSTB)	Predicted rate (MSTB/d)	Residuals/ Errors (MSTB/d)	%Error
0	1131.68	0.00	1111.70	19.98	-1.80
31	1089.70	35.08	1081.11	8.59	-0.79
60	985.58	66.68	1053.26	-67.68	6.43
91	1078.75	97.24	1024.28	54.47	-5.32
121	1006.39	129.60	996.99	9.40	-0.94
152	1001.86	160.80	969.56	32.29	-3.33
182	946.56	190.85	943.74	2.83	-0.30
213	937.37	220.20	917.77	19.60	-2.14
244	902.79	249.26	892.52	10.27	-1.15
274	826.11	276.34	868.74	-42.63	4.91
305	807.24	301.95	844.84	-37.60	4.45
336	766.52	326.97	821.59	-55.07	6.70
367	761.12	350.73	798.99	-37.87	4.74
398	752.16	374.33	777.00	-24.85	3.20
427	661.85	396.14	756.99	-95.14	12.57
458	714.01	416.66	736.16	-22.15	3.01
488	667.97	438.08	716.55	-48.58	6.78
519	674.44	458.79	696.83	-22.39	3.21
549	618.81	479.02	678.27	-59.46	8.77
580	633.96	498.20	659.61	-25.64	3.89
611	640.95	517.86	641.46	-0.51	0.08
641	630.83	537.08	624.37	6.46	-1.03
672	635.48	556.64	607.19	28.28	-4.66
703	612.96	576.34	590.49	22.47	-3.81
734	621.34	595.34	574.24	47.10	-8.20
Average					1.41

Table 3 - Residuals and relative errors from the Yates production history

The 1st and 2nd serial autocorrelations between residuals are plotted in Figure 10 and Figure 11, respectively. It is obviously no correlation between the two successive residuals in both 1st and 2nd serial autocorrelations. Therefore, the statement that the errors are uncorrelated is valid.

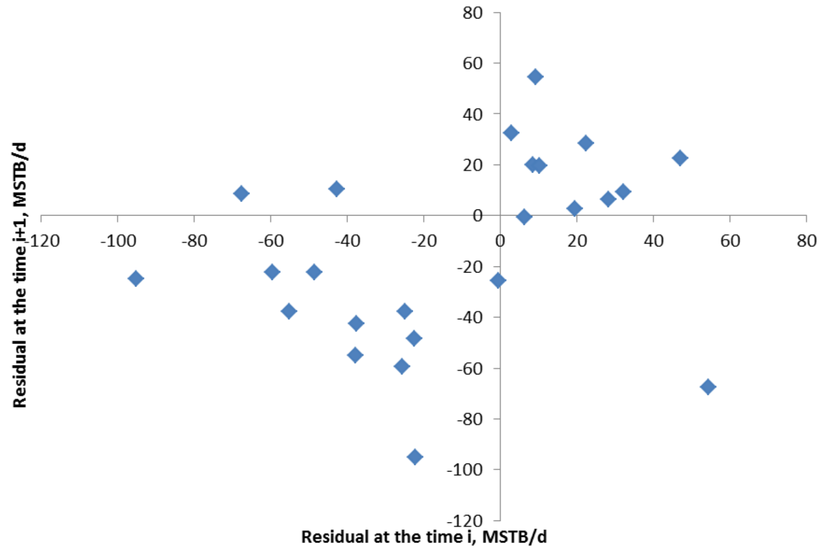

Figure 10 - Plot of successive residuals from Yates production history (1st serial autocorrelation)

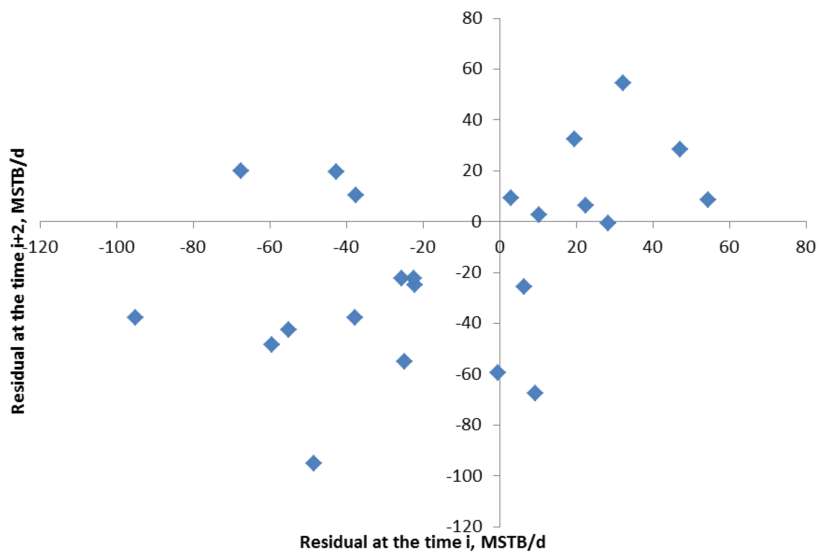

Figure 11 - Plot of successive residuals from Yates production history (2nd serial autocorrelation)

In short, the field data is consistent to the Gauss-Markov theorem that the errors have an expected value of zero, are uncorrelated, and have constant variance.

3.3 ERROR BOUNDS OF THE X-VARIABLE VS. P10/P90 OF MCS RESULTS

The confidence interval of the X-axis intercept, or error bounds, is developed from the standard error of the estimated slope with a certain level of confidence. This confidence interval contains the true intercept when the samples are repeatedly taken at the same times. We hypothesize that the error bounds estimation could be used to estimate the variance of the cumulative production. It is then tested whether it can be applied to estimate the spread. The test starts by following the procedure of MCS from step 1 to 5. The CDF can then be generated from the X-intercepts, the 10^{th} and 90^{th} percentile values are, accordingly, determined to compare with the error bounds.

For the error bounds determination, (Eq. 4), the estimated slope, $\hat{\beta}_1$, from the regression trend of each realization is bracketed to a confidence interval. The confidence level is at 80% because its limits are equivalent to the 90^{th} and 10^{th} percentile values. Figure 12 shows the CDFs of possible recoveries. We then select error bounds from the 50^{th} percentile values of the upper and lower bounds. These error bounds are then compared to the MCS results. The table of the comparison is in Table 4.

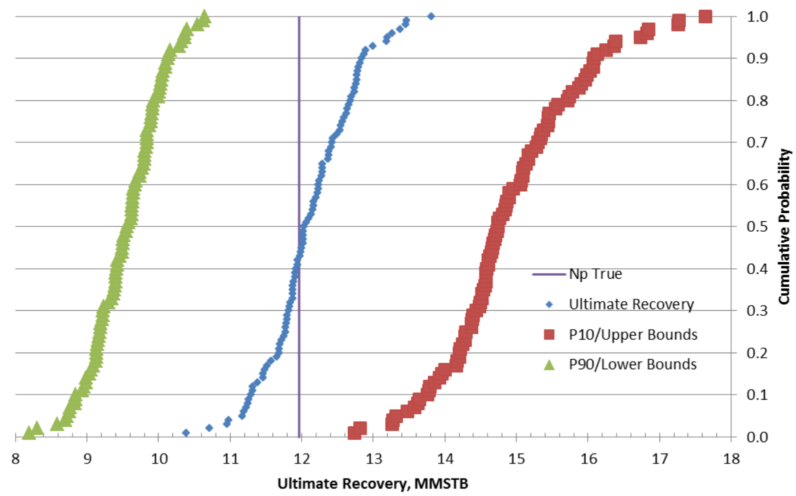

Figure 12 - Cumulative distribution function of the X-axis intercepts and its error bounds

Ultimate Recovery, MMSTB	**Spread**	**P10**	**P50**	**P90**
True Value			11.96	
MC results	0.51	11.63	12.13	12.51
Error Bounds	2.75	9.38	12.13	15.07

Table 4 - Comparison between P10/P90 of MCS and error bounds at equivalent levels of confidence

The confidence interval can be determined from a population mean or future (predicted) observations. The confidence interval of the population mean is often narrower than of the future values as depicted in Figure 13. The confidence interval of the population mean underestimates the uncertainty in $\hat{Y}_i$ because it does not take into account the variability of the future observations. Generally, the confidence interval of the population mean is infrequently required, most of the inferences being based upon the estimation of a distinct, predicted $\hat{Y}_i$, which is not known at the time when the regression is calculated. The error bounds determination is based upon the confidence interval for future observations.

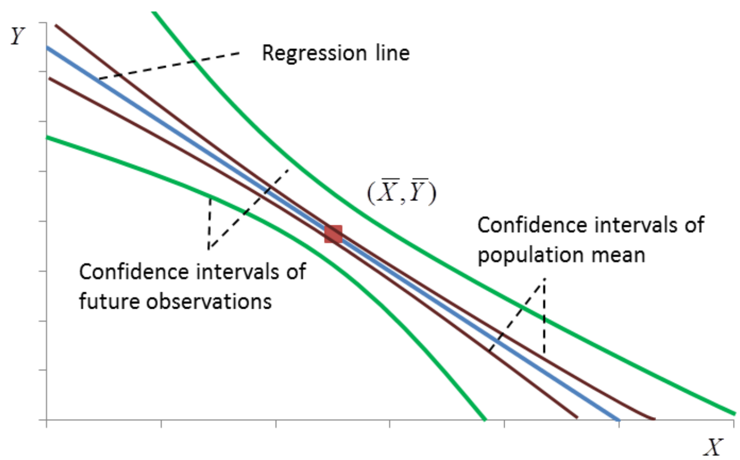

Figure 13 - Example of the confidence intervals for the regression line

The band formed by the confidence interval for all X values is called the Working-Hotelling confidence band. The confidence intervals form hyperbolic lines, as seen in Figure 13, meaning that the confidence interval depends on the value of X. The farther the value of X departs from $\overline{X}$, the larger is the confidence interval. This shortfall is the reason why the variance estimated by the error bounds is overestimated.

In addition, the error bounds determination is based on the principle of the standard error, while the standard deviation is of our interest. The standard error is the uncertainty from the mean and describes the accuracy of the mean. On the other hand, the standard deviation is how far an individual point is from the sample mean and also describes the variability of the individual values. If the sample size increases, the standard error will decrease because the estimator becomes more precise with more information. Alternatively, an increase in sample size will bring the standard deviation closer to the population standard deviation. This means that there is a difference between the standard error and the standard deviation. Thus, the error bounds determination should not be applied to estimate the standard deviation, or the spread.

3.4 Analytical Formulation and Derivation

The key to our problem is to determine the standard deviation or the variance of the cumulative production; consequently an analytical approach for the determination must be developed. We have identified the problem, which is the variability of the cumulative production. Then we must fully understand all relevant parts of the problem. The sources of the variability are the keys and must be investigated. The insights are the foundation for developing an analytical formula for estimating the variability of each element.

The cumulative hydrocarbon produced is an integration of rate over a period of time. The time at each point is deterministic and remains unchanged regardless of any events. The rate, therefore, is the only variable causing the variability in the cumulative production. The true rate is difficult to measure and, most of the time, the observed rate is subject to errors from measurement error and/or rate fluctuation by reservoir itself or operating conditions.

We can then show the measured rate in terms of a deterministic component of the true rate, q, and a random error term of value, ε. The observed rate, $\tilde{q}$, for time interval i is written as

$$\tilde{q}_i = q_i \left(1+\varepsilon_i\right) \qquad \text{(Eq. 28)}$$

(Eq. **28**) defines the error model for this problem. In what follows $\tilde{q}_i$ is stochastic and q_i is deterministic. We assume the model and the model parameters for q_i are determined without error. The model parameters, i.e. initial rate, decline rate and b-component, are not subject to error. The rate q_i generated from such a model is then free of error. The q_i is, literally, a consequence of the reservoir depletion or the true rate. All the errors that may occur are aggregated in the epsilon term.

According to the Gauss-Markov theorem in section 3.2, the error term or epsilon, ε, has zero mean and is independent of everything and uncorrelated to others.

$$E(\varepsilon_i) = E(\varepsilon_i \varepsilon_j) = 0 \quad \textit{when } i \neq j \qquad \text{(Eq. 29)}$$

where i and j are time interval

The expected value of the measured rate is therefore

$$\begin{aligned} E(\tilde{q}_i) &= E\left(q_i(1+\varepsilon_i)\right) \\ &= E(q_i) + E(q_i \varepsilon_i) \\ &= E(q_i) + q_i E(\varepsilon_i) \\ &= E(q_i) \end{aligned} \qquad \text{(Eq. 30)}$$

In other words, the expected value of the deterministic rate and the measured rate are the same.

The cumulative production over a time interval from $i = I$ to $i = J$ is

$$N_{pI} = \Delta t \sum_{i=I}^{J} \tilde{q}_i = \Delta t \sum_{i=I}^{J} \left(q_i(1+\varepsilon_i)\right) \qquad \text{(Eq. 31)}$$

I is the initial and J is the terminal time interval, as illustrated in Figure 14. The extrapolation begins at $I = 0$, where the last measured data is, and continues to J, which can be at any rate but is often at an economic limit (for economic reserves) or zero (for mobile reserves).

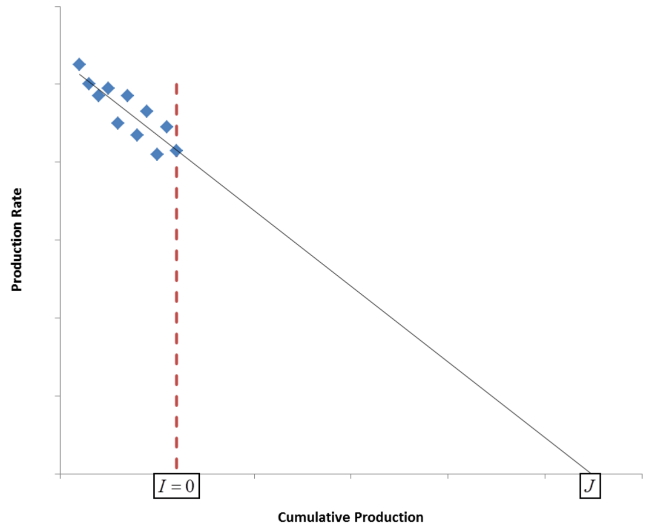

Figure 14 - Predicted production decline from the last measured data, $I=0$, to the economic limit, J

Now the variance of the cumulative production from $i=I$ to $i=J$ is

$$Var(N_{pJ}) = Var\left(\Delta t \sum_{i=I}^{J} \tilde{q}_i\right) = \Delta t^2 Var\left(\sum_{i=I}^{J} \left(q_i(1+\varepsilon_i)\right)\right) \qquad \text{(Eq. 32)}$$

Since q_i is deterministic, it behaves like a constant and

$$Var(X+c) = Var(X) \qquad \text{where } c \text{ is any constant} \qquad \text{(Eq. 33)}$$

Thus, we get

$$Var(N_{pJ}) = \Delta t^2 Var\left(\sum_{i=I}^{J} q_i \varepsilon_i\right) \qquad \text{(Eq. 34)}$$

Recall a definition of variance,

$$Var(X) = E(X^2) - E(X)^2 \qquad \text{(Eq. 35)}$$

Thus,

$$Var(N_{pJ}) = \Delta t^2 \left[E\left(\left(\sum_{i=I}^{J} q_i \varepsilon_i \right)^2 \right) - E\left(\sum_{i=I}^{J} q_i \varepsilon_i \right)^2 \right] \qquad \text{(Eq. 36)}$$

Since $E(\varepsilon_i)$ is zero, the last term becomes zero.

$$\begin{aligned} Var(N_{pJ}) &= \Delta t^2 E\left(\left(\sum_{i=I}^{J} q_i \varepsilon_i \right)^2 \right) \\ &= \Delta t^2 E\left(\left(\sum_{i=I}^{J} q_i \varepsilon_i \right) \left(\sum_{i=I}^{J} q_i \varepsilon_i \right) \right) \\ &= \Delta t^2 E\left(q_I \varepsilon_I \left(\sum_{i=I}^{J} q_i \varepsilon_i \right) + q_{I+1} \varepsilon_{I+1} \left(\sum_{i=I}^{J} q_i \varepsilon_i \right) + \ldots + q_J \varepsilon_J \left(\sum_{i=I}^{J} q_i \varepsilon_i \right) \right) \end{aligned} \qquad \text{(Eq. 37)}$$

Since the errors are uncorrelated, (Eq. 29), the error terms associated with two different intervals, $i \neq j$, become zero. Thus, (Eq. 37) becomes

$$Var(N_{pJ}) = \Delta t^2 E\left(q_I^2 \varepsilon_I^2 + q_{I+1}^2 \varepsilon_{I+1}^2 + \ldots + q_J^2 \varepsilon_J^2 \right) \qquad \text{(Eq. 38)}$$

The final result is

$$Var(N_{pJ}) = Var(\varepsilon_i) \Delta t^2 \sum_{i=I}^{J} q_i^2 \qquad \text{(Eq. 39)}$$

Since the epsilon is assumed to be uniformly distributed over +/- a certain number, the variance of epsilon can be determined using a following equation.

$$Var(X) = \frac{(b-a)^2}{12} \qquad \text{(Eq. 40)}$$

where X is uniformly distributed over a finite interval [a, b]

This analytical formula, (Eq. 39), can estimate the variance of the cumulative production at any time, J, as long as a deterministic production profile from I to J is known. This means that the formula can be applied to any models or production histories.

Chapter 4: Results

4.1 Comparison between the Monte Carlo simulation and the analytical estimation

4.1.1 Variance

Before applying the analytical solution, (Eq. 39), to the study, it must be tested for its robustness. Assuming a single production decline as a baseline or true values, errors are perturbed to the true values in the Monte Carlo simulation and variance of the cumulative production is determined for 60 months. The variance at each time interval is plotted against variance estimation from analytical formula. Because the magnitude of the variances can be large, the relative error, (Eq. 41), is calculated to help clarifying the comparison.

$$Relative\ error = \frac{Var(N_p)_{MCS} - Var(N_p)_{Analytics}}{Var(N_p)_{MCS}} \qquad \text{(Eq. 41)}$$

Figure 15 shows the results of the cumulative production variances and compared to analytical solution. The results from MCS in blue dots mostly agree with analytical estimations. The discrepancy at the very beginning is considerably large but it gradually drops as time interval increases. At last time intervals, relative error between MCS and analytical solution is about zero. This agreement proves that the analytical formula is applicable to the estimation of the variance of the cumulative production. The bias of less than 5% relative error is acceptable. The relative error decrease as time interval goes by. Therefore, the analytical estimation of the variance will be used to represent actual variances for further study.

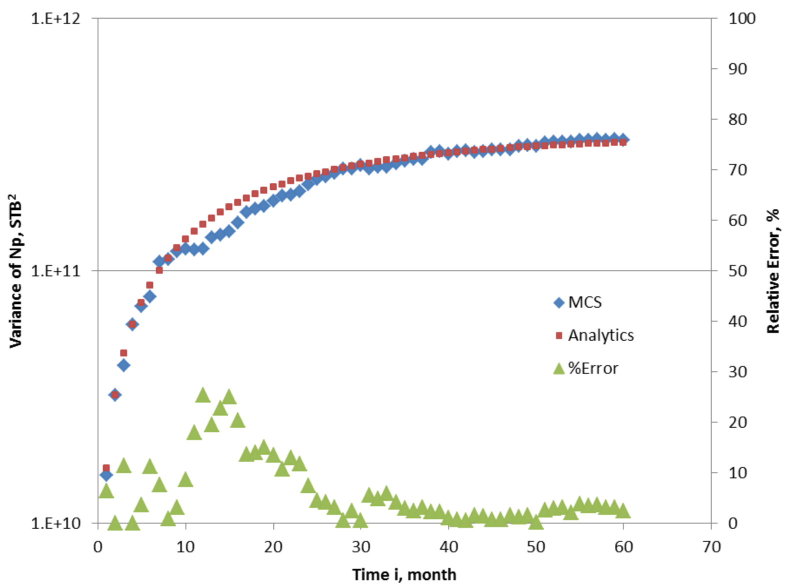

Figure 15 - Variance of cumulative production from exponential decline by time interval after 100 realizations

The previous result is based on the exponential decline model. To further test this analytical formula, the production model is changed to hyperbolic and harmonic models. The model parameters remain at the initial rate of 10,000 STB/d, the decline rate of 0.025/month, and b=0.5 for hyperbolic and b=1 for harmonic cases. The production profiles generated by the two models are now the true values for the MCS. Following the rest procedure of the MCS, we can compare the results between the MCS and analytics in Figure 16 for the hyperbolic case and in Figure 17 for the harmonic case.

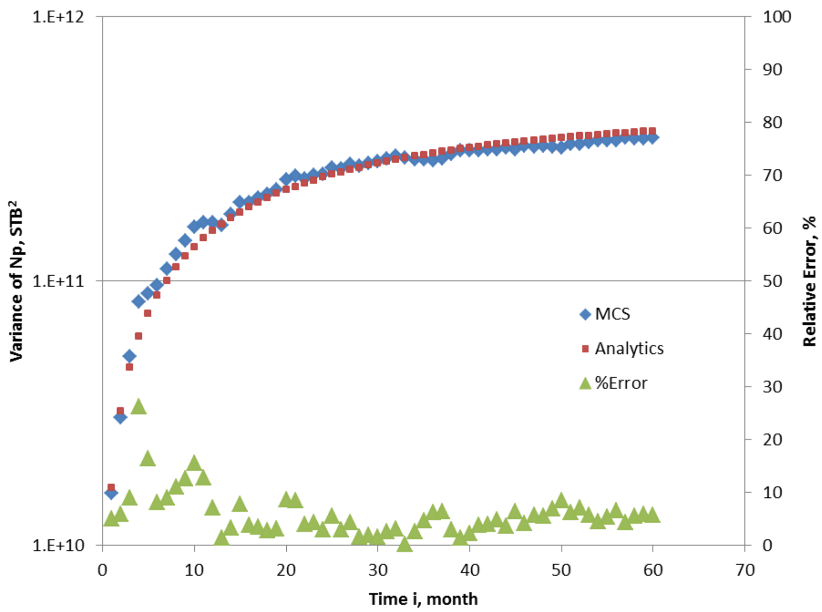

Figure 16 - Variance of cumulative production from hyperbolic (b=0.5) decline by time interval after 100 realizations

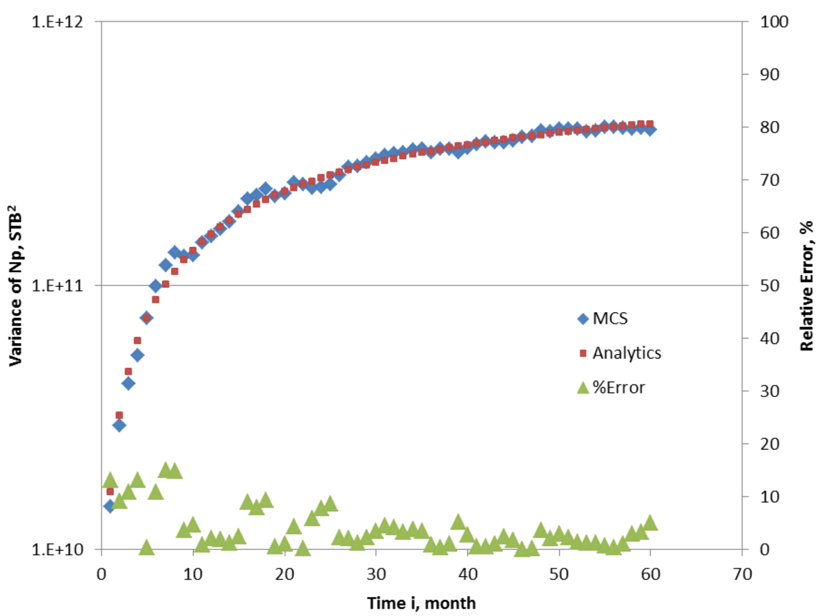

Figure 17 - Variance of cumulative production from harmonic decline by time interval after 100 realizations

The analytical estimates still agree with the MCS results even though the decline model is changed. This suggests that the robust analytical formula is applicable to any events as long as a deterministic production profile is known for the time period in the interest of estimating the variance.

4.1.2 Spread

The analytical formula is now applied to estimate the variance of the ultimate recovery, or the cumulative production at the economic limit. MCS is used to forecast possible production profile and generate a CDF, including the P50 of the ultimate recovery. Since the ultimate recovery is assumed to be lognormal distributed, $lognormal(\mu_{\ln UR}, \sigma_{\ln UR})$, the standard normal score, z-score, can be applied to the analytical variance and the P50 from MCS to determine the P90 and P10, using the following formulas.

$$\mu_{\ln x} = \ln \mu_x - \frac{1}{2}\sigma_{\ln x}^2 \quad \text{(Eq. 42)}$$

$$\sigma_{\ln x} = \sqrt{\ln\left(1 + \frac{\sigma_{\ln x}^2}{\mu_x^2}\right)} \quad \text{(Eq. 43)}$$

$$x_p = e^{Z_p \sigma_{\ln x} + \mu_{\ln x}} \quad \text{(Eq. 44)}$$

where x_p is the value of x corresponding to the percentile p, and z_p is the standard normal score value at the percentile p.

In our base case of exponential decline the least square method is used to determine the best fit straight line and extrapolation of the trend is performed to find the ultimate recovery in each realization. We then assume the 50th percentile value as the mean of the ultimate recovery, μ_x, and the standard deviation of the ultimate recovery is what determined by our analytical formula. The standard score associated with the 90th and 10th percentile values are +/- 1.28. We can then calculate the ultimate recovery at the 90th and 10th percentile, and associated spread (P90-P50).

Ultimate Recovery, MMSTB	Spread	P90	P50	P10
True Value			11.96	
MC results	0.51	11.63	12.13	12.51
Analytics	0.48	11.65	12.13	12.62

Table 5 - Results of ultimate recovery from the MCS and the analytical solution (Exponential model)

The P90/P10 ultimate recoveries estimated from the analytical formula are compared to the MCS results as in Table 5. The spread from MCS is 0.51 while the spread estimated by the analytical formula is 0.48; the discrepancy seems insignificant, about 4% relative error. However, if we consider in term of P90, the analytics gives 11.65 MMSTB, which is only 0.2% different from the MCS result of 11.63 MMSTB; the discrepancy between the MCS and analytics is, thus, insignificantly small. Using the analytical variance estimation in P90 determination gives a reasonably good result.

The P90 by MCS (11.63 MMSTB) is 4.17% less than the predicted P50 (12.13 MMSTB) and 2.80% less than the true reserves (11.96 MMSTB); while the analytic P90 (11.56 MMSTB) is 4.00% less than the predicted P50 (12.13 MMSTB) and 2.63% less than the true reserves (11.96 MMSTB).

Again, our study is extended to other models, i.e. hyperbolic and harmonic models. Even though the results of the hyperbolic model (b=0.5) in Table 6 show a larger spread and higher discrepancy, 11% relative error, the analytical P90 estimation is satisfactory with only 0.33% less than the MCS. In the harmonic case, the discrepancy of the spread between MCS and analytics is larger as shown in Table 7. The relative error of P90 between MCS and analytics is approximately 2%. Therefore, the analytical solution can be used for the spread estimation with any production model as long as a deterministic production profile can be determined. Moreover, the results validate the assumption that the cumulative production is normally distributed.

Ultimate Recovery, MMSTB	Spread	P90	P50	P10
True Value			22.63	
MC results	0.64	21.79	22.44	23.11
Analytics	0.57	21.87	22.44	23.01

Table 6 - Results of ultimate recovery from MCS and analytical solution (Hyperbolic model, b=0.5)

Ultimate Recovery, MMSTB	Spread	P90	P50	P10
True Value			64.51	
MC results	2.02	62.48	64.51	66.05
Analytics	0.71	63.80	64.51	65.21

Table 7 - Results of ultimate recovery from MCS and analytical solution (Harmonic model)

Although the spread in the hyperbolic (b=0.5) and harmonic models are only 2.5% and 1.1% of the P50, respectively, both models are not frequently used for the reserves estimation, unless there is a history of pressure supported production, as they provide quite optimistic forecasts. The exponential decline is normally applied for reserves calculations as it provides conservative forecasts and reflects a "reasonable certainty" standard. Figure 18 shows all three decline equations fit nearly exactly to the production history. The exponential decline forecast will produce the most declines in rates; hence provide the most conservative forecast. The spread from exponential model is solely discussed further.

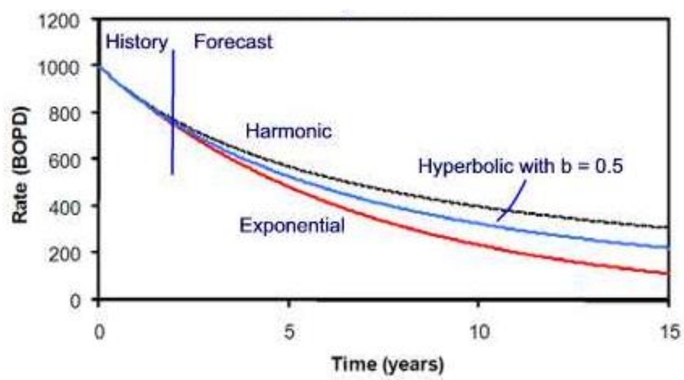

Figure 18 - Production forecast with different decline models (petrocenter.com)

According to Table 5, we generally book proved reserves from the best fit regression extrapolation, which is 12.13 MMSTB. Instead, if the P90 by the analytical solution is booked as the proved reserves, about 4% of the original figure will have to be removed from the proved reserves and moved to probable reserves. In the aspect of finance, half of a million of oil barrels is pulled from the primary asset of a company; equivalently, a value of the company is lessened by $40 million (based on oil price of $80/STB).

According to the 2013 SEC report, ExxonMobil's proved developed reserves were 27.7 billion oil-equivalent barrels. Assuming 5% of the reserves were determined from the DCA, if we adopted our method for the reserves booking, 55.4 million oil-equivalent barrels would have been removed from the proved reserves category. The 4% reduction in proved reserves may be unsubstantial but it would definitely impact the company's value.

4.2 Sensitivity Analysis

Sensitivity analysis is a technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. The procedure of the sensitivity analysis in this study is very similar to those most often employed in the engineering literature. It is based on the idea of varying one uncertain parameter value at a time. The various parameters are assigned high and low values. The system model is then run with the various parameters, one at a time, to evaluate the impact of those changes in various sets of parameter values on the spread.

According to the analytical formulation, the spread is derived from production rates, which are assumed to follow an arbitrary decline model. Each model requires initial rate, decline rate and terminal time interval. The initial rate depends on the starting time, previously denoted by I. The terminal time interval, or terminal rate, depends on the economic limit, denoted by EL. These parameters, as well as error variability of the production rates, are studied their influences on the spread.

4.2.1 Sensitivity to number of additional measured data

The last measured data is where we start estimating the variance of the estimated cumulative production. We denote the starting point as $I = 1$ to be the first point after the last measured data. Assuming a constant time step, the time interval when $I = 1$, $t_{I=1}$,is the time at last measured data plus one. If there is an additional datum, $t_{I=1}$ would change. The starting point is then varied by adding additional measured data.

Impacts of the additional measured data can be determined in 2 cases whether or not the additional data still follow the previous trend $(i < I)$. If they do, the starting point is changing but the rate model remains the same. If they do not follow the trend, not only the starting point is changing but the rate model is also varying.

In case that the additional data do follow the previous trend, only I is varying to simulate the cases for additional measured data of 0-150 while the rate model remains the same (fixed true values). The economic limit remains constant for all the cases; therefore, the terminal point, J, is fixed. The interval between I and J, or the variance prediction interval, is smaller as the number of additional points increases.

Another case is that the additional data do not follow the previous decline. Now the rate model will change as the measured data increases. We may consider this case as varying the rate model, or the changed true values (varied true values). When the true values change, the analytical variance estimation definitely changes. The spread will then be affected by not only the changing prediction interval but also the varying decline model.

Table 8 demonstrates that the analytics with varying true values are slightly different from the analytics with fixed true values; suggesting that the varying of true values slightly affects the spread.

Number of additional data	Spread P90-P50 (MMSTB)		
	MCS	Analytics	
		Fixed true values	Varied true values
0	0.4138	0.4975	0.4975
20	0.2785	0.3017	0.3072
40	0.2008	0.1830	0.1832
60	0.1177	0.1110	0.1107
80	0.0630	0.0673	0.0654
100	0.0424	0.0407	0.0390
120	0.0231	0.0246	0.0240
150	0.0148	0.0114	0.0108

Table 8 - Comparison of the sensitivity of spreads to number of additional measured data by MCS vs. Analytics with fixed and varied true values

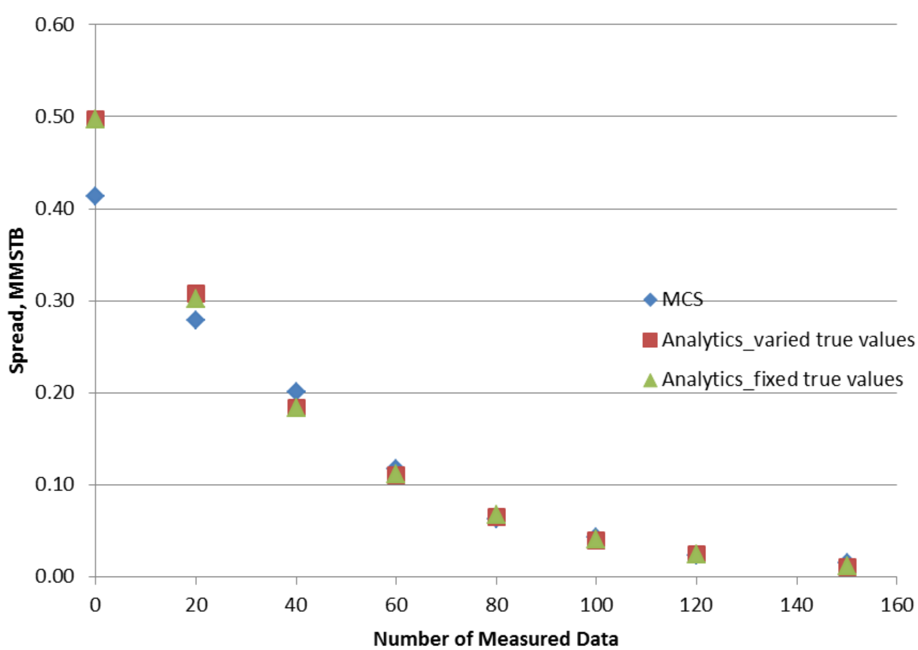

Figure 19 - Sensitivity of the spread to the number of additional measured data

Figure 19 displays the results of the 2 cases compared to the MCS results. Whether or not the additional data follow the previous trend; the spread decreases as the number of additional data increases. The spread exponentially decreases and approaches zero as the number of data increases. The variance of the cumulative production reduces as the prediction interval gets smaller; which means that more information can reduce the uncertainty. The influence of the more data on the spread is greater than that of the varying true values.

4.2.2 Sensitivity to economic limits

Economic limit, EL, is the value where the production just equals the cost of keeping the well or wells on production. The EL is then the rate at which the ultimate economic recovery can be determined. Varying EL can be treated as varying the terminal point, J while I is kept constant. Therefore, the prediction interval increases as the EL lowers. EL is assumed at 50 STB/d in the previous sections; now the EL is varied from 1 to 500 STB/d. The results of the sensitivity of the spread are in Figure 20.

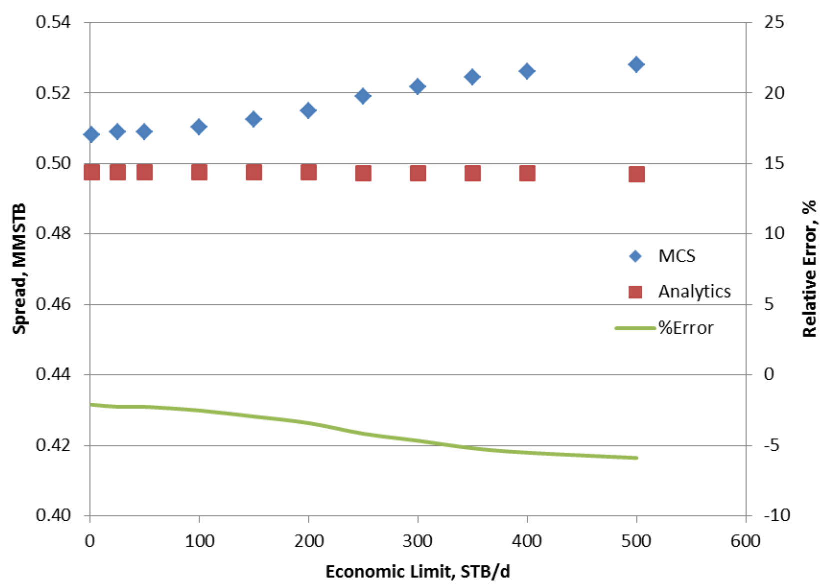

Figure 20 - Sensitivity of the spread to economic limits

The spread variation is trivial. Considering the trend of the analytical spread, the analytical spread slightly increases as EL reduces. Theoretically, lowering EL is the same as increasing J and the interval from I to J is larger; leading to more uncertainty in the estimation because the expected trend line must be extrapolate further to reach the EL. However, the analytical results conflict with the MCS results in that the spread decreases with lowering EL. The MCS spread does not reduce because the terminal rate at higher EL is higher than the terminal rate at the lower EL. The uncertainty associated with higher rate is definitely more than that associated with the lower rate, resulting in the higher spread. This statement is consistent with the lower EL cases that the ultimate recovery is higher but the spread is lower than the higher EL cases.

4.2.3 Sensitivity to error variability

The error variability, or variance of the error, causes observed values to deviate from actual values. In the MCS, the possible observed values are simulated by perturbing production rates. By the word "perturbation", an error term, epsilon, is manipulated (Eq. 28) and so the perturbed rates are produced. The error term is added on the basis of percentage of actual value, which is 50% in the previous sections. The larger the error term, the broader the range of the observed values.

Since the error variability is a direct result of the %error adding to perturbation, (Eq. 40), the sensitivity analysis is conducted for the %error from 0-100%. Figure 21 shows a positive correlation between the spread and the %error, i.e., the spread increases as the %error increases. The spread is zero when there is no perturbation, when the perturbed rates equal to the true rates and so no variance is produced. The linear proportion between the increase in the spread and %error is consistent to the analytical formula, (Eq. 39), that the variance of the cumulative production is linearly proportional to the variance of the error, as shown below.

$$perturbation \quad \propto \quad \sigma_{\varepsilon}^{2}$$

And from (Eq. 40), the uncertainty is assumed to be uniformly distributed over +/- %error, then $\sigma_{\varepsilon}^{2} = \dfrac{(\%error - (-\%error))^{2}}{12}$

Thus, $perturbation \quad \propto \quad \%error$

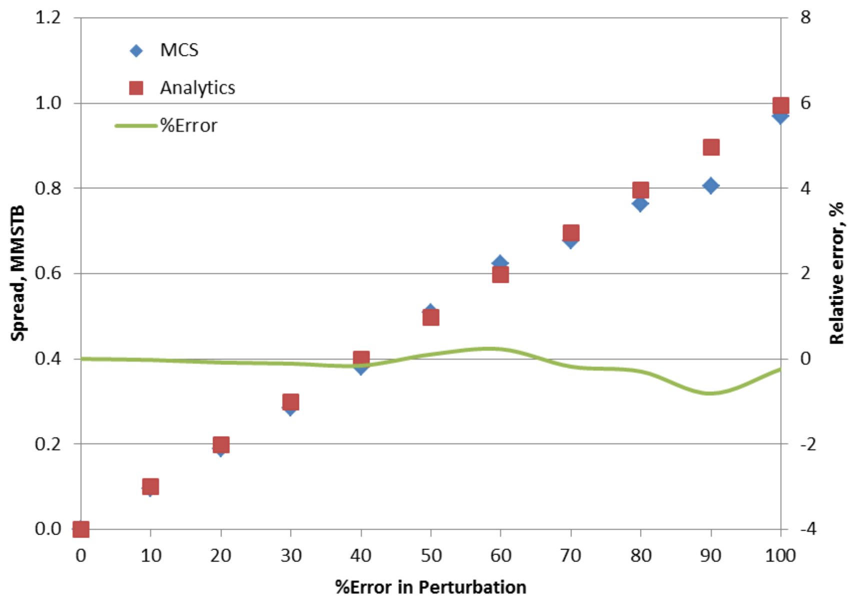

Figure 21 - Sensitivity of the spread to the %error adding to perturbation

Alternatively, the sensitivity of the spreads is plotted against the error variability in Figure 22. The results show a non-linear relationship. Even though there is a linear relationship between the variance of the cumulative production and the variance of the error, (Eq. 39), the spread is estimated from a standard deviation, or a square root of the variance of the cumulative production. The spread is, therefore, linear proportional to the standard deviation of the error and non-linear proportional to the error variability, which is the variance of the error.

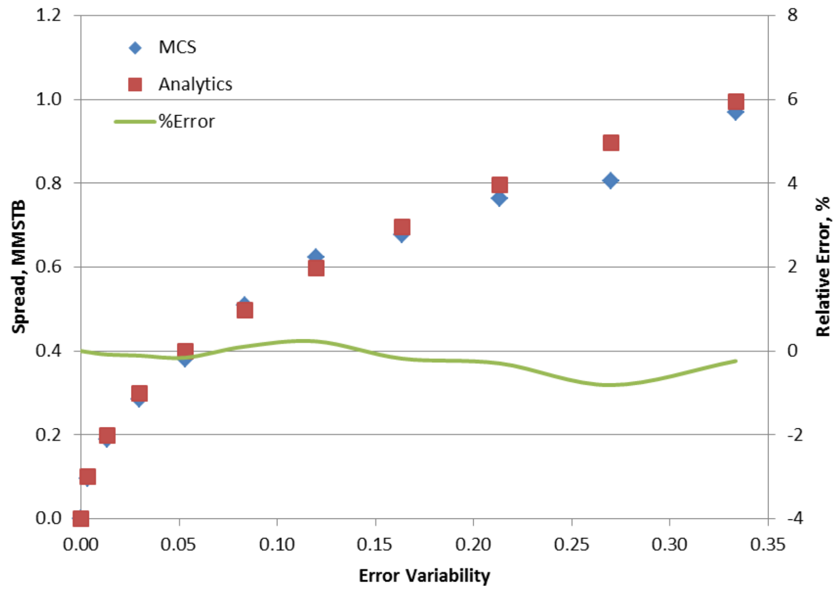

Figure 22 - Sensitivity of the spread to the error variability

The higher error variability; the greater the difference between the observed data and the error-free data. Consequently, the spread increases as the error variability increases. In other words, the higher error variability leads to higher uncertainty in the data and results in larger spread.

4.2.4 Sensitivity to decline rate

After the impact of the initial rate on the spread has been investigated in section 4.2.1, it is interesting to see the influence of the other model parameters. This section studies the sensitivity of the spread to the decline rate, D. The decline rate defines how fast the production drops over a time period. The higher D, the faster decline is. However, if the decline rate is too small, a decline of the production rate may not be observed. In such a case, a model would not be able to define the decline as there is no correlation between rate and time; therefore, the spread limit at zero decline rate is from independent rates. As long as the production forecast can be generated, the analytical formula is applicable even in the case of zero decline rate. The sensitivity analysis here is conducted for D varying from 0.01 to 0.20 /month.

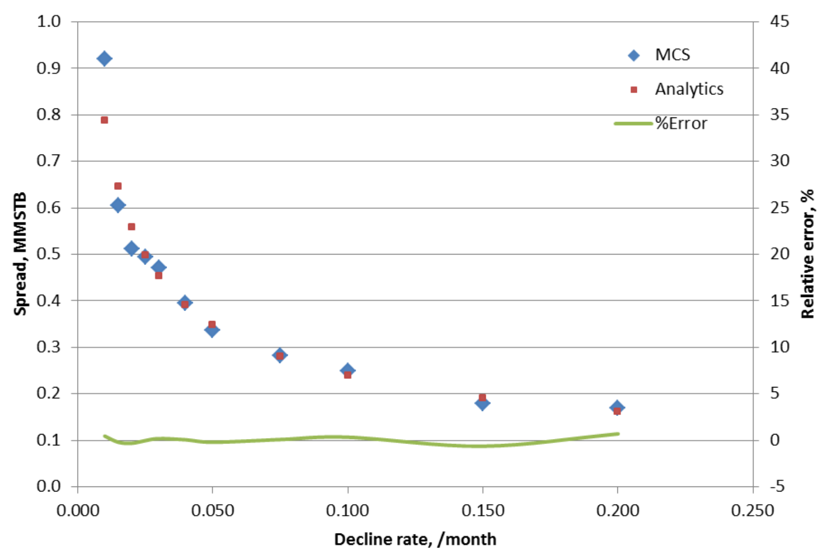

Figure 23 - Sensitivity of the spread to the exponential decline rates

The spread drops as the decline rate increases as shown in Figure 23. When the production rate drops faster from a high decline rate, the cumulative production is lower as well as its spread. The decreasing cumulative production is obviously illustrated in Figure 24 where P90 reserves are plotted against the decline rates. Assuming error term is not changed, the uncertainty associated with the higher expected value is always higher than that with the lower one. The spread undoubtedly drops with the decreasing ultimate recovery.

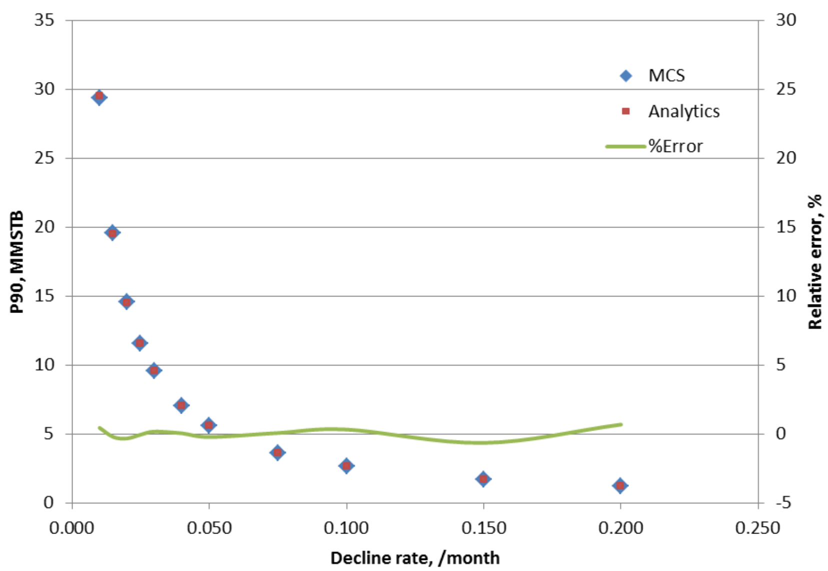

Figure 24 - Sensitivity of the P90 reserves to the decline rates

4.3 APPLICATION

Yates Field

In this section, the analytical solution is applied to determine the spread of the ultimate recovery predictions of actual fields. We first use the production history of the Yates oil field in West Texas, Table 3. The production history forms a straight line in a rate-cum coordinate plot as shown in Figure 9; the decline parameters can be determined using the exponential model and the future production profile is generated accordingly (Figure 25). However, the important problem of the field application is the error term. Even though we can use a formula to quantify the variance of the error (Eq. 40), the required input is how much uncertainty is in the data.

We can determine the error term from the production history. As seen in Table 3, the error term, or relative error, of the Yates oil field varies from -8% to 12% of the predicted model decline. The analytical solution is then used to determine the spread, Table 9. The spread of the Yates field is 5.42 MMSTB, or 0.45% of the estimated ultimate recovery (P50, 1,208 MMSTB). Considering the P50 reserves of the field, 612 MMSTB, if we like to book P90, 0.88% will be removed from the proved reserves. These results are based on the uncertainty of 10%. The study is extended to determining the spread at various level of uncertainty and the results are shown in Table 10.

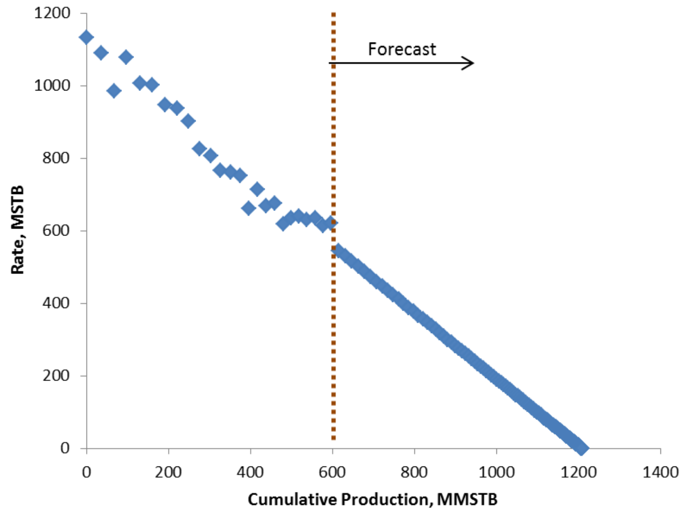

Figure 25 – Production history and forecast of Yates oil field

Prior Cum	595.34	MMSTB
P50 Reserves	612.42	MMSTB
Spread	5.42	MMSTB
P90 EUR	1202.35	MMSTB
P50 EUR	1207.76	MMSTB

Table 9 - Estimated spread and P90/P50 of the Yates oil field with 10% uncertainty

Uncertainty of the data from the forecast model (%)	Spread			Reserves (MMSTB)		EUR (MMSTB)	
	MMSTB	% of reserves	% of EUR	P90	P10	P90	P10
10	5.42	0.88	0.45	607.01	617.84	1,202.35	1,213.18
20	10.83	1.77	0.90	601.59	623.25	1,196.93	1,218.60
30	16.25	2.65	1.35	596.17	628.67	1,191.52	1,224.01
40	21.66	3.54	1.79	590.76	634.09	1,186.10	1,229.43
50	27.08	4.42	2.24	585.34	639.50	1,180.68	1,234.84

Table 10 - Estimated spread at various levels of uncertainties of the Yates data

Constitution Field

This conventional oil field is located in Jefferson County in Texas. The production commenced in 1981. The production history and the production forecast by the DCA in rate-time semi-log and rate-cum plots are in Figure 26 and Figure 27, respectively. The extrapolating trend is well fitted with the history; resulting in lower than 10% of the uncertainty. From Table 11, the spread of the Constitution field is 276 STB, or 0.69% of the P50 reserves (16,295 STB). This spread is small relatively to the EUR. Again, the spread is estimated at various levels of the uncertainty as shown in Table 12 and Figure 28.

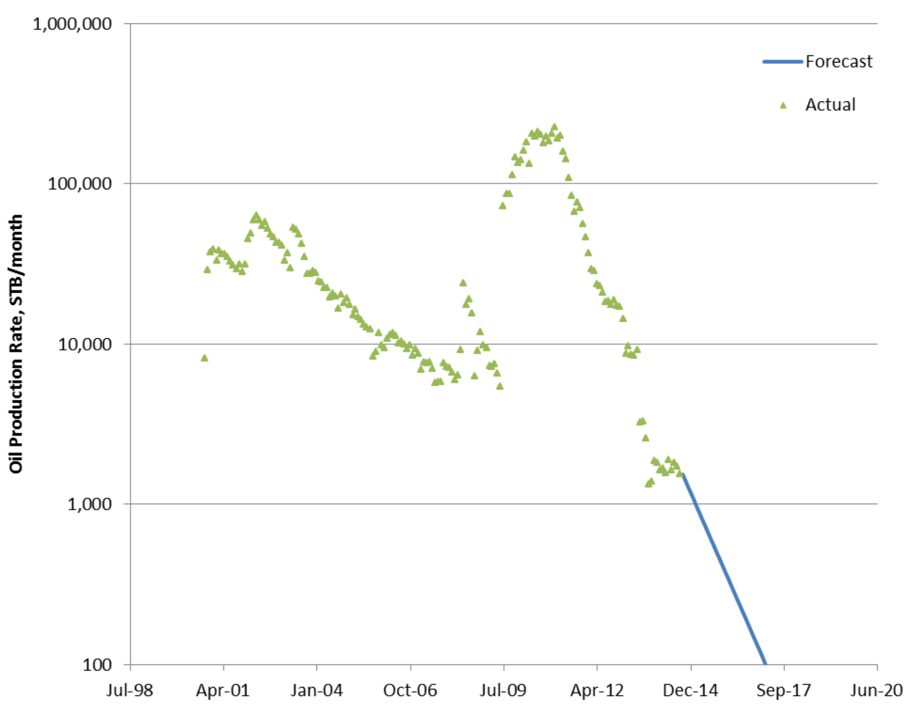

Figure 26 - Production history and forecast of the Constitution field in a rate-time semi-log plot

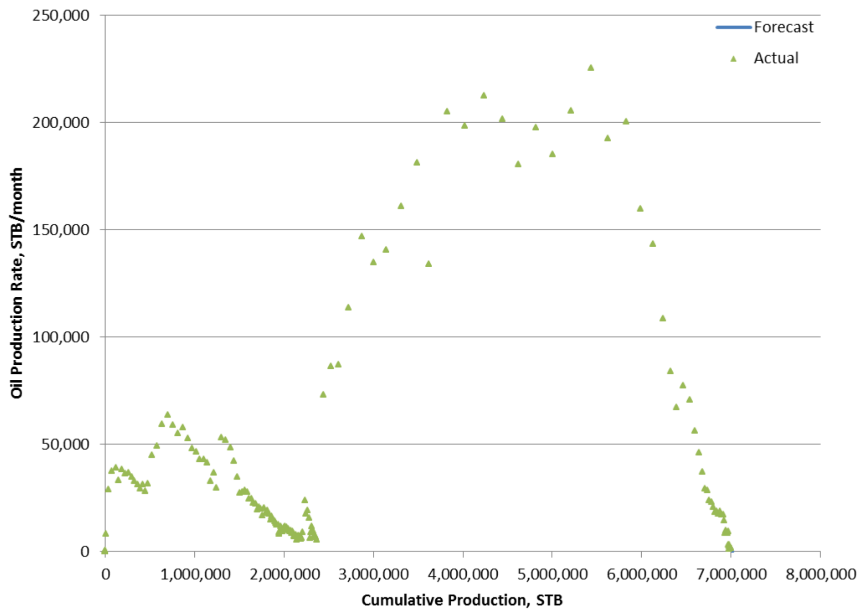

Figure 27 - Production history and forecast of the Constitution field in a rate-cum plot

Prior Cum	7,000,113	STB
P50 Reserves	16,295	STB
Spread	276	STB
P90 EUR	16,019	STB
P50 EUR	7,016,408	STB

Table 11 - Estimated spread and P90/P50 of the Constitution field with 10% uncertainty

Uncertainty of the data from the forecast model (%)	Spread			Reserves (STB)		EUR (STB)	
	STB	%of reserves	%of EUR	P90	P10	P90	P10
10	276	1.69	0.00	16,019	16,571	7,016,132	7,016,684
20	551	3.38	0.01	15,744	16,846	7,015,857	7,016,959
30	827	5.07	0.01	15,468	17,122	7,015,581	7,017,235
40	1,102	6.76	0.02	15,193	17,397	7,015,306	7,017,510
50	1,378	8.46	0.02	14,917	17,673	7,015,030	7,017,786

Table 12 - Estimated spread at various levels of uncertainties of the Constitution data

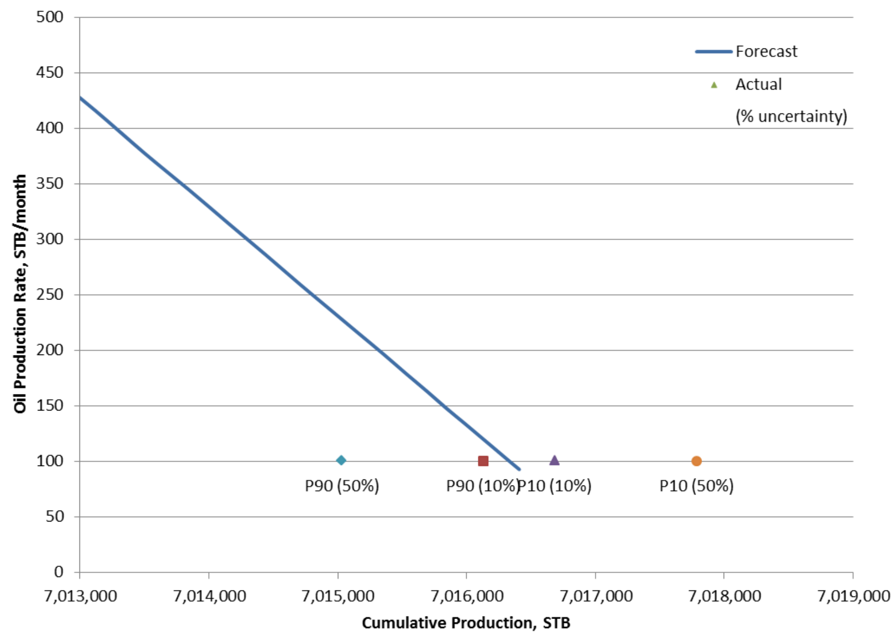

Figure 28 - Estimated P90/P10 of the Constitution field at 10% and 50% uncertainty

Gini Field

Gini field (Wilcox reservoir) is located in Fayette County in Texas. The production commenced in 1985. The production history and forecast in rate-time semi-log and rate-cum plots are in Figure 29 and Figure 30, respectively. The spread and P90/P10 are determined by the analytical solution at 10% uncertainty. From Table 13, the spread of the Gini field is 1,698 STB, or 0.63% of the P50 reserves (269,916 STB). This spread is small relatively to the EUR. The spread is then estimated at various levels of the uncertainty as shown in Table 14 and Figure 31.

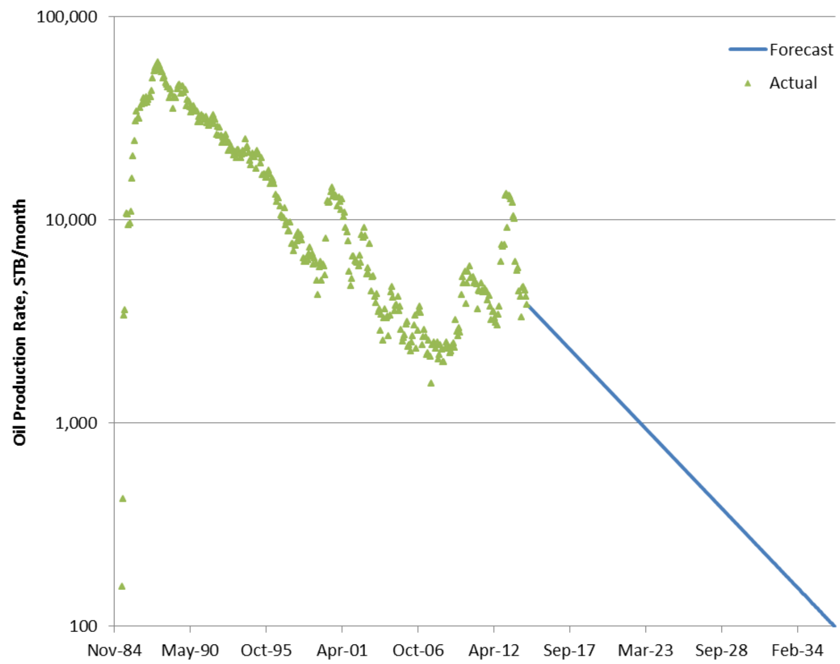

Figure 29 - Production history and forecast of the Gini field in a rate-time semi-log plot

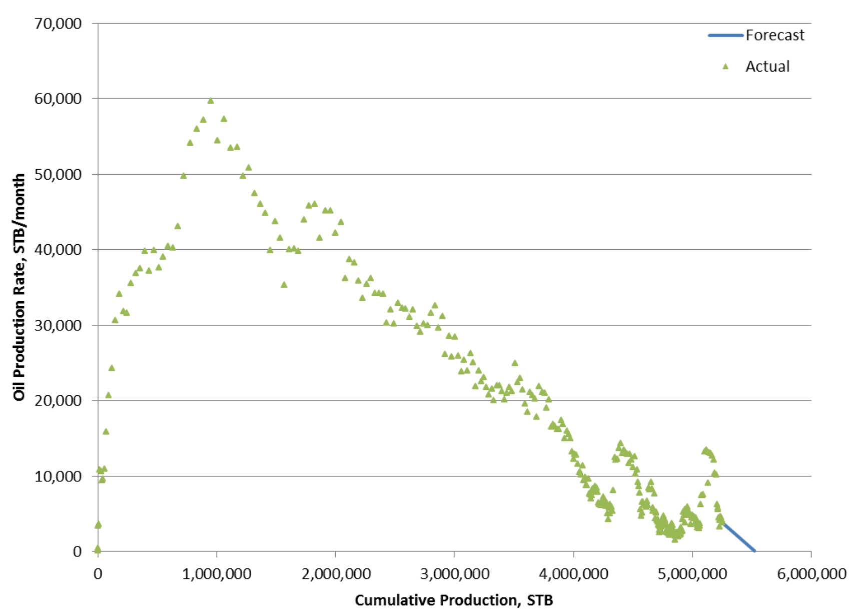

Figure 30 - Production history and forecast of the Gini field in a rate-cum plot

Prior Cum	5,252,894	STB
P50 Reserves	269,916	STB
Spread	1,698	STB
P90 EUR	268,218	STB
P50 EUR	5,522,810	STB

Table 13 - Estimated spread and P90/P50 of the Gini field with 10% uncertainty

Uncertainty of the data from the forecast model (%)	Spread			Reserves (STB)		EUR (STB)	
	STB	%of reserves	%of EUR	P90	P10	P90	P10
10	276	1.69	0.00	16,019	16,571	7,016,132	7,016,684
20	551	3.38	0.01	15,744	16,846	7,015,857	7,016,959
30	827	5.07	0.01	15,468	17,122	7,015,581	7,017,235
40	1,102	6.76	0.02	15,193	17,397	7,015,306	7,017,510
50	1,378	8.46	0.02	14,917	17,673	7,015,030	7,017,786

Table 14 - Estimated spread at various levels of uncertainties of the Gini data

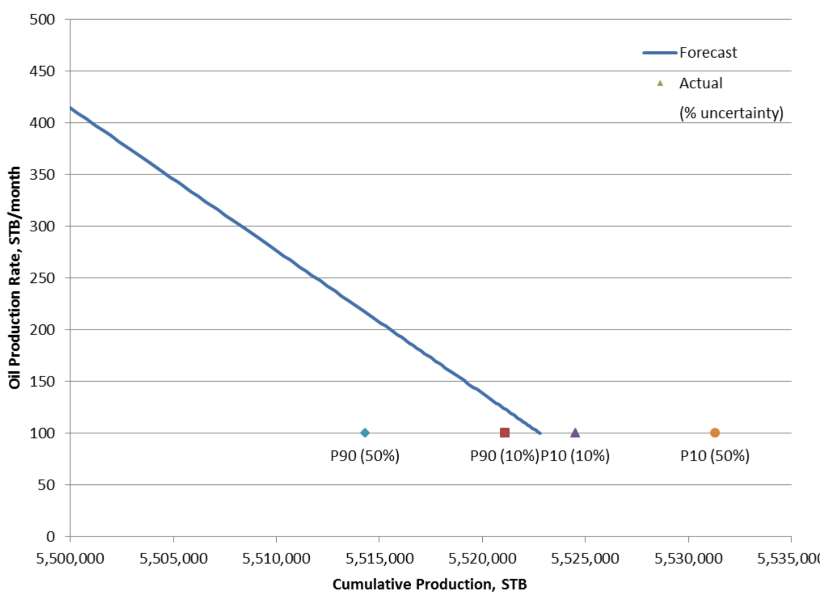

Figure 31 - Estimated P90/P10 of the Gini field at 10% and 50% uncertainty

Fashing Field

The Edwards limestone in the Fashing field is a conventional gas reservoir located in Atascosa County, Texas. The production commenced in 1957. The production history and forecast in rate-time semi-log and rate-cum plots are in Figure 29 and Figure 30, respectively. The spread and P90/P10 are determined by the analytical solution at 10% uncertainty. From Table 13, the spread of the Gini field is 1,698 STB, or 0.63% of the P50 reserves (269,916 STB). This spread is small relatively to the EUR. The spread is then estimated at various levels of the uncertainty as shown in Table 14 and Figure 31.

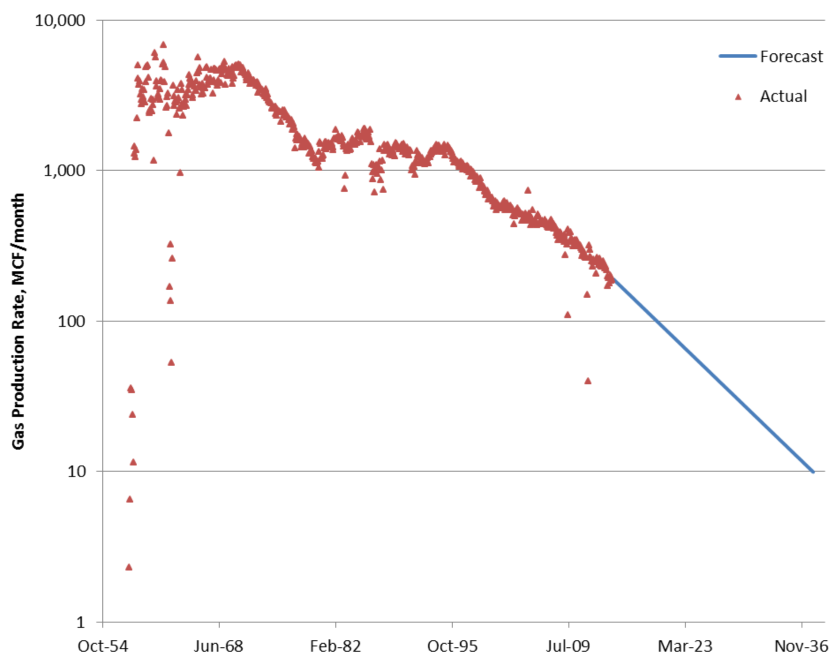

Figure 32 - Production history and forecast of the Fashing field in a rate-time semi-log plot

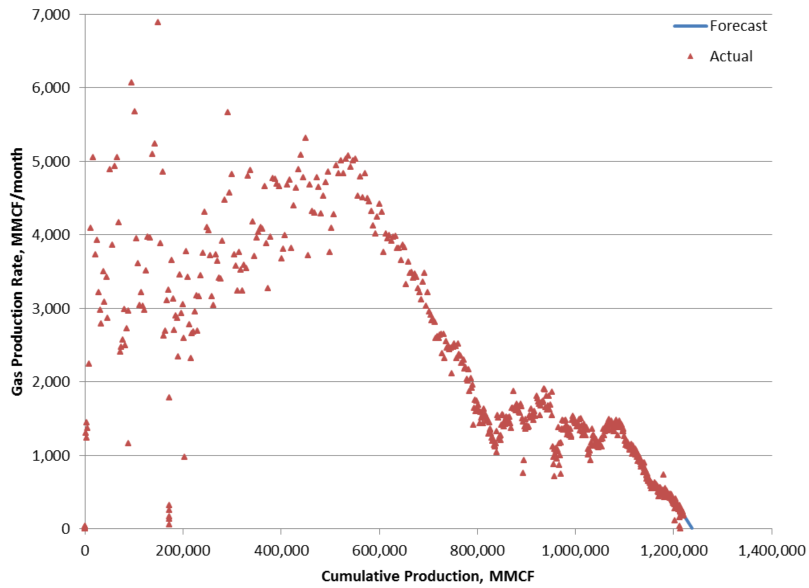

Figure 33 - Production history and forecast of the Fashing field in a rate-cum plot

Prior Cum	1,219,889	MMCF
P50 Reserves	17,376	MMCF
Spread	98	MMCF
P90 EUR	17,278	MMCF
P50 EUR	1,237,265	MMCF

Table 15 - Estimated spread and P90/P50 of the Fashing field with 10% uncertainty

Uncertainty of the data from the forecast model (%)	Spread			Reserves (MMCF)		EUR (MMCF)	
	MMCF	%of reserves	%of EUR	P90	P10	P90	P10
10	98	0.56	0.01	17,278	17,474	1,237,167	1,237,363
20	196	1.13	0.02	17,180	17,572	1,237,069	1,237,461
30	294	1.69	0.02	17,082	17,670	1,236,971	1,237,559
40	392	2.26	0.03	16,984	17,768	1,236,873	1,237,657
50	490	2.82	0.04	16,886	17,866	1,236,775	1,237,755

Table 16 - Estimated spread at various levels of uncertainties of the Fashing data

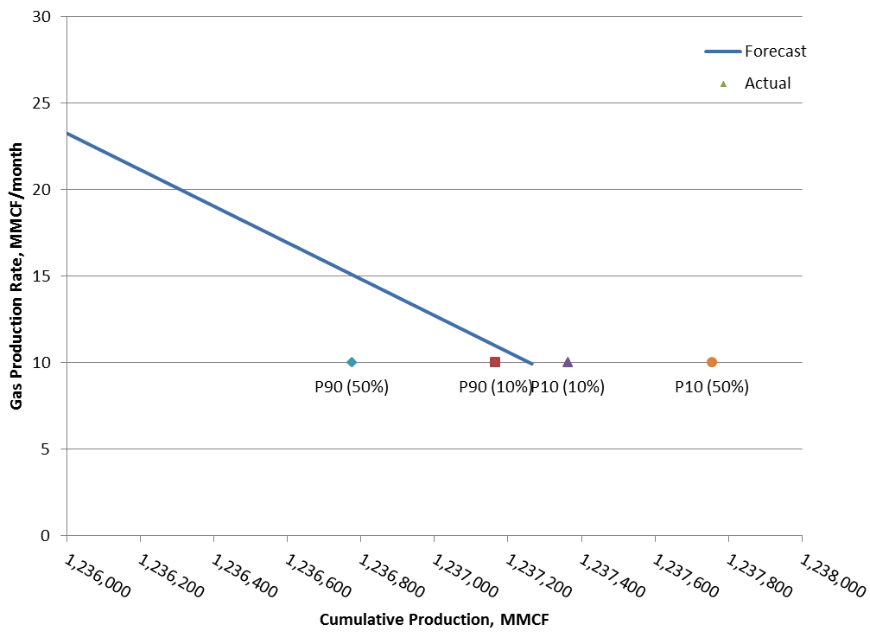

Figure 34 - Estimated P90/P10 of the Fashing field at 10% and 50% uncertainty

Overall the field application results at 50% uncertainty that is consistent to our base case model, except the Constitution field that has few reserves. In term of EUR, apart from the Yates field, the application affects less than 0.2%, as a result of high cumulative production of the fields.

In general, the P90s by the analytical solution at 50% uncertainty are booked as the proved reserves, about 2-4% of the P50 reserves will have to be removed from the proved reserves and moved to probable reserves. There is no a certain percentage of the spread where we could apply for every field. The analytical solution must be applied field by field.

Chapter 5: Conclusions and Recommendations

5.1 Conclusions

When decline curve analysis, DCA, is applied to estimate proved reserves, the normal practice is to find the best fit trend that will be representing the well behavior and which will be projected to forecast the ultimate recovery. We can call this extrapolation the 50th percentile trend corresponding to its "best estimate" trend. The forecast reserves from this trend are normally booked as proved reserves. This practice conflicts with the definition of proved reserves, which requires a high level of confidence, at least 90% probability that the actual recovered amount equals or exceeds the estimate.

The DCA reserves estimation must then be more conservative in compliance with the proved reserves definition. One way to determine the P90 is to quantify a spread, or a difference between the P50 and P90 estimates, and then subtract the spread from the best estimate (P50).

First we looked for a conventional estimation. The traditional error bounds determination seem to be applicable to our problem. It is based on the principle of confidence intervals to determine the variability of the variable (X or Y). However, this conventional method does not work out as expected. Its overestimation of the spread is because the confidence interval increases as it goes farther from the average values. The conventional estimation is, therefore, not applicable to our problem.

Subsequently, the analytical solution is formulated to extrapolate the variance of the cumulative production. The formula is developed from an error model and the integration of the rate over a period of time. The analytical results of exponential decline agree with the MCS results in terms of both extrapolated variance and spread. Even though the production model is changed from exponential to hyperbolic and harmonic models, the analytical results are acceptably accurate. The results prove that the analytical solution for spread estimation works with any production model as long as a deterministic production profile can be determined.

In the base case, the spread can be ranged from less than a percent to over 4% depend on how much uncertainty is in the data. The uncertainty may be as large as 50%; therefore, the spread is approximately 4% of the expected ultimate recovery. In compliance with the proved reserves definition, we would book only 96% of the original proved reserves. The 4% reduction may seem unsubstantial; however, this definitely reduces overall proved reserves and inevitably impacts company value.

The influences of parameters on the spread are determined from the sensitivity analysis. We study the effects of number of additional data, economic limits, error variability, and changing true values of the model. If the changing parameters cause more uncertainty in the data, the spread will increase; and vice versa. The following list is how parameters affect the uncertainty in the extrapolations.

More uncertainty	Less uncertainty
- Decreasing additional data	- Increasing additional data
- Lowering the economic limit	- Raising the economic limit
- Increasing error variability	- Reducing error variability

A changing decline rate affects the ultimate recovery; the higher the decline rate, the lower the ultimate recovery. The spread of the lower ultimate recovery is certainly less than of the higher one. Consequently, the spread reduces as the decline rate increases.

The analytical solution works well with the field data and the results are consistent to our experimental model. However, the analytical result depends on the accuracy of the error and decline model used. For example if the decline curve changes during the estimation period the estimated spread will be inaccurate. Both the error and decline model can be determined from the production history.

In field application, some reserves are definitely removed from the proved reserves and moved to probable reserves and there is no a certain percentage of the spread. The analytical solution need to be applied field by field to determine the spread.

5.2 RECOMMENDATIONS

During the process of decline curve analysis, DCA, reserves determination, a production forecast must be generated. Together with the error term acquired from the production history, an analytical solution is in hand and not restricted to any particular situation. The analytical solution provides a satisfying spread as long as the production profile is accurately forecasted; thus, it can be applied to the DCA for proved reserves estimation. This spread can be applied to lower the reserves figure; resulting in more conservative reserve estimates. When the error term is determined from the production history, the uncertainty in the data may be small and the spread is small, accordingly. This is because a decline model is generally selected from a trend that is best fit with the history. The error between the fitted trend and the history is then small. This tradition may underestimate the spread. It would be better to assume high uncertainty rather than low one.

As the United States Securities and Exchange Commission, SEC, adopts the definition of proved reserves from the SPE, the reported proved reserves, if determined by DCA, must take into account the spread in compliance with the SPE definition. However, if the SEC enforces this method, the world proved reserves will be significantly reduced because many U.S. oil companies have operations abroad; leading to the application of this method worldwide. More studies on impacts of this method must be carried out prior to the SEC official enforcement.

Appendix

Appendix A: Petroleum Reserves Definitions

<u>Development of Reserves Definitions</u>

Petroleum[1] is the world's major source of energy and is a key factor in the continued development of world economies. It is essential for future planning that governments and industry have a clear assessment of the quantities of petroleum available for production and quantities which are anticipated to become available within a practical time frame through additional field development, technological advances, or exploration. To achieve such an assessment, it is imperative that the industry adopt a consistent nomenclature for assessing the current and future quantities of petroleum expected to be recovered from naturally occurring underground accumulations. Such quantities are defined as reserves, and their assessment is of considerable importance to governments, international agencies, economists, bankers, and the international energy industry.

The terminology used in classifying petroleum substances and the various categories of reserves have been the subject of much study and discussion for many years. Attempts to standardize reserves terminology began in the mid 1930s when the American Petroleum Institute considered classification for petroleum and definitions of various reserves

[1]Petroleum: For the purpose of these definitions, the term petroleum refers to naturally occurring liquids and gases which are predominately comprised of hydrocarbon compounds. Petroleum may also contain non-hydrocarbon compounds in which sulfur, oxygen, and/or nitrogen atoms are combined with carbon and hydrogen. Common examples of non-hydrocarbons found in petroleum are nitrogen, carbon dioxide, and hydrogen sulfide.

categories. Since then, the evolution of technology has yielded more precise engineering methods to determine reserves and has intensified the need for an improved nomenclature to achieve consistency among professionals working with reserves terminology. Working entirely separately, the Society of Petroleum Engineers (SPE) and the World Petroleum Council (WPC, formerly World Petroleum Congresses) produced strikingly similar sets of petroleum reserve definitions for known accumulations which were introduced in early 1987. These have become the preferred standards for reserves classification across the industry. Soon after, it became apparent to both organizations that these could be combined into a single set of definitions which could be used by the industry worldwide. Contacts between representatives of the two organizations started in 1987, shortly after the publication of the initial sets of definitions. During the World Petroleum Congress in June 1994, it was recognized that while any revisions to the current definitions would require the approval of the respective Boards of Directors, the effort to establish a worldwide nomenclature should be increased. A common nomenclature would present an enhanced opportunity for acceptance and would signify a common and unique stance on an essential technical and professional issue facing the international petroleum industry.

As a first step in the process, the organizations issued a joint statement that presented a broad set of principles on which reserves estimations and definitions should be based. A task force was established by the Boards of SPE and WPC to develop a common set of definitions based on this statement of principles. The following joint statement of principles was published in the January 1996 issue of the SPE Journal of Petroleum Technology and in the June 1996 issue of the WPC Newsletter:

There is a growing awareness worldwide of the need for a consistent set of reserves definitions for use by governments and industry in the classification of petroleum reserves. Since their introduction in 1987, the Society of Petroleum Engineers and the World Petroleum Council reserves definitions have been standards for reserves classification and evaluation worldwide.

SPE and WPC have begun efforts toward achieving consistency in the classification of reserves.

As a first step in this process, SPE and WPC issue the following joint statement of principles.

> *SPE and WPC recognize that both organizations have developed a widely accepted and simple nomenclature of petroleum reserves.*
>
> *SPE and WPC emphasize that the definitions are intended as standard, general guidelines for petroleum reserves classification which should allow for the proper comparison of quantities on a worldwide basis.*
>
> *SPE and WPC emphasize that, although the definition of petroleum reserves should not in any manner be construed to be compulsory or obligatory, countries and organizations should be encouraged to use the core definitions as defined in these principles and also to expand on these definitions according to special local conditions and circumstances.*

SPE and WPC recognize that suitable mathematical techniques can be used as required and that it is left to the country to fix the exact criteria for reasonable certainty of existence of petroleum reserves. No methods of calculation are excluded, however, if probabilistic methods are used, the chosen percentages should be unequivocally stated.

SPE and WPC agree that the petroleum nomenclature as proposed applies only to known discovered hydrocarbon accumulations and their associated potential deposits.

SPE and WPC stress that petroleum proved reserves should be based on current economic conditions, including all factors affecting the viability of the projects. SPE and WPC recognize that the term is general and not restricted to costs and price only. Probable and possible reserves could be based on anticipated developments and/or the extrapolation of current economic conditions.

SPE and WPC accept that petroleum reserves definitions are not static and will evolve.

A conscious effort was made to keep the recommended terminology as close to current common usage as possible to minimize the impact of previously reported quantities and changes required to bring about wide acceptance. The proposed terminology is not intended as a precise system of definitions and evaluation procedures to satisfy all situations. Due to the many forms of occurrence of petroleum, the wide range of characteristics, the uncertainty associated with the geological environment, and the constant evolution of evaluation technologies, a precise classification system is not practical. Furthermore, the complexity required for a precise system would detract from its understanding by those involved in petroleum matters. As a result, the recommended definitions do not represent a major change from the current SPE and WPC definitions which have become the standards across the industry. It is hoped that the recommended terminology will integrate the two sets of definitions and achieve better consistency in reserves data across the international industry.

Reserves derived under these definitions rely on the integrity, skill, and judgment of the evaluator. They are affected by the geological complexity, stage of development, degree of depletion of the reservoirs, and amount of available data. Use of these definitions should sharpen the distinction between the various classifications and provide more consistent reserves reporting.

Petroleum Reserves Definitions

Reserves are those quantities of petroleum which are anticipated to be commercially recovered from known accumulations from a given date forward. All reserve estimates involve some degree of uncertainty. The uncertainty depends chiefly on the amount of reliable geologic and engineering data available at the time of the estimate and the interpretation of these data. The relative degree of uncertainty may be conveyed by placing reserves into one of two principal classifications, either proved or unproved. Unproved reserves are less certain to be recovered than proved reserves and may be further sub-classified as probable and possible reserves to denote progressively increasing uncertainty in their recoverability.

The intent of the Society of Petroleum Engineers (SPE) and World Petroleum Council (WPC, formerly World Petroleum Congresses) in approving additional classifications beyond proved reserves is to facilitate consistency among professionals using such terms. In presenting these definitions, neither organization is recommending public disclosure of reserves classified as unproved. Public disclosure of the quantities classified as unproved reserves is left to the discretion of the countries or companies involved.

Estimation of reserves is done under conditions of uncertainty. The method of estimation is called deterministic if a single best estimate of reserves is made based on known geological, engineering, and economic data. The method of estimation is called probabilistic when the known geological, engineering, and economic data are used to generate a range of estimates and their associated probabilities. Identifying reserves as proved, probable, and possible has been the most frequent classification method and gives an indication of the probability of recovery. Because of potential differences in uncertainty, caution should be exercised when aggregating reserves of different classifications.

Reserves estimates will generally be revised as additional geologic or engineering data becomes available or as economic conditions change. Reserves do not include quantities of petroleum being held in inventory, and may be reduced for usage or processing losses if required for financial reporting.

Reserves may be attributed to either natural energy or improved recovery methods. Improved recovery methods include all methods for supplementing natural energy or altering natural forces in the reservoir to increase ultimate recovery. Examples of such methods are pressure maintenance, cycling, waterflooding, thermal methods, chemical flooding, and the use of miscible and immiscible displacement fluids. Other improved recovery methods may be developed in the future as petroleum technology continues to evolve.

Proved Reserves

Proved reserves are those quantities of petroleum which, by analysis of geological and engineering data, can be estimated with reasonable certainty to be commercially recoverable, from a given date forward, from known reservoirs and under current economic conditions, operating methods, and government regulations. Proved reserves can be categorized as developed or undeveloped.

If deterministic methods are used, the term reasonable certainty is intended to express a high degree of confidence that the quantities will be recovered. If probabilistic methods are used, there should be at least a 90% probability that the quantities actually recovered will equal or exceed the estimate.

Establishment of current economic conditions should include relevant historical petroleum prices and associated costs and may involve an averaging period that is consistent with the purpose of the reserve estimate, appropriate contract obligations, corporate procedures, and government regulations involved in reporting these reserves.

In general, reserves are considered proved if the commercial producibility of the reservoir is supported by actual production or formation tests. In this context, the term proved refers to the actual quantities of petroleum reserves and not just the productivity of the well or reservoir. In certain cases, proved reserves may be assigned on the basis of well logs and/or core analysis that indicate the subject reservoir is hydrocarbon bearing and is analogous to reservoirs in the same area that are producing or have demonstrated the ability to produce on formation tests.

The area of the reservoir considered as proved includes (1) the area delineated by drilling and defined by fluid contacts, if any, and (2) the undrilled portions of the reservoir that can reasonably be judged as commercially productive on the basis of available geological and engineering data. In the absence of data on fluid contacts, the lowest known occurrence of hydrocarbons controls the proved limit unless otherwise indicated by definitive geological, engineering or performance data.

Reserves may be classified as proved if facilities to process and transport those reserves to market are operational at the time of the estimate or there is a reasonable expectation that such facilities will be installed. Reserves in undeveloped locations may be classified as proved undeveloped provided (1) the locations are direct offsets to wells that have indicated commercial production in the objective formation, (2) it is reasonably certain such locations are within the known proved productive limits of the objective formation,

(3) the locations conform to existing well spacing regulations where applicable, and (4) it is reasonably certain the locations will be developed. Reserves from other locations are categorized as proved undeveloped only where interpretations of geological and engineering data from wells indicate with reasonable certainty that the objective formation is laterally continuous and contains commercially recoverable petroleum at locations beyond direct offsets.

Reserves that are to be produced through the application of established improved recovery methods are included in the proved classification when (1) successful testing by a pilot project or favorable response of an installed program in the same or an analogous reservoir with similar rock and fluid properties provides support for the analysis on which the project was based, and, (2) it is reasonably certain that the project will proceed. Reserves to be recovered by improved recovery methods that have yet to be established through commercially successful applications are included in the proved classification only (1) after a favorable production response from the subject reservoir from either (a) a representative pilot or (b) an installed program where the response provides support for the analysis on which the project is based and (2) it is reasonably certain the project will proceed.

Unproved Reserves

Unproved reserves are based on geologic and/or engineering data similar to that used in estimates of proved reserves; but technical, contractual, economic, or regulatory uncertainties preclude such reserves being classified as proved. Unproved reserves may be further classified as probable reserves and possible reserves.

Unproved reserves may be estimated assuming future economic conditions different from those prevailing at the time of the estimate. The effect of possible future improvements in economic conditions and technological developments can be expressed by allocating appropriate quantities of reserves to the probable and possible classifications.

Probable Reserves

Probable reserves are those unproved reserves which analysis of geological and engineering data suggests are more likely than not to be recoverable. In this context, when probabilistic methods are used, there should be at least a 50% probability that the quantities actually recovered will equal or exceed the sum of estimated proved plus probable reserves.

In general, probable reserves may include (1) reserves anticipated to be proved by normal step-out drilling where sub-surface control is inadequate to classify these reserves as proved, (2) reserves in formations that appear to be productive based on well log characteristics but lack core data or definitive tests and which are not analogous to producing or proved reservoirs in the area, (3) incremental reserves attributable to infill drilling that could have been classified as proved if closer statutory spacing had been approved at the time of the estimate, (4) reserves attributable to improved recovery methods that have been established by repeated commercially successful applications when (a) a project or pilot is planned but not in operation and (b) rock, fluid, and reservoir characteristics appear favorable for commercial application, (5) reserves in an area of the formation that appears to be separated from the proved area by faulting and the geologic interpretation indicates the subject area is structurally higher than the proved area, (6) reserves attributable to a future workover, treatment, re-treatment, change of equipment, or other mechanical procedures, where such procedure has not been proved successful in wells which exhibit similar behavior in analogous reservoirs, and (7) incremental reserves in proved reservoirs where an alternative interpretation of performance or volumetric data indicates more reserves than can be classified as proved.

Possible Reserves

Possible reserves are those unproved reserves which analysis of geological and engineering data suggests are less likely to be recoverable than probable reserves. In this context, when probabilistic methods are used, there should be at least a 10% probability that the quantities actually recovered will equal or exceed the sum of estimated proved plus probable plus possible reserves.

In general, possible reserves may include (1) reserves which, based on geological interpretations, could possibly exist beyond areas classified as probable, (2) reserves in formations that appear to be petroleum bearing based on log and core analysis but may not be productive at commercial rates, (3) incremental reserves attributed to infill drilling that are subject to technical uncertainty, (4) reserves attributed to improved recovery methods when (a) a project or pilot is planned but not in operation and (b) rock, fluid, and reservoir characteristics are such that a reasonable doubt exists that the project will be commercial, and (5) reserves in an area of the formation that appears to be separated from the proved area by faulting and geological interpretation indicates the subject area is structurally lower than the proved area.

Reserve Status Categories

Reserve status categories define the development and producing status of wells and reservoirs.

Developed: Developed reserves are expected to be recovered from existing wells including reserves behind pipe. Improved recovery reserves are considered developed only after the necessary equipment has been installed, or when the costs to do so are relatively minor. Developed reserves may be sub- categorized as producing or non-producing.

Producing: Reserves subcategorized as producing are expected to be recovered from completion intervals which are open and producing at the time of the estimate. Improved recovery reserves are considered producing only after the improved recovery project is in operation.

Non-producing: Reserves subcategorized as non-producing include shut-in and behind-pipe reserves. Shut-in reserves are expected to be recovered from (1) completion intervals which are open at the time of the estimate but which have not started producing, (2) wells which were shut-in for market conditions or pipeline connections, or (3) wells not capable of production for mechanical reasons. Behind-pipe reserves are expected to be recovered from zones in existing wells, which will require additional completion work or future recompletion prior to the start of production.

Undeveloped Reserves: Undeveloped reserves are expected to be recovered: (1) from new wells on undrilled acreage, (2) from deepening existing wells to a different reservoir, or (3) where a relatively large expenditure is required to (a) recomplete an existing well or (b) install production or transportation facilities for primary or improved recovery projects.

Approved by the Board of Directors, Society of Petroleum Engineers (SPE) Inc., and the Executive Board, World Petroleum Council (WPC), March 1997

Nomenclature

English symbols

b	b-component in Arps empirical equations
CDF	Cumulative distribution function
D_i	Initial decline rate
D_t	Decline rate at time t
DCA	Decline curve analysis
E(x)	Expected value of x
EIV	Error in variables
EL	Economic limiting rate
$F_{1,n-2}^{\alpha/2}$	The critical value of F-distribution with significance level α and n-2 degrees of freedom
MCS	Monte Carlo simulation
MMSTB	Million stock tank barrels
MS_ε	Mean squared error
n	number of observations
$\tilde{N}_p$	Uncertain/perturbed cumulative production
N_{p_t}	Cumulative production at time t
OLS	Ordinary least square
P10	Possible reserves or the 10^{th} percentile value
P50	Probable reserves or the 50^{th} percentile value
P90	Proved reserves or the 90^{th} percentile value
$\tilde{q}$	Uncertain/perturbed production rate

q_i	Initial production rate
q_t	Production rate at time t
S_{YY}	Variance of y
S_{XY}	Covariance of x and y
S_{XX}	Variance of x
SEC	United States Securities and Exchange Commission
SPE	Society of Petroleum Engineers
STB	Stock tank barrel
STB/d	Stock tank barrel per day
$t_{n-2}^{\alpha/2}$	The critical value of two-sided t-distribution with significance level α and n-2 degrees of freedom
U[a,b]	Uniform distribution on the interval [a,b]
UR	Ultimate recovery
WPC	World Petroleum Congresses
$\bar{Y}$	Average value of y - $E(y)$
Y_i	Observed value of y
$\bar{X}$	Average value of x - $E(x)$
X_i	Observed value of x

Greek symbols

$\hat{\beta}_0$	Estimate of intercept
$\hat{\beta}_1$	Estimate of slope
δ	A ratio of measurement variances
ε	Error term
μ	Expected value
σ	Standard deviation
σ_{N_p}	Standard deviation of cumulative production

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