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Evaluation and Comparison of Management Strategies

by Data Envelopment Analysis with an Application to Mutual Funds

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Evaluation and Comparison of Management Strategies by Data Envelopment Analysis with an Application to Mutual Funds

by

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Dedication

This dissertation is dedicated to the memory of my father,

Joe Foster Wilson,

MSgt USAAC/USAF (Purple Heart, Pearl Harbor),

PhD (History, University of Georgia),

and to my mother,

Mary Elizabeth Richards Valdez Wilson,

MA (Spanish and French, Pan American University).

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Evaluation and Comparison of Management Strategies

by Data Envelopment Analysis with an Application to Mutual Funds

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A new categorical schema for strategic management is developed; a methodology for its implementation is elaborated; an application to mutual funds based on micro-economic theory is demonstrated; and results which establish quantitative measures for evaluating strategies, improve measures of managerial performance, and establish a new

method of evaluating portfolio performance with guidance for potential mutual fund shareholders is presented.

The evaluation of strategies themselves depends fundamentally on distinguishing them from their execution, from their realization in practice. The accounting definition of strategy, "a plan of action used to guide or control other plans of action" finds an observable, indeed measurable, example in the strategic choices of mutual funds, which are required by law to declare and conform to the general strategy by which they conduct investment management.

The methodology to exploit the declared strategies and performance data of mutual funds is Data Envelopment Analysis (DEA), a nonparametric linear programming method of analysis for use with empirical data. By producing a piecewise linear frontier based on the Pareto-Koopmans efficient performers, DEA provides a basis for measuring performances and facilitates sensitivity analysis. Data Envelopment Analysis measures assume no prior, underlying functional form (such as regression equations or production functions) to relate input to output or to other variables.

An evaluation of a selected group of mutual funds illustrates the general DEA method and evaluates the actual performance of the funds. Then a new application involving an extended, three-stage Data Envelopment Analysis separates the performance of the investment strategies from the effects of managerial shortcomings and abilities to implement the strategies. This makes it possible to separately identify and evaluate what a strategy can accomplish. It also makes it possible to evaluate separately short-run from long-run performance. Finally, DEA identifies benchmarking possibilities for removing these short-run deficiencies.

This new method for evaluating strategies and shortcomings in performance is demonstrated by application to mutual funds, which display striking contrasts in managerial performance and strategic potential. Although demonstrated with mutual funds, this method is not restricted to such applications. Indeed, the methods in this thesis provide a new way of evaluating investment potentials by distinguishing between actual short-run performance and long-run potentials.

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Chapter 1 Introduction

Section 1 Overview

Strategy signifies the intended course and directions by which actions are guided to their ends, aspirations piloted to attainment. Because strategic management concerns the fundamental organization, direction, and production of economic and social activity, its study can have profound and far-reaching consequences for human welfare. It is, therefore, worthy of serious and exacting scholarly investigation. This dissertation undertakes to offer contributions in four areas related to strategic management: (1) a new conceptual framework; (2) a new method of analysis; (3) the identification of an important industry (mutual funds) wherein strategic choice is publicly declared; and (4) an application of these elements to the evaluation of the strategies of a high risk and a low risk sector of this industry.

To begin, the foundation of a categorical schema or conceptual framework is developed for the fundamental concepts and basic factors of strategic management. This framework is based on identifying and analyzing the essential meaning of the idea of strategy which can thus distinguish and disentangle it from the other, related, basic concepts in this area of management. A review of the origins and uses of the term *strategy* in the literature displays a wide range of definitions which confuse and compound the basic idea of strategy—the fundamental approach to attaining a goal—with related concepts: for example, the concomitant and implicit concepts of goals and purpose; the conditional concepts of environment and resources; the ensuing concepts of implementation and tactics. Disencumbering the concept of strategy improves the clarity of strategic models and the cogency of their results. A clearer concept of strategy should facilitate the formulation of strategies which are more precise and suited to their purposes.

Without complete, detailed knowledge of the current state and future development of the domain in which a strategy is to be applied, *ex ante* evaluations of strategic alternatives are constrained, not just by the variability of the factors they contemplate, but even more so, by the degree of uncertainty in assessments of then-current conditions and future projections. Although such uncertain forecasts may be a necessary part of strategy formulation in practice, the final judgement of strategic choices is rendered on the results of strategy implementation. Based on the facts of actual performance relative to goals and objectives, *ex post* evaluations are decisive. However, just as many theoretical explanations confound the concept of strategy with related concepts (especially those regarding its implementation), empirical evaluations based on strategy in practice may confuse strategy itself, that is, its *potential*, the best it might have attained, with its *execution*, that is, the often limited or failed results of attempts to realize strategic purpose.

The second contribution of this study is the application of a mathematical method, based on data envelopment analysis (DEA), for analyzing performance results in such a manner as to separate the realizable potentials of strategies from the limitations or shortcomings of their implementations in practice. Such an analysis can provide a direct and relevant basis for evaluating strategies themselves, as distinct from the possibly misleading complications of inadequate implementations. The combination of a clarified concept of strategy with a computationally tractable methodology for the empirical evaluation of its full potential can make strategic management research more germane to managers charged with executing strategies and to directors to whom they report.

Furthermore, separating the potential of strategies from the variations in their implementation clarifies the performance of management. By controlling for the effect of the strategy which managers are responsible for executing, evaluations of management activity against the standard of strategic potential are relevant, meaningful, and efficacious. The complementary nature of strategy and implementation is especially evident in those situations where an excellent strategy poorly implemented is overcome by the effective implementation of a mediocre one. An evaluation which distinguishes strategy from implementation is relevant both for managers monitoring their own progress and for those who need to appraise management performance.

For issues of strategic management, in particular, such analyses can separate the potential of the strategy from the effectiveness of management efforts to realize its goals. This is of especial importance to strategic management since the purpose of strategy is to produce desired results in practice. The issues—the tasks, means, and problems—of the implementation of strategy are necessarily as consequential as those of strategy formulation, and, indeed, more urgent because of the high rate and great costs of strategic failure in corporate practice. Since strategic issues concern fundamental questions of the business firm, the response to such issues will have a substantial impact on the firm's value, perhaps even its existence. The costs of failure are significant for all stakeholders. For large enterprises, success or failure in strategic implementation can have effects throughout the economy.

As in any developing science, the data accumulate and concepts and theories evolve over time with experience and practice. Economics and management science have been compared to geology, astronomy, or evolutionary biology—as distinct from physics or chemistry—because of their retrospective character and predominant reliance on observation without laboratory control. Indeed, comprehensive theories and precise laws may be more distant for economics than for the natural observational sciences both because of the greater complexity of economic and social phenomena and because of the greater remove or inaccessibility of its fundamental object, human action, which, moreover, may present with greater degrees of freedom in a market economy. However, just as in the more physical observational sciences, there are sometimes natural settings in which the phenomena of interest are particularly evident and their processes more clearly revealed. Thus, a third contribution of this dissertation is the explanation of how particular characteristics of the mutual funds market can be advantageously employed for a tractable analysis of their strategic attributes. Moreover, the mutual funds market is significant for more than just its susceptibility to technical strategic analysis. Because of its size and nature of its activities, it has a significant economic and financial impact in the United States and, because of high investor participation rates, it is important to the financial status of many individual Americans and their families.

The fourth, and final, contribution of this study is the application of these foregoing elements to the analysis and evaluation of two competing strategies for equity mutual funds. This evaluation differs from the usual case in the finance literature in several important ways. First, it is based on the standpoint of management, rather than that of shareholders: that is, the variables are chosen to reflect the issues relevant to evaluating the performance of management rather than those of portfolio performance for shareholders (although they prove to be related). Second, the method does not rely on the mean-variance models of risk and return for financial evaluation. Instead, risk is modeled by more direct and specific constructs. In the initial case evaluated here, one of the two sectors is typically identified in the finance literature as "high risk" and the other as "low risk." The new methodology makes it possible to evaluate management performance within each sector separately, as well as to evaluate separately the overall performance of the sectors themselves. Then the two strategies are compared to each other and performance of the two groups of managers with respect to their own strategies can be compared. Exhaustive evaluation of each separate aspect of performance in this industry is greatly facilitated by the fact of publicly declared and legally binding strategies for its firms and by the public reporting of detailed and standardized financial information.

The results strikingly demonstrate an unambiguous difference between the strategies compared. The analysis also provides specific guidance for the evaluation of the managers of the various funds and grounds for differentials in performance-based consequences, especially remuneration. However, the analysis also has an unexpected consequence for shareholder consideration, since the results can be interpreted as another kind of portfolio evaluation, with implications for shareholder investment decisions.

Finally, since the number of strategies evaluated, the number of variables in the model, and the time period of the evaluation are all of limited scope, this investigation may be regarded the initial steps in a new way to study and evaluate strategies. The last section explores several major directions for extending the research in the future.

Section 2 Plan of Presentation

The first task is to set out and elucidate the key issues of strategic management, which are the definition of strategy and the evaluation of management performance with respect to strategy. Chapter 2 begins the investigation with a view to recasting some of the fundamental concepts which have been employed in the strategic management literature. The focus in this reformulation is the concept and definition of strategy, which serves as the organizing principle for the overall management of action directed to a specifiable end. Discussions of meaning can become abstruse, especially in clarifying a concept about which there is so much confusion, disagreement, and uncertainty. The notion of strategy has many uses and formulations; it lies at the center of complex relations and processes; it balances constancy in goal seeking against a context of change; it guides the pursuit of a future whose attainment is not just uncertain, but often unlikely, without the exertions prescribed, at least implicitly, by the strategy. These considerations motivate two approaches in the next chapter in pursuit of the material and concrete associations of the concept: the first is historical and the second is functional.

First, to maintain grounding in the concrete associations of the idea of strategy, the analysis begins with a review of the historical origins of the word and some of its subsequent uses. An etymological account reviews the material conditions of the creation and uses of the concept and the word which denotes it. Unlike many words, even of much shorter history, *strategy* today retains much of the original denotation invoked by the Greeks who coined it 2500 years ago to signify the function of overall military command. Moreover, modern connotations from its use in economics and politics are not present-day innovations, but were established in both specific, technical use by the Greeks and by their recognition of similar functions in other realms of human activity, particularly in commerce. The Athenian military campaign against Sicily not only illustrates the Greek origins of the word *strategy*, but it also exemplifies a distinction crucial to its comprehension and evaluation: that is, an attempt to execute a strategy is an instance of its realization. It is not the strategy itself, nor is it even a specific strategy of a general class. It is an example, an instantiation which may be more or less representative of what the strategy purports to achieve. The Sicilian campaign also demonstrates the importance of a management which is both competent for and committed to the execution of the strategy.

Examples of the military use of the concept of strategy continue with the explication by von Clauswitz, the Napoleonic-era progenitor of contemporary military theory. The examination of the military concept concludes with the views of Liddell Hart, the British military theorist of the first half of the twentieth century.

During the period that Liddell Hart was writing, von Neumann and Morgenstern applied the term *strategy* to their mathematical model of conflict and cooperation. Although "mathematical game theory" has been usefully applied to a wide range of disciplines, including economics and management, its definition of strategy is unsuitable and infeasible for use in the practical application of strategic management because it requires an explicit specification of all possible courses of action and their outcomes. This is precisely what is generally lacking in just the situation of making a strategic choice.

With the advance in the complexity of business practice and the development of business policy studies, especially after the Second World War, the issues of strategy arose in the management literature during the 1950s, but the first use of the term appears in Chandler [1962]. As early as 1978, Hofer and Schendel review and analyze 13 different formulations of the concept as it appears in the first two decades of management literature. The definitions of subsequent authors in the management literature, including Rumelt [1982, 1991], Porter [1980, 1996] and Hamel and Prahalad [1994], further broaden the concept.

Most of these efforts are concerned to leave nothing of relevance to strategic management out of their definitions of the basic concept. Consequently, strategy is confused with other central but implicit, complementary, associated, ancillary, or ensuant concepts; and its essential meaning, its cybernetic character, is obscured. However, the accounting literature provides a definition which is both general and precise: general because it is applicable in any business setting, for example, at the functional, business, or corporate levels, as well as in other disciplines, such as military science, political theory, foreign policy, and sociology; and precise because it is concisely formulated and directly and unambiguously identifies the object of interest in any given setting. The accounting definition is appropriated from Cooper and Ijiri [1981], who define strategy as "a plan of action that is used to guide or control other plans of action."

Forced to infer from its behavior the strategy a firm may be following, the literature reveals an inability to distinguish the strategies being employed from their implementation. Many studies which examine empirical data have a tendency to include in the definition of strategy everything of possible relevance to the idea. Thus, goals, objectives, performances, and intents have all been impounded in the definitions typically offered in the strategic management literature.

However, the literature in accounting and auditing offers a definition from a profession which has access to "insider" information that includes knowledge of the strategies being employed. As part of their professional service, auditing and accounting practices are directed to evaluate strategies or report on conformance between plans and actions. This informed perspective is offered by the definition of strategy which is provided in *Kohler's Dictionary for Accountants*, edited by Cooper and Ijiri [1981, p. 489-490]: "strategy: a plan of action used to govern or guide other plans of action." This definition is also consistent with other usages, such as are to be found in the military (*e. g.*, the relation between strategies and tactics), as well as in the history literature and in political practice (as when all advertising programs must conform to a campaign strategy).

This definition provides a functional approach to the meaning of strategy. It not only captures the essential meaning of the core concept of strategy, but it also functions to organize the several related concepts of strategic management. The essential meanings of complementary concepts (such as, goals, objectives, environment, resources) become clear in relation to this notion of strategy and distinct from each other. It establishes an overall conceptual order elucidating the logical connections and dependencies of these concepts in relation to strategic management. Most importantly for purposes of clarity, it facilitates distinguishing between the strategies and their implementation.

This clarification of management with respect to strategy is especially important. Strategies are evaluated in both *ex ante* and *ex post* settings. The *ex ante* evaluations are a necessary part of the strategy formulation and selection process in practice; but the forward-looking character of *ex ante* assessments necessarily makes them uncertain with respect to actual outcomes. Based on empirical results, *ex post* evaluations resolve this uncertainty but introduce another. In retrospect, it is (relatively) clear what was done and what resulted. However, is what was done the only way it might have been done, and were those results inevitable? An *ex post* evaluation has the advantage of relying on the facts of empirical outcomes. However, it has the disadvantage of only *indirectly* assessing a "strategy" since it analyzes the performance of one or more implementations, which are then taken to represent the strategy itself.

Distinguishing between strategy and its realization has several benefits. First, it makes explicit the indirect nature of *ex post* evaluations based on specific implementation data. Second, it establishes a standard for the evaluation of management, that is, it enables the comparison of actual performance relative to the potential outcome of the strategy undertaken. Finally, with the full strategic potential as context for management performance, a suitable evaluation helps identify those actions which were critical to its implementation, whether fully exploited or inadequately executed.

Chapter 3 presents the case for the mutual funds industry as an especially advantageous setting for the investigation of strategies. One of the most fundamental and often most difficult issues for strategic analysis is the identification of a firm's strategy. Firms rarely declare their strategies, typically to preserve competitive advantage, or perhaps because no strategy has been explicitly formulated. In some situations, the inference of strategies from, say, firm behavior or organizational form, may seem uncomplicated, but even in these cases there may be aspects of strategy that are not obvious and the imputed strategy may be a distortion of the firm's intent. Of course, the inability of management successfully to execute a strategy may obscure the reflection of it in firm behavior. Finally, many situations of interest are complex and involve intricacies which make inferences about a strategy uncertain or even questionable.

The mutual fund industry is not a typical setting for investigations in strategic management, but it offers several benefits. Its most distinctive advantage is the legal requirement that every mutual fund publicly declares its investment strategy and follows a rigorous and publicly observable process in order to change its strategy. This makes the determination of strategy unambiguous and the identification of strategic groups equally direct and unequivocal.

A further advantage, concomitant with the requirement of public declaration of strategies and part of the more general governmental regulation of the financial industry, is the public availability, both in market sources and in regulatory filings, of extensive, detailed data on mutual fund activities and performance. Moreover, this data is reported according to standardized, financial definitions. Thus the meanings of the numbers are well established and, because applicable to all firms in the market, comparisons among firms is unproblematic. Finally, analysis is greatly facilitated by the ready availability of the data and other evaluations of performance, both in the financial (and general) press and especially by firms which specialize in its collection, analysis, and provision as their main business functions.

Besides its analytic advantages, the mutual fund industry warrants thorough study because of its large size and influence in the general economy. Because of the broad participation by U. S. citizens (and foreigners) in this market and the substantial proportion of wealth invested in it by many participants (individuals and institutions), mutual funds also represent a socially significant market. This has been a major reason for government oversight and regulation.

After making the case for mutual funds' peculiar advantages for strategic management research, Chapter 3 presents a brief review of the mutual funds market. The historical account really begins with the most basic financial instruments even before the onset of human history with the Mesopotamian civilizations. The sophistication of such instruments evolved in response to changing economic and market conditions, especially in Europe, until the modern form emerged in the United States in the 1920s. A fundamental line in the history of this modern form has been the succession of government laws and regulations which have constrained its evolution and determine the conditions under which it operates today. A description of the general types of funds and their typical corporate organization follows the history. Finally, Chapter 3 explains the common measures of fund activity and performance used in the subsequent mathematical model for evaluating fund performances.

Although mutual funds have not typically been subject to analysis in the strategic management literature, they nonetheless have been extensively analyzed in the finance literature. Chapter 4 briefly examines the basic mathematical models applied in this literature to the evaluation of mutual funds performance. The standard paradigm is the Markowitz "mean-variance" model of portfolio selection, which evaluates portfolios based on the joint criteria of average return and risk. However, as subsequently adopted in much of the finance literature, this model identifies risk as the variance of the stream of returns.

Two problems, theoretical and empirical, arise with this approach. Risk is more properly viewed as the likelihood and possible extent of loss, whether as loss of principal or as opportunity cost when compared to a "risk-free" rate or the rate of return from a superior investment. Investments with returns greater than average are universally desired and are offered as the principal justification for the additional costs of actively managed funds. Indeed, a chief concern in strategic management, both in theory and practice, is the identification, attainment, and assurance of rents, *i. e.*, greater than normal market returns. Therefore, the "mean-variance" or "risk-return" model has the theoretical problem of treating outcomes which are desired and sought after as if they were those which are avoided and against which great effort and expense are directed. It equates the opposite ends of the range of results.

This theoretical difficulty is compounded by empirical results. A corollary of this hypothesis is that, *ex ante*, high prospective returns generally result from investments which bear high risk. Bowman [1980] and others have shown the opposite correlation in their analyses of corporate, as distinct from financial market, performance. In this so-called "Bowman paradox," which is revealed *ex post*, higher returns persistently tend to accrue to low-risk activities and high-risk activities tend to result in low returns or losses.

In addition to the "mean-variance" models, Chapter 4 discusses other models from the finance literature (such as regression, benchmarking, and simulation). In contrast, the chapter also explains the two ways in which the model developed in this investigation treats risk. First, the model for the performance of mutual funds does not include an explicit measure of risk. However, with respect to the definition of risk as the likelihood and potential amount of loss, whether of principal or as opportunity cost (as opposed to its definition as the variance in the stream of returns), the present model does reflect risk indirectly or implicitly. A smaller likelihood of loss would generally result in a smaller *ex post* frequency of losses; and a smaller extent of loss (*i. e.*, amount at stake) would result in a smaller *ex post* amount of losses. Therefore, for the fixed time period examined in this study, less risky strategies present as those with fewer occasions and lower amounts of losses, or, equivalently, those with greater frequencies of positive returns.

The evaluation of risk also appears in a second, more fundamental way. The two strategies compared here, equity income and aggressive growth, are generally considered to represent and are marketed as more and less risk averse investment strategies, respectively. Therefore, the comparison of the two strategies represents an evaluation of the performance of two different strategies with respect to risk. The final chapter on future directions describes extensions to the model to include explicit treatments of risk measures not based on variance and discusses the risk characteristics of other pairs of mutual fund investment strategies for further investigation.

Chapter 4 closes with a review of a type of mutual fund analysis which may but does not necessarily employ the mean-variance/risk-return model. These studies are based on (and here introduce) the method of data envelopment analysis (DEA). Data envelopment analysis is the basic analytical tool employed in this investigation (i) to evaluate strategies and (ii) to distinguish between strategies and their implementation. DEA is a specialized version of linear programming; it is designed for the analysis of empirical performance data for any kind of decision making organization (decision making unit, or DMU) which transforms inputs into outputs. In the present case, the DMUs are mutual funds. DEA is easy to compute, provides actionable goals, and facilitates sensitivity analyses.

Data envelopment analysis measures efficiency based on the ratio of outputs to inputs by generalizing the single-output/single-input technical efficiency measures of engineering and economic production theory to a multidimensional multi-output/multi-input measure. It is nonparametric, that is, it does not assume (but does not preclude) a prior, underlying distribution function of the data nor specific functional form (such as regression equations or production functions) to relate input to output or to relate independent to dependent variables. By optimizing on each individual DMU in the analysis, DEA produces a piecewise linear frontier based on the actual performances of the Pareto-Koopmans efficient DMUs, whereas the typical statistical analysis of a single moment value or regression plane collapses all individual performance data into a single measure of "central tendency."

The basic mathematical model of DEA is developed in Chapter 5, which then presents the new, special application of DEA which effects the identification and evaluation of strategies as distinct from their implementations. The chapter begins by reviewing the modern development of the definition of efficiency in economic analysis and the development of DEA from the basic idea of efficiency. Then the method for strategic analysis is fully elaborated. This includes a review of two sets of DEA–based studies that develop techniques which are instrumental to the procedure for identifying and evaluating strategies. The first set demonstrates the use of a nonparametric, rank-based test for the categorical classification of DEA results. The studies of the second set apply DEA to identify the economic states of short-run and long-run performance. This is accomplished by demonstrating that different categories of DEA results exhibit the characteristics which are defined for the two economic states without requiring recourse to long-term economic time series to measure explicitly the transformation of economic performance over time from the short run to the long run. This approach exploits the theoretical economic characterization of long-run performance as having eliminated the inefficiencies of short-run performance.

These two techniques are combined with DEA into a two-stage procedure for the identification and evaluation of strategies. Each stage comprises two steps. Briefly, in stage one, the first step involves separately generating the DEA efficiency frontier for each of two strategies. In the second step of stage one, the inefficient performers are projected to their respective frontiers and the data for each fund transformed to represent an efficient point on the fully realized strategic frontier. Each point now represents, not the actual performance of an individual fund, but the potential performance of a fund were it fully to realize its strategic possibilities.

In the second stage, step three combines into one group, the projected, transformed funds representing the two separate strategies and a DEA evaluation is computed for the joint group of the new, exemplary "funds" (that is, not actual funds, but examples of the strategic potentials for similarly endowed funds). In the fourth step, to determine if one strategy is significantly superior to the other, a rank test is applied to the efficiency scores resulting from step three. Stage one (steps one and two) evaluates the funds of each strategy separately and describes the maximum potential for each individual strategy. Stage two (steps three and four) compares two strategies and determines whether one strategy, at its best, outperforms the other, at its best.

Chapter 6 presents the data and results of applying this procedure to actual mutual funds. The two strategies evaluated were Aggressive Growth (a high-risk category) and Equity Income (a low-risk category) as represented by mutual funds pursuing those strategies and reporting five years of data from 1993 through 1997. A standard data envelopment analysis of the performance data for the mutual funds in the two strategic groups yields clear evidence of the superiority of Equity Income funds over Aggressive Growth funds. However, the new, strategic evaluation procedure presented here provides even more striking results of the differences between the two strategies. It also gives a better picture of the differences in the performance of the managers within each strategic group and explains why the standard DEA evaluation does not reveal the same discrimination evidenced by this new, strategy-comparison method. Finally, although this method was designed to evaluate strategies based on management performance measures and does not include certain variables of particular interest to shareholders, such as tax liabilities or net shareholder returns (that is, after charges and fees), the results are still useful for shareholders contemplating investment in either of these two types of funds.

This dissertation closes with Chapter 7, which takes up two topics. In the first, the contributions to strategic management and financial studies are critiqued and appraised. The second topic proposes numerous suggestions for extensions, modifications, and alternatives to the new method presented here. The specific model of mutual fund activity to which this method has been applied here is limited in the number and scope of the factors included, the time period evaluated, and the number of strategies examined. Not only are other variables suggested for a fuller depiction of fund activities and performance, but non-discretionary variables are also suggested to reflect more general market, financial, and economic factors which are not within the control of managers but which may constrain or enhance their ability to execute strategy.

Furthermore, there are some factors, such as risk for individual funds, developed during the preceding discussion, which are not explicitly represented in the fund performance model of this study. Model formulations which explicitly include risk factors are also suggested. Also indicated are other kinds of analyses with which the method developed here may be compared and combined for confirmation of the general results and as a check against model bias. Finally, the development here employs the most common DEA model. More of the characteristics (such as the time-varying behavior) of performance may be revealed by employing other forms of the DEA model or other, more elaborate models, such as chance constrained programming formulations.

Chapter 2 Strategy

Section 1 Origins of the Meaning of Strategy

The term "strategy" has its origins in classical Greek history. After a long history in the struggle against the Athenian tyranny and the subsequent Spartan-installed oligarchy, Cleisthenes eventually won election as archon and led the establishment of a democratic constitution in Athens at the end of sixth century. His organizational reforms instituted (probably in 501 BCE) ten *phylae*, the largest political divisions in the state, each based on kinship and incorporating a complete system of priest, official, administrative, and military organization. From each was elected a general to command the troops from his *phylae* and join the war council to the highest elected leader.[1]

The English word "strategy" comes from the Greek ***στρατηγία*** [***strategeia***]—office or command of a general, generalship. *Strategeia*, in turn, comes from ***στρατηγός*** [***strategos***, plural ***strategoi***]—a commander in chief or chief magistrate. *Strategos* derives from the combination of ***στρατ - ός***, army, and ***αγ***, ***άγειν***, to lead.[2]

In Athens, the Assembly elected *strategoi* for one-year terms to the *strategeia*. Many were reelected, some for many years. Trials for misconduct or unsatisfactory performance could result in fines, exile, or execution.[3] The first *strategoi* were generals,

[1]Arnold Wycombe Gomme, "Cleisthenes," p. 199, in *The Oxford Classical Dictionary* (Oxford, UK: Clarendon Press of the Oxford University Press, 1949). For a brief account with modern management interpretations, see Stephen Cummings and David Wilson, "Images of Strategy," p. 9, in Stephen Cummings and David Wilson, eds., *Images of Strategy* (Oxford, UK: Blackwell Publishing, 2003).
[2] *The Compact Edition of the Oxford English Dictionary*, 2 Vols. (New York, NY: Oxford University Press, 1971), vol. 2, p. 3084.
[3] Arnold Wycombe Gomme, "Strategi," p. 863, in *The Oxford Classical Dictionary* (Oxford, UK: Clarendon Press of the Oxford University Press, 1949); Iain G. Spence, *Historical Dictionary of Ancient Greek Warfare,* Historical Dictionaries of War, Revolution, and Civil Unrest, No. 16 (Lanham, MD: Scarecrow Press, 2002), pp. 315-6.

leaders of armies; later the term was used for naval command and magistrates. In the fifth century, when leading politicians were elected to command, "the *strategeia* was the most important elected office in the state."[4]

Later, during the fourth century, politicians were rarely elected *strategoi*, who had by then become specialists, elected to specific military functions at home and abroad, including the *administration* of the system whereby Athens extracted service and money from the richest citizens to maintain its fleet.[5]

During military campaigns, often two or three (once, eight) *strategoi* were appointed to the command of an expeditionary force; sometimes one would have supreme command. One such joint command offers a revealing example of the analytical distinction this dissertation seeks to elucidate between "man and plan," that is, between the execution of a plan by the person so charged and the purpose and potential of a plan as proposed. In his history of the Peloponnesian War (431-404 BCE), Thucydides describes the Sicilian Expedition of 415. Alcibiades used the occasion of a dispute between two Sicilian city-states to urge that Athens launch a large expedition against Syracuse, the capture of which would interdict a major grain supply to Sparta. When Nicias, a prominent general who had been a leader of the movement which had secured a peace treaty with Sparta in 421, warned against the expedition because of its risk, the Assembly increased the size of the force, so it became "by far the most costly and

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[4] Spence, *Ancient Greek Warfare,* pp. 315. For relations to modern management paradigms, see Roger Evered, "So What *is* Strategy?" *Long Range Planning,* Vol. 16, No. 3 (1983), pp. 58-59.
[5] Spence, *Ancient Greek Warfare,* pp. 315-6. Also, Gomme, "Strategi," p. 863, in *Oxford Classical Dictionary*.

splendid Hellenic force that had ever been sent out by a single city up to that time."[6]

In the struggle with Sparta, the Assembly of Athens faced fundamental decisions of management. It met the choice of a goal by approving the pursuit of victory through war. It met the choice of a strategy by authorizing the plan of Alcibiades, who led the only faction with a plan. To confront risk, it sought to reduce uncertainty by doubling its stake. Finally, to address the questions of agency, it chose management by all factions: Alcibiades and Nicias, who had led the two opposing factions in the debate, and Lamachus, who had vacillated between them, were given joint command, including extraordinary powers, of this expensive and risky expedition.

Had the plan of Alcibiades, who was designated supreme commander and who had demonstrated strategic brilliance before (and would again after) this campaign, been executed, the expedition might have changed the final outcome of the war. However, political intrigue forced the recall back to Athens of Alcibiades to stand trial for sacrilege, and rather than face death, he deserted to Sparta. Thus, "[t]he stubborn opponent of the plan, Nicias, was left in command to carry it out, and by his obstinate stupidity, carried it to ruin."[7] The entire expeditionary force, including two later expeditions of reinforcements, was destroyed and the stage set for the final defeat of Athens in its long war against Sparta. For Thucydides, there was a clear distinction between the potential of

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[6] Thucydides; Richard Crawley, trans. and Robert B. Strassler, ed., *The Landmark Thucydides: A Comprehensive Guide to the Peloponnesian War* (New York, NY: Free Press, 1996), p. 377 (Bk. 6, Ch. 31, Sec. 2).

[7] B. H. Liddell Hart, *Strategy*, 2nd rev. ed. (New York, NY: Frederick A. Praeger, 1967), p. 32. See also Henry Dickinson Westlake, "Alcibiades," p. 31 in *The Oxford Classical Dictionary*. This example may also represent a central issue in agency theory, since, according to Thucydides (Bk. 7, Ch. 8, Sec. 11-15), fear of punishment for failure may have caused Nicias' overcautious decision making. Thucydides, *Landmark Thucydides*, pp. 432-435.

the strategy and the actual outcome of its execution, which depended on the abilities of those responsible for its realization.[8]

Section 2 Military Definitions

In the modern European tradition, at the close of the Napoleonic era, Carl von Clausewitz presented, in his treatise *Vom Kriege*, the first comprehensive, modern strategic military doctrine, the fundamental principle of which was that the "political object is the goal, war is the means of reaching it, and means can never be considered in isolation from their purpose." Thus,

> [s]trategy is the use of the engagement for the purpose of the war. The strategist must therefore define an aim for the entire operational side of the war that will be in accordance with its purpose. In other words, he will draft the plan of the war, and *the aim will determine the series of actions* intended to achieve it: he will, in fact, shape the individual campaigns and, within these, decide on the individual engagements.[9]

For von Clausewitz, "pure strategy" is "the problem of the war as a whole." No one can start "without first being clear in his mind what he intends to achieve…and how he intends to conduct" his efforts. Fixing the goals of action

> is the *governing principle* which will set its course, prescribe the scale of means and effort which is required, and make its influence felt throughout

[8] For an analysis which finds the plan of Alcibiades so fundamentally flawed that it entailed not only disaster for the expedition, but also the city-state of Athens, see Barbara Garson, "…Unless It's All Greek to Him," *Los Angeles Times*, September 23, 2004, p. B.11.

[9] Carl von Clausewitz; Michael Howard and Peter Paret, trans. and eds., *On War* (Princeton, NJ: Princeton University Press, 1976), pp. 87, 177 [emphasis added]. Based on work begun as early as 1806 and continued to 1830, *Vom Kriege* was published posthumously in 1832. As a caution against overly intricate definitions of strategy, note that von Clausewitz also remarks that "[e]verything in strategy is very simple, but that does not make everything easy," p. 177. See also Azar Gat, *A History of Military Thought: From the Enlightenment to the Cold War* (Oxford, UK: Oxford University Press, 2001), pp. 170-265.

down to the smallest operational detail.[10]

In the twentieth century, responding to the application of technology to warfare and the consequent mass slaughters of Word War I resulting from the failure to adapt and reform military practice,[11] B. H. Liddell Hart also saw strategy as "the art of distributing and applying military means to fulfil the ends of policy."[12] However, he defined strategy more narrowly and more specifically than von Clausewitz. Stressing the primacy of the political object in military strategy, he criticized von Clausewitz for "intrud[ing] on the sphere of policy, or the higher conduct of the war, which must necessarily be the responsibility of the government and not of the military leaders it employs as its agents in the executive control of operations."[13]

This led Liddell Hart to a view of layered, top-down controlled strategic plans.

> As tactics is an application of strategy on a lower plane, so strategy is an application on a lower plane of 'grand strategy'. While practically synonymous with the policy which guides the conduct of war, as distinct from the more fundamental policy which should govern its object, the term 'grand strategy' serves to bring out the sense of 'policy in execution'. For the role of grand strategy—higher strategy—is to co-ordinate and direct all the resources of a nation, or band of nations, toward the attainment of the political object of the war—the goal defined by fundamental policy.[14]

Consider a specific example from the history and practice of war (the activity, for many popular writers, most comparable to the practice of management). For the goal of

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[10] von Clausewitz, *On War*, pp. 577, 579 [emphasis added].
[11] Gat, *A History of Military Thought*, pp. 643-783.
[12] Liddell Hart, *Strategy*, p. 335.
[13] Liddell Hart, *Strategy*, p. 333.
[14] Liddell Hart, *Strategy*, p. 335-336.

winning World War II, Churchill and Roosevelt agreed on what was later called a “grand strategy”: first, defeat Germany and then, defeat Japan. This decision guided or controlled all of the subsequent plans developed to prosecute the war, as, for instance, in the allocation of military manpower, with the commitment of 10 million men to the European theater and only 2 million to the Pacific until the defeat of Germany permitted a reallocation.

Today, however, the term “strategy” is no longer restricted to a military context. It “is also widely used in other fields, such as mathematical and business gaming, but these usages are essentially derivative or metaphorical….”[15] In addition, there is a large literature regarding the concepts of strategy and strategic management in the public sector—including areas of defense; legislative, executive, and judicial politics; and public administration—and in the non-profit sector.[16] Also, these concepts have figured prominently in the literature of futures research,[17] that is, research on the likely state of the world in the future.

Despite its modern currency, this metaphor is not a new coinage. Xenophon related an encounter of his teacher Socrates, who was also the teacher of Alcibiades. When Nichomachides, an experienced military commander, lost an election for *strategos* to Antisthenes, a businessman whom he thought unqualified, to mollify his resentment, Socrates drew a detailed parallel between the candidates based on the need of each to

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[15] Paul Wilkinson, “strategy” in Alan Bullock and Stephen Trombley, eds., *The Harper Dictionary of Modern Thought,* rev. ed. (New York, NY: Harper & Row, 1988), p. 817.

[16] Jack Rabin, Gerald J. Miller, and W. Bartley Hildreth, *Handbook of Strategic Management*, 2nd ed., rev. and exp. (New York, NY: Marcel Dekker, 2000); Sharon M. Oster, *Modern Competitive Analysis*, 2nd ed. (New York, NY: Oxford University Press, 1994).

[17] Evered, “So What *is* Strategy?” pp. 57, 66-72.

plan and manage the use of resources and personnel to meet his objectives.[18] Moreover, this was not the singular insight of a uniquely perspicacious sage; it was cultural knowledge. As the role of *strategoi* evolved and specialized, especially by the fourth century, some were responsible for financial and logistical administration of Athenian forces.

Twenty-two centuries later, von Clausewitz made a similar observation:

> Rather than comparing [war] to art we could more accurately compare it to commerce, which is also a conflict of human interests and activities; and it is *still* closer to politics, which in turn may be considered as a kind of commerce of a larger scale.[19]

Section 3 Mathematical Definitions

Around the same time that Liddell Hart was developing his criticisms of British military practice, John von Neumann published, in 1928, his proof of the "minimax theorem," as it is now called in the "mathematical theory of games" and began his development of mathematical applications to economics and social theory.[20] The modern

[18] Xenophon; Charles Anthon, ed., *Xenophon's Memorabilia of Socrates* (New York, NY: Harper & Brothers Publishers, 1854), pp. 74-77 (Bk. 3, Ch. 4, Sec. 1-12); Xenophon; J. S. Watson, trans. and ed., *Xenophon's Anabasis, or Expedition of Cyrus and the Memorabilia of Socrates* (London, UK: George Bell and Sons, 1907), pp. 430-433 (Bk. 3, Ch. 4, Sec. 1-12). This chapter of Book 3 is the fourth successive dialog concerned with Socratic instruction on military duty and command and is followed by three more chapters on the proper administration of the state; see Raphael Kühner; George B. Wheeler, trans., "Prolegomena," as reprinted in Xenophon; Anthon, ed., *Memorabilia*, especially p. xiii.

[19] von Clausewitz, *On War*, p. 149.

[20] Robert J. Leonard, "Creating a Context for Game Theory," pp. 29-76; Urs Rellstab, "New Insights into the Collaboration between John von Neumann and Oskar Morgenstern on the *Theory of Games and Economic Behavior*," pp. 77-93; Andrew Schotter, "Oskar Morgenstern's Contribution to the Development of the Theory of Games," pp. 95-112; and Philip Mirowski, "What Were von Neumann and Morgenstern Trying to Accomplish?" pp. 113-147, all in E. Roy Weintraub, ed., *Toward a History of Game Theory*, Annual Supplement to Volume 24, History of Political Economy (Durham, NC: Duke University Press, 1992).

application of the concept of strategy to business began with the game theory of von Neumann and Morgenstern[21], who define strategy as "a plan which specifies what choices [the player] will make in every possible situation, for every possible actual information which he may possess at that moment in conformity with the pattern of information which the rules of the game provide for him."[22]

The requirement of "a plan which *specifies* choices," in a level of detail that stipulates the tactics for *every* contingency, is neither feasible for strategic analysis nor reflective of the realities in practice, which need rather a plan which *guides* choices. The situation is not improved in subsequent applications of game theory to economics. One of the benefits for the mathematical theory of games is a difficulty for strategic management:

> Perhaps the most important aspect of the theory of games as applied to political economy is that the methodology it provides for constructing the mathematical models for the study of conflict and cooperation forces is of an explicitness found rarely even in the many mathematical investigations of political economy. In particular, the extensive form of a game calls for a complete process description.[23]

To extend and develop the basic notions of game-theoretic strategy, Thomas C.

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[21] Jeffrey Bracker, "The Historical Development of the Strategic Management Concept," *The Academy of Management Review,* Vol. 5, No. 2 (April 1980), pp. 219-224; H. Igor Ansoff, *Corporate Strategy: An Analytic Approach to Business Policy for Growth and Expansion* (New York, NY: McGraw-Hill, 1965), p. 180; and Robert J. Leonard, "Creating a Context for Game Theory," pp. 29-76, in Weintraub, *History of Game Theory*.

[22] John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior*, 3rd ed. (Princeton, NJ: Princeton University Press, 1953), p. 79

[23] Martin Shubik, "Game Theory Models and Methods in Political Economy," Chapter 7, in Kenneth J. Arrow and Michael D. Intriligator, eds., *Handbook of Mathematical Economics*, Volume 1 (Amsterdam, NL: North Holland, 1981), p. 285.

Schelling formulated a more detailed and nuanced notion of games and strategy. He argues that "games of mutual dependence" should be the focus of investigation. These are more useful models with practical application to the most typical types of conflict, whereas "zero-sum games"—cases of pure conflict—fall at one extreme and "pure-collaboration" games at the other.[24] However, despite the elaboration of moves and the ramification of game structure to create models more reflective of real-world conflict situations, Schelling's strategy concept still requires the prior, complete, and detailed elaboration of moves, countermoves, and payoffs, as established by von Neumann and Morgenstern.

This same sort of difficulty appears in another mathematical approach. The mathematics of probability, note Dubins and Savage, has an "influence on practical affairs." One motivation for their mathematical analysis of gambling is that "theoretical problems of how to make the best of a bad and risky situation also may eventually lead to applications."[25] Although their formulation initially appears to reflect the situation of an investor, they make similarly stringent demands on the concept of strategy as does game theory:

> The gambler, constrained by his initial fortune f_0 and subject to the rules imposed by the gambling house Γ, must decide how to play; he must choose one among all available strategies. Just what is a strategy? How wide is the domain of the gambler's choice? Loosely, a strategy ought to be a rule—not

[24] Thomas C. Schelling, *The Strategy of Conflict* (Cambridge, MA: Harvard University Press, 1960; reprint ed. New York, NY: Oxford University Press, 1963), p. 84. See also R. Duncan Luce and Howard Raiffa, *Games and Decisions: Introduction and Critical Survey* (New York, NY: John Wiley & Sons, 1957).
[25] Lester E. Dubins and Leonard J. Savage, *How to Gamble If You Must: Inequalities for Stochastic Processes* (New York, NY: McGraw-Hill Book Co., 1965), p. vii.

> necessarily a good one—specifying what gamble the gambler is to choose in every possible contingency.[26]

At the same time that von Neumann was collaborating with Morgenstern on game theory, he was also working with others, including Norbert Wiener, who was investigating questions of feedback in control theory, information and entropy in communication theory, and statistical mechanics. To express the essential unity of this set of problems, Wiener designated the field of communication and control theory, whether in machine or animal, by the term *cybernetics*, from the Greek ***χυβερνήτης*** or *steersman*. He noted that Clerk Maxwell in 1868 had written about feedback mechanisms in a paper on governors, which term derives from the Latin for the same Greek source as cybernetics[27]. Here again is the "governing principle" emphasized by von Clausewitz.

Section 4 Strategy for Management

A strategy is a principle or rule which guides activity toward an end. Strategy is inherently teleological, *i. e.*, it is purposeful, it has an objective. Indeed, Rumelt, Schendel, and Teece define the content of strategic management *qua* discipline as "the *purposeful* direction and natural evolution of enterprises."[28] Andrews also notes that strategy implies "a conscious purpose."[29] Clearly, strategy is distinct from goal, since

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[26] Dubins and Savage, *How to Gamble*, p. 11.
[27] Norbert Weiner, *Cybernetics, or Control and Communication in the Animal and the Machine*, 2nd. ed. (Cambridge, MA: The M. I. T. Press, 1948, 1961), pp. 1-29.
[28] Richard P. Rumelt, Dan E. Schendel, and David J. Teece, eds., *Fundamental Issues in Strategy: A Research Agenda* (Boston, MA: Harvard Business School Press, 1994), p. xi [emphasis added].
[29] Kenneth R. Andrews, *The Concept of Corporate Strategy*, 3rd ed. (Homewood, IL: Irwin, 1987), p. xi.

different strategies may pursue the same end.[30] Nor is strategy the same as the activities undertaken to realize its end, since the same principle may be implemented by different actions. Finally, it is distinct from the context or environment in which the activities take place and from those in which the end is assumed to exist, since the same strategy may be used at different times and under (somewhat) different circumstances. (Useful in this context is Churchman on the teleology of whole systems.[31]) These are all categorical distinctions which, when clearly and rigorously made, help clarify and organize the examination of strategic management and its related concerns.[32]

Strategy is here understood in a general sense, and, as befits strategic management research, which closely examines measures of economic and financial performance, the definition is taken from the accounting literature. In *Kohler's Dictionary for Accountants*, Cooper and Ijiri define strategy as "a plan of action [that] is used to govern or guide other plans of action."[33] Moreover, in this definition, the goal or object of the activity is presumed to be, or explicitly stated as, given and distinguishable from the

[30] "The objective is, first, what the strategy is designed to achieve. Second, it should be a measure of success, a way of knowing when the objective has been reached. Both must be clear and precise. They often are not." George Edward Thibault, "Military Strategy: A Framework for Analysis," in George Edward Thibault, ed., *The Art and Practice of Military Strategy* (Washington, DC: National Defense University, 1984), p. 3.
[31] C. West Churchman, *Prediction and Optimal Decision: Philosophical Issues of a Science of Values*. (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1961) and *The Design of Inquiring Systems: Basic Concepts of Systems and Organization* (New York, NY: Basic Books, Inc., 1971).
[32] For a classical view, Kühner describes Xenophon's characterization of the doctrine of Socrates: "The Good, which should be the object of man's pursuit, is *the useful*…. The useful is defined to be the 'end of action,' or the result which we expect by action. Every thought and act of man should be *useful*, *i. e.*, should have reference to some special end. Independently, then, and in itself, nothing is good, but only becomes such by special reference to its object." Kühner, "Prolegomena," pp. xiv-xv, in Xenophon; Anthon, ed., *Memorabilia*.
[33] W[illiam] W. Cooper and Yuji Ijiri, eds., *Kohler's Dictionary for Accountants*, 6th ed. (Englewood Cliffs, NJ: Prentice-Hall, 1981), pp. 489-490. This idea is not entirely new to strategic management; *e. g.*, Andrews, *Concept of Corporate Strategy*, p. xi, characterizes strategy as a "plan of action" but does not identify its relation to other plans nor distinguish it from other concepts related to strategy.

strategy.

Thus, an overall (corporate) strategy may give rise to business, marketing, financial, and production strategies which represent tactical plans from the overall, corporate standpoint, but these plans, in turn, are also used to guide and control other, more subordinate plans, such as the separate plans used to guide each of the several product lines of a diversified firm or the activities in different local marketing districts.[34] Hence, this definition of strategy can be used flexibly to fit the context in which the term is applied, especially the hierarchical levels of strategies typically defined in the literature: institutional or enterprise, corporate, business, and functional.[35]

Especially after the Second World War, with the increase in the complexity of business practice and its reflection in academic business policy studies, the issues of strategy arose in the management literature during the 1950s, but the first occurrence of the term itself is in Chandler's *Strategy and Structure*,[36] a foundation work for the field of strategic management in business. Since that initial appearance, however, the strategic management literature has not yet established a consistent, verifiable definition of

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[34] Such levels of control and increasing detail in subordinate levels are explicit in Cooper and Ijiri: "**strategy** 1. A *plan* of action. 2. A plan used to govern or guide other plans…. Sense 2 distinguishes different layers of *control* and supporting detail as in a *corporate strategy* which governs a marketing strategy, and so on. The sense 1 usage is general, or generic, and covers all of these possibilities and others as well." Cooper and Ijiri, *Kohler's Dictionary*, pp. 489-490 [emphasis in the original].

[35] For example, Dan E. Schendel and Charles W. Hofer, *Strategic Management: A New View of Business Policy and Planning* (Boston, MA: Little, Brown and Co., 1979), pp. 11-13; Edward H. Bowman, "Strategy Changes: Possible Worlds and Actual Minds," p. 30, in James W. Fredrickson, ed., *Perspectives on Strategic Management* (New York, NY: Harper & Row, 1990); and for an example of the coordination of business strategy by corporate strategy, Alfred D. Chandler, Jr., "The Functions of the HQ Unit in the Multibusiness Firm," Chapter 12, pp. 323-360, in Rumelt, Schendel, and Teece, eds., *Fundamental Issues in Strategy*.

[36] Alfred D. Chandler, *Strategy and Structure: Chapters in the History of the American Industrial Enterprise* (Cambridge, MA: MIT Press, 1962).

strategy, the fundamental concept of the discipline. Strategy research includes many approaches to what strategy is, how it should be formulated, and how it should be pursued.

The classical or standard definitions usually offered in strategic management studies are very different from the accounting definition cited here. For example, many authors quote Chandler, a pioneer of the field: strategy is "…the determination of the basic long-term goals and objectives of an enterprise, and the adoption of courses of action and the allocation of resources necessary for carrying out these goals."[37] However, this definition includes goals and their determination (which are logically prior to a strategy for their achievement), the issue of a strategy's adoption (a question of implementation), and resource allocation (execution and tactical issues), without focusing on the fundamental "courses of action" intended to achieve the goals.

Another foundational writer, Andrews defines corporate strategy as

> …the pattern of decisions in a company that determines and reveals its objectives, purposes or goals, produces the principal policies and plans for achieving those goals, and defines the range of business the company is to pursue, the kind of economic and human organization it is or intends to be, and the nature of the economic and non-economic contribution it intends to make to its shareholders, employees, customers, and community.[38]

The key concept—"the principal policies and plans for achieving those goals"—lies at the core of this statement, but it includes goals, blurs the distinctions between strategy

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[37] Chandler, *Strategy and Structure*, p. 13.
[38] Andrews, *Concept of Corporate Strategy*, p. 13.

and tactics, and ends with a characterization of the organization and its role among all groups of stakeholders. Indeed, Andrews asserts that "to separate goals from the policies designed to achieve those goals" is a crucial mistake.[39] This is consistent with von Clauswitz's insistence that military strategies are meaningless outside the context of their political aims. Nonetheless, goals and strategies are not the same things and their definitions should distinguish one from the other.

Among early writers, Ansoff comes close to the accounting definition above. Proceeding from an elaboration of the classes of decisions made by managers, the explicit separation of strategy from goals and objectives, and the concept of risk as a function of the amount and extent of knowledge, he defines strategy as a "rule for making decisions" "under conditions of partial ignorance…relating to [a] firm's match to its environment."[40] Thus, whereas Ansoff advances the core concept ("rule for making decisions"), he also stresses a general *condition* (uncertainty or "partial ignorance") of strategy *implementation* and loses the sense of strategic purpose by restricting the objective to an indefinite condition (to match a firm to its environment), which is irrelevant, for example, to a firm which seeks to pursue a disruptive strategy or to change its environment.

Reviewing the first 20 years of strategic management literature, Hofer and Schendel analyze 13 different formulations of the concept of strategy. Their analysis identifies three major areas of disagreement: (i) "the breadth of the concept of strategy" (whether strategy includes goals); (ii) whether strategy (exclusive of goals) has

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39 Andrews, *Concept of Corporate Strategy*, p. 14.
40 Ansoff, *Corporate Strategy*, pp. 119-121.

components; and (iii) whether the strategy formulation process includes goal setting.[41]

They define strategy as "a statement of the *fundamental means* [an organization] will use, subject to a set of environmental constraints to try to achieve its objectives" and, because action is required for the attainment of objectives, the statement must include "a description of the most important patterns of…resource deployments" and "a description of the most critical…environmental interactions." They define the components of strategy as (i) scope or domain, the extent of interaction with the environment; (ii) resource deployment or distinctive competences; (iii) competitive advantages; and (iv) synergy. Thus, their definition is narrow (that is, exclusive of goals), but with components. To the broader, more inclusive notion, they apply another military term, "grand design," which comprises goals, strategies, and policies.[42]

More recent writers are no more succinct or focused than their predecessors. In his effort to establish an analytical basis and generalized rules for the field of strategic management, Porter provides a three-part characterization of strategy—creating a unique position, choosing among trade-offs, and integrating the firm's many activities[43]—which is less a general definition of strategy than a prescription of three (among several) concerns that successful strategies typically undertake. In Porter's major works, there are

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[41] Charles W. Hofer and Dan Schendel, *Strategy Formulation: Analytical Concepts* (St. Paul, MN: West Publishing Co., 1978), pp. 17-20.

[42] Hofer and Schendel, *Strategy Formulation*, pp. 23-25 [emphasis in the original].

[43] "Strategy is the creation of a unique and valuable position, involving a different set of activities…. Strategy is making trade-offs in competing. The essence of strategy is choosing what not to do. Without trade-offs, there would be no need for choice and thus no need for strategy…. Strategy is creating fit among a company's activities. The success of a strategy depends on doing many things well—not just a few and integrating among them. If there is no fit among activities, there is no distinctive strategy and little sustainability." Michael E. Porter, "What Is Strategy?" *Harvard Business Review*, Vol. 74, No. 6 (Nov/Dec 1996), pp. 66, 68, 70.

only three "generic strategies." Low cost leadership, product differentiation, and narrow market or segment focus are the three exclusive, general means of securing sustainable rents ("competitive advantage") in a necessarily competitive context. The chief concern is the implementation of these strategies to confront the "five forces" (buyers, suppliers, new entrants, substitutes, and competitors) in the context of the particulars of a given industry structure.[44]

Hamel and Prahalad[45] and Hamel[46] restrict strategy to the process by which a firm anticipates radically different future needs, transforms itself to gain a preemptive position in that future, and thereby shapes the structure of future industry. For them, the fundamental strategy is the development of superior capabilities, "core competence," in anticipation of future demand. None of these is untrue or unimportant, but they are all pieces of the story and pieces of related stories. Most researchers define not strategies in general but what they believe to be *good* strategies for firms in a competitive market environment. Most (Porter is an important example) also implicitly anticipate a stable industry structure (or one predictably evolving, such as the classical industry maturation cycle), although Hamel and Prahalad stress preparation for (exogenously given) disruptive changes in the future.

In 1990, Mintzberg offered a typology of ten "schools of thought" about strategy,

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[44] Michael E. Porter, *Competitive Strategy, Techniques for Analyzing Industries and Competitors* (New York, NY: Free Press, 1980) and *Competitive Advantage: Creating and Sustaining Superior Performance*, with a new Introduction, (New York, NY: Free Press, 1985, 1998).

[45] Gary Hamel and C. K. Prahalad, *Competing for the Future* (Boston, MA: Harvard Business School Press, 1994).

[46] Gary Hamel, "Strategy As Revolution," *Harvard Business Review*, Vol. 74, Issue 4 (Jul/Aug 1996), p. 69.

but his “schools” focus on ten types of strategy formulation process. This necessarily mixes with concepts of strategy the associated concepts of goals, environmental context, resource evaluations, implementation style and timing, and organizational culture. These ten schools are further categorized into three groups according to the kind of theory they represent. The first three schools constitute the group of normative or prescriptive theory, that is, how strategies should be formulated. The second group comprises six schools of positivist or descriptive theory, that is, how strategies are in practice formulated. This group includes the “entrepreneurial school,” which includes firms which do not have strategies, and the “learning school,” which also may include firms without strategies and which is especially susceptible to confusing strategy and implementation, since its strategy is only revealed (and, indeed, apprehended by the firm itself) *ex post facto* in the succession of its actions. In the end, Mintzberg endorses the tenth school of his schema and the only member of the third group of integrative theory, the “configuration school,” which combines every view, every process, every ancillary problem in staged or life cycle models.[47]

In the strategic management literature, many efforts to define strategy leave out no idea related to the basic concept. Their conceptual frameworks lack the parsimony of analytic discernment. Consequently, strategy is confused with other significant but implicit, complementary, associated, ancillary, or ensuant concepts; and its essential meaning, its cybernetic character, is obscured. Moreover, such catchall definitions

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[47] Henry Mintzberg, “Strategy Formation: Schools of Thought,” pp. 105-235, in Fredrickson, *Perspectives on Strategic Management*. For an expanded updating of “Schools of Thought,” see Henry Mintzberg, Bruce Ahlstrand, and Joseph Lampel, *Strategy Safari: A Guided Tour Through the Wilds of Strategic Management*, (New York, NY: Free Press, 1998).

appropriate aspects or characteristics of putatively good strategies in specific or narrow industrial settings. In contrast, the definition from accounting is both general and precise: general, because it is applicable in any business setting, such as, functional, business, or corporate, as well as in other disciplines, such as military science, political theory, foreign policy, and sociology; and precise because it is concisely formulated and directly and unambiguously identifies the object of interest in any given setting.

The definition provided by Cooper and Ijiri can function as an organizing concept for strategic management. The essential meanings of complementary concepts become clear in relation to it and distinct from each other. It distinguishes itself from the ideas related to strategy; and by establishing their relationship to itself, it distinguishes each from the others. It establishes an overall conceptual order elucidating the logical connections and dependencies of the concepts of strategic management.

Another difference facilitated by clear categorical distinctions is the evaluation of whole strategies. For example, the classic and seminal series by Rumelt,[48] which relies on Chandler's definition, is concerned with the partial strategy or sub-strategy of diversification and its relation to firm performance. Furthermore, these strategies can only be inferred by means of an elaborate classification methodology. Indeed, many studies in strategic management investigate substrategies or aspects of strategies. Such restrictions to substrategies may be imposed by the fact that only some aspects of firms' whole strategies may be observable. Even limited strategies must often be inferred

[48] Richard P. Rumelt, *Strategy, Structure, and Economic Performance* (Boston, MA: Harvard Business School, 1974, 1986); "Diversification strategy and profitability," *Strategic Management Journal*, Vol. 3, No. 4 (Oct-Dec 1982), pp. 359-369; "How much does industry matter?" *Strategic Management Journal*, Vol. 12, No. 3 (Mar 1991), pp. 167-185.

indirectly rather than measured as the result of direct observation.

The difficulty in assessing an overall strategy, either corporate or business, is closely related to another fundamental difficulty of strategic management research: the inaccessibility or unobservability of the strategy or any aspect of it at all. Three fundamental factors contribute to this situation. First, a firm may not have a strategy. This was probably more prevalent in the past, before the advent of sophisticated strategic management theories and procedures available today and before the decline in post-World War II economic growth and the erosion of the temporary competitive advantage of U. S. firms due to the destruction of their industrial competitors in the war. Besides manager preference for an extemporaneous style, the firm may face conditions (*e. g.*, rapidly growing markets, slowly changing environment, and lack of competition) which permit "organic unmanaged adaptation"[49] or which at least mitigate the pressures for goal setting, strategy formulation, environment and resource evaluation, and in-depth planning.

In addition, Ansoff suggests there may be reduced scope for strategic management among portfolio companies, that is, holding companies and investment trusts, such as mutual funds.[50] Porter, on the contrary, criticizes the tendency among diversified firms to reduce corporate strategy to the portfolio management of their ensemble of business units. He argues that the most important strategic issue for diversified firms is the development of a "horizontal strategy" by which business unit

[49] H. Igor Ansoff, *Implanting Strategic Management* (Englewood Cliffs, NJ: Prentice-Hall International, 1984), pp. 33-34, 459-462 and Ansoff, *Corporate Strategy,* pp. 112-118. See also, Andrews, *Concept of Corporate Strategy,* pp. 17-18

[50] Ansoff, *Corporate Strategy,* pp. 116-117.

strategies and activities are coordinated to produce interaction or synergy effects.[51] Moreover, history indicates that a passive, portfolio management style may not be efficient and, in periods of intense economic competition, not viable. Shleifer and Vishny[52] argue that the merger and divestiture wave of the 1980s unwound the conglomerate merger wave of the 1960s because such a unrelated diversified corporate form is generally inefficient and could not withstand the increased pressures of heightened competition in the 1980s.

A second factor obscuring strategies is that firms may intentionally hide them. By maintaining confidentiality, the firm may hope to avoid provoking either internal or external resistance to planned actions.[53] Moreover, if the formulation of the strategy (particularly in its content or detail) is especially revealing of a firm (for example, its capabilities or intents), its disclosure may provide a business intelligence advantage to competitors. Finally, if the strategy is perspicuous in the assessment of business conditions or opportunities, it represents a resource, the inimitability of which is protected by nondisclosure.

This last concern leads directly to the third factor. From the viewpoint of the philosophy of science, the study of strategic management presents several theories, the fundamental concepts of which are either measurement unobservable (the means for direct, measurable observation in principle cannot exist) or state unobservable (the act of

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[51] Porter, *Competitive Advantage*, pp. 317-442.

[52] Andrei Shleifer and Robert W. Vishny, "Takeovers in the '60s and the '80s: Evidence and Implications," *Strategic Management Journal*, Vol. 12, Special Issue: Fundamental Research Issues in Strategy and Economics (Winter 1991), pp. 51-59; reprinted in Rumelt, Schendel, and Teece, eds., *Fundamental Issues in Strategy*, pp. 403-418.

[53] Andrews, *Concept of Corporate Strategy,* pp. 17-18.

observing changes the state of the phenomenon). The opportunism of transaction cost economics is both measurement and state unobservable; the utility functions of agency theory (and mathematical game theory and economics) are measurement unobservables; and, in the resource-based view of the firm, the sustainability of the competitive advantage from a relatively inimitable resource is enhanced by increasing its unobservability.[54] The difficulties for research in strategic management posed by these kinds of issues resulted in studies which provoked early criticisms of a mechanistic application of logical positivist reductionism and of their lack of usefulness for practitioners.[55] Indeed, strategy itself may often be characterized as measurement unobservable. This dissertation proposes a context, the mutual fund industry, and a method, a special application of data envelopment analysis, which together can provide for strategic management research results which are both phenomenologically grounded and significantly consequential for practice.

Section 5 Implementation of Strategy

Because strategy cannot be observed directly, inevitable uncertainty characterizes its evaluation. An *ex ante* evaluation of intent is uncertain because incomplete knowledge and inaccurate beliefs of relevant present and future events preclude an exact calculation of a strategy's outcome. Complimentarily, an *ex post* evaluation of strategy

[54] Paul C. Godfrey and Charles W. L. Hill, Jr., "The Philosophy of Science and the Problems of Unobservables in Strategic Management Research," pp. 227-248, in Rabin, Miller, Hildreth, *Handbook of Strategic Management*.

[55] Bowman, "Strategy Changes," pp. 25-27, in Fredrickson, ed., *Perspectives on Strategic Management*; Roger Evered, "Strategic Management: A New View of Business Policy and Planning," *Administrative Science Quarterly*, Vol. 25, No. 3, (Sep 1980), pp. 541-542, also cited in Bowman, "Strategy Changes," p. 25.

depends on the assessment of its implementation as proxy for its potential attainment. With an estimate of the potential of a strategy separate from the actual outcomes of its implementation, not only is strategy clarified, but the understanding of its implementation is greatly improved in two ways of practical importance. A separate estimate of a strategy's potential provides an appropriate standard for the evaluation of the management effort which implemented the strategy. Further, evaluation of managers' practice against the standard of full strategic potential can identify the activities which contributed to the strategy's successful or inadequate implementation.

Since the *sine qua non* of strategy is the guiding of action to the attainment of goals, methods for improving its effective implementation are of as significant practical importance as methods for improving the quality of strategies formulated. Moreover, the current status of practice in strategic management makes such improvement urgent. The high rates and great costs of failure among corporate efforts to implement strategy present an opportunity for significant impact in the practice of strategic management.

A 1998 study in the United States presented two findings directly relevant to this issue. First, a survey of 275 portfolio managers and analysis of 300 investment reports revealed that better than 33 percent of investment decisions rely on the assessment of intangibles like corporate strategy, management credibility, innovation, and ability to recruit and retain talent. In addition to financial data, investors expend much effort seeking information with which to assess those intangibles which best predict future earnings. The predictions of analysts who used such measures showed significant improvement in accuracy over predictions by those who did not.[56]

Second, although there was variation by industry on the most revealing of the 39 nonfinancial indicators evaluated, overall, the most important was the ability to execute strategy, even more than the quality of the strategy formulated.[57] A report in 2000 replicated these results in the United Kingdom, where, on average, almost 45 percent of the investment decision depended on nonfinancial measures.[58] In 2001, a study of the United States financial services sector reported similar results, including the finding that financial data alone explained less than 50 percent of the variance of share price, whereas the addition of measures for intangibles accounted for over 80 percent of the variance.[59] In both replications, successful strategy execution was the most important factor: "The fact that a company had a reputation for fulfilling its strategy seemed more important than whether its strategy was sound."[60] In 2002, the senior executives from the Global Most Admired Companies, as selected by a survey of 10,000 directors, executives, and analysts from around the world, reported a 71 percent success rate in implementing their strategies, against 47 percent for peer companies not among the "Most Admired."[61]

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[56] Ernst & Young, *Measures that Matter*, (Boston, MA; 1998); cited in Jonathan Low and Tony Siesfeld, "Measures that matter: Wall Street considers more than you think," *Strategy & Leadership*, Vol. 26, No. 2 (March-April 1998), 24-28; David A. Light, "Performance Measurement," *Harvard Business Review*, (Nov.-Dec. 1998), p. 17; and Robert S. Kaplan and David P. Norton, *The Strategy-Focused Organization: How Balanced Scorecard Companies Thrive in the New Business Environment*, (Boston, MA: Harvard Business School Press, 2001), p. 1.

[57] Ernst & Young, *Measures that Matter.*

[58] Andrew Tivey, *et al.*, *Measures That Matter: An outside-in perspective on shareholder value recognition*, (London, UK: Ernst & Young, 2000).

[59] Robert Holman and Richard Flavell, "The measures that matter," *The Banker*, Vol. 151, Issue 900, (February 2001), pp. 76-77.

[60] Holman and Flavell, "The measures that matter," p. 76.

[61] "FORTUNE/Hay Group Study of the Global Most Admired Companies: What defines a great enduring company?" Toronto, ON: Canada News Wire, May 13, 2002, [http://web.lexis-nexis.com/universe/document?_m=404ea3e5447f5984f1635e906c5f2163&_docnum=77&wchp=dGLbVlb

The consequences for managers of failed execution of strategy can lead to career failure. A 1982 article on the changes overcoming the limited practice of strategic management[62] reported that "a survey of management consultants reported that fewer than 10 percent of effectively formulated strategies were successfully implemented."[63] Seventeen years later, the situation had not improved. A review of the careers of more than 65 CEOs reported that, while the average tenure of chief executives remained at nearly eight years, poor performers were fired more quickly—in one-third the time of a generation before. In 70 percent of these dismissals, the major cause was failure to execute strategy.[64]

The cost of strategy implementation failure is large both in the individual case and in the cumulative loss from the high failure rate throughout industry. A detailed analysis of merger activity among manufacturing and minerals companies in the 1960s and 1970s found that, despite the occasional successful performer, on average one third of the 6,000 acquired firms were subsequently sold off. For those cases where the acquisition was retained, profitability for pooling-of-interest acquisitions declined sharply from pre-acquisition levels down to industry norms. For purchase and tender offer acquisitions, pre-merger profitability at industry averages declined post-merger to well below the levels of no-merger peers.[65] In a review of the acquisitions by 33 large American

-zSkVb&_md5=2e2b4e427449a358dec826efa2f5558e]; and Christine Y. Chen and Matthew Boyle, "The World's Most Admired Companies 2002," *Fortune*, Vol. 145, Issue 5 (Mar. 4, 2002), pp. 26-31.

[62] Walter Kiechel III, "Corporate strategists under fire," *Fortune*, Vol. 106, (Dec. 27, 1982), pp. 34-38.

[63] Kaplan and Norton, *The Strategy-Focused Organization*, p. 1.

[64] Ram Charan and G. Colvin, "Why CEOs Fail," *Fortune*, Vol. 139, (June 21, 1999), pp. 68 ff.

[65] David J. Ravenscraft and F. M. Scherer, *Mergers, Sell-Offs, and Economic Efficiency*, (Washington, DC: The Brookings Institution, 1987), pp. 192-195. The same general results, less sharply drawn, appear in Rumelt, *Strategy, Structure, and Economic Performance*.

companies from 1950 to 1980, 53 percent overall were subsequently divested, from the low of 44 percent for acquired start-ups to the high of 74 percent for acquisitions in unrelated new fields.[66] Porter warned that, in consequence of these failures, many blamed strategic thinking itself rather than correcting the flaws in their strategic conceptions.

The correction for all those bungled acquisitions represented by the 1980s merger and divestiture wave did not end problems in acquisitions activity. From 1995 to 2001, of 302 major acquisitions (each at least $500 million, requiring at least 15 percent and averaging 47 percent of the buyer's market capitalization), 61 percent resulted in a significant loss of shareholder value. "A year after their deals, the losers' average return was 25 percentage points below their industry peers'…. The average return for all buyers was 4.3% below their peers and 9.2% below the S&P 500."[67] In February, 2004, in the wake of the collapse of Enron, WorldCom, and others, a report of the International Federation of Accountants based on 27 case studies (16 failures and 11 successes) from 10 countries found that failures of major corporations was often due to poorly implemented or articulated strategies, the most significant type of which was failed acquisition. The primary recommendation in this report was the establishment of a board of directors committee—analogous to the audit committee for conformance issues—specifically to guide and monitor performance in strategic implementation.[68]

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[66] Michael E. Porter, "From competitive advantage to corporate strategy," *Harvard Business Review*, Vol. 65, Issue 3 (May/June 1987), pp. 43-59.
[67] David Henry and Frederick F. Jespersen, "Mergers: Why Most Big Deals Don't Pay Off," *Business Week*, Issue 3803 (Oct. 14, 2002), pp. 60 ff.
[68] Bill Connell, Richard Mallett, Patrick Rochet, Edward Chow, Luca Savino, Prisilla Payne, *Enterprise Governance: Getting the Balance Right*, New York, NY: International Federation of Accountants, 2004.

Chapter 3 Mutual Funds Industry

Section 1 Advantages of Mutual Funds for Strategic Analysis

The basis of the present day structure of mutual funds was established by a series of hearings, reports, and legislation that culminated in the Investment Company Act of 1940 (ICA 1940), which, among other things, requires that investment companies publicly declare in the prospectus and SEC filings their investment objectives, styles, and policies; severely restricts how changes in those declared policies may be made; and establishes the responsibility of mandated, independent directors to insure conformance to those policies.[1]

This dissertation distinguishes between the fund's "objective," defined as the fund's purpose or goal and the fund's "strategy," defined as a plan to guide the actions implemented to achieve the goal; that is, in particular, between the intent to make profits by operating an investment fund and, as defined in regulations and the prospectus, the policies, styles, and limitations—publicly declared and legally binding—which guide the investment management activity. Thus, an advantage of this industry is that clearly defined strategies and changes in strategy are definitively ascribable to firms and need not be guessed at or otherwise inferred. Examining the mutual funds which have declared one of 36 strategies provides performance exemplars for that strategy class or category.[2]

In addition, the structure and constraints in the mutual fund industry provide a

[1] 15 USCS §80a–8, 13, 15, 24.
[2] Morningstar reports 36 categories for the data series analyzed here. Currently, the Investment Company Institute (ICI) uses 33 categories.

simplified and more accessible version of the issues and processes at work in other industries in less obvious and more involved forms. Much of portfolio and options analysis has been carried over to product firms' corporate and business level strategy considerations as evidenced by the "real options" approach.[3] Also, outsourcing of firm functions (mutual funds typically contract with third parties for most or all of their functions) is increasingly common in many operating companies. Furthermore, the performance evaluations of the funds within a given strategy class will involve asset valuations, rates of return, risk measures, and opportunity costs, all of which are given directly, immediately, publicly, and objectively by the market in this industry and directly associated with the managers responsible for executing the strategies of each firm. Because the values of variables in the models here developed measure directly, or nearly so, the processes of interest and the results of the evaluations have direct consequences for management practice, this analysis thus hopes to withstand E. H. Bowman's criticism of some strategic management research as the desiccations of logical positivist reductionism.[4]

The mutual funds industry is larger than most other industries of any kind. There are important economic and social consequences of this now $7 trillion (and growing) industry, which represents the second largest financial market (after banking) in the United States and with which are placed the investments of 95 million shareholders in 54

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[3] Martha Amram and Nalin Kulatilaka. *Real Options: Managing Strategic Investment in an Uncertain World* (Boston, MA: Harvard Business School Press, 1999).
[4] E. H. Bowman, "Strategy Changes," in Fredrickson, ed., *Perspectives on Strategic Management*, pp. 17-18, 22, 25-29.

million (almost 50 per cent of all) US households.[5]

This chapter reviews the origin, development, and structure of the mutual fund industry, the legal and regulatory constraints, and the organizational structure and operational management of mutual funds in the United States. In order to clarify the financial and economic functions of mutual funds and the settings from which they arise, Section 2 begins with an account of the prehistoric origins of financial instruments. This history places the development of the mutual fund type of investment in the context of economic evolution and helps dispel pretensions of modernist or American parochialism.[6] Section 3 takes up a more proximate history in the "second financial revolution" with the rise of the European nation state and the origin of predecessor instruments. Section 4 traces the development of the modern form of mutual funds from their origins in the late 19th century through the developments in the beginning of the 21st century. It reviews the initial and subsequent legal constraints on the industry and the several stages of the industry's growth. Finally, Section 5 briefly describes an aspect of the typical organizational structure and operational management of modern American mutual funds that determines the relationship between the fund management company and the fund shareholders.

Section 2 Ancient Forms of Financial Instruments

To understand the origin and development of mutual funds requires recognition of

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[5] Investment Company Institute, *2003 Mutual Fund Fact Book* (Washington, DC: Investment Company Institute, 2003).

[6] The thorough and insightful book by Lee Gremillion, *Mutual Fund Industry Handbook: A Comprehensive Guide for Investment Professionals* (Hoboken, NJ: John Wiley & Sons, Inc. and The National Investment Company Service Association, 2005) is the second edition of a work originally titled, *A Purely American Invention: The U. S. Open-End Mutual Fund Industry* (NICSA, 2001).

the fundamental characteristics of this form of investment. A mutual fund is a pooling, organized by an investment intermediary, of assets from shareholders for investment in financial instruments, the returns from which provide a stream of revenue to the shareholders. This general definition includes a broad range of specific instruments besides mutual funds, but these are the basic components. The distinguishing characteristics developed subsequently.

For investors, the suppliers of capital, the benefits of this type of instrument are several. First among them is the fundamental purpose of investment: the increase of assets in their conversion to a stream of future payments. A second benefit is risk reduction through the diversification of the pool portfolio. The pooling of assets provides individual shareholders a scope of diversification, and thus, a reduction of risk, often unattainable in the portfolios of individual means. A third is also a consequence of the scaling effect of pooling: participation in markets and investments inaccessible to smaller commitments. Fourth, and especially relevant for smaller investors, the project is managed by investment professionals.

With respect to the demand for capital, this type of instrument also has several advantages. First, it brings together amounts of capital which are individually inadequate for many requirements and, through pooling, creates a greater effective supply. Second, this instrument makes capital available to illiquid investments through securitization, that is, the intermediation of the fund allows investors to loan funds to illiquid activities through a security which is relatively liquid for the investor. Third, but by no means last, the increased mobilization of capital improves the efficiency of the financial markets and therefore reduces the interest cost to borrowers.

Besides the parties representing forces of supply of and demand for capital, there is a third class of party to the transactions inherent in mutual funds: the fund sponsor and those providing services to the fund in support of its activities. The intermediation they provide generates several revenue streams, the number and nature of which have evolved with the development and refinement of the mutual fund as an investment instrument.

Mutual funds are the product of longer than historical development. The more elementary features of financial instruments were developed first; and, over time, with experience and the emergence of new and more complex economic conditions, subsequent features were devised and combined. The most basic aspect of these instruments exemplifies a fundamental concept of finance, namely, the exchange rate between present and future use of resources—interest.

The warmth and increased rain at the end of the Pleistocene ice age about 13,000 years ago and the resulting increase in the wild plant and animal species which supported the hunter-gatherer societies in the regions of the Fertile Crescent in turn promoted the increase in human population in these areas. About 10,000 years ago began the two-millennial period during which these plants and animals were first domesticated and agriculture emerged.[7] Early in the fourth milliennium BCE, climate change reduced the rainfall in lower Mesopotamia and opened up the area for greater permanent human habitation and the introduction of irrigation-based farming, which was intensified with

[7] Bruce D. Smith, *The Emergence of Agriculture* (New York, NY: Scientific American Library, 1995), pp. 48-89.

the further decrease in precipitation by the beginning of the third milliennium.[8]

After the emergence of agriculture, the need to manage and account for the increased economic product for the growing population led to the development of a system of tokens (small artifacts of clay, shaped in specific forms) which provided a persistent record of the types and quantities produced and exchanged, as well as claims to services (labor). Over time, as the economy developed other crops, herding, fishing, and crafts manufacture, the society developed specialized production and elaborated complex hierarchies, first in the religious organizations (which produced the first monumental architecture around 3350) and then the state organizations.[9] The accounting system of tokens was expanded and adapted to the new requirements until it led eventually to the development of writing (around 3150)[10] and counting devices (around 3100).[11] The subsequent, written accounting methods evolved in complexity until they exhibited a kind

[8] Hans J. Nissen, Peter Damerow, and Robert K. Englund; Paul Larsen, trans., *Archaic Bookkeeping: Early Writing and Techniques of Economic Administration in the Ancient Near East* (Chicago, IL: The University of Chicago Press, 1993), p. 1.

[9] Marc Van De Mieroop, "The Invention of Interest: Sumerian Loans," pp. 17-30, in William N. Goetzmann and K. Geert Rouwenhorst, eds., *The Origins of Value: The Financial Innovations That Created Modern Capital Markets* (New York, NY: Oxford University Press, 2005).

[10] Denise Schmandt-Besserat, *Before Writing, Volume I: From Counting to Cuneiform* (Austin, TX: University of Texas Press, 1992); Nissen, Englund, Damerow, *Archaic Bookkeeping*, pp. 11-18; Richard Mattessich, "Recent Insights into Mesopotamian Accounting of the 3rd Millennium B. C.—Successor to Token Accounting," *Accounting Historians Journal*, Vol. 25, No. 1 (June 1998), pp. 1-27, reprinted with corrections in Richard Mattessich, *The Beginnings of Accounting and Accounting Thought: Accounting Practice in the Middle East (8000 B.C. to 2000 B.C.) and Accounting Thought in India (300 B.C. and the Middle Ages)* (New York, NY: Garland Publishing, 2000), pp. 99-128; Tom Mouck, "Ancient Mesopotamian Accounting and Human Cognitive Evolution," *Accounting Historians Journal*, Vol. 31, No. 2 (December 2004), pp. 97-124. For an account framed in the concepts of Foucault and Derrida, see Mahmoud Ezzamel and Keith Hoskin, "Retheorizing Accounting, Writing and Money with Evidence from Mesopotamia and Ancient Egypt," *Critical Perspectives on Accounting*, Vol. 13, Issue 3 (June 2002), pp. 333-367. For an account of the argument that writing developed from pre-dynastic Egyptian inventory accounting, see Richard Mattessich, "The Oldest Writings, and Inventory Tags of Egypt," *Accounting Historians Journal*, Vol. 29, No. 1 (June 2002), pp. 197-208, which reviews Günter Dreyer, *Umm el-Qaab I: Das Prädynastische Königsgrab U-j und seine frühen Schriftzeugnisse* [Umm el-Qaab I: The Predynastic Royal Tomb U-j and its Early Writing Evidence], (Mainz, DE: Philip von Zabern, 1998).

of double-entry record keeping, with applications to distribution and production cost accounting, labor costing, and long-term budgetary planning.[12]

The centralization of production and exchange and the maintenance of reserves were initially organized through the religious institutions, which were based on self-supporting estates,[13] but over the first half of the third millennium, these were somewhat superseded by state institutions, which also were based on self-supporting estates.[14] Credit seems first to have developed because the vicissitudes of agriculture forced some producers into arrears for their obligatory contributions to the religious or state institutions. In addition to institutional credit, loans (with interest) from other individuals appeared. As both state and religious institutions increased the use of intermediaries, or tax farmers, the two forms of credit (institutional and individual) became increasingly mixed.

The forms of credit expanded. Loans of silver or loans in kind with repayment in silver are also represented in the archaeological evidence. Among the many thousands of clay tablets, which recorded, *inter alia*, contracts, inventories, and court decisions, and

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[11] Denise Schmandt-Besserat, "Tokens and Counting," *Biblical Archaeologist*, Vol. 46 (1983), pp. 117-120; Nissen, Englund, Damerow, *Archaic Bookkeeping*, pp. 25-29.
[12] Orville R. Keister, "Commercial Record-Keeping in Ancient Mesopotamia," *The Accounting Review*, Vol. 38, No. 2 (April 1963), pp. 371-176; Richard Mattessich, "Counting, Accounting, and the Input-Output Principle: Recent Archeological Evidence Revising Our View on the Evolution of Early Record Keeping," in O. Finley Graves, ed., *The Costing Heritage—Studies in Honor of S. Paul Garner* (Harrisonburg, VA: Academy of Accounting Historians, Monograph No. 6, 1991), pp. 25-49, reprinted in Mattessich, *Beginnings of Accounting and Accounting Thought*, pp. 45-69; Richard Mattessich, "Recent Insights into Mesopotamian Accounting"; Nissen, Englund, Damerow, *Archaic Bookkeeping*, pp. 30-104.
[13] Richard L. Sterba, "The Organization and Management of the Temple Corporations in Ancient Mesopotamia," *The Academy of Management Review*, Vol. 1, No. 3 (July 1976), pp. 16-26.
[14] Norman Yoffee, "Political Economy in Early Mesopotamian States," *Annual Review of Anthropology*, Vol. 24 (1995), pp. 281-311; Guillermo Algaze, "Initial Social Complexity in Southwestern Asia: The Mesopotamian Advantage," *Current Anthropology*, Vol. 42, No. 2 (April 2001), pp. 199-233; A. H. Pruessner, "The Earliest Traces of Negotiable Instruments," *The American Journal of Semitic Languages and Literatures*," Vol. 44, No. 2 (January 1928), pp. 88-107.

which survive from the several city-states, regional kingdoms, and empires of the Mesopotamian civilizations, some from as early as the 24th century appear to be loan contracts. Early examples of Babylonian contracts for silver loans at interest have been dated back to ca. 1820 BCE.[15] Credit is also attested by loans for overland and maritime trade ventures.[16] Thus, even before writing was developed in human history, the Mesopotamian civilizations had forms of credit and interest. Once the economy had become more diverse and complex, both equity and debenture participation in merchant financing was recorded in cuneiform tablets.

Moreover, the evidence of credit and interest exists not only in the many examples remaining from individual financial transactions, but also in the codification of social practice as represented by the promulgated laws or collected legal summaries of the major rulers. Laws controlling or protecting property interests were prominent in the succession of Mesopotamian law codes,[17] which also included regulations regarding loans and interest. There is substantial similarity (although the specific focus varied) among the legal pronouncements, from the earliest and most fragmentary evidence of Urukagina (Uru-Inimgina) of Lagash (ca. 2400 BCE),[18] through Ur-Nammu of Ur (ca.

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[15] Van De Mieroop, "The Invention of Interest," pp. 23-25.

[16] Audrey Bossuyt, Laurence Broze, and Victor Ginsburgh, "On Invisible Trade Relations between Mesopotamian Cities during the Third Millennium B.C.," *The Professional Geographer*, Vol. 53, No. 3 (August 2001), pp. 374-383; A. L. Oppenheim, "The Seafaring Merchants of Ur," *Journal of the American Oriental Society*, Vol. 74, No. 1 (January-March 1954), pp. 6-17.

[17] Martha T. Roth, *Law Collections from Mesopotamia and Asia Minor* (Atlanta, GA: Scholars Press, 1995); Russ VerSteeg, *Early Mesopotamian Law* (Durham, NC: Carolina Academic Press, 2000); G. R. Driver and John C. Miles, eds., trans. and commentary, *The Babylonian Laws, Volume 1: Legal Commentary* (Oxford, UK: The Clarendon Press at the Oxford University Press, 1952); Raymond Westbrook, *A History of Ancient Near Eastern Law, Volume 1*, Volume Seventy-Two in the Handbook of Oriental Studies (Leiden, NL: Brill, 2003).

[18] VerSteeg, *Early Mesopotamian Law*, pp. 18-19.

2100)[19], Lipit-Ishtar of Isin (ca. 1930),[20] the laws of Eshnunna (attributed to Dadusha, ca. 1770)[21], Hammurabi of Babylon (ca. 1750),[22] and the edict of Ammisaduqa of Babylon (17th century).[23]

One of the continuing concerns of the rulers of these early civilizations was the persistent tendency for land, labor, and other assets to become concentrated in the hands of wealthy individuals who accumulated the holdings of others through the inability of debtors to meet high interest requirements.[24] (In Sumer, from 3000 to 1800, the official interest rate on grain was $33^{1}/_{3}\%$, while that on silver was 20%.[25]) Periodically, rulers voided the debts incurred by producers who fell into arrears in their obligatory contributions or through other impoverishment. Lands and homes lost in this manner

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[19] J. J. Finkelstein, "The Laws of Ur-Nammu," *Journal of Cuneiform Studies*, Vol. 22, No. 3/4 (1968-1969): pp. 66-82; Roth, *Law Collections*, pp. 13-22; VerSteeg, *Early Mesopotamian Law*, pp. 19-24, ff.

[20] Francis R. Steele, "The Lipit-Ishtar Law Code," *American Journal of Archaeology*, Vol. 51, No. 2 (April-June 1947), pp. 158-164; Roth, *Law Collections*, pp. 23-35; VerSteeg, *Early Mesopotamian Law*, pp. 24-27.

[21] Reuven Yaron, *The Laws of Eshnunna*, 2nd rev. ed. (Jerusalem, ISR: The Magnes Press, The Hebrew University, 1988); Roth, *Law Collections*, pp.57-70; VerSteeg, *Early Mesopotamian Law*, pp.27-30.

[22] M. E. J. Richardson, *Hammurabi's Laws: Text, Translation and Glossary* (Sheffield, UK: Sheffield Academic Press, 2000); Roth, *Law Collections*, pp. 71-142; VerSteeg, *Early Mesopotamian Law*, pp. 30-41. See also Leroy Waterman, "Business Documents of the Hammurabi Period," *American Journal of Semitic Languages and Literatures*, Vol. 29, No. 3 (April 1913): pp. 145-204, Leroy Waterman, "Business Documents of the Hammurabi Period. II," *American Journal of Semitic Languages and Literatures*, Vol. 29, No. 4 (July 1913): pp. 288-303, Leroy Waterman, "Business Documents of the Hammurabi Period. III," *American Journal of Semitic Languages and Literatures*, Vol. 30, No. 1 (October 1913): pp. 48-73.

[23] J. J. Finkelstein, "Ammisaduqa's Edict and the Babylonian 'Law Codes'," *Journal of Cuneiform Studies*, Vol. 15, No. 3 (1961), pp. 91-104.

[24] Michael Hudson, "The mathematical economics of compound interest: a 4,000-year overview," *Journal of Economic Studies*, Vol. 27, No. 4/5 (November 2000): pp. 344-363; Michael Hudson, "Mesopotamia and Classical Antiquity," *American Journal of Economics and Sociology*, Vol. 59, No. 5 (November 2000): pp. 3-26; Michael Hudson and Marc Van De Mieroop, eds., *Debt and Economic Renewal in the Ancient Near East* (Bethesda, MD: CDL Press, 2002).

[25] These rates are explicitly given in, for example, the unattributed Laws of X (ca. 2050-1800 BCE), Roth, *Law Collections*, p. 38. See also Sidney Homer and Richard Sylla, *A History of Interest Rates*, 3rd ed., revised (New Brunswick, NJ: Rutgers University Press, 1996), pp. 25-31; Michael Hudson, "How Interest Rates Were Set, 2500 BC-1000 AD: *Máš*, *tokos* and *fœnus* as Metaphors for Interest Accruals," *Journal of the Economic and Social History of the Orient*, Vol. 43, No. 2 (May 2000): pp. 132-161.

were returned to their previous owners, and debt slaves were freed. However, debts incurred in other ways, such as trade investments, remained valid and were not forgiven.[26]

A pooling of assets and sharing in results appears throughout the course of human history. Again, the Babylonian record describes, for example, the pooling of investments for the provisioning of trading ships, the sharing in the profits and losses, and the sale of interests in such ventures.[27] Similar arrangements facilitated trade overland, especially to the other Mesopotamian cities and to their chief trading centers in surrounding countries. Hammurabi attempted unsuccessfully to proscribe investment in trading ventures that did not participate in its risk, that is, to enforce equity and bar pure debt participation.[28] Thus, at least by four thousand years ago, human intercourse had developed to a stage of production and exchange that required sophisticated accounting and financial instruments, including pooled investments with transferable shares and bearer bonds.

Section 3 Mutual Fund Precursors

The preceding section discussed the rise of literate culture from the means and instruments of material accumulation in human society. This culture was promoted and sustained by a level of production and exchange which required accounting and financial

[26] Such remissions were made through a proclamation known as a *mīšarum*, as in the edict of Ammisaduqa, although they are also known for Ur-Nammu, Lipit-Ishtar, and Hammurabi. Finkelstein, "Ammisaduqa's Edict," pp. 99-104; Niels Peter Lemche, "Andurarum and Misarum: Comments on the Problem of Social Edicts and Their Application in the Ancient near East," *Journal of Near Eastern Studies*, Vol. 38, No. 1 (January 1979): pp. 11-22; Jacob Rosenberg and Avi Weiss. "Clean Slate Proclamations, The Jubilee, and Anti-Monopoly Laws." Paper for the Biblical Economics Conference, Jerusalem, June 2000 [http://faculty.biu.ac.il/~weissa1/wp/jubilee.pdf].

[27] Pruessner, "The Earliest Traces of Negotiable Instruments," pp. 88-107; Keister, "Commercial Record-Keeping," p. 373; Oppenheim, "Seafaring Merchants of Ur," pp. 8-9.

[28] Oppenheim, "The Seafaring Merchants of Ur," pp. 6-17.

instruments, including transferable shares in pooled investments and bearer bonds. These instruments were, however, all tied to real objects, *e. g.*, baskets of barley, ingots of silver, or the cargoes of caravans and ships. They were intended to facilitate production and commerce of real goods and no evidence of a separate market in financial instruments has been adduced.

The next major stage in the development of financial instruments which more closely approached the form of modern mutual funds did not occur until the advent of the European financial revolution in the 16th century. These financial innovations resulted from and facilitated the struggles of the rising nation-states throughout the continent and their competition for wealth from distant overseas sources of trade and plunder, initiated by the inflows of gold and silver from Portuguese and Spanish trade, conquest, and extraction in Africa, the East and West Indies, and Central and South America.[29] Such far distant sources of wealth "required financial intermediaries capable of mobilizing larger sums, waiting for longer periods, and dealing with greater numbers of clients spread over greater distances than ever before."[30] In addition to the impetus to financial intermediation from the expanding commercial opportunities and from the requirements of governments for military campaigns, this economic and political activity took place in the context of the "price revolution of the 16th century," when prices were generally rising and the rate of increase in returns to land and capital (rents and interest) exceeded

[29] For a broad overview, see Larry Neal, "How it all began: the monetary and financial architecture of Europe during the first global capital markets, 1648-1815," *Financial History Review*, Vol. 7, No. 2 (October 2000), pp.117-140.
[30] Larry Neal, *The rise of financial capitalism: International capital markets in the Age of Reason* (New York, NY: Cambridge University Press, 1990): p. 4.

the rate for the returns to labor (wages).[31]

In these circumstances, two financial innovations have been proposed as initiating the financial revolution in early modern Europe. The earlier was the issuance of life and of heritable (perpetual but redeemable) annuities which were funded by the revenues from excise and property taxes levied by Charles V on the provinces of the Netherlands in 1542.[32] Although life annuities have been attested since 205 BCE,[33] for 16th century Europe, these new instruments were relatively safe and transferable long-term securities which supported a large and growing market, although the secondary market was more limited. The other innovation was the development and refinement of negotiable bills of exchange in the international financial and commercial market of Antwerp later in the

[31] This was the second of the four great price revolutions in the period since the 11th century continuing through the 20th. In some places, it began as early as 1470 and extended to 1650, the longest of the long-term, increasing price trends in modern history. David Hackett Fischer, *The Great Wave: Price Revolutions and the Rhythm of History* (New York, NY: Oxford University Press, 1996), esp. pp. 65-116. For an early characterization of this price trend, see Earl J. Hamilton, *American Treasure and the Price Revolution in Spain, 1501-1650* (Cambridge, MA: Harvard University Press, 1934) and "American Treasure and the Rise of Capitalism (1500-1700)," *Economica*, No. 27 (November 1929), pp. 338-357. Hamilton attributes the rise in prices to the inflationary effects of the influx of silver and gold. Neal (in *Rise of financial capitalism*, pp. 3-5, and ff.) attributes much of the price rise and certain interest rate anomalies to the demand of governments for military expenditures exceeding the increased supply of bullion. Fischer (in *Great Wave*, pp. 70-75) attributes the price rise to population growth, which recovered after the devastations of the 14th century and relative stability of the Renaissance. See also Fernand Braudel and Frank Spooner, "Prices in Europe from 1450 to 1750," pp. 374-486, in E. E. Rich and Charles Wilson, eds., *The Cambridge Economic History of Europe*, Vol. 4, *The Economy of Expanding Europe in the Sixteenth and Seventeenth Centuries* (Cambridge, UK: Cambridge University Press, 1967), cited by Neal, *Rise of financial capitalism*, p. 3, and by Fischer, *Great Wave*, pp. 70 endnote 14 (pp. 332-333), 241. For the demand of governments for finance, see Neal, *Rise of financial capitalism*, pp. 3-5, 9-13, and Earl J. Hamilton, "Origin and Growth of the National Debt in Western Europe," *American Economic Review*, Vol. 37, No. 2 (May 1947), pp. 118-130. For the trends in interest rates, see also Homer and Sylla, *History of Interest Rates*, pp. 104-143.

[32] James D. Tracey, *A Financial Revolution in the Habsburg Netherlands: Renten and Rentiers in the County of Holland, 1515-1565* (Berkeley, CA: University of California Press, 1985), also cited by Neal, *Rise of financial capitalism*, pp. 5, 8-9. For loan rates in Antwerp and Bruges, especially for the Habsburgs, who were the largest debtors in Europe at this time, see Homer and Sylla, *History of Interest Rates*, pp. 113-121.

[33] K. Geert Rouwenhorst, "The Origins of Mutual Funds," pp. 249-270, in William N. Goetzmann and K. Geert Rouwenhorst, eds., *The Origins of Value: The Financial Innovations That Created Modern Capital Markets* (New York, NY: Oxford University Press, 2005).

same century.[34] After Portuguese Jews and Protestants were expelled from Antwerp in 1585, the mechanism of bills of exchange was refined in Amsterdam. Whereas domestic bills had a smaller market and were repaid in installments, foreign bills faced a larger potential market, were paid in full when due, and provided a convenient means of transferring wealth internationally.[35]

By 1609, investors could find in the shares of the Dutch East India Company (VOC for *Vereenigte Oost-Indische Compagnie*) the advantages of both these instruments. The structure of the VOC (permanent capital fund and separation of operating control from ordinary share ownership) facilitated the successful management of its financial operations. Because its shares offered high and stable dividends, carried rights of transfer which were easy and cheap to exercise, and were available to foreign investors, they sustained a strong and active secondary market.[36] Although the VOC share represented an interest in the diversified commercial ventures of a single company, rather than in a diversified portfolio of equities of several companies, yet this early investment represented an important step in financial development, which eventually produced the modern closed-end mutual fund.

English innovation proceeded later and more slowly than in the Netherlands and France. For example, the English East India Company (EIC) required the entire 17th

[34] Herman van der Wee, "Monetary, Credit and Banking Systems," pp. 290-392, in E. E. Rich and Charles Wilson, eds., *The Cambridge Economic History of Europe. Vol. 5: The Economic Organization of Early Modern Europe* (Cambridge, UK: Cambridge University Press, 1977), also cited in Neal, *Rise of financial capitalism*, p. 5.
[35] Neal, *Rise of financial capitalism*, pp. 7-8, 10
[36] Neal, *Rise of financial capitalism*, pp. 8-9; William Robert Scott, *The constitution and finance of English, Scottish and Irish joint-stock companies to 1720*, (Cambridge, UK: The University Press, 1911), Vol. 1, p. 331; Vol. 3, pp. 275-277.

century to adopt just two of the key features of the VOC, and still faced shareholder revolts throughout the 18th century.[37] The pace of English financial evolution quickened with the arrival of William III of Orange, through the Dutch methods introduced with the capital and specialists accompanying his ascension to the English throne in the Revolution of 1688. However, especially because of the onerous restrictions on their purchase, transfer, and receipt of payments, the English government-backed annuities experienced less demand than similar earlier offerings had in the Netherlands and France.[38] More successful was the Million Adventure lottery, which sold large numbers of tickets (100,000) at low prices (£10) for a payoff of £1000 per year for 16 years for winners and £1 per year over that period for losers.[39]

To reduce the burdens incurred by war financing throughout the 17th and into the 18th centuries, the English and French governments attempted to repeat the successes of the VOC and EIC by converting their high interest, long-term annuity debts into equity in large monopoly trading companies. The immediate need came from the expenses of the War of the League of Augsburg, 1688-1697, (or the War of the Grand Alliance, once England and William III entered against France and Louis XIV in 1689). The Bank of England was created to provide a loan to the government in 1694 (the same year the government launched the Million Adventure) and again later in 1697 converted more government debt.

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[37] The EIC instituted a permanent capital fund in 1650 and foreign ownership (without rights to directorships) in 1698; see Neal, *Rise of financial capitalism*, p. 9.
[38] Neal, *Rise of financial capitalism*, pp. 14-16, 45-46.
[39] Neal, *Rise of financial capitalism*, pp. 14, 51; Edward Chancellor, *Devil Take the Hindmost: A History of Financial Speculation* (New York, NY: Farrar, Straus, Giroux, 1999), pp. 40-41.

The following year, a group of private bankers[40] initiated a similar undertaking, formally called "the Bank on tickets of the Million adventure" and based, not on a new loan to the government, but on the outstanding tickets of the Million lottery. Subsequently the Bank was continued beyond its scheduled date of expiration in 1710 by a second subscription and by the purchase of existing life annuities, "reversions" (or extensions on existing annuities), and new annuities purchased directly by the Bank.[41]

Neal summarizes Scott's assessment of the Million Bank thusly:

> First, it was the first example of a reverse-leveraged buy-out of government debt, that is, an operation in which underpriced government debt was directly exchanged at a favorable price for shares in a joint-stock company. …Second, it was essentially a well-managed *mutual fund* in government annuities, and as such it gives us as accurate an account of the average rate of return on risk-free assets as we can find before the innovation of the South Sea annuities in 1723.[42]

Furthermore, during the earliest period of the London stock market (1698-1708), the Million Bank, before its original expiration date, changed from a closed-end to an open-end fund. It also expanded the type of assets it held to include securities of other companies, eventually also the South Sea Company, although it sold its holdings before

[40] "…the Million Bank Company is no corporation, though it has a joint-stock, being only a legal partnership entered into in the reign of King William III. for dealing in irredeemable government securities." Adam Anderson, *An historical and chronological deduction of the origin of commerce, from the earliest accounts*, 4 Vols. (London: Logographic Press, 1787, 1789), Vol. 1, p. lxiv.
[41] Scott, *Joint-stock companies*, Vol. 3, pp. 275-287; Neal, *Rise of financial capitalism*, pp. 49-52; Chancellor, *Devil Take the Hindmost*, pp. 40-41.
[42] Neal, *Rise of financial capitalism*, p. 51 [emphasis added]; Scott, *Joint-stock companies*, Vol. 3, pp. 279-280. Scott wrote (1910-1912) prior to the modern argument that "irrational [market] bubbles" do not exist. See the following footnote 43.

the bubble's collapse.[43] Another example of the mutual fund form of investment was the bubble company promoted by the banker Matthew West, a "[c]ompany for buying and selling South Sea stock and all other public stocks."[44] The Million Bank was eventually liquidated in 1797, having paid dividends of 6% from 1710 through 1716, then 5% through 1727, and finally 4% after 1728.[45]

The next example of an early form of mutual fund was the investment trust *Eendragt Maakt Magt* established by an Amsterdam broker and merchant in 1774. This instrument relied on the experience with the two main instruments in the Dutch capital market, bonds from Dutch and other governments which it combined and securitized pooled plantation loans, called negotiaties. The trust thus securitized the stream of income from illiquid plantation mortgages and substituted the shares of the trust for the portfolio of foreign government and plantation loans. This fund was issued with a fixed number of shares with a plan for repurchase and eventual liquidation. At least two other

[43] Scott, *Joint-stock companies*, Vol.3, pp. 285-287; Neal, *Rise of financial capitalism*, pp. 59, 77-117, Chancellor, *Devil Take the Hindmost*, pp. 68, 71, 93. The original financial analysis in Neal's *Rise of financial capitalism* is also marshaled in defense of his thesis that the spike in South Sea Company share prices and their collapse in 1720 did not represent irrational investor behavior. Rather, the market was inadequate to support the huge volume of shares demanded. Such an account neglects the bribery and private and government corruption that was fundamental to the South Sea scheme. "Neal's account of the South Sea Bubble, like Garber's version of the Tulip Mania, is intended to bolster the modern theory of efficient markets and rational investors. In its most extreme form, this theory denies the possibility of irrationality in speculative bubbles…. The theory of the 'rational bubble' appears to be nothing more than an elaborate restatement of the 'greater fool' investment strategy…. The exponents of the 'rational bubble' appear to overlook the fact that the success of this strategy is dependent on liquidity (i.e., the constant presence of both buyers and sellers in the market) and that in a panic buyers vanish at the very moment when 'rational bubble' speculators are seeking to unload their shares." Chancellor, *Devil Take the Hindmost*, pp. 94-95, in reference to the arguments in Neal, *Rise of financial capitalism* and Peter M. Garber, *Famous First Bubbles: The Fundamentals of Early Manias* (Cambridge, MA: MIT Press, 2000). However, many in the business community did not succumb to the mania, including the Million Bank, and thus its third point of interest. Chancellor, *Devil Take the Hindmost*, pp. 58-95; Julian Hoppit, "Financial Crises in Eighteenth-Century England," *The Economic History Review*, New Series, Vol. 39, No. 1 (Feb. 1986), pp.39-58; Scott, *Joint-stock companies*, Vol.3, pp. 285-286.
[44] Scott, *Joint-stock companies*, Vol. 3, p. 454; also quoted in Chancellor, *Devil Take the Hindmost*, p. 70.
[45] Scott, *Joint-stock companies*, Vol. 3, p. 287, fn. 1.

trusts of this type were established, one was a "fund of funds" since it invested in the first, and the other was not dissolved until 1893.[46] In 1788, Dutch bankers organized negotiaties for liquidated American revolutionary war debt. This venture proved so successful that eventually 29 such funds were placed in the Amsterdam market between 1787 and 1804.[47]

In the early 19th century, there were some isolated examples of this type of investment. In the United States, the Massachusetts Hospital Life Insurance Company organized a trust in 1823, from which were distributed to the contributing beneficiaries their share of the income from all the assets in the fund.[48] In 1822, William of Orange organized a trust in Brussels and another, similar fund was also started in Brussels.[49] A fund was organized in Switzerland in 1849.[50]

This form of investment finally took firm root in England, although it started slowly. The first companies loosely identified as trusts were London Financial

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[46] K. Geert Rouwenhorst, "The Origins of Mutual Funds," Chapter 15, pp. 249-270, in William N. Goetzmann and K. Geert Rouwenhorst, eds., *The Origins of Value: The Financial Innovations that Created Modern Capital Markets* (New York, NY: Oxford University Press, 2005).
[47] P. J. van Winter, , cited in Rouwenhorst, "Origins," pp. 262-265. For discussion of early U. S. capital markets and their integration with markets in Europe, see Richard Sylla, "U.S. Securities Markets and the Banking System, 1790-1840," Federal Reserve Bank of St. Louis *Review*, May/June 1998, pp. 83-98; Richard Sylla, Jack W. Wilson, and Robert E. Wright, "Integration of Trans-Atlantic Capital Markets, 1790-1845," paper for Centre for Economic Policy Research conference Early Securities Markets, 2004, [http://www.vwl.uni-freiburg.de/fakultaet/erwien/conference/CEPR%20Conference%20papers2/Trans-Atlantic%20Capital%20Market%20Integration%20ver4b.pdf]; and Peter L. Rousseau and Richard Sylla, "Emerging Financial Markets and Early U.S. Growth," NBER Working Paper No. 7448, November 1999.
[48] William Howard Steiner, *Investment Trusts: American Experience* (New York, NY: Adelphi Co., 1929), pp. 43-44;
[49] Theodore J. Grayson, *Investment Trusts: Their Origins, Development, and Operation* (New York, NY: John Wiley & Sons, Inc., 1928), p. 11; Hugh Bullock, *The Story of Investment Trusts* (New York, NY: Columbia University Press, 1959), p.1.
[50] Steiner, *Investment Trusts*, p. 17; Benjamin R. Chabot and Christopher Kurz, "Trust Me with Your Money: English Investors and the Precursor of the Modern Mutual Fund," p. 2 [http://www.icf.yale.edu/pdf/hist_conference/Ben_Chabot.pdf].

Association and the International Financial Society of London, both formed in 1863, just after the enactment of the first Joint Stock Companies Act of 1862, designed to prevent fraudulent and risky operations which had earlier led to large losses, followed by the Act of 1867. In addition to proscribing fraudulent activities, these laws allowed shareholders to participate in the profits of companies while their liabilities were limited to their equity participation.[51] The fund usually reported as the first true investment trust was the Foreign and Colonial Government Trust, founded in 1868, and followed by 17 more by 1875.[52] In 1873, the major Scottish investment trusts began to form, beginning with the Scottish American Investment Trust and the Oregon and Washington Investment, Co., Ltd., which was the earliest of the trusts which were eventually merged in the Alliance Trust in 1888.[53] These funds set the proximate example for establishment of the American investment trusts.

Section 4 Development of Mutual Funds in the United States

The history of the mutual fund industry in the United States can be considered in three major periods: from its inception to 1940; after the completion of its reformation in 1940 until about the end of the 1970s; and from its accelerated growth beginning around 1980 until its prominent position in the financial markets today.[54] Furthermore, the first

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[51] Bullock, *Story of Investment Companies*, pp. 2-4; Grayson, *Investment Trusts*, pp. 11-15, 22-32; Steiner, *Investment Trusts*, pp. 17-30.

[52] Bullock, *Story of Investment Companies*, pp. 4-5; George Glasgow, *The English Investment Trust Companies* (New York, NY: John Wiley & Sons, Inc., 1931), presents detailed financial reports on the British trusts through 1930.

[53] J. C. Gilbert, *A History of Investment Trusts in Dundee*, 1873-1938 (London, UK: P. S. King & Son, Ltd., 1939) and George Glasgow, *The Scottish Investment Trust Companies* (London, UK: Eyre and Spottiswoode (Publishers) Ltd., 1923).

[54] William J. Baumol, Steven M. Goldfeld, Lilli A. Gordon, and Michael F Koehn, *The Economics of Mutual Fund Markets: Competition Versus Regulation* (Boston, MA: Kluwer Academic Publishers, 1990), pp. 8-12, defines the second period as 1940-1970 and the third as 1970-1990.

period (of the industry's emergence) can be viewed in two subperiods. In the first phase, investment trusts were introduced and gradually grew into a rapid proliferation in the years before the 1929 crash. Open-end funds were also introduced late in this phase, which was dominated by the closed-end form of fund organization. The profits from operating closed-end funds during the rising market of the 1920s were multiplied by abusive practices in many of the funds. The second phase began with the stock market crash, through which many of the fraudulent practices of the closed-end funds and the advantages for investors of open-end funds were revealed. Following the market collapse, open-end funds continued to grow slowly, but closed-end funds were virtually stagnant. The second phase ended with a succession of reforming and regulatory legislation culminating in the Investment Company Act of 1940.

The British and Scottish investment trusts were emulated in the United States, beginning with the New York Stock Trust in 1889 and the Boston Personal Property Trust in 1893. In the first decade of the 20th century, the Railway and Light Securities Company was formed in 1904 and the Alexander Fund in 1907; six funds were formed in the 1910s; and another eight before 1924.[55] "Although inconspicuous until 1924, American investment trusts already contained the germs of nearly all the forms and varieties which later came…into being."[56] By the first half of 1928, at least 199

[55] Brief descriptions of the first and some prominent early trusts are presented in Bullock, *Story of Investment Trusts*, pp. 14-24; but detailed descriptions and analyses of numerous early funds and detailed evaluations of the industry in general are developed throughout in Leland Rex Robinson, *Investment Trust Organization and Management*, (New York, NY: Ronald Press Co., 1926, Revised Edition, 1929); Steiner, *Investment Trusts*; and Grayson, *Investment Trusts*.
[56] John Francis Fowler, Jr., *American Investment Trusts* (New York, NY: Harper & Brothers Publishers, 1928), p. 5.

investment trusts, managing $1.2 billion of assets, had been established.[57] The early investment trusts were organized as closed-end funds (the shares of which, once issued, are not redeemed, but traded in the secondary market) and unit investment trusts (for which portfolio allocations tended to be fixed, the fund often had a limited life span, and redemption was often permitted).

However, the Alexander Fund (1907) was established as a private, open-end fund, until converted into a public fund in 1928.[58] The first publicly offered, open-end mutual fund, the Massachusetts Investors Trust, formed in 1924,[59] was still operating as of 2004 with $6 billion in assets as part of the MFS family of funds. Other early, still operating funds include the State Street Research Investment Trust, which began as a private fund but converted to a public fund in 1924 and managed $1.4 billion in assets in 2004, and the Pioneer Fund, which formed in 1928 as the Fidelity Investment Trust and in 2004 held over $7 billion in assets.[60] By 1929, 19 open-end funds had been established with $140 million of assets.

The lack of transparency in the operation of closed-end funds had made them especially susceptible to abusive practices, including (1) outright ponzi schemes; (2) since holdings were undisclosed, any value could be claimed for the fund; (3) excessive debt; and (4) trading in questionable securities, including as personal favors. While the

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[57] Fowler, *American Investment Trusts*, pp. 1-19. Gremillion, *Handbook*, p. 16, cites 89 closed-end funds with $3 billion assets in 1929, when the NYSE listed $87 billion in stocks. C. P. Keane, *Keane's Manual of Investment Trusts*, Second Annual Number (Boston, MA: Financial Publishing Co., 1929), "presents statements of 391 companies operating 414 investment trusts" (Preface) and a listing of the British investment trusts through the first quarter of 1929.
[58] Grayson, *Investment Trusts*, pp. 253-258; Bullock, *Story of Investment Trusts*, pp. 16-17, ff.
[59] Dwight P. Robinson, Jr., *Massachusetts Investors Trust: Pioneer in Open-End Investment Trusts* (New York, NY: The Newcomen Society in North America, 1954), pp.8-25.
[60] Gremillion, *Handbook*, pp. 18-19.

Dow Jones Industrial Average (DJIA) fell 34 percent from the end of 1920 to the end of 1930, "the closed-end funds fell from an average premium of 47 percent *above* net asset value to an average discount of 25 percent *below* net asset value—a drop of 72 percent (not counting the simultaneous drop in net asset value itself.)"[61] No new closed-end funds opened during the 1930s.

The typical practices of open-end funds had spared their investors from the worst consequences. Disclosure of holdings and redemption on demand at net asset value had precluded these funds from (1) holding unmarketable securities; (2) undertaking excessive leverage; and (3) inflating asset values or making them subject to speculation. Open-end funds continued to grow in number and size during the 1930s.[62]

The first period of the mutual fund industry culminated in the series of securities legislation beginning with the Securities Act of 1933[63] which regulated the public offering of securities by (1) requiring registration of the securities; (2) requiring provision of a prospectus; (3) explicitly prohibiting fraud; (4) regulating advertising; and (5) requiring the provision of a Statement of Additional Information. The following year the Securities Exchange Act of 1934[64] established the Securities and Exchange Commission (SEC) to regulate exchanges and dealers and to institute standards for record-keeping, reporting, financial responsibility, qualifications for those engaged in securities trading, and practices.[65] The Revenue Act of 1936[66] included open-end funds among those

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[61] Gremillion, *Handbook*, p. 17; Bullock, *Story of Investment Trusts*, pp. 41-61.
[62] Gremillion, *Handbook*, p. 17; C. Russell Doane, *Investment Trusts and Funds: From the Investor's Point of View*, Economic Education Bulletin, Vol. VI, No. 3 (April 1966), pp. 64-83.
[63] 15 USC § 77a *et seq.*
[64] 15 USC § 78a *et seq.*
[65] Gremillion, *Handbook*, p. 19; Fortune, *Mutual Funds*, p. 50-53.

granted tax pass-through status so long as the fund distributed to shareholders all taxable income.[67]

The Investment Company Act of 1940[68] was the product of hearings and reports by the SEC[69] pursuant to the Public Utilities Holding Company Act of 1936. It targeted the major investment trust practices considered detrimental to investor and public interest and is the basis of all mutual fund specific regulation: Inadequate disclosure was addressed by requiring registration with the SEC, a statement of policies and structure, annual reports, and other specific accounts and records. Against the pursuit of management company over shareholders interests were raised the requirements that 40 percent of the board of directors not be affiliated with the management company; that written contracts with major advisors and underwriters be approved by shareholders; and that affiliates of fund service providers be prohibited from financial transactions with the fund. To prevent the subversion of shareholder value, the Act required all shares to have equal voting rights and that shareholders approve investment policies and auditors. To

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[66] 26 USC § 4982a *et seq.*

[67] Gremillion, *Handbook*, pp. 20-21.

[68] 15 USC § 80a-1 *et seq.*

[69] United States Securities and Exchange Commission, *Investment Trusts and Investment Companies*, Report of the Securities and Exchange Commission, *Part One: The Nature, Classification, and Origins of Investment Trusts and Investment Companies*, House Document No. 707, 75th Congress (1938); *Part Two: Statistical Survey of Investment Trusts and Investment Companies*, House Document No. 70, 76th Congress (1939); *Part Three: Abuses and Deficiencies in the Organization and Operation of Investment Trusts and Investment Companies*, House Document No. 279, 76th Congress and House Document No. 136, 77th Congress (1939, 1940, 1941); *Part Four: Control and Influence over Industry and Economic Significance of Investment Companies*, and *Part Five: Conclusions and Recommendations* (1941); and six supplemental reports, *Investment Trusts in Great Britain*, House Document No. 380, 76th Congress (1939); *Investment Counsel, Investment Management, Investment Supervisory, and Investment Advisory Services*, House Document No. 477, 76th Congress (1939); *Commingled and Common Trust Funds Administered by Banks and Trust Companies*, House Document No. 476, 76th Congress (1939); *Companies Sponsoring Installment Investment Plans*, House Document No. 482, 76th Congress (1939); *Fixed and Semifixed Investment Trusts*, House Document No. 567, 76th Congress (1940); and *Companies Issuing Face Amount Installment Certificates*, House Document No. 659, 76th Congress (1940).

preclude pyramiding and other mismanagement practices, the Act restricted ownership of other funds (modified in 1996); stipulated custodianship of securities; required bonding of employees with access to cash or securities; and required that dividends be paid from undistributed income or their source be disclosed. Misleading and fraudulent accounting practices were to be prevented by requiring independent auditors. Changing fund structure for management exploitation was to be prevented by requiring shareholder approval for changes in structure or investment practices and policies. Excessive debt was prohibited by restricting practices equivalent to borrowing or the issuance of senior securities. Adequate reserves were promoted by requiring a minimum net worth of $100,000 for funds covered by the Act.[70]

Finally, the Investment Advisors Act of 1940[71] required registration with the SEC of mutual fund advisors (except banks, which were regulated under separate banking laws), limited the term of advisory contracts to two years, terminable on 60 days' notice, and required approval by a majority of outside directors for advisory contract renewal.[72]

The second period of the history of mutual funds began after the series of hearings and legislation, and it was primarily a period of growth and establishment. Although investor protections had improved substantially, the mutual fund market grew slowly during World War II, the financing of which absorbed most investment capital, and the number of funds remained fixed at 68 for its duration. See Table 1.1 for the number of

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[70] Alfred Jaretzki, Jr., "The Investment Company Act of 1940," Washington University *Law Quarterly*, Vol. 26, No. 3 (April 1941), pp. 303-347; Walter P. North, "A Brief History of Federal Investment Company Legislation," *Notre Dame Lawyer*, Vol. 44 (June 1969), pp. 677-698; Bullock, *Story of Investment Companies*, pp. 74-96; Fortune, *Mutual Funds*, pp. 46-48, 50-53; Gremillion, *Handbook*, pp. 19-22; Baumol, *et al.*, *Economics of Mutual Funds*, pp. 48-54.

[71] 15 USC § 80b-1 *et seq.*

funds, total net assets, and number of shareholder accounts by year from 1940 to 2004. By the end of 1945, total net assets grew by more than 45 percent over the previous year and, for the first time in the United States, the 70 open-end funds with over $1 billion surpassed in total assets the 40 closed-end funds with just under $1 billion. The market continued to grow in number of funds, total assets, and number of shareholder accounts. After two decades of steady growth (the only years recording declines in net assets from the previous year were 1941 and 1957), two types of problems emerged. In 1962, 1963, and 1966, SEC sponsored reports investigated the complaints about excessive advisory fees, loads, and conflicts of interest. Congressional bills were first introduced in 1967 and eventually enacted as the Investment Company Amendments Act of 1970, amending the 1933, 1934, and Advisors 1940 Acts as well.[73] The new requirements stipulated that outside directors must be "disinterested" (a stronger exclusion than the previous "unaffiliated") and clarified the fiduciary responsibility they had in supervising contracts.

The other problem reflected a slowdown in the growth of mutual funds. Three years in the 1960s (1962, 1966, 1969) and later four in the 1970s (1970, 1973, 1974, 1977) showed declines in net assets from the previous years. Similarly, for three year in the 1960s, the number of funds declined from previous years and fell again in 1975. Finally, the number of accounts fell every year from 1972 through 1978. Fluctuating stock market prices and increasing inflation rates had eroded the value of mutual fund holdings. Small investors were caught between the high interest rates of the 1970s and

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[72] Gremillion, *Handbook*, p. 20.

[73] Walter P. North, "The Investment Company Amendments Act of 1970," *Notre Dame Lawyer*, Vol. 46 (1971), pp. 712-732; Baumol, *et al.*, *Economics of Mutual Fund Markets*, pp. 54-60; Gremillion, *Handbook*, p. 22-24.

YEAR	FUNDS	ASSETS[1]	ACCOUNTS
1940	68	448.0	296,056
1941	68	401.6	293,251
1942	68	486.8	312,609
1943	68	653.7	341,435
1944	68	882.2	421,675
1945	73	1,284.2	497,875
1946	74	1,311.1	580,221
1947	80	1,409.2	672,543
1948	87	1,505.8	722,118
1949	91	1,973.5	842,198
1950	98	2,530.6	938,651
1951	103	3,129.6	1,110,432
1952	110	3,931.4	1,359,000
1953	110	4,146.1	1,537,250
1954	115	6,109.4	1,703,846
1955	125	7,837.5	2,085,325
1956	135	9,046.4	2,580,049
1957	143	8,714.1	3,110,392
1958	151	13,242.4	3,630,096
1959	155	15,818.0	4,276,077
1960	161	17,025.7	4,897,600
1961	170	22,788.8	5,319,201
1962	169	21,270.7	5,910,455
1963	165	25,214.4	6,151,935
1964	160	29,116.3	6,301,908
1965	170	35,220.2	6,709,343
1966	182	34,829.4	7,701,656
1967	204	44,701.3	7,904,132
1968	240	52,677.2	9,080,168
1969	269	48,290.7	10,166,788
1970	361	47,618.1	10,690,312
1971	392	55,045.3	10,900,952
1972	410	59,830.6	10,635,287
1973	421	46,518.5	10,330,862
1974	431	35,776.8	9,970,439
1975	426	45,874.4	9,876,082
1976	452	51,267.6	9,060,089
1977	477	49,036.9	8,692,601
1978	505	55,837.7	8,658,354
1979	522	94,193.6	9,790,018
1980	564	134,760.9	12,087,646
1981	665	241,365.4	17,498,938
1982	857	296,678.1	21,448,409
1983	1,026	292,985.1	24,604,659
1984	1,243	370,680.0	27,635,660
1985	1,528	495,385.1	34,098,401
1986	1,835	715,667.8	45,373,627
1987	2,312	769,171.9	53,717,241
1988	2,737	809,370.5	54,056,016
1989	2,935	980,671.1	57,559,770
1990	3,079	1,065,190.2	61,947,955
1991	3,403	1,393,185.3	68,331,800
1992	3,824	1,642,536.7	79,931,440
1993	4,534	2,069,963.2	93,213,698
1994	5,325	2,155,324.9	114,383,364
1995	5,725	2,811,292.2	131,219,221
1996	6,248	3,525,800.8	150,042,149
1997	6,684	4,468,200.6	170,264,389
1998	7,314	5,525,209.3	194,073,595
1999	7,791	6,846,339.2	226,412,794
2000	8,155	6,964,667.0	244,748,546
2001	8,307	6,974,975.9	248,759,332
2002	8,256	6,391,570.8	250,981,045
2003	8,126	7,414,400.0*	260,882,000 †
2004	8,044	8,106,870.0*	267,363,000 †

[1] million of dollars   * source reports in billion of dollars   † source reports in thousands

NOTE: break in series data after 1983, when funds investing in other funds were excluded

SOURCES: 1940-1970, 1970-1987—Investment Company Institute, *Mutual Fund Fact Book*, 1984-1988, and *Trends in Mutual Fund Activity* (March 1985), as quoted in Baumol, *et al*, *Economics of Mutual Fund Markets*, Table 1.6, p. 29; 1970-1987, Table 1.8, pp. 33-34; 1987-2002—Investment Company Institute, *2003 Mutual Fund Fact Book*, p. 63; 2003-2004—Investment Company Institute, *2005 Investment Company Fact Book*, p.59. All figures are year-end.

Table 1.1 Annual Sizes of Mutual Fund Industry, 1940-2004:
Number of Funds, Total Net Assets, Number of Shareholder Accounts

the limits imposed by Regulation Q[74] on the interest rates payable by banks on depository accounts. Money market funds were introduced in 1974, and were followed by a proliferation of specialized funds. With minimum requirements a tenth or less than those of most money market instruments and interest rates above the bank deposit rate, money market funds drew large flows of new money into the mutual fund market. During 1979, total assets in money market funds grew more than four times their end-1978 level; and the number of accounts grew to almost five times the previous year's.[75] From 1977 to 1982, when new legislation allowed banks to compete with the new funds, savings deposits fell by $125 billion while the assets of money market funds grew by over $200 billion.[76]

The renewed and accelerated growth at the end of the 1970s marks the beginning of the third period in the history of the U. S. mutual fund industry. The changed regulations allowing banks to compete with mutual funds and other money market instruments stemmed bank deposit losses but did not reverse them. Moreover, as the money market rates receded, the rise in stock market prices made the equity funds attractive again and total net assets grew through appreciation as well as new inflows. This growth was reinforced by the shift of retirement funds into mutual funds. The types of funds continued to proliferate: whereas the Investment Company Institute (ICI) had tracked five categories of funds in 1970, seven in 1975, their number quickly grew to 22

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[74] The limitations of Regulation Q have also been cited as an important factor in the growth of the Eurodollar market. See, for example, E. Wayne Clendenning, *The Euro-Dollar Market* (Oxford, UK: Oxford University Press, 1970), pp. 23, 24-8, 186-187, and Paul Einzig, *The Euro-Dollar System: Practice and Theory of International Interest Rates*, Fifth Edition (London, UK: Macmillan, 1973), pp. 27-28, 57-59, 79-80, 96-97, ff.

[75] Baumol, *et al.*, *Economics of Mutual Fund Markets*, pp. 31-40; Gremillion, *Handbook*, pp. 25-30.

in 1987 and 33 soon after. Along with the new types of funds, new means of distribution (including fund supermarkets, wrap programs, offerings from advisors, banks, and the Internet) were developed to sell them.[77] In 1979, the SEC had adopted rule 12b-1, which allowed funds to charge some kinds of distribution costs to assets, thereby providing an independent means of marketing.[78] Finally, in addition to new types of funds, new channels of distribution and intensified marketing, new features and services for mutual funds were introduced, including check writing, electronic funds transfers, automatic investment and reallocation plans, and reporting of tax consequences. These factors also promoted the proliferation of fund complexes or families, in which fund managements offer a range of fund types and special facilities for transfers among the funds within a family.[79]

Despite the collapse of the stock market in 2001 and corporate scandals, when asset growth was nearly flat for 2001 and declined by over $580 billion during 2002, mutual funds generally continued to grow in number, assets, and accounts. Although the number of funds fell each year from 2002 through 2004, net assets rose over $1 trillion in 2003 and almost another $700 billion in 2004, and the number of accounts grew by almost 10 million in 2003 and another 6.5 million in 2004. The mutual fund scandals of 2003 involving abusive market timing and late trading, contributed to the decrease in the number of funds and resulted in new regulations to improve investor protection, fund

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[76] Baumol, *et al.*, *Economics of Mutual Fund Markets*, p. 37.
[77] Gremillion, *Handbook*, pp. 28-32; Baumol, *et al.*, *Economics of Mutual Fund Markets*, pp. 37-46
[78] Gremillion, *Handbook*, pp. 24-25, 26.
[79] Gremillion, *Handbook*, pp. 30-32; Michael Lounsbury and Hayagreeva Rao, "Sources of Durability and Change in Market Classifications: A Study of the Reconstitution of Product Categories in the American Mutual Fund Industry, 1944-1985," *Social Forces*, Vol. 82, No. 3 (March 2004), pp. 969-999.

governance, and transparency.[80]

Section 5 The Relation of Management Companies and Shareholders

The essential constituents of a mutual fund are the fund as a separate, legal entity (whether organized as a corporation or a business trust), the board of directors or trustees (who supervise fund activities with fiduciary responsibility for shareholder interests as explicitly defined in laws and regulations), the shareholders (individual and institutional), and the portfolio of securities (purchased with the proceeds of investor share purchases).[81] As a result, the

> typical open-end mutual fund has very limited internal resources, contracting out almost all of its activities. Thus, an open-end mutual fund can be seen as a set of contracts between the trustees and other organizations which provide specific services. Among the parties to a mutual fund are the *sponsor*, which organizes the fund at inception; the *distributor*, a registered broker-dealer serving the role of investment banker and responsible for issuing new shares; the *advisor*, responsible for the fund's portfolio decisions and for its borrowing and lending decisions; the *administrator*, responsible for accounting and monitoring of cash flows and transactions; the *custodian*, usually a bank, responsible for holding the records of securities held and traded, for establishing the prices of those securities, and for general accounting; and the *transfer agent*, responsible for maintaining records of who owns the fund's shares, for receiving or paying cash from sales or redemptions of the fund's shares, and for distributing cash dividends or capital gain distributions. A number of independent corporations provide

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[80] James L. Bicksler, "Mutual Fund Debacle: Economic Foundations, Fundamental Problems and First Step Governance Reforms," Comments before the SEC on Proposed Rule: Investment Company Governance [Release Nos. IC-26323; File No. S7-03-04], March 10, 2004 [http://www.sec.gov/rules/proposed/s70304/jlbicksler031004.htm].

these services, but at large fund complexes these agents are often affiliates of the mutual fund's advisor."[82]

In addition, other prominent parties are independent auditors, consultants and legal representation, brokers executing the portfolio trades, and brokers and advisors selling fund shares.[83]

This elaborate delineation of mutual fund structure may obscure the chief parties in practice. "*Mutual funds* are investment companies that issue and sell redeemable securities that represent an undivided interest in the assets held by the fund. They are operated by companies that manage fund investments…."[84] Thus, only two are primary parties, namely, the shareholders, who provide the capital and own the shares of the company, and the management company, into the trust of which that capital is placed for the generation of returns. "The motivation to sponsor, organize, and manage funds is for the purpose of attracting, pooling, and investing shareholder dollars for *asset-based fees*."[85] For 12 publicly held management companies, 2003 after-tax profits were about 0.16 percent of over $2 trillion in assets under management, that is, better than $3.2 billion or more than $267 million each.[86] "Mutual fund management has been a

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[81] John A. Haslem, *Mutual Funds: Risk and Performance Analysis for Decision Making* (Oxford, UK: Blackwell Publishing, 2003), pp. 16-22.
[82] Peter Fortune, "Mutual Funds, Part I: Reshaping the American Financial System," *New England Economic Review* (July/August 1997), p. 47-48 [original emphasis]. See also Haslem, *Mutual Funds*, pp. 16-22, ff.
[83] Gremillion, *Handbook*, pp. 35-56, ff.
[84] Richard C. Dorf, *The New Mutual Fund Advisor: Everything You Need to Know about Investing in No-Loads*, Revised Edition (Chicago, IL: Probus Publishing Company, 1991), p. 3 [original emphasis].
[85] Haslem, *Mutual Funds*, p.20 [emphasis added].
[86] Strategic Insight, "Moving On, with Reflections and Optimism," *Strategic Insight Overview*, Issue 5 (2004), as quoted in Gremillion, *Handbook*, p. 44.

profitable business…."[87]

Although the boards of directors or trustees are explicitly charged as fiduciaries of shareholder interest, that responsibility did not prevent the abuses which prompted attempts to strengthen the boards' stewardship of shareholder interests in 1970 and again in 2004. Warren Buffet argues that even the independent, disinterested directors are selected by the management company and are subservient to it.[88] Indeed, the only exception to this arrangement is The Vanguard Group, Inc., a management company organized by John Bogle and owned by the funds which it manages. Several benefits of this form of organization include: (1) service to shareholders as owners rather than customers; enhancement of shareholder value by (2) reducing costs related to expensive portfolios to attract new money; (3) educational marketing rather than expensive advertising to attract new money; (4) greater reliance on low cost investment approaches, like index funds; (5) less pressure for risky investments to compensate for higher expenses; and (6) provision of fund services at cost.[89] In addition to this list of advantages, this organizational form is noteworthy for its unique representation in the Vanguard Group. These considerations underlie the standpoint adopted in this study, namely, the evaluation of mutual fund performance from the viewpoint of management, the exclusion of some variables which represent investor performance criteria, and the use of fund manager performance evaluations by higher management and their directors.

[87] Gremillion, *Handbook*, p. 44. Steiner, *Investment Trusts* (1929), p. 5, n. 4, quotes three early demurrers at the emphasis on fund directors as "entrusted" with the investments of shareholders.
[88] Berkshire Hathaway Inc., *2003 Annual Report*, March 4, 2004, as quoted in Gremillion, *Handbook*, p. 40-42. See also Haslem, *Mutual Funds*, pp. 112-129.
[89] Haslem, *Mutual Funds*, pp. 121-122.

Chapter 4 Evaluation of Mutual Funds

Section 1 Introduction

Although mutual funds have not typically been subject to strategic management analysis, they nonetheless have been extensively analyzed in the finance literature. Chapter 4 briefly examines the mathematical models applied in this literature to the evaluation of mutual funds performance. The standard paradigm is the Markowitz mean-variance model of portfolio selection, which evaluates portfolios based on the joint criteria of average return and risk. However, it measures risk as the variance of the stream of returns. Two problems, one conceptual and one empirical, arise with this approach.

Conceptually, risk, to those who actually have value at stake, has two components: (i) the likelihood and (ii) possible extent of loss. In financial investments, such loss may be either loss of principal or opportunity cost as compared to a risk-free rate or a superior investment. Investments with returns greater than average are universally desired and the principal justification for actively managed funds. Indeed, a chief concern in strategic management, both in theory and practice, is the identification, attainment, and assurance of rents, *i. e.*, greater than normal market returns. Therefore, the "risk = variance" model makes the fundamental, conceptual confusion of treating outcomes which investors desire and actively pursue the same as outcomes which they avoid and seek to avert. It equates the opposite ends of the range of results.

The asymmetry of reactions to gains and losses by real persons was one of the fundamental results of prospect theory, first proposed in Kahneman and Tversky [1979], which sought to account for violations of the implications of expected utility theory in the results of a series of psychological experiments. In these experiments, subjects demonstrated risk-averse behavior with respect to gains and risk-seeking behavior with respect to losses. These different behaviors were both driven by a more fundamental "loss aversion." The consequences of these and related behaviors include inefficient portfolio selection and asset prices which deviate from fundamental values. A large number of subsequent studies have documented these anomalous (from the view of expected utility theory and efficient market hypothesis) behaviors among individual and institutional investors and professional market traders.[1]

Another theoretical problem with the variance characterization of risk is apparent when variance is taken as the deviation from a trend line, for example, the residuals in regression analysis. Consider two series of returns, both of which begin equal to the risk-free rate. In the first series, returns decrease by a fixed number of basis points each successive period; whereas, in the second series, returns remain constant for, say, two periods then jump the same fixed number of basis points in the third period, remain constant for two periods, then jump again, etc. The first series of returns, although

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[1] See also Tversky and Kahneman [1991, 1992] for extensions to the original theory. Shefrin and Statman [1985] documented the "disposition effect," the tendency of investors to sell winners too early and hold losers too long; Heisler [1994] found the effect among futures traders; Odean [1998], Barber and Odean [1999], and Grinblatt and Keloharju [2000] found it among individual investors; and Shapira and Venezia [2001] demonstrated the effect among individual and institutional investors. Benartzi and Thaler [1995] used prospect theory to explain why equity prices are higher than predicted by standard finance theory; Olsen [1997] found those effects among professional money managers. DeBondt and Thaler [1985] found that investors inappropriately extrapolate past performance; Chopra, Lakonishok, and Ritter [1992] and Lakonishok, Shleifer, and Vishny [1994] documented inappropriate extrapolation behavior with regard to long-term returns, while Jegadeesh and Titman [1993] found it for short-term returns. List [2003] shows, however, that "the endowment effect" (Thaler [1980]) is eliminated among those with substantial market experience.

positive for several periods, is, in each succeeding period, increasingly below the risk-free rate and, therefore, represents an increasing opportunity cost until it reaches zero and becomes increasingly negative (that is, registering loss of principal) thereafter. This series is represented by a straight line with negative slope and the regression of the series data points coincides with the original line. Thus the deviation from the trend line, that is, the residual sum of squares, is zero: there is no variance of the actual return values from the trend of return values. The second series, however, is a non-decreasing, stair-step function which is not identical with the regression line based on the second data series. Thus the variance of the actual return values from the regressed values, the residual sum of squares, is positive. The second series can be constructed with a large variance, but it registers no loss, whereas the first series, despite its small or zero variance, represents continuous and increasing loss.

Finally, there is a more fundamental problem of using two moments of the same process to represent different characteristics of the investment activity. Ruefli [1990a] demonstrates that modeling return and risk by the mean and variance of the series of returns results in inestimable relations or spurious results.

These conceptual and theoretical difficulties are compounded by empirical problems. A corollary of the "risk = variance" hypothesis is that high returns are only available to investments which bear high risk. However, Bowman [1980] and others have shown the opposite correlation: that higher returns tend to accrue to low risk activities and high risk activities tend to result in low returns or losses.

This chapter discusses models of this type as well as others (such as distributional, regression, and benchmarking) in the finance literature. It also explains how the model of this dissertation only indirectly reflects risk. Possible explicit treatments of nonvariance-based risk in the type of model here developed are described in the final chapter on future directions. By way of introducing data envelopment analysis, the basic methodology of the technique developed in Chapter 6, the final section of this chapter reviews the literature of applications of data envelopment analysis to the evaluation of mutual fund performance and describes the two general directions of that work.

The problem of performance evaluation depends on the standard of measurement. There are two general approaches: (i) the absolute standard, a physical or material optimum, extreme, or limit behavior; and (ii) the relative standard, comparison against peers or other referents, benchmarking.

An absolute standard is given by theory. In engineering applications, *e. g.*, the efficiency of energy conversion or the work efficiency of heat engines, the theoretical limits are given by scientific theory. Some theories, especially some physical theories, are more detailed, comprehensive, and well tested than others, which, like economics and financial theories, are more limited, inexact, and weakly conforming with practice and experience. Less rigorous theories are therefore more problematic and debatable and less capable of rendering definitive judgments about theoretical maxima.

Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM) can be used in both ways. They have been interpreted as establishing the theoretically maximum returns and minimum risks possible for portfolios and capital assets and therefore providing the prices (evaluations) of such assets based on their performance relative to the theoretical limits. These measures, however, are given in the context of market conditions. Such evaluations are often "opportunity cost" assessments, which are necessarily contingent and relative. Moreover, the market may not only set the conditions of transformation and performance, but may also become a direct, or by means of proxies, an indirect, benchmark, as, for example, when comparing mutual fund performance against the overall market or an index fund. Even common financial measures of risk exhibit the distinction. For example, risk as variability, or variance, is an internal measure dependent on the behavior of returns and as revealed in data of past fluctuations. However, risk, as volatility, or beta or correlation, is a measure of how returns depend on the market of which they are a part.

The mean-variance measures of Markowitz [1952, 1959], Treynor [1965] (who is explicit), and Sharpe [1966] are ostensibly measures of performance, and efficiency is a fundamental standard of performance. The variables of their analyses are designed to express the performance of a whole ensemble or collection in a single value. This is a consequence of all the moments being defined in terms of the first moment, central tendency, and the subsequent moments being further extensions in the description of the dataset as a whole based on its mean.

Thus, Markowitz, Treynor, and Sharpe seek to provide an objective measure of performance. Their methods, because statistical, describe the average performance of the whole, or the extent of deviations from the average, or the asymmetries in the deviations from the average, successively more detailed descriptions of the space defined by the dataset reduced to a single value. Thus, these measures are abstract, since their values are equal to the values of any actual individual only fortuitously.

DEA, on the other hand, provides answers much less succinct than the list of moments of a parametric family, but, because more comprehensive, also more useful. The values of variables of each individual represent the empirical, actual, historical attainment relative to all other individuals in the comparison group. DEA scores measure the results of each individual's effort and express it in relation to the corresponding measures of all others' efforts. Its measures report the performance of each individual and its status relative to others, not merely the numbers expressing a statistical artifact.

Performance is standards based evaluation. The standard may be taken, for example, in terms of theoretical extrema. Alternatively, the standard may be based on the performance of all other examples of the same class (where class is determined by the context or domain of discourse) and, in particular, the "best" performers, those who achieved their goals and at the best rates of transformation.

Section 2 Modern Portfolio Theory

The papers of Roy [1952] and Markowitz [1952], which model the portfolio selection problem in terms of returns and risk as given by the expectation and variance of the portfolio returns, initiated the era of modern finance theory and modern portfolio theory in particular.[2] Markowitz [1956, 1959] corrected and expanded the model. These were soon followed by Sharpe [1963, 1964] (and independently by Lintner [1965a] and Mossin [1966]), who applied the mean-variance theory of portfolio selection to establish

[2] Merton H. Miller, "The History of Finance," *Journal of Portfolio Management*, Vol. 25, Issue 4 (Summer 1999), pp. 95-101, and William F. Sharpe, "Mutual Fund Performance," *Journal of Business*, Vol. 39, Issue 1 (Jan. 1966), p.119.

the equilibrium price of assets; and Fama [1965], who rescued the random walk model of stock prices by demonstrating that the departure of empirical stock price series from true randomness was not sufficient to compensate the transactions costs and changes in risk of attempting to exploit that difference. These tools were applied to the modern analysis and evaluation of mutual funds in the finance literature beginning with Treynor [1965], Sharpe [1966, 1975], and Jensen [1968, 1969, 1972a, 1972b]. These were fixed period, static models with one or both of the assumptions that (i) investor utility functions are quadratic and that (ii) investor subjective probability distributions for expected returns are normal.

In the standard mean-variance portfolio model, the object is to compose a portfolio from n assets which have random rates of return, $r_1, r_2, \ldots, r_n$. The expected rate of return of each is $\bar{r}_i \equiv \mathrm{E}[r_i]$, $i = 1, \ldots, n$. The proportions in which the assets appear in the portfolio are given by the corresponding weights $w_1, w_2, \ldots, w_n$. Because the returns are random, that is, their values uncertain, they are assumed to have a distribution for which their variances are given by σ_i^2 and their covariances by σ_{ij}, where $\sigma_{ii} = \sigma_i^2$, for $i, j = 1, \ldots, n$. Thus, the portfolio rate of return, $r = w_1 r_1 + w_2 r_2 + \ldots + w_n r_n$, has mean

$\mathrm{E} = \bar{r} = \Sigma_{i=1}^{n} w_i \bar{r}_i$ and variance $\mathrm{V} = \sigma^2 = \Sigma_{i=1}^{n} \Sigma_{j=1}^{n} w_i w_j \sigma_{ij} = \Sigma_{i \neq j} w_i w_j \sigma_{ij} + \Sigma_{i=1}^{n} w_i^2 \sigma_i^2$

(E for expected return and V for variance). The variability of the portfolio returns as given by the variance σ^2 is identified as the risk, which has two components, (i) systematic risk, $\Sigma_{i \neq j} w_i w_j \sigma_{ij}$, which results from the correlations among all the assets in the portfolio, and (ii) specific risk, $\sum_{i=1}^{n} w_i^2 \sigma_i^2$, which depends on the variances—risks—of the individual assets. Every vector of weights $\boldsymbol{w} \equiv (w_1, w_2, \ldots, w_n)^{\mathsf{T}}$, where $\sum_{i=1}^{n} w_i^2 = 1$, defines a portfolio. A prohibition of short sales is specified by the additional constraints w_i □ 0. Every portfolio which satisfies all constraints is said to be feasible, obtainable, or legitimate.

Let $w_i = 1/n$, then $\sum_{i=1}^{n} w_i^2 \sigma_i^2 = (\sum_{i=1}^{n} \sigma_i^2)/n^2 \le \sigma_{\max}^2/n$, where $\sigma_{\max}^2 \equiv \max\{ \sigma_i^2 \mid i = 1, \ldots, n\}$. Thus, as n increases, that is, as the portfolio is increasingly diversified, the specific risk decreases. However, suppose all the returns have the same variance s^2 and the return correlations are a constant z, then, again for $w_i = 1/n$, the variance for the portfolio rate of return r is $\sigma^2 = \sum_{i \ne j}(zs^2/n^2) + \sum_{i=1}^{n}(s^2/n^2) = n(n-1)[zs^2/n^2] + [s^2/n] = zs^2 + (1-z)s^2/n$, which is bounded below by the average covariance $zs^2 = \sigma_{ij}$. This property, the Markowitz "law of the average covariance," means that the systematic risk cannot be diversified away.

The set of all obtainable mean-standard deviation combinations forms a convex set (a parabola opening to the right) in the mean-standard deviation space. The boundary of this convex set from the minimum variance point (the point of the parabola) extending in the direction of increasing mean returns (upward and rightward) defines the efficiency frontier of portfolios. For any given degree of risk (standard deviation), the portfolio with the greatest return belongs to this frontier, and, for any given level of expected return, the portfolio with the least risk belongs to this frontier. Some point on the mean returns axis represents the risk free rate of returns (since the standard deviation is 0 for every point on the means axis). The ray from the risk free rate on the mean return axis and tangent to the efficient portfolio frontier represents all efficient portfolios when borrowing or lending of the risk free asset is permitted. If such borrowing is not allowed, then all efficient portfolios lie on the segment from the risk free rate to the tangent point. The segment represents all convex combinations of the two extremes, the risk free asset and the efficient portfolio (at the tangent) of risky assets. This is the one-fund theorem due to Tobin [1958].

If all investors observe mean-variance optimization and assess returns and risk with the same probability distributions (so that they all optimize based on the same expectations, variances, and covariances) and if transactions have zero cost, then all investors will hold positions on this ray. The tangential portfolio is the market portfolio, since only it holds all risky assets, and the ray, called the capital market line, represents the relation between the expected rate of return and risk for all efficient portfolios. Therefore, prices of efficient assets and portfolios fall on this line and individual risky assets and inefficient portfolios fall below the capital market line. This is the Capital Asset Pricing Model (CAPM) due to Sharpe [1964], Lintner [1965a], and Mossin [1966].[3] The capital market line gives the expected rate of return $\overline{r}$ and the risk σ of any

[3] Craig W. French, "The Treynor Capital Asset Pricing Model," *Journal of Investment Management*, Vol. 1, No. 2 (2003), pp. 60-72, attributes some of the earliest work in developing the CAPM to Treynor in unpublished, mimeographed papers "Market Value, Time, and Risk" (1961) and "Toward a Theory of Market Value of Risky Assets" (1962).

efficient asset as $\bar{r} = r_F + [(\bar{r}_M - r_F)/\ \sigma_M]\ \sigma$, where r_F is the risk free rate, and $\bar{r}_M$ and σ_M are the market expected rate of return and standard deviation, respectively. The slope of the capital market line, $(\bar{r}_M - r_F)/\ \sigma_M$, also called the Sharpe ratio, gives the increase in expected rate of return for every unit increase in risk.

The expected rate of return for an individual asset *i* with respect to its individual risk depends on its correlation with the efficient market portfolio return, namely, $\bar{r}_i = r_F + \beta_i\ (\bar{r}_M - r_F)$, where $\beta_i \equiv \sigma_{i,M} / \sigma_M^2$ and $\sigma_{i,M} \equiv \text{Cov}\ [r_i, r_M]$.

An asset's rate of return in excess of the risk free rate is proportional, by the factor β, to the market's rate of return in excess of the risk free rate. The return rate for asset *i* is $r_i = r_F + \beta_i\ (r_M - r_F) + \varepsilon_i$, where $E[\varepsilon_i] = 0$ and $\text{Cov}[\varepsilon_i, r_M] = 0$. Therefore, $\sigma_i^2 = \beta_i^2\ \sigma_M^2 + \text{Var}[\varepsilon_i]$. Again, the individual asset variance, or total risk, has two components, namely, (i) $\beta_i^2\ \sigma_M^2$, the systematic, or market, risk, which arises from the correlation of the asset with the market as a whole and which, for assets with nonzero betas, cannot be reduced by diversification, and (ii) $\text{Var}[\varepsilon_i]$, the specific risk, the impact of which on the portfolio, because $\text{Cov}[\varepsilon_i, r_M] = 0$, can be reduced by diversification. For the portfolio, beta is the weighted average of the individual asset betas. Since the expected returns-beta relation is linear, the pairs of betas and their associated expected returns define the security market line in beta-expected return space.

Section 3 Portfolio and Capital Theory Evaluation of Mutual Funds

Among the first efforts in finance literature to use the then newly developed tools for the evaluation of mutual fund performance appeared in Treynor [1965], who sets the problem thusly:

> In order to reward management for good performance…it is necessary to be able to recognize it…. [T]o the extent that [funds] are heavily invested in common stocks, the return achieved in any one period is subject to wide fluctuations which are beyond the control of investment management …. [N]o one has devised a satisfactory way to measure [management's] impact on performance.

The Treynor measure ranks funds according to both return and risk, separates market from specific risk, and is invariant under changing market return assumptions. For each of 20 funds, his first step is to regress fund returns against market return, generating the "characteristic line," the slope of which (or beta) is the measure of volatility or risk. He takes the deviations from the regression as the measure of specific risk. For characteristic lines with equal slopes (volatility), the one with higher fund return intercept is the superior performer. Next, he generates the "portfolio possibility line" by plotting fund expected excess returns at a given market return against risk, using the returns and volatility values previously determined. Although the fund returns change according to the market return assumed, the relative ranking of the funds is invariant. Finally, the funds are ranked by the ordering of the market return at which their characteristic lines intersect the horizontal at the risk free rate, that is, by the market rate of return needed to produce a fund rate of return equal to the risk free rate.

Sharpe [1966] explicitly applied the newly developed theories to the evaluation of mutual funds. Based on the (near) random walk character of stock prices and the efficient market hypothesis, he proposes that the scope of investment management is restricted to security analysis (the evaluation of individual security performance and covariances) and to portfolio analysis (the evaluation of portfolios based on security performance projections and the specification of efficient portfolios for different risk classes). Given optimal portfolios, best net returns performance depends on avoiding unnecessary expenses like the "fruitless search for incorrectly valued securities."[4]

This view is tested by several methods. Using historical return averages and variances as substitutes for *ex ante* projections of expected return and return variance, the lines generated by applying the Tobin effect are compared and summarized by their (inverse) slopes. The measure of evaluation is the "reward-to-variability ratio" (R/V), which has become known as the Sharpe ratio, namely, for fund i, average returns A_i in excess of the risk free rate p (for "pure rate of interest," is the risk free rate r_F) divided by the standard deviation V_i of the annual rates of return: $(A_i - p)/V_i$. This is called the "reward per unit of variability." Evaluating the same funds for the preceding decade and plotting the fund ranks from the latter decade against the earlier decade (to test for the ability of past R/V to predict future R/V) results in a scattered diagram, albeit with an upward trend. Thus, an imperfect persistence of performance is demonstrated.

This procedure is followed by a second, which substitutes volatility (beta) for total variability (variance or standard deviation) in the R/V ratio to form the Treynor Index (TI): $(A_i - p)/B_i$, where B_i (calculated for a period prior to the period for which

[4] Sharpe, "Mutual Fund Performance," p. 121.

TI is calculated) is the historical change in a fund's rate of return per unit change in the market rate, with the Dow Jones Industrial Average (DJIA) used as a proxy for the market return. Although the two rankings (R/V against TI), provide similar results (rank correlation coefficient .974), Sharpe attributes the similarity to the fact that all the funds evaluated were well diversified and suggests that, for relatively undiversified funds, the results would have exhibited greater difference since the Treynor Index captures only the systematic, market risk and excludes the specific risk. He finds the exclusion of specific risk in the TI a deficiency for evaluating historical performance but an advantage in predicting future performance. In a test of predictability, the plotting of R/V based ranks for one decade against the TI based ranks for the prior decade results in a slightly better correlation than the same test of later R/V ranks against prior R/V ranks.

Testing ranks based on R/V against ranks based on expense to net asset ratios produced results with predictive power comparable to the results of TI based ranks: low expense ratios were associated with better reward to variability ratios. Fund size (net asset value) showed little predictive power.

According to the three measures of assessment (rank correlation coefficient, odds of remaining in the upper/lower half of performers, and regression correlation coefficient and regression slope t-value), the best predictor was rank based on variability in the earlier decade for rank based on variability in the later decade. Despite the deviations from the predicted trend, which are attributed to presumed changes in management philosophy and inadvertent performance, Sharpe revealingly comments "that fund managers fulfill remarkably well the obligation to stay within their selected risk classes."

One of the purposes of the present study is to test for performance differences according to risk classes as signified by type of strategy.

As measured by the R/V ratio, the 34 funds in the Sharpe study were statistically significantly inferior performers when compared to the DJIA (as market proxy). Because not all costs (namely, loads) were included in computing the fund returns and no costs were considered for the DJIA returns, a second test based on the gross fund returns to variability ratios found that the average mutual fund was statistically not inferior to the DJIA. One advantage of the method of data envelopment analysis developed in the present study is that expenses (and *any* other inputs into the transformation process) can be taken into account and are reflected in the efficiency results without having to diminish the (output) results to evaluate fund performance fairly.

Finally, Sharpe concludes that

> performance can be evaluated with a simple yet theoretically meaningful measure that considers both average return and risk. This measure precludes the 'discovery' of differences in performance due solely to differences in objectives (*e. g.*, the high average returns typically obtained by funds who consciously hold risky portfolios.)[5]

Again, it is precisely the consequences of differences in "objectives," that is, strategies, that the present study reveals. This study tests the very proposition that Sharpe assumes, and evaluates whether funds with risky portfolios do provide higher average returns.

Despite the theoretical significance, broad-based and long-term application, and continued (including dynamic and stochastic) development of the Markowitz mean-

[5] Sharpe, "Mutual Fund Performance," p. 137.

variance portfolio theory and the Sharpe-Lintner-Mossin Capital Asset Pricing Model based on it, a growing literature has investigated limitations of these theories. Starting with Markowitz [1959], and including Quirk and Saposnik [1962], Borch [1963, 1968, 1969, 1974], Bierwag and Grove [1966], Samuelson [1967, 1970], Feldstein [1969], Alderfer and Bierman [1970], Tsiang [1972, 1974], Chipman [1973], Klevorick [1973], Bierwag [1974], Levy [1974], Samuelson and Merton [1974], these investigations develop restrictions on the probability distributions of returns or on the utility functions of investors. The investigations of the failure of the market portfolio to attain mean-variance efficiency or of other deficiencies in the model (insufficiency in short selling mechanism or risk-free rate instruments) also began early, as in Friend and Blume [1970], Black, Jensen, and Scholes [1972], Miller and Scholes [1972], Blume and Friend [1973], and Fama and MacBeth [1973], among others.

In particular, problems with the assumptions regarding risk were noted early. Because market timing activities introduce nonlinearities in the β of the basic regression equation, Treynor and Mazuy [1966] propose the addition of a quadratic term. Subsequent adjustments to the regression equations include the switching methods of Kon and Jen [1978, 1979] and the addition of a term dependent on market return exceeding the risk free rate by Merton [1981] and Henriksson and Merton [1981]. Ferson and Schadt [1996] propose a conditional model to capture the effects of portfolio betas which vary because of varying betas of constituent assets as well as their varying portfolio weights.

These studies also indicated the importance of additional information in making evaluations. Fund manager forecasts in the nonparametric model of Henriksson and Merton [1981], the fund asset proportions in the conditional model of Ferson and Schadt [1996], and investor knowledge of future dividends for variance bounds in volatility tests in LeRoy and Porter [1981], Shiller [1981], and LeRoy and Steigerwald [1995] are examples of the additional information evaluation models often require.

Although the theory originally required a returns distribution for which the first two moments are sufficient for a complete description, the normal distribution is commonly assumed. However, Lau, Lau, and Wingender [1990], Turner and Weigel [1992] and Campbell and Hentschel [1992] show that portfolio returns are generally not Gaussian. In particular, Aggarwal, Rao, and Hiraki [1989], Kritzman [1994], and Corrado and Su [1996] find that the sample skewness and kurtosis imply non-normality. Leland [1999] shows that hedging with options also causes non-normal distributions of returns.

Samuelson [1970] argues for inclusion of moments beyond variance to improve the accuracy of the mean-variance portfolio analysis. Arditti [1971] proposes adding third moment terms to the portfolio regression. Kraus and Litzenberger [1976] argue that the model extended to include nondiversifiable skewness is consistent with empirical data. Ang and Chau [1979] also reformulate the portfolio regression with a skewness term to account for active trading by fund managers. Similarly, Martin and Spurgin [1998] argue that, although skewed distributions of component assets may be diversified away, trading activity may produce a skewed portfolio distribution. Prakash and Bear [1986] and Stephens and Proffitt [1991] include both coskewness and cokurtosis terms in their formulations.

One motivation for capturing the skewness of the returns distributions reflects the view that investors prefer skewness to the upside, which implies non-quadratic investor utility functions, as argued by Arditti [1971, 1975], Kane [1982], and Ho and Cheung [1991]. Harvey and Siddique [2000] demonstrate that such an investor preference is consistent with the characterizations of risk aversion by Arrow [(1963) 1971] and Pratt [1964]. In defense of the standard mean-variance theory, Tsiang [1972, 1974] concedes that the empirical demand for skewness, as evidenced in limited liability organization, stop-loss orders, options, etc., can only be ignored in those cases where the relevant risk is very small relative to expected total wealth.

Moreover, the preference for skewness is closely related to the reformulation of the concept of risk to include only the downside dispersion of returns, whatever the shape of their distributions. An early focus on the lower distributional tail was the concept of regret in Savage [1954]. Indeed, Markowitz [(1959) 1970] derives the portfolio optimization based on semi-variance, of which he notes that

> [a]nalyses based on S [semi-variance] tend to produce better portfolios than those based on V [variance]. Variance considers extremely high and extremely low returns equally undesirable. An analysis based on V seeks to eliminate both extremes. An analysis based on S_E, on the other had, concentrates on reducing losses. [1970, p.194]

Mao [1970a, b], Hogan and Warren [1972, 1974], and Porter [1974] followed, based on the observation that managers generally view risk as the failure to attain target rates of return, as surveyed in Libby and Fishburn [1977], and that mean-semivariance optimal portfolios (generally) are efficient by stochastic dominance criteria. These analyses are generalized by Fishburn [1977] beyond mean-lower semivariance to a model of mean-risk dominance in which risk is measured by a probability weighted function of deviations below a specified target return. In an earlier work, Stone [1973, p. 675]

> defines two related three-parameter risk measures and shows that common risk measures of variance, semi-variance, mean absolute deviation, and probability of an outcome worse than some disaster level are all special cases of one…and that standard deviation is a special case of the other. …any of these risk measures has implicitly involved decisions about: (1) a reference level of wealth…; (2) the relative importance of large versus small deviations; (3) the outcomes that should be included in the risk measures.

Bawa and Lindenberg [1977], and Harlow and Rao [1989] show that optimal asset allocation and CAPM, respectively, can be reformulated with downside deviations. Sortino and van der Meer [1991], Marmer and Ng [1993], Merriken [1994], Miller and Leiblein [1996], and Ogryczak and Ruszczynski [1998] continue this line of research. Sortino and Price [1994] particularize the general model of Fishburn [1977] by fitting a lognormal distribution to deviations below the minimally acceptable return.

Since certainly a major component of portfolio risk is market risk, the development, broad application, and institutional adoption of the methodology of value at risk is another form of the focus on downside variation. Although based on the standard theory, value at risk estimates the maximum expected loss for a given confidence level within a given period, assuming normal market conditions. Chung [1999] reviews and Jorion [2001] details the value at risk methodologies.

In addition to the axiomatic development of the mean-variance model, there are two kinds of other factor models. An important alternative to the CAPM is the arbitrage pricing theory of Ross [1976, 1977] and its many extensions, according to which "the random return of each security is a linear combination of a small number of common, or pervasive, factors, plus an asset-specific random variable."[6] In this theory, the underlying factors are determined by statistical factor analysis. Fung and Hsieh [1997] apply factor analysis to hedge funds and Mitev [1998] to commodity trading advisors; both find returns due to general investment and trading approaches, such as following trends, trading on spreads and other systems.

In another approach, hypothesized, potential factors are examined for their effects through multi-factor regression analyses. To identify the source of differential net new mutual fund investment, Allerdice and Farrar [1967] use cross-sectional analysis over a three annual periods to examine both external, *i. e.*, general economic, factors as well as internal, or fund-specific, factors, of which portfolio turnover and performance have the greatest positive and expense ratio has the greatest negative effects. Fama and French [1992, 1995] correlate stock earnings and returns to size and book-to-market ratios. Brinson, Singer, and Beebower [1991] find that portfolio asset allocation determines over 90% of fund returns. Sharpe [1992] also finds as much as 98% of portfolio returns due to investment "style" factors, such as growth and income stocks, value stocks, high yield bonds, etc. Some of these factors are closely related to the strategy categories

[6] Gregory Connor and Robert A. Korajczyk "The Arbitrage Pricing Theory and Multifactor Models of Asset Returns," Chapter 4, pp. 87-144, in R. A. Jarrow, V. Maksimovic, and W. T. Ziemba, eds., *Finance*, Handbooks in Operations Research and Management Science, Vol. 9 (Amsterdam, NL: Elsevier, 1995), p. 88.

investigated in the present study. Brown and Goetzmann [1995] determines that returns depend on common "strategies" beyond style analysis. Carhart [1997] uses the three factors of Fama and French [1993] and the momentum effect of Jegadeesh and Titman [1993] to find that actively traded, equity mutual fund performance persistence depends primarily on common stock return factors and fund expenses and costs, rather than fund manager skills.

Daniel, Grinblatt, Titman, and Wermers [1997] develops hypothetical comparison portfolios as the benchmarks for the evaluation of funds of similar characteristics (based on size, book-to-market, and prior year returns). They find that the small difference in performance by active funds approximately equals the difference in expenses between active and passive funds. These benchmarks are quite different from the benchmarks of market index funds, abstract indices, characteristic regression factors, or specially selected comparison funds. An example of this last case is the generalization of the Sharpe ratio presented in Sharpe [1994]. The original ratio of returns in excess of the risk-free rate over the standard deviation of those returns becomes, in the *ex ante* case, the ratio of the expected differences between the fund being evaluated and the rate of any benchmark (*i. e.*, comparison) security or portfolio over the predicted standard deviation of those differences. Among other techniques used to examine fund performance are Kothari and Warner [2001], which uses simulations, and Kwon [1991] and Brockett, Charnes, Cooper, Kwon, and Ruefli [1992] which develop a chance-constrained programming model to evaluate fund strategies. Data envelopment analysis also provides benchmarks, but, as will be demonstrated, these are determined specifically for each fund under evaluation by those among the best performers which are closest to it.

Because of their fundamental nature, the economic concepts of uncertainty, risk, and return prove of concern not just in the financial markets, but equally for firms and industries for real products. These issues are central to the view of strategic management, which studies questions of major commitments in uncertain environments, whether at strategic, corporate, or functional levels. The signal challenge to the notion that profit is the premium for entrepreneurial risk taking appears in Bowman [1980], which demonstrates that business risk and return are negatively correlated across companies within industries. In this setting, risk is the probability distribution of the outcomes of resource commitments; the variance in the profit consequences of those commitments is the measure of risk; and profit is measured by the return on equity, i. e., after-tax profit divided by stockholders' equity. Bowman finds "that in the majority of industries studied, higher average-profit companies tended to have lower risk, i.e., variance, over time."[7]

Numerous studies have pursued a clarification and an understanding this phenomenon. Bowman [1982, 1984] finds that support for the suggestion that already troubled companies may attempt to compensate by undertaking riskier activities. In investigating the relation of strategy, market structure, and risk-return, Cool, Dierickx, and Jemison [1989] find market share, and therefore, an inferred use of market power, are significant factors. Miller and Bromiley [1990] use factor analysis to identify three kinds of firm risk: income stream risk, stock returns risk, and strategic risk. They find that

[7] Bowman [1980], p. 19.

prior income stream or strategic risk reduces subsequent performance, and that prior performance and subsequent income stream risk were inversely related.

Kwon [1991], Brockett, Charnes, Cooper, Kwon, and Ruefli [1993], and Brockett, Cooper, Kwon, and Ruefli [1997] adopt a different approach which uses chance-constrained programming. In contrast to the semi-variance, which is concerned with deviations below the mean, the chance-constrained approach can direct attention to the tails where large losses can occur with small probabilities. Moreover, their model formulation separates risk and return, which are defined independently of each other rather than as two moments of the same process. These investigations also confirm a negative relation between risk and returns when they apply this model to empirical mutual fund data and compare the chance constrained results with both published evaluations and their results of a survey of brokers and newsletter editors.

A very different approach to these issues is presented by Ruefli [1990a], in which a fundamental theoretical difficulty in the risk-return investigations of strategic management is examined. Those studies which use mean and variance measures of the same process to represent the phenomena of returns and risks, respectively, employ constructs which render statements of their relationship unverifiable, ungeneralizable, and irremediable through the application of additional equations. This conclusion is demonstrated by deriving a negative relationship between mean and variance for a given period when the corresponding relationships over the succession of subperiods are all positive.

The return-risk/mean-variance approach had been criticized for misrepresenting the concept of risk. Risk is not the single valued, second moment of a distribution (which in practice, was narrowly defined or restricted), but the tuple of the probability of events below a specified value and the amount exposed (the potential loss). Then Bowman and others argued that the relationship which was presumed to be described by this construct was in fact the reverse in practice, that is, that high return firms exhibited low risk (as measured by variance), not the high risk supposedly required to achieve high returns. Then Ruefli demonstrated that the construct was theoretically invalid, that the purported relationship was an artifact of the partitioning of the values of two functions of the same variable.

An alternative approach to the questions of performance and risk is offered in Collins and Ruefli [1992], which develops a new concept of risk as in industry or market state-defined terms and a new measure of risk in nonparametric, ordinal statistics.[8] These measures are extended with entropy measures and Kolmogorov-Smirnov statistics in the subsequent Collins and Ruefli [1996] and Ruefli and Wiggins [2000]. A review of prior practice appears in Ruefli, Collins, Lacugna [1999]. The attraction of the established, standard analysis is strong and the debate continues, for one thread of which see Bromiley [1991], Oviatt and Bauerschmidt [1991], Wiseman and Bromiley [1991], Ruefli [1991], Brockett, Charnes, Cooper, Kwon, Ruefli [1992], Ruefli and Wiggins [1994], Núñez and Cano [2002], Brockett, Cooper, Kwon, Ruefli [2003a], Cano and Núñez [2003], Brockett, Cooper, Kwon, Ruefli [2003b].

[8] An early basis for this approach was presented in Ruefli and Wilson [1987], Ruefli [1988], and Ruefli [1990b].

Section 4 DEA Evaluations of Mutual Funds Performance

This final section of Chapter 4—a review of prior literature which has applied data envelopment analysis to mutual funds—provides a transition from the discussion of the established financial models, "modern portfolio theory," to a production theoretic approach for strategy evaluation. The basis of the method developed in this dissertation for evaluating strategic performance (in the present instance, of mutual funds) is data envelopment analysis. While the application of DEA to the evaluation of strategies is new, DEA has been previously applied to mutual fund performance in several studies, most of which model the fundamental risk and performance concepts of standard finance theory. This section will also as serve as an introduction to DEA itself prior to its development in Chapter 5.

The prior literature exhibits two directions in the type of research undertaken. The first was initiated by the pioneering paper of Murthi, Choi, and Desai [1997]. This paper established three essential themes: methodological adaptability, extension of standards and agents, control for strategy.

It displayed the DEA methodology resolving a data analysis issue which was intractable by previous methods. The standard measure of evaluation for mutual fund performance is the Sharpe ratio, R/σ, and among the many proposed adjustments to the measure is $(R - C_I)/\sigma$, where C_I are investor costs. (Note that C_S would represent the costs to any given stakeholding group S.) The objection to this adjustment is that the diminished result does not fully represent the return to *risk*. Murthi, Choi, and Desai demonstrated that data envelopment analysis could account jointly and simultaneously for the effect of additional costs while evaluating returns at their full reported values, without netting those costs.

In doing so, they offer their contribution as an extension to the Sharpe measure (and standard finance theory), by applying DEA to include additional variables without having to "adjust" the returns data. However, this inclusion makes a fundamental change. The dimensions added by these additional variables—costs to investors—represent values of concern to other economic agents. The Sharpe ratio exists as a measure dependent solely on the distribution of returns, and therefore, it represents an internal measure of performance reflecting the interests solely of the fund as an economic agent. The DEA model includes those additional factors and thus allows other groups easily to adjust the analysis to reflect more closely their interests as investors in the fund, or potentially, for example, the interests of the fund family management. Moreover, exogenously given factors (for example, interest rates, foreign exchange rates, weather, natural disasters, political turmoil) may be included without prejudice to the evaluation. Indeed, the method can indicate those who did well under extraordinarily difficult circumstances. In the mathematics of the model, the interests of all these factors are represented by the multiple input variables and multiple output variables.

DEA introduces other economic agents in another way. The efficiency surface that forms the standard of evaluation is determined by the actual performance values of the best among those being evaluated. That is, DEA is a relative measure. It computes the best possible performance score for each fund, consistent with how all other funds performed. The ones with the best scores are the benchmarks for the others. Thus, DEA is a comparative, a relative method. It computes performance in the management of factors of conflicting interests within the fund (tradeoffs among factors) by comparison against competing funds, conflicting interests of an external nature. In the mathematical model, the performances of these interests are represented by constraint equations.

The third theme established by Murthi, Choi, and Desai was the DEA evaluation of mutual funds in (seven) separate groups of "investment objectives" (strategic categories). They then compared the average results among category groups. Finally, they compared group correlations with other performance measures, like the Sharpe ratio and Jensen's alpha.

The paper of McMullen and Strong [1998] was the second in this thread. It demonstrated the use of DEA to support investor decision making by including as costs (inputs), along with risk (standard deviation of returns), sales charges, expenses, and (as a barrier to investment) initial minimums. It also re-evaluated the funds excluding the minimum investment requirement to demonstrate the ease of adjusting the model to fit particular investor interests. Their paper demonstrated the multidimensionality of DEA by using three reward (output) measures, namely, returns over three increasing time horizons. It also took an investor focus in the selection of the sample—the funds, recommended in leading business magazines, represent a mixed group as to strategic category and size. It explained several of the many different evaluations included in a data envelopment analysis, especially the interpretation of slack values in supporting investors' choices.

The next paper in this sequence was the work of Basso and Funari [2001]. They included subscription and redemption fees (front-end and back-end loads, which are investor concerns) among the costs (inputs). However, the greater part of their innovations extends the other line of march in the literature.

In the fourth paper of this line, Choi and Murthi [2001] return to their analysis by employing a DEA model which exhibits variable returns to scale. They apply the new model to evaluate again the funds in seven strategic categories. They also examine the DEA results for implications for different financial market hypotheses, such as the better performance of smaller funds.

The most recent paper of this trend is Haslem and Scheraga [2003]. To analyze the large-cap funds from a Morningstar 500 listing, they propose a model whose costs (inputs) suggest a production theoretic analysis: asset allocations, expenses, value style of assets, and total fund assets; but five of the six costs are expressed as ratio measures. However, their single output measure is the Sharpe ratio; thus, the model can be rewritten with returns as the single output and the classic risk measure, σ, as a multiplicative factor of the linear combination of production costs. Murthi and Choi discussed multiplicative models, but the model which Haslem and Scheraga evaluate is equivalently multiplicative in the inputs. They then compute the mean scores of the funds partitioned into the three groups of efficient, “almost efficient,” and inefficient funds with respect to 29 input and output variables and other “profile” financial measures and determine the 12 which are significantly different across the efficiency partitions.

The historically second paper to apply the methods of data envelopment analysis to mutual funds also initiated the second approach in this field of research. The paper of

Morey and Morey [1999] established two themes in this second line of investigation. First, they apply the multidimensionality and methodological adaptability of DEA to resolve difficulties in the classical models of financial analysis by expanding the Sharpe analysis "within itself," and, thus, maintain the scope and standpoint of the Sharpe and Treynor analyses. Morey and Morey describe how DEA solves the portfolio selection problem by modeling the standard Sharpe ratio of the Markowitz model, R/σ. Their DEA approach facilitates the computation of the Sharpe scores and the results include realizable, individualized benchmark portfolios to guide the improvement of inefficient funds.

Second, they address the question of an appropriate time horizon, which is an issue for the use of the Sharpe measure. They demonstrate how DEA can treat different time horizons simultaneously or can evaluate them sequentially and conditionally, if a preference ordering is exogenously given. This exogenous ordering can subsequently be used to discriminate among all the funds initially found to be efficient when different time horizons are evaluated simultaneously.

These extensions by Morey and Morey are "within" the classic model in that they introduces no variables representing other factors or agents; the rewards (outputs) are all returns and the costs (inputs) are standard deviations (risk proxies) of the returns. The extensions involve an elaboration of the distribution of returns by the introduction of a preference ordering among different time horizons. However, other agents are still introduced by the comparative nature of DEA and efficiency scores based on performance relative to all the members of the sample analyzed. They test a sample of funds all from the same strategic category, aggressive growth.

In the next paper, Wilkens and Zhu [2001] extended this line of research by adding other characteristics of the returns distribution to the DEA model. They recognized that variance of returns (or equivalently, standard deviation) treated rewards and losses the same and that investors exhibited preferences for upside variances, or positive skewness in the returns distribution. They exploit the multicriteria capabilities of DEA to better characterize the returns distribution investors prefer. They include skewness and minimum return among the outputs along with returns; they include the percent of negative returns, as a measure of downside risk, among the inputs along with risk (standard deviation). Thus, their paper extends the Sharpe ratio/Markowitz model by adding variables representing other characteristics of the returns distribution, not other factors of production or stakeholders in the fund's performance. Again, other agents are represented in the constraints and introduced by the relative nature of the scores. With their model, they examine commodity trading advisors, rather than mutual funds.

The previously mentioned paper by Basso and Funari [2001] was also the next to contribute to this second line of analysis. To the fund returns, their analysis added as output the relative number of time intervals for which the fund's performance placed it among those stochastically dominant over competing funds according to the criterion of declining absolute risk aversion, a characterization of investor utility functions. This measure is a consequence of the fund's returns distribution relative to those of other funds. In addition to the factor costs (investor subscription and redemption fees) included among the inputs, their model used the multicriteria capability of DEA to add another characterization of the risk (besides variance) based on the returns distribution. They compare the results of their DEA models with the standard measures in the literature. For the single output of excess returns and single input of standard deviations, they show that their DEA results equal the traditional ratio measures normalized by the largest ratio score of the sample.

Two related papers are the last to apply DEA to mutual funds in this line of research. Joro and Na [2001] also employ the multicriteria capability of DEA to include additional characterization of the distribution of returns. Also recognizing the weakness of the assumption of symmetry in this distribution, they follow Wilkens and Zhu to add a measure of the distribution's third moment, the sample standardized skewness. They demonstrate how the different forms of the convexity constraint can model the different stipulations which permit lending and borrowing, lending but not borrowing, and neither lending nor borrowing. They also indicate that the model can be adjusted to change the focus of the evaluation, for example, to minimize variance for a given level of returns and skewness, or to maximize return and skewness while minimizing variance simultaneously. They examine large-cap, Morningstar five-star rated funds representing a mix of strategic categories.

In a subsequent paper, Joro and Na [2002] extend their earlier analysis to examine the same issues in greater detail. Additionally, they examine the effects of including among inputs variables (expense ratio and loads) which are not higher moments or other distributional characterizations. Finally, they compare DEA results with the results from direct computation of theoretical efficiency scores based on the standard mean-variance and mean-variance-skewness models.

The remaining discussion examines each paper individually in chronological order and further detail. In the first application of DEA to mutual fund performance, Murthi, Choi, and Desai [1997, p. 408] state that "[t]he three major issues in portfolio performance evaluation are the appropriate benchmark to be used for comparison, the role of market timing and the endogeneity of transactions costs." Then they review difficulties with three widely used measures of mutual fund performance. For Jensen's alpha, results vary according to the model used for the benchmark (*e. g.*, CAPM or APT); and market timing, which violates the model's assumption of constant beta, produces bias. The Sharpe ratio excludes transactions costs and therefore neglects their connection to performance, as does the Grinblatt and Titman [1993] measure based on the correlation between returns and changes in portfolio weights.

Instead, they propose an extension to the Sharpe ratio (R/σ) by adding a weighted sum of the transactions costs to the denominator: $R/(\Sigma\, \omega_i X_i + \nu\sigma)$, where R is the portfolio returns in excess of the risk-free rate, σ is the standard deviation of the returns, X_i are the transactions costs, and ω_i and ν are appropriate weights. Their measure is a ratio of output to input (specifically, a single output to multiple inputs), the form of the fractional program at the basis of DEA. They use the CCR[9] model of DEA to compute the weights and the index of efficiency.

[9] See the DEA discussion in Chapter 5. The name derives from the authors of Charnes , Cooper, and Rhodes [1978]; see also Cooper, Seiford, and Tone [2000], Chap.1, pp. 2-14, Chap. 2, pp. 21-39, and Chap. 3, pp. 41-83, for a full discussion of the CCR model.

Their new measure addresses the main issues in portfolio performance evaluation. (i) DEA is a non-parametric method which presupposes no theoretical model to provide the benchmark, which, instead, is given by the best performances among those funds being compared. (ii) Transactions costs are not neglected since DEA can simultaneously evaluate multiple inputs and multiple outputs. (iii) The results of DEA indicate relative importance among the inputs. Murthi and Choi find that almost all funds are efficient with respect to standard deviation of returns (their measure of risk), but that inefficiencies in costs vary according to the category of fund. Note that (i) also addresses the market timing issue since the DEA analysis obviates the theoretical requirement for a stable beta. Moreover, (iii) may inform market-timing analysis by results on variables which reflect such activity. For example, their finding that asset allocation funds are especially inefficient with respect to turnover suggests that further research might determine whether asset allocation fund managers are poor market timers or asset allocation is an investment approach not well suited to market timing.

Unlike most subsequent studies, Murthi, Choi, and Desai analyze mutual funds separately according to their "investment objective" (what is identified as the strategic category in this study). Then they correlate average DEA results for the separate categories with the Jensen, Sharpe, Morningstar rating, and beta measures, as well as with average costs and average net asset value.

In undertaking to "demonstrate the DEA methodology rather than to prescribe an alternative to mean-variance optimization," McMullen and Strong [1998, p.1] take the point of view of an individual investor, both with regard to the sample of mutual funds evaluated and the variables used to determine performance. Their sample comprises 135 common stock funds featured in magazine articles for individual investors. Furthermore, although Capital Market Theory asserts the sufficiency of the information in expected returns, variances, and covariances for the optimization of portfolios, McMullen and Strong observe that individual investors have other concerns which are not addressed by the standard theory. However, the methods of data envelopment analysis can simultaneously incorporate the numerous concerns of typical investors, such as recent and historical returns, volatility of returns, expenses and sales charges, and minimum initial investment requirements. Thus, the DEA model can be particularized to incorporate the multiple criteria of individual interests. In addition, it provides a single measure of evaluation for ease of interpretation and additional information to facilitate investor portfolio selection.

To reflect these investor concerns, they employ a model of variable returns to scale with three outputs—one-, three-, and five-year annualized returns—and four inputs—standard deviation of three-year returns (as a proxy for risk), sales charge, minimum initial investment, and expense ratio. They also note that the multicolinearity of the output measures, which would complicate multiple regression or factor analysis, is not a difficulty for DEA. However, they normalize the variable values on the sample means and standard deviations and rescale from zero by adding the absolute value of the minimums.

Their results indicate that 12 of the 135 funds popular with individual investors are efficient. They also identify those funds which are "near efficient" and display the relatively small amounts of input reductions or output augmentations which would be required to bring them to the efficiency frontier. To demonstrate the adaptability of DEA for individual needs, they re-evaluate the sample from the standpoint of large investors not constrained by minimum investment requirements with a model that excludes the initial investment variable.

Morey and Morey [1999] apply DEA to mutual funds to address two problems in assessing mutual fund performance: (i) conflicting and subjective weights for combining fund performances over different periods to yield the rankings provided by commercial services, such as Lipper Analytical Services and Morningstar; and (ii) rankings which vary according to the benchmark portfolio used for evaluation in CAPM- and APT-based procedures. Their sample of mutual funds comprised the 26 funds with 10 years' data (and, therefore, with five and three years' data, as well) as of 1995 from the aggressive growth category of funds.

For each fund which demonstrates less than optimal performance, DEA provides quantitative measures relating its performance to that of the optimal performers closest to it, measures which are based on weights computed to give that fund the best possible evaluation with respect to optimal performance. Thus, because DEA provides performance benchmarks from among the sample and, indeed, identifies individualized benchmarks for each sub-performing fund based on evaluating that fund in the best possible terms, the problem of benchmark bias is overcome. The potential improvements in performance for less-than-optimal funds are proportional in all measures of output or input simultaneously.

For situations in which performance during one time period is more significant than in another, it is possible to determine the maximum improvement in the most critical period, then, holding those results fixed, establish the possible improvements in the next most important period, and so on, for however many periods may be of interest. To address by means of data envelopment analysis the problem of conflicting and subjective weights in assessing the periods of performance evaluation requires a rank ordering of the importance or significance of the variables in the analysis. Accepting the Morningstar ranking of the importance of the time horizons (ten years more important than five years, and five years more than three), Morey and Morey apply the lexicographic, pre-emptive procedure in DEA to obtain a unique ranking and the maximum slacks for each fund.

In reviewing the succession since Markowitz of finance models designed to account for risk in the evaluation of portfolio returns, Wilkens and Zhu [2001] critique models addressed to the problem of time-varying risk due to market timing activities, which cause changes in portfolio betas or other proxies for risk. Ignoring the nonstationarity of the processes[10], rather, such models, which assume nonsymmetric returns distributions, regress on measures of skewness and kurtosis, regress on "style" (*i. e.*, allocation in the portfolio among asset types), or regress on factors for "strategy" (*i. e.*, general investment or trading approaches, such as, following trends, seeking spreads, or other systems). They argue that the factors in these models are all proxies for

[10] William Feller, *An Introduction to Probability Theory and Its Applications*, Volume II, Second Edition (New York: John Wiley & Sons; 1971), pp. 87-98; J. L. Doob, *Stochastic Processes* (New York: John Wiley & Sons; 1953), pp. 94-99; James D. Hamilton, *Time Series Analysis* (Princeton, NJ: Princeton University Press; 1994), pp. 45-46; Peter Kennedy, *A Guide to Econometrics*, Fifth Edition (Cambridge, MA: The MIT Press; 2003), pp. 319-328.

risk; that the models assume that the same functional relationships apply to all portfolios; and that regression models, which describe average behavior, are not appropriate for performance evaluation, which requires best performance benchmarks.

Against these inadequacies, they propose a DEA model which takes, as inputs, two risk measures (standard deviation and percentage of negative returns—a downside risk proxy) and, as outputs, return, skewness, and minimum return. Such a model presupposes no functional relationship among the factors, evaluates each individual portfolio against best actual performance (that is, benchmarks are efficient securities rather than risk factor proxies), and classifies portfolios according to the segment of the efficiency frontier containing their benchmarks.

Basso and Funari [2001] begin by drawing a parallel between the fractional program form (virtual outputs/virtual inputs) of data envelopment analysis and the ratio forms (output/input) of the standard portfolio performance measures: Treynor [1965] (r_j/β_j, that is, reward, or mean returns in excess of the risk free rate, $r_j = E(R_j) - r_F$, to volatility, or $\beta_j = \mathrm{Cov}(R_j, R_M)/\mathrm{Var}(R_M)$); Sharpe [1966] (r_j/σ_j, reward to variability, or $\sigma_j = \sqrt{E[R_j - E(R_j)]^2}$); and Ang and Chua [1979] ($r_j/\sqrt{HV_j}$, reward to the square root of the downside half-variance, or $HV_j = E(\min[R_j - E(R_j), 0]^2)$). Unlike the preceding ratio forms, Jensen's alpha [1968] is the intercept with the rate of return axis of a regression based on the CAPM model, $E(R_j) - r_F = \alpha + \beta[E(R_M) - r_F]$.

To begin, Basso and Funari show that the DEA equivalents of each of the standard ratio measures are generalized or normalized versions of the respective ratios.

For example, for a given portfolio, the DEA measure based on the single output of portfolio rate of excess returns and single input of standard deviation of portfolio returns equals the Sharpe ratio for that portfolio divided by the maximum of Sharpe ratios among all portfolios in the tested sample.

In their first set of analyses, Basso and Funari expand on Murthi, Choi, and Desai [1997] by including an additional risk measure as input. Their DEA models use a single output, portfolio returns, and multiple inputs—two risk components (the portfolio beta and either the standard deviation or the square root of half-variance of portfolio returns) and two cost components (subscription and redemption costs). DEA evaluations are computed for three groups, 24 stock funds (and an exchange index), 9 balanced funds, and 15 bond funds (and a treasury bill for the riskless investment). In a second set of analyses, Basso and Funari include an additional output, the number of subperiods in which a fund appears in the stochastic dominance efficient set relative to the total number of subperiods. They compute stochastic dominance based on decreasing absolute risk aversion, a subset of the third-order stochastic dominance rule.[11]

Although the standard measures of portfolio performance are sensitive to the time horizon for which performance is evaluated, Basso and Funari show that, under assumptions of stationarity and independence, instantaneous (as opposed to compounded) rates of return result in performance measures for one period which are proportional to those of another period according to the ratio of the periods. This consequence follows

[11] They employ an algorithm for stochastic dominance which is due to Vickson [1975] and their application of it is detailed in Basso and Pianca [1997]. See also Tehranian [1980]; for a review of stochastic dominance, see Levy [1992].

from the fact that for instantaneous (or logarithmic) return rates, the overall rate of return for a given period is the sum of the rates for equal-length subperiods. Thus, if for two periods, t_1 and t_2, $t_2 = \tau\, t_1$, then (Sharpe index for t_2) = ($\sqrt{\tau}$) (Sharpe index for t_1), and similarly for the Ang and Chua reward-to-half-variance index. Furthermore, (Treynor index for t_2) = τ (Treynor index for t_1), and similarly for the Jensen measure. Therefore, *rankings* based on these standard financial measures are unaffected.

They invoke the "units invariance" theorem[12] for the CCR model to show that the DEA results of their first set of evaluations (single output) are unaffected by the base time period used, and, therefore, the rankings also are unchanged. However, for the second set of evaluations, they surmise that DEA results may vary because the additional output variable for relative stochastic dominance may vary according to the subdivision of the overall time horizon.

As in the prior DEA mutual fund studies by Murthi, Choi, and Desai [1997], Choi and Murthi [2001] approach the task of evaluating performance differences according to the "objectives" (or strategic) category of funds. To begin, they also argue against the exclusion of transactions costs in the assessment of portfolio performance, citing, in particular, Grinblatt and Titman [1989] that such data reflects fund managers' ability to generate above 'normal' returns. Moreover, they argue for analysis of costs in order to evaluate economies of scale in fund returns. As in preceding studies, they note that DEA does not require a particular functional form or theoretical assumption (such as market

[12] For a demonstration of the "units invariance" theorem for CCR models in data envelopment analysis, see Cooper, Seiford, Tone [2000], p. 24; for other models with a "units invariance" property, see pp. 61, 97, 111, 228.

equilibrium) beyond convexity and monotonicity of the estimated production function and allows for the simultaneous evaluation of gross returns, costs and risk.

In order to capture a measure of scale effects, Choi and Murthi augment their extended Sharpe ratio model with the free variable u which effects the scale evaluation in the BCC[13] model of data envelopment analysis: $(\mathrm{R} - u)/(\Sigma\, \omega_i \mathrm{X}_i + \nu\sigma)$, where R is return, X_i are the costs, σ is the standard deviation of returns as a measure of risk, ω_i and ν are optimizing weights. The augmenting term u makes the efficient production surface piece-wise linear and causes the linear extension of the facet closest to the fund being evaluated to intersect the returns axis R in one of three possible cases: at a point less than zero, indicating the facet of the efficiency production surface represents increasing returns to scale; zero, indicating the facet represents constant returns to scale; or a point greater zero, indicating decreasing returns to scale. For each fund evaluated, the maximized ratio (the optimum of the objective function) is the efficiency of the fund relative to the best performance in the group with which it is evaluated; the optimized weights ω_i and ν determine the slope of the production surface facet nearest the fund; and the sign of u (negative, zero, positive) determines the returns to scale of the efficient production surface nearest the fund.

Choi and Murthi analyze their sample both in separate groups according to category, and jointly, in one group together. In examining the seven category efficiency

[13] See the DEA discussion in Chapter 5. The name derives from the authors of Banker, Charnes, and Cooper [1984]; see also Cooper, Seiford, and Tone [2000], Chap. 4, pp.85-113, for a development of the BCC model.

averages resulting from the joint analysis, they find them similar (.723 – .628) except for the exceptional average of the income fund group (.876). They attribute the significantly best return rate by aggressive growth funds (32.69% vs. the other groups, 14.93% – 20.95%) to the increasing returns to scale of the efficient production surface closest to 90% of the funds in that category.

They demonstrate that the DEA evaluation of the single output, return R, to the single input, risk σ, for a given fund equals the Sharpe ratio for that fund normalized by the ratio of the best performing fund in the group (assuming zero risk-free rate). In the separate category analysis, they find high correlation with Sharpe and Jensen measures and even higher correlation, when the DEA scores are computed with only return as output and risk as input. They argue that no significant correlation between category mean DEA scores and category mean net asset values, except for growth-income funds, explains the performance advantage of small funds by controlling for transactions costs and scale economies.

Finally, they report three conclusions based on slack values. First, the small average slack values for standard deviation among all the categories imply that most funds have mean-variance efficient portfolios. Second, the low average slacks for expense ratio among growth and aggressive growth funds imply efficient expenditures to produce high returns among the funds of those categories[14]. Third, "[l]oads and turnover ratios show very large slack variables across all categories, indicating that loads and

[14] Asset allocation funds may also qualify: see Choi and Murthi [2001] Table 6, p. 872

turnover ratios are the main sources of inefficiency scores."[15] Within each category, they find that no-load funds consistently exhibit higher average efficiency scores than load funds.

In Haslem and Scheraga [2003], the sample comprises the 80 mutual funds without missing data of the originally 84 large-cap funds in the Morningstar 500 for 1999. The objective is to identify the efficient and inefficient funds and to identify which financial variables differ significantly between the two resulting sets. Indeed, the funds are divided into three groups according to their DEA scores: efficient (1.00), "least inefficient" (0.90–0.99), and inefficient (less than 0.90).

To evaluate these funds, two groups of variables are defined: (i) the input and output variables of the DEA model which generates the performance-efficiency scores and (ii) the "profile variables" which are correlated with the scores of the resulting efficient, almost efficient, and inefficient groups. The input variables suggest a production theoretic model: (1) cash, as percentage of fund assets, "reflects differences in liquid and earning assets, including any fund efforts at market timing"; (2) expense ratio; (3) stocks, as percentage of assets, "reflects differences in basic stock/bond asset allocations"; (4) price/earnings ratio and (5) price/book ratio, both "reflect differences in value/growth investment style"; and (6) fund total assets, to reflect scale efficiency and as "surrogates for the market value of investor paid-in capital" which "represent opportunity costs in alternative uses."[16] However, the output variable is the Sharpe index. Thus, the

[15] Choi and Murthi [2001], p. 872.
[16] Haslem and Scheraga [2003], p. 42.

DEA results indicate how the input variables contribute to the widely used risk-return performance ratio.

Indeed, Haslem and Scheraga give all the variables, except total assets, as dimensionless ratio measures, although these may be reformulated as absolute quantities. Their model is (Sharpe index)/(%C+%X+%S+(P/E)+(P/B)+A), which, by using variable definitions, can be rewritten as $R/\sigma\ [(C/A)+(X/A)+(S/A)+(S/E_S)+(S/B_S)+A] = (RAE_SB_S)/\sigma\ [E_SB_S(C+X+S+A^2)+SB_S+SE_S]$, where R is return rate, σ is standard deviation of returns, C is total dollar amount of cash and liquid assets, X is total dollar amount of expenses, S is total dollar amount (market value) of assets in stocks, E_S is the total earnings of stocks, B_S is the total book value of stocks, and A is the total market value of assets. Although Choi and Murthi [2001] discussed multiplicative models, all previous models had been linear in the inputs and linear in the outputs.

Of the 80 large-cap funds, 27 (33%) are efficient, 22 (27%) are almost efficient, and 31 (39%) are inefficient. Of the efficient funds, 17 are classed as value funds, 9 as blend funds, and only 1 as a growth fund. Then, for each of the three groups, they examine the means of the fund values on 29 input, output, or other "profile" variables for significant differences among the groups.[17] Twelve of the variables exhibit significant differences among the groups. Four of these variables (bear market rank, three-year earnings growth, price/earnings ratio, and price/book ratio, the last two of which are input variables) are significantly different among all three groups. The eight remaining

[17] Based on the Tukey-Kramer test at the 0.05 level; see Haslem and Scheraga [2003], Kramer [1956], Duncan [1955].

variables (Sharpe index, Morningstar three-year risk rating, Jensen's alpha, beta, standard deviation of returns, portfolio turnover, bonds and other as percent of assets, and stocks as percent of assets—the first is the output variable, the last is an input variable, and σ is a component of the Sharpe index) are only significantly different between the efficient and inefficient groups. They find that the variables associated with the efficient large-cap funds characterize a conservative, value-oriented rather than an aggressive, growth style of investing.

The paper by Joro and Na [2001] applies DEA to compensate for some of the shortcomings of the models based on the mean-variance theory of Markowitz [1952]. They begin with a review of the studies which established that the utility theory of von Neumann and Morgenstern requires that either the returns distributions be Gaussian or that investors exhibit quadratic utility functions. On the one hand, portfolio returns are generally not normally distributed; and, on the other, investor preference for (positively) skewed returns distributions implies non-quadratic utility functions. Because the standard portfolio performance measures, like those of Sharpe, Treynor, and Jensen, derive from the CAPM, they inherit the same theoretical weaknesses.

Joro and Na endeavor to rehabilitate the mean-variance paradigm for portfolio performance evaluation by using DEA to effect a straightforward extension of the model from its expression as the ratio of the first to the second central moment of the returns distribution to the inclusion of the third central moment, skewness. Since investors prefer positive skewness (more variance to the upside), the measure is added to the outputs of portfolio performance and the new model becomes $(R+skw)/\sigma$. They motivate the need to account for skewness by citing empirical studies and by a theoretical development. The third order approximation based on a Taylor's series expansion of a generalized utility function around the mean of the portfolio returns exhibits the three desirable properties for utility functions proposed by Arrow and Pratt[18], namely, "(i) positive marginal utility for wealth, *i. e.*, nonsatiety with respect to wealth, (ii) decreasing marginal utility for wealth, *i. e.*, risk aversion, and (iii) non-increasing absolute risk aversion, *i. e.*, risky assets are not inferior goods."[19]

DEA provides a tractable method for making a natural extension of mean-variance analysis to include skewness. With DEA, both the output orientation (for a fixed level of variance (risk), maximize return and skewness) and the input orientation (for a fixed level of reward (return and skewness), minimize variance) are easily computed. In conformance with standard mean-variance practice, Joro and Na use the variance minimizing approach. Moreover, the CAPM model may variously permit or prohibit borrowing or lending at the risk-free rate. They also demonstrate that the varying stipulations on risk-free borrowing or lending are easily established in DEA by the nature of the convex combination constraint on the virtual weights. If the virtual weights, for which the optimizing program solves, are constrained only to be non-zero, then lending and borrowing are permitted; if the weights are also constrained to sum to not greater than 1, then lending, but not borrowing, is permitted; and if the virtual weights are constrained to sum to exactly 1, then neither lending nor borrowing is permitted.

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[18] See especially the essays "Exposition of the Theory of Choice under Uncertainty," pp. 44-89, and "Theory of Risk Aversion," pp. 90-120, in Kenneth J. Arrow, *Essays in the Theory of Risk-Bearing* (Amsterdam, NL: North-Holland Publishing Co., 1971); and John W. Pratt, "Risk Aversion in the Small and in the Large," *Econometrica*, Vol. 32, No. 1/2 (January/April 1964), pp. 122-136.

They test the group of 54 Morningstar five-star rated funds for which complete data are available. Thus, their sample represents a mixture of strategic categories. To their sample, they also add data on four major index funds which are frequently used as "benchmarks." They use the 90-day T-bill rate as a proxy for the risk-free rate. Their model assumes lending but not borrowing at the risk-free rate. Their mean-variance-skewness model yields two additional efficient funds as compared to the simple mean-variance model. The model is derived in terms of expectations of given probability distributions, but is realized in terms of sample statistics computed from the historical data of the sample funds. Because the model involves second power (variance) and, more so, third power (skewness) functions, it proves to be computationally expensive. Moreover, maximizing the skewness causes the feasible set to be nonconvex

In a subsequent paper, Joro and Na [2002] extend their earlier analysis to compare the results of several DEA formulations of the portfolio performance problem with the results of standard mean-variance formulations. They examine five DEA models which include, as outputs—mutual fund rates of returns (above the risk-free rate, for which they use T-bill rates); or returns and (sample standardized) skewness—and, as inputs—variance (of mutual fund rates of returns); variance and expense ratio; or variance, expense ratio, and loads.

They draw two main conclusions. As the models increase in variables, efficiency scores cannot deteriorate: they can only remain unchanged or improve. In some cases, this represents a benefit. Adding skewness to the model improves the results for a fund

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[19] Joro and Na [2001], p. 7.

which otherwise “gets penalized for its upside potential if just variance is used to measure risk.”[20] However, their methodology is indiscriminate and will accommodate less justifiable adjustments, such as allowing the tradeoff of more risk for good performance in less relevant or critical factors, like loads.

Their second main conclusion is that DEA results are always greater than the “true” mean-variance or mean-variance-skewness efficiency scores (except when both are efficient). This follows from the piece-wise linear character of the DEA frontier which approximates, and therefore lies below and to the right of, the true, curved frontier. Thus, equal scores result only where the DEA surface vertices coincide with the curved mean-variance frontier.

[20] Joro and Na [2002], p. 21.

Chapter 5 Methodology for Evaluation of Strategies by Data Envelopment Analysis

Section 1 Introduction

The methods of mutual fund analysis developed here are not based on the mean-variance/risk-return model. The methodologies to be demonstrated are instead based on data envelopment analysis (DEA). DEA is a specialized form of linear programming. Unlike other forms of linear programming, however, it is not directed to the formulation of *ex ante* plans. It is instead designed for the *ex post* analysis of empirical performance data for any kind of decision making organization (Decision Making Unit, or DMU) which transforms inputs into outputs. In the present study, the DMUs are mutual funds.

This method of analysis is based on the data of actual performance and does not require explicit statement of functional forms or specification of families of underlying statistical distributions. In this sense, it is an "empirically oriented" technique, as opposed to the customary statistical methods, which require formal, prior statement of the functional forms that are postulated or hypothesized to relate inputs to outputs. Moreover, while the usual statistical approaches emphasize "averages," which are statistical artifacts, the DEA measures are derived in a manner which provides evaluations individually for each entity and its associated observations.

Data envelopment analysis measures efficiency based on the ratio of outputs to inputs in a form that generalizes the single-output/single-input technical efficiency measure of engineering and economic production theory to a multi-output/multi-input measure without requiring that all factors be treated in the same units of measure. It is nonparametric; that is, it assumes no prior, underlying distribution function of the data, nor production function, nor specific functional form (such as are required in regression equations) to relate inputs to outputs or independent to dependent variables. In a sequence of optimizations, one for each individual in the analysis, DEA produces a piecewise linear production frontier based on the Pareto-Koopmans[1] efficient performers, rather than an estimate of central tendency of the group as a whole, as in a regression plane or moment value.

DEA is a specialization of mathematical programming based on a transformation of computationally difficult fractional programs into computationally tractable and

[1] "[A] Pareto optimum is a state where no consumer can be made better off without making another consumer worse off." Gerard Debreu, "Valuation Equilibrium and Pareto Optimality," *Proceedings of the National Academy of Sciences of the United States of America*, Vol. 40, No. 7 (1954), p. 588. Arrow and Hahn "use the term 'Pareto efficient' instead of the more common 'Pareto optimal' because the latter term conveys more commendation that the concept should bear…in any sense in which distributional ethics are involved. …[A]n allocation is efficient if there is no way of making anyone better off. The present definition leads to simpler results and avoids some special, odd cases." Kenneth J. Arrow and F. H. Hahn, *General Competitive Analysis* (San Francisco, CA: Holden-Day, Inc., 1971), p. 91. "[A]n optimum point, in [Pareto's] sense, is not a unique point. If transfers of income from one individual to another are arbitrarily imposed, there will be a new optimum point, and there is absolutely no way of deciding whether the new point is better or worse than the old." Paul A. Samuelson, *Foundations of Economic Analysis* (Cambridge, MA: Harvard University Press, (1947) enlarged ed., 1983), p. 214. Throughout Part I, Samuelson discusses in depth Pareto's work in the context of the historical development of economic theory. See also Gerard Debreu, "The Coefficient of Resource Utilization," *Econometrica*, Vol. 19, No. 3, (July 1951), pp. 273-292, esp. pp. 278-279; Werner Hildenbrand, "Introduction," pp. 1-29, esp. pp. 8-16, in Gerard Debreu, *Mathematical economics: Twenty papers of Gerard Debreu* (New York, NY: Cambridge University Press, 1983); O. Lange, "The Foundations of Welfare Economics," *Econometrica*, Vol. 10, No. 3/4 (July-October 1942): pp. 215-228. For the role of this concept in a Marxist characterization of the "subjectivist" character of economic analysis, O. Lange, *Political Economy, Vol. 1: General Problems* (New York: The Macmillan Company, 1963), esp. pp. 226-277. Pareto applied the concept to consumers; Koopmans adapted it to production. For Koopmans' reformulation, see Tjalling C. Koopmans, "Analysis of Production as an Efficient Combination of Activities," Chapter III, pp. 33-97, in Koopmans, ed., *Activity Analysis of Production and Allocation* (New Haven, CT: Yale University Press, 1951); T. C. Koopmans, "Allocation of Resources and the Price System," *Three Essays on the State of Economic Science* (New York, NY: McGraw-Hill Book Company, 1957), pp. 1-126; and A. Charnes and W. W. Cooper, "Chapter IX. Theory and Computations for Delegation Models of Activity-Analysis Type: K-Efficiency, Functional Efficiency, and Goals," pp. 288-325, of Charnes and Cooper, *Management Models and Industrial Applications of Linear Programming*, Vol. I (New York, NY: John Wiley & Sons, Inc., 1961), which develops a linear programming formulation of Koopmans' "activity analysis," the coordination through a price system of decentralized decision makers.

conceptually revelatory linear programs. The technique was first presented in 1978[2] as an optimized generalization of the Farrell[3] single-output/multi-input measure of efficiency.

Although it is possible to trace the notions of efficiency in the Western tradition back to Aristotle[4], the modern concepts stem from the work of Sadi Carnot in 1824 on heat engines and the second law of thermodynamics.[5] He determined that no conversion of heat energy could completely recover the total energy in the original state without ideal components or infinite devices. Efficiency is achieved by the most complete conversion of energy from one state to another, with the least loss of amounts not realized in the output of the conversion. Thus, to the extent that the total potential energy available in a given state is calculable and the energy of the resultant state is measurable, the ratio of resultant to potential energy is a measure of the rate of conversion. Since, except for limiting or ideal conditions, the resultant is less than the potential, the ratio in actual circumstances is always less than 1 and efficiency improves as resultants in practice approach their theoretical limit. In engineering, the definition is frequently given as efficiency = (work) / (input energy). Since both work and input are measured in the

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[2] A. Charnes, W. W. Cooper, E. L. Rhodes, "Measuring the efficiency of decision making units," *European Journal of Operational Research*, Vol. 2, No. 6 (November 1978), p. 429-444.
[3] M. J. Farrell, "The Measurement of Productive Efficiency," *Journal of Royal Statistical Society*, Series A (General), Vol. 120, Part 3 (1957), pp. 253-281.
[4] Aristotle, *Posterior Analytics*, Book II, Section 11, pp. - in Richard McKeon, ed., *The Basic Works of Aristotle*, New York, NY: Random House, 1941; see also *Physics*, Book II, Section 3, and *Metaphysics*, Book I, Section 3.
[5] For a discussion of the thermodynamic principles, see, for example, J. D. Fast, *Entropy: The Significance of the Concept of Entropy and Its Applications in Science and Technology*, 2nd ed. (Eindhoven, NL: Philips Technical Library, 1968), pp. 6-39; for discussion of the relation to economics, see Nicholas Georgescu-Roegen, *The Entropy Law and the Economic Process* (Cambridge, MA: Harvard University Press, 1971), pp. 127-130, 276-283.

same units (typically, joules), the efficiency ratio is dimensionless.

Because engineering applications often seek to build devices or change to a state of matter with a known quantity of energy in the resultant, the notion of efficiency is sometimes restricted to the minimization of the input energy required for the target resultant energy. However, the fundamental objective is the minimization of the per unit energy cost of conversion, which may also be attained by the maximization of the resultant energy of conversion from a given level of input energy. In either case, an improvement in efficiency is given, on a per unit basis, by an increase in the ratio of output to input.

By analogy, in the economic case, for a specified target of output (for example, the optimum level of production for firms in a perfectly competitive market, *i. e.*, production at the level where short-run marginal cost equals short-run average cost), profit is maximized by the minimization of cost, which is achieved by reducing the amounts of inputs in the face of fixed factor prices. Alternatively, if input amounts are restricted below the level needed for optimum output amounts, the firm improves efficiency by producing at the maximum rate of conversion for the level of inputs available.

Section 2 Early Definitions of Economic Efficiency

In the period following the Second World War, the advances in the models and methods of economic analysis which accompanied the growth in the levels, extents, and complexities of economic activity made possible the development of new concepts and improved measures of economic productivity and efficiency. Farrell[6] decried the inadequacies of indices (at the time, generally based on the ratio of the money value of production to the money value of inputs) and especially the distorting effects of over-reliance on the partial productivity measure, average labor productivity, which related money value of total output to the single input, labor. The economic and political competition between the socialist and capitalist economies also motivated the development of measures which could be used to evaluate the economic efficiencies of the competing systems. Both Debreu[7] and Farrell asserted the applicability of their measures to the comparison of different economic systems.

In the context of welfare economics and utility theory, Debreu developed an abstract and general measure of economic efficiency, which he called "the coefficient of resource utilization," ρ. By ignoring the cases of saturation (*e. g.*, satiation or congestion[8]), Debreu could establish the equivalence of the problem in terms of utility satisfaction to one in terms of quantities of commodities. By relying on the convexity of sets of vectors of resources feasible for the production of the set of vectors of commodities which provide at least the minimum required level of utility, without using continuity or differentiability conditions, he derived ρ, "the smallest fraction of the actually available physical resources [actual activity vectors] that would permit the achievement of" the required level of utility. This construct also produces vectors **p**

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[6] Farrell, "Measurement of Productive Efficiency," pp. 253-254.
[7] Debreu, "Coefficient of Resource Utilization," pp. 273-292.
[8] For a DEA approach to the condition of congestion, see William W. Cooper, Honghui Deng, and Lawrence M. Seiford, "Chapter 7: Congestion," pp. 177-201, in William W. Cooper, Lawrence M. Seiford, and Joe Zhu, *Handbook on Data Envelopment Analysis* (Norwell, MA: Kluwer Academic Publishers, 2004). For the technical definition, see p. 179.

normal to the supporting hyperplanes through the optimal points on the frontiers of the feasible sets in commodity space. For optimal activity vector $\mathbf{z}^0$ and actual activity vector $\mathbf{z}^*$, the actual loss is given by $\mathbf{z}^0 - \mathbf{z}^* = \mathbf{z}^0 (1 - \rho)$, and its value is given approximately by $\mathbf{p}^0 \cdot \mathbf{z}^0 (1 - \rho)$, where $\mathbf{p}^0$ is an actual price vector used in place of the intrinsic vector $\mathbf{p}^*$.

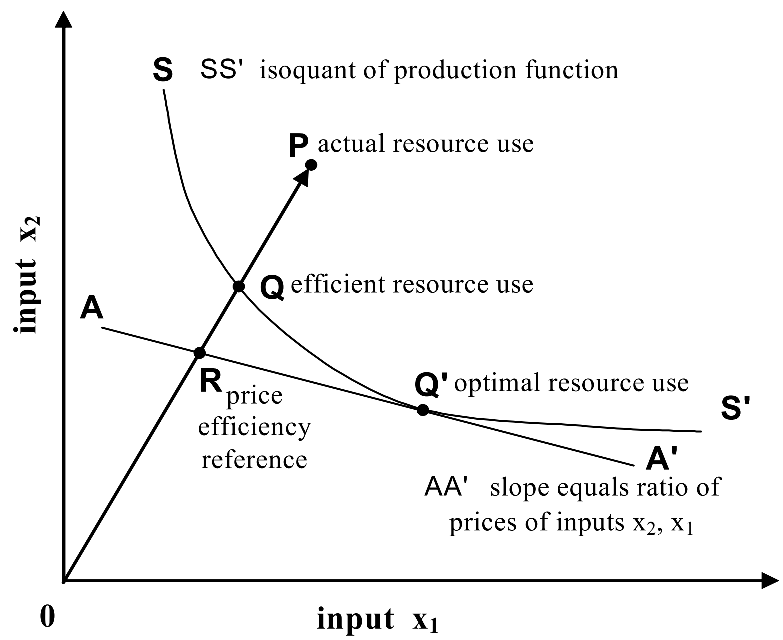

Figure 1 Farrell efficiency example

After Debreu, Farrell approached the problem with the same basic idea of a ratio of an optimal production vector to an actual production vector. In Figure 1, which represents the initial example from Farrell, all points to the right of the efficient production function isoquant **SS'** produce the same output as that represented by the isoquant. Technical efficiency of production at the point **P** is defined by the ratio **0Q/0P**. The point **Q** represents the same output level as **P** but with a proportional reduction of the inputs. The family of lines all with a slope equal to the ratio of the prices of the two inputs x_2 and x_1, has a member **AA'** which is tangent to the isoquant **SS'** at the point **Q'**. Both **Q** and **Q'** represent the same output and both are technically efficient, but the cost of production at **Q'** is the fraction **0R/0Q** of the cost at **Q**. This ratio is defined as the price efficiency of **Q**. Then the overall efficiency of the firm at **P** is the product of the technical efficiency and the price efficiency, given by **0R/0P** = (**0Q/0P**) (**0R/0Q**).

This initial model is based on a number of assumptions which Farrell successively relaxes in the subsequent development. Most importantly, it assumes that the efficient production function is known. Furthermore, it assumes constant returns to scale, a single output, and (only) two inputs. In replacing the assumed production function, Farrell rejects a function theoretically derived from engineering considerations—the manufacturing process is, in general, too complex to account for all factors (many of which are indirect) and their interactions—in favor of one based on the best empirical performances. This empirical function is determined by plotting the points of firm performance in a scatter diagram and connecting those points in the convex hull which meet two assumed requirements—the resulting piecewise linear isoquant (i) is convex to the origin and (ii) nowhere has positive slope (*i. e.*, there is no saturation or congestion)—while retaining (temporarily) the original assumption of constant returns to scale. To assure the second requirement, he added to the points of observation two points at infinity, $(0, \infty)$ and $(\infty, 0)$, so that the segments at each end of the isoquant are virtually parallel to an input axis, *i. e.*, they have infinitesimal slopes.

Farrell cautions against an over reliance on the motivating diagrams, noting that they must be abandoned to extend the method into higher dimensions. He emphasizes that the key to his method is the mathematical derivation which will allow the efficiency of a firm to be determined by comparison to an efficient hypothetical firm which uses factors in the same proportion as the firm under evaluation because they both belong to the same ray from the origin. Such a hypothetical firm is constructed as a weighted average of two observed firms on the isoquant, lying on either side of the intersecting ray, the weights chosen so that the hypothetical benchmark has the necessary factor proportions.

When generalized for n inputs and m outputs, the efficient isoquant of the two input example, constructed from line segments defined by pairs of observed points (and points at infinity) becomes a piecewise-linear efficiency surface S defined by facets—segments of $n+m-1$-dimensional hyperplanes. Each facet is defined by $n+m$ of the observed points and corresponding points at infinity, $(\infty, 0,\ldots, 0)$, $(0, \infty,\ldots, 0)$, …, $(0,\ldots, 0, \infty)$, and the origin, in the case of constant returns to scale. Any observed point P_k is defined by its n-vector of inputs $\boldsymbol{x}_k$ and m-vector of outputs $\boldsymbol{X}_k$ as $P_k = (\boldsymbol{X}_k, \boldsymbol{x}_k)^{\mathsf{T}}$ and can be represented by $n+m$ points in the $m+n$ linear equations

$$\begin{aligned} [\boldsymbol{X}_i, \boldsymbol{X}_{i+1},\ldots, \boldsymbol{X}_{i+m+n-2}, \mathbf{0}]\,\boldsymbol{\lambda} &= (\boldsymbol{\lambda}^{\mathsf{T}}\mathbf{e})\,\boldsymbol{X}_k \\ [\boldsymbol{x}_i, \boldsymbol{x}_{i+1},\ldots, \boldsymbol{x}_{i+m+n-2}, \mathbf{0}]\,\boldsymbol{\lambda} &= \boldsymbol{x}_k \end{aligned} \tag{1}$$

where $\mathbf{e}$ is the summation vector (that is, the vector all the components of which are 1). These points define a facet of the efficient surface if and only if $\boldsymbol{\lambda}^{\mathsf{T}}\mathbf{e} \geq 1$ for every P_k in the set of observed points. The efficiency of P_k is defined as max $[1/(\boldsymbol{\lambda}^{\mathsf{T}}\mathbf{e})]$ as $\boldsymbol{\lambda}$ varies over all facets of S.

The system of equations (1) includes the origin **0** because it still assumes constant returns to scale. In the section called "Increasing and Diminishing Returns to Scale," Farrell describes two cases which can result from the final generalization which relaxes this assumption, namely, economies of scale and diseconomies of scale. For diseconomies, he says, the only adjustments required are in the definition of a facet, namely, (i) disallow negative weights for the origin, and (ii) allow any set of $n+m$ points, not necessarily including the origin.

For economies of scale, his solution is much more complex: (i) divide "the observations into groups of roughly equal output"; (ii) apply the method to each group separately (on the assumption "that returns are constant within a group to a sufficient degree of approximation") to "yield a different efficient isoquant for each level of output"; and (iii) compare the several isoquants to "show the extent and nature of the economies of scale."[9] Farrell concedes that this procedure requires a substantially greater number of observations than the original method. However, he argues, these greater number of observations are likely to be available in those cases where economies of scale are important—measuring efficiency of firms or plants. Moreover, when greater numbers of observations are not available—as in measuring the efficiency of an industry—economies of scale are less likely to be important, because

> the relevant economies of scale are those where a larger industry permits greater specialization by firms; for those 'economies of large-scale industry' that take the form of cheaper inputs are irrelevant, while it is unlikely that the method will be applied to industries that are too small to permit firms of

[9] Farrell, "Measurement of Productive Efficiency," p. 259.

optimum size.[10]

This approach is insufficient in a number of ways. Perhaps most importantly, in fact, Farrell's device does not relinquish the assumption of constant returns to scale. It only transforms the original problem into a sequence of like problems. Moreover, the nonlinear production function with varying returns to scale will not in general be well represented by a succession of constant returns to scale problems, especially for coarse partitions of the output range or of the input range over which they manifest.

In another type of difficulty, his diagrams of (single input, single output) production functions which illustrate the deviations from constant returns to scale are labeled "diseconomies of scale" and "economies of scale" and show curves enveloping a convex area and a concave area, respectively. This formulation results from his argument that constant returns to scale imply a linear production function which intersects the origin.[11] First, economies of scale and diseconomies of scale must be distinguished from

[10] Farrell, "Measurement of Productive Efficiency," p. 259.

[11] Farrell's view is explicated in, for example, Gerard Debreu, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium* (New York: John Wiley & Sons, Inc., 1959), pp. 39-42, where definitions are given in the setting of a commodity space, $\mathbf{R}^l$, which comprises a dimension for each output and each input, with the outputs represented by positive numbers and inputs by negative numbers. For any possible production plan y_j belonging to Y_j, the production possibility set of the *j*th producer, and for $t \in \mathbf{R}$, then Y_j exhibits *non-decreasing returns to scale* if $t > 1$ and $t y_j \in Y_j$; *non-increasing returns to scale* if $0 \leq t < 1$ and $t y_j \in Y_j$; and *constant returns to scale* if $t \geq 0$ and $t y_j \in Y_j$. This last condition leads to a production function defined by hyperplanes through the origin. The significance of this type of production function is due to its role in equilibrium theory: "…if factor prices are singularly given, so that the firm can just break even, then its maximum-profit scale is indifferent albeit its input proportions used are the same cost-minimizing ones at every scale. Perfect competition is most robustly viable when constant returns to scale obtains." Samuelson, *Foundations*, pp. 464-465. See also Ronald W. Shephard, *Theory of Cost and Production Functions* (Princeton, NJ: Princeton University Press, 1970), in which the model is more detailed and elaborated. Shephard's analysis employs a homothetic transform $F(\cdot)$ of the production function $\Phi(\mathbf{x})$ which is homogeneous degree 1 [see note 12 below] (pp. 20-36) and the major development on returns to scale (pp. 255-260) is based on the cost and benefit functions. One difficulty of a homogeneous, constant returns to scale production function *f* is through the profit function. Given the vector $(p, \mathbf{w})$ comprising the components of a single output price, p, and multiple input costs, $\mathbf{w}$, such that optimal profits, π^*, are positive, that is, $p f(\mathbf{x}^*) - \mathbf{w} \cdot \mathbf{x}^* = \pi^* > 0$, then, together with the other assumptions of the model, constant

increasing returns to scale and decreasing returns to scale, respectively. The former refer to the rates of change in the *cost* of production associated with proportional expansions in the scale of production, whereas the latter refer to the rates of change in the *amounts* of output associated with proportional expansions in the scale of production.[12] Returns to scale—as the source of internal economies of scale and the basis of technical, productive efficiency—have a major impact on total economies of scale, but they are not identical. Economies of scale usually include factors which Farrell explicitly excludes in assessing the efficiency of industries, namely, the factor price advantages which accrue to buying in greater volume by individual firms which increase their scale of production and the external economies of scale which shift the entire industry production function downward.

Second, both of Farrell's illustrative diagrams of production functions (convex for diseconomies and concave for economies of scale) appear to exhibit increasing, decreasing, and constant returns to scale over different ranges of input. In contrast, the following definition relates the response of the production technology in output amounts to a proportional change in the amounts of inputs: If output increases (or decreases) in the same proportion as a proportional increase (or decrease) in inputs, the returns to scale are constant; if output changes in a greater proportion than the proportional change in inputs, returns to scale are increasing; and if output changes in a smaller proportion than

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returns to scale of production function f implies that profits are unbounded, since, for real $t > 1$, $p\,f(t\mathbf{x}^*) - \mathbf{w}\cdot(t\mathbf{x}^*) = t\pi^* > \pi^*$; see Hal R. Varian, *Microeconomic Analysis* (New York: W. W. Norton & Co., 1978), pp. 12-20, and Shephard, *Cost and Production Functions*, pp. 21-22, 29-30.

[12] G. Bannock, R. E. Baxter, and R. Rees, "Diseconomy," p. 119, "Economies of scale," pp. 135-137, and "Returns to scale," pp. 354-355, in *The Penguin Dictionary of Economics* (London, UK: Allen Lane, (1972) 1977).

the proportional change in inputs, returns to scale are decreasing.[13] A nonlinear (for example, piece-wise linear) production function may exhibit all three types of returns over different ranges of inputs.

Moreover, according to Farrell's definitions, a linear production function (even one intersecting the origin) need not exhibit constant returns to scale, since its rate of returns to scale will depend instead on the slope of the production function. Although Farrell formally relaxes the single output and constant returns assumptions at the conclusion of generalizing his method, the entire prior development relies on them, as does his major example. Furthermore, the analysis of output production and input factor functions must be clearly distinguished from the profit maximization and production cost minimization functions, especially when all outputs are combined into one product and the aggregate measured in money terms, as in Farrell's major example of United States agricultural production.

Finally, for Farrell, the relaxation of the single output and constant returns assumptions also entails a choice between two types of technical efficiency measures. A specified level of output would have been efficiently produced with only the fraction e_1 of the inputs actually consumed; or a specified level of inputs would have efficiently yielded $1/e_2$ times the amount of output actually produced. In data envelopment analysis, these two approaches are called input orientation and output orientation, respectively.

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[13] James M. Henderson and Richard E. Quandt, *Microeconomic Theory: A Mathematical Approach*, 2nd ed. (New York: McGraw-Hill, Inc., 1971), pp.79-81. In the most simple case, a production function f, homogeneous of degree k, that is, $f(t\boldsymbol{x}) = t^k f(\boldsymbol{x})$ for input vector $\boldsymbol{x}$ and non-negative real t, exhibits returns to scale which are increasing if $k > 1$, constant if $k = 1$, and decreasing if $0 \leq k < 1$.

Furthermore, since any kind of production function may be evaluated from an input or output orientation, this question was at issue from the initial development of Farrell's method, that is, even under "constant returns to scale" assumptions. However, the open choice was obscured by the single output, constant returns isoquant formulation of his problem, which made the reduction of excess input usage, e_1, computationally easier and seemingly "natural" in his simplified problem setting. In DEA, the direction of efficiency improvement (input reduction or output augmentation) is stipulated during problem formulation and is informed by the nature of the activity under evaluation and the purpose of the analysis. Indeed, such a choice must be settled whether the DEA model determines a linear production function with a fixed rate of return to scale, as in the CCR model, or a piece-wise linear function with varying returns to scale, as in the BCC model. (Both models are explained in the following sections.) Furthermore, DEA also supports other orientations, such as, for example, the shortest distance to the efficiency surface, which may require simultaneous adjustments of both reduced input use and increased output production.

The final adjustment Farrell makes is the inclusion of "quasi-factors," which are inputs to the production process not usually counted among the factors of production. Farrell cites air, water, climate, and location as examples of one type of quasi-factor and thickness of coal seam as another. He advocates explicitly including the first type (zero-price inputs like air and water) because they function as ordinary factors and because such a model is ready to accommodate increases in their prices (as is common today when air and water, especially of stipulated quality, are often costly, not free, inputs).

Besides a vague suggestion of *ad hoc* treatment, he does not discuss the examples of climate and location. In data envelopment analysis, however, these types of factors can be included in a standard manner, called non-discretionary factors, which appear in the constraint system but not in the functional to be optimized. The DEA method can account for all significant factors which affect performance but over which the manager has no control or discretion, including physical factors like weather, economic factors like interest rates, or social factors like the primary language of students.

For the example of coal seam thickness, Farrell advocates an approach similar to his adaptation for economies of scale: "divide the observations into groups homogenous (to a desired degree of approximation) in the quasi-factor."[14] If the quasi-factor varies continuously in quality, he proposes that a method analogous to his treatment of diseconomies of scale be employed. Data envelopment analysis can easily comprehend both discrete and continuous discretionary and nondiscretionary factors without resort to anfractuous *ad hoc* devices.

After the generalization of his method, Farrell offers interpretations and caveats regarding its application. In particular, he is concerned that differences among firms in the average quality or the distribution of qualities of factors will affect technical efficiency scores attributed to management. He proposes that measurable differences in factor quality, in analogy with economies of scale and quasi-factors, can be used to define a larger number of relatively homogeneous factors of production. As indicated previously, data envelopment analysis obviates many of the problems faced by Farrell's

[14] Farrell, "Measurement of Productive Efficiency," p. 259.

method and is much more capable of appropriately modeling heterogeneity of factor quality. However, financial factors are uniform in quality because money is fungible and shares or bonds of the same type and class from a given firm are equivalent, that is, because none of their essential qualities depends on the physical forms which signify or represent them. Thus, the use of financial factors, which are available in the same quality to all mutual funds, makes this issue moot for the present study.

Farrell also remarks on the difficulties in applying his method to the evaluation and comparison of whole industries. His first concern is for the effect of using aggregated industry data on the estimation of the true production function. Again, he asserts that industry analysis can forego the special treatment for economies and diseconomies of scale since an industry's efficiency encompasses and reflects the extent to which it accommodates firms of optimum size. However, he does feel that evaluations of industries will be more difficult to accomplish than those of firms because of the lack of availability and comparability of data and because of the much smaller number of observations. He proposes, therefore, a new measure called the "industry structural efficiency" which compares an industry's overall performance with the efficient production surface of its best practice firms. (The exact procedure is not detailed.) He acknowledges some disadvantages; for example, an industry which comprises uniformly inefficient firms will score a higher "structural efficiency" than one which contains both efficient and inefficient firms. Nevertheless, it provides a comparison of some aspects of efficiency even when the lack of comparable data precludes the isoquant comparison method; it can supplement comparisons of isoquants generated from poor quality data;

and, most importantly, it allows the comparison of efficiencies of different industries and, furthermore, the comparison of different economic systems. While not fully implementing this idea, Chapter 6 of this study does analyze several summary measures of how each investment strategy category performed overall relative to the performance of its best practice funds.

Farrell is also concerned with the degree to which measurement errors will bias results. He argues that if efficiencies vary more than the size of measurement errors, the bias will be negligible. Another advantage of data envelopment analysis is the extremely powerful techniques of sensitivity analysis, which can determine precisely how much data values may vary before affecting efficiency results. These techniques are not demonstrated in this study but will contribute to more comprehensive, future research.

In reviewing previous methods, Farrell criticizes index of efficiency measures because combining the components of input (and output) vectors into a weighted average scalar index is equivalent to valuing the components at prices proportional to the weights. If prices are not uniform throughout the industry, the technical efficiency of at least some firms will be evaluated by prices which they do not face. This is another limitation which is overcome by data envelopment analysis, since in DEA the combination of input and output components is accomplished by virtual weights which are optimized for each firm individually.

Before the presentation of a detailed example of his method, Farrell discusses the computational issues. His major example involves multiple inputs, but only one output and an assumption of "constant returns to scale." His general computational procedure is a direct implementation on an early computer of the method previously described: (i) solve for $\boldsymbol{\lambda}$ with all possible combinations of n points ($n = 4$) $[\boldsymbol{x}_i, \boldsymbol{x}_{i+1}, \ldots, \boldsymbol{x}_{i+n-1}]\, \boldsymbol{\lambda} = \boldsymbol{x}_k$, and (ii) test each set of n points to determine if it satisfies $\boldsymbol{\lambda}^{\mathsf{T}} \mathbf{e} \geq 1$, for every point P_k belonging to the set of observations. For m observed points ($m = 48$), the ${}_{n+m}C_n$ matrix inversions and the same number of matrix multiplications proved a significant computational load for the early computer EDSAC.

More problematic were the computational difficulties with points at infinity: the use of sufficiently large values for infinity was beyond the capacity of the computer and the use of appropriately bordered lower order matrices for facets with infinite elements was too complex to program for the available computing resources. Farrell avoided these difficulties by computing facets for observed points only (thereby also greatly reducing the overall computational load) and then using a technique of computing in lower dimensional space those facets with points at infinity. Farrell then discusses two other computational approaches, which he calls more sophisticated because they involve a reduced computational burden. In the first, start with an individual, efficient facet (available by inspection of the data) and successively compute the adjacent facets until the entire efficiency surface is determined. In the second method (which avoids matrix inversions altogether), start with the efficient surface associated with a small number of observed points and adjust the surface for each additional point successively introduced.

At the beginning of his paper, Farrell acknowledges a debt of inspiration to the methods of activity analysis,[15] but does not actually employ any of them. Moreover,

[15] He cites in particular Koopmans, ed., *Activity Analysis*.

following the paper's presentation, discussant A. J. Hoffman offers two revealing comments. He identifies the second of the more sophisticated, proposed computational procedures as the "double description method," attributable to Motzkin, Raiffa, Thompson, and Thrall.[16] The first of the more sophisticated methods, suggests Hoffman, is most efficiently implemented as a linear programming problem using the (then-recent) dual simplex method of C. E. Lemke.[17] As will become apparent in the review of data envelopment analysis, the methods of linear programming, and DEA in particular, solve or avoid many of the modeling and computational problems which made implementation of Farrell's theoretical approach difficult in practice.

Section 3 Data Envelopment Analysis

Consistent with the notion of Pareto-Koopmans efficiency, as described in economics, DEA evaluates each entity, called a decision making unit (DMU), in a manner that accords its performance the best possible evaluation, determined relative to the performances of all other DMUs but without requiring *a priori* choices of or subjective evaluations for weights of combination. A sample of n entities, or DMUs, to be analyzed, where for each, m is the number of inputs and s is the number of outputs, is

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[16] T. Motzkin, "The Double Description Method of Maximization," *Notes of Seminar on Linear Programming at the Institute for Numerical Analysis, National Bureau of Standards* (Los Angeles, CA, December 1950); and T. S. Motzkin, H. Raiffa, G. L. Thompson, and R. M. Thrall, "The Double Description Method," pp. 51-73, in H. W. Kuhn and A. W. Tucker, eds., *Contributions to the Theory of Games II*, Annals of Mathematics Studies, No. 28 (Princeton, NJ: Princeton University Press, 1953). This method, which was first proposed in Motzkin's dissertation (T. S. Motzkin, *Beiträge zur Theorie der linearen Ungleichungen*, Basel, 1933), was the first explicit description of an incremental, vertex enumeration algorithm for convex polyhedra. It was also overlooked later by many researchers in "computational geometry," but, rediscovered and refined, it is used in many fields today. See David Avis, David Bremner, and Raimund Seidel, "How good are convex hull algorithms?" *Computational Geometry*, Vol. 7, Issues 5-6 (April 1997), pp. 265-301.

[17] C. E. Lemke, "The Dual Method of Solving Linear Programming Problems," *Naval Research Logistics Quarterly*, Vol. 1, No. 1 (1954), pp. 36-47.

represented as a cluster of *n* points in an ($m + s$)-dimensional vector space. Those DMUs which define the upper surface, or efficiency frontier, of the cluster are the best performers. Those away from that surface lie at the shortest distance (which represents their relative inefficiency) from the efficient DMUs they most resemble and which constitute their peer group for comparison. In a series of evaluations, one for each DMU, DEA determines multipliers (*i. e.*, weights) which combine a DMU's multiple outputs and multiple inputs into a ratio of a single "virtual output" to a single "virtual input" which gives its performance a best value relative to the performances of all other DMUs.

By projecting the points representing the inefficient performers onto the efficiency frontier, the surface thus generated represents an approximation of the surface of most efficient production possibilities. This representation has, *inter alia*, two distinct advantages. First, since it is based on actual performance, the potentials it represents are realizable and the prescriptions that may be derived for inefficient performers are operationalizable. Second, again since it is based on actual performance and not forced to fit an *a priori* functional form or probability distribution, the shape and other characteristics of the surface reflect the actual economic and technological conditions of the activity under investigation.

Farrell closes the introduction to his paper with the assessment that the "measures developed are intended to be quite general, applicable to any productive organization…."[18] The first readily practicable methodology which encompassed Farrell's general approach and realized such broad applicability was reported in the paper

[18] Farrell, "Measurement of productive efficiency," p. 254.

by Charnes, Cooper, and Rhodes in 1978.[19] Their work established a new definition of efficiency and demonstrated optimization solution procedures for the evaluation of not-for-profit entities in public programs. However, the paper also demonstrated that their methodology, by subsuming the work of Farrell on efficiency and of Shephard (and others) on cost and production functions, applied to commercial firms and industries as well.

Analogous to the concept of efficiency in engineering, Charnes, Cooper, and Rhodes proposed a "measure of the efficiency of any DMU…as the maximum of a ratio of the weighted outputs to weighted inputs subject to the condition that the similar ratios for every DMU be less than or equal to unity."[20] This is formalized as

[19] Their paper, "Measuring the efficiency of decision making units," was preceded by A. Charnes, W. W. Cooper, and E. Rhodes, "Expositions, interpretations, and extensions of Farrell efficiency measures," Management Sciences Research Group Report (Pittsburgh, PA: Carnegie-Mellon University School of Urban and Public Affairs, 1975); A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units with some new production functions and estimation methods," Center for Cybernetic Studies Research Report CCS 276 (Austin, TX: University of Texas Center for Cybernetic Studies, 1977); and E. Rhodes, *Data Envelopment Analysis and Related Approaches for Measuring the Efficiency of Decision Making Units with an Application to Program Follow Through in U. S. Education* (Pittsburg, PA: Ph. D. dissertation, Carnegie-Mellon University School of Urban and Public Affairs, 1978). Note also A. Charnes and W. W. Cooper, "The non-Archimedean CCR ratio for efficiency analysis: A rejoinder to Boyd and Färe," *European Journal of Operational Research*, Vol. 15, No. 3 (March 1984), pp. 333-334, for a clarification of a sometimes misunderstood aspect of the methodology. See also A. Charnes, W. W. Cooper, and E. Rhodes, "Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through," *Management Science*, Vol. 27, No. 6 (June 1981), pp. 668-697. For the history of the development of DEA, see A. Charnes and W. W. Cooper, "Preface to Topics in Data Envelopment Analysis," *Annals of Operations Research*, Vol. 2 (1985), pp. 59-94; Abraham Charnes, William W. Cooper, Arie Y. Lewin, and Lawrence M. Seiford, "Introduction," pp. 3-21, in *Data Envelopment Analysis: Theory, Methodology, and Applications* (Norwell, MA: Kluwer Academic Publishers, 1994); and William W. Cooper, Lawrence M. Seiford, and Kaoru Tone, *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software* (Norwell, MA: Kluwer Academic Publishers, 2000), esp. pp. 33, 68-70, ff.

[20] Charnes, Cooper, Rhodes, "Measuring the efficiency of decision making units," p. 430.

$$\max\ \zeta = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

subject to (2)

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1; \quad j = 1, \ldots, n,$$

$$u_r, v_i \ \ge\ 0; \quad r = 1, \ldots, s; \quad i = 1, \ldots, m,$$

where the y_{rj} are the outputs and x_{ij} the inputs of the jth DMU, *i. e.*, the known performance data, and the u_r and v_i are the virtual weights provided by the solution of the program. Such a program needs to be solved for each of the n DMUs. The constraint system remains the same for all n solutions, but the functional changes as each DMU is evaluated in turn, with the subscript 0 in the functional used to indicate the specific DMU being evaluated. Although this form is felicitous for its direct representation of the notion of efficiency as a ratio of outputs to inputs and for its specification of the maximum possible ratio as 1, it entails computational difficulties.

However, the solution to these computational problems by means of variable transformations had already been demonstrated by Charnes and Cooper[21] before the development of data envelopment analysis. Assuming the regularity of the constraints of

[21] A. Charnes and W. W. Cooper, "Programming with Linear Fractional Functionals," *Naval Research Logistics Quarterly*, Vol. 9, Nos. 1 & 4 (September-December 1962), pp. 181-186, and A. Charnes and W. W. Cooper, "An explicit general solution in linear fractional programming," *Naval Research Logistics Quarterly*, Vol. 20, No. 3 (1973), pp. .

program (2) above, that is, in vector notation, for

$$\max \ \zeta = \mathbf{u}^{\mathsf{T}}\mathbf{y}_0 \,/\, \mathbf{v}^{\mathsf{T}}\mathbf{x}_0$$

$$\text{subject to} \qquad (3)$$

$$\mathbf{u}^{\mathsf{T}}\mathbf{y}_j \,/\, \mathbf{v}^{\mathsf{T}}\mathbf{x}_j \le 1, \quad j = 1,\ldots, n$$

$$\mathbf{u}, \mathbf{v} \ge \mathbf{0}\ ,$$

the solution set $\mathbf{W} \equiv \{\mathbf{w}^{\mathsf{T}} = (\mathbf{u}^{\mathsf{T}}, \mathbf{v}^{\mathsf{T}})\mathbf{:}\ \mathbf{u}^{\mathsf{T}}\mathbf{y}_j \,/\, \mathbf{v}^{\mathsf{T}}\mathbf{x}_j \le 1, \ \ j = 1,\ldots, n;\ \mathbf{u}, \mathbf{v} \ge 0\}$ is nonempty and bounded (or, failing which, the problem may be regularized), then the fractional program (3) may be replaced by the equivalent linear program

$$\max \ \zeta = \mathbf{u}^{\mathsf{T}}\mathbf{y}_0$$

$$\text{subject to} \qquad (4)$$

$$\mathbf{v}^{\mathsf{T}}\mathbf{x}_0 = 1$$

$$\mathbf{u}^{\mathsf{T}}\mathbf{y}_j \le \mathbf{v}^{\mathsf{T}}\mathbf{x}_j\ , \quad j = 1,\ldots, n$$

$$\mathbf{u}\, , \mathbf{v} \ge \mathbf{0}\ .$$

The new program (4), together with its dual and their fractional program equivalents, constitute the CCR (for Charnes, Cooper, Rhodes) model in data envelopment analysis. Efficiency of an evaluated DMU_0 depends on two conditions: (i) $\zeta^* = 1$, and (ii) $\exists$ an optimal ($\mathbf{u}^*$, $\mathbf{v}^*$) such that $\mathbf{u}^* > \mathbf{0}$ and $\mathbf{v}^* > \mathbf{0}$. If, on the other hand, DMU_0 has $\zeta^* < 1$, then for at least one constraint (that is, for at least one $\mathrm{DMU}_j \ne \mathrm{DMU}_0$), $\mathbf{u}^{\mathsf{T}}\mathbf{y}_j = \mathbf{v}^{\mathsf{T}}\mathbf{x}_j$. The set of all such DMU_js which are also efficient constitutes the reference set or peer group for DMU_0, and the convex combinations of the members of the reference set constitute the efficient frontier of DMU_0.

The dual of (4) is

min θ

subject to (5)

$$\theta \mathbf{x}_0 - \mathbf{x}_j^{\mathsf{T}} \boldsymbol{\lambda} \geq 0, \quad j = 1, \ldots, n$$

$$\mathbf{y}_j^{\mathsf{T}} \boldsymbol{\lambda} \geq \mathbf{y}_0, \quad j = 1, \ldots, n$$

$$\boldsymbol{\lambda} \geq \mathbf{0}.$$

If DMU_0 is inefficient, then $\theta^* < 1$, which means that some combination of DMUs were able to produce the outputs $\mathbf{y}_0$ with less inputs than used by DMU_0, namely, $\mathbf{x}_0$. In that case, the excess consumption of inputs, $\mathbf{s}^- \in \mathbf{R}^m$, and the shortfall in production of outputs, $\mathbf{s}^+ \in \mathbf{R}^s$, are given by $\mathbf{s}^- = \theta \mathbf{x}_0 - \mathbf{X}\boldsymbol{\lambda} \geq \mathbf{0}$ and $\mathbf{s}^+ = \mathbf{Y}\boldsymbol{\lambda} - \mathbf{y}_0 \geq \mathbf{0}$, respectively, where $\mathbf{X}$ is the $m \times n$ matrix of inputs and $\mathbf{Y}$ is the $s \times n$ matrix of outputs of all n DMUs in the evaluation. Thus, efficiency in the CCR model requires that (i) $\theta^* = 1$ and (ii) $\mathbf{s}^+ = \mathbf{0}$ and $\mathbf{s}^+ = \mathbf{0}$, that is, that the DMU lies on the frontier and all slacks are zero.

Figure 2 presents a hypothetical, single input/single output, example of a CCR evaluation. The hyperplane defining the production possibilities set and efficient frontier is, in this case, a ray from the origin through those DMUs such that all other DMUs are either on the hyperplane (efficient) or below it (inefficient). Two arrows emanate from each inefficient DMU to the hyperplane, one vertically and one horizontally. The vertical arrow indicates the amount and direction of improvement necessary for the inefficient DMU to attain efficiency in the output orientation, that is, the additional output necessary to overcome the shortfall in production. The horizontal arrow indicates the input

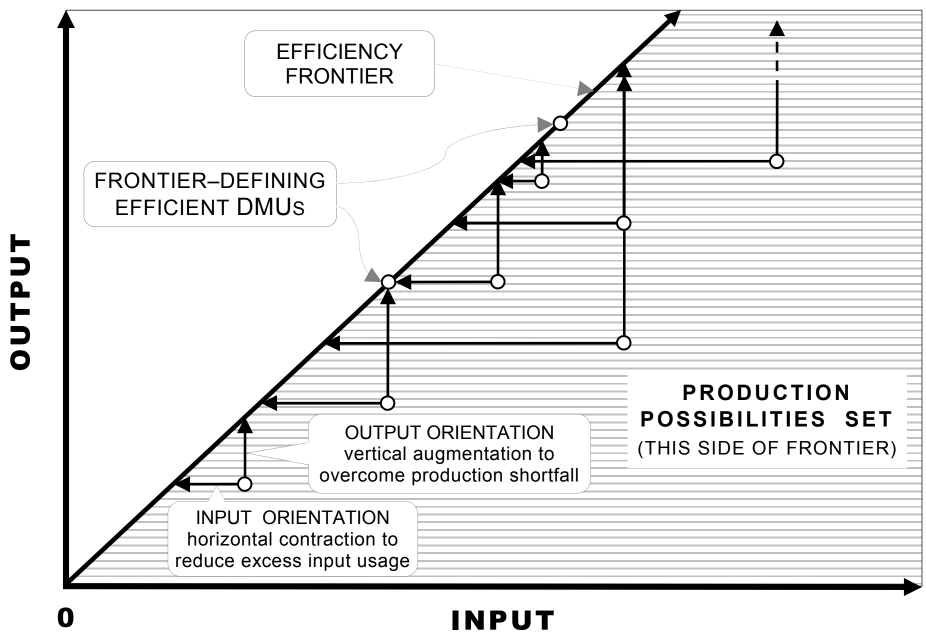

Figure 2 CCR Model: Frontier-defining Efficient DMUs, Efficiency Frontier, Production Possibilities Set, and Output and Input Adjustments for Inefficient DMUs

orientation, that is, the reduction in input usage necessary to achieve efficiency.

Section 4 BCC Mathematical Model

This section provides a brief description of the mathematical model and some of the facts that are used in this study.[22] Various DEA models have been developed since the original formulation by Charnes, Cooper, and Rhodes in 1978. The model used for this analysis, and described in this section, due to Banker, Charnes, and Cooper,[23] is known as the BCC model of DEA. This study uses the BCC model with an "output

[22] For an up-to-date and comprehensive presentation of all major DEA models, see William W. Cooper, Lawrence M. Seiford, and Kaoru Tone, *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software* (Boston, MA: Kluwer Academic Publishers, 2000).

[23] R. D. Banker, A. Charnes, W. W. Cooper, "Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis," *Management Science,* Vol. 30, No. 9 (Sep. 1984), pp. 1078-1092.

oriented" objective, which is given as follows:

$$
\begin{aligned}
\max \quad & \phi_0 \quad + \quad \varepsilon\left(\sum_{r=1}^{s} s_r^{+} + \sum_{i=1}^{m} s_i^{-} \right) \\
\text{subject to} \quad & \\
& \phi_0\, y_{r0} = \sum_{j=1}^{n} y_{rj}\lambda_j \; - \; s_r^{+}, \qquad r = 1,\ldots,s \\
& x_{i0} = \sum_{j=1}^{n} x_{ij}\lambda_j \; + \; s_i^{-}, \quad i = 1,\ldots,m \\
& 1 = \sum_{j=1}^{n} \lambda_j \\
& \lambda_j, s_r^{+}, s_i^{-} \geq 0 \quad \forall\, i, j, r.
\end{aligned} \qquad (6)
$$

The y_{rj} and x_{ij} are the observed values (given in the performance data) of the $r = 1,\ldots, s$ outputs and $i = 1,\ldots, m$ inputs, respectively, for each of the $j = 1,\ldots, n$ DMUs and $j = 0$ refers to the DMU_j being evaluated.

In this study, each DMU is a mutual fund of a given strategic category. The typical evaluation of mutual funds in the financial literature is based on performance as measured by the levels of returns from investment activity. In this study, however, the orientation is toward managerial performance and strategy evaluation; so the model uses different criteria, including three measures of return and a measure of the total value of the fund (fund size) for the outputs y_{rj} to be considered. The inputs x_{ij} for this study are measures of management activity which include the fees and other expenses charged to shareholders.

Since the inputs reflect management activity rather than costly raw materials or other scarce resources and since the chief concern is the returns (outputs) generated by that activity, this study employs the output oriented version of the BCC model. Although the fees, inputs in this model, are typically small percentages of the amounts invested, they can represent large sums for large investments. Inefficient DMUs may be projected to the efficiency frontier in one of three directions. In the output orientation, the projection is made by increasing at least some outputs until the DMU moves up to the efficiency frontier without decreasing any other outputs and without increasing any inputs. In contrast, in the input orientation at least some inputs are reduced until the DMU moves back toward the efficiency frontier without increasing any other inputs and without decreasing any outputs. Finally, the third approach both reduces input excesses and increases output shortfalls to project the inefficient DMU to the efficiency frontier at some point between the points of output-oriented and input-oriented projection. The output orientation is established by the presence of ϕ, which is to be maximized, as a factor in the constraints relating to outputs but not in those relating to inputs.

The objective of (6) maximizes ϕ_0 and thus maximizes all of the outputs without changing their proportions in the constraints where ϕ appears. The value of ϕ defines what is called in economics "weak technical efficiency," and a DMU for which $\phi_0 = 1$ exhibits weak technical efficiency. If a DMU lacks weak technical efficiency, then it does not lie on the efficiency frontier and $\phi_0 > 1$ is the factor by which the outputs of that DMU must be multiplied to increase them sufficiently so that efficiency is achieved. The point would then lie on the efficiency frontier, that is, that point is projected to the "weak" part of the efficiency frontier.

It is possible for a DMU to lie on the frontier but still be dominated by another

DMU, that is, another DMU may produce more output for the same inputs (output slack for the dominated DMU) or the same output for less inputs (input slack for the dominated DMU). This occurs when the DMU lies in a face or edge of the frontier which is parallel to an axis of an input or output dimension. Such a dominated DMU would have a value of $\phi = 1$ (because it lies on the frontier surface), but it would also have non-zero slacks (because its performance was dominated by another DMU). Therefore, the criterion for *full technical efficiency* requires that both

$$\begin{aligned} &\text{(i) } \phi^* = 1, \quad \textit{i. e.}, \text{ the DMU lies on the frontier, and} \\ &\text{(ii) } s_r^{+*} = 0, r = 1,\ldots, s; \; s_i^{-*} = 0, i = 1,\ldots, m, \quad \textit{i. e.}, \text{ all slacks are zero.} \end{aligned} \tag{7}$$

The ε in (6) is non-Archimedean[24] and is realized computationally in a two-stage process. Stage one maximizes ϕ_0 in (6) without reference to the slacks, s_r^+ and s_i^-. Then, stage two sets maximum $\phi_0 = \phi_0^*$, the optimum value established in stage one, in the constraints. The sum of the slacks is then maximized without changing the value of ϕ^* as determined in stage one. This accords ϕ_0 the property of preemptive priority and

[24] That is, ε is positive and smaller than any positive *real* number. Utilizing the two-stage computational process makes it unnecessary to specify its value. "An ordered field F is said to be *Archimedean ordered* if for all $a \in F$ and all $b \in P$ [*i. e.*, b positive] there exists a positive integer n such that $nb > a$. In intuitive language, this definition means that no matter how large a is and how small b is, successive repetitions of b will eventually exceed a." Edwin Hewitt and Karl Stromberg, *Real and Abstract Analysis* (New York, NY: Springer-Verlag, 1965), p. 37. A simpler version is the "Theorem of Eudoxus. If a and b are any two positive rational numbers, then a natural number n always exists such that $nb > a$." Also "[t]his theorem is usually, but incorrectly, ascribed to *Archimedes*; it is already to be found in *Euclid*, Elements, Book V, Def. 4." Konrad Knopp, *Theory and Application of Infinite Series* (New York, NY: Hafner Publishing Co., 1971), pp. 7, 7 fn. 7, 11. For the use of non-Archimedean elements in linear programming see A. Charnes, "Optimality and Degeneracy in Linear Programming," *Econometrica*, Vol. 20, No. 2 (April 1952), pp. 160-170, and A. Charnes and W. W. Cooper, "Chapter XII. Degeneracy, Perturbation, and Complete Regularization," pp. 413-447 and also pp. 52-54, in *Management Models and Industrial Applications of Linear Programming*, Vol. I (New York, NY: John Wiley & Sons, Inc., 1961).

assures that its value will not be reduced by any choice of the slacks.

With optimum values denoted by *, the solution of (6) yields new, projected values for the y_{r0} and x_{i0} which are denoted by $\hat{y}_{r0}$ and $\hat{x}_{i0}$ in the following expressions:

$$\begin{aligned} \hat{y}_{r0} &= \phi_0^* \, y_{r0} + s_r^{+*} = \sum_{j=1}^{n} y_{rj} \, \lambda_j^* \geq y_{r0}, \quad r = 1, \ldots, s \\ \hat{x}_{i0} &= x_{i0} - s_i^{-*} = \sum_{j=1}^{n} x_{ij} \, \lambda_j^* \leq x_{i0}, \quad i = 1, \ldots, m. \end{aligned} \tag{8}$$

The inefficiencies in each output (shortfall in production of output) and each input (excess consumption of input)—that is, the differences between the original values and the projected, efficient values—are given by

$$\begin{aligned} \Delta y_{r0} &= \hat{y}_{r0} - y_{r0} \geq 0, \quad r = 1, \ldots, s \\ \Delta x_{i0} &= x_{i0} - \hat{x}_{i0} \geq 0, \quad i = 1, \ldots, m. \end{aligned} \tag{9}$$

Thus, $\Delta y_{r0} = \Delta x_{i0} = 0, \ \forall \ i$ and r, together with $\phi_0^* = 1$ define *full technical efficiency*, which is distinguished from "weak" technical efficiency, where the non-zero slacks are not considered as part of the criterion of efficiency.

The values in (9) are the corrections each inefficient DMU must make in order to be projected to the efficiency frontier. Once the projections are effected, all the mutual funds lie on the efficiency surface, both the originally efficient funds and the projected, efficiency-adjusted, originally inefficient funds. The resultant surface represents the best production possibilities for that category of strategy.

Technical efficiency, therefore, refers to performance which places a DMU on the production frontier. In weak technical efficiency, the DMU may also have non-zero slacks, that is, although it is on the frontier, it may be dominated by another DMU which has the same level of output for less input (the weakly efficient DMU has positive input slacks, $s_r^{+*} > 0$, an excess consumption of inputs by the amount of the slacks) or which has more output for the same level of input (the weakly efficient DMU has positive output slacks, $s_i^{-*} > 0$, a shortfall in outputs by the amount of the slacks). On a piecewise linear frontier, the first case occurs when the weakly efficient DMU lies in a facet or edge which is parallel to an input dimension and the second case when it lies in a facet or edge which is parallel to an output dimension.

Figure 3 displays a simplified, hypothetical, one-output/one-input example, showing the output-oriented projection to the efficiency frontier of three types of inefficient funds, F to $\hat{F}$, G to $\hat{G}$, and E to Ê. The efficiency frontier is defined by the originally efficient funds, A, B, C, and D.

The first type of inefficiency is represented by the point labeled F, which has an efficiency score given by $1/\phi^*$, where ϕ^* is the first stage optimization variable in Equation 6. The value of ϕ^* is the factor by which the outputs in the linear constraint equations in which it appears [Equation 6] must be increased and the inputs must be decreased to project the point to the frontier, namely, $\hat{y}_{r0} = \phi_0^* y_{r0} \geq y_{r0}$, $r = 1, \ldots, s$, where the $\hat{y}_{r0}$ are the new output coordinates of the projected point. For a technically efficient point, $\phi^* = 1$ and all $s_r^+ = 0$ so that $\hat{y}_{r0} = y_{r0}$; but for a technically inefficient point, $\phi^* > 1$ and $\hat{y}_{r0} > y_{r0}$. For point F, $\phi_F^* = \hat{y}_{1F} / y_{1F}$ and its (in)efficiency score is $1/\phi_F^* = y_{1F} / \hat{y}_{1F}$. Thus, in Figure 3, the vertical dotted arrow indicates the projection from the original data point F to $\hat{\mathrm{F}}$, its projection on the frontier, and the bracket indicates the length of that arrow, that is, the amount of shortfall in output relative to what an efficient fund would have achieved, $\Delta y_{1F} = \hat{y}_{1F} - y_{1F} > 0$.[25]

The fund E exhibits another type of inefficiency and demonstrates the significance of the two-part definition of efficiency given by Equation 2, which required both (i) that the point lies on the frontier ($\phi^* = 1$) and (ii) that face of the frontier is not dominated, (slacks are zero, $s_r^{+*} = 0$ and $s_i^{-*} = 0$). The point E is on the frontier (therefore, $\phi^* = 1$) but clearly the point D represents the same amount of output with substantially less input (therefore, the input slacks are positive). Thus, for weakly efficient E,

$\hat{y}_{rE} = \phi_E^* y_{rE} + s_r^{+*} = y_{rE}$ because $\phi_E^* = 1$ and $s_{rE}^{+*} = 0$, so that

$\Delta y_{1E} = \hat{y}_{1E} - y_{1E} = 0$, that is, there is no shortfall in output. However,

$\hat{x}_{iE} = x_{iE} - s_i^{-*} < x_{iE}$ because $s_1^{-*} > 0$. Therefore, $\Delta x_{1E} = x_{1E} - \hat{x}_{1E} > 0$, that is, there is an excess consumption of input. This excess, or input slack, is represented by a horizontal dotted arrow, horizontal because it represents an adjustment in the input dimension, and its length is indicated by the horizontal bracket. Note also that the

[25] Since the Figure represents a one-output/one-input example, the numerical subscripts are superfluous, but are retained for consistency with the notation of the general model in the text.

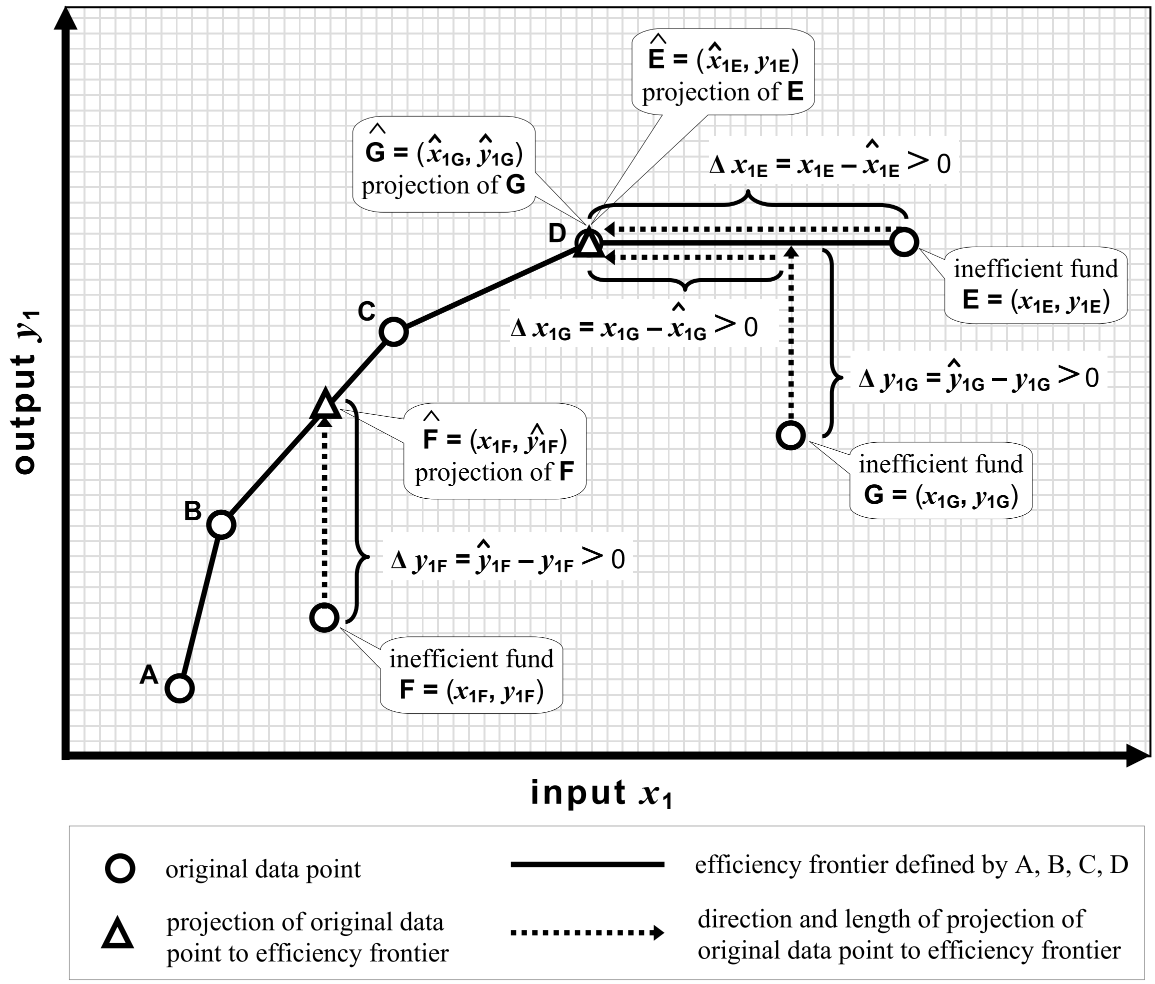

Figure 3 Examples of Projection of Inefficient Mutual Funds to Efficiency Frontier in a Hypothetical, Output-oriented, One-output/One-input Case

projection of E coincides with the efficient, frontier-defining point D.

Finally, point G represents both kinds of inefficiency defined in Equation 7. For G, a vertical projection represented by the vertical dotted arrow intersects the face of the frontier between D and G. Such a projection results in a point that also has positive slack in the input and therefore must also be projected horizontally (back along the input dimension) until it also coincides with D.

As discussed previously, this procedure has several practical benefits. Such projections provide managers *specific* and *concrete* targets for improvement. Furthermore, because such projections are located on a surface generated from the actual performance of best practitioners, such targets are *realistic* performance goals relative to what the evidence shows to be possible by reference to all performances. Moreover, the DEA projection process specifies the efficient DMUs closest to the point of projection, thereby providing a *peer group* of efficient exemplars to guide managers' adjustments to improved performance.

Such a collection of projected points answers an important question about underperformers: how might they have performed had they been able to overcome the difficulties which hindered their efforts? In other words, what are the production possibilities of a strategy *per se* if they are not obscured or confounded by the contingent results of the strategy *in usu*? Thus, the efficiency frontier estimates how the *strategy* performs once the limitations of managerial practice are removed. DEA provides a means of separating actual practice from potential achievement. Once all the inefficient funds have been projected to the efficiency frontier, which the evidence of actual performance establishes as achievable, the resulting points can be taken to represent fully efficient realizations of the corresponding strategy.[26]

Section 5 Mann–Whitney Rank Analysis

When comparing two strategies, if the efficient surface of one completely dominates the other, then that one strategy is superior. However, in general, such selfevident superiority cannot be expected. For the more general case, a statistical test is needed to distinguish whether some members of a sample exhibit significantly different characteristics from the remaining members. To determine whether the funds of one strategy category perform significantly better than those of another, the DEA evaluations of this investigation are analyzed by a nonparametric method which does not require the sizes of the two groups be equal.

The test is based on the Mann-Whitney U statistic, which is a variation of the Wilcoxon rank sum test[27] and which is equivalent to a two-group specialization of nested orthogonal contrast models.[28] This test is used to assess the relationship between two qualities or characteristics, one of which produces a ranking and the other of which produces a dichotomy. For mutual funds, the dichotomy is given by the two strategies being compared and the ranking is given by the rank ordering of the DEA efficiency scores of the individual mutual funds when all the funds from both strategy categories are evaluated as a single, joint sample. If neither of the strategies is better than the other, then, on average, the ranks of the funds from one strategy will be neither greater nor smaller than the ranks of the funds from the other strategy.

In this study, for the rank ordering of the sample which combines the funds from both categories, all ties are assigned mid-rank values. Then the sum R of ranks for those

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[26] The efficiency frontier is a lower bound for the "ideal" production possibilities surface, because in some cases, in principle at least, even the best performers might have done better. In cases where the efficient performers did as well as could possibly have been done, then this surface is also the upper bound of actual performance possibilities.

[27] Myles Hollander and Douglas A. Wolfe, *Nonparametric Statistical Methods* (New York, NY: John Wiley & Sons, 1973), pp. 68-75; John W. Pratt and Jean D. Gibbons, *Concepts of Nonparametric Theory* (New York, NY: Springer-Verlag, 1981), pp. 249-272; Sir Maurice Kendall, *Rank Correlation Methods*, 4th ed. (London, UK: Charles Griffin & Co., 1975), pp. 41-43, 165.

funds in either *one* of the strategy categories is used to compute the Mann-Whitney rank statistic U, where

$$U = n_1 \cdot n_2 + \left[\frac{n_1 \left(n_1 + 1\right)}{2}\right] - R, \tag{10}$$

where n_1 and n_2 are the numbers of funds in the two strategy categories. For sufficiently large subsample sizes (that is, $n_1, n_2 > 10$)

$$Z = \frac{U - \frac{n_1 \cdot n_2}{2}}{\sqrt{\frac{n_1 \cdot n_2 \left(n_1 + n_2 + 1\right)}{12}}}, \tag{11}$$

is approximately normally distributed, so that a standard two-sided test is applied to determine if the funds from one strategy dominate those from the other. Thus, the null hypothesis H_0 (there is no difference between the two strategies) is accepted whenever $-Z_{\alpha/2} \le Z \le Z_{\alpha/2}$, where Z_α is the αth centile of the standard normal probability distribution. The null hypothesis is rejected and the alternative H_A (one strategy is superior to the other) is accepted when either $Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$.

Section 6 Comparison of Strategies

An early application of this projection technique together with the nonparametric (rank sum) test for differences between distinguishing groups or categories of DMUs appears in Brockett and Golany [1996], who evaluate school program performance. Similarly, Brockett, *et al.* [1998] use this method to evaluate forms of ownership (stock

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[28] John I. Marden, *Analyzing and Modeling Rank Data* (London, UK: Chapman & Hall, 1995), p. 130; Kendall, *Rank Correlation Methods*, p. 165.

versus mutual) and types of marketing (agency versus direct) in the property liability insurance industry.

This approach also lends itself to an interpretation in terms of the micro-economic theories of short-run and long-run behavior. This is the basis of the application by Kao [2000], for instance, in which, following production economics theory, the short-run production frontiers of the several plants of multi-plant firms are used to estimate an enveloping, long-run production frontier. Barua, *et al.* [2004] carry this further to specifically identify characteristics of a two-stage DEA analysis with economic properties theoretically required of a long-run equilibrium, namely: (i) technical inefficiencies (that is, waste) which are typical in short-run production are eliminated in the long-run; and (ii) the long-run production frontier is at least as technically efficient as the corresponding short-run frontier. By associating efficiency evaluations with the micro-economic properties of a future, long-run equilibrium state, they avoid the problems of specifying how long is long enough for the long-run as well as avoiding the need to specify the form of the production functions as functions of time.

These same properties of DEA can be exploited to evaluate strategies as distinct from their execution. Instead of future states, here "ideal" states—*i. e.*, performances that fully realize the strategy potential, unrestrained by shortcomings in practice—are identified with fully efficient frontiers. The two concepts—future states, as economic categories, and ideal states, as strategic management categories—may be identified by appeal to the same micro-economic principles adduced in Kao [2000] and in Barua, *et al.* [2004], that is, in the long run the market process will eliminate the individual short-run inefficiencies.

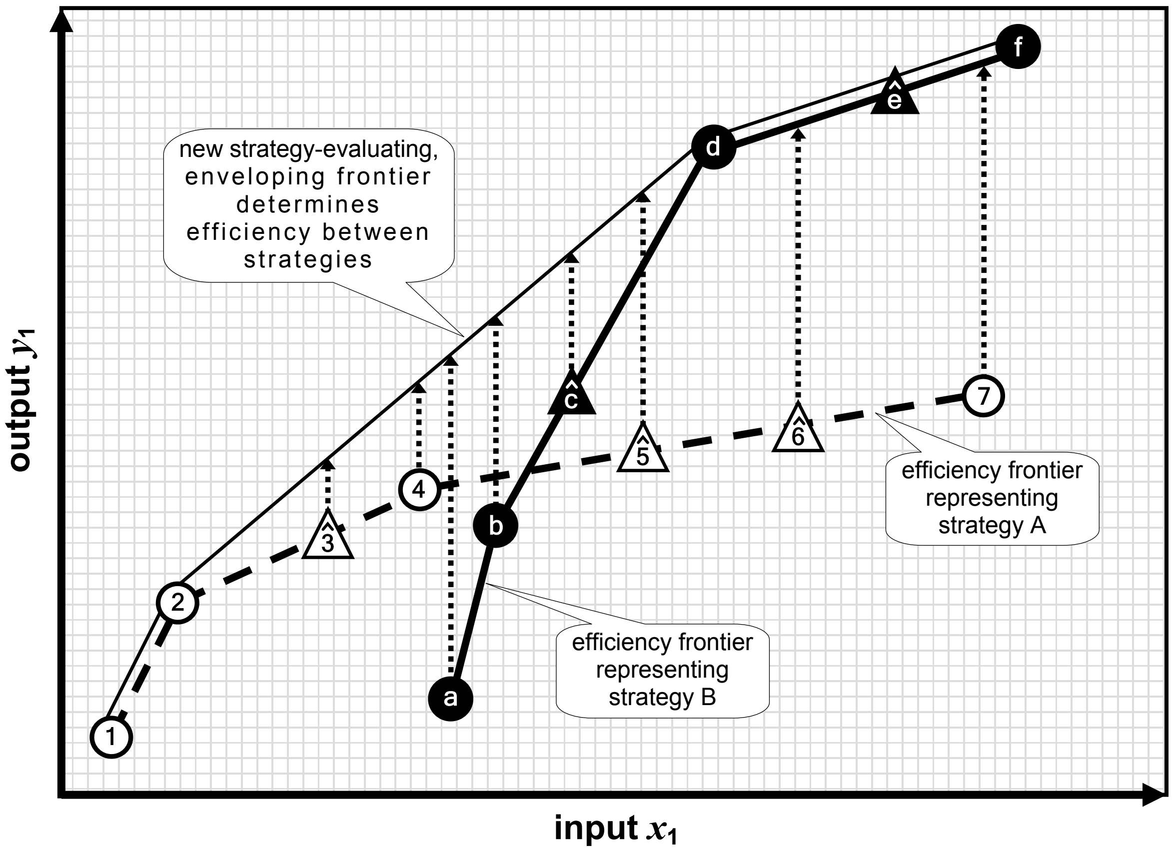
The joint analysis reported in the Chapter 6, Section 1 evaluates how *funds* executing the two strategies performed in practice. However, to evaluate the *strategies* themselves, the actual performance data of inefficient funds must be replaced by the values of their projections to the efficiency frontiers of their respective strategy groups. This identifies what the strategies can accomplish and estimates managerial shortcomings. Each of these two, new sets of projected data represents the surface of best possible performances for its strategy. Then, the two, new, projected-data sets are again combined into a joint group so that the strategies themselves can be evaluated by DEA methods without being confounded by short-run managerial deficiencies in performance.

The application of DEA in this case is not to funds evaluated on their actual performances, but rather to (projected) "funds" as exemplars of the full possibilities of their respective strategies. This third-stage analysis evaluates not the actual performances of the several funds with all the confounding inadequacies of practice, the vectors $(\boldsymbol{x}_0, \boldsymbol{y}_0)$, but rather the strategies as represented by the vectors $\left(\hat{\boldsymbol{x}}_{0,}\ \hat{\boldsymbol{y}}_0\right)$, all of which are points on the efficiency frontiers and which estimate the best possible performances which the data showed could have been accomplished by their respective strategies.

Figure 4 presents a hypothetical example of two, one-output/one-input projected-data efficiency frontiers (thick, solid line and thick, dashed line) which represent the strategies as evaluated separately in the manner illustrated in Figure 3. Here they are evaluated by a third DEA frontier (thin, solid line) which, as generated by applying DEA to the two separate frontiers, envelops both. In this figure, the thick, solid and dashed surfaces represent the results of the stage one, separate DEA evaluations of the two competing strategies. The circled points at the vertices of the piecewise linear surfaces represent the DMUs which were evaluated as efficient and which determined the efficiency frontier for their respective strategies. The triangular points represent DMUs which were evaluated as inefficient in stage one and projected to the efficiency frontier in stage two.

The thin, solid, piecewise linear surface represents the result of the third stage application of DEA, with respect to which the superiority of one or the other strategy will be determined. Some faces of this frontier are coincident with faces of one or the other strategy frontiers and contain both DMUs which were originally efficient (namely, DMUs **1** and **2** from strategy A, and DMUs **d** and **f** from strategy B), as well as DMUs which appear because they were projected to their respective frontiers (DMU **ê** from strategy B). (Although they are separated in the diagram for clarity, the faces illustrated by parallel segments represent coincident faces.)

However, the face from DMU **2** to DMU **d** does not exist in either of the original strategy frontiers, but results from using DEA to evaluate the two strategies together. Moreover, DMUs **a** and **b**, which were originally efficient for strategy B, are dominated by the frontier of strategy A and do not appear on this new efficiency frontier. However, DMU **4**, although originally efficient for strategy A and not dominated by the frontier of strategy B, nonetheless does not appear on the new frontier because it is dominated by the linear combinations of DMUs **2** and **d**, both of which are on the frontier. This new face



Results from Step One and Step Two

- efficiency frontier representing strategy A (dashed line)
- originally efficient DMUs which defined strategy A frontier (open circle)
- originally inefficient DMUs projected to strategy A frontier (open triangle)

Results from Step One and Step Two

- efficiency frontier representing strategy B (thick solid line)
- originally efficient DMUs which defined strategy B frontier (filled circle)
- originally inefficient DMUs projected to strategy B frontier (filled triangle)

Results from Step Three DEA Comparing Strategies

- enveloping efficiency frontier defined by efficient portions of competing strategy frontiers, includes faces coincident with faces from original strategy frontiers ([1, 2] and [d, f]) and new faces ([2, d]) (thin solid line)
- projection of inefficient strategy performances to the strategy-evaluating frontier (dotted arrow)

Figure 4: Example of a Strategy-Evaluating Frontier for Two Hypothetical, One-output/One-input Frontiers which Represent Separate Strategies and the Projection of Strategy Inefficient Funds to the Strategies-Efficient Frontier

of the frontier shows what would be possible for a DMU employing both strategies in different proportions.

Chapter 6 Analysis and Results

Section 1 Data Source, Population, Variables, Software

Evaluations of mutual funds are greatly facilitated by the ready availability of extensive data series. Legal and regulatory requirements provide detailed and comparable information on all the funds in the industry. Fund activities are thus revealed for close investigation. In particular, the primary objects of this study, fund strategies, must be publicly declared and continue to govern fund activities until they are changed through a public and legally mandated process.

The data in this study are derived from Morningstar, Inc.'s *Principia Pro Plus for Mutual Funds*, for November 1998. This database has a universe of 10,352 mutual funds, tracked for the ten years from the five year period, 1988 through 1997 along more than 150 variables, with data reported as of October 31, 1998. Coverage also includes 101 indexes and index funds. The analysis here is based on data from 1993 through 1997. The analysis studies those funds which report a prospectus "objective"—that is, a strategy—of either "aggressive growth" or "equity income" in their investments in primarily domestic equities. Of the resulting 376 funds, 226 follow a strategy of investing to generate "equity income," while the investment strategy for 150 funds seeks returns from "aggressive growth." The analysis requires complete data for each of the evaluated funds. Therefore, primarily because of null values for the 3-year and 5-year compounded average total annual return data for funds not yet 3 or 5 years old as of 1997, the sample is reduced to 120 funds—70 remaining from the "equity income" strategy and 50 from the "aggressive growth" strategy.

The variables on which the analysis is based are listed in Table 1, which displays the six input variables and the four output variables that were employed. Table A1, in the appendix, presents the detailed Morningstar definitions of the variables used in this analysis. Of the output variables, listed at the bottom of this table, the first three—AnRet97, TRA3Y, TRA5Y—represent, respectively, short-, mid-, and longer-term return performance, and the fourth, NetAss, represents size attainment performance.

The input variables reflect costs incurred in order to achieve these four output results. Thus, %Cash represents liquidity paid for by "investment income foregone," although in a declining market, the low return from cash may exceed falling or negative

TYPE	CODE	NAME
input	%Cash	percent of net assets held as cash or cash equivalents
input	TurnoverRatio	turnover ratio, as percent of net assets
input	ExpnRatio	expense ratio, as percent of net assets
input	FrontLoad	front-end load, as percent of initial share purchase
input	DfrrdLoad	deferred load, as percent of share redemption
input	12b-1	12b-1 fees, as percent of net assets
output	AnRet97	annual return for 1997, as percent, + 100%
output	TRA3Y	3 year compounded average annual returns, as percent, + 100%
output	TRA5Y	5 year compounded average annual returns, as percent, + 100%
output	NetAss	net assets, month-end, in millions of dollars

Table 6.1: Input and Output Variables for DEA of Mutual Funds

returns from investments. Turnover ratio reflects the level of management activity (and associated costs) necessary to achieve the investment performance. Expense ratio reflects the operating costs and management fees, the direct charge for the management activities which generate the returns. These first three inputs are viewed as charges to or subtractions from the pool of funds available for investment and revenue generation.

The next three inputs are not direct charges to the fund; but as charges to shareholders, they are viewed as investor disincentives. Front-end load is a sales charge to the shareholder at the time of share purchase and deferred load is a charge to the shareholder at share redemption. The 12b–1 fees are distribution and marketing charges to shareholders. These three charges are incurred to draw shareholders to the fund, but they also represent a reduction of the amount of investor capital actually invested and, therefore, as an added cost to investors, a disincentive for potential shareholders to place assets with the fund.

The data source, Morningstar's *Principia Pro*, is affected by survivor bias, because data series are removed from the database when the funds they represent fail or dissolve. However, this study is retrospective and the period of analysis is coterminous with the period of fund performance. Therefore, the time-period criterion used for this study—namely, funds which had been operating for at least five years as of the end of 1997—avoids this limitation by including all funds which would have met the requirement. Nevertheless, studies which, for example, sought to evaluate mutual fund performance over a twenty-year span based on five-year returns would be severely biased by this data set. Moreover, survivor bias prevents analysis of the failure likelihood associated with different strategies.

The software used to effect the data envelopment analyses was DEA-Solver PRO, which takes data in the form of a Microsoft Excel worksheet and produces ten worksheets of results. Except for three sheets, the summary and two graphs, the resulting worksheets report information in a list of the DMUs and their associated data, with a different sheet for each type of data. The worksheets are labeled according to the type of data reported for each DMU: RTS (returns to scale), Slack (the slacks—excess inputs, shortfall outputs—for each variable), Weight (the virtual multipliers, $\boldsymbol{v}_i$ and $\boldsymbol{u}_r$, for each variable as given by the dual form), WeightedData (the products of the virtual multipliers and their corresponding input or output variable values), Projection (the values of the efficiency projection points, the differences between the original values and the projection values, and the differences expressed as percentage changes), Rank (the DMUs and scores listed in efficiency score rank order), and Score (the DMUs and efficiency scores with their associated reference set DMUs and corresponding coefficients, λ). The two graphs are bar graphs of DMUs against the efficiency scores; one graph lists the DMUs in their original order and the other lists them in efficiency rank order. The final worksheet includes summary statistics of the input/output values and of the efficiency scores. Saitech, Inc.[1] markets the DEA-Solver PRO software. All other analyses were effected with Microsoft Excel native Functions or, as in the case of the Mann-Whitney tests, for example, VBA for Excel procedures.

Section 2 Standard Data Envelopment Analysis of Mutual *Funds* from Both Strategy Groups Evaluated Together

The first analysis is comparable to previous DEA evaluations of mutual funds and

[1] Saitech, Inc., 1 Bethany Road, Suite 54, Hazlet, New Jersey 07730, USA. The url for their Internet home page is http://www.saitech-inc.com/; the url for the DEA-Solver PRO page is http://www.saitech-inc.com/Products/Prod-DSP.asp/. A learning version, with samples, of the DEA-Solver software is distributed on the CD-ROM included with William W. Cooper, Lawrence M. Seiford, and Kaoru Tone, *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, Boston, MA: Kluwer Academic Publishers; 2000, which also includes the software manual as an appendix.

employs one of the widely used DEA models, the BCC model, which was explained in the preceding, methodology chapter. All 120 funds of the sample, even though they represent two different strategies, are analyzed together. The results of the analysis provide evaluations of each fund's performance relative to all of the other funds in the combined sample. Relative efficiency scores are based on each fund's performance data, which are the result of all contributing factors, including the fund's strategy and its managerial performance. Table 2 presents summary statistics from the DEA analysis of the funds in the combined sample (column 2), as well as the statistics for the funds in the two subsamples decomposed according to the two strategies (column 3 for the Aggressive Growth subsample and column 4 for the Equity Income subsample).

As Table 2 shows, only about one sixth (21 of 120 [column 2, row 7 and row 2,

0	1	2		3		4	
1	SUMMARY STATISTICS DEA EVALUATION OF FUNDS	ageq97BCCO[1] joint sample		aggrw97[2] as subsample		eqinc97[3] as subsample	
2	number of DMUs (funds) =	120		50		70	
3	average of efficiency scores =	0.963529		0.941127		0.979531	
4	standard deviation of scores =	0.039424		0.049268		0.018866	
5	maximum score =	1		1		1	
6	minimum score =	0.754144		0.754144		0.923233	
7	number of efficient DMUs =	21	17.50%[4]	4	19.05%[5]	17	80.95%[5]
8	number of inefficient DMUs =	99	82.50%[4]	46	46.45%[6]	53	53.55%[6]

[1] the joint sample of **aggressive growth** and **equity income** funds combined, 1997 data, BCC Output-oriented DEA model [column 2]
[2] the **aggressive growth** funds [column 3], as a subsample of the joint sample
[3] the **equity income** funds [column 4], as a subsample of the joint sample
[4] as a percentage of the total, joint sample size, i. e., 120 [column 2, row2]
[5] the subsample's percentage of the joint total number of *efficient* funds [column 2, row 7]
[6] the subsample's percentage of the joint total number of *inefficient* funds [column 2, row 8]

Table 6.2: Descriptive Statistics of DEA Evaluations of Mutual *Funds*
from Two Strategies Combined into One Sample,
with Statistics for the Subsamples of Each Strategy within the Joint Sample

respectively], or 17.5%) of all mutual funds in the joint sample are efficient. Of these, only 4 out of 50 [column 3, row 7 and row 2, respectively] (or 8%) of the Aggressive Growth funds are efficient, whereas 17 of 70 [column 4, row 7 and row 2, respectively] (or 24.3%) of the Equity Income funds are efficient. Alternatively, within the joint sample, 81% [column 4, row 7] of the efficient funds pursue the Equity Income strategy, whereas only 19% [column 3, row 7] pursue the Aggressive Growth strategy.

The funds from the two subsamples also score very differently in other respects. For example, the average efficiency score, nearly 0.98 [column 4, row 3] for the Equity Income funds is greater than the average of 0.94 [column3, row 3] for the Aggressive Growth funds. Moreover, the dispersion in efficiency scores for Equity Income funds with a standard deviation of nearly 0.019 [column 4, row 4] is more than two and a half times smaller than that for Aggressive Growth funds at more than 0.049 [column 3, row 4], despite the fact that the Equity Income subsample is 40% larger (70 versus 50 [row 2, column 4 and column 3, respectively]). Since the maximum score is 1 (signifying full technical efficiency), the difference in variances of efficiency scores is reflected in the difference of minima between the two subsamples. A minimum efficiency score of 0.92 [column 4, row 6] among Equity Income funds means that the worst performer in that subsample produces 92% of the output needed to achieve efficiency when compared with all other funds in the joint sample, whereas the worst performer among the Aggressive Growth funds produces only 75% [column 3, row 6] of the output required to achieve efficiency.

These observations can be evaluated more rigorously by the nonparametric

Wilcoxon rank sum test or the equivalent test based on the Mann–Whitney rank sum statistic **U** (as discussed in Section 5 of Chapter 5) to decide between the hypotheses:

H_0: The two types of funds are equally efficient.
H_A: One type of fund is less efficient than the other.

In this case, the test is two sided, since with respect to one strategy group, the other may be more or less efficient. As described in the chapter on methodology, the test is based on the sum of the ranks (midranks, in case of ties) of the DEA efficiency scores of the funds from the two groups when evaluated jointly. Informally, if the two groups are from the same population, then neither should, on average, have ranks greater or smaller than the other.

The results of applying the Mann–Whitney test are presented in Table 3. These results are unambiguous: the probability that Aggressive Growth funds are as efficient as Equity Income funds is almost infinitesimal. The α significance level [bottom row, last two columns] is 0.0000305%; that is, the conclusion that Equity Income funds perform

MANN-WHITNEY TEST RESULTS	symbol	aggrw[1]	eqinc[2]
strategy subsample size	n	50	70
rank sum of strategy subsample	**R**	3987	3273
Mann-Whitney rank statistic of strategy subsample	**U**	788	2712
Z score of strategy subsample	**Z**	– 5.120814	5.120814
α confidence level for difference in funds performance by strategy subsample	α	0.000000305	0.000000305

[1] subsample of **aggressive growth** [2] subsample of **equity income**

Table 6.3: Mann–Whitney Test Results on DEA Scores of Actual Performance Data, Evaluating *Funds* from Both Strategy Groups Together

better than Aggressive Growth funds can be asserted with 99.9999695% confidence.

As compelling as these results are, this analysis must still be considered as only suggestive of the relative merits of the competing *strategies* because it is based on the actual performance of *funds* and therefore depends not only on the respective strategies but also on the efforts and abilities of managers facing varying conditions. That is, these results do not reveal what the strategies being followed are capable of achieving after managerial deficiencies are eliminated. In general, how can it be determined whether the demonstrated effect is primarily the result of the power of the strategies or the abilities of the managers who implemented them or the conditions in which they acted or the goals to which they were directed?

As explained previously, exploitation of special features of the mutual funds market and of the sample selection criteria simplifies these issues for analysis by comparing funds which (i) all sought the same goal (*i. e.*, maximizing returns from investment activities) and which (ii) faced the same conditions (by operating in the same markets, primarily domestic equities) for (iii) the same time period (1993-1997). The only differences among market conditions faced by the evaluated funds were imposed by the constraints of their strategies. Thus, goals and conditions are fairly constant among the tested subjects.

Section 3 Performance of Mutual Fund *Strategies*

The remaining, fundamental question of separating the effects of the strategy from those of the manager can be addressed by employing a two-stage, four-step application of DEA. Stage one, in two steps, evaluates the individual funds separately within their respective strategy groups. Stage two, in two more steps, evaluates the strategies by using the results from the first stage.

Stage one involves multiple group analyses, in each of which individual funds are evaluated based on their actual performance measures compared to the other funds within their own strategy group. In the first step of stage one, the funds are separated and analyzed within their respective strategy groups to determine how actual managerial performance related to the possibilities afforded by their respective strategies, as defined by the efficient funds of that group. In the next step of stage one, again within each strategy group separately, the shortcomings in fund performance as actually managed are eliminated from consideration by projection of inefficient funds to their respective efficiency frontiers, which represent the full potential of the strategies. These projection values also represent the amounts of improvement needed for inefficient funds to have attained full efficiency performance and indicate which efficient funds represent the closest targets or benchmarks to guide their improvement.

Stage two involves a single, combined group analysis, in which the data are taken to represent the strategies under evaluation. Thus, in the first step of stage two, the strategies themselves are compared by a data envelopment analysis of all the funds combined into one sample and evaluated together; however, their actual performances are replaced by the projections produced in the stage one, step two analysis. In this third step, strategies are evaluated not on actual fund performances, but rather on idealized fund performances, using the projection data as exemplars of their strategies' fully realizable potentials. Finally, in the second step of the second stage, the results of the preceding step are subjected to rank tests for significance, that is, to determine if the representatives of the two strategies exhibit statistically significant differences. The results will prove to reach farther than just the original objective of evaluating strategies *per se* and will also provide a means for evaluating portfolios, with consequences for investment decisions by prospective mutual fund shareholders.

Section 3.1 Step One: Performance of Mutual *Funds* within Their Separate Strategy Groups

The first step in the four-step process to evaluate the competing strategies requires a separate data envelopment analysis of the funds for each strategy. Table 4 presents the summary statistics of the DEA evaluations of each, separate strategy group, presented together in parallel columns for ease of comparison. Table 4 has two columns of statistics [column 2 and column 3], one for each DEA evaluation of the two strategies considered as separate samples. These two separate evaluations are necessary to establish the potential of each strategy and to generate the projected data which will

0	1	2		3	
1	**SUMMARY STATISTICS OF DEA EVALUATIONS**	aggrw97BCCO[1]		eqtyinc97BCCO[2]	
2	number of DMUs (funds) =	50		70	
3	average of efficiency scores =	0.958206		0.980567	
4	standard deviation of scores =	0.051246		0.018192	
5	maximum score =	1		1	
6	minimum score =	0.778588		0.923233	
7	number of efficient DMUs =	22	44.00%[3]	20	28.57%[4]
8	number of inefficient DMUs =	28	56.00%[3]	50	71.43%[4]

[1] the **aggressive growth** strategy group funds, 1997 data, BCC Output-oriented DEA
[2] the **equity income** strategy group funds, 1997 data, BCC Output-oriented DEA
[3] percent of the total funds for **aggressive growth** strategy group [column 2, row 2]
[4] percent of the total funds for **equity income** strategy group [column 3, row 2]

Table 6.4: Comparison of Descriptive Statistics of Separate DEA Evaluations for Aggressive Growth Strategy Funds and Equity Income Strategy Funds

represent the strategies *per se* and replace the actual performance values of inefficient funds.

When the mutual funds are evaluated separately, according to strategy groups, the results demonstrate both similarities to and differences from the results of funds considered as strategy subsamples within a single, joint sample. Thus, Table 4 shows that the Aggressive Growth strategy group, evaluated separately, is close to evenly split (44% to 56%) between efficient and inefficient funds (22 versus 28, [column 2, row 7 and row 8, respectively]), whereas the comparable numbers were very different (4 versus 46 [Table 2: column 3, row 7 and row 8, respectively], or 8% to 92%) for Aggressive Growth strategy funds as a subsample of the original, combined sample. Despite this very different result, Aggressive Growth funds evaluated separately continue to display a large dispersion in efficiency scores, over 0.051 [column 2, row 4], as they did when examined as a subsample, over 0.049 [Table 2: column 3, row 4], and to exhibit a minimum efficiency score, 0.778588 [column 2, row 6], and an average efficiency score, 0.958206 [column 2, row 3], nearly as low as the 0.754144 minimum score and the 0.941127 average efficiency score [Table 2: column 3, row 6 and row 3, respectively] when viewed as a subsample of the original, combined sample.

On the other hand, the Equity Income strategy funds, when evaluated separately, have two and a half times as many inefficient as efficient funds (50 versus 20, or 71.4% to 28.6% [column 3, row 8 and row 7, respectively]), which ratio is comparable to that revealed when they are examined as a subsample of the larger, combined sample (53 versus 17 [Table 2: column 4, row 8 and row 7, respectively], or 75.7% to 24.3%). The values for standard deviation, minimum score, and average efficiency score also change little in the separate DEA evaluation from their values for those funds as a subsample of the combined sample. (Compare Table 4: column 3, rows 4, 6, and 3 with Table 2: column 4, rows 4, 6, and 3, respectively.)

However, when the statistics of the separate DEA evaluations of the two strategy groups are compared in Table 4, although Aggressive Growth funds have a higher proportion of efficient funds, 44% versus 28.6% [row 7, column 2 and column 3, respectively], nonetheless, the Equity Income inefficient funds tend to be closer to their efficient peers. This is shown by a higher average efficiency score (0.98 versus 0.96 [row 3, column 3 and column 2, respectively]), a much lower standard deviation (approximately one third, or 0.018 versus 0.051 [row 4, column 3 and column 2, respectively]), and a much higher minimum (in)efficiency score (0.92 versus 0.77 [row 6, column 3 and column 2, respectively]) than those of the Aggressive Growth group.

These results are similar to those reported in Table 2 for the funds as subsamples within the larger, joint sample. Thus, *within their own groups*, the Equity Income funds generally perform closer to their strategy frontier than do the Aggressive Growth funds to their own strategy production frontier.

Comparisons with more detail are tabulated in the Appendix, where Tables A2 (for inputs) and A3 (for outputs) compare the maxima, minima, averages, and standard deviations of each variable across the separate strategy groups. Appendix Table A4 compares the correlations among the variables for the two strategy groups.

Each of these two data envelopment analyses, then, provides a measure of the maximum performance possibilities afforded by its respective strategy as revealed by the actual performances of the best practitioners of that strategy. In addition, these analyses also provide evaluations of the performances of the various fund managements, evaluations which are especially pertinent because they are made relative to the possibilities within the scope of their respective strategies. However, for the purpose of evaluating strategies *per se*, the important contributions of this first stage are (i) the pair of efficiency frontiers representing the best performance achievable by their respective strategies and (ii) the data necessary to effect the projections for step two.

Section 3.2 Step Two: Projection of Inefficient Funds to Their Respective Efficiency Frontiers

For the second in this multi-step process, recall from the methodology exposition in Section 4 of Chapter 5, that a given fund under evaluation is represented by the vector of its (actual) performance data, $(\boldsymbol{x}_0, \boldsymbol{y}_0)$, where the subscript 0 signifies the particular fund being evaluated, and that the solution of the BCC output-oriented model [Chapter 5, Equation 6] yields optimal values (denoted by *) which determine [cf. Chapter 5, Equation 8]

$$\begin{aligned} \hat{y}_{r0} &= \phi_0^* y_{r0} + s_r^{+*} \geq y_{r0}, \quad r = 1,\ldots,s \\ \hat{x}_{i0} &= \quad x_{i0} - s_i^{-*} \leq x_{i0}, \quad i = 1,\ldots,m. \end{aligned} \qquad (1)$$

The values $\left(\hat{\boldsymbol{x}}_{0,}\ \hat{\boldsymbol{y}}_0\right)$ represent how the fund, were it efficient, should have performed relative to the other funds in the comparison. The difference between how it should have performed and how it did in fact perform is given by [cf. Chapter 5, Equation 9]:

$$\Delta y_{r0} = \hat{y}_{r0} - y_{r0} \geq 0, \qquad r = 1, \ldots, s$$
$$\Delta x_{i0} = x_{i0} - \hat{x}_{i0} \geq 0, \qquad i = 1, \ldots, m. \tag{2}$$

The existence and amount of managerial inefficiency, relative to the strategic potential, is given by Equation 2 whenever strict inequality obtains in any input or any output.

These calculations are performed for each inefficient fund identified in the previous DEA evaluations. The Δy_{r0} and Δx_{i0} are obviously zero for each efficient fund. Finally, step two is completed when two new data sets are constructed, using the projected values $\left(\hat{\boldsymbol{x}}_{0,} \ \hat{\boldsymbol{y}}_0\right)$ in place of the actual values of inefficient funds.

Section 3.3 Step Three: Evaluation of Performance of Mutual Fund *Strategies* by Comparison of Efficient Frontiers—Joint Exemplar DEA

The joint analysis that is reported in Section 2 above (the standard application of DEA) evaluated how *funds* executing the two strategies performed in practice. However, to evaluate the *strategies* themselves, the actual performance data of inefficient funds must be replaced by the values of their projections to the efficiency frontiers of their respective strategy groups. Each of these two, new sets of projected data represents the surface of best possible performance for its strategy. With the results of the first stage, steps one and two, the second stage proceeds to the analysis of the strategies, based on data unobscured by the shortcomings of individual performance. The two, new, projected-data sets are now combined into a joint sample so that the strategies themselves can be evaluated by data envelopment analysis based on data which now represent the full potential of each strategy.

The application of DEA in this step is not to actual funds evaluated on their observed performances, but rather to hypothetical "funds" as exemplars of the full possibilities of their respective strategies. This third-step analysis evaluates not the vectors ($\boldsymbol{x}_0, \boldsymbol{y}_0$) of actual practice (as in step one, separately within their respective strategies, or as in Section 2, combined into a joint sample regardless of strategy), but rather the vectors $\left(\hat{\boldsymbol{x}}_{0,}\ \hat{\boldsymbol{y}}_0\right)$ of best practice for their strategies. These best practice vectors are all points on their respective efficiency frontiers, which estimate the best possible performances for each strategy. Because they are determined from observed data, they represent realizable, practicable performance goals.

Summary results of applying this third-step DEA to the *projected* performance data for the two mutual fund strategies are presented in Table 5, which has the same structure as Table 2 for the joint analysis of the *actual* performance data. Since the data all represent points on their respective efficiency frontiers, the increase in the average efficiency scores, which approach 1, that is, full technical efficiency [row 3, columns 2, 3, and 4], and the sharp reduction in dispersion [row 4, columns 2, 3, and 4] as compared to the corresponding DEA results from the joint analysis using actual data [cf. Table 2] are not surprising. Whereas only one-sixth of all combined sample funds were efficient based on actual data, almost three-fifths (69 of 120, or 57.5% [column 2, row 7]) of all combined sample funds based on projected data are efficient. Interestingly, within the Aggressive Growth strategy subsample, the numbers of efficient and inefficient funds (4 and 46 [column 3, row 7 and row 8], respectively), despite being based on projected data, have not changed from the results based on actual performance data.

However, the numbers for the Equity Income subgroup are very different. After projection to their efficiency frontier, almost all of the Equity Income funds evaluate as efficient in this third-step DEA of the combined sample with projected Aggressive Growth funds. In particular, 65 of the 70 Equity Income funds [column 4, row 7 and row 2, respectively] evaluate as efficient based on projected data as compared to 17 of 70 [Table2: column 4, row 7 and row 2, respectively] based on actual data when evaluated

0	1	2		3		4	
1	SUMMARY STATISTICS DEA EVALUATION OF STRATEGIES	ageq97BCCO[1] joint sample		aggrw97[2] as subsample		eqinc97[3] as subsample	
2	number of DMUs (funds) =	120		50		70	
3	average of efficiency scores =	0.993029		0.983407		0.999901	
4	standard deviation of scores =	0.013644		0.017136		0.000500	
5	maximum score =	1		1		1	
6	minimum score =	0.937127		0.937127		0.996254	
7	number strategically superior DMUs =	69	57.50%[4]	4	5.80%[5]	65	94.20%[5]
8	number strategically inferior DMUs =	51	42.50%[4]	46	90.20%[6]	5	9.80%[6]

[1] the joint sample of *projected* **aggressive growth** and **equity income** funds combined, 1997 data, BCC Output-oriented DEA model [column 2]
[2] the *projected* **aggressive growth** funds [column 3], as a subsample of the joint sample
[3] the *projected* **equity income** funds [column 4], as a subsample of the joint sample
[4] as a percentage of the total, joint sample size, *i. e.*, 120 [column 2, row2]
[5] the strategy's percentage of the joint total number of *strategically superior* funds [column 2, row 7]
[6] the strategy's percentage of the joint total number of *strategically inferior* funds [column 2, row 8]

Table 6.5: Descriptive Statistics of DEA Evaluation of *Strategies* Based on Projected Data, with Statistics for the Subsamples of Each Strategy within the Joint Sample

together with funds in the Aggressive Growth strategy. The Equity Income strategy accounts for over 94% of the efficient funds [column 4, row 7], while the Aggressive Growth strategy accounts for less than 6% of the efficient funds [column 3, row 7]. Conversely, the Aggressive Growth strategy results in over 90% of inefficient funds [column 3, row 8], while the Equity Income strategy results in less than 10% of such funds [column 4, row8].

Section 3.4 Step Four: Performance of Mutual Fund *Strategies* by Comparison of Efficient Frontiers—Rank Analysis

This assessment of superior performance by the Equity Income strategy is next subjected to the Mann–Whitney rank sum test. In this last step, the test is applied to the combined sample of projected data in order to evaluate statistically the relative performance of the strategies. Here it is proper to speak of the performance of the two *strategies*, whereas the previous joint analysis of Section 2 referred to the performance only of *funds* which pursued the two strategies [cf. Tables 2 and 3]. These results, presented in Table 6, reaffirm and greatly strengthen the conclusion of the previous test. Moreover, the method of separating the strategy potential from the observed performances makes it possible to attribute the source of the difference to strategy rather than execution. The α confidence levels of virtually 0 reported in Table 6 mean that the hypothesis that Equity Income is a *strategy* superior to Aggressive Growth can be asserted with almost 100% confidence.

MANN-WHITNEY RESULTS	symbol	ag	eq
strategy subsample size	n	50	70
rank sum of strategy subsample	R	4604	2656
Mann-Whitney rank statistic of strategy subsample	U	171	3329
Z score of strategy subsample	Z	– 8.405161	8.405161
α confidence level for difference in strategy performance	α	≈ 0*	≈ 0*

*to 50 000 digits of precision as calculated with *Mathematica 3.0*

Table 6.6: Mann–Whitney Test Results on DEA Evaluations of Projected Data Comparing Two *Strategies*

However, the results do not mean that Aggressive Growth *funds* necessarily under perform Equity Income funds. For example, the DEA results reported in Table 5 indicated that four Aggressive Growth funds, indeed the same four funds identified in the DEA results for actual performance data in Table 2, were efficient. Moreover, when not handicapped by being compared with funds with a superior strategy, that is, when evaluated separately, recall that Table 4 revealed that nearly half of the Aggressive Growth funds were efficient as compared to only just over a quarter of the Equity Income funds. Thus, within their strategic limitations, a larger percentage of Aggressive Growth funds realize their potential than do Equity Income funds.

Section 4 Risk–Return Characteristics of Strategic Categories

Since much of the research in support of the Bowman[2] "paradox" is based on analysis of manufacturing firms, a natural question is whether the same phenomenon holds in the financial markets. In particular, does the greater liquidity contribute to an increased market efficiency which obviates the paradox? Although the model in this study of the mutual fund production function includes neither specific measures of revenue variability (variance, mean squared error, or β from the Capital Asset Pricing Model) nor other non-dispersion measures of risk, the comparison of strategic categories is designed to evaluate the performance of two strategies with putatively different risk characteristics. If the Bowman "paradox" does not hold in the mutual funds market, then the riskier strategic category should present individual funds which outperform the less risky category.

The data representing fund performance present neither variability nor dispersion

[2] Edward H. Bowman, "A Risk/Return Paradox for Strategic Management," *Sloan Management Review*, Spring 1980, pp. 17-31.

information nor time series from which they might be computed, thus no mean-variance model can be evaluated. However, Table 7 presents some summary information on the rank performance of efficient funds for each of the three returns variables: Total Annual Return 1997 (AnRet97), 3-Year Annualized Total Return (TRA3Y), and 5-Year Annualized Total Return (TRA5Y). Also included are data on the worst and best performers and the variances among the funds for each of the three variables within each strategy category. The data in Table 7 are summarized from tables in the Appendix.

The three source tables in the Appendix, Tables A8, A9, and A10, report, in parallel columns for the two strategic categories, the data for each fund in rank order for the three returns variables AnRet97, TRA3Y, and TRA5Y, respectively. The funds of each strategy category are identified by symbol for easier reference to the complete data tables (Tables A6 and A7 of the Appendix). The three tables display for each fund within each strategy, the following information, in column order: **RNK** (rank by the respective return variable within the strategy category); **FUND** (ticker symbol); the observed value of the return variable of interest—**AnRet97** (Total Annual Returns for 1997) in Table A8, **TRA3Y** (3-Year Annualized Total Returns) in Table A9, **TRA5Y** (5-Year Annualized Total Returns) in Table A10; the same return variable expressed in units of sample standard deviation—**ARSD** (AnRet97 in units of standard deviation) in Table A8, **3YSD** (3-Year Annualized Returns in units of standard deviation) in Table A9, and **5YSD** (5-Year Annualized Returns in units of standard deviation) in Table A10; and **EFF** (BCC-O efficiency score). At the foot of each table is a summary of the information on the performance of the efficient funds. These summaries also appear in Table 7 at the top

	Total Annual Return 1997			3-Year Annualized Total Return			5-Year Annualized Total Return		
	AnRet97	**AGGRW**	**EQINC**	**TRA3Y**	**AGGRW**	**EQINC**	**TRA5Y**	**AGGRW**	**EQINC**
	Efficient Funds			**Efficient Funds**			**Efficient Funds**		
1	TOTAL	22	20	TOTAL	22	20	TOTAL	22	20
2	AVG RNK	18.8	14.8	AVG RNK	19.1	19.8	AVG RNK	17.0	21.4
3	AVG/N	0.38	0.21	AVG/N	0.38	0.28	AVG/N	0.34	0.31
	Funds with Losses			**Funds with Losses**			**Funds with Losses**		
4	NUMBER	4	0	NUMBER	8	0	NUMBER	4	0
5	MIN	86.97	118.55	MIN	79.47	110.07	MIN	81.53	109.05
	Best Performer			**Best Performer**			**Best Performer**		
6	MAX	175.00	136.41	MAX	120.92	123.68	MAX	120.18	118.15
7	RNK TRA3Y	47	34	RNK AnRet97	11	6	RNK AnRet97	11	8
8	RNK TRA5Y	50	54	RNK TRA5Y	1	11	RNK TRA3Y	1	3
9	AVG RTN RNK	32.7	29.7	AVG RTN RNK	4.3	6.0	AVG RTN RNK	4.3	4.0
10	AVG RTN RNK/N	0.65	0.42	AVG RTN RNK/N	0.09	0.09	AVG RTN RNK/N	0.09	0.06
	Dispersion			**Dispersion**			**Dispersion**		
11	RANGE	88.03	17.86	RANGE	41.45	13.61	RANGE	38.65	9.10
12	STD DEV	12.925	3.817	STD DEV	8.460	2.536	STD DEV	7.042	2.105

Table 6.7: Returns Ranks for Efficient Funds, Funds with Losses, and Best Performing Funds, and Dispersions within Each Strategy Category for Each Returns Variables

[rows 1, 2, and 3] under the columns for the corresponding return variables.

Table 7 presents the summary information in three columnar sections, one for each returns variable. The data within each columnar section are further separated into groups of successive rows. The first group of data (labeled rows 1, 2, and 3) in each columnar section reports on the efficient funds for each strategic category. The second group (labeled rows 4 and 5) in each columnar section shows the funds reporting losses (that is, returns not greater than 100) for each strategic category. The third group (labeled rows 6 through 10) reports on the best performer in each strategic category. Finally, the fourth group in each columnar section, labeled rows 11 and 12, reports two dispersion measures, range and standard deviation, within each strategic category.

In the first group of each columnar section, row 1 reports the number of efficient funds for each of the two strategy categories. These are the same in each columnar section, that is, for each of the three return variables. Row 2 reports the average rank of the efficient funds for the respective return variable within each strategic category. Because the two strategy category samples are different sizes (50 and 70), average ranks are not directly comparable. Therefore, row 3 of each columnar section reports the average rank of efficient funds for the corresponding return variable normalized by the sample size. This provides a relative measure of where the average rank of efficient funds falls in the range of ranks for its strategic category sample. For example, the Equity Income (EQINC) strategic category (with a sample size of 70) has 20 efficient funds. Were they all to rank in the top 20 for a given return variable, then their average rank would be 10.5 and the normalized average rank would be 0.15. However, the

Aggressive Growth (AGGRW) category (with a sample size of 50) has 22 efficient funds. Were they all to rank in the top positions, their average rank would be 11.5 and their normalized average rank would be 0.23. This procedure does not solve the comparability problem, but it can be suggestive.[3]

The next group (rows 4 and 5) in each columnar section of Table 7 reports on the funds reporting losses for the given return variable. Row 4 displays the number of funds which reported losses for the respective return variable. Row 5 displays the actual value of the return variable for the lowest ranked fund in each of the two strategic categories.

The third group (rows 6 through 10) reports on the best performer within each strategic category for the respective return variable. Row 6 displays the value of the return variable for the best performer (top ranked firm) for the respective return variable for each strategy category. To indicate the consistency of strong performance, row 7 and row 8 report what rank that best performing fund achieved with respect to the other two returns variables. Row 9 shows the average of rank 1 (top performer for that variable)

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[3] This is a crude measure of the number and rank distribution of efficient funds. The greatest value the normalized average rank can attain for *any* sample size is 1, which always occurs when there is only one efficient fund and it ranks last. If N is the sample size, then such a singular efficient fund has rank N, its average rank is N, and its normalized average rank is $N/N = 1$. The smallest value the normalized average rank can attain varies depending on sample size, and, for any given sample size, it always occurs when there is only one efficient fund and it ranks first. Therefore, such a fund has rank 1, its average rank is 1, and its normalized average rank is $1/N > 0$. (In the present study, the smallest possible normalized average ranks are $1/50 = 0.02$ and $1/70 \cong 0.014$.) Normalized average ranks fall in approximately the middle of these ranges (that is, within $[1/N, 1]$) in several circumstances. In particular, if every fund in the sample were efficient, then their average rank would be $\Sigma n/N$, $n = 1,\ldots, N$, and the normalized average rank would be $(\Sigma n/N)/N = \Sigma n/N^2 = N(N+1)/(2N^2) = (N+1)/(2N) = ½+(1/(2N))$ which approaches ½ as N increases. The same value for normalized average rank obtains for any number of efficient funds $m < N$ whenever the m ranks are symmetrically distributed about the sample midrank. Thus, the normalized average rank is a measure of the rank distribution of a distinguished subsample within a sample. The closer the value approaches 1, then the fewer the number of distinguished items and the more they are distributed toward the bottom of ranks. Conversely, the closer the normalized average rank approaches $1/N$, the fewer the number of distinguished items and the more they are distributed toward the top of ranks.

and that fund's ranks for the remaining two return variables, as listed in rows 7 and 8. Row 10 shows the normalized value of the average rank shown in row 9.

Finally, the fourth group shows measures of dispersion in the values. Row 11 lists the range for each variable within each strategy category, that is, the difference between the MAX (row 6) and the MIN (row 5) of the respective returns variables for each strategy category. The last row 12 lists the sample standard deviations for the respective returns variables within each strategy category.

The data on the three returns variables in Table 7 and the source tables in the Appendix show two different kinds of extreme value performance. An individual fund may report an extraordinary return for one period, but if the extreme (high) values in the streams of returns from a group of funds are produced by different funds in each succeeding period, for a *shareholder* to seek consistently high returns would require shifting the shareholder's investment from period to period in anticipation of the top performer of the upcoming period. If, however, the same fund can produce successive extraordinary returns, the shareholder investment problem becomes picking the fund that will generate the series of high returns. Despite the many theoretical and empirical problems with the mean-variance model of risk-return,[4] if that model were to hold, then at least some funds from the supposedly high risk strategy group should produce returns significantly greater than those of the low risk strategy group even though the high risk group may perform more poorly in general, that is, in the mean.

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[4] In addition to Bowman, "Risk/Return Paradox," see, for example, Timothy R. Ruefli, "Mean-Variance Approaches to Risk-Return Relationships in Strategy: Paradox Lost," *Management Science*, Vol. 36, No. 3 (March 1990): pp. 368-380; and Eugene F. Fama and Kenneth R. French, "The Cross-Section of Expected Stock Returns," *The Journal of Finance*, Vol. 47, No. 2 (June 1992): pp. 427-465.

According to Table 7 and Tables A8, A9, and A10, Aggressive Growth seems like a high-risk investment strategy compared to Equity Income. In terms of spread, AGGRW exhibits almost five times the range of EQINC for AnRet97, three times for TRA3Y, and more than four times for TRA5Y. Furthermore, the sample standard deviation of the AGGRW strategy funds is greater than 3 ⅓ times that of the EQINC strategy funds for each of the three returns variables.

The category AGGRW also appears more risky when viewed in the more specific, pertinent, and common terms of risk, namely, with regard to the probability of loss and the amount of probable loss[5]. In those terms, at least four AGGRW funds exhibit losses for each of the three variables, the worst performers losing between 15 and 20 per cent of investments, whereas no EQINC fund suffered losses during the period of study. Therefore, the frequency of loss is certainly higher in the AGGRW category and the amount of loss is substantial. In 1997, the worst performer in the AGGRW category lost over $19 million. In another sense of frequency of loss, the worst performer from the AGGRW category for TRA5Y lost on average almost 18.5 per cent every year for the five years 1993-1997.

In terms of extraordinary (high) returns, these simple summary data are suggestive but not conclusive. The top AnRet97–ranked Aggressive Growth fund had a much greater return than the top AnRet97–ranked Equity Income fund: 175.00 versus 136.41 [first columnar section, row 6] (that is, 75.0% returns versus 36.4% returns).

[5] P. Fishburn, "Foundations of Risk Measurement. I. Risk as Probable Loss," *Management Science*, Vol. 30, No. 4 (April 1984), pp. 396-406; James M. Collins and Timothy W. Ruefli, "Strategic Risk: An Ordinal Approach," *Management Science*, Vol. 38, No. 12 (December 1992), pp. 1707-1731.

However, the top seven EQINC funds had returns exceeding the second ranked AGGRW fund. Also, the top nine EQINC funds had greater ARSD (AnRet97 as measured in units of standard deviation) than did the second ranked AGGRW fund as measured in units of standard deviation (ARSD column), that is, Equity Income had heavier upside tails. Furthermore, all 70 EQINC funds had AnRet97 exceeding the returns of the 23rd (heavy dashed overline in Table A8) and all lower ranked AGGRW funds. Thus, all 70 EQINC funds had returns exceeding the average returns of AGGRW funds (means for each strategy category are marked by heavy solid line in Tables A8, A9, A10). Finally, for AnRet97, the only Aggressive Growth fund (American Heritage, AHERX) reporting higher returns than the first-ranked Equity Income fund (175.00 to 136.41, or almost 38.6 percentage points of greater return) also reported the lowest TRA5Y (5-year annualized return) of all AGGRW funds. It was also the third smallest AGGRW fund in terms of Total Net Assets.

The two categories also differ in the returns performance rank of efficient funds. For example, 7 (32%) of the 22 efficient (light gray fill in the Appendix tables) AGGRW funds score below their category mean for AnRet97, whereas only 2 (10%) of the 20 efficient EQINC funds score below their AnRet97 mean.

In terms of the average five-year returns, two AGGRW funds score higher than the top two EQINC funds but the differences are small: 120.18 and 118.78 versus 118.15 and 117.89. Moreover, the top 40 EQINC funds score higher than the 10th ranked AGGRW fund for TRA5Y. However, for the average three-year returns (TRA3Y), the top 11 EQINC funds score higher than the first-ranked AGGRW fund and the top 56

EQINC funds score as well or better than the fourth-ranked AGGRW fund for TRA3Y.

Section 5 Strategic Analysis

Section 3 demonstrated the application of data envelopment analysis to answer a broad or general question: is there a difference among the general strategies that govern the practices of the thousands of investment firms which manage trillions of dollars in assets for millions of investors in this country and the world? In this initial, pair-wise comparison, which also represents a first demonstration of a new methodology for the evaluation of strategies, the answer is an unequivocal yes. The nonparametric Mann-Whitney test confirms the interpretation of the DEA results. However, there is a great deal more to the DEA results than just the summary answer to the basic question, a conclusion which follows from the description of two efficient production surfaces and their relation to each other. These more complex results provide not only the summary judgment but also the basis for detailed analysis of the factors underlying the general conclusion. Moreover, the examination of this extensive data can inform the evaluation and revision of the model specification for strategic analysis.

The extensive results of the data envelopment analysis provide the material for two kinds of detailed strategic analysis. The primal, envelopment form of the DEA model provides the coefficients of convex combinations of the efficient peers which serve as benchmarks for the improvement of inefficient funds to their efficient potential. Analysis of the frequencies and values of these benchmark coefficients helps identify the funds which represent the strongest performance, and analysis of their performance (or exemplar) values facilitates the identification of the combinations and extents of performance required for success. In addition to the members of the best practice peer group and their proportions in benchmarking inefficient funds, the primal DEA formulation also provides the projection values of efficient performance. The examination of the projection values and, in particular, of the differences between the projection and original, performance values, or such differences viewed as percentage changes, clearly and explicitly identifies how far from efficiency each inefficient fund lies and how much it must improve. Such examination also identifies the particular dimensions in which performance was best and in which it was worst for each inefficient fund.

The dual or multiplier form of the DEA formulation, as given in (5) of Chapter 5, provides another set of resultant data. For each fund, it provides a complete listing of the weights which produce the best evaluation for that fund. The variables of the dual optimization problem represent those weights which combine the fund's performance values, amongst all the performances of the sample, to yield the best possible appreciation. The optimization drives the selection of weights so that the ratio of the combination of outputs to the combination of inputs approaches or attains 1.0, consistent with the performance values of the other DMUs in the sample, that is, consistent with the constraint equations (and variable restrictions). The analysis of these weights reveals the relative importance of the several dimensions represented in the data. Thus, the DEA result is not just a summary conclusion, but also provides an analytic product which serves as the means for investigating the grounds of the summary result. The dual variables actually show the rate of increase in the efficiency score per unit of increase of each input and each output.

Table 5 summarized the results of step three, the data envelopment analysis of the efficiency-projected data for mutual funds from both strategy categories evaluated in a joint sample. The Mann-Whitney results were prefigured by the simple classification of Table 5, which showed that, of 120 funds in the combined sample, each evaluated as if it were efficient in its use of its own strategy, 69 strategy-exemplars were efficient and 51 were inefficient. Of those efficient, 65 were Equity Income exemplars (of a total 70 Equity Income funds, or almost 93%). Four efficient exemplars were Aggressive Growth (of 50, or 8%). The worst performers for each strategy category also exhibit substantial differences: 6.3% inefficiency for the lowest scoring Aggressive Growth fund versus less than 0.4% for the lowest Equity Income fund. Thus, the strategies perform very differently with respect to supporting efficient exemplars for mutual fund investing and with respect to the nature of poor performance.

However, all four efficient Aggressive Growth funds were originally efficient with respect to their own strategy category (results from step one); whereas, of the 65 efficient Equity Income exemplars, only 17 (or 26.2%) were originally efficient. The remaining 48 Equity Income funds which evaluated as efficient in the strategy comparison (step three) were all projections from their originally inefficient performances, as adjusted in step two.

Section 5.1 Benchmarking Peer Groups: Reference Sets and Their Convex Combinations

For the 51 inefficient fund exemplars, DEA results also provide the efficient projection values and the coefficients of convex combinations of efficient points which generate them. Such combinations of efficient points constitute the reference sets, the best practices peer groups, for the inefficient points. The proportions in which the reference set values are combined to produce the efficient projection of an inefficient point are given by the lambda variables of the envelopment form of the DEA program. Of the 69 efficient exemplars, only 18 appear in the reference set of other (inefficient) funds. The remaining 51 efficient funds do not appear in the best practices, benchmark group for inefficient funds and are reference points only for themselves. Tables A15, A16, and A17 of the Appendix present summary comparisons of the 18 efficient funds which do appear in these reference sets. Table A15 reports the results for the Equity Income funds which appear in more than one reference set; Table A16 reports on Equity Income funds which appear in only one reference set; and Table A17 reports on the three Aggressive Growth funds which appear in reference sets.

Table 8 displays the number of inefficient funds according to the size of their reference set, that is, according to the number of efficient funds the convex combination of which equals the efficient projection value of the inefficient funds. Thus, only 18

Size of Reference Set	1	2	3	4	5	Totals
number of inefficient funds with this size reference set	8	21	14	6	2	51
number with AgGrw reference funds	0	3	12	5	2	22
number with EqInc reference funds	8	21	14	6	2	51

Table 6.8: Number of Strategically Inferior Exemplars According to the Size of the Reference Set
and the Strategy Categories of Funds in Their Reference Sets (from the Results of Step Three)

efficient funds represent the efficient projections of all 51 inefficient funds and constitute the 126 reference set participations (8∗1+21∗2+14∗3+6∗4+2∗5=126). Moreover, as is clear from the Appendix Tables, reference set participation among these 18 funds is not evenly distributed.

One fund, GAEI^ (Table A15, first data column), appears in the reference set of 43 (84.3%) of the 51 inefficient funds. Mutual fund stock ticker symbols are used in this study as DMU identifiers for DEA computations, and all such symbols end in X. The trailing caret (^), which replaces the final X in GAEIX, serves to indicate the use of efficiency-projected values in place of original observations for the stage three (strategy comparison) evaluations. Thus, the exemplar which figures so prominently in providing guidance to other, inefficient and strategically dominated funds is itself a projection of a fund which was originally inefficient within its own, Equity Income, strategy class. Furthermore, the contributions it makes to the efficiency projections of inefficient funds, summarized in Table 9, are also significant. In 29 of those 43 cases (67.4%), the fund exemplar GAEI^ appears with a coefficient greater than 0.5. Note also, that of the eight strategically dominated fund exemplars which have a single fund in their reference set (first data column, Table 8), GAEI^ is the singleton reference set member of three of them (last column, Table 9).

λ	$\lambda<.1$	$.1\leq\lambda<.2$	$.2\leq\lambda<.3$	$.3\leq\lambda<.4$	$.4\leq\lambda<.5$	$.5\leq\lambda<.6$	$.6\leq\lambda<.7$	$.7\leq\lambda<.8$	$.8\leq\lambda<.9$	$.9\leq\lambda<.1$	$\lambda=1$	$\lambda>0$
#	3	2	3	3	3	5	6	6	3	6	3	43

λ – ranges of lambda values for GAEI^ in the convex combinations for efficient projection points
– number of reference sets in which GAEI^ appears with a lambda value in the column range

Table 6.9: Distribution of Lambda Values for GAEI^, an *Equity Income* Fund, in the Convex Combinations Representing the Efficient Projections of Inefficient Exemplars.

All of the other six Equity Income exemplars which appear in multiple reference sets represent original values, that is, those points reflect the actual performances of mutual funds (see Appendix Table A15). Two of these, VEIPX and PRFDX, appear in 18 and 16 reference sets, respectively, but their contributions are more limited. The lambda values for VEIPX attain a maximum at 0.636364 and average 0.206369 (Appendix Table A15, column 3), while those for PRFDX are more limited still (Table A15, column 4). Of the remaining four, HWEQX (Table A15, column 6), which belongs to five reference sets, has a maximum lambda value of 1.0 and an average of 0.802758. These high values result from the fact that it is the singleton member of the reference set (therefore with a lambda value of 1) for three inefficient exemplars, and, in the fourth set, its coefficient has a value of 0.777778.

The remaining eight Equity Income exemplars which appear in reference sets contribute to the convex combination of an efficient projection point only once each (see Appendix Table A16). Of these eight, three represent original performance measures, while the other five are efficient projection exemplars, the status of which is indicated by the ^ in place of a final X in the ticker symbol.

The results in Table 5 indicate that only four of the Aggressive Growth mutual funds appear at the strategy efficiency frontier (the results of step three). All four also represent originally efficient funds as determined in step one, that is, with respect to their own strategic category. Three of these funds (AHERX, FMILX, and PAGRX) appear in the reference sets for other funds (see Appendix Table A17) and, together, belong to 22 reference sets, 14, 5, and 4, respectively (23 reference set memberships in 22 reference sets, that is, two of these funds appear in the same set only once). However, only AHERX and FMILX belong to the 5 of these 22 reference sets which represent improvements of Equity Income funds. Moreover, their contributions are limited: for AHERX, its three lambda values average 0.027336, and for FMILX, its two values average 0.078439.

Besides the limited contributions of efficient Aggressive Growth funds in the reference sets of inefficient Equity Income funds, they also have limited participation in most of the reference sets of inefficient Aggressive Growth funds. Table 8 (row 3) shows the number of Aggressive Growth funds which appear in the reference sets of different cardinalities. All eight inefficient funds with singleton reference sets are Aggressive Growth funds, but in every case, the single reference fund is an Equity Income fund (Table 8, column 2, row 3 and row 4).

For the reference sets with two elements each (Table 8, column 3), Aggressive Growth funds appear in only three sets (14.3%), although all 21 of the inefficient funds corresponding to these sets are Aggressive Growth funds. In these three cases, however, the contributions of the Aggressive Growth exemplars are substantial: 0.847514 and 0.322835 for FMILX; 0.384615 for PAGRX.

For the group of funds for which reference sets contain three efficient funds (Table 8, column 4), Aggressive Growth funds appear in 12 of the 14 sets. Of the 14 inefficient funds, only one is Equity Income and the one Aggressive Growth fund in its reference set, AHERX, has a lambda value of 0.065540. For the remaining 13 inefficient Aggressive Growth funds in this group, the Aggressive Growth fund in the reference set has a lambda value greater than 0.5 in only three cases. The only inefficient fund with two Aggressive Growth funds in its reference set also belongs to this group, but the sum of their lambda values is less than 0.5 (0.444444 = 0.443651 + 0.000793, for PAGRX and AHERX, respectively). The fund FMILX appears in eight reference sets with substantial weights: a maximum of 0.944589, a minimum of 0.278365, and an average of 0.466060.

In the group of six funds for which reference sets have cardinality of four (Table 8, column 5), two are Equity Income funds, but the Aggressive Growth fund (AHERX in both cases) has reference coefficients of only 0.001348 and 0.015119. Three of the inefficient Aggressive Growth funds in this group have significant Aggressive Growth referents: lambdas equal to 0.507676, 0.367048, and 0.316157, PAGRX and FMILX twice, respectively. The reference set of the last fund (also Aggressive Growth) of this group comprises only Equity Income exemplars.

Finally, the two inefficient funds with reference sets containing five members are both Equity Income, and each has one Aggressive Growth exemplar in its reference set (FMILX in both cases). In one instance, FMILX appears with a small coefficient (0.030185) and in the other with a moderate coefficient, 0.126694, the largest for an Aggressive Growth fund in the reference set of an inefficient Equity Income fund. Thus, in only one case (of five) does an Aggressive Growth fund provide even moderate guidance for the improvement of an inefficient Equity Income fund. Further, for 29 of the 46 inefficient Aggressive Growth funds (63%), their reference sets do not include efficient Aggressive Growth exemplars. For 16 of the remaining 17 funds, efficient

Aggressive Growth funds appear with lambdas between 0.278365 and 0.944589. These are summarized in Table 10.

λ	λ<.1	.1≤λ<.2	.2≤λ<.3	.3≤λ<.4	.4≤λ<.5	.5≤λ<.6	.6≤λ<.7	.7≤λ<.8	.8≤λ<.9	.9≤λ<.1	λ=1	λ>0
#	6	1	1	8	2	2	1	0	1	1	0	23

λ **-** lambda value ranges of FMILX, AHERX, and PAGRX in reference sets
 # - number of reference sets in which FMILX, AHERX, and PAGRX appear with the given lambda values

Table 6.10: Distribution of Lambda Values for the Three *Aggressive Growth* Funds, FMILX, AHERX, and PAGRX, Which Appear in Reference Sets

Section 5.2 Targets: Efficiency Projection Values

In addition to examination of the efficient peer groups and the relative significance of the two strategies in providing benchmarks, another approach to the comparison of strategies based on the extensive results from DEA is through the analysis of the projection values of inefficient funds. An important caveat in this regard is the difference between the entities (DMUs) in steps one and two and the entities in step three. The entities in the early stages are actual mutual funds, the values evaluated are their actual performance measures, and the projection values for inefficient funds are given by reference to the actual performance of their efficient peers within their own strategic categories. However, many of the entities under evaluation in the third step do not represent actual mutual funds, many of which have already been transformed by the earlier process so that they represent the strategy potential without the complicating factor of individual management. As was seen in the case of GAIEX, these exemplars of strategic potential need not represent actual funds to be important indicators of strategically dominating performance.

In the present case, the projection value results are very different for the two strategies. As indicated in Table 5, since the Aggressive Growth strategy contributed only 4 of its 50 funds as efficient strategy exemplars, the remaining 46 funds would all have to decrease inputs or increase outputs to reach the production surface. On the other hand, since 65 of the efficient exemplars represented Equity Income, only five remaining funds from this strategy would be subject to projections different from their initial values. Tables 11 and 12 compare the two strategies based on summary statistics regarding the projection values (efficient funds are considered to project to themselves), the differences between initial and projection values, and those differences expressed as percentage changes for each of the four output and six input measures. The summary values for each strategy group are presented in apposing columns beneath each output and input column heading. The labels for the statistics are self-explanatory (**min**[imum], **max**[imum], **mean**, **st**[andard]**dev**[iation]), except perhaps, **min > 0** and **max < 0**. Since efficient exemplars exhibit no changes in their values, their incorporation in the scope of the statistic **min** forces the minimum change in an output dimension to be 0.00. The inclusion of the **min > 0** statistics indicate the smallest positive changes required to bring *inefficient* funds to the production surface along output dimensions. Similarly, the **max < 0** statistics indicate the least non-zero change along input dimensions.

A detailed examination would analyze, *inter alia*, the transformations of the inefficient exemplars separately from their efficient peers; however, these two aggregated tables offer some interesting considerations, nonetheless. For example, with respect to the three output measures, AnRet97, TRA3Y, and TRA5Y, the means of the final, or projection, values for the two strategies are nearly equal: between only 0.35 and 1.37 percentage points apart (row of **mean** in the block of **projection values** under the three output columns in Table 11). However, the standard deviations for Aggressive Growth funds range from about 3.5 to 4.8 times those of the Equity Income funds (row of **stdev** in the block of **projection values** under the three columns). Because almost all of the Equity Income funds were efficient, they exhibit primarily zero differences between initial and projection values, whereas, since almost all of the Aggressive Growth funds were inefficient, they exhibit primarily positive differences. Thus, the means and standard deviations of the *differences* in the aggregated tables are all between one and two orders of magnitude greater for Aggressive Growth than for Equity Income exemplars. A similar contrast is exhibited by the minima and maxima, although the differences are not as pronounced: the **min > 0** for TRA5YN are nearly identical for the two strategies. Within strategy group differences between **min** and **max** are also striking. For example, the largest differences among Aggressive Growth funds range from over 8 times to over 36 times those of the Equity Income funds for these three output measures (rows of **min** and **max** in the block of **differences** between projections and observations).

With respect to NetAss, the fourth output measure, the results are somewhat different. The largest Equity Income fund is more than 70% larger than the largest Aggressive Growth fund and the range of Equity Income total net assets is also more than 70% larger than that of Aggressive Growth. (See the **min** and **max** rows of the **projection**s block under the NetAss column of Table 11.) Between the strategy groups, the difference between their **min > 0** values for NetAss is greater than four orders of

	outputs ▶	AnRet97		TRA3Y		TRA5Y		NetAss	
	strategies ▶	AGGRW	EQINC	AGGRW	EQINC	AGGRW	EQINC	AGGRW	EQINC
projection values[1]	min	114.220000	126.640000	91.330000	116.760000	81.530000	113.130000	8.400000	36.200000
	min>0								
	max	175.000000	136.410000	122.510000	123.680000	120.180000	118.150000	12999.740574	22277.100000
	max<0								
	mean	131.416047	131.770993	120.303140	121.676509	116.871988	117.346937	1941.277110	2026.780326
	stdev	7.023311	1.768666	4.882361	1.399272	5.448912	1.135905	2864.072253	3876.348144
differences between projections and observations[2]	min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	min>0	0.266240	0.018331	0.083874	0.016934	0.015305	0.016327	0.181064	0.000006
	max	31.195455	3.219982	16.355993	0.451839	7.428338	0.885735	4924.814931	558.171675
	max<0								
	mean	6.968254	0.067634	4.839449	0.011956	1.948158	0.017962	364.171020	10.489304
	stdev	6.714264	0.398564	4.110176	0.060411	2.009755	0.108719	746.310607	68.076594
differences as percentage changes	min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	min>0	0.002100	0.000100	0.000700	0.000100	0.000100	0.000100	0.000100	0.000300
	max	0.308000	0.025300	0.154100	0.003800	0.067100	0.007700	9.999000	5.046800
	max<0								
	mean	0.059384	0.000524	0.042882	0.000099	0.017178	0.000154	1.142898	0.077594
	stdev	0.063444	0.003124	0.038125	0.000504	0.018082	0.000943	2.113076	0.603658

[1] Efficient funds are considered to project to themselves, thus the underlying values are the projection values for 51 inefficient strategy exemplars and the starting (same as ending) values for the 69 efficient strategy exemplars.

[2] The values for observations in this case are not necessarily original performance data, since the starting values for step three are the result of step two: the original values of efficient funds and the projections of inefficient funds as evaluated within their respective strategy categories

Table 6.11: Comparison between Strategy Categories Based on Summary Statistics of the Projection Values, Differences between Projection and Observation Values, and Differences as Percentage Changes, for Outputs, Based on Results from the Step Three DEA Strategies Evaluation

inputs ▶		%Cash		TurnoverRatio		ExpnRatio		FrontLoad		DfrdLoad		12b-1	
strategies ▶		AGGRW	EQINC	AGGRW	EQINC	AGGRW	EQINC	AGGRW	EQINC	AGGRW	EQINC	AGGRW	EQINC
projections	min	0.000000	0.000000	2.000000	5.000000	0.650000	0.450000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	min>0	0.200000	0.200000					0.835096	0.029711		5.000000	0.300000	0.001292
	max	4.145455	22.300000	1180.000000	192.000000	5.850000	1.690000	5.500000	5.750000	0.000000	5.000000	0.300000	1.000000
	max<0												
	mean	1.552422	2.235984	60.529141	27.721486	1.053298	0.936039	0.599922	0.505922	0.000000	0.071429	0.006000	0.040736
	stdev	1.037691	2.959445	165.153336	29.588599	0.710470	0.164273	1.209890	1.494081	0.000000	0.597614	0.042426	0.140453
differences	min	-7.776325	-25.957615	-195.703416	-87.723786	-1.055308	-0.679706	-5.000000	-5.369918	-5.000000	-1.000000	-1.000000	-1.000000
	min>0												
	max	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	max<0	-0.054331	-0.308032	-1.414955	-2.315288	-0.003991	-0.679706	-0.063049	-1.273168	-0.129695	-1.000000	-0.023026	-0.059292
	mean	-1.492697	-0.513113	-25.426811	-1.656195	-0.146478	-0.009710	-0.894966	-0.094901	-0.182594	-0.014286	-0.107038	-0.018704
	stdev	1.747608	3.198389	39.560094	10.887662	0.244539	0.081240	1.664005	0.657471	0.896134	0.119523	0.232498	0.122860
percentages	min	-0.869800	-0.927100	-0.923100	-0.456900	-0.535700	-0.338200	-1.000000	-0.933900	-1.000000	-1.000000	-9.999000	-1.000000
	min>0												
	max	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	max<0	-0.034000	-0.220000	-0.036500	-0.117300	-0.004200	-0.338200	-0.052900	-0.933600	-1.000000	-1.000000	-1.000000	-1.000000
	mean	-0.349912	-0.032419	-0.334460	-0.013643	-0.105248	-0.004831	-0.222702	-0.026679	-0.080000	-0.014286	-0.587564	-0.042857
	stdev	0.276381	0.145757	0.322286	0.071677	0.146620	0.040423	0.373726	0.156685	0.274048	0.119523	1.446728	0.203997

Table 6.12: Comparison between Strategy Categories Based on Summary Statistics of the Projection Values, Differences between Projection and Observation Values, and Differences as Percentage Changes, for Inputs, Based on Results from the Step Three DEA Strategies Evaluation

magnitude. Unsurprisingly, both the mean and standard deviation for Equity Income projected values are larger than those for Aggressive Growth. However, the largest change between initial and projection value is almost nine times greater for Aggressive Growth than for Equity Income (cf. the **max** row of the **differences** block under the NetAss column).

Table 12 presents similar information with respect to the six input measures. The input variable %Cash represents a cost to the fund in interest and appreciation earnings foregone over the interest from cash management, even though a substantial cash reserve may be needed to pursue an investment style which relies heavily on market timing. The Equity Income category exhibits the larger maximum value at 22.3% (the efficient fund CEIFX) over the Aggressive Growth category maximum at 4.15% (for CSTIX) [cf. row **max** of the **projections** block under %Cash column of Table 12]. The Equity Income mean is 40% larger than that for Aggressive Growth and the standard deviation is almost three times as great. However, although Equity Income also has the largest absolute contraction to bring a fund to the efficient surface (25.96 versus 7.78 percentage points [row **max** of the **differences** block]), Aggressive Growth displays the larger average relative contraction, 1.49 percentage points versus 0.51 percentage points (row **mean** of the **differences** block). Thus, in general, the Equity Income funds were able to maintain their higher cash reserves because they were efficient in other respects, whereas the Aggressive Growth funds (although the highest *initial* value was only 10.00%) were forced to make adjustments because so many were inefficient. Of the 50 Aggressive Growth funds, 35 of the 46 inefficient funds (or about 76%) required reductions in

%Cash, while, of the 70 Equity Income funds, 4 out of 5 inefficient funds (or 80%) required contractions.

A similar relationship obtains with respect to the 12b–1 input variable. Again, Equity Income funds have a maximum projection value 3 ⅓ times and an average projection value 6 ¾ times that of Aggressive Growth funds (rows **max** and **mean**, respectively, **projections** block, under 12b–1 column of Table 12). However, the mean contraction for Aggressive Growth funds along the 12b–1 input dimension to bring inefficient funds to the efficient strategy surface was almost 5 ¾ times that for Equity Income funds (row **mean**, **differences** block).

For DfrdLoad, only five funds had non-zero initial values, three Aggressive Growth and two Equity Income funds, and the four inefficient funds were contracted to zero. Only one, efficient, Equity Income fund remained with a non-zero value for DfrdLoad after projection.

For FrontLoad, Aggressive Growth and Equity Income funds have similar, minimum, maximum, and mean projection values (0.00 and 0.00, 5.50 and 5.75, 0.60 and 0.51, respectively [rows **min**, **max**, and **mean**, in the **projections** block under the FrontLoad column of Table 12]). However, Aggressive Growth funds averaged almost nine times the contraction from initial values as the average of Equity Income funds (row **mean** in the **differences** block). This larger average reduction in FrontLoad was again due to the large number of Aggressive Growth funds which were inefficient (only two of the efficient funds have positive FrontLoads), whereas the Equity Income funds with positive FrontLoad were all efficient except for two, which required some reduction in this input.

For TurnoverRatio, again the imbalance in results is influenced by the disparity in the number of efficient funds in the two strategy categories: 32 Aggressive Growth funds must reduce this input to attain efficiency, while only 3 Equity Income funds require reductions. The maximum projection value for Aggressive Growth is more than six times that of the Equity Income maximum; and the mean of the final, efficient, projected value is more than twice as large for Aggressive Growth than for Equity Income (rows **max** and **mean**, **projections** block, TurnoverRatio column of Table 12). That is, even after contraction of this input, Aggressive Growth funds, on average, have twice the turnover ratio of Equity Income funds. The mean reduction in TurnoverRatio for Aggressive Growth funds to reach efficiency is more than 15 times that for Equity Income funds. Indeed, the largest *reduction* for an Aggressive Growth fund is greater than the largest, final efficient projection value for an Equity Income fund.

Finally, for ExpnRatio, as in the other input dimensions, the differences in changes in values to attain efficiency between the two strategy groups depends in large part on the great difference in number of efficient funds within each category. Only one Equity Income fund requires reduction in this dimension to attain efficiency, whereas 29 Aggressive Growth funds do. Thus, while the means of final efficient values for the two strategies are close (1.05 for Aggressive Growth and 0.94 for Equity Income [row **mean**, block **projections**, column ExpnRatio of Table 12]), the mean *reduction* to attain these values is 15 times greater for Aggressive Growth than the mean reduction for Equity Income (row **mean**, block **differences**).

Section 5.3 Dual Multipliers: Optimal Weights

The preceding discussion has reviewed some of the several means for detailed analysis of the factors underlying the efficiency scores provided by data envelopment analysis. For each fund, in addition to the summary measure of performance (the efficiency score), the primal program (the envelopment form) of DEA directly represents and provides the values for the individualized, efficient peer groups for inefficient funds, the coefficients of combination ($\boldsymbol{\lambda}_j$s) among the efficient peers for benchmarking the potential performance of inefficient funds, and the target values of potential performance for guiding the improvement process. Alternatively, the dual program, or multiplier form, of DEA directly represents the weights ($\boldsymbol{v}_i$s for inputs, $\boldsymbol{u}_r$s for outputs) according to which the values of observed performance are combined to provide the best evaluation and, thus, the highest possible efficiency score for each fund in the sample under analysis. An examination of these virtual weights also yields insight into the basis of efficient performance.

A theorem of DEA states that any DMU with a unique minimum for any input or a unique maximum for any output is efficient in the BCC model.[6] This may result from placing all the weight on the unique value so that no convex combination of other DMUs can equal the extreme value. Table 13 illustrates this property by presenting the first-ranked mutual funds for each of the four outputs and the six inputs. The table has two parts: the upper left-hand corner exhibits the ranks, together with their associated data,

[6] Cooper, Seiford, Tone, *DEA: Comprehensive Text*, p. 90.

for each top ranked fund with respect to the four outputs and two of the inputs. In each of these cases, the output or input value represents a unique extreme and the corresponding fund is efficient.

The first listed fund (because it is the top-ranked fund for the first listed output value, AnRet97) is the Aggressive Growth fund AHERX. However, for all of the remaining five outputs and inputs of this section of the table (that is, for outputs and inputs which exhibited unique maxima and minima, respectively), AHERX ranked 120, last in the combined sample of funds from both strategy categories. It is a stark example of the theorem: a unique maximum in one output allows this fund with bottom ranks in the remaining outputs and inputs to evaluate as efficient. The result from the DEA computations is

$$\text{eff} = (\boldsymbol{v}\,\boldsymbol{x}_{\text{AHERX}} - v_0) \,/\, \boldsymbol{u}\,\boldsymbol{y}_{\text{AHERX}}$$

$$= \frac{(0.015493,\, 0.0,\, 0.050316,\, 0.0,\, 0.0,\, 0.0)\,(0.0,\, 1180.00,\, 5.85,\, 0.0,\, 0.0,\, 0.0)^{\mathrm{T}} - (-\,0.705654)}{(0.0,\, 0.0,\, 0.005714,\, 0.0)\,(91.33,\, 81.53,\, 175.00,\, 8.4000)^{\mathrm{T}}}$$

$$= \frac{(0.00 + 0.00 + 0.294346 + 0.00 + 0.00 + 0.00) + 0.705654}{(0.00 + 0.00 + 1.00 + 0.00)} = \frac{1}{1} = 1,$$

where $\boldsymbol{v}$ is the vector of optimal input weights, $\boldsymbol{u}$ is the vector of optimal output weights, $\boldsymbol{x}_{\text{AHERX}}$ is the vector of input observations for fund AHERX, $\boldsymbol{y}_{\text{AHERX}}$ is the vector of output observations for fund AHERX, and v_0 is the unrestricted scalar multiplier associated with the primal (envelopment form) convexity constraint, $\boldsymbol{e}\,\lambda = 1$, in which $\boldsymbol{e}$ is the summation vector (every component equals 1) and λ is the vector of coefficients for the corresponding reference set. Since AHERX is efficient, the components of λ are all 0

	fund[1]		efficiency score	Outputs (with unique maxima)				Inputs (with unique minima)		Inputs (non-unique minima)			
				AnRet97	TRA3Y	TRA5Y	NetAss	Turnover Ratio	Expn Ratio	%Cash	Front Load	Dfrd Load	12b-1
unique extremes	agAHERX	rank[2]	1	1	120	120	120	120	120	12	39	39	38
		data	1.000000	175.00	91.33	81.53	8.40	1180.00	5.85	0.00	0.00	0.00	0.00
	eqCEIFX	rank	1	7	1	73	111	111	95	119	69	69	68
		data	1.000000	133.35	123.68	117.07	73.80	119.00	1.10	22.30	0.00	0.00	0.00
	agFMILX	rank	1	98	54	1	30	114	44	78	100	91	71
		data	1.000000	124.63	120.92	120.18	1416.00	142.00	0.94	2.90	3.00	0.00	0.00
	eqFEQIX	rank	1	65	58	71	1	62	2	54	43	43	42
		data	1.000000	129.98	120.86	117.10	22277.10	23.00	0.65	1.80	0.00	0.00	0.00
	agPAGRX	rank	1	9	99	94	119	1	112	1	34	34	33
		data	1.000000	132.68	115.43	115.47	16.90	2.00	1.46	0.00	0.00	0.00	0.00
	eqVEIPX	rank	1	59	32	62	19	60	1	109	62	62	61
		data	1.000000	131.17	122.48	117.46	2530.00	22.00	0.45	6.00	0.00	0.00	0.00
non-unique extremes	agPAGRX[3]	rank	1	9	99	94	119	1	112	1	34	34	33
		data	1.000000	132.68	115.43	115.47	16.90	2.00	1.46	0.00	0.00	0.00	0.00
	eqSBCI^	rank	1	32	10	32	47	34	40	57	1	7	2
		data	1.000000	132.24	122.55	118.01	864.14	15.96	0.92	1.95	0.00	0.00	0.00
	agINAG^	rank	85	108	80	59	51	86	33	86	19	1	88
		data	0.996686	123.25	119.74	117.61	724.8974	59.03	0.88	3.55	0.00	0.00	0.00
	eqPLIN^	rank	1	24	11	21	71	15	84	38	2	8	1
		data	1.000000	132.51	122.52	118.14	608.57	14.99	1.00	0.95	0.00	0.00	0.00

[1] Each fund identifier (ticker symbol) is prefixed with **ag** or **eq** to indicate the strategy categories Aggressive Growth and Equity Income, respectively. Those ending in **X** indicate that the values are original performance observations, while those ending in **^** indicate projected values.

[2] The ranks are based on the values of the combined sample of 120 fund strategy exemplars used in the step three analysis.

[3] Although this fund is the first ranked for the corresponding input variable, **%Cash**, its value, 0.00, is not unique: there are 12 funds with the same value, 8 of which are efficient and 4 inefficient. Its rank of 1 is due to the secondary ranking criterion, **TurnoverRatio**, for which it does have a unique minimum, as exhibited two rows above. Similarly for the ranks of funds based on the last three inputs—**FrontLoad**, **DfrdLoad**, and **12b–1**: all funds in the table (except FMILX) exhibit 0.00 for each of these inputs, so the rankings, despite the secondary and tertiary ranking conditions (the other three inputs of the group) are spurious and depend on peculiarities of the ranking algorithm (Microsoft Excel).

Table 6.13: Ranks and Associated Data for Funds Which Exhibit Extreme Values in Outputs and Inputs

except for that associated with AHERX itself, which must equal 1.

However, the attainment of a unique minimum input value or unique maximum output value does not imply that the associated weight will always bring the corresponding product to 1, while the remaining components of the weights vector are all zero to yield products equal to zero for the remaining input or output values. This point is demonstrated by the next fund in Table 13, CEIFX, an Equity Income fund. In this case, non-zero weights are associated with both TRA3Y, the input for which CEIFX exhibits a unique maximum, and AnRet97, the input for which it ranked seventh.

Furthermore, the existence of a unique extreme value in the inputs or outputs is not even guaranteed to have a non-zero weight. The Equity Income fund FEQIX, fourth fund row of Table 13 shows a unique maximum for the output NetAss, which, however, has an associated optimal weight of 0.00; and its only non-zero output weight is associated with TRA3Y, for which it ranked 58. The details of virtual multipliers and the associated data values for the six Table 13 funds which exhibit unique maxima or minima are presented in the Appendix, Table A20. Appendix Table A18 lists the optimal virtual weights for all 120 funds and is followed by a summary Table A19.

For the remaining four input dimensions, the top ranked funds are separated at the bottom and far right of Table 13. Although these "rank 1" funds all attain the minimum value, their minima are not unique. Since distinct ranks based on identical values are indeterminable, the given ranks are based on secondary and tertiary criteria. In the case of %Cash, 12 funds exhibit the minimum value of 0.00; of these, 8 are efficient and 4 inefficient. These four inefficient funds (SECU^, BBSE^, DGTIX, and DELTX) exemplify the requirement of a *unique* input minimum to imply efficiency. The assignment of rank 1 for %Cash to PAGRX is based on the secondary sorting criterion of TurnoverRatio, for which PAGRX does have a unique minimum, as shown in the upper left section of Table 13. (The tertiary criterion is ExpnRatio.)

The same difficulty is especially evident in the case of the last three inputs (bottom three table rows, last three table columns of Table 13). In each of these three cases, the supplementary criteria are the values of the other two inputs. Thus, for FrontLoad, the secondary criterion is DfrdLoad and the tertiary is 12b–1. For the input FrontLoad, 90 funds exhibit the minimum value of 0.00; for DfrdLoad, 115 funds exhibit the minimum 0.00; and for 12b–1, 91 funds present the same minimum value. Thus, the top ranks displayed in the bottom section of Table 13 are spurious and depend on peculiarities of the sorting algorithm (in this case, Microsoft Excel, which makes recourse to the order prior to sorting, which is alphabetic by fund names—not ticker symbol—for Aggressive Growth and then Equity Income). Any of the other candidate funds might have been assigned rank 1, including the other funds listed in the table (except FMILX) which all present 0.00 values for these three inputs. Indeed, for the input DfrdLoad, the algorithm assigned rank 1 to INAG^, which is an inefficient Aggressive Growth fund (efficiency score of 0.996686 and efficiency rank of 85) and which again demonstrates the necessity of a unique minimum or maximum for the implication of efficiency.

The next tables, Table 14 and Table 15, present summary statistics on the optimal multipliers or virtual weights. Table 14 presents the summaries regarding the input variables, with the information for the strategic categories in apposing columns under the

weights	v_0		v_1		v_2		v_3		v_4		v_5		v_6	
variables	convexity		%Cash		TurnoverRatio		ExpnRatio		FrontLoad		DfrdLoad		12b–1	
strategies	ag	eq	ag	eq	ag	eq	ag	eq	ag	eq	ag	eq	ag	eq
min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
min > 0	0.705654	0.810573	0.000207	0.000109	0.000075	0.000025	0.010661	0.004428	0.000811	0.000078			0.028078	0.001796
max	1.067091	1.002432	72.786843	7.951257	0.500000	0.026695	0.149522	1.428571	0.006281	0.196467	0.000000	0.000000	32.714574	0.013192
avg	0.977032	0.793720	1.459722	0.248920	0.010149	0.002512	0.010339	0.167259	0.002801	0.002984	0.000000	0.000000	0.656561	0.000445
stdev	0.152431	0.293869	10.293042	1.259691	0.070691	0.004419	0.024891	0.276363	0.002898	0.023470	0.000000	0.000000	4.626218	0.001670
# > 0	49	62	14	45	15	58	20	69	27	17	0	0	5	11
% > 0	98.00%	88.57%	28.00%	64.29%	30.00%	82.86%	40.00%	98.57%	54.00%	24.29%	0.00%	0.00%	10.00%	15.71%

Table 6.14: Summary of Optimal Input Weights for *Aggressive Growth* and *Equity Income* Exemplars, from the Dual Program, or Multiplier Form, of Step Three, DEA of Strategy Categories

weights	u_1		u_2		u_3		u_4			
variables	TRA3Y		TRA5Y		AnRet97		NetAss		efficiency	
strategies	ag	eq	ag	eq	ag	eq	ag	eq	ag	eq
min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.937127	0.996254
min > 0	0.008663	0.000612	0.001597	0.002776	0.000822	0.003344	0.00000001	0.00000033		
max	0.008663	0.008423	0.009126	0.008663	0.006479	0.007617	0.005921	0.000641	1.000000	1.000000
max < 1									0.999873	0.999862
avg	0.000173	0.001964	0.007764	0.001634	0.000426	0.004154	0.000119	0.000019	0.983407	0.999901
stdev	0.001225	0.002892	0.002403	0.003201	0.001437	0.003383	0.000837	0.000102	0.017136	0.000500
# > 0	1	28	47	16	5	45	20	18	50	70
% > 0	2.00%	40.00%	94.00%	22.86%	10.00%	64.29%	40.00%	25.71%	100.00%	100.00%

Table 6.15: Summary of Optimal Output Weights and of Efficiency Scores for *Aggressive Growth* and *Equity Income* Exemplars, from the Dual Program, or Multiplier Form, of Step Three, DEA of Strategy Categories

multiplier (and associated input) heading, while Table 15 presents the same information for output variables, with a summary column of the efficiency scores for the two strategies. The second from the last row of each table, labeled **# > 0**, reports for each of the strategies the number of funds for which the virtual multiplier is greater than zero for the given input or output. The last row of each table, labeled **% > 0**, reports the same values as the row above but as percentages of the total number of funds in the respective strategy categories.

An interesting fact appears in the first data column of Table 14, labeled "convexity." As presented in the last row of that column, the number of funds for which the v_0 multiplier is non-zero is 49 (98.00%) for Aggressive Growth and 62 (88.57%) for Equity Income funds, the highest number for any multiplier among the Aggressive Growth funds and the second highest among Equity Income. This multiplier is a consequence of the fact that the efficiency surface bends, in a piece-wise linear fashion, around those points closest to the origin along input dimensions and those farthest from the origin along output dimensions. This surface is part of the convex hull of the cluster of n points located in $(m + s)$-dimensional space, where n is the number of entities (decision making units, or funds, in the present study) and m and s are the numbers of inputs and outputs, respectively. Were it not for the convexity constraint, the efficiency surface would not include as many of the efficient points. For example, in the CCR model of DEA, the efficiency surface is a hyperplane through the origin and intersects the point with the steepest angle relative to the input dimensions. (This is why the CCR model can never have more efficient points than the BCC model, but the BCC model typically does have more than the CCR model applied to the same data.) The funds for which $v_0 = 0$ are all efficient funds: Aggressive Growth PAGRX, and Equity Income DELDX, FEQIX, FISEX, HWEQX, MAPOX, MSIVX, COIG^, and VEIPX. (See Appendix Table A20 for the complete listing.)

At the other extreme is the multiplier $\boldsymbol{v}_5$ associated with the input variable DfrdLoad. The columns of data for both Aggressive Growth and Equity Income show that this multiplier is zero for every fund evaluated. No fund could exploit this input positively in order to maximize its performance score. In the next column of Table 14, for multiplier $\boldsymbol{v}_6$ associated with input 12b–1, the number of positive values is almost as low. Only 5 (10.00%) of the Aggressive Growth funds and 11 (15.71%) of the Equity Income funds exploit this input to maximize their efficiency scores.

The remaining input multipliers display an interesting pattern. When more than half of the funds of one strategic category exhibit positive multipliers associated with a given input, then fewer than half of the funds in the other category exhibit positive multipliers for that input. Equity Income funds exploit (that is, have positive associated multipliers $\boldsymbol{v}_3$ and $\boldsymbol{v}_2$ for) the inputs ExpnRatio and TurnoverRatio at high rates, 98.57% and 82.86%, respectively, versus 40.00% and 30.00%, respectively, for Aggressive Growth funds. Only for the multiplier $\boldsymbol{v}_4$ associated with input FrontLoad, is the relationship reversed: 27 (54.00%) of Aggressive Growth funds against 17 (24.29%) of Equity Income funds have positive values for this weight.

Table 15 presents similar data for the multipliers associated with the four output measures, as well as a final column summarizing efficiency scores. In the case of the multiplier for TRA3Y, only PAGRX among the Aggressive Growth funds has a positive $\boldsymbol{u}_1$. It is also the only Aggressive Growth fund to have a zero valued $\boldsymbol{v}_0$; its only positive input multiplier is $\boldsymbol{v}_2$, associated with TurnoverRatio, for which it exhibits a unique minimum (see Table 13, fifth fund row **agPAGRX**, TurnoverRatio column); and $\boldsymbol{u}_1$ (corresponding to TRA3Y) is its only positive output multiplier. However, 28 (40.00%) of the Equity Income funds establish their efficiency ratings with positive multipliers for TRA3Y.

The input AnRet97 also is infrequently exploited by Aggressive Growth funds, only five of which have positive multipliers $\boldsymbol{u}_3$. Of these, only AHERX—which exhibits, among all 120 funds, the unique maximum for AnRet97 (and, therefore, ranks first) but ranks last for the other inputs and outputs for which unique minima or maxima exist—is efficient and $\boldsymbol{u}_3$ is its only positive output multiplier. The other four are also among the 47 Aggressive Growth funds with positive multipliers $\boldsymbol{u}_2$ for TRA5Y, the output measure for which the largest number of Aggressive Growth funds have a positive multiplier. However, AnRet97 is the output measure for which the largest number of Equity Income funds has a positive multiplier. Moreover, Equity Income funds exploit the output variables more evenly than do the Aggressive Growth funds. Whereas among the Aggressive Growth funds, the number of funds with corresponding positive multipliers ranges from 2% for TRA3Y to 94% for TRA5Y, among the Equity Income funds, the numbers range from 22.86% for TRA5Y to 64.29% for AnRet97 (cf. bottom row, labeled **% > 0**, Table 15).

The following Table 16 displays, by strategy category, the number of exemplars

Distribution of Nbrs of Positive Weights		1 multiplier			2 multipliers			3 multipliers			4 multipliers			5 multipliers			6 multipliers			7 multipliers		
		v	*u*	*v+u*	*v*	*u*	*v+u*	*v*	*u*	*v+u*	*v*	*u*	*v+u*	*v*	*u*	*v+u*	*v*	*u*	*v+u*	*v*	*u*	*v+u*
Aggressive Growth	**eff**	1	3	0	0	1	1	2	0	0	1	0	1	0	0	2	0	0	0	0	0	0
		2.0%	6.0%			2.0%	2.0%	4.0%			2.0%		2.0%			4.0%						
	ineff	0	25	0	21	20	0	23	1	6	2	0	32	0	0	7	0	0	1	0	0	0
			50.0%		42.0%	40.0%		46.0%	2.0%	12.0%	4.0%		64.0%			14.0%			2.0%			
	both	1	28	0	21	21	1	25	1	1	3	0	33	0	0	9	0	0	1	0	0	0
		2.0%	56.0%		42.0%	42.0%	2.0%	50.0%	2.0%	2.0%	6.0%		66.0%			18.0%			2.0%			
Equity Income	**eff**	0	39	0	4	24	0	25	2	1	16	0	9	20	0	32	0	0	20	0	0	3
			55.7%		5.7%	34.3%		35.7%	2.9%	1.4%	22.9%		12.9%	28.6%		45.7%			28.6%			4.3%
	ineff	0	0	0	0	1	0	5	4	0	0	0	0	0	0	1	0	0	4	0	0	0
						1.4%		7.1%	5.7%							1.4%			5.7%			
	both	0	39	0	4	25	0	30	6	1	16	0	9	20	0	33	0	0	24	0	0	3
			55.7%		5.7%	35.7%		42.9%	8.6%	1.4%	22.9%		12.9%	28.6%		47.1%			34.3%			4.3%

Table 6.16: Number of Fund Strategy Exemplars, by Strategy and by Efficiency, Which Exhibit Positive Input and Output Virtual Weights, Step Three DEA of Strategy Categories

Numbers of Positive Weights		*v*			*u*			*v + u*		
		min	max	avg	min	max	avg	min	max	avg
Aggressive Growth	**eff**	1	4	2.75	1	2	1.25	2	5	4.00
	ineff	2	4	2.59	1	3	1.48	3	6	4.07
	both	1	4	2.60	1	3	1.46	2	6	4.06
Equity Income	**eff**	2	5	3.80	1	3	1.43	3	7	5.23
	ineff	3	3	3.00	2	3	2.80	5	6	5.80
	both	2	5	3.74	1	3	1.53	3	7	5.27

Table 6.17: Minimum, Maximum, and Mean Number of Positive Input and Output Virtual Weights, by Strategy and by Efficiency, for Fund Strategy Exemplars, in Step Three DEA of Strategy Categories

which exhibit positive virtual weights for inputs and outputs. Table 17 presents the minimum, maximum, and average number of positive weights for inputs and outputs by fund strategy exemplars as broken out by strategy and efficiency status. Table 16 breaks out the totals for each multiplier and Table 17 for each statistic according to efficiency or inefficiency status to show the different distributions of the two strategies. Not noted in the tables, but controlling the distribution range for the number of positive multiplier values, is the fact that the first vector of dual multipliers, $\boldsymbol{v}$, has seven components (one for the convexity constraint and six for each input) and the second vector, $\boldsymbol{u}$, has four components (one for each output).

The main difference between the strategies evident from Table 16 is the general rightward shift of distribution among Equity Income as compared to Aggressive Growth funds. The largest number of positive $\boldsymbol{v}$ components among Aggressive Growth is four, exhibited by only three funds (bottom row **both** in Aggressive Growth block, column **4**/$\boldsymbol{v}$, Table 16). (This largest realized number of positive components is not the maximum theoretically possible, which is seven, one for each of six inputs plus one for the convexity constraint.) However, the great majority of funds in this strategy exhibit either two (21 funds or 42.0%) or three (25 funds or 50.0%) positive $\boldsymbol{v}$ components (same row, columns **2**/$\boldsymbol{v}$ and **3**/$\boldsymbol{v}$). The lower tail of this distribution (same row, column **1**/$\boldsymbol{v}$) is represented by PAGRX, which, as discussed previously, has a single positive input multiplier and a single positive output multiplier. Thus, for the input and convexity multipliers, Aggressive Growth funds demonstrate a range of four, from 1 to 4, in a single peaked distribution, skewed to the high end.

In contrast, the Equity Income category has a large number of funds (all efficient) which exhibit, for their range, the largest number of positive $\boldsymbol{v}$ components: 20 funds (28.6%) have five positive input multipliers (Table 16, block Equity Income, row **both**, column **5**/$\boldsymbol{v}$). Moreover, 16 funds (22.9%) exhibit four positive $\boldsymbol{v}$ components; and 30 funds (42.9%), including all five inefficient Equity Income funds, show three positive $\boldsymbol{v}$ components (same row, columns **4**/$\boldsymbol{v}$ and **3**/$\boldsymbol{v}$). The least number of positive input multipliers is two, which are exhibited by four of the funds (same row, column **2**/$\boldsymbol{v}$). Thus, for Equity Income funds, the range of four is the same as for Aggressive Growth, but it extends from two through five in a double-peaked distribution. These differences are also reflected in the average number of positive convexity and input multipliers—2.60 versus 3.74 (Table 17, column $\boldsymbol{v}$/**avg**, row **both** of Aggressive Growth block and row **both** of Equity Income block, respectively).

As was noted in the discussion based on Table 15, the funds of the two strategies tend to exploit different output multipliers in maximizing their efficiency scores. However, the distributions of the *number* of positive output multipliers are similar. For both strategies, the range of three extends from one to three positive components. Also, for both strategies, the majority of funds have only one positive output multiplier: 28 or 56% for Aggressive Growth and 39 or 55.7% for Equity Income (Table 16, column **1**/$\boldsymbol{u}$, row **both** of Aggressive Growth block and row **both** of Equity Income block, respectively). All but one of the remaining funds among the Aggressive Growth category have two positive weights, while all but six of the remaining Equity Income funds exhibit two positive output weights (same rows, columns **2**/$\boldsymbol{u}$ and **3**/$\boldsymbol{u}$).

Of course, the distributions of the total number of positive virtual weights are conditioned on the two sets of distributions just examined. Since Aggressive Growth presents only one fund with just one positive $\boldsymbol{v}$ component, there could be at most one fund with only two—the least possible number—total positive components, and, since only one fund has three positive $\boldsymbol{u}$ components, there could be at most one fund with seven—the largest possible number—total positive components. Indeed, the first case obtains but the second does not. For the Aggressive Growth strategy category, the distribution of the total number of positive virtual weights has a range of five (from two through six) and is somewhat symmetric about the 66% of funds which exhibit a total of four positive weights (Table 16, row **both** of the Aggressive Growth block, columns **1**/ $\boldsymbol{v} + \boldsymbol{u}$ through **6**/ $\boldsymbol{v} + \boldsymbol{u}$).

For Equity Income, the least possible total number of positive virtual weights is three and the greatest possible total is eight. Again, the first case is realized (by only one fund) but the second is not. The distribution has a single mode at five (the middle of the range) total positive components with 33 (or 47.1%) of the funds, but it is skewed to the upper tail with 34.3% of funds exhibiting a total of six positive virtual weights (Table 16, row **both** of the Equity Income block, columns **3**/$\boldsymbol{v} + \boldsymbol{u}$ through **7**/$\boldsymbol{v} + \boldsymbol{u}$).

This section has only introduced the possibilities for detailed analysis of the factors underlying efficiency performance results from data envelopment analysis. The extensive and detailed results from DEA provide for each inefficient fund exemplar the benchmark of a best practices peer group, specific values to target for performance improvement, and the weights which combined its actual performance to best effect.

This is one of the great practical advantages of DEA for those charged with managing an individual fund or family of funds: concrete, specific, and realizable guidelines for the improvement of the performance of any individual fund or group of funds. However, by examining the overall patterns of best practices peers, the extents of their superior performances, and the reliance on or significance of the input and output measures which established their dominance, the industry and strategy analyst can begin to identify those factors which contribute to sustainable competitive advantage.

Section 6 Conclusions

The chief result of this investigation is the demonstration of a method by which strategies themselves may be evaluated relative to one another as distinguished from the use made of those strategies by management. The conceptual clarifications of management strategies offered in Chapter 2 now have practical implications with a tractable method that permits the evaluation of strategies *per se* as separated from the varying abilities of managers to implement them. In general, the analysis of strategies, designs, policies, plans, or other species of decision-making has been limited and problematic whenever an empirical, *ex post facto* evaluation was undertaken, because an empirical evaluation is necessarily based on the data of actual performance, which confounds the intent or purpose with other factors, chief among which are the abilities of managers to realize those decisions. The application of the four-step DEA procedure here described permits an empirically based estimation of the full potential of strategies unobscured by the shortcomings and inefficiencies of their implementation.

Economic theory provides an additional interpretation of these results. The short-run performance of managers in implementing strategies is marked by shortcomings and inefficiency, which the discipline of the market will eliminate to produce long-run performance equal to the potential of the strategy. Having a readily applicable method of evaluating individual performance against a practicable and relevant standard of best performance helps shortens the time to realize long-run improvements and reduces the accumulation of losses.

This new method was applied to the mutual funds because special characteristics of that industry facilitate strategic analysis. First, the legal requirements of publicly declaring and conforming to an investment strategy make the specification of the strategy unambiguous. In addition, comprehensive data on the activities of all funds provides the basis for quantitative evaluation of how managers perform with respect to the strategies pursued. Beyond the characteristics which facilitate the technical analysis, models set in the context of financial markets can contribute to and benefit from the extensive mathematical investigations developing in this area, which then serve to represent many of the activities of firms involved in production of real outputs by employment of real assets and other resources. The work of Black, Merton, and Scholes, among others,[7] in the pricing of financial options as applied to the evaluation of real or nonfinancial investments is an example of how financial theory is used to model the essence of

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[7] Robert C. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, Vol. 4, No. 1 (Spring 1973), pp. 141-183; Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, Vol. 81, No. 3 (May-June 1973), pp. 637-654; Robert C. Merton, "Applications of Option-Pricing Theory: Twenty-Five Years Later," *American Economic Review*, Vol. 88, No. 3 (June 1998), pp. 323-349; Gordon Sick, "Real Options, " Chapter 21, pp. 631-691, in R. A. Jarrow, V. Maksimovic, and W. T. Ziemba, eds., *Finance*, Handbooks in Operations Research and Management Science, Vol. 9 (Amsterdam, NL: Elsevier, 1995); Martha Amram and Nalin Kulatilaka, *Real Options: Managing Strategic Investment in an Uncertain World*, Boston, MA: Harvard Business School Press, 1999.

capitalist production. Finally, since the mutual fund industry represents a significant part of the wealth holdings of many persons and institutions and serves important functions in the U. S. economy, it merits study by strategic management as a significant industry in national and international economic development.

Although the particular characteristics of mutual funds make them conducive to this type of strategic analysis, the same methodology may be applied, albeit with more effort, to other industries. For firms involved in real production, *ex post facto* (that is, empirical) evaluation must still separate the potential of strategy from the limitations of its realization.

The evaluations of managerial performance can now be set in the appropriate context of strategic performance, that is, these managerial performance measures show the extent to which managers were able to realize the potential afforded by the strategy. Thus of two managers, one with higher absolute numbers may prove to have underperformed the potential of the strategy defining his purpose and scope, while another with lower absolute numbers may prove to have fully exploited the potential of a more limited strategy.

In addition, since mutual funds are invested portfolios, this method also provides a means of comparing and evaluating different portfolios in a clear and quantitative manner as an alternative, or at least an adjunct, to the standard methods and models of portfolio evaluation. This method evaluates mutual funds for both their short-run and long-run relative performances. Thus, the application of this methodology to mutual funds may provide guidance to potential mutual fund shareholders with respect both to the strategic type of fund and to the particular fund managements within a strategy group under consideration.

Chapter 7 Conclusions and Future Directions

Section 1 Contributions and Limitations

This dissertation has sought to open a new direction in the understanding and evaluation of strategic management through two complementary approaches. The first approach has been to seek conceptual clarity by means of a new categorical schema or conceptual framework for the fundamental concepts or factors of strategic management. The second approach has been the development of a computationally tractable, mathematical methodology which evaluates empirical performance data to test the conceptual distinctions established in the new framework. These two approaches are exemplified in the four contributions offered in this dissertation.

The first contribution is the introduction of a definition—for the basic concept of *strategy*—adopted from accounting. It provides the basis for an analytical framework which facilitates making distinctions and organizing relationships among the various concepts related to strategic management. That the discipline of strategic management should discover the definition for its fundamental concept, strategy, in another, albeit related, field of study and practice is not unusual. The field of strategic management, which evolved from business policy, has marshaled theory and practice from many disciplines in the process of formulating an understanding of what strategic management is and has been and what it might be. Scholars in the field have borrowed from systems theory, economics and industrial organization, cognitive psychology, learning theory, political science, cultural anthropology, and ecology. That the essential meaning of its primary concept should be so suitably formulated in the accounting literature is especially natural in view of the function accounting plays in the detailed, material evaluation of management efforts to execute strategy.

The definition of strategy as "a plan of action that is used to guide or control other plans of action" is both general and precise. As a general definition, it expresses the fundamental cybernetic quality of strategy so that it applies to any circumstance in which activity must be directed to achieve a goal. Thus, it is appropriate for military and political settings, non-profit or planned economic organizations, individual aspirations in daily life and mass movements in historic, social struggle, as well as for business in a relentlessly competitive market. This general applicability is the result of a definition which has captured the essential quality of the concept.

The definition is explicitly hierarchical; it reflects a layered approach to the formulation and implementation of strategy. Therefore, strategy depends on the setting or domain of discourse. Corporate strategy subsumes the strategies of its business units and its functional departments. This general approach to goal attainment is also seen in very different disciplines, such as the practice of top-down, structured programming, which is designed to maintain the primary purpose of the project as the driver and as the constraining context for every particular, subsidiary, or detailed development. Within the domain of discourse, the emphasis remains on the whole or overall strategy, which establishes the context for substrategies, tactics, and related factors, like goals, resources, and environment.

This definition is also precise. Its referent is unambiguous, so that in any situation with adequate information, the strategy can be identified or it can be determined that no strategy exists. It is a ready and understandable standard for identifying strategies. Moreover, the clarity and precision of this definition facilitate distinguishing all the related concepts, such as goals, resources, environments, and competitors, from strategy itself. With such analytical discrimination, the associated concepts can be organized by their relations to strategy and therefore to each other. This definition also does not prescribe how the strategy is formulated or emerges, and therefore, it may be applied by analysts from the planning as well as the learning school.

Finally, an important consequence of this definition is the distinction between strategy, as a *plan* of action, and the *implementation* of strategy, an instance of attempting to execute the planned actions and to realize the strategy's purpose. This difference is well recognized among practitioners and boards of directors and its importance can be measured by the huge values lost to failures of strategic implementation. The methods developed here rely on this distinction, by using the data of actual execution to infer the full potential of the intent, in a procedure that keeps one distinct from the other. This distinction is commonly lost in many definitions of strategy offered in the strategic management literature. Especially when undeclared, strategies must be inferred by observers from the results of efforts to implement them. However, to say that gradual disclosure, or emergence, through the accretion of practical result, is the same thing as strategy is to confuse the observer with the actor.

The second contribution of this study is the explication and demonstration of a methodology for evaluating and comparing strategies. This method is an application of data envelopment analysis, which extrapolates from the empirical results of strategic implementations, with all their variations and limitations, an estimate of the best that the strategy could offer, which then provides the grounds for comparing strategies according to their fullest realizations. Within the setting of the mutual fund industry, where strategies are explicitly declared and where comprehensive, detailed, and standard performance data are readily available, a two-stage process for the analysis of strategies is detailed. This method has four major benefits.

Based on data envelopment analysis, the methodology analyzes the performance data of funds within the same strategy group to produce a production frontier, a surface generated by the Pareto-Koopmans efficient performers of the group. The procedure is a peer-based comparison wherein only best performing peers are used to estimate the maximum possible production in place of a theoretically given optimum. Thus, the first major benefit of this method is an empirically based estimate of the maximum potential of executing the given strategy.

The resulting production frontier provides the best standard against which to measure the performance of managers responsible for executing that strategy. The optimum potential for a given strategy is the most pertinent and consequential basis for management evaluation. Moreover, DEA not only provides the best standard for evaluation, but its evaluation of management effort is both realistic and actionable: realistic, because it is based on the data of actual practice; and actionable, because poor performers are referred to the efficient performers whose practices and results are closest to their own, providing peer guides for improvement. The primary attention of academic researchers in strategic management has been the formulation of effective strategies, but the primary concern of those in the practice of strategic management has been the successful implementation of strategies. The results of DEA applied to strategic evaluation provide the second major benefit of this method to the evaluation of management and the improvements of its efforts, namely, the most relevant standard of evaluation and quantitative, realistic, and actionable measures for performance improvement. Furthermore, the method is computationally tractable and easy to use software is readily available.

Once the efficiency frontiers of strategies have been determined, the methodology developed in this study then provides a third benefit, the comparison of competing strategies. The comparison is based on the best estimates of the strategies themselves, unobscured by the limitations of inadequate implementations. In the example presented in this study, the funds are all primarily equity funds; all participate in the same market; the data represent the same periods and the same economic conditions; and the sample selection criteria are the same. Thus, the comparison of strategies is made *ceteris paribus*, and the resulting evaluations reveal the performance of one strategy relative to the other. Moreover, the comparison results are not just qualitative; they are quantitative and measure the extent to which one strategy is superior to the other. This methodology provides an objective means, with controls for relevant factors, for assessing the relative merits of one strategy against another.

A fourth benefit of this methodology is that it can reveal new, blended strategies. In the hypothetical example and in the test case comparison of Aggressive Growth versus Equity Income, when the production frontier of one strategy does not completely dominate the other, the two surfaces cross and part of each strategic surface dominates part of the other surface. The DEA evaluation of strategies in this case generates a new efficiency surface against which points representative of each of the two strategies are evaluated. At least part of this new surface does not belong to either of the original surfaces in the comparison but is instead generated as convex combinations of points representing the two strategies. Therefore, the procedure described in this study not only evaluates strategies and provides a measure of how much one is better than the other, it also indicates, in appropriate circumstances, how strategies superior to either of the two, for some range of the inputs, may be developed by blending the two in stipulated convex combinations.

Conceptual perspicuity and precision and powerful mathematical techniques, however, do not resolve all problems in the analysis and evaluation of management strategies. Even with a clear idea of what strategy is in general, the strategies in specific instances are often difficult to discern. They may be intentionally concealed or disguised; they may be poorly specified or ill formed; they may change over time. The third contribution of this dissertation, then, is the demonstration of the peculiar suitability of the mutual funds industry for testing concepts and methods of strategic evaluation.

The foremost characteristic of mutual funds that facilitates strategic analysis is the legal requirement that funds publicly declare their investment strategies and that they may change their strategies only through an elaborate and public process. Therefore, in this industry, strategies are certain, definitive, stable, and complete. Without necessitating speculation or inference of their strategies, mutual funds serve as a laboratory for methods of strategic evaluation.

The gauge of strategic potential must be performance in practice. Mutual funds constitute a public financial market. Legal and practical requirements provide comprehensive and detailed data, the meaning and precision of which are known and uniform for all participants in the market. The data to evaluate fund performance are provided through free and fee-based financial information services. Many of these services, as well as analysts, periodicals, and journals, provide evaluations of mutual funds against which results of this method may be compared. Not every fund activity is reported in the available data. However, data reflecting so many activities is readily accessible that, at least initially, the performance measurement process is reduced to acquiring the large stores of publicly available financial data.

Of course, there are differences between physical product and financial companies. Whereas mutual funds can convert and reallocate assets relatively quickly, easily, and completely, for a manufacturing firm, conversion requires more time and effort and is subject to the loss of sunk costs. Nonetheless, mutual funds capture essential features of the firm in a capitalist market. This similarity is reflected in the growing trend of managing material product firms with methods derived from the financial markets, such as the portfolio approach for diversified corporations and project evaluation by means of real options.

The fourth and final contribution of this dissertation is given in the evaluation of the Aggressive Growth and Equity Income strategies for mutual fund investing. This demonstration of the methodology provides a concrete evaluation of two investment strategies with the unambiguous conclusion that Equity Income is a superior strategy compared to Aggressive Growth over most of the range of inputs.

The comparison also reveals, however, that, within the scope of their own strategies, the Aggressive Growth group of funds had a higher percentage of efficient funds than did the Equity Income group. That is, the Aggressive Growth group had a 1 ½ times greater rate of efficiency than the rate of the Equity Income group, each with respect to its own strategy. This means that, on average, the Aggressive Growth managers realized more of the potential of their strategies than did the Equity Income managers, but that the Equity Income managers pursued a strategy that had much more potential return. It is often observed that a mediocre strategy, well implemented, can produce better results than a superior strategy, poorly implemented. In the present case, however, the Equity Income managers did well enough in implementation and their strategy offered enough more potential, that the superior effort of Aggressive Growth managers could not overcome the others' advantage.

Another difference between the two strategic groups may indicate the relative difficulties in executing the two strategies. The range of efficiency scores for the Aggressive Growth funds is almost three times that of the range for Equity Income funds. Equivalently, since a full efficiency score (of 1) represents an upper bound for scores of each group, the worst performer among Aggressive Growth funds was almost three times farther from its efficient frontier than was the worst performer among Equity Income funds. Therefore, while only slightly more than one quarter of the Equity Income funds was efficient, nonetheless, the inefficient funds were not far from optimal performance.

Conversely, while almost half of Aggressive Growth funds were efficient, those which failed to attain efficiency generally exhibited much greater shortcomings. The evaluation presented in this study reveals one strategy to be harder to implement, to involve greater costs, and still, at its best, with rare exceptions, to produce smaller returns. Clearly, this analytical method can provide comprehensive and nuanced evaluations of strategic performance.

One more consequence follows from this analysis. Although the returns to funds as modeled in this application did not reflect risk in the mean-variance or mean-volatility sense, that is, the DEA evaluations were not based on "risk adjusted" returns, risk, nevertheless, is central to the analysis. First, the comparison of the two strategies can be considered a comparison of strategies involving different degrees of risk. In this test, the riskier strategy, consistent with other empirical studies, fares worse. Second, characteristics of risk appear in the results of the weaker strategy, namely, a significantly greater downside variation in performance and higher average costs, especially as represented by greater turnover. Therefore, even without explicitly modeling risk in the funds returns, this methodology can be used to investigate the nature and effects of risk.

Finally, although the method of analysis developed here has little in common with the methods of portfolio evaluation standard among the finance literature, nonetheless the results represent evaluations of two different kinds of portfolios. Despite the management orientation of the present application and the lack in the model of important factors of concern to fund shareholders, such as fees- and tax liability-adjusted returns, the results must surely be of interest to shareholders as they assess the impact of the two types of mutual funds on their personal wealth portfolios.

Whatever the contributions of this study, there remain certain limitations. Some limitations are general or intrinsic to the methodology; some are specific to the implementation of this new method made here. Chief among the general limitations, of the method itself, is the requirement that the strategies be given or at least that the individual entities (DMUs) within the group to be evaluated are known to pursue the same strategy. The public declaration and persistence of strategies for mutual funds make them ideal for testing concepts and methods of strategic analysis and a thorough evaluation of the entire industry can be effected with this new method. However, once other industries are to be analyzed, the strategy researcher using this method will face the same problem that previous researchers have encountered, namely discerning what strategy a firm follows and which firms follow the same strategy.

Another problem is sample size. The mathematics of the method require that the number of DMUs be at least two or three times the sum of the number of inputs and outputs. Monopoly and oligopoly situations present limitations on the detail of the model describing the transformation process and, in extreme cases, may even preclude the use of this method.

Section 2 Future Directions

Many of the limitations specific to the study presented here, rather than to the method in general, indicate immediate directions for future research. The first type of limitation regards the limited application of the model developed in this study. The first, and perhaps the easiest to address, is the limited number of test cases, of strategies evaluated. There are 36 total categories of strategy among mutual funds (according to the Morningstar reports), so the extension of this analysis to all of them is an immediate task. Among these categories, many will likely prove to be much more similar to each other than the two examined here.

Such an extension will also provide a test of the description of blended strategies, which result when comparing strategies, neither of which completely dominates the other. It remains to investigate whether Growth and Income funds represent the blending of Aggressive Growth and Equity Income. Such a result would buttress the argument for the efficiency of financial markets, in that such markets actually produce examples of theoretically efficient formations.

For Morningstar's 36 strategies, exhaustive pairwise comparisons will require 630 evaluations. Nonparametric multiple-comparison tests of corresponding multiple-group DEA evaluations can be evaluated against the prior pairwise analyses.

These evaluations may also be extended by more detailed analysis of the shortcomings of inefficient funds. Such an extension would be especially pertinent to the evaluation of manager performance and the identification of those measures most likely to improve the results of low performance funds. This direction of research would be enriched by the inclusion of variables for the more detailed representation of manager activity, such as tenure and remuneration.

Another type of limitation involves the specific transformation process modeled in this study, that is, the particular inputs and outputs and the variables chosen to represent them. In this regard, several extensions are possible. One direction is a fuller description of factors involved in generating returns: for example, data on number of assets held and percentage of assets (by number or by value) held among the top ten assets provide measures of how concentrated the fund is. Data on manager tenure and terms of compensation might be included to examine questions of agency, a central issue for evaluations focused on manager performance.

In addition to ways in which the current model already addresses risk, another extension will include variables which directly reflect the risk of the investment strategies. The present mid-period and full-period growth rates are effectively compound annual growth rates and, as such, refer to the entire period for which each is computed. Since an infinite number of patterns of losses and gains, or returns variance, can result in a given CAGR, it is not an appropriate measure of returns for any intermediate term short of the full period to which it refers. Instead of the conventional variance or volatility, other measures of risk, as well as measures of extraordinary returns, can be incorporated into the model. Such variables would measure the frequency and extent of downside performance—loss—both with respect to absolute loss (loss of principle) and opportunity cost (performance below the common standards, like the Treasury bill rate, as a proxy for the risk-free rate, or the fund category index rate, as a proxy for the fund category performance, or the Dow Jones Industrial Average or the S&P500, as a proxies for the market as a whole).

Another type of variable to be examined in this model is the class of nondiscretionary variables. These include measures of factors not within the control of the manager or the result of management action, but which, nonetheless may promote or retard management efforts or represent other constraints. These variables may include interest rates, inflation rates or price index changes, stock market averages or trends, and other measures of economic conditions. Other nondiscretionary variables may reflect legislative or regulatory constraints.

Another direction of future research will be the application of different DEA models. The output-oriented BCC model used here is an easy to understand, commonly employed DEA model. It is a variable returns to scale model and the scale results must be more closely evaluated and explicated for their implications on the relation of the size of mutual funds to their performance. Size and concentration are related through their market effects and through regulatory restrictions. Their joint investigation may also have relevance for regulatory issues.

Other DEA models permit the investigation of allocative efficiency, an immediately interesting problem in the context of portfolio asset allocation. However, several of the allocation models are not units invariant, even when objectives are expressed in terms of money value. The range adjusted measure (RAM) model, which expresses the objectives as ratios of performance relative to the ranges of the data, can be employed to overcome this limitation.

The analysis here and the proposed extensions evaluate the strategies of groups of individual funds and, therefore, business or competitive strategies. Such strategies are substrategies, lower level strategies, when viewed from the perspective of the management company, which typically establishes and manages several funds. The possibility of analyzing fund families permits the evaluation of corporate strategies in a financial market. The fact that many mutual funds are members of families of funds provides the opportunity to adapt and extend the approach presented here to examine issues related to industry, corporate, and business level activity and the question of diversification, which has figured prominently in the strategic management literature.

Although the preceding discussion has involved only static models, DEA can be applied to effect dynamic analysis. With the long time series available for mutual fund performance, DEA window analysis can provide a dynamic evaluation of fund performance.

An extension employing both nondiscretionary variables and dynamic window analysis may prove especially revealing for policy and regulatory issues. The history of legislative amendments to the Investment Company Act of 1940 is associated with periods of problems and exposure of wrongdoing in the industry and by affiliated or contracted fund services providers, as witness the investigations beginning in the latter half of 2003. Dynamic analysis can establish the changes in mutual fund industry and strategic category performance subsequent to regulatory amendments.

Between these instances of amendments to the law (which usually increase regulatory restrictions), there often appear arguments for greater deregulation. A re-analysis based on the relaxation of some model constraints representing regulatory restrictions (non-discretionary variables), such as discussed by Brockett, Cooper, and Lasdon [2003], may provide insight into the consequences of some deregulatory proposals.

One of the strengths of data envelopment analysis is the ease of performing sensitivity analysis. All the preceding extensions should include sensitivity analyses of their results. Such results help reveal how limited or robust are the conclusions and subsidiary analyses of DEA evaluations.

Furthermore, instead of defining the strategy classes based on the objectives declared in the prospectus, the strategy classes may be based on the investment categories as defined by other mutual fund data providers or the 33 Investment Companies Institute (ICI) categories and those results compared with the results presented here. It may be that the other category definitions have more explanatory power and, if so, may have regulatory implications.

Other categorizations are also possible. For example, higher fees and remuneration for actively managed funds are justified as costs and compensation, respectively, for the additional activities and skills of astute managers. Constructing classes of funds based on fund managers (an implicit imputation of strategy) can offer a test of this justification. These investigations of alternative specifications of strategic classes and comparison with results based on the declared strategies within the mutual fund industry can provide a controlled test of the use of the methodology developed here with implicit or inferred strategic categories.

Outside the mutual fund market, where a major problem is determining what the strategies are, or at least which firms follow the same strategies, the DEA methodology developed here can be used together with the work of other researchers. For example, Porter has a very specific schema for identifying and categorizing strategies. His categorizations can be used to provide the strategies evaluated by the present method.

Similarly, schemata of other researchers and theorists may be so tested and the results examined for the explanatory power of alternate characterizations of strategy.

Finally, the model developed here can be linked with other kinds of models. Of particular interest are regression models, because of their prevalence in the literature, and stochastic dominance models, because of their applicability in assessing relationships among production surfaces.

APPENDIX: TABLE A1

Table A1: Morningstar, Inc. Definitions of Variables
As Reported in *Principia Pro Plus for Mutual Funds*, November 1998

NOTE: Except for comments and notes in italics, the following definitions of variables are quotations from the *Principia Pro Plus* Help File, Morningstar Principia Pro, Version 4.02, Build 62, copyright 1996-1998, Morningstar Inc., 225 West Wacker Drive, Suite 400, Chicago IL 60606.

The heading of each entry consists of the variable name as used in this study followed by the variable name as used by Morningstar:
<var name> <Morningstar name>

INPUT VARIABLES

%Cash Composition

A breakdown of the fund's portfolio holdings, as of the date listed, into general investment classes. Cash encompasses both actual cash and cash equivalents (fixed-income securities with a maturity of one year or less) held by the portfolio plus receivables minus payables. The composition is obtained from quarterly fund surveys and fund portfolios.

Composition breakdown allows investors to glean information about the portfolio's investment strategy. A portfolio with a large percentage of its assets in cash, for example, might indicate a defensive position. Investors should note that negative percentages of cash indicate that the portfolio is leveraged, i.e.; has borrowed against its own assets to buy more securities.

Composition information is obtained from the more recent of quarterly fund surveys or portfolios submitted by the funds.

A cash position can act as a good indicator of how a fund manager feels about the market at present. It proves, however, an even more accurate indicator of how different fund groups feel about the market. It's an easy task to assess a fund group's aggregate cash position in equities by simply exporting data into a spreadsheet. But its imperative that the average weighted cash position be determined. To do so, begin by exporting all the net asset amounts and cash position amounts for the specific fund group. Multiply each cell in the assets column by $1 million. Next, divide each cell in the cash position column by 100. Multiply the two columns together. The resulting column will contain each fund's total cash position in dollars. This new column may be labeled "total cash." Add together each total cash column cell, and divide the resulting figure by the total sum of the net assets column. The resulting percentage tells you the actual percentage of all the fund group's assets sitting in cash. The cash positions can vary widely among fund families.

TurnoverRatio Turnover Ratio

This is a measure of the fund's trading activity which is computed by taking the lesser of purchases or sales (excluding all securities with maturities of less than one year) and dividing by average monthly net assets. A turnover ratio of 100% or more does not necessarily suggest that all securities in the portfolio have been traded. In practical terms, the resulting percentage loosely represents the percentage of the portfolio's holdings that have changed over the past year.

A low turnover figure (20% to 30%) would indicate a buy-and-hold strategy. High turnover (more than 100%) would indicate an investment strategy involving considerable buying and selling of securities.

Morningstar does not calculate turnover ratios. The figure is culled directly from the financial highlights of the fund's annual report.

This figure is calculated on the lesser of purchases or sales. Therefore, a $100 million fund that is rapidly growing may buy another $100 million in assets, but have a zero percent turnover if it does not sell any of its holdings.

Turnover is important for several reasons. First, it's an indication of management strategy: buy-and-hold vs. trading on short-term fluctuations. Second, funds with higher turnover (implying more trading activity) incur greater brokerage fees for affecting the trades. Third, funds with higher turnover tend to distribute more capital gains than low turnover funds, because high-turnover funds are constantly realizing the gains. Studies show, however, that funds must have very low turnovers (specifically 10% or less) to make appreciable differences in the capital gains distributions. A change in a fund's general turnover pattern can indicate changing market conditions, a new management style, or a change in the fund's investment objective.

ExpnRatio Expense Ratio

The percentage of fund assets paid for operating expenses and management fees, including 12b-1 fees, administrative fees, and all other asset-based costs incurred by the fund, except brokerage costs. Fund expenses are reflected in the fund's NAV. Sales charges are not included in the expense ratio.

The expense ratio is useful because it shows the actual amount that a fund takes out of its assets each year to cover its expenses. Investors should note not only the current expense-ratio figure, but also the trend in these expenses; it could prove useful to know whether a fund is becoming cheaper or more costly. When considering high expenses vs. low expenses, potential investors must also consider the fund's objective and its size. Certain objectives, such as foreign equity funds, have higher costs and therefore, higher expense ratios. As for size, smaller funds are normally costlier than larger funds, as they do not have the benefits of economies of scale.

Morningstar does not calculate expense ratios. The figure is culled directly from the most recent audited annual report.

Domestic equity funds tend to have lower expense ratios than international stock funds, in part because they are cheaper to run.

An expense ratio is accrued on a daily basis by taking the fee rates and multiplying 1/365 of the rates against the average daily net assets of the fund. For example, if the fund expense ratio was determined to be 1.00%, the daily fee would be 1.00% of average daily net assets multiplied by 1/365. This causes the expense ratio to be accrued evenly, with little daily effect to the fund's NAV. Most companies associated with the fund, such as the investment advisor, are actually paid on a monthly basis.

FrontLoad Front-End Load

The initial, or front-end, sales charge is a one-time deduction from an investment made into the fund. The amount is generally relative to the amount of the investment, so that larger investments incur smaller rates of charge. The sales charge serves as a commission for the broker who sold the fund.

APPENDIX: TABLE A1 (continued)

A fund's potential fees and sales charges are an important factor to consider before making an investment. The load fee compensates the broker or financial planner for the service of providing professional investment advice. Sound advice is, of course, an investment itself, and one worth paying for.

This information is taken directly from the fund's prospectus.

While most A share classes of funds in the Federated family have a direct front load of 5.50%, the Eagle Growth fund has a front-end load of 8.50%. The front-end load for this particular fund decreases depending on the amount of money you initially invest:

0-9,999	8.50%
10,000-24,999	7.75%
25,000-49,999	6.25%
50,000-99,999	4.00%
100,000-and up	0.00%

(Data as of 12/31/97)

A fund may have a sliding-scale maximum initial sales charge of 5.50% that is reduced to 4.50% if 200,000 is invested and reduced to 3.50% if 400,000 is invested. Its minimum may be 0.50% if at least $5 million is invested. If 0% is listed, then the fund does not have a front-end sales charge.

DfrrdLoad Deferred Load

These are also known as back-end sales charges and are imposed when investors redeem shares. The percentage charged generally declines the longer shares are held. This charge, often coupled with 12b-1 fees as an alternative to a traditional front-end load, diminishes over time.

Understanding a fund's fee structure is essential in determining whether or not a fund is appropriate for your portfolio or investment plan. With a deferred fee an investor has the advantage of getting the full financial power of their investment from the on set.

This information is taken directly from the fund's prospectus.

Nearly all B shares, no matter what the fund, have a deferred load. Most B share classes of funds in the Federated family, for example, have a deferred load of 5.50%. The deferred fee for this particular fund decreases by a certain amount the longer you remain invested in the fund:

0-1 year	5.50%
1-2 years	4.75%
2-3 years	4.00%
3-4 years	3.00%
4-5 years	2.00%
5-6 years	1.00%
6 years and up	0.00%

(Data as of 12/31/97)

Investors should note that there are combinations of deferred loads and 12b-1 fee that are costlier than a (typically higher) front-end load. Although contingent deferred sales charges usually decline to zero after a specified number of years, the cumulative 12b-1 fee always compensates any possible loss the fund company might incur by long-term shareholder holding onto their shares until the stated load is zero. The amount invested, and how long it will be invested, should be paramount in the share-class investment decision. Very long-term investors normally do not benefit from deferred load funds, especially deferred load funds that do not convert to the less-expensive front load shares.

12b-1 12b-1 Fees

The maximum annual charge deducted from fund assets to pay for distribution and marketing costs. Although usually set on a percentage basis, this amount will occasionally be a flat figure. Only active 12b-1 plans are represented here. This information is taken directly from the fund's prospectus. (Morningstar lists the maximum amount).

Some 12b-1 fees are something of a hidden charge, because they are taken out of the NAV. Morningstar breaks the 12b-1 amount out of the expense ratio so investors know how much they're paying.

This information is taken directly from the fund's prospectus.

Many funds have no 12b-1 fee; B share classes and C share classes often charge the highest allowed by law--1.00% annually. Of the funds that currently charge 12b-1 fees, the highest is 1.00% while the lowest is 0.01%. (Data as of 12/31/97)

Brokers who sell funds with 12b-1 fees get a sweet deal. They receive a commission which is based on the amount of money the investor puts into the fund and continues as long as the money remains in the fund. Also, as the investment grows, the broker makes more money (in the form of a commission) on the investment.

As of July, 1993, no member of the NASD is allowed to sell shares of a mutual fund whose asset-based sales charges are greater than 75 basis points per year. However, a maximum additional 25 basis points under the title "service fees" is permitted. This effectively raises a fund's fee limit to 1.00% per year. Services fees, also known as trailing commissions, are fees paid by funds to brokers for continuing liaison services to clients, such as providing investment information or addressing general inquiries.

These NASD rules place different limitations on funds with different fee structures. If a fund charges service fees, then its combined fee structure (including asset-based fees, front-end loads, and deferred loads) may not exceed 6.25% of total new gross sales. If, however, the fund does not charge service fees, yet has asset-based ones, the total sales charge cannot exceed 7.25%. Another stipulation prohibits any broker from describing (orally or in writing) a mutual fund as being "no-load" if it has either 12b-1 fees (including service fees), or loads that exceed 25 basis points annually. Funds that have distribution fees in excess of.25%, but are not liable for broker compensation, may call themselves "no commission" funds.

NASD rules prohibit 12b-1 fees from being higher than 1%. This 1% is comprised of two parts: a distribution fee and a shareholder service fee, both of which are paid to the distributor. The distribution portion of the fee cannot exceed more than 75 basis points, while the shareholder service fee limit is 25 basis points.

In a typical multi-class situation, the class A fund has a front-end load and either a 0.25% distribution fee or a 0.25% service fee. Class B shares usually have a contingent deferred sales charge and a corresponding 0.75% 12b-1 fee, plus a maximum 0.25% service fee. Frequently, deferred loads have a conversion feature which automatically converts class B shares to class A status after a period of seven or eight years. Class C shares customarily charge a level load with the same fee structure found in a class B share, minus the conversion feature.

Investors should be aware, however, that these class designations are simply a noticeable industry trend, and not an enforced rule for fund structure. Consequently, investors should note that fee structure rules surrounding fund classes often can be complex and convoluted when it comes to fee structures and applications. Fee structure rules are designated differently among various fund families—lending further credence to the wisdom of reading a fund prospectus thoroughly before making an investment decision.

APPENDIX: TABLE A1 (continued)

OUTPUT VARIABLES

AnRet97n Annual Returns, 1997

Total returns calculated on a calendar-year basis. The annual return for a fund will be the same as its trailing 12-month total return only at year-end.

See next item for further definition.

NOTE: In this study, these values are added to 100%, expressing the return as principle plus (or minus) earnings (or losses), in percentage terms. This represents the total return from the investment and obviates difficulties with negative output values.

TRA3Y 3 Year Annualized Total Return
TRA5Y 5 Year Annualized Total Return

Expressed in percentage terms, Morningstar's calculation of total return is determined each month by taking the change in monthly net asset value, reinvesting all income and capital-gains distributions during that month, and dividing by the starting NAV. Reinvestments are made using the actual reinvestment NAV, and daily payoffs are reinvested monthly.

Unless otherwise noted, Morningstar does not adjust total returns for sales charges (such as front-end loads, deferred loads and redemption fees), preferring to give a clearer picture of a fund's performance. The total returns do account for management, administrative, 12b-1 fees and other costs taken out of fund assets. Total returns for periods longer than one year are expressed in terms of compounded average annual returns (also known as geometric total returns), affording a more meaningful picture of fund performance than non-annualized figures.

Morningstar calculates total returns, using the raw data (NAVs, dividends, capital gain distributions) collected from fund companies.

Morningstar determines year-to-date, one-, three-, and 12-month, three-, five-, 10-, and 15-year total returns by calculating the increase in the NAV over that time period including the reinvestment of distributions. We calculate monthly total returns using the following formula:

TRm = {[Ending NAV (1+ Distribution/Reinvestment NAV) - Beginning NAV] / Beginning NAV} x 100

where: **Ending NAV** = current month-end NAV
Beginning NAV = previous month-end NAV
Distribution = amount of distribution
Reinvestment NAV = the price per share on the day the distribution is reinvested.

Monthly returns can be compounded to achieve longer time period returns.

$$\text{TRcumulative} = \left[\left[\left(1+\frac{\text{TRm1}}{100}\right)\left(1+\frac{\text{TRm2}}{100}\right)\cdots\left(1+\frac{\text{TRmn}}{100}\right)\right]-1\right]*100$$

where: **TRm1** = total return for first month
TRmn = total return for month n
n = number of months in time period

In order to generate more usable figures, Morningstar annualizes total returns spanning more than one year, using the following formula:

TRannual = {[1 + TRcumulative/100]1/n] -1 } x 100

where **n** = the number of years

These compounded average annual returns are also known as geometric total returns.

APPENDIX: TABLE A1 (continued)

NOTE: In this study, these values are added to 100%, expressing the return as principle plus (or minus) earnings (or losses), in percentage terms. This represents the total return from the investment and obviates difficulties with negative output values.

NetAss **Net Assets**

The month-end net assets of the mutual fund, recorded in millions of dollars. Net-asset figures are useful in gauging a fund's size, agility, and popularity. They help determine whether a small company fund, for example, can remain in its investment-objective category if its asset base reaches an ungainly size.

Morningstar lists the month-end assets, as they have been reported by the fund.

It's important to keep in mind that the size of the fund as measured by net assets has little or no correlation to the size of the companies in which the fund invests.

One caveat to this rule: Very large funds usually find it difficult to invest in small companies. Because [*a very large fund which invests in small companies*] has so much money to invest--yet can't invest a large amount in any one company--the number of holdings in its portfolio [*may*] balloon. This also makes the fund much less liquid, as it would have to sell off a large number of holdings to make a significant change in its current investment strategy.

APPENDIX: TABLE A2, TABLE A3, TABLE A4

Table A2: Descriptive Statistics of the Input Variables of Aggressive Growth (**ag**) and Equity Income (**eq**) Mutual Funds

SUMMARY STATISTICS	%Cash		TurnoverRatio		ExpnRatio		FrontLoad		DfrrdLoad		12b-1	
	ag	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**
max	29.3	28	1180	192	5.85	2.14	5.75	5.75	5	5	1	1
min	0	0	2	5	0.65	0.45	0	0	0	0	0	0
average	4.958	6.587143	121.22	56.59657	1.4834	1.108	1.92	1.996429	0.56	0.307143	0.291	0.209714
stnd def	4.991036	6.978742	172.5344	39.83832	0.837094	0.352917	2.488694	2.480457	1.47187	1.112567	0.349527	0.292497

Table A3: Descriptive Statistics of the Output Variables of Aggressive Growth (**ag**) and Equity Income (**eq**) Mutual Funds

SUMMARY STATISTICS	TRA3Yn		TRA5Yn		AnRet97n		NetAss	
	ag	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**
max	120.92	123.68	120.18	118.15	175	136.41	12424.3	22277.1
min	79.47	110.07	81.53	109.05	86.97	118.55	4.7	6.9
average	108.28	118.6631	109.9516	114.8746	116.5686	127.2006	1384.46	1500.703
stnd dev	8.460048	2.53642	7.042158	2.105071	12.92495	3.817443	2791.849	3770.873

Table A4: Correlations among Input and Output Variables of Aggressive Growth (**ag**) and Equity Income (**eq**) Mutual Funds

correlations	%Cash		TurnoverRatio		ExpnRatio		FrontLoad		DfrrdLoad		12b-1		TRA3Yn		TRA5Yn		AnRet97n		NetAss	
	ag	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**	**ag**	**eq**
%Cash	1	1	-0.05808	-0.10671	-0.12195	-0.05199	-0.07261	0.297505	-0.00905	0.085789	-0.03335	0.099496	0.095641	0.059242	0.148459	0.042577	-0.07451	-0.21239	-0.00207	0.115404
TrnRtio	-0.05808	-0.10671	1	1	0.659591	0.21793	-0.12976	0.04057	-0.12594	-0.00607	-0.14891	0.160838	-0.26096	-0.03628	-0.52665	-0.00074	0.544934	0.069874	-0.1266	-0.12556
ExpRtio	-0.12195	-0.05199	0.659591	0.21793	1	1	-0.19557	-0.0652	0.210614	0.498198	0.284636	0.618742	-0.28714	-0.14237	-0.56255	-0.2879	0.447789	-0.01887	-0.13916	-0.32343
FrntLd	-0.07261	0.297505	-0.12976	0.04057	-0.19557	-0.0652	1	1	-0.29353	-0.2222	-0.06003	0.110116	0.149334	0.098973	0.129582	0.140718	-0.01904	-0.05063	0.2697	-0.06326
DfrrdLd	-0.00905	0.085789	-0.12594	-0.00607	0.210614	0.498198	-0.29353	-0.2222	1	1	0.723169	0.685534	0.074188	-0.05178	0.083328	-0.08596	0.069249	0.048512	0.10954	-0.01488
12b-1	-0.03335	0.099496	-0.14891	0.160838	0.284636	0.618742	-0.06003	0.110116	0.723169	0.685534	1	1	0.028475	0.010363	0.047316	-0.02928	-0.06179	0.09199	0.198936	-0.08202
TRA3Y	0.095641	0.059242	-0.26096	-0.03628	-0.28714	-0.14237	0.149334	0.098973	0.074188	-0.05178	0.028475	0.010363	1	1	0.897541	0.868797	0.408951	0.632496	0.223886	0.19664
TRA5Y	0.148459	0.042577	-0.52665	-0.00074	-0.56255	-0.2879	0.129582	0.140718	0.083328	-0.08596	0.047316	-0.02928	0.897541	0.868797	1	1	0.089379	0.503006	0.310138	0.256191
AnRt97	-0.07451	-0.21239	0.544934	0.069874	0.447789	-0.01887	-0.01904	-0.05063	0.069249	0.048512	-0.06179	0.09199	0.408951	0.632496	0.089379	0.503006	1	1	0.104651	0.05841
NetAss	-0.00207	0.115404	-0.1266	-0.12556	-0.13916	-0.32343	0.2697	-0.06326	0.10954	-0.01488	0.198936	-0.08202	0.223886	0.19664	0.310138	0.256191	0.104651	0.05841	1	1

APPENDIX: TABLE A5

Table A5: Morningstar Prospectus Objectives*—Strategy Categories

▶	1	Aggressive Growth
	2	Asset Allocation
	3	Balanced
	4	Convertible Bond
	5	Corporate Bond—General
	6	Corporate Bond—High Quality
	7	Corporate Bond—High Yield
	8	Diversified Emerging Markets
▶	9	Equity Income
	10	European Stock
	11	Foreign Stock
	12	Government Bond—Adjustable Rate Mortgage
	13	Government Bond—General
	14	Government Bond—Mortgage
	15	Government Bond—Treasury
	16	Growth
	17	Growth and Income
	18	Multi-Asset Global
	19	Multi-Sector Bond
	20	Municipal Bond—California
	21	Municipal Bond—National
	22	Municipal Bond—New York
	23	Municipal Bond—Single State
	24	Pacific Stock
	25	Small Company
	26	Specialty
	27	Specialty—Commercial
	28	Specialty—Financial
	29	Specialty—Health
	30	Specialty—Natural Resources
	31	Specialty—Precious Metals
	32	Specialty—Real Estate
	33	Specialty—Technology
	34	Specialty—Utilities
	35	World Bond
	36	World Stock

* The term "objective" is commonly used in the mutual fund industry, but this study refers to activities such as "investing in growth and income securities" as a *strategy* used in pursuit of the *objective* of maximum positive returns from investments in financial markets. [▶ indicates the *strategies*, Aggressive Growth and Equity Income, examined in this study.]

APPENDIX: TABLE A6

Table A6: Aggressive Growth Funds and Input/Output Variables
As Reported in *Principia Pro Plus for Mutual Funds*, November 1998

	Fund Name	Ticker	AnRet97	TRA3Y	TRA5Y	NetAss	% Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
	AGGRW category sample averages		116.24	109.87	110.56	819.4	8.20	135.00	1.64	1.34	1.05	0.44
1	AIM Aggressive Growth	AAGFX	112.24	104.05	113.65	2641.3	5.60	73.00	1.06	5.50	0.00	0.25
2	AIM Constellation A	CSTGX	112.92	108.91	113.12	12424.3	6.70	67.00	1.11	5.50	0.00	0.30
3	AIM Constellation Instl [1]	CSTIX	113.45	109.43	113.66	189.1	6.70	67.00	0.65	0.00	0.00	0.00
4	Alliance Quasar A	QUASX	117.24	113.71	112.18	571.0	4.60	135.00	1.67	4.25	0.00	0.30
5	Alliance Quasar B	QUABX	116.29	112.86	111.32	663.6	4.60	135.00	2.51	0.00	4.00	1.00
6	American Cent-20thC VistaInv	TWCVX	91.32	90.05	101.86	895.3	2.10	96.00	1.00	0.00	0.00	0.00
7	American Heritage	AHERX	175.00	91.33	81.53	8.4	0.00	1180.00	5.85	0.00	0.00	0.00
8	Bull & Bear Special Equities	BBSEX	105.23	98.07	99.57	33.0	0.00	260.00	2.53	0.00	0.00	1.00
9	Crabbe Huson Special Prim	CHSPX	111.28	90.10	98.16	112.4	0.50	33.00	1.50	0.00	0.00	0.25
10	Delaware Pooled Aggress Grth	DPAGX	113.00	110.57	110.16	4.7	5.70	95.00	0.90	0.00	0.00	0.00
11	Delaware Trend A	DELTX	119.43	110.58	109.58	370.7	0.00	114.00	1.34	4.75	0.00	0.30
12	Delaware Trend Instl	DGTIX	119.86	110.84	109.85	46.8	0.00	114.00	1.09	0.00	0.00	0.00
13	Dreyfus Premier Aggr Grth A	DRLEX	86.97	79.47	87.81	128.1	0.30	76.00	1.20	5.75	0.00	0.00
14	Enterprise Capital Apprec A	ENCAX	120.27	113.62	112.68	105.5	9.60	61.00	1.65	4.75	0.00	0.45
15	Evergreen Aggressive Grth A	EAGAX	107.66	111.29	110.87	119.9	1.60	56.00	1.25	4.75	0.00	0.75
16	Evergreen Omega A	EKOAX	124.53	116.76	113.53	142.4	9.90	76.00	1.31	4.75	0.00	0.25
17	Evergreen Small Co Growth B [2]	EKABX	113.39	94.29	102.58	172.1	3.10	70.00	1.77	0.00	4.00	1.00
18	Fidelity Capital Apprec	FDCAX	126.52	114.18	112.94	2290.7	2.80	176.00	0.66	0.00	0.00	0.00
19	Fidelity Emerging Growth	FDEGX	119.45	114.93	117.15	2297.8	3.20	212.00	1.05	0.00	0.00	0.00
20	Fidelity New Millennium	FMILX	124.63	120.92	120.18	1416.0	2.90	142.00	0.94	3.00	0.00	0.00

APPENDIX: TABLE A6 (continued)

	Fund Name	DMU	AnRet97	TRA3Y	TRA5Y	NetAss	% Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
	AGGRW category sample averages		116.24	109.87	110.56	819.4	8.20	135.00	1.64	1.34	1.05	0.44
21	Founders Special	FRSPX	116.43	103.76	106.34	223.6	2.40	110.00	1.30	0.00	0.00	0.25
22	FundManager Agg Grth Fin Adv	FTAGX	116.65	111.01	111.03	22.2	11.30	51.00	1.59	4.50	0.00	0.50
23	IDS Strategy Aggressive B	INAGX	114.72	110.88	111.14	685.0	11.40	95.00	1.77	0.00	5.00	0.75
24	Invesco Dynamics	FIDYX	124.90	115.60	114.85	1261.8	7.40	178.00	1.08	0.00	0.00	0.25
25	Kaufmann	KAUFX	112.56	107.84	112.98	4239.8	10.00	65.00	1.88	0.00	0.00	0.75
26	Matterhorn Growth	FWLEX	113.66	108.82	107.55	7.9	3.90	137.00	4.00	0.00	0.00	0.25
27	MFS Emerging Growth B	MEGBX	119.73	112.50	115.88	5317.3	3.20	21.00	1.97	0.00	4.00	1.00
28	Oppenheimer Capital Ap A	OPTFX	126.32	120.86	118.78	1461.8	5.60	66.00	1.01	5.75	0.00	0.25
29	Pacific Horizon Aggr Grth A	PHAGX	114.22	107.10	106.89	168.9	0.00	83.00	1.46	5.50	0.00	0.00
30	Permanent Port Aggr Growth	PAGRX	132.68	115.43	115.47	16.9	0.00	2.00	1.46	0.00	0.00	0.00
31	Phoenix Aggressive Growth A	PHSKX	119.37	112.15	113.86	212.6	8.50	518.00	1.20	4.75	0.00	0.25
32	PIMCo Opportunity A	POPAX	95.96	99.44	104.53	121.3	4.90	86.00	1.31	5.50	0.00	0.25
33	PIMCo Opportunity C	POPCX	95.25	98.73	103.76	335.6	4.90	86.00	2.06	0.00	1.00	1.00
34	Pin Oak Aggressive Stock	POGSX	101.30	108.23	111.72	40.9	3.90	17.00	0.99	0.00	0.00	0.00
35	Principal MidCap A	PEMGX	122.94	111.45	113.33	332.9	7.50	10.00	1.26	4.75	0.00	0.25
36	Putnam New Opportunities A	PNOPX	122.55	113.61	115.93	8320.1	2.40	65.00	0.98	5.75	0.00	0.35
37	Putnam Voyager A	PVOYX	125.98	116.04	116.06	12167.2	4.30	60.00	0.96	5.75	0.00	0.35
38	Putnam Voyager B	PVOBX	125.07	115.17	115.17	6347.9	4.30	60.00	1.71	0.00	5.00	1.00
39	Security Ultra A	SECUX	117.86	111.59	109.13	67.6	0.00	68.00	1.71	5.75	0.00	0.00
40	Shelby	SHELX	106.23	105.58	109.15	35.9	0.20	177.00	1.29	0.00	0.00	0.00
41	Smith Barney Aggr Growth A	SHRAX	128.58	114.99	114.36	387.9	1.70	6.00	1.21	5.00	0.00	0.25
42	Smith Barney Aggr Growth B	SAGBX	127.59	114.09	113.48	231.8	1.70	6.00	2.01	0.00	5.00	1.00

APPENDIX: TABLE A6 (continued)

	Fund Name	DMU	AnRet97	TRA3Y	TRA5Y	NetAss	% Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
	AGGRW category sample averages		116.24	109.87	110.56	819.4	8.20	135.00	1.64	1.34	1.05	0.44
43	State St Research Capital S	SCFCX	106.40	101.57	107.34	113.2	5.00	231.00	0.96	0.00	0.00	0.00
44	Stein Roe Capital Opport	SRFCX	106.15	106.14	110.72	640.5	7.10	35.00	1.17	0.00	0.00	0.00
45	Strong Discovery	STDIX	110.85	103.64	107.02	297.3	29.30	170.00	1.40	0.00	0.00	0.00
46	USAA Aggressive Growth	USAUX	107.56	109.56	113.41	669.4	2.60	83.00	0.71	0.00	0.00	0.00
47	Value Line Leveraged Gr Inv	VALLX	123.80	120.43	117.68	498.4	3.60	37.00	0.86	0.00	0.00	0.00
48	Value Line Spec Situations	VALSX	132.10	114.84	114.24	145.2	8.70	146.00	1.08	0.00	0.00	0.00
49	Wasatch Aggressive Equity	WAAEX	119.23	105.62	109.72	128.9	5.90	48.00	1.50	0.00	0.00	0.00
50	WPG Tudor	TUDRX	111.11	101.37	103.68	90.1	16.30	106.00	1.24	0.00	0.00	0.00

footnotes

[1] AIM Constellation Instl — CSTIX = ticker symbols are used for DMU identification; this fund has no ticker symbol

[2] Evergreen Small Co Growth B — 70.00 = 1997 turnover ratio not available: historical rate used instead:
1996 second highest, 94%, 1995 second lowest, 38%;
1981 highest 100%, 1979 lowest 15%

VARIABLE DEFINITIONS (For detailed definitions, see Appendix Table A1.)

OUTPUT VARIABLES

AnRet97 = Annual Return 1997 + 100.00
TRA3Y = Total Return Annualized 3 Year + 100.00
TRA5Y = Total Return Annualized 5 Year + 100.00
NetAss = Net Assets $MM; most as of 1998-09-30

INPUT VARIABLES

% Cash = Percentage of Holdings as Cash
TurnoverRatio = Total Return Annualized 5 Year + 100.00
ExpnRation = Percentage of Assets Paid to Expenses and Fees
FrontLoad = Initial Sales Charge as Percentage of Investment
DfrrdLoad = Redemption Charge as Percentage of Redemption
12b-1 = Annual Marketing Charge as Percentage of Assets

APPENDIX: TABLE A7

Table A7: Equity Income Funds and Input/Output Variables
As Reported in *Principia Pro Plus for Mutual Funds*, November 1998

	Fund Name	Ticker	AnRet97	TRA3Y	TRA5Yn	NetAss	% Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
	EQINC category sample averages		126.56	118.59	114.93	990.3	6.30	60.00	1.33	1.27	1.04	0.40
1	Accessor Value & Income	AVAIX	132.95	121.95	117.28	110.6	6.50	68.00	1.05	0.00	0.00	0.00
2	Amana Income	AMANX	124.54	117.65	112.26	20.0	11.90	8.00	1.36	0.00	0.00	0.00
3	American National Income	AMNIX	122.72	115.16	113.57	202.0	17.00	39.00	1.05	5.75	0.00	0.00
4	BNY Hamilton Equity-Inc Inv	BNEIX	125.85	117.95	113.87	33.4	1.70	58.00	0.97	0.00	0.00	0.25
5	Capital Income Builder	CAIBX	123.33	117.74	113.95	8064.0	23.70	28.00	0.65	5.75	0.00	0.30
6	Chase Equity-Income Instl	RIEIX	131.05	123.33	117.45	105.5	6.50	14.00	1.00	0.00	0.00	0.00
7	Cutler Equity-Income	CEIFX	133.35	123.68	117.07	73.8	22.30	119.00	1.10	0.00	0.00	0.00
8	Delaware Decatur Income A	DELDX	129.71	120.51	116.53	1811.9	0.00	90.00	0.88	4.75	0.00	0.30
9	Enterprise Equity Income A	ENGIX	128.08	119.68	115.99	100.5	14.80	33.00	1.50	4.75	0.00	0.45
10	Evergreen Fund for Tot Ret A	EKTAX	124.07	120.76	115.16	45.4	21.30	66.00	1.21	4.75	0.00	0.75
11	Evergreen Income & Growth Y	EVTRX	125.58	112.51	109.05	749.5	0.40	133.00	1.25	0.00	0.00	0.00
12	Excelsior Income & Growth	UMIGX	122.10	111.54	110.22	83.2	1.30	32.00	1.02	0.00	0.00	0.00
13	Federated Equity-Income A	LEIFX	125.11	120.73	115.71	806.9	4.70	69.00	1.09	5.50	0.00	0.00
14	Fidelity Adv Eqty Inc Instl	EQPIX	126.64	118.73	117.69	473.3	2.00	55.00	0.67	0.00	0.00	0.00
15	Fidelity Adv Eqty Inc T	FEIRX	125.90	118.06	116.88	2531.3	2.00	55.00	1.21	3.50	0.00	0.75
16	Fidelity Equity-Income	FEQIX	129.98	120.86	117.10	22277.1	1.80	23.00	0.65	0.00	0.00	0.00
17	Fidelity Equity-Income II	FEQTX	127.17	121.58	117.05	17659.7	8.40	77.00	0.68	0.00	0.00	0.00
18	First American Equity-Inc A	FFEIX	127.53	120.33	115.95	11.7	5.50	23.00	1.00	4.50	0.00	0.25
19	Franklin Equity Income I	FISEX	127.21	116.76	113.13	409.1	0.00	29.00	0.97	5.75	0.00	0.25
20	Gabelli Equity-Income	GABEX	127.85	119.98	115.66	84.1	0.30	43.00	1.78	0.00	0.00	0.25
21	Galaxy Equity Income Ret A	GAEIX	125.51	119.13	116.39	198.9	14.90	37.00	1.39	3.75	0.00	0.00
22	Harbor Value	HAVLX	131.20	119.85	116.90	157.8	1.30	146.00	0.83	0.00	0.00	0.00
23	HighMark Income-Equity Fid	HMIEX	127.29	119.32	115.92	626.0	2.10	46.00	0.99	0.00	0.00	0.00

APPENDIX: TABLE A7 (continued)

	Fund Name	DMU	AnRet97	TRA3Y	TRA5Yn	NetAss	% Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
	EQINC category sample averages		126.56	118.59	114.93	990.3	6.30	60.00	1.33	1.27	1.04	0.40
24	Hotchkis & Wiley Eqty-Inc	HWEQX	131.15	118.72	115.58	154.0	0.00	23.00	0.87	0.00	0.00	0.00
25	IDS Diversified Equity-Inc A	INDZX	120.08	116.67	112.97	1826.1	4.20	81.00	0.88	5.00	0.00	0.18
26	Invesco Industrial Income	FIIIX	126.45	118.37	114.35	4704.6	4.60	58.00	0.90	0.00	0.00	0.25
27	Mairs & Power Balanced	MAPOX	128.04	119.69	115.32	36.2	0.70	5.00	0.92	0.00	0.00	0.00
28	Managers Income Equity	MGIEX	127.19	118.20	115.72	63.5	4.90	96.00	1.32	0.00	0.00	0.00
29	Merrill Lynch Strat Div A	MADVX	128.51	121.51	116.46	23.2	10.20	26.00	1.04	5.25	0.00	0.00
30	Merrill Lynch Strat Div B	MBDVX	127.32	120.30	115.23	70.4	10.20	26.00	2.08	0.00	4.00	1.00
31	Monitor Income-Equity Tr	MIEFX	125.99	120.03	115.73	233.2	1.60	24.00	0.81	0.00	0.00	0.00
32	Morgan Stanley Inst Val Eq A	MSIVX	129.20	118.10	115.42	60.9	0.00	36.00	0.70	0.00	0.00	0.00
33	Nations Equity-Income Inv A	NEQIX	125.71	116.04	113.32	54.3	0.90	74.00	1.11	0.00	0.00	0.25
34	Nations Equity-Income Inv C	NEINX	125.21	115.56	112.78	6.9	0.90	74.00	1.69	0.00	0.50	0.75
35	Nations Equity-Income Prim A	NEQUX	126.13	116.34	113.60	703.0	0.90	74.00	0.86	0.00	0.00	0.00
36	Old Dominion Investors'	ODIFX	123.83	115.49	113.67	10.8	16.10	86.00	1.16	4.00	0.00	0.25
37	One Group Income Equity A	OIEIX	132.17	122.16	117.82	113.8	0.90	15.00	1.25	4.50	0.00	0.25
38	One Group Income Equity Fid	HLIEX	132.52	122.51	118.15	613.3	0.90	15.00	1.00	0.00	0.00	0.00
39	Oppenheimer Equity-Income A	OPPEX	129.68	119.66	115.32	3133.6	25.90	24.00	0.88	5.75	0.00	0.25
40	Parkstone Eq Income Instl	PKHEX	125.43	117.91	112.19	242.0	0.60	19.76	1.33	0.00	0.00	0.00
41	Parnassus Income Equity Inc	PRBLX	120.15	110.07	109.08	35.0	20.10	34.00	1.05	0.00	0.00	0.00
42	Pillar Equity-Income A	PLINX	124.69	119.32	115.06	17.2	8.10	77.00	1.05	4.00	0.00	0.25
43	Pillar Equity-Income I	PLIAX	125.04	119.64	115.32	91.1	8.10	77.00	0.80	0.00	0.00	0.00
44	PIMCo Equity Income Instl	PEIIX	131.38	119.68	115.90	124.7	3.10	45.00	0.71	0.00	0.00	0.00
45	PIMCo Renaissance A	PQNAX	135.92	121.08	115.61	75.7	3.50	192.00	1.26	5.50	0.00	0.25
46	PIMCo Renaissance C	PQNCX	134.90	120.16	114.73	391.0	3.50	192.00	2.01	0.00	1.00	1.00
47	Pioneer Equity-Income A	PEQIX	134.89	121.38	116.43	585.4	0.60	18.00	1.11	5.75	0.00	0.25
48	Prudential Equity-Income A	PBEAX	136.41	118.84	114.06	638.7	2.30	36.00	0.94	5.00	0.00	0.30

APPENDIX: TABLE A7 (continued)

	Fund Name	DMU	AnRet97	TRA3Y	TRA5Yn	NetAss	% Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
	EQINC category sample averages		126.56	118.59	114.93	990.3	6.30	60.00	1.33	1.27	1.04	0.40
49	Prudential Equity-Income B	PBQIX	135.34	117.98	113.22	1299.7	2.30	36.00	1.69	0.00	5.00	1.00
50	Putnam Equity Income A	PEYAX	126.46	120.17	117.26	993.1	3.90	82.00	1.06	5.75	0.00	0.35
51	Riverfront Income Eqty Inv A	RFIEX	125.97	118.23	116.12	70.4	0.20	157.00	1.75	4.50	0.00	0.25
52	Safeco Income No Load	SAFIX	126.43	117.68	114.57	461.1	6.00	52.00	0.85	0.00	0.00	0.00
53	SEI Instl Equity-Income A	SEEIX	127.96	120.00	116.34	123.7	9.80	40.00	0.85	0.00	0.00	0.00
54	SG Cowen Income+Growth A	COIGX	122.90	114.32	111.66	44.7	0.00	75.00	1.21	4.75	0.00	0.25
55	Smith Barney Lrg Cap Val A	SBCIX	127.86	119.59	115.16	775.6	3.00	40.00	0.92	5.00	0.00	0.25
56	Smith Barney Lrg Cap Val L	SBGCX	126.85	118.64	114.07	59.3	3.00	40.00	1.69	0.00	1.00	1.00
57	Smith Barney Prem Tot Ret A	SOPAX	125.20	117.42	114.58	885.9	12.40	43.00	1.11	5.00	0.00	0.25
58	Smith Barney Prem Tot Ret B	SOPTX	124.55	116.84	114.02	3117.2	12.40	43.00	1.60	0.00	5.00	0.75
59	Stagecoach Divr Eqty Inc A	SDINX	120.21	116.55	113.87	172.8	1.70	59.00	1.12	5.25	0.00	0.05
60	State St Research Alpha A	SSEAX	127.54	118.73	114.53	110.7	12.50	53.00	1.24	4.50	0.00	0.25
61	T. Rowe Price Equity-Income	PRFDX	128.82	120.15	117.87	12436.6	5.00	24.00	0.79	0.00	0.00	0.00
62	U.S. Global Inv Income	USINX	123.08	116.17	109.59	10.2	3.70	29.00	2.14	0.00	0.00	0.00
63	UAM DSI Disc Value Instl	DSIDX	123.42	118.28	115.52	69.7	0.40	126.00	1.05	0.00	0.00	0.00
64	United Income A	UNCMX	127.42	121.72	117.89	6958.0	28.00	34.00	0.84	5.75	0.00	0.25
65	USAA Income Stock	USISX	126.99	116.97	113.41	2367.5	8.30	22.00	0.65	0.00	0.00	0.00
66	Value Line Income	VALIX	118.55	116.71	112.72	162.5	11.10	54.00	0.87	0.00	0.00	0.00
67	Van Kampen Equity-Income A	ACEIX	124.13	118.74	115.51	729.3	13.40	86.00	0.86	5.75	0.00	0.25
68	Van Kampen Equity-Income B	ACEQX	123.23	117.83	114.68	1014.8	13.40	86.00	1.64	0.00	5.00	1.00
69	Vanguard Equity-Income	VEIPX	131.17	122.48	117.46	2530.0	6.00	22.00	0.45	0.00	0.00	0.00
70	Westcore Growth & Income	WTEIX	127.25	116.00	110.57	12.1	1.40	40.00	1.15	0.00	0.00	0.00

VARIABLE DEFINITIONS (Same variables and definitions as reported in Appendix Table A6; for detailed definitions, see Appendix Table A1.)

APPENDIX: TABLE A8

Table A8: Total Annual Return 1997 Mean-Variance Characteristics
Ranked by Total Annual Return 1997

	AGGRESSIVE GROWTH 1997				EQUITY INCOME 1997			
RNK	FUND	AnRet97	ARSD[1]	EFF	FUND	AnRet97	ARSD[1]	EFF
strategy avg[2]		116.24			strategy avg	126.56		
sample avg[3]		116.57	12.925[4]	0.958206	sample avg	127.20	3.8174[4]	0.980567
1	AHERX	175.00	4.52	1.000000	PBEAX	136.41	2.41	1.000000
2	PAGRX	132.68	1.25	1.000000	PQNAX	135.92	2.28	1.000000
3	VALSX	132.10	1.20	1.000000	PBQIX	135.34	2.13	1.000000
4	SHRAX	128.58	0.93	1.000000	PQNCX	134.90	2.02	1.000000
5	SAGBX	127.59	0.85	0.985076	PEQIX	134.89	2.01	1.000000
6	FDCAX	126.52	0.77	1.000000	CEIFX	133.35	1.61	1.000000
7	OPTFX	126.32	0.75	1.000000	AVAIX	132.95	1.51	1.000000
8	PVOYX	125.98	0.73	1.000000	HLIEX	132.52	1.39	1.000000
9	PVOBX	125.07	0.66	1.000000	OIEIX	132.17	1.30	0.997319
10	FIDYX	124.90	0.64	0.993041	PEIIX	131.38	1.09	0.998601
11	FMILX	124.63	0.62	1.000000	HAVLX	131.20	1.05	0.998505
12	EKOAX	124.53	0.62	0.975037	VEIPX	131.17	1.04	1.000000
13	VALLX	123.80	0.56	1.000000	HWEQX	131.15	1.03	1.000000
14	PEMGX	122.94	0.49	0.981671	RIEIX	131.05	1.01	1.000000
15	PNOPX	122.55	0.46	1.000000	FEQIX	129.98	0.73	1.000000
16	ENCAX	120.27	0.29	0.950963	DELDX	129.71	0.66	1.000000
17	DGTIX	119.86	0.25	1.000000	OPPEX	129.68	0.65	0.979920
18	MEGBX	119.73	0.24	1.000000	MSIVX	129.20	0.52	1.000000
19	FDEGX	119.45	0.22	1.000000	PRFDX	128.82	0.42	1.000000
20	DELTX	119.43	0.22	1.000000	MADVX	128.51	0.34	0.988580
21	PHSKX	119.37	0.22	0.951333	ENGIX	128.08	0.23	0.981718
22	WAAEX	119.23	0.21	0.938734	MAPOX	128.04	0.22	1.000000
23	SECUX	117.86	0.10	0.990877	SEEIX	127.96	0.20	0.986251
24	QUASX	117.24	0.05	0.940417	SBCIX	127.86	0.17	0.975840
25	FTAGX	116.65	0.01	0.939252	GABEX	127.85	0.17	0.999972
26	FRSPX	116.43	-0.01	0.911279	SSEAX	127.54	0.09	0.969361
27	QUABX	116.29	-0.02	0.946435	FFEIX	127.53	0.09	0.981767
28	INAGX	114.72	-0.14	0.944961	UNCMX	127.42	0.06	1.000000
29	PHAGX	114.22	-0.18	1.000000	MBDVX	127.32	0.03	0.978420
30	FWLEX	113.66	-0.23	0.914785	HMIEX	127.29	0.02	0.981230
31	CSTIX	113.45	-0.24	1.000000	WTEIX	127.25	0.01	0.959954
32	EKABX	113.39	-0.25	0.880863	FISEX	127.21	0.00	1.000000
33	DPAGX	113.00	-0.28	0.936098	MGIEX	127.19	-0.00	0.979433
34	CSTGX	112.92	-0.28	1.000000	FEQTX	127.17	-0.01	1.000000

APPENDIX: TABLE A8 (continued)

RNK	FUND	AnRet97	ARSD	EFF	RNK	FUND	AnRet97	ARSD	EFF
35	KAUFX	112.56	-0.31	1.000000		USISX	126.99	-0.06	0.964801
36	AAGFX	112.24	-0.33	0.958325		SBGCX	126.85	-0.09	0.966850
37	CHSPX	111.28	-0.41	0.847892		EQPIX	126.64	-0.15	1.000000
38	TUDRX	111.11	-0.42	0.884449		PEYAX	126.46	-0.19	0.992544
39	STDIX	110.85	-0.44	0.909415		FIIIX	126.45	-0.20	0.969618
40	EAGAX	107.66	-0.69	0.943899		SAFIX	126.43	-0.20	0.971246
41	USAUX	107.56	-0.70	1.000000		NEQUX	126.13	-0.28	0.967764
42	SCFCX	106.40	-0.79	0.912135		MIEFX	125.99	-0.32	0.985781
43	SHELX	106.23	-0.80	0.960779		RFIEX	125.97	-0.32	0.993413
44	SRFCX	106.15	-0.81	1.000000		FEIRX	125.90	-0.34	0.989637
45	BBSEX	105.23	-0.88	0.891494		BNEIX	125.85	-0.35	0.964082
46	POGSX	101.30	-1.18	1.000000		NEQIX	125.71	-0.39	0.959120
47	POPAX	95.96	-1.59	0.877309		EVTRX	125.58	-0.42	0.953676
48	POPCX	95.25	-1.65	0.881713		GAEIX	125.51	-0.44	0.985104
49	TWCVX	91.32	-1.95	0.883492		PKHEX	125.43	-0.46	0.972480
50	DRLEX	86.97	-2.29	0.778588		NEINX	125.21	-0.52	0.954549
					51	SOPAX	125.20	-0.52	0.969843
					52	LEIFX	125.11	-0.55	0.982447
					53	PLIAX	125.04	-0.57	0.978125
					54	PLINX	124.69	-0.66	0.973901
					55	SOPTX	124.55	-0.69	0.965551
					56	AMANX	124.54	-0.70	0.973091
					57	ACEIX	124.13	-0.80	0.979111
					58	EKTAX	124.07	-0.82	0.979869
					59	ODIFX	123.83	-0.88	0.962082
					60	DSIDX	123.42	-0.99	0.989700
					61	CAIBX	123.33	-1.01	0.968781
					62	ACEQX	123.23	-1.04	0.970715
					63	USINX	123.08	-1.08	0.945086
					64	COIGX	122.90	-1.13	0.959960
					65	AMNIX	122.72	-1.17	0.961236
					66	UMIGX	122.10	-1.34	0.932882
					67	SDINX	120.21	-1.83	0.963775
					68	PRBLX	120.15	-1.85	0.923233
					69	INDZX	120.08	-1.87	0.957388
					70	VALIX	118.55	-2.27	0.955360

[1] ARSD — AnRet97 in units of standard deviation, as given by strategy category sample standard deviation

[2] strategy average — average of the Morningstar population for each strategy category

[3] sample average — average of each strategy category sample used in this study

[4] sample standard deviation for each strategy category; used to compute ARSD

Efficient Funds	AGGRW	EQINC
Total	22	20
Avg Rnk AnR97	18.8	14.8
Avg/N	0.3755	0.2114

	mean performance for each category
	all EqInc funds better than all lower-ranked AgGrw funds
	all above-mean (and next 5 below-mean) EqInc funds perform better than all lower-ranked AgGrw funds

APPENDIX: TABLE A9

Table A9: 3-Year Annualized Total Return Mean-Variance Characteristics Ranked by 3-Year Annualized Total Return 1995-1997

	AGGRESSIVE GROWTH 1997				EQUITY INCOME 1997			
RNK	FUND	TRA3Y	3YSD[1]	EFF	FUND	TRA3Y	3YSD[1]	EFF
	strategy avg[2]	109.87			strategy avg	118.59		
	sample avg[3]	108.28	8.460[4]	0.958206	sample avg	118.66	2.5364[4]	0.980567
1	FMILX	120.92	1.49	1.000000	CEIFX	123.68	1.98	1.000000
2	OPTFX	120.86	1.49	1.000000	RIEIX	123.33	1.84	1.000000
3	VALLX	120.43	1.44	1.000000	HLIEX	122.51	1.52	1.000000
4	EKOAX	116.76	1.00	0.975037	VEIPX	122.48	1.50	1.000000
5	PVOYX	116.04	0.92	1.000000	OIEIX	122.16	1.38	0.997319
6	FIDYX	115.60	0.87	0.993041	AVAIX	121.95	1.30	1.000000
7	PAGRX	115.43	0.85	1.000000	UNCMX	121.72	1.21	1.000000
8	PVOBX	115.17	0.81	1.000000	FEQTX	121.58	1.15	1.000000
9	SHRAX	114.99	0.79	1.000000	MADVX	121.51	1.12	0.988580
10	FDEGX	114.93	0.79	1.000000	PEQIX	121.38	1.07	1.000000
11	VALSX	114.84	0.78	1.000000	PQNAX	121.08	0.95	1.000000
12	FDCAX	114.18	0.70	1.000000	FEQIX	120.86	0.87	1.000000
13	SAGBX	114.09	0.69	0.985076	EKTAX	120.76	0.83	0.979869
14	QUASX	113.71	0.64	0.940417	LEIFX	120.73	0.81	0.982447
15	ENCAX	113.62	0.63	0.950963	DELDX	120.51	0.73	1.000000
16	PNOPX	113.61	0.63	1.000000	FFEIX	120.33	0.66	0.981767
17	QUABX	112.86	0.54	0.946435	MBDVX	120.30	0.65	0.978420
18	MEGBX	112.50	0.50	1.000000	PEYAX	120.17	0.59	0.992544
19	PHSKX	112.15	0.46	0.951333	PQNCX	120.16	0.59	1.000000
20	SECUX	111.59	0.39	0.990877	PRFDX	120.15	0.59	1.000000
21	PEMGX	111.45	0.37	0.981671	MIEFX	120.03	0.54	0.985781
22	EAGAX	111.29	0.36	0.943899	SEEIX	120.00	0.53	0.986251
23	FTAGX	111.01	0.32	0.939252	GABEX	119.98	0.52	0.999972
24	INAGX	110.88	0.31	0.944961	HAVLX	119.85	0.47	0.998505
25	DGTIX	110.84	0.30	1.000000	MAPOX	119.69	0.40	1.000000
26	DELTX	110.58	0.27	1.000000	PEIIX	119.68	0.40	0.998601
27	DPAGX	110.57	0.27	0.936098	ENGIX	119.68	0.40	0.981718
28	USAUX	109.56	0.15	1.000000	OPPEX	119.66	0.39	0.979920
29	CSTIX	109.43	0.14	1.000000	PLIAX	119.64	0.39	0.978125
30	CSTGX	108.91	0.07	1.000000	SBCIX	119.59	0.37	0.975840
31	FWLEX	108.82	0.06	0.914785	HMIEX	119.32	0.26	0.981230
32	POGSX	108.23	-0.01	1.000000	PLINX	119.32	0.26	0.973901
33	KAUFX	107.84	-0.05	1.000000	GAEIX	119.13	0.18	0.985104
34	PHAGX	107.10	-0.14	1.000000	PBEAX	118.84	0.07	1.000000

APPENDIX: TABLE A9 (continued)

RNK	FUND	TRA3Y	3YSD	EFF		FUND	TRA3Y	3YSD	EFF
35	SRFCX	106.14	-0.25	1.000000		ACEIX	118.74	0.03	0.979111
36	WAAEX	105.62	-0.31	0.938734		EQPIX	118.73	0.03	1.000000
37	SHELX	105.58	-0.32	0.960779		SSEAX	118.73	0.03	0.969361
38	AAGFX	104.05	-0.50	0.958325		HWEQX	118.72	0.02	1.000000
39	FRSPX	103.76	-0.53	0.911279		SBGCX	118.64	-0.01	0.966850
40	STDIX	103.64	-0.55	0.909415		FIIIX	118.37	-0.12	0.969618
41	SCFCX	101.57	-0.79	0.912135		DSIDX	118.28	-0.15	0.989700
42	TUDRX	101.37	-0.82	0.884449		RFIEX	118.23	-0.17	0.993413
43	POPAX	99.44	-1.04	0.877309		MGIEX	118.20	-0.18	0.979433
44	POPCX	98.73	-1.13	0.881713		MSIVX	118.10	-0.22	1.000000
45	BBSEX	98.07	-1.21	0.891494		FEIRX	118.06	-0.24	0.989637
46	EKABX	94.29	-1.65	0.880863		PBQIX	117.98	-0.27	1.000000
47	AHERX	91.33	-2.00	1.000000		BNEIX	117.95	-0.28	0.964082
48	CHSPX	90.10	-2.15	0.847892		PKHEX	117.91	-0.30	0.972480
49	TWCVX	90.05	-2.15	0.883492		ACEQX	117.83	-0.33	0.970715
50	DRLEX	79.47	-3.41	0.778588		CAIBX	117.74	-0.36	0.968781
					51	SAFIX	117.68	-0.39	0.971246
					52	AMANX	117.65	-0.40	0.973091
					53	SOPAX	117.42	-0.49	0.969843
					54	USISX	116.97	-0.67	0.964801
					55	SOPTX	116.84	-0.72	0.965551
					56	FISEX	116.76	-0.75	1.000000
					57	VALIX	116.71	-0.77	0.955360
					58	INDZX	116.67	-0.79	0.957388
					59	SDINX	116.55	-0.83	0.963775
					60	NEQUX	116.34	-0.92	0.967764
					61	USINX	116.17	-0.98	0.945086
					62	NEQIX	116.04	-1.03	0.959120
					63	WTEIX	116.00	-1.05	0.959954
					64	NEINX	115.56	-1.22	0.954549
					65	ODIFX	115.49	-1.25	0.962082
					66	AMNIX	115.16	-1.38	0.961236
					67	COIGX	114.32	-1.71	0.959960
					68	EVTRX	112.51	-2.43	0.953676
					69	UMIGX	111.54	-2.81	0.932882
					70	PRBLX	110.07	-3.39	0.923233

[1] 3YSD — TRA3Y in units of standard deviation, as given by strategy category sample standard deviation

[2] strategy average — average of the Morningstar population for each strategy category

[3] sample average — average of each strategy category sample used in this study

[4] sample standard deviation for each strategy category; used to compute 3YSD

Efficient Funds	AGGRW	EQINC
Total	22	20
Avg Rnk TRA3Y	19.1	19.8
Avg/N	0.3827	0.2836

(solid line)	mean performance for each category
(heavy dashed line)	all EqInc funds better than all lower-ranked AgGrw funds
(light dashed line)	all above-mean (and first 11 below-mean) EqInc funds perform better than all lower-ranked AgGrw funds

APPENDIX: TABLE A10

Table A10: 5-Year Annualized Total Return Mean-Variance Characteristics Ranked by 5-Year Annualized Total Return 1993-1997

	AGGRESSIVE GROWTH 1997				EQUITY INCOME 1997			
RNK	FUND	TRA5Y	5YSD[1]	EFF	FUND	TRA5Y	5YSD[1]	EFF
	strategy avg[2]	110.56			strategy avg	114.93		
	sample avg[3]	109.95	7.042[4]	0.958206	sample avg	114.87	2.105[4]	0.980567
1	FMILX	120.18	1.45	1.000000	HLIEX	118.15	1.56	1.000000
2	OPTFX	118.78	1.25	1.000000	UNCMX	117.89	1.43	1.000000
3	VALLX	117.68	1.10	1.000000	PRFDX	117.87	1.42	1.000000
4	FDEGX	117.15	1.02	1.000000	OIEIX	117.82	1.40	0.997319
5	PVOYX	116.06	0.87	1.000000	EQPIX	117.69	1.34	1.000000
6	PNOPX	115.93	0.85	1.000000	VEIPX	117.46	1.23	1.000000
7	MEGBX	115.88	0.84	1.000000	RIEIX	117.45	1.22	1.000000
8	PAGRX	115.47	0.78	1.000000	AVAIX	117.28	1.14	1.000000
9	PVOBX	115.17	0.74	1.000000	PEYAX	117.26	1.13	0.992544
10	FIDYX	114.85	0.70	0.993041	FEQIX	117.10	1.06	1.000000
11	SHRAX	114.36	0.63	1.000000	CEIFX	117.07	1.04	1.000000
12	VALSX	114.24	0.61	1.000000	FEQTX	117.05	1.03	1.000000
13	PHSKX	113.86	0.56	0.951333	HAVLX	116.90	0.96	0.998505
14	CSTIX	113.66	0.53	1.000000	FEIRX	116.88	0.95	0.989637
15	AAGFX	113.65	0.53	0.958325	DELDX	116.53	0.79	1.000000
16	EKOAX	113.53	0.51	0.975037	MADVX	116.46	0.75	0.988580
17	SAGBX	113.48	0.50	0.985076	PEQIX	116.43	0.74	1.000000
18	USAUX	113.41	0.49	1.000000	GAEIX	116.39	0.72	0.985104
19	PEMGX	113.33	0.48	0.981671	SEEIX	116.34	0.70	0.986251
20	CSTGX	113.12	0.45	1.000000	RFIEX	116.12	0.59	0.993413
21	KAUFX	112.98	0.43	1.000000	ENGIX	115.99	0.53	0.981718
22	FDCAX	112.94	0.42	1.000000	FFEIX	115.95	0.51	0.981767
23	ENCAX	112.68	0.39	0.950963	HMIEX	115.92	0.50	0.981230
24	QUASX	112.18	0.32	0.940417	PEIIX	115.90	0.49	0.998601
25	POGSX	111.72	0.25	1.000000	MIEFX	115.73	0.41	0.985781
26	QUABX	111.32	0.19	0.946435	MGIEX	115.72	0.40	0.979433
27	INAGX	111.14	0.17	0.944961	LEIFX	115.71	0.40	0.982447
28	FTAGX	111.03	0.15	0.939252	GABEX	115.66	0.37	0.999972
29	EAGAX	110.87	0.13	0.943899	PQNAX	115.61	0.35	1.000000
30	SRFCX	110.72	0.11	1.000000	HWEQX	115.58	0.34	1.000000
31	DPAGX	110.16	0.03	0.936098	DSIDX	115.52	0.31	0.989700
32	DGTIX	109.85	-0.01	1.000000	ACEIX	115.51	0.30	0.979111
33	WAAEX	109.72	-0.03	0.938734	MSIVX	115.42	0.26	1.000000
34	DELTX	109.58	-0.05	1.000000	MAPOX	115.32	0.21	1.000000

APPENDIX: TABLE A10 (continued)

RNK	FUND	TRA5Y	5YSD	EFF	FUND	TRA5Y	5YSD	EFF
35	SHELX	109.15	-0.11	0.960779	OPPEX	115.32	0.21	0.979920
36	SECUX	109.13	-0.12	0.990877	PLIAX	115.32	0.21	0.978125
37	FWLEX	107.55	-0.34	0.914785	MBDVX	115.23	0.17	0.978420
38	SCFCX	107.34	-0.37	0.912135	EKTAX	115.16	0.14	0.979869
39	STDIX	107.02	-0.42	0.909415	SBCIX	115.16	0.14	0.975840
40	PHAGX	106.89	-0.43	1.000000	PLINX	115.06	0.09	0.973901
41	FRSPX	106.34	-0.51	0.911279	PQNCX	114.73	-0.07	1.000000
42	POPAX	104.53	-0.77	0.877309	ACEQX	114.68	-0.09	0.970715
43	POPCX	103.76	-0.88	0.881713	SOPAX	114.58	-0.14	0.969843
44	TUDRX	103.68	-0.89	0.884449	SAFIX	114.57	-0.14	0.971246
45	EKABX	102.58	-1.05	0.880863	SSEAX	114.53	-0.16	0.969361
46	TWCVX	101.86	-1.15	0.883492	FIIIX	114.35	-0.25	0.969618
47	BBSEX	99.57	-1.47	0.891494	SBGCX	114.07	-0.38	0.966850
48	CHSPX	98.16	-1.67	0.847892	PBEAX	114.06	-0.39	1.000000
49	DRLEX	87.81	-3.14	0.778588	SOPTX	114.02	-0.41	0.965551
50	AHERX	81.53	-4.04	1.000000	CAIBX	113.95	-0.44	0.968781
51					BNEIX	113.87	-0.48	0.964082
52					SDINX	113.87	-0.48	0.963775
53					ODIFX	113.67	-0.57	0.962082
54					NEQUX	113.60	-0.61	0.967764
55					AMNIX	113.57	-0.62	0.961236
56					USISX	113.41	-0.70	0.964801
57					NEQIX	113.32	-0.74	0.959120
58					PBQIX	113.22	-0.79	1.000000
59					FISEX	113.13	-0.83	1.000000
60					INDZX	112.97	-0.90	0.957388
61					NEINX	112.78	-1.00	0.954549
62					VALIX	112.72	-1.02	0.955360
63					AMANX	112.26	-1.24	0.973091
64					PKHEX	112.19	-1.28	0.972480
65					COIGX	111.66	-1.53	0.959960
66					WTEIX	110.57	-2.04	0.959954
67					UMIGX	110.22	-2.21	0.932882
68					USINX	109.59	-2.51	0.945086
69					PRBLX	109.08	-2.75	0.923233
70					EVTRX	109.05	-2.77	0.953676

[1] 5YSD — TRA5Y in units of standard deviation, as given by strategy category sample standard deviation

[2] strategy average — average of the Morningstar population for each strategy category

[3] sample average — average of each strategy category sample used in this study

[4] sample standard deviation for each strategy category; used to compute 5YSD

Efficient Funds	AGGRW	EQINC
Total	22	20
Avg Rnk TRA5Y	17	21.45
Avg/N	0.34	0.3064

Line	Meaning
solid line	mean performance for each category
heavy dashed line	all EqInc funds better than all lower-ranked AgGrw funds
light dashed line	all above-mean EqInc funds perform better than all lower-ranked AgGrw funds

APPENDIX: TABLE A11

Table A11: Aggressive Growth Funds with Efficiency Projected Input/Output Values
[Results of Strategy Comparison Methodology, Step 2]
(listed alphabetically by fund name)

nbr	fund	AnRet97	TRA3Y	TRA5Y	NetAss	%Cash	Turnover Ratio	Exp Ratio	Front Load	Dfrrd Load	12b-1
1	AAGFX	126.11	120.28	118.59	2756.16	5.17	73.00	1.00	5.47	0.00	0.24
2	CSTGX	112.92	108.91	113.12	12424.30	6.70	67.00	1.11	5.50	0.00	0.30
3	CSTIX	113.45	109.43	113.66	189.10	6.70	67.00	0.65	0.00	0.00	0.00
4	QUASX	124.79	120.91	120.05	1420.22	3.15	135.00	0.95	3.25	0.00	0.02
5	QUABX	123.31	119.81	117.62	701.16	3.55	56.72	0.88	0.00	0.00	0.00
6	TWCVX	127.57	116.41	115.29	1013.37	2.10	78.96	1.00	0.00	0.00	0.00
7	AHERX	175.00	91.33	81.53	8.40	0.00	1180.00	5.85	0.00	0.00	0.00
8	BBSEX	124.05	112.34	111.69	37.02	0.00	77.35	1.21	0.00	0.00	0.00
9	CHSPX	131.31	115.99	115.77	132.56	0.50	11.57	1.38	0.00	0.00	0.00
10	DPAGX	123.80	120.43	117.68	498.40	3.60	37.00	0.86	0.00	0.00	0.00
11	DELTX	119.43	110.58	109.58	370.70	0.00	114.00	1.34	4.75	0.00	0.30
12	DGTIX	119.86	110.84	109.85	46.80	0.00	114.00	1.09	0.00	0.00	0.00
13	DRLEX	124.52	113.28	112.78	164.53	0.30	76.00	1.20	0.25	0.00	0.00
14	ENCAX	126.47	120.45	118.49	1272.72	4.96	61.00	1.02	4.75	0.00	0.20
15	EAGAX	128.43	118.16	117.46	578.61	1.60	56.00	1.18	1.03	0.00	0.00
16	EKOAX	127.72	119.75	117.97	1210.45	4.63	63.00	1.12	4.75	0.00	0.21
17	EKABX	128.73	117.66	116.45	231.29	1.60	17.58	1.19	0.00	0.00	0.00
18	FDCAX	126.52	114.18	112.94	2290.70	2.80	176.00	0.66	0.00	0.00	0.00
19	FDEGX	119.45	114.93	117.15	2297.80	3.20	212.00	1.05	0.00	0.00	0.00
20	FMILX	124.63	120.92	120.18	1416.00	2.90	142.00	0.94	3.00	0.00	0.00
21	FRSPX	127.77	118.20	116.69	283.38	1.99	21.37	1.13	0.00	0.00	0.00
22	FTAGX	125.02	120.64	118.21	963.49	4.57	51.00	0.93	2.78	0.00	0.12
23	INAGX	123.25	119.74	117.61	724.90	3.55	59.03	0.88	0.00	0.00	0.00
24	FIDYX	125.78	116.41	115.65	1270.64	2.49	103.98	0.97	0.00	0.00	0.00
25	KAUFX	112.56	107.84	112.98	4239.80	10.00	65.00	1.88	0.00	0.00	0.75
26	FWLEX	124.25	120.18	117.57	474.12	3.42	35.24	0.89	0.00	0.00	0.00
27	MEGBX	119.73	112.50	115.88	5317.30	3.20	21.00	1.97	0.00	4.00	1.00
28	OPTFX	126.32	120.86	118.78	1461.80	5.60	66.00	1.01	5.75	0.00	0.25
29	PHAGX	114.22	107.10	106.89	168.90	0.00	83.00	1.46	5.50	0.00	0.00
30	PAGRX	132.68	115.43	115.47	16.90	0.00	2.00	1.46	0.00	0.00	0.00
31	PHSKX	125.48	120.34	119.68	1268.87	2.60	127.28	0.99	2.68	0.00	0.00
32	POPAX	125.88	120.88	119.15	1449.75	4.89	86.00	0.99	5.03	0.00	0.18
33	POPCX	123.80	120.43	117.68	498.40	3.60	37.00	0.86	0.00	0.00	0.00
34	POGSX	101.30	108.23	111.72	40.90	3.90	17.00	0.99	0.00	0.00	0.00
35	PEMGX	129.58	116.13	115.45	339.12	1.28	10.00	1.26	1.90	0.00	0.10

APPENDIX: TABLE A11 (continued)

nbr	fund	AnRet97	TRA3Y	TRA5Y	NetAss	%Cash	Turnover Ratio	Exp Ratio	Front Load	Dfrrd Load	12b-1
36	PNOPX	122.55	113.61	115.93	8320.10	2.40	65.00	0.98	5.75	0.00	0.35
37	PVOYX	125.98	116.04	116.06	12167.20	4.30	60.00	0.96	5.75	0.00	0.35
38	PVOBX	125.07	115.17	115.17	6347.90	4.30	60.00	1.71	0.00	5.00	1.00
39	SECUX	126.45	112.62	112.57	68.22	0.00	29.35	1.46	1.86	0.00	0.00
40	SHELX	126.86	113.59	113.61	76.17	0.20	42.23	1.29	0.00	0.00	0.00
41	SHRAX	128.58	114.99	114.36	387.90	1.70	6.00	1.21	5.00	0.00	0.25
42	SAGBX	131.40	115.82	115.70	235.31	0.45	6.00	1.42	0.00	0.13	0.03
43	SCFCX	123.80	120.43	117.68	498.40	3.60	37.00	0.86	0.00	0.00	0.00
44	SRFCX	106.15	106.14	110.72	640.50	7.10	35.00	1.17	0.00	0.00	0.00
45	STDIX	123.80	120.43	117.68	498.40	3.60	37.00	0.86	0.00	0.00	0.00
46	USAUX	107.56	109.56	113.41	669.40	2.60	83.00	0.71	0.00	0.00	0.00
47	VALLX	123.80	120.43	117.68	498.40	3.60	37.00	0.86	0.00	0.00	0.00
48	VALSX	132.10	114.84	114.24	145.20	8.70	146.00	1.08	0.00	0.00	0.00
49	WAAEX	127.01	118.62	116.88	324.27	2.30	24.34	1.08	0.00	0.00	0.00
50	TUDRX	125.63	119.40	117.23	399.38	2.86	29.80	0.98	0.00	0.00	0.00

VARIABLE DEFINITIONS (For detailed definitions, see Appendix Table A1.)

n	<SYMBOL>	*heavy end borders indicate funds which were originally efficient within their own strategic category*

Summary Statisitcs for Efficiency Projected Input/Output Values
of Aggressive Growth Funds

stat	AnRet97	TRA3Y	TRA5Y	NetAss	%Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
min	101.30	91.33	81.53	8.40	0.00	2.00	0.65	0.00	0.00	0.00
max	175.00	120.92	120.18	12424.30	10.00	1180.00	5.85	5.75	5.00	1.00
avg	124.45	115.46	114.92	1577.11	3.05	85.96	1.20	1.49	0.18	0.11
std dev	9.7775	5.5620	5.6184	2759.5445	2.2998	164.1589	0.7247	2.2062	0.8961	0.2335

APPENDIX: TABLE A12

Table A12: Equity Income Funds with Efficiency Projected Input/Output Values
[Results of Strategy Comparison Methodology, Step 2]
(listed alphabetically by fund name)

nbr	fund	AnRet97	TRA3Y	TRA5Y	NetAss	%Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
1	AVAIX	132.95	121.95	117.28	110.60	6.50	68.00	1.05	0.00	0.00	0.00
2	AMANX	129.04	120.90	116.03	59.30	2.63	8.00	0.95	0.00	0.00	0.00
3	AMNIX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
4	BNEIX	132.45	122.51	118.11	717.85	1.18	15.38	0.97	0.00	0.00	0.00
5	CAIBX	129.56	121.53	117.62	8323.86	10.48	25.46	0.65	1.36	0.00	0.06
6	RIEIX	131.05	123.33	117.45	105.50	6.50	14.00	1.00	0.00	0.00	0.00
7	CEIFX	133.35	123.68	117.07	73.80	22.30	119.00	1.10	0.00	0.00	0.00
8	DELDX	129.71	120.51	116.53	1811.90	0.00	90.00	0.88	4.75	0.00	0.30
9	ENGIX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
10	EKTAX	131.21	123.24	117.53	160.61	5.89	14.11	1.00	0.00	0.00	0.00
11	EVTRX	131.68	120.29	116.65	785.91	0.40	19.77	0.92	0.00	0.00	0.00
12	UMIGX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
13	LEIFX	131.66	122.89	117.78	821.32	3.85	16.17	0.99	0.00	0.00	0.00
14	EQPIX	126.64	118.73	117.69	473.30	2.00	55.00	0.67	0.00	0.00	0.00
15	FEIRX	131.91	122.12	118.10	2557.81	1.57	16.48	0.97	0.00	0.00	0.00
16	FEQIX	129.98	120.86	117.10	22277.10	1.80	23.00	0.65	0.00	0.00	0.00
17	FEQTX	127.17	121.58	117.05	17659.70	8.40	77.00	0.68	0.00	0.00	0.00
18	FFEIX	132.42	122.56	118.10	579.44	1.27	14.93	1.00	0.00	0.00	0.00
19	FISEX	127.21	116.76	113.13	409.10	0.00	29.00	0.97	5.75	0.00	0.25
20	GABEX	131.61	119.98	116.44	307.10	0.30	20.33	0.91	0.00	0.00	0.00
21	GAEIX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
22	HAVLX	131.40	120.99	117.08	5704.06	1.30	19.89	0.83	0.00	0.00	0.00
23	HMIEX	132.50	122.51	118.14	648.15	0.99	15.13	0.99	0.00	0.00	0.00
24	HWEQX	131.15	118.72	115.58	154.00	0.00	23.00	0.87	0.00	0.00	0.00
25	INDZX	131.97	122.32	118.00	1907.38	2.18	17.03	0.88	0.00	0.00	0.00
26	FIIIX	131.78	122.08	117.93	4852.02	1.65	17.29	0.90	0.00	0.00	0.00
27	MAPOX	128.04	119.69	115.32	36.20	0.70	5.00	0.92	0.00	0.00	0.00
28	MGIEX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
29	MADVX	131.80	122.91	117.81	363.30	3.66	14.51	1.00	0.00	0.00	0.00
30	MBDVX	131.73	122.95	117.77	338.75	3.93	14.46	1.00	0.00	0.00	0.00
31	MIEFX	131.29	121.76	117.64	10530.74	1.60	19.02	0.81	0.00	0.00	0.00
32	MSIVX	129.20	118.10	115.42	60.90	0.00	36.00	0.70	0.00	0.00	0.00
33	NEQIX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
34	NEINX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
35	NEQUX	130.57	120.70	117.38	726.42	0.90	27.79	0.86	0.00	0.00	0.00

APPENDIX: TABLE A12 (continued)

nbr	fund	AnRet97	TRA3Y	TRA5Y	NetAss	%Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
36	ODIFX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
37	OIEIX	132.53	122.50	118.14	612.26	0.90	15.00	1.00	0.03	0.00	0.00
38	HLIEX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
39	OPPEX	132.34	122.18	117.68	3197.81	1.87	17.49	0.88	0.78	0.00	0.03
40	PKHEX	132.06	121.25	117.29	460.20	0.60	17.67	0.96	0.00	0.00	0.00
41	PRBLX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
42	PLINX	132.51	122.52	118.14	608.57	0.95	14.99	1.00	0.00	0.00	0.00
43	PLIAX	132.03	122.50	117.90	1310.28	2.75	17.55	0.80	0.00	0.00	0.00
44	PEIIX	131.56	121.62	117.23	1411.99	3.10	20.16	0.71	0.00	0.00	0.00
45	PQNAX	135.92	121.08	115.61	75.70	3.50	192.00	1.26	5.50	0.00	0.25
46	PQNCX	134.90	120.16	114.73	391.00	3.50	192.00	2.01	0.00	1.00	1.00
47	PEQIX	134.89	121.38	116.43	585.40	0.60	18.00	1.11	5.75	0.00	0.25
48	PBEAX	136.41	118.84	114.06	638.70	2.30	36.00	0.94	5.00	0.00	0.30
49	PBQIX	135.34	117.98	113.22	1299.70	2.30	36.00	1.69	0.00	5.00	1.00
50	PEYAX	132.40	122.43	118.14	1000.56	1.03	15.29	0.99	0.00	0.00	0.00
51	RFIEX	130.33	120.95	116.89	1545.54	0.20	73.33	0.91	3.69	0.00	0.23
52	SAFIX	132.15	122.50	117.96	1136.04	2.29	16.91	0.85	0.00	0.00	0.00
53	SEEIX	132.15	122.50	117.96	1136.04	2.29	16.91	0.85	0.00	0.00	0.00
54	COIGX	130.03	120.11	116.32	1440.73	0.00	75.00	0.88	3.69	0.00	0.23
55	SBCIX	132.24	122.55	118.01	864.14	1.95	15.96	0.92	0.00	0.00	0.00
56	SBGCX	132.17	122.71	117.98	490.80	2.25	14.76	1.00	0.00	0.00	0.00
57	SOPAX	132.43	122.45	118.14	913.45	1.00	15.23	0.99	0.00	0.00	0.00
58	SOPTX	131.70	121.99	118.09	3228.42	1.81	16.99	0.95	0.00	0.00	0.00
59	SDINX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
60	SSEAX	132.52	122.51	118.15	613.30	0.90	15.00	1.00	0.00	0.00	0.00
61	PRFDX	128.82	120.15	117.87	12436.60	5.00	24.00	0.79	0.00	0.00	0.00
62	USINX	131.79	122.92	117.80	359.40	3.70	14.50	1.00	0.00	0.00	0.00
63	DSIDX	131.76	120.40	116.72	358.13	0.40	19.44	0.93	0.00	0.00	0.00
64	UNCMX	127.42	121.72	117.89	6958.00	28.00	34.00	0.84	5.75	0.00	0.25
65	USISX	131.62	122.47	117.66	2453.87	4.59	22.00	0.65	0.00	0.00	0.00
66	VALIX	132.20	122.50	117.99	1066.34	2.11	16.65	0.87	0.00	0.00	0.00
67	ACEIX	132.18	122.50	117.97	1101.19	2.20	16.78	0.86	0.00	0.00	0.00
68	ACEQX	132.38	122.42	118.14	1045.42	1.05	15.33	0.99	0.00	0.00	0.00
69	VEIPX	131.17	122.48	117.46	2530.00	6.00	22.00	0.45	0.00	0.00	0.00
70	WTEIX	132.56	122.46	118.07	568.42	1.40	19.73	1.00	0.00	0.00	0.00

VARIABLE DEFINITIONS (For detailed definitions, see Appendix Table A1.)

n	<SYMBOL>	*heavy end borders indicate funds which were originally efficient within their own strategic category*

APPENDIX: TABLE A12 (continued)

Summary Statisitcs for Efficiency Projected Input/Output Values
of Equity Income Funds

stat	AnRet97	TRA3Y	TRA5Y	NetAss	%Cash	Turnover Ratio	Expn Ratio	Front Load	Dfrrd Load	12b-1
min	126.64	116.76	113.13	36.20	0.00	5.00	0.65	0.00	0.00	0.00
max	136.41	123.68	118.15	22277.10	22.30	192.00	2.01	5.75	5.00	1.00
avg	131.78	121.51	117.16	2005.63	2.49	31.54	0.97	0.58	0.12	0.07
std dev	1.9983	1.5465	1.2956	4244.7983	3.5924	39.3591	0.2212	1.6268	0.7183	0.2087

APPENDIX: TABLE A13

Table A13: Summary Comparison of Changes among Aggressive Growth Funds between Original Observations and Efficiency Projection Values within Their Strategic Category

OUTPUT		AnRet97			TRA3Y			TRA5Y			NetAss		
Efficient	(no change)			22			22			22			22
Inefficient	no change			0			0			0			0
	change			28			28			28			28
Total funds				50			50			50			50
		projctd	obsrvtn	diff	projctd	obsrvtn	diff	projctd	obsrvtn	diff	projctd	obsrvtn	diff
min		101.30	86.97	14.33	91.33	79.47	11.86	81.53	81.53	0.00	8.4000	4.7000	3.7000
max		175.00	175.00	0.00	120.92	120.92	0.00	120.18	120.18	0.00	12424.3000	12424.3000	0.0000
average		124.45	116.57	7.88	115.46	108.28	7.18	114.92	109.95	4.97	1577.1061	1384.4600	192.6461
range		73.70	88.03	–14.33	29.59	41.45	–11.86	38.65	38.65	0.00	12415.9000	12419.6000	–3.7000

INPUT		%Cash			TurnoverRatio			ExpnRatio		
Efficient	(no change)			22			22			22
Inefficient	no change, obs=0			2			0			0
	no change, obs≠0			5			9			4
	change			21			19			24
TOTAL funds				50			50			50
		projctd	obsrvtn	diff	projctd	obsrvtn	diff	projctd	obsrvtn	diff
min		0.00	0.00	0.00	2.00	2.00	0.00	0.65	0.65	0.00
max		10.00	29.30	–19.30	1180.00	1180.00	0.00	5.85	5.85	0.00
average		3.05	4.96	–1.91	85.96	121.22	–35.26	1.20	1.48	–0.28
range		10.00	29.30	–19.30	1178.00	1178.00	0.00	5.20	5.20	0.00

APPENDIX: TABLE A13 (continued)

INPUT		FrontLoad			DfrrdLoad			12b-1		
Efficient	(no change)			22			22			22
Inefficient	no change, obs=0			17			23			8
	no change, obs≠0			2			0			0
	change			9			5			20
Total funds				50			50			50
		projctd	obsrvtn	diff	projctd	obsrvtn	diff	projctd	obsrvtn	diff
	min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	max	5.75	5.75	0.00	5.00	5.00	0.00	1.00	1.00	0.00
	average	1.49	1.92	–0.43	0.18	0.56	–0.38	0.11	0.29	–0.18
	range	5.75	5.75	0.00	5.00	5.00	0.00	1.00	1.00	0.00

VARIABLE DEFINITIONS (For detailed definitions, see Appendix Table A1.)

APPENDIX: TABLE A14

Table A14: Summary Comparison of Changes among Equity Income Funds
between Original Observations and Efficiency Projection Values within Their Strategic Category

OUTPUT		AnRet97			TRA3Y			TRA5Y			NetAss		
Efficient	(no change)			20			20			20			20
Inefficient	no change			0			0			0			0
	change			50			50			50			50
TOTAL funds				70			70			70			70
		projctd	obsrvtn	diff	projctd	obsrvtn	diff	projctd	obsrvtn	diff	projctd	obsrvtn	diff
min		126.64	118.55	8.09	116.76	110.07	6.69	113.13	109.05	4.08	36.2000	6.9000	29.3000
max		136.41	136.41	0.00	123.68	123.68	0.00	118.15	118.15	0.00	22277.1000	22277.1000	0.0000
average		131.70	127.20	4.50	121.66	118.66	3.00	117.33	114.87	2.45	2016.2910	1500.7029	515.5882
range		9.77	17.86	–8.09	6.92	13.61	–6.69	5.02	9.10	–4.08	22240.9000	22270.2000	–29.3000

INPUT		%Cash			TurnoverRatio			ExpnRatio		
Efficient	(no change)			20			20			20
Inefficient	no change, obs=0			1			0			0
	no change, obs≠0			13			4			18
	change			36			46			32
TOTAL funds				70			70			70
		projctd	obsrvtn	diff	projctd	obsrvtn	diff	projctd	obsrvtn	diff
min		0.00	0.00	0.00	5.00	5.00	0.00	0.45	0.45	0.00
max		28.00	28.00	0.00	192.00	192.00	0.00	2.01	2.14	–0.13
average		2.75	6.59	–3.84	29.38	56.60	–27.22	0.95	1.11	–0.16
range		28.00	28.00	0.00	187.00	187.00	0.00	1.56	1.69	–0.13

APPENDIX: TABLE A14 (continued)

INPUT		FrontLoad			DfrrdLoad			12b-1		
Efficient	(**no change**)			20			20			20
Inefficient	**no change, obs=0**			19			44			7
	no change, obs≠0			0			0			0
	change			31			6			43
TOTAL funds				70			70			70
		projctd	**obsrvtn**	**diff**	**projctd**	**obsrvtn**	**diff**	**projctd**	**obsrvtn**	**diff**
	min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	max	5.75	5.75	0.00	5.00	5.00	0.00	1.00	1.00	0.00
	average	0.60	2.00	–1.40	0.09	0.31	–0.22	0.06	0.21	–0.15
	range	5.75	5.75	0.00	5.00	5.00	0.00	1.00	1.00	0.00

VARIABLE DEFINITIONS (For detailed definitions, see Appendix Table A1.)

APPENDIX: TABLE A15, TABLE A16, TABLE A17

Table A15: Summary Comparison of Extent to Which Efficient Equity Income Funds Appear in the Envelopment Form Reference Sets of Inefficient Funds, (Funds which Appear in the Reference Set of More Than One Inefficient Fund) Step Three, DEA of Strategy Category Exemplars

fund	GAEI^[1]	VEIPX	PRFDX	FEQIX	HWEQX	PLIA^	CEIFX
$\lambda > 0$[2]	43	18	16	7	5	3	3
min	0.051277	0.005185	0.004134	0.001696	0.236014	0.173621	0.007791
max	1.000000	0.636364	0.498267	0.469793	1.000000	0.592954	0.124252
avg	0.600516	0.206369	0.141784	0.235931	0.802758	0.397410	0.073127

Table A16: Comparison of Extent to Which Efficient Equity Income Funds Appear in the Envelopment Form Reference Sets of Inefficient Funds, (Funds which Appear in the Reference Set of Only One Inefficient Fund) Step Three, DEA of Strategy Category Exemplars

fund	DELDX	GABE^	SOPT^	MIEF^	EQPIX	MAPOX	HAVL^	EKTA^
$\lambda > 0$	1	1	1	1	1	1	1	1
$\lambda =$	1.000000	1.000000	0.651337	0.619338	0.504638	0.248387	0.165988	0.026736

Table A17: Summary Comparison of Extent to Which Efficient Aggressive Growth Funds Appear in the Envelopment Form Reference Sets of Inefficient Funds, (Funds which Appear in the Reference Set of More Than One Inefficient Fund) Step Three, DEA of Strategy Category Exemplars

fund	FMILX	AHERX	PAGRX
$\lambda > 0$	14	5	4
min	0.030185	0.000793	0.384615
max	0.944589	0.065540	0.693351
avg	0.409922	0.026618	0.507323

footnotes

1 All mutual fund ticker symbols end in **X**. However, for those funds which were inefficient within their own strategic category (step one) and whose values were projected to their efficient frontier (step two) for subsequent analysis, the final **X** has been replaced with **^** to indicate the use of projected values rather than original observations.

2 The values in this row indicate the number of inefficient funds (actually, strategic category exemplars) in the reference set of which the respective efficient fund (exemplar, identified in the column head) participates, that is, for which the λ values are greater than zero in the envelopment form of the data envelopment analysis.

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