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To my wife, Pınar

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Business Cycles and Labor Market Reallocation

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This dissertation studies the behavior of labor markets over the business cycle. The chapters extend existing literature on the cyclical change in labor markets by considering alternative environments such as on-the-job search and costly screening as well as by estimating a structural labor market search model to understand the effects of different exogenous forces. Chapter 2 uses a standard labor market search model to uncover the cyclical properties of unobserved forcing variables that determine the exogenous state of the aggregate labor market. A structural estimation of the model implies that labor market reallocation as well as implied job separations is strongly procyclical. This result and the recent literature emphasizing the lack of volatility in the standard search models provide the motivation for stressing the role of on-the-job search in creating more volatility for vacancies and unemployment in Chapter 3. A model of on-the-job search with match specific learning is developed in this chapter. Simulations of the model show that job-to-job transitions significantly improve the volatility of vacancies and unemployment. The model implies that firms are more likely to meet employed workers in expansions and that those

they meet are more likely to accept firm's job offer because they are more likely to be employed in a low quality match. This introduces strongly procyclical labor market reallocation through procyclical job-to-job transitions. Chapter 4 seeks to determine the quantitative importance of costly screening for the observed excess volatility. A model similar to the one used in Chapter 3 is utilized to emphasize the symmetric incomplete information about match quality. In the model, employers are endowed with a costly screening technology, which enables them to meet with potentially better quality workers. Since this technology is costly, employers decide to use it when aggregate productivity is low, i.e. when the opportunity cost of not producing is low. These countercyclical changes in screening investment improve the cyclical behavior of vacancies. Therefore, this dissertation provides possible answers for the observed high volatility of key labor market aggregates by emphasizing the significance of recruitment behavior and a more through modeling of workers' search behavior.

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Chapter 1

Introduction

This dissertation studies the behavior of labor markets over the business cycle. The chapters extend existing literature on the cyclical change in labor markets by considering alternative environments such as on-the-job search and costly screening as well as by estimating a structural labor market search model to understand the effects of different exogenous forces. Although each chapter has a different focus, they are all motivated by similar questions and facts and they all use the same theoretical approach to answer the questions posed in each chapter. Our modeling choice for each chapter is a version of standard labor market search model, originally pioneered by Dale Mortensen and Christopher Pissarides [1].

In the last two decades, these models have been used frequently to understand aggregate labor market phenomena, such as equilibrium unemployment and vacancies (Mortensen and Pissarides (1994), Pissarides (2000)). This theoretical framework also proved to be useful in analyzing the effects of various labor market policies including unemployment insurance and labor turnover costs. However, search models have recently been criticized for their business cycle implications. In particular, Shimer (2005a) and Hall (2005) argue that standard models of labor market search

[1] A comprehensive and extensive treatment of these models are presented in Pissarides (2000).

require implausibly large shocks to generate substantial variation in key variables; unemployment, vacancies and market tightness (vacancy to unemployment ratio). Standard deviations of unemployment and vacancies are 10 times, market tightness is 19 times as large as the standard deviation of the average product per worker in the U.S. This leads to a puzzle since a standard calibration of Mortensen-Pissarides model implies that the variations in all these variables is basically the same as labor productivity. [2] This puzzle forms the main focus of this dissertation. In particular, Chapters 3 and 4 address possible ways to solve this puzzle whereas the following chapter aims to understand more about the implications of the standard labor market search models about business cycles.

Chapter 2 is based on a joint work with Kenneth Beauchemin and uses a standard labor market search model to uncover the cyclical properties of unobserved forcing variables that determine the exogenous state of the aggregate labor market. We aim to understand whether matching function instability can be a source of missing volatility. Aggregate matching function is a critical feature of labor market search models and determine the mechanism with which unemployed workers and vacancies form productive matches. Therefore, by focusing on the matching technology, this chapter combines the standard model of labor market search with observed U.S. time series measures on employment, vacancies, and aggregate output to uncover the cyclical properties of three unobserved forcing variables that comprise the exogenous state of the aggregate labor market: labor productivity, the rate of job separation, and the allocational efficiency of the labor market. We posit the latter variable to

[2] Results of the representative simulations appear in Chapter 3 .

be inversely related to the degree of mismatch in the pool of searching workers and vacancies, given numbers of each, and identify its movements as scalar shifts in the standard matching function. Our procedure is based on Ingram, Kocherlakota and Savin (1994), and involves simulated method of moments estimation of the joint process governing the three exogenous variables. We show that, in order to explain the U.S. data with the standard search model, all three exogenous variables should be significantly procyclical and job separations and allocational efficiency should be significantly volatile. This counterfactual finding is due to the fact that, in this environment, all of the worker reallocation required to accommodate cyclical variations in employment should involve an unemployment spell.

This counterfactual implication about job separations and the recent literature emphasizing the lack of volatility in the standard search models provide the motivation for stressing the role of on-the-job search in creating more volatility for vacancies and unemployment in Chapter 3. This chapter studies amplification of productivity shocks in labor markets through on-the-job-search. There is incomplete information about the quality of the employee-firm match which provides persistence in employment relationships and the rationale for on-the-job search. Amplification arises because productivity changes not only affect firms' probability of contacting unemployed workers but also of contacting already employed workers. Since higher productivity raises the value of all matches, even low quality matches become productive enough to survive in expansions. Therefore the measure of workers in low quality matches is greater when productivity is high, implying a higher probability of switching to another match. In other words, firms are more likely to meet employed workers in expansions and those they meet are more likely to accept firm's job offer because they are more likely to be employed in a low quality match. This introduces strongly procyclical labor market reallocation through procyclical job-to-job transitions. Simulations with a productivity process that is consistent with average labor productivity in the U.S. show that standard deviations for unemployment, vacancies and market tightness (vacancy-unemployment ratio) match the U.S. data. The model also reconciles the presence of endogenous separation with the negative correlation of unemployment and vacancies over business cycle frequencies (i.e. it is consistent with the Beveridge curve).

Chapter 4 seeks to determine the quantitative importance of costly screening for the observed excess volatility. A model similar to the one used in Chapter 3 is utilized to emphasize the symmetric incomplete information about match quality. In the model, employers are endowed with a costly screening technology, which enables them to meet with potentially better quality workers. Since this technology is costly, employers decide to use it when aggregate productivity is low, i.e. when the opportunity cost of not producing is low. These countercyclical changes in screening investment improve the cyclical behavior of vacancies.

In conclusion, this dissertation provides possible answers for the observed high volatility of key labor market aggregates by emphasizing the significance of recruitment behavior and a more through modeling of workers' search behavior.

Chapter 2

On the Cyclicality of Labor Market Mismatch and Aggregate Employment Flows

Economists have long acknowledged that buyers and sellers of labor market services are challenged by search frictions to an extent not experienced by participants in other markets – frictions that originate in the wide cross-sectional variation in worker and job qualities, and the consequent burden placed upon labor market participants to process this information. With information frictions at the heart of labor market analysis, it is unfortunate that they are extraordinarily difficult, or even impossible, to observe in a systematic fashion. Attempts to understand measured labor market phenomena and their relation to business cycle fluctuations are doubtlessly frustrated by the limitation. This chapter proposes an indirect measure of these hidden labor market qualities by combining information from the observable labor market aggregates with the information derived from a dynamic general equilibrium (DGE) articulation of the standard labor market search model.

We conceptualize these hidden qualities as an exogenous state vector, the key dimension of which gauges the degree to which the populations of searching workers and vacant positions are suited to each other. This then captures the efficiency with which labor markets allocate workers to jobs, and vice versa. Two additional dimensions, the average productivity of workers and the average rate at which workers separate from their current positions, complete the unobserved labor market state. Our procedure constructs a unique history of this unobserved state vector that simultaneously satisfies the restrictions imposed by DGE search and matching theory and the observed histories of aggregate output, employment, and vacancies. Given the history of the exogenous state vector, the internal logic of the search and matching model also implies realized time series observations on the gross flows of workers into and out of employment.

We intend this study to serve a dual purpose: one of measurement and the other of diagnosis. In the measurement domain, we gauge the cyclical characteristics of the unobserved exogenous variables and gross employment flows, measuring their variability and comovements. In doing so, we provide answers to a number of intriguing questions. Does the degree of labor market mismatch vary systematically over the business cycle, and if so, is it procyclical or countercyclical? Are the movements of workers into and out of employment procyclical, countercyclical or neither? Are the cyclical patterns of the employment inflows and outflows significantly different, i.e. are they asymmetric? The answers to these questions subsequently carry implications for the pattern of labor force reallocation over the business cycle, providing a basis for further theoretical speculation into the nature of business cycles. A strict Schumpeterian view of business cycle dynamics, for example, implies countercyclical labor force reallocation and a nearly teleological interpretation of recessions as periods of 'creative' job destruction and factor reallocation. Our results will clearly inform such discussions.

In its simultaneous treatment of theory and measurement, the techniques that we apply in this chapter not only allow us a glimpse of the unobserved, they also serve a diagnostic purpose designed to engender a deeper understanding of existing theory. By allocating all of the cyclical variation present in the observed endogenous variables to either theoretical variation, i.e. variation that is understood by a stable theoretical framework, or to exogenous variation in the unobserved forcing variables, the procedure separates cyclical phenomena that are understood through the lens of economic theory, from those that are not. From a pure measurement perspective, there is nothing further to be understood: the theory is perfect and so the exogeneity is well-defined and truly exogenous. Of course, we are not so sanguine. Some of the measured exogenous cyclical variation inevitably contains some part model misspecification. Indeed, we intend this chapter to partly serve a 'pre-theoretical' function that informs future modeling efforts to enrich the current understanding of labor market dynamics and their linkages to aggregate economic fluctuations more generally. We expect such efforts to include theoretical structures that help explain the comovements between our exogenous variables.[1]

In recognition of the information problems on both sides of labor markets, Mortensen and Pissarides (1994) and Pissarides (2000) have provided researchers with an analytically convenient and intuitively pleasing framework to capture the costly search process induced by the informational complexity of labor markets –

[1]The literature reveals substantial interest in this endeavor. In addition to Merz (1995), Andalfatto (1996), Cole and Rogerson (1999), and Shimer (2005a), a short list includes Lilien (1982), Abraham and Katz (1986), Blanchard and Diamond (1989), Blanchard and Diamond (1990), Caballero and Hammour (1994), Hall (1995), Den Haan, Ramey and Watson (2000), Gomes, Greenwood, and Rebelo (2001), Pries (2004).

one which is readily amenable to macroeconomics analysis. Our theoretical identifying assumptions spring from a DGE implementation of their framework, at the heart of which is a 'matching function' which determines the number of job matches formed in a given period – the gross employment inflow – as an increasing function of total job vacancies and the number of searching workers.[2] In its simplest form, the Mortensen-Pissarides framework determines the gross employment outflow as an exogenous constant fraction of total employment – the rate of job separation.

We adopt the Mortensen-Pissaides framework as our instrument of measure primarily for its ability to reconcile net employment changes with gross employment flows using well-articulated dynamic economic theory. By construction, the cyclical properties of the three exogenous forcing variables in our analysis – aggregate labor productivity, the rate of job separation, and allocational efficiency of labor markets – are mutually consistent with this framework and the time series observations on employment, vacancies, and aggregate output. The aggregate labor productivity measure (Z) follows quickly from the model economy's resource constraint and possesses cyclical properties nearly identical to conventional labor productivity measures. In keeping with the most basic Mortensen-Pissarides model, the rate of job separation (σ) is exogenous and simply gives the fraction of employed persons that will separate from their jobs, for whatever reason, during a particular period and must be determined simultaneously with the exogenous allocational efficiency variable.

Our third characteristic of the hidden labor market state captures the ef-

[2]See Petrongolo and Pissarides (2001) for a comprehensive review of the literature regarding the matching function and its role in search and matching models and in empirical studies.

ficiency with which existing labor market institutions pair searching workers with available jobs. This is not as transparent as the first two and merits further discussion. We take as axiomatic that the matching function, say $M(V,U)$ – where the flow into employment is a function of the number of vacancies and searching workers – owes its existence to the notion of mismatch, "an empirical concept that measures the degree of heterogeneity in the labor market across a number of dimensions, usually restricted to skills, industrial sector, and location" (Petrongolo and Pissarides, 2001). That is, in the absence of mismatch, jobs and workers would match instantly. Accordingly, an exogenous increase in labor market mismatch, given the matching inputs (V,U), decreases the number of matches formed, or equivalently, decreases the 'allocative efficiency' of the aggregate labor market. Since we are interested in exploring the cyclical properties of mismatch and allocational efficiency, we relax the structural stability of the standard matching function by allowing exogenous multiplicative shifts in the rate of match formation given the levels of matching inputs. Thus, we write $\chi M(V,U)$, where $\chi > 0$ measures the allocational efficiency of the labor markets.[3]

Given the three exogenous components of the hidden labor market state (Z, σ, and χ), we are able to derive measures of job finding and job separation. Job finding is mediated by the matching function and is given by the expression $\chi M(U,V)$; job separation is simply the product of the job separation rate and employment, or

[3]In principle, χ would also pick up lower frequency instability in the matching function contributed by technological advances in matching, changes in government policy, and similar changes. By removing a low-frequency trend from all of the observed endogenous variables prior to analysis, we effectively filter out these movements in allocational efficiency, *a priori*.

σN. To measure the exogenous shocks, we first derive the complete set of independent theoretical restrictions implied by the socially efficient allocation of the DGE search model. The model is essentially borrowed from Merz (1995) and Andalfatto (1996), but abstracts from physical capital accumulation.[4] This provides us with three conditions that characterize the equilibrium allocation of employment, vacancies, and output: 1) a resource constraint defining the feasible allocations, 2) an Euler equation implied by an intertemporally efficient program of vacancy-creation, and 3) the equation of motion for employment reconciling gross employment flows with net employment flows. To solve the model, we log-linearize these conditions and specify a general VAR(1) process to govern the joint shock process. With knowledge of the parameters defining the VAR(1) process, the entire system is easily inverted to obtain a history of exogenous shocks conditional on these parameters. We have no such prior knowledge, of course, and so we follow the simulated method of moments procedure proposed by Lee and Ingram (1991) to obtain these parameters.

The measurement function of this chapter shares the aim of numerous predecessors that measure unobserved time series characteristics from existing evidence and theoretical restrictions, with the works of Solow (1956) and Prescott (1986) perhaps the most famous of these. In the context of DGE environments, this process is initially formalized by Ingram, Kocherlakota, and Savin (1994) who advocate the use of singular models to produce inferred shock series that are unique. This ap-

[4]Unlike their models, ours abstracts from physical capital accumulation. This simplification not only allows us to economize on the number of parameters in the joint distribution of shocks that must be estimated, but also facilitates comparisons with studies of the aggregate labor market that rely more closely on the original Mortensen-Pissarides framework (e.g. Mortensen and Pissarides (1994), Cole and Rogerson (1999), and Shimer (2005a)).

proach was subsequently extended by Chari, Kehoe, and McGratten (2002) who use maximum likelihood to estimate the parameters governing the distribution of shocks. They also emphasize the diagnostic role of the procedure. Our technique strongly resembles theirs, except for the method used to extract the parameters governing the exogenous forcing process; we use simulated method of moments rather than maximum likelihood.

In its specific attention paid to labor market dynamics, the measurement function shares the goal of more direct attempts to infer the aggregate cyclical characteristics of gross employment flows using partial evidence offered by inherently incomplete data sets. Blanchard and Diamond (1990) analyze the gross worker flow data from the Current Population Survey (CPS) compiled by the Bureau of Labor Statistics (BLS). Unfortunately, these observed worker flows do not reconcile period-to-period aggregate net employment changes and often display large discrepancies, even of opposite sign. Davis and Haltiwanger (1992) and Davis, Haltitwanger, and Schuh (1996) construct and analyze gross job flow data in U.S. manufacturing based on plant-level changes in employment. Given that manufacturing is a small and declining fraction of U.S. employment, drawing inferences regarding aggregate job and worker flows from their results is problematic. In contrast to these works, our approach is more 'top-down' than 'bottom-up'. Rather than accepting the limitations imposed by the incompleteness and inaccuracies inherent in existing direct observations of gross employment flows, our work accepts the restrictions of existing economic theory as an identifying assumption. Because dynamic general equilibrium theory is central to our approach, our results provide an internally consistent view of labor market dynamics and their relationship to economic activity at large.

The view of aggregate employment flows received from the afformentioned studies can be summarized in three broad strokes. First, gross worker and job flows are large compared to the corresponding net flows. Second, the average amplitude of fluctuations in the employment outflow is larger than that of the employment inflow. That is, employment reductions during recessions are more the consequence of an increase in the outward flow from employment than a decrease in the inward flow. Third, this cyclical pattern in the employment flows partly reflects a marked asymmetry in gross job flows: job destruction rises sharply during recessions and job creation is nearly acyclic. Together, these observations point to a pronounced countercyclical pattern in labor force reallocation; the available data imply that worker and job flows increase in recessionary periods and decrease during booms.

Our results support only the first strand of the received view; beyond that, they imply a strikingly different picture of aggregate labor market dynamics. All unobserved forcing variables – labor productivity, allocative efficiency, and the job separation rate – turn out to be strongly procyclical. There is little surprise regarding labor productivity; our implied measure is quite similar to traditional definitions. By contrast, our procedure implies that both allocational efficiency and the job separation rate are highly variable. More importantly, the structure of the model passes along the strong procyclical variation to gross employment flows and this delivers the startling conclusion that both the flow into employment and flow out of employment are strongly *procyclical*. Furthermore, the employment flows are *symmetric* – a property that follows from the requirement that the jointly large gross employment flows reconcile the comparatively small period-to-period observed changes in aggregate employment, i.e. the net employment flow. In marked contrast to the conventional wisdom, our results imply that the bulk of labor force reallocation occurs during booms, not recessions. Interestingly, a recent and quite systematic analysis of the CPS by Shimer (2005b), concludes that the job finding probability of a representative searching worker is strongly procyclical and the probability of separation faced by a representative employed worker is approximately acyclical. These findings are consistent with the strongly procyclical employment inflows and procyclical labor force reallocation found here, but not with the strong cyclical symmetry of the emploment inflows and outflows implied by our procedure.

In the diagnostic realm, the current chapter inevitably intersects with explicit efforts – of which ours is not – to validate or invalidate the Mortensen-Pissarides framework. Next chapter provides one such study. As discussed in the next chapter, Cole and Rogerson (1999) and Shimer (2005a) could be regarded as part of this group of studies. The Cole and Rogerson study documents some of the quantitative successes and failures of the Mortensen-Pissarides framework using a reduced-form approach, but do so with an eye toward replicating some of the more salient gross job flow facts in U.S. manufacturing (e.g. Davis and Haltiwanger, 1992) that we, along with Shimer (2005b), find to be an unreliable guide to aggregate employment dynamics.

Shimer (2005a) casts doubt on the quantitative applicability of the Mortensen-Pissarides framework in the form of a data puzzle. He shows that a general form of the model, which includes structural stability in matching, cannot produce the wide procyclical variation observed in the vacancy-unemployment ratio in response to quantitatively reasonable labor productivity and job destruction shocks. Our results show that with matching function instability, the socially efficient allocations implied by the Mortensen-Pissarides framework are consistent with procyclical matching efficiency and labor market mismatch. We subsequently ask whether the shocks, and their cyclical properties, provide a reasonable source of aggregate labor market fluctuations. The current study complements Shimer's as we provide a resolution to the existence of simultaneously large fluctuations in the vacancy-unemployment ratio and small fluctuations in aggregate labor productivity.

The remainder of the chapter is organized as follows. Section 2.1 outlines our version of the Mortensen-Pissarides model and derives the theoretical restrictions that allow us to identify the unobserved shocks. In section 2.2, we briefly describe the observed data and its basic statistical properties. Section 2.3 presents the simulated method of moments procedure for determining the VAR(1) process that governs the shocks. Section 2.4 presents the results and analyzes the cyclical properties of job creation and destruction as well as those of the underlying shocks. Section 2.5 interprets these findings in the context of recent literature. We briefly outline our conclusions and set a direction for future research in Section 2.6.

2.1 The Model

The economy is inhabited by a continuum of infinitely-lived worker/households distributed uniformly along the unit interval; there is also a continuum of firms. At the beginning of each period, a worker is considered either employed or unemployed.

The measure of employed workers is denoted N_t; the measure of unemployed workers is the complement $U_t \equiv 1 - N_t$. The representative household has preferences over state-contingent consumption and employment given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right), \qquad 0 < \beta < 1, \tag{2.1}$$

where β is the subjective discount factor. Following Merz (1995), the period utility function is separable in consumption and employment, with

$$U\left(C_t, N_t\right) = \log C_t - \frac{N_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \qquad \gamma > 0,$$

where γ defines the wage elasticity of labor supply at a constant marginal utility of wealth (the "Frisch elasticity" of labor supply).

Both workers and firms must undergo a costly search process before jobs are created and output is produced. At the beginning of each period, each unemployed worker searches for a job expending ϕ consumption units in the process. Aggregate period-t search costs incurred therefore equal $\phi\left(1 - N_t\right)$ consumption units. Firms create job vacancies, but only by expending κ units of output per vacancy per period, generating aggregate "recruiting" costs equal to κV_t. Here, as in the traditional Mortensen-Pissarides framework, all jobs must be posted as vacancies before they can be filled. Once a job is filled, it produces output equal to Z_t generating aggregate output

$$Y_t = Z_t N_t \tag{2.2}$$

where $Z_t > 0$ is the exogenously determined productivity of labor.

The matching function captures the labor market search frictions. The typical formulation determines the number of job matches formed in a given period, $M(V_t, U_t)$, as an increasing function M of job vacancies, V_t, and the number of job seekers, U_t, where M exhibits constant returns to scale. With search costs ultimately arising from heterogeneity-induced information problems, we interpret the matching function as a mapping from the labor market's informational state in a given period – which implicitly includes the degree of mismatch between the characteristics of vacant jobs and searching workers – to the number of job matches formed. To allow for fluctuations in mismatch, we generalize the matching function to include a multiplicative shock term, χ_t. Hence, the number of matches formed in period t is given by

$$M_t = \chi_t M(V_t, U_t) = \chi_t V_t^{\alpha} (1 - N_t)^{1-\alpha} \tag{2.3}$$

where $0 < \alpha < 1$ and χ_t is the period-t realization of an unobserved shock process. Increases in χ_t raise the number of matches formed given the numbers of searching workers and available positions. Consequently, fluctuations in χ_t signify improvements or deteriorations in the 'allocational efficiency' of the labor market.

While job matches are being formed, others are dissolved. We assume that the fraction of existing matches dissolved during period-t, σ_t, is also determined as the realization of an exogenous stochastic process. The period-t change in aggregate employment, i.e. the net employment flow, is defined as the difference between the period gross employment inflow and gross employment outflow:

$$N_{t+1} - N_t = M_t - \sigma_t N_t. \tag{2.4}$$

Note that each flow is directly impacted by unobserved shocks: the flow into employment by the allocational efficiency term, χ_t, and the outflow by the rate at which workers separate from jobs, σ_t.

The state of the economy in a given period, or (N_t, e_t), consists of the beginning-of-period employment level N_t, and values of the unobserved and exogenous state vector $e_t = (Z_t, \chi_t, \sigma_t)$. We make the standard Markovian assumption which allows agents to form expectations of future-period quantities using knowledge of the current state only. Given the current state, the socially efficient allocation of employment, vacancies, and consumption, $\{N_{t+1}, V_t, C_t\}$, solves the following recursively-defined social planner's problem:

$$\upsilon\left(N_t, e_t\right) = \max_{N_{t+1}, V_t, C_t} \left\{U\left(C_t, N_t\right) + \beta \mathrm{E}_t \upsilon\left(N_{t+1}, e_{t+1}\right)\right\} \tag{2.5}$$

subject to

$$C_t + \phi\left(1 - N_t\right) + \kappa V_t \leq Z_t N_t. \tag{2.6}$$

$$N_{t+1} = \left(1 - \sigma_t\right) N_t + \chi_t M\left(V_t, 1 - N_t\right). \tag{2.7}$$

where $\upsilon\left(N_t, e_t\right)$ is the future discounted social value of employment level N_t and the exogenous state e_t. Equation (2.6) represents the period-t resource constraint prohibiting the sum of current expenditures on consumption, job search, and vacancy creation to exceed current output, and equation (2.7) describes the trajectory of employment (2.4) with the matching function (2.3) determining the current-period flow into employment.

The corresponding first-order and envelope conditions imply an Euler equation describing an intertemporally efficient vacancy-posting scheme for the economy.

Suppressing arguments and letting primes denote one-period-ahead quantities, we write

$$U_C \frac{\kappa}{\chi M_V} = \beta \mathrm{E}_t U'_C \left\{ Z' + \phi + \frac{U'_N}{U'_C} + \frac{\kappa}{\chi' M'_V} \left[(1 - \sigma') - \chi' M'_U \right] \right\} \tag{2.8}$$

equating the loss in welfare resulting from the generation of an additional vacancy with the expected future social benefit. In this expression,

$$\frac{1}{\chi M_V} = \alpha^{-1} \frac{V}{\chi M}$$

gives the average duration of vacancies multiplied by the elasticity of vacancies in matching, $\alpha = \frac{V M_V}{M}$. The left-hand side of (2.8), therefore, represents the utility loss associated with a marginal increase in vacancies. The expected gain of the marginal vacancy, given by the right-hand side of (2.8), derives from many sources. The expression $Z' + \phi + \frac{U'_N}{U'_C}$ gives the period-$(t + 1)$ net social benefit flowing from an additional match formed in the current period t. The term Z' equals the output flowing from the match; ϕ gives the (constant) search costs foregone by the worker in the match. The final term in the sum, $\frac{U'_N}{U'_C}$ – negative since $U'_N < 0$ and $U'_C > 0$ – represents the consumption value of the leisure foregone by the newly matched worker. In the basic Mortensen-Pissarides setup, this quantity is a constant; here it is allowed to vary over the business cycle according to the worker's preferences.

The final term in braces represents the net future social benefit arising from the expected persistence of a job match. Given that any single current-period match survives with probability $1 - \sigma'$, future social welfare will increase simply by reducing expected future recruiting costs by the quantity $\frac{\kappa(1-\sigma')}{\chi' M'_V}$. The second term in this sum, $-\chi' M'_U$, represents the future reduction in the future job-finding rate $\frac{\chi M}{U}$ due to the current depletion of the unemployment stock; the expected recruiting cost in future consumption units equals $\frac{\kappa M'_U}{M'_V}$.

As a system, equations (2.6)–(2.8) characterize the socially-optimal allocation of employment, vacancies, and consumption given a joint distribution for the exogenous forcing variables or shocks: Z_t, χ_t and σ_t. The traditional Mortensen-Pissarides approach determines these quantities in a market equilibrium with a real wage emerging as the outcome of Nash bargaining between firms and households. The socially optimal allocation characterized above is supported by a similar market allocation mechanism provided that: 1) asset markets are rich enough for households to diversify away employment risk, and 2) the relative bargaining power between households and firms is such that the positive and negative search externalities net out to zero.[5] Although we do not take a position on the precise nature of the allocation mechanism, we maintain that existing market and institutional arrangements direct the realized allocation sufficiently close to the social optimum to establish equations (2.6)–(2.8) as a useful instrument of measure.

2.2 The Data

Before proceeding to shock measurement, we briefly review some of the well-known facts regarding the observed aggregate U.S. labor market measures that bear on our analysis. Given that the model presented in the previous section does not

[5]Hosios (1990) determines the conditions under which the Pareto-optimum is supported as a decentralized market equilibrium in a static evironment; Merz (1995) and Andalfatto (1996) do the same in dynamic general equilibrium settings. The market equilibrium in the current work closely follows those of Merz and Andalfatto.

require a labor market participation decision for worker/households, we must choose whether to express our employment and unemployment variables, N_t and $U_t \equiv 1-N_t$ relative to labor force or the age 16 and over population. Although there are valid arguments in favor of both normalizations, we find that the choice little affects our results, and choose the labor force (employment plus unemployment) as our reference population. In the absence of a long time series on actual job vacancies, we follow standard practice and construct vacancies from the Conference Board's help-wanted advertising index. The resulting vacancy series, V_t, is also expressed per member of the labor force. Also, since our model abstracts from the capital accumulation decision, we must choose between aggregate output and aggregate consumption – a choice that reflects our desire to preserve a consistent and well-understood labor productivity measure and one that can be more readily compared to those in other studies. Since the aggregate labor input N_t produces all goods and services, including private investment goods and those purchased by government, real GDP provides the appropriate output measure. All of our time series are constructed at the quarterly frequency and run from 1948:1 to 2003:4.

Although we are chiefly interested in the cyclical properties of these variables, it is useful to first compare their magnitudes as measured by the sample first moments: mean employment (N) equals .944 or 94.4% of the labor force, mean unemployment (U) equals .056 (5.6% of the labor force), and mean vacancies (V) equals .047 (4.7% of the labor force). The average vacancies-unemployment ratio $\left(\frac{V}{U}\right)$ equals .944. We use these values to assist in preference and technology parameter calibration.

To describe the business-cycle variation in these quantities, we follow Shimer (2005a) and remove the low-frequency trend in all variables implied by the Hodrick-Prescott filter under a smoothing parameter of 10^5. We apply this procedure to remove movements in the aggregates induced by institutional and technological changes associated with job-matching, so that they are not spuriously assigned to matching function instability arising from cyclical movements in labor market mismatch. The cyclical characteristics of the observed variables are summarized in Table 1. Employment, vacancies, and the vacancies-unemployment ratio are all strongly procyclical and persistent; unemployment is strongly countercyclical and persistent. Employment and unemployment both lag output slightly with peak correlations lagging aggregate output by one quarter. Note as well, the extreme volatility of the vacancy-unemployment ratio with a standard deviation of 37 percent around its trend. These data also affirm the Beveridge curve with a strong contemporaneous correlation between vacancies and unemployment of $-.920$.

Given that our methods imply measures of the bidirectional worker flows between employment and unemployment (or nonemployment), we briefly review some of the existing evidence regarding gross job and worker flows here. Direct evidence on the aggregate employment flows arises primarily from two sources: the gross job flow data from the U.S. manufacturing sector constructed and analyzed by Davis and Haltiwanger (1992) and Davis, Haltiwanger, and Schuh (1996), and the monthly gross flow of workers between employment, unemployment, and "not in the labor force" derived from the Current Population Survey (CPS) analyzed most extensively by Blanchard and Diamond (1990). In three broad strokes, the following picture

Table 1. Cyclical Behavior of Observed Labor Market Variables.

		Cross-correlation of output with:						
Variable (x)	%*SD*	$x\,(t-3)$	$x\,(t-2)$	$x\,(t-1)$	$x\,(t)$	$x\,(t+1)$	$x\,(t+2)$	$x\,(t+3)$
Output (Real GDP)	2.15	.520	.724	.884	1.000	.882	.716	.510
Employment (N)	1.01	.264	.485	.700	.853	.872	.789	.635
Unemployment (U)	17.1	−.257	−.485	−.699	−.850	−.869	−.790	−.641
Vacancies (V)	20.6	.396	.606	.768	.863	.832	.730	.557
V/U	37.0	.340	.562	.752	.875	.867	.775	.609

of gross worker and job flows emerges from these works. First, gross worker and job flows are large compared to the corresponding net flows. For example, Davis et. al. (1996) report that annual manufacturing job destruction averages 10.3 percent of total manufacturing employment, and a corresponding figure for job creation of 9.1 percent. The difference, approximately the average net change in manufacturing employment, reflects the declining importance of manufacturing during their sample period. In addition, they report an average quarterly employment inflow of 9.7 percent of employment and average quarterly outflow of 9.4 percent.[6] Second, the average amplitude of fluctuations in the employment outflow (into unemployment or out of the labor force) is larger than that of the employment inflow (from unemployment and outside the labor force). That is, employment decline during recessions is more the result of an increase in the outward flow from employment than a decrease in the inward flow. Third, this cyclical pattern in the employment flows partly reflects a sharp asymmetry in gross job flows with job destruction rising more sharply during recessions than job creation falls. That is, job destruction is countercyclical and job creation is nearly acyclic. Together, these observations point to a countercyclical pattern in labor force reallocation; worker and job flows increase in recessionary periods and decrease during booms.

[6]Job flow averages are based on the 1972:2 - 1988:4 period; worker flow averages are based on the 1972:1 - 1886:4 sample period. Their results on worker flows rely heavily on corrected CPS measures gathered by Blanchard and Diamond (1990).

2.3 Measuring the Shocks

In this section, we present our procedure for measuring the unobserved exogenous shocks to labor productivity, matching efficiency, and the job destruction rate: $\{Z_t, \chi_t, \sigma_t\}$.

2.3.1 Identification and Estimation

To uniquely identify each of the shock series, the observations on employment, vacancies, and consumption $\{N_t, V_t, C_t\}$ are substituted into the theoretical restrictions comprised of equations (2.6), (2.7), and (2.8). We begin with the observation that the labor productivity shocks $\{Z_t\}$ are computed directly from the planner's resource constraint (2.6), given the histories of the three observed, endogenous variables:

$$Z_t = \frac{C_t + \phi\,(1 - N_t) + \kappa V_t}{N_t}.$$

Given our calibration of the technology parameters ϕ and κ (discussed below), the aggregate search and recruiting costs (the latter two terms of the numerator) sum to only one percent of steady state output. Coupled with our simplification allowing measured real GDP to proxy model consumption, Z_t is nearly identical to the traditional average product of labor definition of labor productivity.

With $\{Z_t\}$ so computed and substituted into the intertemporal efficiency condition (2.8), only equations (2.8) and (2.7) remain in play. These equations along with inferred labor productivity and the observed endogenous variables jointly imply realizations of allocational efficiency and the job separation rate: $\{\chi_t\}$ and $\{\sigma_t\}$. Although computing these series requires surmounting the usual technical hurdle imposed by evaluating the conditional expectation characteristic of the intertemporal efficiency condition, it is instructive to gather some intuition regarding the procedure by first examining the perfect foresight case: with the unobserved, exogenous forcing variables treated as deterministic sequences, the conditional expectation is vanquished from equation (2.8).

First, we examine the implications of the equation of motion (2.7) reconciling net employment changes $N_{t+1} - N_t$ as the difference between the employment inflow, $\chi_t M(V_t, 1 - N_t)$, and the employment outflow, $\sigma_t N_t$. Suppose for the moment that the matching function is structurally stable, or equivalently, allocational efficiency is acyclical, i.e. $\chi_t = \chi$ all t. Under this assumption, the job separation rate is computed as

$$\sigma_t = \frac{\chi M(V_t, 1 - N_t) - (N_{t+1} - N_t)}{N_t}$$

using the observations on vacancies and employment. The result, although not apparent from the expression, contradicts conventional wisdom: the job separation rate turns out to be procyclical, rising during booms and falling during recessions. Instead, suppose that χ_t is allowed to vary under the assumption that the job separation rate is constant or acyclical: $\sigma_t = \sigma$ all t. Then, χ_t, computed as

$$\chi_t = \frac{N_{t+1} - N_t + \sigma N_t}{M(V_t, 1 - N_t)},$$

turns out to be countercyclical, falling during booms and rising during recessions. This signals higher degrees of labor market mismatch during recoveries than recessions – a view that is consistent with recessions as periods of 'cleaning up,' but also one which is potentially inconsistent with an intertemporally efficient allocation of vacancies.

To investigate this possibility, we next turn to the perfect foresight version of the intertemporal efficiency condition (2.8). For the sake of analysis, the equation is expressed as

$$\kappa \left[\frac{U_C}{\beta U_C'} \frac{1}{\chi M_V} - \frac{1-\sigma'}{\chi' M_V'} \right] = Z' + \phi + \frac{U_N'}{U_C'} - \frac{\kappa M_U'}{M_V'} \tag{2.9}$$

separating the expressions containing the unobserved exogenous shock terms, χ and σ, from those containing exclusively observed variables. The right-hand side expression, containing only observed variables, is sharply countercyclical in spite of the fact that labor productivity (Z') is procyclical. Ignoring the constant search-cost term, ϕ, the final two terms on the right-hand side are both countercyclical. The intuition behind the countercyclical behavior of $\frac{U_N'}{U_C'}$ – the rate at which the representative worker/household demands consumption units in exchange for additional labor time – is straightforward. During booms, or periods of high employment and high consumption, the marginal disutility of work increases and the marginal utility of consumption decreases; the opposite is true during recession. Given that the term is negative, it displays countercyclical behavior. The final term, given parametrically by

$$\frac{\kappa M_U'}{M_V'} = \kappa \frac{1-\alpha}{\alpha} \frac{V'}{U'}, \tag{2.10}$$

represents the future vacancy costs imposed by the current draining of the unemployment pool to fill available positions. Given that it directly inherits the strongly procyclical nature of the vacancy-unemployment ratio, its negative sign makes it strongly countercyclical. And, though our calibration of κ and α implies considerable damping of the extreme variation in the vacancy-unemployment ratio, the remaining cyclical variation in $\frac{\kappa M'_U}{M'_V}$ strongly dominates that generated by $\frac{U'_N}{U'_C}$, so that even if the latter term is held constant (as the traditional Mortensen-Pissarides framework implies), the right-hand side of (2.9) remain strongly countercyclical.

To see how the unobservables χ and σ must respond to maintain the equality in (2.9), it is useful to approximate the persistence in the marginal utility of consumption and allocational efficiency by equating current-period variables with the corresponding one-period-ahead variables: $U_C = U'_C$ and $\chi = \chi'$. With these approximations, the perfect foresight intertemporal efficiency condition (2.9) reduces to

$$\frac{\kappa}{\chi' M'_V}\left[\frac{1}{\beta} - 1 + \sigma'\right] \approx Z' + \phi + \frac{U'_N}{U'_C} - \frac{\kappa M'_U}{M'_V} \tag{2.11}$$

Since increases in allocational efficiency correspond to decreases in the left-hand side of this relation, a constant rate of job separation σ' implies countercyclical allocational efficiency χ', large in recessions, small during booms. Alternatively, given that increases in the rate of job separation σ' produce increases in the left-hand side, fixing allocational efficiency implies a procyclical job separation rate, small in booms and large during recessions. Therefore, the perfect foresight approximation of our model economy does not lead us to a quick answer regarding the broad cyclical properties of allocational efficiency and the job separation rate. Given average labor productivity inferred from the aggregate resource constraint (2.8), the equation-of-motion for employment and the deterministic Euler equation, taken separately, imply opposing comovements for each. Whereas the equation-of-motion for employment requires procyclical allocational efficiency and a countercyclical job separation rate, the deterministic Euler equation implies countercyclical allocational efficiency and a procyclical job separation rate.

We now proceed to the complete measurement procedure to produce a unique realization of unobserved shocks that jointly satisfy the observed data and the theoretical restrictions of the model. To overcome the usual analytical hurdles introduced by solving (2.8), we proceed by log-linearizing the system (2.6)–(2.8) around its steady state. Dropping the time subscript to denote steady-state values and using lower-case letters to represent the corresponding log-deviation from steady-state, we define the endogenous variables as follows: $n_t \equiv \ln\left(\frac{N_t}{N}\right)$, $v_t \equiv \ln\left(\frac{V_t}{V}\right)$, and $c_t \equiv \ln\left(\frac{C_t}{C}\right)$. The log-deviations of exogenous variables are similarly defined: $z_t \equiv \ln\left(\frac{Z_t}{Z}\right)$, $\widetilde{\chi}_t \equiv \ln\left(\frac{\chi_t}{\chi}\right)$, and $\widetilde{\sigma}_t \equiv \ln\left(\frac{\sigma_t}{\sigma}\right)$. To complete the conditional evaluation of expectations, we must complement the log-linearized efficiency conditions with a VAR(1) structure to the exogenous shocks:

$$\widetilde{e}_{t+1} = A\widetilde{e}_t + \varepsilon_{t+1} \tag{2.12}$$

where $\widetilde{e}_t = (z_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)'$, A is a 3×3 matrix of constants, and $\varepsilon_t = (\varepsilon_{zt}, \varepsilon_{\widetilde{\chi}t}, \varepsilon_{\widetilde{\sigma}t})'$ is trivariate normal with $\mathrm{E}\varepsilon_t = 0$ and $\mathrm{E}[\varepsilon_t \varepsilon_t'] = \Sigma$.

Given values for the nine parameters comprising the VAR(1) matrix of coefficients A, the decision rules mapping the period-t state $(n_t, s_t, \widetilde{e}_t)$ into values for the endogenous variables (n_{t+1}, v_t, c_t) are required to be log-linear as follows:

$$\begin{bmatrix} n_{t+1} \\ v_t \\ c_t \end{bmatrix} = \Pi \begin{bmatrix} n_t \\ z_t \\ \widetilde{\chi}_t \\ \widetilde{\sigma}_t \end{bmatrix}, \qquad \Pi = \begin{bmatrix} \pi_{nn} & \pi_{nz} & \pi_{n\widetilde{\chi}} & \pi_{n\widetilde{\sigma}} \\ \pi_{vn} & \pi_{vz} & \pi_{v\widetilde{\chi}} & \pi_{v\widetilde{\sigma}} \\ \pi_{cn} & \pi_{cz} & \pi_{c\widetilde{\chi}} & \pi_{c\widetilde{\sigma}} \end{bmatrix} \tag{2.13}$$

where the π parameters are expressions comprised of technology and preference parameters. Easy manipulation segregates the observed variables from the unobserved exogenous variables:

$$\begin{bmatrix} n_{t+1} - \pi_{nn} n_t \\ v_t - \pi_{vn} n_t \\ c_t - \pi_{cn} n_t \end{bmatrix} = \widehat{\Pi} \begin{bmatrix} z_t \\ \widetilde{\chi}_t \\ \widetilde{\sigma}_t \end{bmatrix}, \qquad \widehat{\Pi} = \begin{bmatrix} \pi_{nz} & \pi_{n\widetilde{\chi}} & \pi_{n\widetilde{\sigma}} \\ \pi_{vz} & \pi_{v\widetilde{\chi}} & \pi_{v\widetilde{\sigma}} \\ \pi_{cz} & \pi_{c\widetilde{\chi}} & \pi_{c\widetilde{\sigma}} \end{bmatrix}. \tag{2.14}$$

Given data series for employment, vacancies, and consumption, the left-hand side of this expression is a vector of constants in any given period. With values of all model parameters in hand, the matrix $\widehat{\Pi}$ is easily inverted to yield the period-t realization of the forcing process: $(z_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)$. Also, substituting the sequence of shock realizations $\{z_t, \widetilde{\chi}_t, \widetilde{\sigma}_t\}_{t=0}^{T}$ into the VAR(1) process (2.12) determines the underlying realizations of innovations: $\{\varepsilon_{zt}, \varepsilon_{\widetilde{\chi}t}, \varepsilon_{\widetilde{\sigma}t}\}_{t=1}^{T}$. Of course, all of this assumes knowledge of the unknown constants in Π. Although there is sufficient independent evidence to calibrate the technology and preference parameters that help comprise these constants, the same cannot be said of the unknown coefficients of matrix A. In the absence of useful *a priori* information concerning the stochastic properties of the forcing variables, the available time series evidence must be filtered through the theoretical identifying restrictions to infer these characteristics.

Assuming values for technology parameters (α, κ, ϕ), preference parameters (β, γ), and unconditional steady-state values $(N, V, U, C, Z, \chi, \sigma)$, we must determine the 15 parameter values of the vector θ comprised of the 9 coefficients of the $3 \times$

3 VAR(1) coefficient matrix, A, and the 6 independent parameters of the 3×3 variance-covariance matrix of innovations, Σ. Given that our model is singular by construction, it yields a large number of moments to serve as parameter selection criteria. Furthermore, since the unobserved forcing variables represent all of the residual variation that is left behind by theory and observation, we choose 15 moment conditions for an exact identification of parameters values. Thus, we define θ as

$$\theta = \arg\min_{\theta} \; [\mathbf{m} - \mathbf{m}(\theta)]' [\mathbf{m} - \mathbf{m}(\theta)]$$

minimizing the distance between a 15-dimensional vector of theoretical moments $\mathbf{m}(\theta)$ and the corresponding 15-dimensional vector of observed data moments, $\mathbf{m}$.[7] The theoretical moments, however, involve unobserved exogenous variables, and so cannot be determined analytically. As a consequence, we apply the simulated method of moments (SMM) procedure advocated by Ingram and Lee (1991), substituting simulated moments for the theoretical moments. We refer the interested reader to Appendix B for further details of the estimation procedure.

2.3.2 Calibration

With a large empirical literature to draw upon and stationary labor market variables at hand, we combine micro-evidence with long-run data averages to calibrate the steady state values of the exogenous shocks and the technology parameters. We begin by setting the steady state values of the labor market variables, N_t, V_t, and U_t, equal to the corresponding data first moments: $N = .944$, $V = .047$,

[7]The vector θ must therefore also solve the 15-equation nonlinear system $\mathbf{m}(\theta) = \mathbf{m}$.

and $U = .056$. Given these values, we observe that the steady-state version of the equation-of-motion for employment (2.7), or

$$\sigma N = \chi V^{\alpha} U^{1-\alpha}, \tag{2.15}$$

sharply restricts the steady state values of the shocks, χ_t and σ_t, and matching technology parameter, α. Based on Blanchard and Diamond's (1989) estimates of the U.S. aggregate matching function, we set α equal to .6. The steady state rate of job separation is chosen to be 10 percent of total employment per quarter, or $\sigma = .10$, based on the CPS worker flow data reported by Davis, et. al. (1996) that uses the correction of Abowd and Zellner (1985). Under these settings, the steady state employment condition (2.15) subsequently pins down steady state allocative efficiency level: $\chi = 1.856$. These values imply steady state gross employment flows of $\sigma N = M = .094$ per quarter, or 9.4 percent of the labor force. Furthermore, the average duration of a vacancy, $(M/V)^{-1}$, implied is .502 quarters or about 45 days, reproducing the value reported by van Ours and Ridder (1992) using data from the Dutch economy (although their number is not explicitly a target in our calibration). The implied unemployment duration is .599 quarters, or 54 days.

Without loss of generality, we normalize the steady state of inferred aggregate output to equal one, $ZN = 1$, yielding steady-state labor productivity $Z = 1/N = 1.06$. Under this assumption, the steady state resource constraint becomes

$$C + \phi U + \kappa V = 1.$$

Note that in the absence of search and recruiting costs, i.e. $\phi = \kappa = 0$, labor productivity reduces to the traditional average product of labor definition. Steady state labor productivity equals C^{-1} in that case. (Recall that we must proxy consumption with aggregate output, or real GDP.) In the presence of search and recruiting costs, our imputed output measure deviates from measured real GDP somewhat, but we anticipate the magnitude of the difference to be small, with the settings of parameters ϕ and κ largely determining the gap. Unlike the model's other parameters, independent evidence regarding these two parameters is scarce. We follow Andalfatto (1996) in assuming steady state recruiting expenditures to be one percent of output, or $\kappa V = .01$, implying $\kappa = .211$; with no better information regarding the cost of search borne by workers, we assume that steady state search costs are likewise one percent of aggregate output, $\phi U = .01$, yielding $\phi = .177$. The steady state value of consumption is therefore $C = .98$, or 98 percent of output.

Finally, we consider the two preference parameters, β and γ, the subjective discount factor and the Frisch elasticity of the labor supply, respectively. We choose $\beta = .99$ consistent with a steady-state risk-free real interest rate of 4 percent. We follow Merz's (1995) interpretation of the empirical literature and choose $\gamma = 1.5$ for the Frisch elasticity.

2.4 Results

In this section we characterize the dynamic properties of the forcing variables – labor productivity, allocational efficiency, and the job separation rate – and those of gross employment flows that follow from the former. We first discuss the properties of these five series as if they are products of pure measurement. In other words, we assume that our version of the Mortensen-Pissarides model suffers no misspecification errors implying accurate time series measurement of the exogenous forcing variables and corresponding gross employment. We subsequently address the possibility of model misspecification.

The simulated method-of-moments procedure discussed in the previous section determines the following point estimates defining (2.12), the joint distribution of shocks:

$$A = \begin{bmatrix} .58083 & .00646 & -.00437 \\ .05589 & .44310 & -.14951 \\ .28616 & -.40217 & .29534 \end{bmatrix}$$

and

$$\Sigma = \begin{bmatrix} .000002 & -.000017 & -.000068 \\ - & .000719 & .001748 \\ - & - & .005290 \end{bmatrix}.$$

We now turn to the time series realizations of these shocks as implied by the realized innovations $\{\varepsilon_{zt}, \varepsilon_{\widetilde{\chi}t}, \varepsilon_{\widetilde{\sigma}t}\}$, (2.12), and the above estimate of the VAR(1) coefficient matrix A.

2.4.1 Cyclical Properties of the Shocks

The statistics reported in Table 2 provide our first glimpse of the dynamic behavior of exogenous shocks; corresponding characteristics of inferred output are also reported for benchmark comparisons. In interpreting these statistics, it should be recalled that inferred output is nearly identical to actual output measured by real GDP due to the relatively small size of aggregate search and recruiting costs (and equivalently, that model consumption is nearly identical to aggregate output). For each shock series, we examine: 1) volatility, or the amplitudes of the fluctuations around trend measured by the percentage standard deviation from trend, 2) the comovements of the variables as measured by the contemporaneous correlations with inferred output, and 3) phase shifts measured by locating peak correlations with output over a domain of four lags and four leads.

We begin by noting the standard univariate properties of aggregate output: its typical deviation from trend is roughly 2 percent and is quite persistent with an autocorrelation function that reveals a steady but inertial decline in the linear dependence upon past values. As anticipated from numerous prior studies, average labor productivity is strongly procyclical and displays roughly one-half the variation of output. It is a bit more persistent than aggregate output and has no tendency to lead or lag the cycle.[8]

Turning our attention to the dynamics of allocational efficiency and the job separation rate, we see that both display conspicuous deviations about trend, especially the job separation rate. The 30.3 percent standard deviation about trend in allocational efficiency is roughly 14 times that of aggregate output and gives clear support to the hypothesis that the matching function is structurally unstable. The job separation rate, with a standard deviation equal to 47.7 percent trend, is approximately 22 times more volatile than output. As the case with labor productivity, allocational efficiency and the job separation rate are strongly procyclical, and both show contemporaneous correlations of about .90. Both are persistent, but less so than aggregate output and labor productivity; job separation is less persistent than

[8] In contrast, labor productivity constructed using an hours measure of the labor input tends to lead the cycle by a quarter or two. See Kydland (1995).

Table 2. Cyclical Behavior of the Forcing Variables.

		Cross-correlation of output with:						
Variable (x)	$\%SD$	$x(t-3)$	$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+3)$
Inferred Output (Y)	2.15	.520	.724	.884	1.000	.882	.716	.510
Exogenous Forcing Variables:								
(Z)	1.40	.609	.766	.859	.925	.724	.521	.317
(χ)	30.2	.455	.676	.843	.905	.844	.707	.528
(σ)	47.7	.376	.598	.785	.896	.874	.767	.595
Gross Employment Flows:								
($U \rightarrow N$)	44.9	.376	.614	.794	.868	.806	.668	.481
($N \rightarrow U$)	44.7	.308	.545	.746	.865	.839	.732	.538

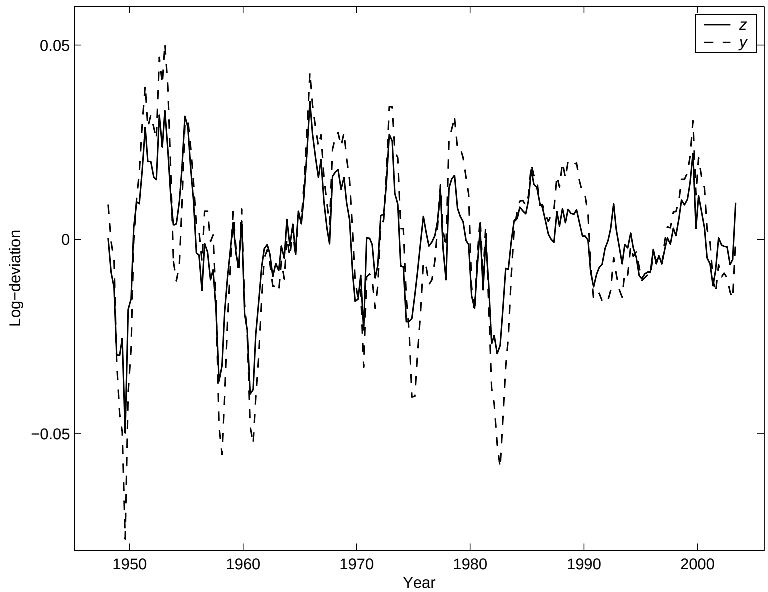

Figure 2.1: Labor productivity shock (solid) and inferred aggregate output (dashed).

allocational efficiency. These properties are plainly evident in Figures 2.1–2.3 which display the time series plot of each forcing variable against inferred output. We defer our discussion of these findings and for now, simply note that they are clearly consistent with the view that much of the interesting behavior of labor markets is buried in the gross employment flows.

2.4.2 Gross Employment Flows

Our measurements of labor market allocational efficiency and the aggregate job separation rate imply time series histories for the gross employment flows between the state of unemployment and employment (or not employed). In model terms, the period-t employment inflow equals $\chi_t M\left(V_t, 1 - N_t\right)$; the period-$t$ outflow equals $\sigma_t N_t$.

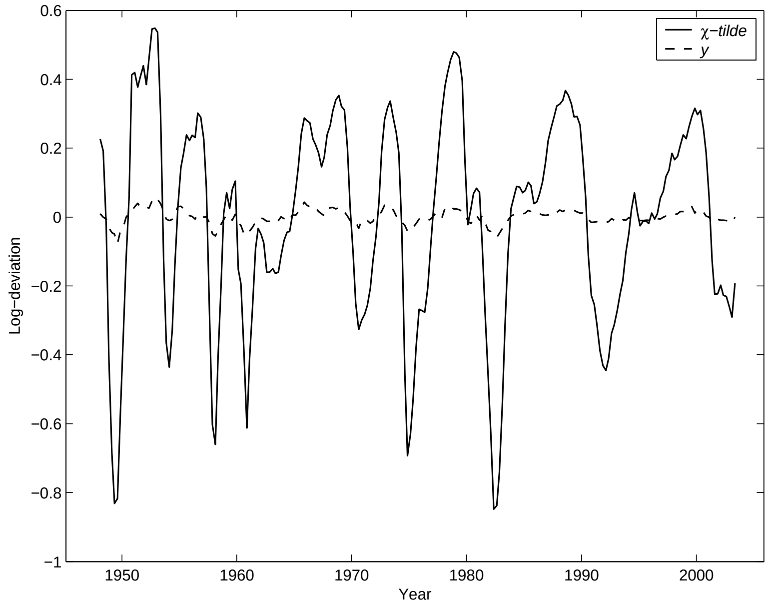

Figure 2.2: Allocational efficiency shock (solid) and inferred aggregate output (dashed).

Before reporting our measures of gross labor flows, we feel it important to caution against using industry-level data on gross job and worker flows to infer corresponding aggregate properties. Since industry-level employment flows are subject to significant leakage – e.g. workers leave manufacturing for other sectors, and vice versa – the cyclical characteristics of the inflows and outflows can differ markedly. Aggregate quantities, of course, are not exposed to intersectoral leakages. Additionally, job flows are conceptually distinct from employment flows: the latter include movements associated with the flow of workers separate from the flow of jobs (e.g. quits and layoffs). Given that aggregate net employment flows are small – the average absolute quarterly flow averages .27 percent of the labor force with a standard

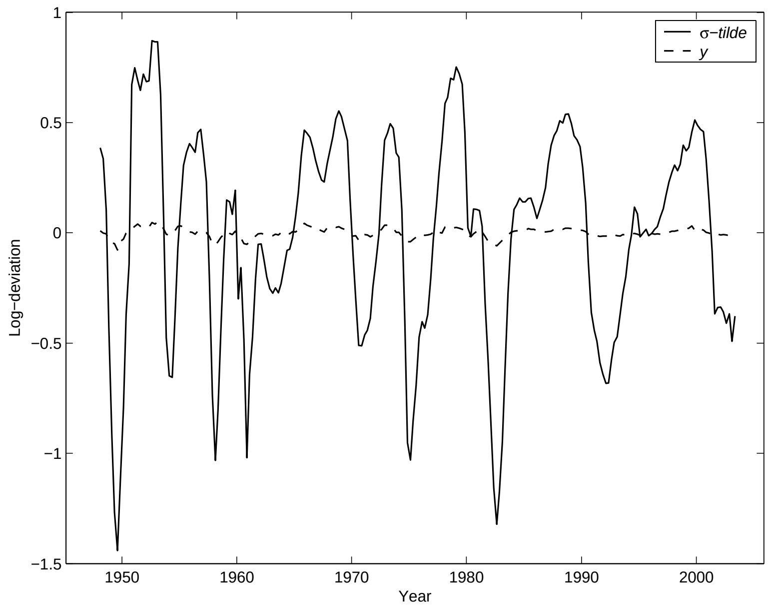

Figure 2.3: Rate of job separation shock (solid) and inferred aggregate output (dashed).

deviation from trend of only .40 percent – it is impossible for the bidirectional gross employment flows to inherit such strong cyclical asymmetry. If gross flows are large in the aggregate, then cyclical properties of the inflows and outflows must be nearly identical. Any sharp increase or decrease in one, must be matched by a similarly sharp increase or decrease in the other, to maintain the narrow difference between the two, i.e. the comparatively small net employment change. By implication, the gross employment flows are fairly symmetric in their cyclical properties and either flow captures well the movements in labor reallocation over the cycle.

The relevant question is then two-fold. First, are the gross flows highly variable? If so, are they jointly procyclical, countercyclical, or neither? Adherence to

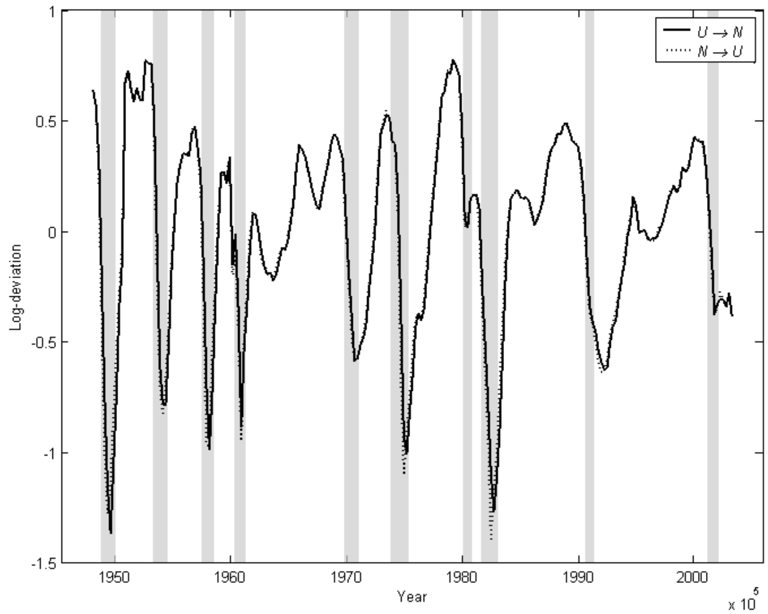

Figure 2.4: Implied employment inflow (solid line) and outflow (dotted line): Log-deviations from trend.

the conventional view that aggregate employment inflows are procyclical (perhaps mildly) and employment outflows are countercyclical is certain to produce disappointment; one pillar must fall. The statistics in Table 2 are revealing. We first note that our procedure produces gross employment flows that are indeed highly variable. Both display standard deviations roughly 45% standard deviation from trend with the inflow slightly more volatile. Next, we see that the correlations of the gross flows with contemporaneous output at three leads and lags reveals both to be strongly *procyclical* and persistent. Thus, it is the cyclical behavior of the inferred employment outflow that defies conventional wisdom. Here, it is procyclical, rising during booms and declining during recessions, along with the flow into employment. The statistics in Table 2 also reveal a pronounced phase separation of one flow from the other, with the employment outflow correlating with output most strongly during the leading periods relative to the inflow. This indicates that the employment outflow tends to lead the inflow. Figure 2.4, plotting the log-deviations from trend of both flows, convincingly illustrates both the tight procyclical relationship and the phase shift. The lagging characteristic of the employment inflow mirrors the well-known tendency for total employment to increase in the wake of recessionary periods. The shaded regions depict recession periods designated by the National Bureau of Economic Research (henceforth, NBER).

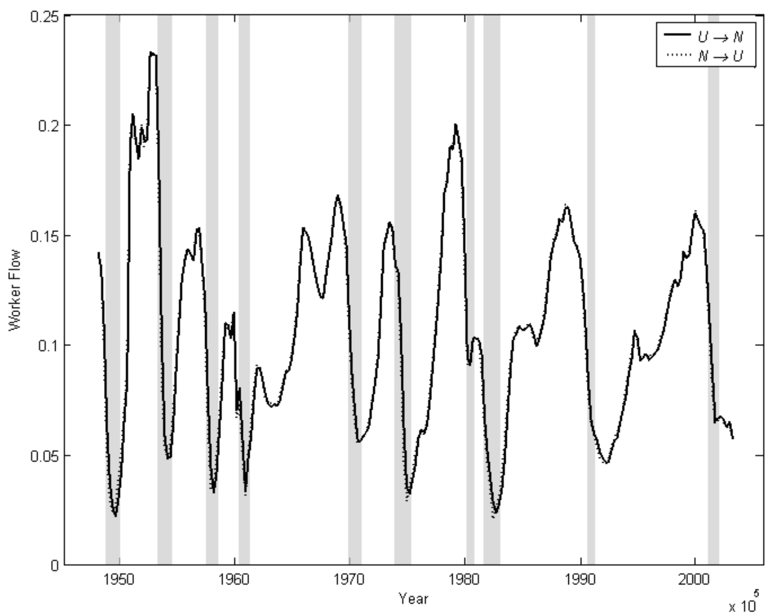

Figure 2.5: Implied employment flows per member of the labor force per quarter: inflow (solid line) and outflow (dotted line).

Finally, we investigate the primitive levels of the gross flows implied by steady state levels and the log-deviations shown in Figure 2.5. As reported earlier, the calibration of our model implies steady state gross flows of .094 workers per member of the labor force per quarter. Figure 2.6 indicates that the imputed employment flows range between 2 percent to 24 percent of the labor force per quarter.

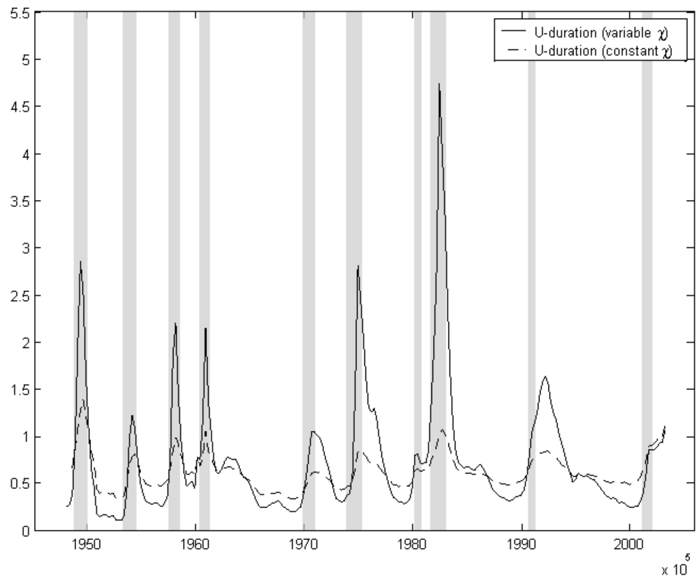

Figure 2.6: Implied unemployment durations in quarters: variable allocational efficiency (solid line) and constant allocational efficiency (dashed line).

This figure clearly shows the procyclicality of the implied gross flows, rising during expansions and reaching a peak before each NBER-defined recession, subsequently falling through the recession-period, occasionally reaching a trough well after the NBER-recession ending date. To gain perspective on the magnitude of this variation, Davis, Haltiwanger, and Schuh (1996) report quarterly job destruction rates (jobs destroyed as a fraction of employment) in their 1972:2–1988:4 quarterly sample period ranging between 3 percent and 11 percent per quarter. In comparing these figures, one must keep in mind that our method allows for worker flows not captured by changes in job flows. Given, that Davis, et. al. also estimate that total job reallocation (roughly the sum of job creation and job destruction) only accounts for between one-third and one-half of total worker reallocation, the variation in their manufacturing job destruction flow is comparable in magnitude to the variation in gross employment flows computed here for the entire economy.

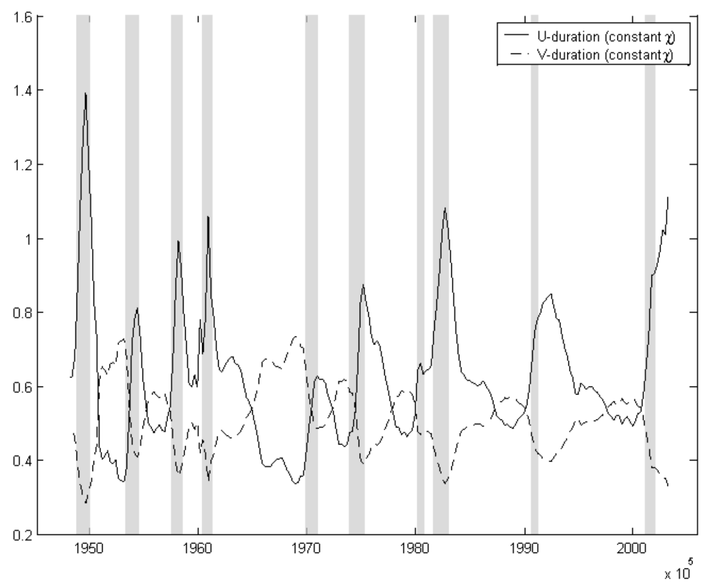

Figure 2.7: Implied average durations in quarters: unemployment (solid line) and vacancies (dashed line).

2.5 Discussion

2.5.1 The mechanics

We begin our analysis of the results by identifying the mechanisms of the model that act in concert with the more salient cyclical properties of the observed data to produce the results highlighted in the previous section. Motivated by the persistent, procyclical movements of labor productivity Z_t (Table 2, Figure 2.1), and the impulse response functions suggesting an independent role of the labor productivity shock (Figures 2.4–2.7), we first trace out the dynamics engendered by our DGE version of Mortensen-Pissarides model in response to a sudden and persistent increase in labor productivity, holding constant allocational efficiency χ_t and the rate of job separation σ_t. Due to persistence, a current shock, i.e. innovation, signals greater future productivity as captured by the term Z' in the intertemporal efficiency condition (2.8), producing a current increase in vacancies as firms respond to the higher anticipated productivity benefits of filled positions. Consequently, additional job matches form in the period of impact – matches that become productive in the ensuing period – thereby increasing employment and reducing unemployment. These effects are summarized by an increasing vacancies-unemployment ratio.

The innovation in labor productivity also sets in motion forces that work to reduce the vacancy-unemployment ratio. To see this, one first notes that the resource constraint (2.6) translates the ensuing anticipated increase in future productivity and employment into an increase future consumption through an augmented flow of output.[9] The increases in employment and consumption subsequently reduce

[9]The sum of search and vacancy-creation costs, $\phi(1-N_t)+\kappa V_t$, small and the increase in

the representative worker's marginal willingness to substitute non-market activities for consumption, i.e. decreases $\frac{U'_N}{U'_C}$ in equation (2.8). This offsets to some extent an individual firm's propensity to create vacancies and the attending increase in employment. Furthermore, the current reduction in the employment pool persists and offsets some of the future benefits of currently high productivity by frustrating future hiring efforts through the term $-\frac{\kappa M'_U}{M'_V}$. This term represents the additional future recruiting costs exacted by the depleted stock of searching workers on the right hand side of (2.8). Recall that this last quantity (or more precisely, its absolute value) is directly proportional to the vacancy-unemployment ratio – a proxy for the 'tightness' of the labor market. The data, as we have seen, displays extremely large procyclical variation in this ratio, and casts doubt on the model's ability to produce the required cyclical variation in response to realistically sized shocks to labor productivity.[10]

By allowing both matching efficiency and the job separation rate to vary over the business cycle, our identification procedure responds to this tension by, in effect, equating the observed vacancy-unemployment ratio with the socially optimal one in each period. The highly variable and procyclical allocative efficiency shock χ_t (Table 2, Figure 2.2) effectively increases the expected gains of vacancy creation in the face of an exogenous increase in labor productivity, thus generating additional vacancies while also increasing the rate at which unemployed workers meet up with them. As a

vacancy-creation costs κV_t counteract the reduction in search costs $\phi(1 - N_t)$.

[10]This point is convincingly demonstrated by Shimer (2005a) using a more conventional Mortensen-Pissarides model with a structurally stable matching function. We are indebted to his work for articulating the opposing forces on the theoretical vacancy-unemployment ratio restraining its response to labor productivity shocks.

result, the flow of workers from unemployment to employment increases, reducing the unemployment pool. The increase in vacancies coupled with the decrease in unemployment, thus gives an additional upward push to the vacancy-unemployment ratio moving the economy along the Beveridge curve in accord with the data. Although the vacancy-employment ratio is moving decidedly in the proper direction, it cannot do so with a sizeable increase in net employment, all else constant. As aggregate employment revealing relatively small period-to-period changes, a complete picture of labor market dynamics requires more employment outflow to restock the unemployment pool depleted by greater efficiency in matching. This element, of course, is provided by the procyclical rate of job separation σ_t (Table 2, Figure 2.3).

Given that these labor market dynamics are largely driven by systematic variation in the allocative efficiency of the labor market, it is instructive to study the implied average durations of vacancies and unemployment spells over the cycle. In the standard setting, the stable matching function forms the basis for monotonic mappings of the vacancy-unemployment ratio into the durations: increasing for vacancies, decreasing for unemployment. Given that the observed vacancy-unemployment ratio is strongly procyclical, a stable matching function produces average vacancy durations that fall during recessionary periods and rise during booms, with opposite movements for the unemployment durations. By contrast, the durations implied by the matching function multiplied by our procyclical allocational efficiency series are both countercyclical, with unemployment durations up much more sharply than vacancy durations during recessionary periods. Figure 2.7 clearly shows this cyclical behavior, with the average unemployment duration reaching nearly 5 quarters during 1982, an extreme event by this measure, and the corresponding vacancy duration reaching roughly 1.5 quarters. Figure 2.8 compares the average vacancy durations implied by a stable matching function (χ set to its steady state value) versus the one implied by the inferred allocative efficiency series; Figure 2.6 show the corresponding comparison for the average unemployment durations. Qualitatively, the counter-cyclical behavior of vacancies implied by the current approach is at odds with the traditional, stable matching function model. Quantitatively, the procedure implies cyclical variation in average employment durations well in excess of those implied by a stable matching function.

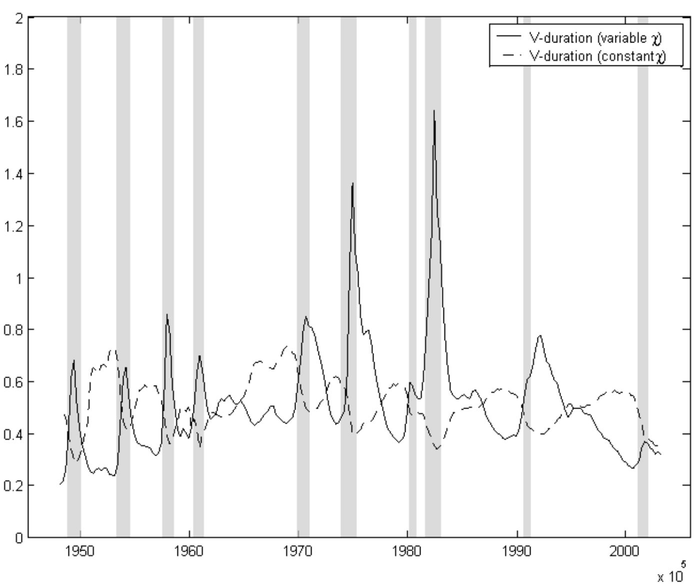

Figure 2.8: Implied vacancy durations in quarters: variable allocational efficiency (solid line) and constant allocational efficiency (dashed line).

The implication of procyclical labor force reallocation is broadly consistent with recent and independent work by Shimer (2005b) who infers aggregate job finding and job separation probabilities from employment, unemployment, and unemployment duration data based on the Current Population Survey of the BLS. Although he finds the job finding probability to be strongly procyclical as in the current study, the job separation probability is nearly acyclical. Additionally, Shimer's strongly procyclical job finding probablility is reflective of our strongly countercyclical average unemployment duration. The combined effect, of course, implies the procyclical labor force reallocation property that is found by our procedure, but not the tight procyclical symmetry between the flows that we derive as a consequence of matching the period-to-period changes in aggregate employment. Both studies, nonetheless, imply a radical change in thinking regarding behavior of employment flows over the business cycle from the wisdom received from the manufacturing job flow studies (Davis and Haltiwanger, 1992 and Davis, Haltwanger, and Schuh, 1996) and earlier analysis of worker flows based on the CPS data (Blanchard and Diamond, 1990).

2.5.2 Measurement: Implications for Business Cycles

Here, we briefly play the devil's advocate role and treat our findings as pure measurement, and discuss the implications for our understanding of labor market dynamics and business cycles.

Perhaps the most striking result is the procyclical behavior of labor force reallocation, with NBER recession periods consistently marked by falling gross employment flows. Adjusted worker flow data derived from the CPS indicates the reverse pattern. In their analyses of these data, Davis, et. al. (1996) conclude that "the countercyclical behavior of both inflows and outflows is consistent with the view that recessions are periods of intense restructuring activity in the economy" (p. 134).[11] In a similar vein, Blanchard and Diamond (1990) conclude that the asymmetrically large cyclical fluctuations in the employment outflow compared to those of the employment inflow "rules out a Schumpeterian view of cyclical fluctuations, with booms as times when inventions are implemented yielding high job creation." The CPS data is instead consistent with the popular view of recessions as 'cleansing' mechanisms, or periods in which unproductive firms, jobs, and techniques are erased from the productive system. In other words, recessions should be marked by substantial factor reallocation, including labor reallocation, in comparison to booms.

At the level of the unobserved shocks, the procyclical pattern of labor reallocation is produced by the twin procyclical forces of allocative efficiency (χ_t) and the rate of job separation (σ_t). According to our interpretation, declining allocative efficiency during recessionary periods is symptomatic of a widening gulf between the locations and skills of unemployed workers, and the locations and required skills of firms with vacancies. That is, recession are periods when the symmetric incomplete information problem of labor market search is aggravated, and hence it is comparatively difficult for firms and workers to form productive matches. The reduced rate of job destruction complements this view. Figures 2.6 and 2.8 offer another perspective

[11] In making this statement, they are mindful of the strength of comovements linking aggregate worker flows and manufacturing job flows. Their measures of manufacturing job destruction and aggregate unemployment inflows both rise sharply during recessions and bear a high contemporaneous correlation (0.71); although employment outflows display less cyclical variation, its contemporaneous correlation to job destruction is nonetheless substantially positive (0.47). The statistical linkage between unemployment ouflows and employment inflows on the one hand, and job creation on the other, is much weaker (contemporaneous correlations of 0.16 and 0.22).

on this phenomenon, showing the strong tendency for *both* average vacancy durations and unemployment durations to rise during recessions, presumably indicating a large number of potential job matches that are foregone. If cleansing is to be temporally concentrated at all, the results indicate that it will occur during booms, when productivity, matching efficiency, and job separation are all on the rise.

Save for the cyclical timing, we maintain that the picture of labor market dynamics and business cycles is decidedly Schumpeterian. Again, we note that labor productivity, the rate of job separation, and allocational efficiency are all procyclical. In the Schumpeterian perspective, productivity improvements stem from innovative flurries and rapid technology adoption that leads to the 'creative destruction' of unproductive jobs and rapid reallocation of the labor force. The comparatively high rates of job separation (σ_t) measured during boom periods reflect not only involuntary separations from creative job destruction, but also increased quits as workers capitalize on better opportunities. The improved allocational efficiency of labor markets (χ_t) during these periods signals an amelioration of the two-sided information problems as more matches are formed from a given number of vacancies and searching workers. It is as if a significant proportion of the work force, queued in either unemployment or unproductive jobs during recessions, are gradually matched in more productive jobs as recession gives way to the 'productivity-storm' of a boom. As new opportunities are created, so are incentives for the reallocation of the labor force across activities and locations. In a descriptive vein, our perspective on business cycle dynamics improves upon the Schumpeterian one as it does not produce labor productivity that is counterfactually countercyclical.

2.5.3 Diagnosis: Implications for Theory

Perhaps the most striking of our results is the implied procyclicality of the gross aggregate employment flows in the face of the conventional wisdom received from incomplete survey data implying countercyclical (and asymmetric) flows. By itself, we do not feel that the aberration is sufficient to declare the model invalid as measurement device. The CPS data on gross worker flows is notoriously unreliable. A number of systematic biases inherent in the measurement procedure have been identified and corrective measures proposed.[12] Even more disturbing from our point of view, however, is that these data do not yield the net employment changes implied by the published data, and often the implied net change is of the wrong sign. As we have already stressed, if we require that the employment flows reconcile with the net changes and accept, *a priori*, the notion that the gross flows are large, asymmetry in the measured flows cannot prevail – inflows and outflows must rise and fall together over the cycle. As for manufacturing job flow data, it represents a small and declining proportion of U.S. economy: manufacturing employment currently accounts for approximately 10 percent of total employment. In contrast, our employment flows exactly reconcile the observed aggregate net employment changes and are, by construction, comprehensive.

Two of three theoretical identifying restrictions imposed by the model are sufficiently transparent and without controversy. The resource constraint (2.6) along with observed data provides an aggregate labor productivity measure Z_t that is

[12] Blanchard and Diamond (1990) and Davis, et. al. (1996) report results using data based on adjustments proposed by Abowd and Zellner (1985).

nearly identical to the standard output per worker definition of aggregate labor productivity. The equation-of-motion for employment (2.7) defines a simple flow-stock reconciliation.

The intertemporal efficiency condition (2.8), by comparison, is rich with content. We have already examined the perfect foresight version of this equation in Section 4 in motivating our measurement procedure. In that analysis, we paid some attention to the role played by the strongly procyclical nature of expression (2.10) which represents the future recruiting costs exacted as a consequence of running down the stock of unemployed persons to fill current positions. This expression shows these costs to be determined solely by the technological aspects of matching, and inherit its strongly procyclical behavior directly from the pronounced procyclicality of the vacancy-unemployment ratio. Much of the identification burden is thus placed on the precise specification of the matching function, i.e. the constant-returns Cobb-Douglas matching function (2.3) which implies a unit constant elasticity of substitution between the two matching inputs of vacancies and unemployment.

As an alternative to Cobb-Douglas, consider the more general constant elasticity of substitution (CES) matching function

$$M\left(V_t, U_t\right) = \left[\alpha V_t^{-\rho} + \left(1-\alpha\right) U_t^{-\rho}\right]^{\frac{\eta}{\rho}},$$

where $-1 < \rho < \infty$ determines the elasticity of substitution, $\frac{1}{1+\rho}$, and $\eta > 0$ determines the returns to scale. Under the CES specification, expression (2.10) becomes

$$\frac{M_U'}{M_V'} = \frac{1-\alpha}{\alpha}\left(\frac{V'}{U'}\right)^{1+\rho}. \tag{2.16}$$

Note that as $\rho \to -1$, the ratio $\frac{M'_U}{M'_V}$ becomes constant and equal to $\frac{1-\alpha}{\alpha}$. That is, as unemployment and vacancies become perfect substitutes in matching, the degree of measured procyclical variation in the term $\frac{M'_U}{M'_V}$ is reduced to zero, and consequently, so is its corresponding influence in the intertemporal efficiency condition for vacancies (2.8). The economics of this result are as follows.[13] In terms of (2.8), an exogenous increase in labor productivity (Z') raises the value of a filled position relative to the value of non-market activities (U'_N/U'_C) and search costs foregone (ϕ), thereby encouraging the substitution of vacancies for unemployment leading to an increase in the vacancies-unemployment ratio. With great ease of substitution between vacancies and unemployment in matching, the magnitude of this response is large. In relation to our results, the exogenous increases required from the allocational efficiency χ and the job separation rate σ in producing a large observed increase in the vacancy-unemployment ratio diminishes with increases in the substitutability of the matching inputs.[14] Note also that the returns to scale parameter η drops from expression (2.16), implying that a resolution is not to be found in the thickness of market externalities.

Finally, we briefly mention a weakness arising not from theory *per se*, but in the inexact mapping between the model variables and observed endogenous variables. In particular, we made the simplifying assumption that output is either consumed or used up in the labor market search process. Given that the latter component is

[13]This intuition mirrors that given by Shimer (2005a) in his diagnostic evaluation of the Mortensen-Pissarides model.

[14]Shimer, however, notes that Blanchard and Diamond's (1989) 0.74 point estimate of the elasticity of substitution goes the other way, but not with enough precision to reject the Cobb-Douglas unit elasticity case. With less substitutability, even more forcing is required from χ and σ.

small, our model is akin to a representative agent asset-pricing model with equilibrium consumption equal to output. To proxy aggregate consumption, we opted for the aggregate output measure of real GDP (per member of the labor force) over the consumption of nondurables and services. The primary advantage of this approach is in keeping our measure of labor productivity as close as possible to the traditional average product of labor definition. Given their strong cyclical similarities – consumption is strongly procyclical and only a bit less variable than real GDP – we do not expect that a switch from an output-based consumption measure to actual consumption would reverse our main results. The alternative is to complicate the model by admitting investment and capital accumulation. By producing another efficiency condition, the set of theoretical restrictions increases from three to four, necessitating the definition of another unobserved exogenous variable and an increase in the number of parameters to be estimated from 15 to 24, significantly increasing the computational burden. This extension is beyond the scope of this chapter, but research is ongoing to shed light on this and the other aforementioned issues.

2.6 Conclusion

We have demonstrated that the Mortensen-Pissarides model of labor market search combined with the observed time series for aggregate output, employment, and vacancies is consistent with considerable procyclical variation in both the allocative efficiency of labor markets and the rate of job separation. Given that the model exactly reconciles observed net employment changes with gross employment flows, and that the data determines the net employment changes from period-to-period, the model and data also imply measures for the employment inflow and outflow. Much of this result is simple arithmetic. The small and procyclical period-to-period changes observed in aggregate employment, combined with large predicted gross employment flows, implies virtually identical cyclical characteristics in inflows and outflows. In the aggregate, no asymmetry in employment flows can be observed. They are either both procyclical or both countercyclical and our results imply that they are procyclical.

Our investigation into the mechanics of the DGE search model that, along with the data, produces these results, echoes Shimer's (2005a) diagnostic exploration of Mortensen-Pissarides framework. He shows that subjecting the more conventional environment, which includes a stable matching function, to reasonably sized shocks in labor productivity and job separation cannot produce the substantial variation in the vacancy-unemployment ratio evinced by the data. In contrast, our procedure allows the allocational efficiency of the labor market to vary along with labor productivity and the job separation rate, so that the search model achieves a perfect fit with the observed data, including the marked variation in the vacancy-unemployment ratio. The simultaneous procyclical variation required of both labor market allocative efficiency and the job separation rate provides an alternative interpretation of Shimer's conclusions regarding the Mortensen-Pissarides model. Additionally, our conclusion that labor force reallocation is procyclical is broadly consistent with Shimer's (2005b) analysis of the CPS data which finds procyclical job finding probabilities and nearly acyclical job separation probabilities.

The results also shed light on the nature of business cycle fluctuations. Perhaps most importantly, they do not support the cleansing hypothesis, or the view that recessions are periods of intense factor reallocation that clears inefficient firms, jobs, and production techniques from the productive system. Conceptually, the cleansing hypothesis has a close kinship with the Schumpeterian notion of creative destruction, wherein innovations and technology adoption provide the catalyst for factor reallocation. Both views imply that factor reallocation is clustered during recessionary periods. Our results deliver the opposite cyclical timing: labor reallocation is concentrated during booms, not recessions. If cleansing is to occur, it is to occur during the expansionary phase of the cycle. Our results do not rule out Schumpeterian creative destruction, only its timing. This modification of the standard Schumpeterian cyclical schematic actually improves its standing with the facts as it does not imply counterfactually countercyclical labor productivity.

Our procedure has forced all of the about-trend variation in aggregate output, unemployment, and vacancies that cannot be understood by the Mortensen-Pissarides framework into the three exogenous variables: labor productivity, the job separation rate, and allocative efficiency. We expect that we have therefore overstated the magnitude of fluctuations in allocative efficiency, the job separation rate, and the implied gross employment flows. With part of the chapter's stated mission as 'pre-theoretical,' i.e. a guide to future theoretical research, we recognize that the exogenous forcing variables may not indeed be truly exogenous. Our hope is to stimulate further research into the nature of our findings to generate even richer theoretical structures which will eventually weaken the measurement content of our exogenous labor market state.

Chapter 3

On-the-Job Search and Labor Market Reallocation

In the last two decades, labor market search models have been used extensively to understand aggregate labor market phenomena, such as equilibrium unemployment and vacancies (Mortensen and Pissarides (1994), Pissarides (2000)). This theoretical framework also proved to be useful in analyzing the effects of various labor market policies including unemployment insurance and labor turnover costs. However, search models have recently been criticized for their business cycle implications. In particular, Shimer (2005a) and Hall (2005) argue that standard models of labor market search require implausibly large shocks to generate substantial variation in key variables; unemployment, vacancies and market tightness (vacancy to unemployment ratio) [1]. Standard deviations of unemployment and vacancies are 10 times, market tightness is 19 times as large as the standard deviation of the average product per worker in the U.S. A puzzle arises since a standard calibration of Mortensen-Pissarides model implies that the variations in all these variables is basically the same as productivity.

[1]Fujita and Ramey (2005) present simulations of the 'standard' Mortensen - Pissarides model that show much more variability. However, their representation of the Mortensen - Pissarides model deviates from the standard version in many respects, including a different timing assumption and different separation rates for new and prevailing matches.

This chapter studies amplification of productivity shocks in labor markets through on-the-job-search. Nagypal (2004a) and Shimer (2005b) argue that job-to-job transitions are crucial for the cyclical worker reallocation. Exploiting dependent interviewing methods introduced in the CPS in 1994, Fallick and Fleischman (2004) find that these flows are large: On average 2.6% of employed workers change employers each month. Moreover, job-to-job transitions turn out to be significantly procyclical. This particular flow cannot be analyzed by standard search models. Thus, on-the-job search seems to be a natural extension of the standard labor market search model.

In the model, workers are allowed to search for another job while employed without incurring any cost. There is also symmetric incomplete information about the quality of the match, which provides persistent employment relationships and a rationale for on-the-job search. Thus, workers in low quality matches have an incentive to search for and accept better quality matches. In equilibrium, workers are distributed over different match qualities at any point in time. Amplification arises in the model because productivity changes not only affect firms' probability of contacting unemployed workers but also of contacting already employed workers. For instance, in expansions, firms are more likely to meet employed workers and those they meet are more likely to accept firm's job offer because they are more likely to be employed in a low quality match. This provides the incentive for the firms to post more vacancies than predicted in the standard model. The logic behind this is simple; since higher productivity raises the value of all matches, even low quality matches become productive enough to survive in expansions. Therefore the measure of workers in low quality matches is greater when productivity is high, implying a higher probability of switching to another match. This introduces strongly procyclical labor market reallocation through procyclical job-to-job transitions. Therefore, the effects of productivity shocks on employment distribution play a key role in generating the desired amplification.

One other contribution of on-the-job search that helps to create amplification is the presence of larger stock of job seekers. In the standard labor market search model, a positive productivity shock leads to higher number of vacancies and lower unemployment by increasing the job finding rate. As the productivity shock persists, since all new workers come from the unemployment pool, firms will expect to find increasingly less number of unemployed workers to fill in the available vacancies. This dampens the positive effect of productivity shock on the supply of vacancies. With on-the-job search, however, this offsetting effect will not be present. To the contrary, due to a substantial number of employed workers at low quality matches who are ready to switch, firms have incentives to post additional vacancies.

The model provides a possible channel for amplification that does not require changing the wage determination process or the information structure to a large extent. In particular, simulations show that the standard deviations for all three labor market variables are matched. The model also successfully predicts that market tightness, defined as the ratio of vacancies to unemployment, is more volatile than both vacancies and unemployment. In addition, the presence of endogenous separation is reconciled with the negative correlation between unemployment and vacancies over business cycle frequencies.

This chapter also has a computational contribution. On-the-job search with match heterogeneity implies that the entire employment distribution becomes a state variable for the recursive problem. It is well known in the literature that this complicates the numerical solution of the equilibrium. I utilize the algorithm used by Krusell and Smith (1998) to address a similar problem. The computational exercise suggests that approximating the worker's acceptance probability of a firm's job offer suffices to characterize the equilibrium. This enables me to numerically solve for the stochastic equilibrium of this economy. In contrast, other studies that modeled on-the-job search either used some simplifying assumptions to get rid of the endogenous effects of heterogeneity or simply restricted the analysis to non-stochastic equilibrium.

The next section discusses the related literature. Section 3.2 describes the U.S. aggregate labor market data. It shows that the variation in average labor productivity is much less than the observed variation in vacancies, unemployment and market tightness. This section also includes some results from a simulation of the standard labor market search model in order to quantify the size of the "amplification puzzle". Section 3.3 describes the economic environment and lays out the dynamic optimization problem of agents. Section 3.4 characterizes the equilibrium of the economy and describes the computational procedure to handle the presence of the employment distribution in the state space. Section 3.5 and Section 3.6 discuss calibration and the solution to the computational problem in detail respectively. Section 3.7 presents the results from the simulation of the model and discusses the implications of the model.

3.1 Related Literature

Early studies of labor market search either failed to address the magnitude of the exogenous forcing process (Mortensen and Pissarides (1994), Cole and Rogerson (1999)) or implied counterfactually positive relationship between unemployment and vacancies (Andolfatto (1996), Merz (1995), Ramey and Watson (1997)).

Shimer (2005a) and Hall (2005) claim that the reason for the lack of amplification in these models is the underlying wage determination mechanism. In search models, an increase in labor productivity raises the labor demand, hence the number of vacancies posted by firms. Since labor markets match vacancies and unemployment as an increasing function of both, more vacancies increase the job-finding probability of workers. Higher job-finding probability reduces unemployment, implying a negative relationship between vacancies and unemployment. However, since workers are now hired at a higher pace, unemployment duration also falls in addition to unemployment. This raises the workers' threat point in bargaining and leads to an offsetting change in terms of higher wages. Therefore, firms' incentive to create vacancies falls (Shimer (2005a), p. 25-26). Hall (2004, 2005), Shimer (2004) and Kennan (2004) build on this presumption and introduce wage rigidity either exogenously or through an endogenous mechanism. As I argue in this chapter, a modification to the wage mechanism is not a necessary condition for amplification. Indeed, a recent study by Mortensen and Nagypal (2005) discusses this point extensively, suggesting that wage rigidity per se is not the answer for amplification. For instance, assuming no bargaining strength for workers leads to constant wages that are equal to the reservation wage (i.e. the value of leisure). Even in this case, the variability of labor market variables relative to productivity are an order of magnitude smaller (Mortensen and Nagypal (2005), p.9).

Several recent studies also aim to provide a mechanism to amplify the effects of business cycle shocks on unemployment and vacancies (Hagedorn and Manovskii (2005), Krause and Lubik (2004), Nagypal (2005) and Silva and Toledo (2005)). Hagedorn and Manovskii (2005) use an unrealistically high value of leisure to generate amplification. Silva and Toledo (2005)'s result depends on a combination of right parameter values for separation, hiring and training costs. Krause and Lubik (2004) and Nagypal (2005) are closer to this study in that both model on-the job search. In general on-the-job search introduces the heterogeneity of job seekers into the picture. Coupled with the aggregate uncertainty, this complicates the problem to a great extent. This might be the reason why Krause and Lubik (2004) assumes a segregated market for different kind of jobs to simplify the potential complexity of the model, whereas Nagypal (2005) only restricts the analysis to non-stochastic equilibrium. In contrast, I handle the heterogeneity that is induced through on-the-job search so that the stochastic equilibrium of the model could be studied.

3.2 U.S. Labor Market Facts

This section presents some of the salient features of the U.S. aggregate labor market data over the business cycle to motivate the questions addressed in the chapter. I focus on three key labor market variables; unemployment, vacancies and market tightness as defined by the ratio of vacancies to unemployment. These are standard variables describing the state of the labor market. Since the mechanism emphasized in this chapter also has implications for transitions between different labor market states, I present two series that proxy transition probabilities between unemployment and employment. These measures are recently constructed by Shimer (2005a and 2005b).

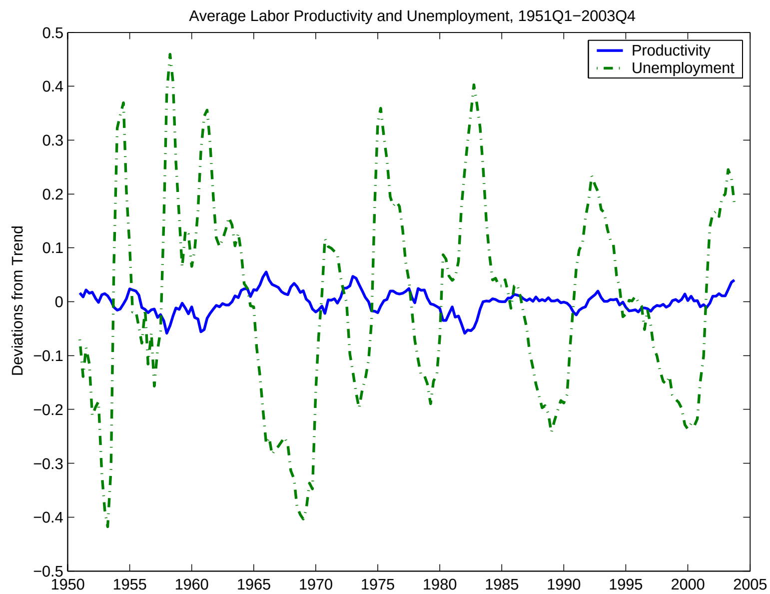

Figure 3.1: Quarterly Unemployment and Labor Productivity: Cyclical components

Unemployment is the quarterly average of seasonally adjusted monthly data constructed by the Bureau of Labor Statistics (BLS) using the Current Population Survey (CPS) data. Vacancies are proxied by quarterly averages of the seasonally adjusted monthly Help-Wanted Advertising Index constructed by the Conference Board. The index is normalized to 100 for 1987. Market tightness variable is constructed using these two and equals the ratio of unemployment to vacancies. In order to determine productivity changes over the cycle, I use real output per person in the non-farm business sector. This particular series is chosen to ensure comparability with the recent body of literature. It is also a natural way to think about productivity in the standard Mortensen-Pissarides model. This series is part of BLS's Major Sector Productivity and Costs program. It is normalized to 100 for 1992.

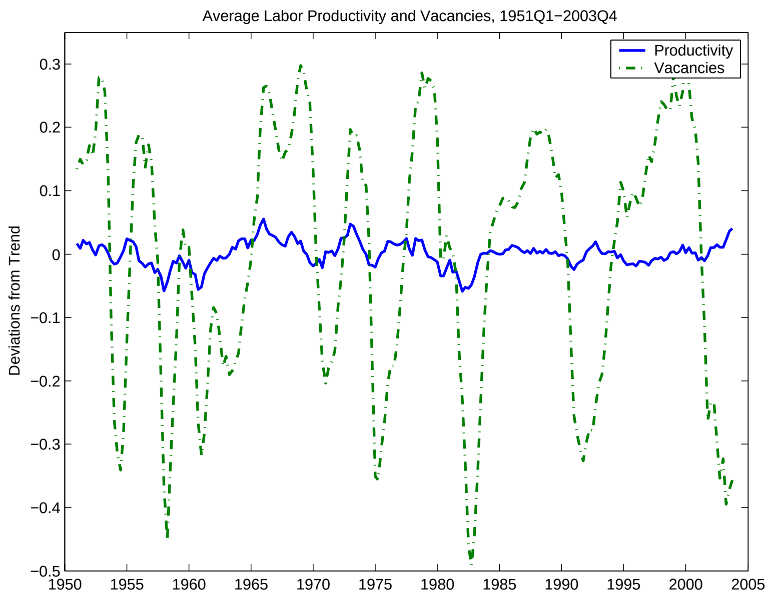

Figure 3.2: Quarterly Vacancies and Labor Productivity: Cyclical components

Job finding and separation probabilities describe the hazard of changing labor market state. For instance, job finding probability is the hazard rate for an unemployed worker of finding a job. Hence, it gives the probability of switching between state of unemployment (u) to state of employment (e). The opposite of this measure is separation probability (e-u transition). Shimer uses short term unemployment data and total unemployment data to pin down these probabilities[2]. Let U_t be the number of unemployed in month t, U_t^s be the number of workers unemployed less than a month in month t, and E_t be the number of workers employed in month t. Then, job finding and separation probabilities are constructed by the following two formulas respectively.

$$f_t = 1 - \frac{U_{t+1} - U_{t+1}^s}{U_t} \tag{3.1}$$

$$s_t = \frac{U_{t+1}^s}{E_t(1 - f_t/2)} \tag{3.2}$$

The separation probability takes into account the possibility of having a short spell of employment in a month to get rid of the time aggregation bias. All of the data reported here are expressed as quarterly averages of monthly data, except the average labor productivity, which is quarterly. The data covers the post-war period, starting from first quarter of 1951 and ending by the end of 2003. All variables are expressed in log deviations from an HP filter with a smoothing parameter 10ˆ5.

First, consider the cyclical variation in unemployment and vacancies relative to the labor productivity. As Figures 3.1 and 3.2 show, both variables show much more variability than the average labor productivity. Same is true for the cyclical variation of market tightness (see Figure 3.3) . The latter two variables show strong procyclicality as opposed to countercyclical unemployment. Figures (3.6) and (3.5)

[2]A detailed discussion of how these are constructed is in Shimer (2005a and 2005b). Data is available through his website: http://home.uchicago.edu/˜shimer/data/

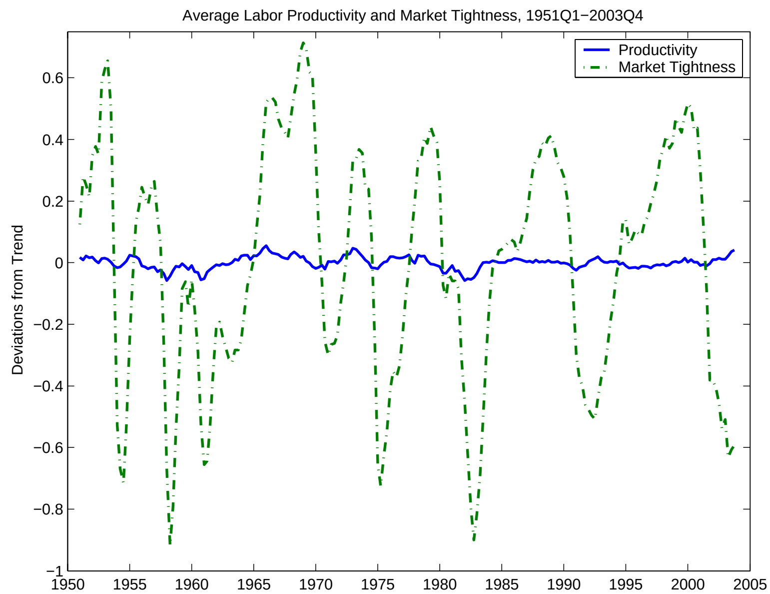

Figure 3.3: Quarterly Market Tightness and Labor Productivity: Cyclical components

complement this picture. It appears from these two figures that job finding probability is strongly procyclical and separation probability is countercyclical. These findings are summarized in Table 3.1. The second row denotes the variables of interest: u for unemployment, v for vacancies, v/u for market tightness, u-e for job finding probability, e-u for separation probability and z for labor productivity. The third row in the table states standard deviations of these variables and the fourth row gives one period auto correlations. Amplification of productivity shocks is clear from the third row. Both unemployment and vacancies are 10 times more volatile than labor productivity. Market tightness is even more volatile, approximately 19 times more. The amplification puzzle, which motivates this chapter, states that the standard labor market search model cannot generate this much amplification

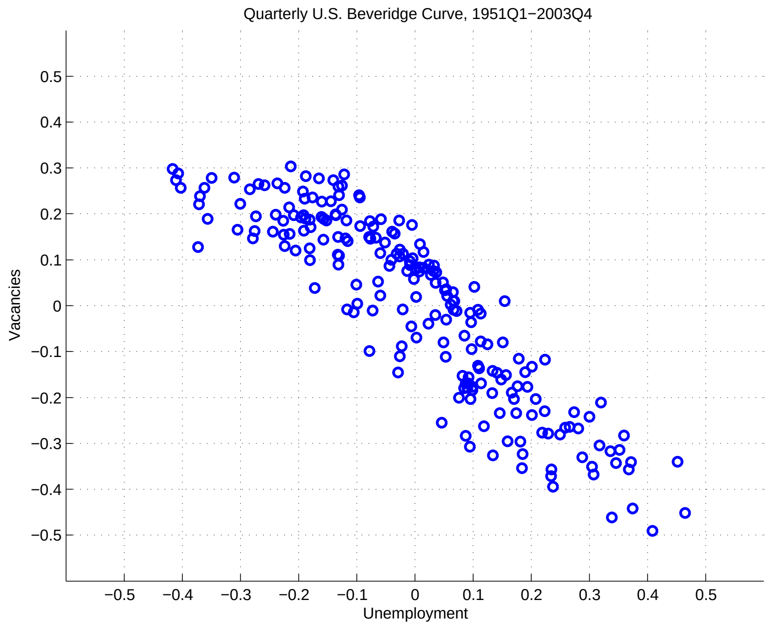

Figure 3.4: Quarterly U.S. Beveridge Curve

based on a productivity process that resembles z in the data. In order to compare data findings in Table 3.1 with the implications of the model, I simulate a standard Mortensen-Pissarides model. Since, this model is well known in the literature, details of it is presented in the Appendix (see Pissarides (2000) for an extensive treatment of the model and its implications).

Table 3.2 presents simulation results from the standard Mortensen-Pissarides model. Productivity process is calibrated to match the actual z series in terms of standard deviation and autocorrelation. In particular, I use a two point Markov process approximated to match the underlying AR(1) process of z according to Tauchen's method (Tauchen, 1986). As Table 3.2 clearly indicates, the standard

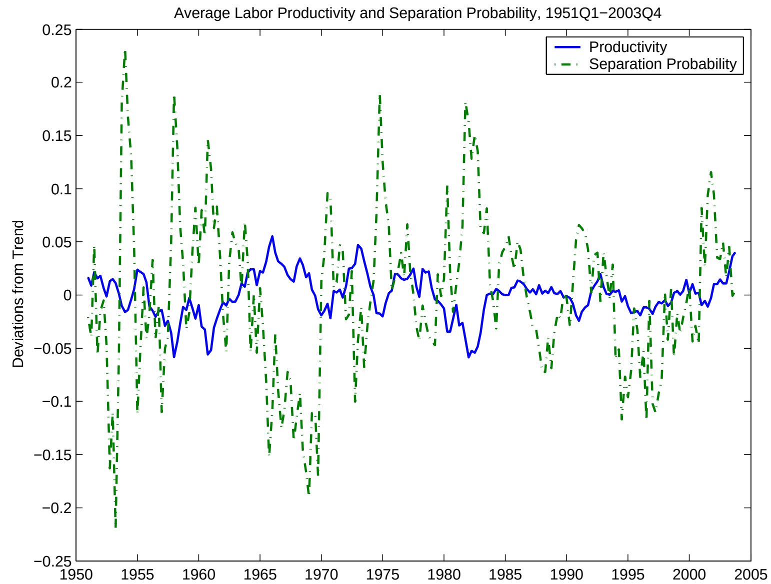

Figure 3.5: Separation Probabilities

model implies almost the same magnitude of variation in all key variables. There is virtually no amplification. Third rows of Table 3.1 and Table 3.2 make this point clear beyond doubt. This discrepancy between the model's implications and the data is referred to as *amplification puzzle* in this chapter.

One key feature of the data is that separations show a countercyclical variation. This is clearly evident in Figure 3.5 and Table 3.1. The standard model, however, assumes constant separations. Studies trying to model endogenous separations as a potential channel to introduce variations in unemployment provides a natural extension to the standard model. However, as argued in the introduction, endogenous separations usually lead to counterfactually positive correlation between unemployment and vacancies.

U.S. DATA (Quarterly, 1951Q1-2003Q4)						
	u	v	v/u	u-e	e-u	z
Std	0.19	0.20	0.38	0.12	0.07	0.02
Auto	0.94	0.95	0.95	0.91	0.73	0.89
Cross Correlations						
u		-0.89	-0.97	-0.95	0.71	-0.42
v			0.97	0.90	-0.69	0.37
v/u				0.95	-0.72	0.40
u-e					-0.58	0.41
e-u						-0.52

Table 3.1: U.S. Quarterly Data

MP Model with Constant Separation						
	u	v	v/u	u-e	e-u	z
Std	0.01	0.02	0.03	0.01	0	0.02
Auto	0.85	0.74	0.81	0.81	1	0.81
Cross Correlations						
u		-0.87	-0.94	-0.99	0	-0.94
v			0.99	0.92	0	0.99
v/u				1	0	1
u-e					0	1
e-u						0

Table 3.2: MP Model with Constant Separation

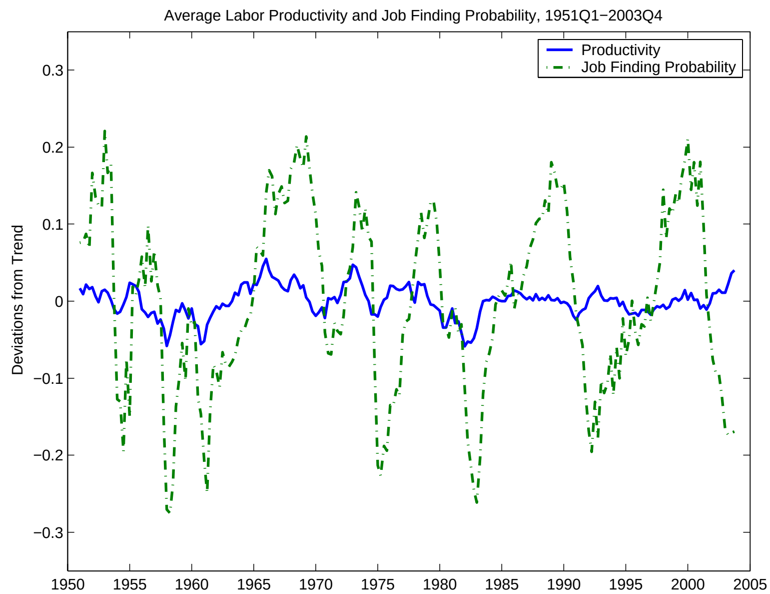

Figure 3.6: Job Finding Probabilities

The negative relationship between unemployment and vacancies has long been recognized by researchers. Indeed, one of the key facts that the standard model was intended to explain was this negative correlation, which has been traditionally named as the Beveridge Curve. The U.S. Beveridge Curve is shown in Figure 3.4. This relationship is also apparent in Table 3.1 in the form of a strong negative correlation of -0.89.

Shimer (2005a) provides a detailed discussion of why separation shocks alone, or separations in general fail to generate the Beveridge curve. In order to emphasize this point, results from the simulation of a standard search model with idiosyncratic match productivity would be helpful. For this, I will present simulations from an extension of the standard model laid out in the next section but does not feature

MP Model with Endogenous Separation						
	u	v	v/u	u-e	e-u	z
Std	0.09	0.03	0.07	0.06	0.11	0.02
Auto	0.78	0.69	0.81	0.82	0.83	0.81
Cross Correlations						
u		0.94	−0.99	−0.99	0.72	−0.99
v			−0.88	-0.88	0.73	−0.88
v/u				0.99	−0.69	0.99
u-e					−0.62	0.99
e-u						−0.69

Table 3.3: MP Model with Endogenous Separation

on-the-job search.

Simulations in Table 3.3 indicate that endogenizing separations imply a positive correlation between unemployment and vacancies. The correlation turns out to be 0.94 in contrast to the empirical counterpart of −0.89. The model proposed in this chapter not only reconciles endogenous separations with a downward sloping Beveridge curve but also increases the amplification lacked in the standard model.

Finally, the U.S. data indicates that most of the variations in unemployment and vacancies are due to more variable job finding probabilities. The artificial probability series constructed by using (3.1) and (3.2) indicate that, it is the hiring which varies more over the cycle. The standard deviation of the job finding probability is almost two times greater than that of separations. Their cyclical variations are presented in Figures 3.6 and 3.5. Although on-the-job search introduced in this chapter implies another possible transition, namely e-e transition, these probabilities are still useful benchmarks to compare.

3.3 The Economic Environment

The model I present here is an extended version of Pries and Rogerson (2005). They study implications of different labor market institutions on hiring policies and labor market flows. Their model incorporates symmetric incomplete information into the standard search model. Symmetric incomplete information motivates agents to learn about their match quality over time by observing idiosyncratic component of their output. This mechanism causes persistent idiosyncratic match specific productivity. Alternatively, one could assume a slightly more complicated persistent exogenous process that governs idiosyncratic component of matches. However, as Nagypal (2004b) argues, learning about match quality is the key determinant of match specific capital especially after first few months of tenure. Hence, learning about match quality provides an empirically relevant story about productivity changes over the job. In addition, I add two key features to this model to explain the amplification puzzle: On-the-job search and aggregate uncertainty.

There is a continuum of risk neutral workers and employers who discount the future at the rate $\beta \in (0,1)$. The measure of workers is normalized to 1. Workers and employers come together in a labor market which is characterized by search frictions.

3.3.1 Learning and Production Technology

Employers are endowed with a production technology that produces $y_t \in Y = \{y^h, y^l | y^h > y^l\} \subset \mathbb{R}_{++}$ when matched with a worker. Hence, when a worker and a firm form a productive match, they produce $z_t y_t$, which depends on the inherent match quality and aggregate state, z_t. Aggregate productivity is governed by a Markov process, $\Psi(z_{t+1}|z_t)$ and is independent of the idiosyncratic component. Even though both workers and firms observe the match specific component of the output, y_t, and the aggregate state, they do not observe their actual match quality, q, which can be *good* or *bad*. Match specific output is determined by the following relationship[3]:

$$\begin{aligned}\Pr(y_t &= y^h|q=g) = \Pi_g > \Pr(y_t = y^l|q=g) = 1-\Pi_g \\ \Pr(y_t &= y^l|q=b) = \Pi_b > \Pr(y_t = y^h|q=b) = 1-\Pi_b\end{aligned} \tag{3.3}$$

Though q is unobservable, agents receive an initial signal $\gamma_0 \in [0,1]$ that corresponds to the probability that the match will be good if formed. It is same for both the worker and the firm. This initial signal is received from a truncated normal distribution, i.e $\gamma_0 \sim \Gamma(\eta,\sigma)$[4]. This distribution is time invariant. After the initial period, both parties start learning abouth their match quality based on output realizations. Since there is no asymmetric information and the output is observed by both, they will have the same posterior belief about the match quality. Let $\Pr(q=g|y_{t-1}) = \gamma$ denote this probability that the current match is a good match conditional on the past output realization on the match, y_{t-1}. Agents need

[3]This is slightly different from Pries and Rogerson (2005) and allows for long term learning. It could be interpreted as a reduced form learning process that in effect is governed as in Jovanovic (1979).

[4]Because γ is restricted to be in the unit interval, $\Gamma(\eta,\sigma)$ represents the cdf of a normal distribution with parameters $\hat{\eta}$ and $\hat{\sigma}$ which is appropriately reweighted to be well defined. Hence, a pair of parameters of the actual distribution $(\hat{\eta},\hat{\sigma})$ implies a corresponding pair for Γ, (η,σ).

to infer $\Pr(q = g|y_t)$ and $\Pr(q = b|y_t)$ for $y_t \in \{y^h, y^l\}$. At this point, it may be useful to compute the posteriors. It follows from simple Bayesian inference.

$$\Pr(q = g|y_t = y) = \frac{\Pr(q = g|y_{t-1})\Pr(y_t = y|q = g)}{\Pr(q = g|y_{t-1})\Pr(y_t = y|q = g) + \Pr(q = b|y_{t-1})\Pr(y_t = Y - y|q = b)}$$

After some algebra, I arrive at the following posteriors implied by prior belief, γ, and time t output realization.

$$\begin{aligned} \Pr(q &= g|y_t = y^h) = \gamma^h = \frac{\gamma\Pi_g}{\gamma\Pi_g + (1-\gamma)(1-\Pi_b)} \\ \Pr(q &= g|y_t = y^l) = \gamma^l = \frac{\gamma(1-\Pi_g)}{\gamma(1-\Pi_g) + (1-\gamma)\Pi_b} \end{aligned} \tag{3.4}$$

The posterior is updated to γ^h after observing a high output and to γ^l after low output. Intuitively, γ^h is expected to be higher than the current state γ. More formally, the current state is related to the future state in a simple way under (4.1).

Remark 3.3.1. If (4.1) holds, $\gamma^h(\gamma) \geq \gamma \geq \gamma^l(\gamma)$ and both $\gamma^h(\gamma)$ and $\gamma^l(\gamma)$ are increasing in γ. In addition, $\gamma^h(\gamma)$ is concave whereas $\gamma^l(\gamma)$ is convex[5].

This remark summarizes the information revelation process over time. An important feature of this learning mechanism is that it rules out any strategic action by both parties to influence the learning process. The evolution of the beliefs is entirely governed by the exogenous process defined in (4.1) and (4.2)[6]. As the firm

[5]The fact that these two mappings are increasing in γ could be established by the positive sign of the first derivatives. Also, comparing γ^h, γ^l and γ, one can easily establish that $\gamma^h(\gamma) \geq \gamma \geq \gamma^l(\gamma)$ is true. Concavity of $\gamma^h(\gamma)$ and convexity of $\gamma^l(\gamma)$ follow from these arguments and the fact that both $\gamma^h$and γ^l maps $[0, 1]$ to itself.

[6]This exogenous process dictated by learning for the evolution of match specific output can be thought of as any exogenous persistent match specific productivity shock where the state space

receives high output realizations, its anticipation that the match is indeed of good quality increases. The opposite is true for a firm that keeps receiving low output realizations. Furthermore, the higher the initial prior about the match quality, the higher the next period's state variable.

Finally for simplicity, I refer to the unconditional probability of observing a high output (low output) at each period as α $(1-\alpha)$ in the rest of the chapter. They are simply defined as linear functions of γ: $\alpha(\gamma) = \gamma\Pi_g + (1-\gamma)(1-\Pi_b)$, $(1-\alpha(\gamma)) = \gamma(1-\Pi_g) + (1-\gamma)\Pi_b$.

The presence of match specific productivity implies that all matches are indexed by their quality (γ) at any point in time. This requires a way of describing the heterogeneity at any point in time. A mapping that gives the measure of the employment at any subset of the match quality state space is a natural way of describing this heterogeneity. Let $n_t(\gamma) \in [0,1]$ be the total number of matches that are believed to be good quality with probability γ. Since, each match employs one worker, this also gives the total number of employed workers with match quality index γ. I also assume that agents are rational in their expectations. In other words, γ fraction of all matches among $n_t(\gamma)$ are actually good quality. How this distribution evolves over time depends both on the endogenous decisions made by agents and the equation of motion for aggregate productivity. I assume that this is summarized by a function G such that:

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and the transition probabilities are defined appropriately. For instance, a process for this match specific component may take values ranging from $\alpha(0)y^h + (1-\alpha(0))y^l$ to $\alpha(1)y^h + (1-\alpha(1))y^l$ and the transition probability from the current state γ to the future state might be defined as $\Pr(\gamma^h|\gamma) = \alpha(\gamma)$ and $\Pr(\gamma^l|\gamma) = 1-\alpha(\gamma)$.

$$n_{t+1} = G(n_t, z_t, z_{t+1}) \tag{3.5}$$

In the discussion of the equilibrium, employment distribution and (4.3) plays a critical role.

3.3.2 Matching Technology and Wage Determination

The meeting process is facilitated by an aggregate matching function, which maps the number of searchers on both sides of the market into meetings. Since this chapter focuses on the importance of the match quality distribution on reallocation over business cycles, search effort will be ignored. This is not an unusual assumption in labor market search models, where search input into the matching function is generally approximated by the number of unemployed. Due to on-the-job search, search input is approximated by the measure of all searchers, which equals the entire labor force[7]. The fact that both employed and unemployed workers meet a vacancy this does not imply that employed workers find jobs at the same rate as unemployed workers do. Idiosyncratic match quality generates endogenously different matching rates for all workers.

The matching technology is summarized by a constant returns to scale matching function, $M(v_t, 1)$, that takes the amount of job seekers, 1, and vacancies, v_t as its arguments[8]. This implies that the rate at which workers meet a job opportu-

[7] Alternatively, one can assume negligible costs for exerting search effort for all workers to ensure that both unemployed and employed workers search for a job.

[8] Since not all meetings result in a match, the term "matching" is used for meeting, in the context of this chapter.

nity is $f(v_t) = M(v_t, 1)/1$. Similarly, a vacancy will meet a worker at the rate $h(v_t) = M(v_t, 1)/v_t$. The meeting rate is not equal to matching rate in this model, because not all meetings end as successful matches. There are two possible types of meetings in this framework; meetings between an unemployed worker and a vacancy, and meetings between an employed worker and a vacancy. Meetings between an unemployed worker and a vacant firm turns into productive matches if their common beliefs about the match quality are above a certain threshold, which is to be determined endogenously in the equilibrium. When an already employed worker meets a vacancy, she has to decide whether to stay in her current match. This decision depends not only on the possible quality of the prospective match (if formed), but also on the quality of her existing match. Agents' initial signal about the quality of the potential match is also drawn from Γ. As a result, if the worker quits and changes her job, the firm becomes idle and can choose to post a vacancy next period.

For simplicity, I assume that there is no recall of past job offers and no wage bidding by firms to attract a worker. Incorporating a strategic interaction between a worker's current employer and a potential employer might change the results presented here at the expense of complicating the wage determination mechanism. This simplifying assumption is not uncommon in the literature (Nagypal, 2005). Furthermore, this chapter aims to provide a mechanism for amplification through the effects of labor market search on the entire employment distribution but not on wage determination.

The firm pays a wage that is determined by a sharing rule over the match surplus which is common in the literature[9]. The sharing rule is such that workers keep $\phi \in (0,1)$ fraction of the match surplus whereas firms get $(1-\phi)$ of it. Wages are renegotiated each period by splitting the surplus with the same rule. This does not preclude persistence in wages because inherent match quality, γ, and aggregate productivity, z, are both persistent. Under these assumptions about wage determination, it is clear that a worker already employed in a match with γ probability of being a good match is willing to switch to a new employer if she faces a higher initial signal. Thus, she experiences a job-to-job transition if new signal, γ_0, is greater than the current match quality, i.e. $\gamma_0 > \gamma$ [10]. If the current employment distribution is n_t, then the probability that an employed worker, conditional on meeting, is willing to switch jobs is a function of this distribution:

$$\int n_t(\gamma)(1-\Gamma(\gamma))d\gamma \tag{3.6}$$

This is the essential feature of the model that introduces the employment distribution into the state space.

Finally, the alternative to a match for a firm is posting a vacancy , which costs $c > 0$ units of consumption per period and generates a possibility of a new match in the next period. Firms have incentive to post vacancy as long as the

[9]This is equivalent to Nash Bargaining when there is no on-the-job search. Shimer (2003) analyses strategic bargaining in a model of on the job search.

[10]I assume, throughout the rest of the analysis, that she retains the match when indifferent. It is intuitive to suggest that not all workers will be willing to change jobs if it is costly enough. This will only require a certain premium over the current match and reduce some job-to-job transitions. However, for empirically plausible values for such costs it does not eliminate job-to-job transitions, hence the mechanism underlined in this chapter.

value of posting one is positive. This is ensured by the free entry of firms and implies that equilibrium value of vacancy is driven to zero. For workers, the outside option is to be unemployed and to consume $b > 0$, which could be interpreted as unemployment benefits or value of leisure. This implies that ongoing matches are destroyed endogenously when the match surplus becomes negative. Because of the particular sharing rule I use, such a decision does not create any disagreements, i.e. both parties agree to end the match jointly. In the equilibrium, this implies a reservation prior, $\bar{\gamma}$. below which the match ceases to be productive and dissolves. On the other hand, a worker may unilaterally end a match, if she meets another vacancy and gets a better initial match quality signal. As explained in the previous paragraph, firms should take the possibility of such a decision into account when they are in a match. Hence, on-the-job search introduces possible match destruction, even though the surplus of the match is strictly positive. Matches are also subject to an exogenous shock in each period that renders the match unproductive. This probability is denoted by λ[11].

3.3.3 Timing of Events

It would be instructive to describe the timing of events within a period to understand agents' information set at each point in time. Events with a time period follows the sequence below, which is also depicted in a chart at the end of the chapter:

- Matches that were productive in the last period start the period t with the

[11]This exogenous probability ensures that in the nonstochastic steady state of this economy we have a non-degenerate employment distribution over match quality space.

information, γ_t, z_t and n_t. Unemployed workers and vacant firms start the period with z_t and n_t.

- Workers and firms within a match decide whether to stay or exit the match. Because of the surplus sharing rule, there is no disagreement between two parties.
  - If the decision is to stay, production occurs, y_t is realized. Workers consume wages, firm consumes net output. Match quality is updated to γ_{t+1}.
  - If the decision is to exit, worker becomes an unemployed searcher and consumes b. Firm becomes idle.
- After production, match quality distribution changes to n_t^+, which is different from n_t due to learning and endogenous separations.
- Firms decide to post vacancy at the cost of c until the value of a vacant position is driven to zero. This pins down the total number of vacancies, v_t.
- Meetings occur according to $M(v_t, 1)$, and initial signals are drawn from Γ.
  - Employed worker who meets a vacancy quits and changes her job if the new signal indicates a higher quality match. This decision is unilateral.
  - Unemployed worker who meets a vacancy decides whether to form a match or stay unemployed.

- New matches are formed, which will be productive in $t+1$. Existing matches are subject to exogenous destruction with probability λ.

- Match quality distribution is updated to n_{t+1}.

3.3.4 Bellman Equations

In order to define the equilibrium of this economy, I start with the Bellman equations that determine values of being in different labor market states. State variables for agents form a list $\{\gamma, z, n\}$, where n is the aggregate employment distribution and time subscripts are dropped for convenience. Aggregate state variables z and n are not correlated with the law of motion for the individual state variable γ, since the learning process is independent of the aggregate state[12]. Equation of motion for γ is given by posteriors defined in (4.2), whereas that of z is governed by the Markov process. The part of the law of motion that concerns $n(\gamma)$ is denoted by G such that $n' = G(n, z, z')$, where the variables with "$'$" denote one period ahead variables. Knowing the aggregate state allows agents to predict future meeting rates.

Let $V_u(z, n)$ be the value of being unemployed for a worker when aggregate productivity is z and the employment distribution is n.

$$V_u(z,n) = b + \beta E_{z'|z}\left\{ f(v) \int V_e(\gamma', z', n') d\Gamma(\gamma') + (1 - f(v))\, V_u(z', n') \right\} \tag{3.7}$$

[12]The matches that will survive in different aggregate states will be a function of the aggregate state in equilibrium. What I mean here is that conditional on survival of the match in the next period posterior is only determined by the exogenous learning process.

An unemployed worker consumes b in this period and expects to come up with a possible match with probability $f(v)$, in which case, she gets the value of having a match, denoted by $V_e(\gamma', z', n')$. Alternatively, she will stay unemployed with probability $1 - f(v)$. Expectation operator takes $\Psi(z'|z)$ into account and n' is governed by $G(n, z, z')$.

The value of having a match which is of good quality with probability γ is slightly more complicated.

$$V_e(\gamma, z, n) = \max \left\{ \begin{array}{c} w(\gamma, z, n) \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\begin{array}{c} \alpha(\gamma)V_e(\gamma^h, z', n') \\ +(1-\alpha(\gamma))V_e(\gamma^l, z', n') \end{array} \right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\begin{array}{c} \Gamma(\gamma^h)V_e(\gamma^h, z', n') \\ +\int_{\gamma^h} V_e(\gamma', z', n')d\Gamma(\gamma') \end{array} \right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\begin{array}{c} \Gamma(\gamma^l)V_e(\gamma^l, z', n') \\ +\int_{\gamma^l} V_e(\gamma', z', n')d\Gamma(\gamma') \end{array} \right] \\ +\beta\lambda E_{z'|z}V_u(z', n'), V_u(z, n) \end{array} \right\} \tag{3.8}$$

The worker compares the returns on retaining the match and not accepting or dissolving it. First five terms within the maximum operator define the discounted expected value of forming (or staying with) the match. Current return from the match equals wage payments, $w(\gamma, z, n)$. Expected value of staying in the match has three components. First, the worker might not meet another vacancy with probability $\beta(1-\lambda)(1-f(v))$ and stay with her current employer. In this case, depending on the current output realization, she will update her belief about the quality of the match according to (4.2). Since a high output is realized with probability $\alpha(\gamma)$, the expected future value of the match becomes $V_e(\gamma^h, z', n')$. Alternatively, a low output realization leads to a lower posterior and a corresponding expected future value of being in a match, $V_e(\gamma^l, z', n')$. The latter two terms state what happens when worker meets a new vacancy. She faces a new vacancy with probability $\beta(1-\lambda)f(v)\alpha(\gamma)$ after recently producing high output. Depending on the result of the new draw from the distribution Γ, either the current match survives or the worker experiences a job-to-job transition. Current match survives, if the new draw falls below γ^h and dissolves otherwise. Once again, note that this separation is initiated by the worker. On the other hand, a new meeting might occur following a low output, which happens with probability $\beta(1-\lambda)f(v)(1-\alpha(\gamma))$. Worker's choice between her current match and the new vacancy is determined similarly. Finally, the match might exogenously dissolve due to an exogenous shock with probability λ.

Firm's problem could be defined in terms of Bellman equations in a similar fashion. Let $J_u(z,n)$ and $J_e(\gamma, z, n)$ be values of having a vacant job and being in a match respectively.

$$J_u(z,n) = -c + \beta E_{z'|z}\left\{h(v)\mu \int J_e(\gamma', z', n')d\Gamma(\gamma') + (1-h(v)\mu)\, J_u(z', n')\right\} \quad (3.9)$$

Posting a vacancy costs c per period and ensures that the firm will meet a worker in the next period with probability $h(v)$. Conditional on meeting with a worker, firm ends up forming a match with probability μ, which is a function of the employment distribution and is defined in detail in the following section.

On the other hand, the position might stay vacant either because the contacted worker does not accept the match (with probability $h(v)(1-\mu)$) or the position could not meet any worker at all (with probability $1-h(v)$).

$$J_e(\gamma,z,n)=\max\left\{\begin{array}{c} z(\alpha(\gamma)y^h+(1-\alpha(\gamma))y^l)-w(\gamma,z,n) \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\begin{array}{c}\alpha(\gamma)J_e(\gamma^h,z',n') \\ +(1-\alpha(\gamma))J_e(\gamma^l,z',n')\end{array}\right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^h)J_e(\gamma^h,z',n') \\ +\left(1-\Gamma(\gamma^h)\right)J_u(z',n')\end{array}\right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^l)J_e(\gamma^l,z',n') \\ +\left(1-\Gamma(\gamma^l)\right)J_u(z',n')\end{array}\right] \\ +\beta\lambda E_{z'|z}J_u(z',n'),\ J_u(z,n)\end{array}\right\} \tag{3.10}$$

A firm that starts the current period with a match that is of good quality with probability γ, has to decide whether to go on with this arrangement and pay $w(\gamma,z,n)$ to the worker or destroy the match (or not start the match at all). In the latter case, payoff to the firm is simply the value of being a vacant job. Current return from the match to the firm is the expected net output which is defined as $z(\alpha(\gamma)y^h+(1-\alpha(\gamma))y^l)-w(\gamma,z,n)$. Once the firm stays with this match, worker's possible meetings with new vacancies should be taken into account to determine the discounted expected future value. For instance, the firm's employee might contact a new vacancy with probability $\beta(1-\lambda)f(v)$. When there is no new meeting in the next period, which happens with probability $\beta(1-\lambda)(1-f(v))$, expected value of the current match only depends on how beliefs and the aggregate state change. However, whenever the firm's employee contacts a new vacant position, the future of the match depends on worker's choice because of the absence of any wage bidding. For instance, a new vacancy is contacted by the worker following a high output with probability $\beta(1-\lambda)f(v)\alpha(\gamma)$, and the match will be destroyed (retained) with probability $\left(1-\Gamma(\gamma^h)\right)$ $(\Gamma(\gamma^h))$. If the current period output turns out to be low, these probabilities change accordingly. Finally, the match may end due to an exogenous shock, leaving the firm with a vacancy.

3.4 Equilibrium

There are four endogenous decisions to be made by the agents in this economy: Workers' and firms' decision as to when to destroy an existing match, workers' choice to unilaterally end the match to make a job-to-job transition, firms' decision on how many vacancies to create and the wage to be paid. Among them the second decision is trivial and has already been substituted in the Bellman equations in the previous section. It simply implies that a worker will not accept any new job match, if the prior about the match specific quality in this new offer falls below her belief about her current match quality. Hence, in what follows, I focus on the other three decisions.

Let $\chi(\gamma, z, n)$ denote the optimal decision rule on match formation (and destruction) and $v(z, n)$ denote the number of vacancies posted in equilibrium as a function of the aggregate state. Then the equilibrium of this economy can be easily defined.

Definition 3.4.1. The **equilibrium** of this economy is a list $w(\gamma, z, n)$, $v(z, n)$, $\chi(\gamma, z, n)$, $J_e(\gamma, z, n)$, $J_u(z, n)$, $V_e(\gamma, z, n)$, $V_u(z, n)$ and $G(n, z, z')$ such that;

1. Given $w(\gamma, z, n)$, $v(z, n)$, $\chi(\gamma, z, n)$ and $G(n, z, z')$, value functions satisfy (4.7)-

(4.6)

2. Given $w(\gamma, z, n)$, $v(z, n)$, $G(n, z, z')$ and value functions, $\chi(\gamma, z, n)$ is optimal.

3. (Free entry of firms) Given $w(\gamma, z, n)$, $\chi(\gamma, z, n)$, $v(z, n)$and $G(n, z, z')$, each firm posts a vacancy as long as $J_u(z, n) > 0$. Hence, aggregate $v(z, n)$ makes the value of posting a vacancy zero, i.e. $J_u(z, n) = 0, \forall z, n$.

4. (Surplus Sharing) Each period: $V_e(\gamma, z, n) - V_u(z, n) = \phi\,[J_e(\gamma, z, n) - J_u(z, n) + V_e(\gamma, z, n) - V_u($ and $J_e(\gamma, z, n) - J_u(z, n) = (1-\phi)\,[J_e(\gamma, z, n) - J_u(z, n) + V_e(\gamma, z, n) - V_u(z, n)]$.

5. Decision rules $w(\gamma, z, n), v(z, n)$, and $\chi(\gamma, z, n)$ indeed generate $G(n, z, z')$ subject to Bayesian updating and equation of motion for z.

The specific surplus sharing rule used in this chapter implies that both workers and firms agree to leave when the surplus of the match falls below zero. Surplus of the match is defined as the quantity, $J_e(\gamma, z, n) - J_u(z, n) + V_e(\gamma, z, n) - V_u(z, n)$. When the match surplus is negative, the share each party gets become negative simultaneously. Hence, in order to describe the decision rule $\chi(\gamma, z, n)$, it is essential to write down the surplus function. Subtracting outside options from $J_e(\gamma, z, n)$ and $V_e(\gamma, z, n)$, and adding them up leads to an expression defining the value of the surplus from a match with quality γ, in aggregate state z and n. Details of the derivation is presented in the appendix. Let this value be denoted by $S(\gamma, z, n)$. The appendix shows that this surplus function has the following recursive form.

$$S(\gamma,z,n)=\max\left\{\begin{array}{c} z(\alpha(\gamma)y^h+(1-\alpha(\gamma))y^l)-b \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\alpha(\gamma)S(\gamma^h,z',n')+(1-\alpha(\gamma))S(\gamma^l,z',n')\right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\Gamma(\gamma^h)S(\gamma^h,z',n')+\phi\int_{\gamma^h}S(\gamma',z',n')d\Gamma(\gamma')\right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\Gamma(\gamma^l)S(\gamma^l,z',n')+\phi\int_{\gamma^l}S(\gamma',z',n')d\Gamma(\gamma')\right] \\ -\beta E_{z'|z}f(v)\phi\int S(\gamma',z',n')d\Gamma(\gamma'), \qquad 0 \end{array}\right\} \tag{3.11}$$

subject to $\Psi(z'|z)$ and $G(n,z,z')$.

This equation is one of the key equations characterizing the equilibrium. For any $v>0$, this equation describes when an existing match should be destroyed bilaterally. Since the right part of the expression within the outermost bracket is constant and the left is increasing in γ, it is easy to show that the decision rule $\chi(\gamma,z,n)$ takes the following form for a given v.

$$\chi(\gamma,z,n)=\begin{cases} 1 \text{ if } \gamma\geq\bar{\gamma}(z,n) \\ 0 \text{ if } \gamma<\bar{\gamma}(z,n)\end{cases} \tag{3.12}$$

The reservation threshold, $\bar{\gamma}(z,n)$, determines whether a match should survive. It also summarizes the hiring decision. In other words, a meeting will turn into a match if the reservation threshold is reached and an ongoing match is terminated if the match quality falls below this threshold. In equilibrium, it turns out that $\bar{\gamma}(z,n)$ is a decreasing function of z. Intuitively, agents become less willing in recessions to undertake matches that are less probable to be good quality. Since all matches are less productive the threshold for a match to survive is higher in recessions. This particular form of the decision rule causes discrete changes in employment distribution across different aggregate productivity levels. For instance, when productivity falls, all prevailing matches that have current priors below the new (and higher) threshold will be destroyed endogenously. Hence, some existing matches that are productive in expansions cease to be so in recessions, causing countercyclical job destruction.

The second important equation determining the equilibrium of this economy comes from the free entry condition and pins down the equilibrium number of vacancies posted. As it is shown in the appendix, the value of vacancy can be written as a function of the surplus function. It takes a simple form:

$$J_u(z,n) = -c + \beta E_{z'|z} J_u(z',n') + \beta h(v)\mu\,(1-\phi)\,E_{z'|z}\int S(\gamma',z',n')d\Gamma(\gamma') \quad (3.13)$$

However, free entry of firms imply that $J_u(z,n) = 0$ for all z, n in equilibrium. Substituting this in (4.10) gives the second key condition describing the equilibrium.

$$\frac{c}{\beta h(v)\mu\,(1-\phi)} = E_{z'|z}\int S(\gamma',z',n')d\Gamma(\gamma') \quad (3.14)$$

Equations (3.11) and (4.11) jointly determine the equilibrium values for $v(z,n)$ and $\bar{\gamma}(z,n)$. These two equilibrium conditions are standard in models of labor market search with endogenous job destruction (Mortensen and Pissarides, 1994). Given the law of motions $G(n,z,z')$ and $\Psi(z'|z)$ for the aggregate state, they characterize part of the equilibrium definition.

The rest of the equilibrium requires describing the endogenous equation of motion for the aggregate match quality distribution. Equilibrium definition requires that the decision rules determined by (3.11) and (4.11) should be consistent with the equation of motion for the match quality distribution, $G(n, z, z')$. The presence of this distribution significantly complicates the numerical solution. Thus, I leave the discussion of this last component of the equilibrium to the following section, which describes the practical challenges of the computational problem and the solution method employed.

3.4.1 Employment Flows

In order to shed more light on the mechanism advocated here, it is essential to understand how match quality distribution evolves over time. Let n_{t-1} be the match quality distribution at the end of time period $t-1$. I assume that agents, both workers and firms, are rational in their expectations about match quality. In other words, among the matches that are currently believed to be good with probability γ, fraction of the good matches are indeed γ.

From any distribution n_{t-1}, decision rules $v(z_t, n_t)$, $\chi(\gamma, z_t, n_t)$ and law of motion $G(n_{t-1}, z_{t-1}, z_t)$, generate the employment distribution for time period t:

$$n_t(\gamma) = \chi(\gamma, z_t, n_t) \left\{ \begin{array}{c} (1-\lambda)\,\alpha(\gamma_1)\,\{1 - f\,(v(z_t, n_t))\,[1-\Gamma(\gamma_1)]\}\,n_{t-1}(\gamma_1) \\ +\,(1-\lambda)\,(1-\alpha(\gamma_2))\,\{1 - f\,(v(z_t, n_t))\,[1-\Gamma(\gamma_2)]\}\,n_{t-1}(\gamma_2) \\ +\,(1-\lambda)\,f\,(v(z_t, n_t))\int_0^{\gamma^1}\alpha(\gamma')n_{t-1}(\gamma')d\Gamma(\gamma') \\ +\,(1-\lambda)\,f\,(v(z_t, n_t))\int_0^{\gamma^2}(1-\alpha(\gamma'))\,n_{t-1}(\gamma')d\Gamma(\gamma') \\ +f\,(v(z_t, n_t))\,(1-\int_0^1 n_{t-1}(\gamma')d\gamma')g(\gamma) \end{array} \right\} \tag{3.15}$$

where $g(\gamma)$ denotes the pdf of the distribution function $\Gamma(\gamma)$, and γ_1 and γ_2 are defined as;

$$\gamma = \frac{\gamma^1 \Pi_g}{\gamma_1 \Pi_g + (1-\gamma_1)(1-\Pi_b)} \text{ and } \gamma = \frac{\gamma_2(1-\Pi_g)}{\gamma_2(1-\Pi_g) + (1-\gamma_2)\Pi_b} \tag{3.16}$$

This recursive definition for employment distribution tracks down employment reallocation across different quality matches over time. To better understand the notation, it is helpful to think where workers should have been in the last period to end up in matches with a particular match quality γ. First of all, some fraction of workers with this match quality constitutes new hires from the unemployed. This corresponds to the last term in brackets in (**??**). Previously unemployed workers meet a vacancy with probability $f\left(v(z_t, n_t)\right)$. Hence, $f\left(v(z_t, n_t)\right)(1-\int_0^1 n_{t-1}(\gamma')d\gamma')$ gives the total measure of unemployed who meet a vacancy this period. Among them, only $g(\gamma)$ of them draws the prior γ, and are candidates for a new match with γ quality in this period. However, the decision to form the match as a result of this meeting depends on the rule, $\chi(\gamma, z_t, n_t)$. This condition implies that overall number of new hires from the unemployment pool will be equal to:

$$f\left(v(z_t, n_t)\right)(1 - \int_0^1 n_{t-1}(\gamma')d\gamma')\left[1 - \Gamma(\bar{\gamma}(z_t, n_t))\right]. \tag{3.17}$$

Flows into $n_t(\gamma)$ might also come from already employed workers. This group of workers have potentially different histories. For instance, some of them end up in $n_t(\gamma)$ after a job-to-job transition. On the other hand, a fraction of these already employed workers constitute participants of matches that endogenously update their posteriors to γ after output realizations in the last period. Let's consider matches which might improve their posteriors to γ, because they have recently experienced a good output realization. This happens to matches with γ_1 probability of being a good match in period $t-1$. Hence, they were part of $n_{t-1}(\gamma_1)$. Only $(1-\lambda)\alpha(\gamma_1)$ of $n_{t-1}(\gamma_1)$ experience high output and do not suffer an exogenous match destruction. Some of the workers in these matches might not meet any other vacancy at all, which happens with the probability $(1 - f(v(z_t, n_t))$. The rest of them, however, might not be willing to change jobs even if they meet a new vacancy, which occurs with the probability $f(v(z_t, n_t))\,\Gamma(\gamma_1)$. This completes the description of the first term within brackets in (**??**). The second term defines the measure of previously employed workers from matches that has experienced a low output in the last period and yet survived exogenous shocks.

The following two terms in (**??**) give the measure of workers who ended up in a match with γ match quality after quitting their previous matches. If these workers have recently experienced a high output in the last period, they could potentially come from the interval $[0, \gamma_1]$. Otherwise, they were part of the employment distribution over $[0, \gamma_2]$. Note that, employment distribution by the end of period t, should always take $\chi(\gamma, z_t, n_t)$ into account. This part creates the separation that is endogenous.

The results in this chapter show that most of the reallocation is undertaken through job-to-job transitions. A job-to-job transition necessarily implies a simultaneous separation and a new hire. Thus, it involves reallocating workers across matches.

3.4.2 Computational Strategy

Match quality distribution is part of the state space in this model. Due to (3.6), firms need to predict this match quality distribution. It appears explicitly in firms' Bellman equations through μ and implicitly in workers' Bellman equations. The challenge posed by the presence of aggregate distribution is not new in the literature. It is well known that numerical solutions of economies with heterogenous agents where aggregate distribution is a state variable is fairly complicated. Fortunately, Krusell and Smith (1998) provides us with a possible computational strategy to solve this type of problems. The novelty in their approach is to approximate the function $G(n, z, z')$ with a finite set of moments in such a way that it is consistent with individual's problem and is the best approximation within a particular class of functional forms. Hence, individuals use this approximate function to predict future prices and their predictions are approximately equal to the actual time series from the simulated model.

The same methodology may be applied to our problem. Krusell and Smith (1998)'s example economy is a standard neoclassical growth model with uninsured idiosyncratic individual risk of being unemployed. Hence, individuals in their economy need to predict aggregate capital stock distribution in the next period to pin down the prices that they will face tomorrow. It turns out that a log linear equation in average capital stock is a sufficient representation of how the entire distribution evolves. In our example however, agents need to predict the future match quality distribution to pin down the probability of a worker's acceptance of a new job offer.

Recall that the beginning of period match quality distribution evolves from n_t

to n_t^+ because of the new information revealed through production and endogenous separations at the beginning of the period. Then the probability that a worker who meets a vacancy accepts firm's job offer is a function of the match quality distribution:

$$\mu = (1 - \int n_t^+(\gamma)d\gamma)(1 - \Gamma(\bar{\gamma})) + \int n_t^+(\gamma)(1 - \Gamma(\gamma))d\gamma \quad (3.18)$$

The first term in (3.18) gives the probability of meeting an unemployed worker and forming a match. Among those whom the firm meets (which happens at the rate $h(v)$) $(1-\int n_t^+(\gamma)d\gamma)$ are unemployed workers and they receive an initial signal. If it is above the reservation threshold, $\bar{\gamma}$, it is worth to form the match. This comes from the equilibrium decision rule $\chi(\gamma, z, n)$, which takes the value 1 for $\gamma \geqslant \bar{\gamma}$ and 0 otherwise. The second term in (3.18) gives the probability of meeting an employed worker and forming a match. Each worker who is in a match indexed by belief γ, will accept the firm's match offer with probability $1 - \Gamma(\gamma)$.

This is why, for practical purposes, it is sufficient to have a simple probability in the state space instead of the match quality distribution. The computational algorithm I use to solve for the equilibrium of this economy involves an approximation of the law of motion for μ. So, even though the match quality distribution is changing over time, agents need to know how a simple moment of this distribution changes over time. For any equation of motion defined by $G(n, z, z')$, there is an implied equation of motion for μ[13]. Let the equation of motion implied be $H(\mu, z, z')$. Given this belief and the stochastic process for z, agents' problem could be solved using

[13]This mapping may not be unique in principle.

equilibrium conditions (3.11) and (4.11). Solution to these equilibrium conditions lead to decision rules $\chi(\gamma, z, \mu)$ and $v(z, \mu)$, which are now defined as a function of μ for practical purposes. Then, these decision rules, an initial condition for employment distribution, equation of motion for employment, (4.3), and the definition for μ determine next period's probability μ'. If this μ' is consistent with $H(\mu, z, z')$, we arrive at the fixed point of the mapping from (μ, z, z') to μ'. The next step involves determining whether $H(\mu, z, z')$ is a 'good' approximation for the underlying equation of motion, $G(n, z, z')$. This ensures that agents lack of knowledge about the evolution of the match quality distribution causes only negligible errors in optimal decisions.

The precise algorithm for the computation of the equilibrium involves following steps:

1. Select N-point grid on μ, 2-point grid on z and M-point grid on γ.

2. Guess on a parameterized functional form for $H(\mu, z, z')$ and on parameters of this function. Call this $\hat{H}$.

3. Given $\hat{H}$, guess a Nx2 vector, $v(z, \mu)$ and solve the decision rule $\chi(\gamma, z, \mu)$ by Iterating over the surplus function defined in recursive equation (3.11) until convergence. Obtain the value of $S(\gamma, z, \mu)$.

4. Given the surplus function, $S(\gamma, z, \mu)$, check whether free entry condition in (4.10) is satisfied. If it is satisfied, $v(z, \mu)$ is equilibrium decision rule and continue to step 5. Otherwise, if free entry condition implies that the value of

vacancy is positive in state (z, μ), increase $v(z, \mu)$, else decrease it and go to step 3 with the new guess on $v(z, \mu)$.

5. Use decision rules $\chi(\gamma, z, \mu)$ and $v(z, \mu)$ and an initial employment distribution to simulate the evolution of employment distribution. From this simulated series, estimate the implied sequence of μ. Use the estimated series of μ, to update the parameters of the functional form guess for $H(\mu, z, z')$. If the initial guess on parameters are confirmed, jump to step 6, otherwise go back to step 1 with a new set of parameter estimates.

6. Having had the parameters converged, check how much error the particular functional form for $\hat{H}$ creates for the agents. If the functional form enables them to predict probabilities with negligible error, stop. Otherwise choose a new functional form and/or new moments and start over.

Computational results show that a linear functional form for μ' is a good guess. The details of this part of the computation is described in Section 3.6 below. Since computing $H(\mu, z, z')$ is an important contribution of this chapter and is not standard in this literature, a separate section within the main body of the chapter is devoted to this computation.

3.5 Calibration

In order to understand the contribution of on-the-job search to amplification, I calibrate a benchmark model, where there is no on-the-job search. This benchmark model is otherwise identical (preferences, production and matching technology) to the model presented in the preceding section. Hence, the benchmark model only has idiosyncratic match quality on top of the standard model. This helps to identify the effect of on-the-job search. The time period is one month and β is calibrated to match 4% annual interest rate. This implies that $\beta = 0.9967$.

First step is to calibrate a productivity process. This is achieved by estimating a two state Markov-Chain approximation for the AR(1) process for the real output per worker in the non-farm business sector. As Table 1 indicates, this productivity data has a standard deviation of 0.02 and a first order autocorrelation of 0.89. Since the standard deviation of this process will be affecting the volatility of other variables directly, matching the exact standard deviation would be desired. Thus I find the following Markov process as the best approximation, which implies a standard deviation of 0.02 and a first order autocorrelation of 0.81.

$$\Psi(z_{t+1}|z_t) = \begin{bmatrix} 0.9682 & 0.0318 \\ 0.0318 & 0.9682 \end{bmatrix}, \; z^h = 1.0259 \text{ and } z^l = 0.9748$$

Several parameters are taken from other studies. The share of the surplus taken by worker, ϕ, is chosen to be 0.36 (Shimer, 2005a). The mean of the truncated normal density Γ is set to 0, i.e. $\eta = 0$. This is from Pries and Rogerson (2005). I also normalize the value of the match specific output when it is low to 1, i.e. $y^l = 1$.

The functional form for the constant returns to scale matching function is usually in Cobb-Douglas form in standard search models. However, since I have on-the-job search with a unit measure of search input, this particular form does not necessarily guarantee us a well defined meeting probability. In other words, $f(v)$

Calibrated Parameters					
Parameter	Value		Parameter	Value	
β	0.9967	4% interest	Π_g	0.56	Match
ϕ	0.36	Shimer (2005)	Π_b	0.56	Restriction
y^l	1	Normalization	c	0.11	Match
z^h	1.0259	U.S. Avg.	b	3.32	Restriction
z^l	0.9748	Output p/w	y^h	7	Match
η	0	PR (2005)	λ	0.0041	DHS (1996)
σ	0.153	PR (2005)	z^{ss}	1	Normalization

Table 3.4: Calibration with On-the-Job Search

and/or $h(v)$ may not be well defined for some v. Thus, I choose a different functional form for the matching function that has been used by Barlevy (2002) and den Haan et.al (2000). Matching function takes the following simple form:

$$M(v_t, 1) = \frac{v_t}{v_t + 1} \tag{3.19}$$

Equation (4.16) defines a constant returns to scale matching function, which obeys the usual regularity conditions. In addition, meeting probabilities within a period, $f(v)$ and $h(v)$ are by definition in $[0, 1]$.

In order to calibrate the rest of the parameters, I target three moments from the model; steady state unemployment rate of 5.68%, steady state job finding probability of 45% and steady state probability of accepting a match conditional on meeting of 50%. The first two statistics are from the data. The latter statistic equals $1 - \Gamma(\bar{\gamma}^{ss})$ and chosen to be in line with Pries and Rogerson (2005). These targets imply a steady state monthly separation probability of 2.71%. This is slightly lower than the average separation probability in the U.S. data, which is 3.4%. However, we cannot simultaneously match an average job finding probability of 45% and an average separation probability of 3.4%. The exogenous job destruction probability, λ, is then calibrated to match the fraction of shutdowns among all job destruction. Davis et al. (1996) estimates that 11% of all job destruction is accounted for by shutdowns. Since all separations in the benchmark model's steady state are either exogenous (due to λ) or due to learning about the quality of the match, a slightly higher value for this fraction is targeted. Specifically, it is assumed that 15% of all separations are exogenous. This pins down λ, which is set to 0.0041.

Since job finding probability is $f(v^{ss})\,(1-\Gamma(\bar{\gamma}^{ss})\,)$ in the steady state equilibrium of this benchmark economy, the target for $1-\Gamma(\bar{\gamma}^{ss})$ implies that $v^{ss}=9$. Given these targets, learning process can be calibrated to match them. For instance, the standard deviation of the distribution for prior signals, σ, implies an equilibrium value for $\bar{\gamma}^{ss}$ if $(1-\Gamma(\bar{\gamma}^{ss})\,)=0.5$. The value of $\bar{\gamma}^{ss}$ does not have any intrinsic value for the purpose of this chapter. Hence, I target a value of 0.1 for this equilibrium value. This target and $(1-\Gamma(\bar{\gamma}^{ss})\,)=0.5$, implies that $\sigma=0.153$. With the targets for $\bar{\gamma}^{ss}$ and v^{ss} and values for λ,σ and η, the model can be easily simulated to generate a stationary match quality distribution. Only determinants of this equilibrium distribution are learning parameters Π_g and Π_b. Recall that these two parameters determine the pace of learning. There is no apriori reason to have a faster learning depending on the inherent quality. Hence, it is assumed in the simulations that both good matches and bad matches reveal the information at the same pace, i.e. $\Pi_b=\Pi_g$. Then, the average unemployment rate requires $\Pi_b=\Pi_g=0.598$. It turns out that the benchmark model with these set of parameters actually imply a tenure distribution that is consistent with the U.S. data. This is not a dimension that I target, but the stationary match quality distribution determines how long each match is likely to survive. Taking this into account, the benchmark model generates a tenure distribution such that 25.71% of the employed are with 1 year of tenure, 21.42% of the employed are with 3-4 years of tenure and 23.49% of the employed are with 5-9 years of tenure. The corresponding values from BLS's Employment Tenure Summary are 20.7%, 19% and 20.7% respectively[14].

Calibrating a value for b is not straightforward. Recall that $y^l = 1$. In this benchmark model, finding an interior solution for $\bar{\gamma}$, requires that the value of surplus is at least 0 for $\gamma = 0$ and strictly positive for $\gamma = 1$. In general, this implies that b should be somewhere between y^h and y^l. Taking this restriction into account and the targets for $\bar{\gamma}^{ss}$ and v^{ss}, I pick the values for b, y^h and c. Implied values for these parameters are 3.32, 7 and 0.11 respectively. This completes the calibration for the benchmark model.

3.6 Computing $H(\mu, z, z')$

The challenging task of computing the equilibrium of this economy is already outlined in Section 3.4.2. The most critical step of the algorithm is to determine a good functional form for the equation of motion of μ'. I posit a linear functional form guess, which depends on both the current productivity and past productiv-

[14] The data could be found in Table 3 of the Summary at http://www.bls.gov/news.release/tenure.t03.htm.

ity. Intuitively, since the model economy undergoes significant discrete changes at the lower end of the distribution when aggregate productivity changes, the equation of motion may very well depend on both states. To illustrate this, consider two stationary distributions of match quality shown in Figure 3.7. The solid line represents the stationary match quality distribution implied by the model outlined in Section 3.3, when aggregate productivity stays constant at z^l forever. On the other hand, the dashed line represents the stationary match quality distribution in expansions. Because the equilibrium reservation threshold $\bar{\gamma}$ is a decreasing function of z, we have a substantial mass of workers with lower match qualities in expansions. Although these thresholds change in the stochastic equilibrium, this feature of the model survives. Thus, when the aggregate state changes, there will be considerable job destruction (or creation) at the lower end of the distribution. This justifies adding z' (in addition to z) to the list of independent variables determining μ'. Ultimately I use the following functional form.

$$
\begin{aligned}
\mu' &= \theta_1 + \theta_2\mu, \text{ if } z = z^l \text{ and } z' = z^l \qquad (3.20)\\
\mu' &= \theta_3 + \theta_4\mu, \text{ if } z = z^l \text{ and } z' = z^l \\
\mu' &= \theta_5 + \theta_6\mu, \text{ if } z = z^l \text{ and } z' = z^l \\
\mu' &= \theta_7 + \theta_8\mu, \text{ if } z = z^l \text{ and } z' = z^l
\end{aligned}
$$

Once there is a functional form guess for $H(\mu, z, z')$ and an initial set of parameter values for $\theta's$, computation of the equilibrium starts by discretization of the state space (γ, z, μ). Aggregate productivity only takes two values, z^h and

z^l. Match quality index is defined on a 250-point grid over the unit interval. I use M = 250 grid points because the grid on γ should be fine enough to capture the underlying individual heterogeneity. This heterogeneity determines the exact value of the equilibrium condition $\bar{\gamma}(z,n)$[15]. Finally, μ's are assumed to take values between 0.13 and 0.19 and are equally spaced on 15 grid points[16]. The upper and lower bounds on μ are chosen such that at the simulation stage none of the realized (actual) values for μ fall out of this range.

Next step is to solve the recursive equations defined in (4.11) and (3.11) using the functional guess for $H(\mu, z, z')$. This step is fairly standard and leads to decision rules $\chi(\gamma, z, \mu)$ and $v(z, \mu)$. Then, the model economy is simulated for 15000 periods starting from an initial distribution. The simulation length should be long enough to create enough artificial samples for states (z, z'), where $z \neq z'$[17]. Simulation of the model generates two separate time series for μ one of which is predicted by $H(\mu, z, z')$ and the other one is the actual. After discarding the initial several hundred periods, actual realizations of μ are used to estimate the regressions in (3.20) via ordinary least squares. The OLS estimates of θ's are used as new parameter guesses until convergence. Once parameters converge, I need to evaluate the goodness of "fit" for the particular functional form for $H(\mu, z, z')$.

[15]I have tried finer grids, but they do not seem to lead to changes on the equilibrium values for $\bar{\gamma}(z,n)$.

[16]Adding more grid points essentailly did not change the results at all. However, lowering the number of grids for μ will reduce the predictive power of the functional forms for states (z, z'), where $z \neq z'$.

[17]Due to the persistence in z, these periods are very rare as opposed to periods when aggregate state does not change at all.

It turns out that agents do infer μ' with considerable precision when only μ,z and z' are explanatory variables. Regression results from the simulations of the model is a standard way of measuring how good an approximation the equilibrium is (Krusell and Smith, 1998). The following four equations show the extent of the fit.

$$\begin{aligned}
\mu' &= 0.05044 + 0.69617\mu, \text{ if } z = z^l \text{ and } z' = z^l \\
R^2 &= 0.9987,\ F = 5279389,\ p = 0 \\
\mu' &= 0.04774 + 0.83172\mu, \text{ if } z = z^l \text{ and } z' = z^h \\
R^2 &= 0.9974,\ F = 53143.60,\ p = 0 \\
\mu' &= 0.06635 + 0.51251\mu, \text{ if } z = z^h \text{ and } z' = z^l \\
R^2 &= 0.9841,\ F = 842.97,\ p = 0 \\
\mu' &= 0.06338 + 0.62260\mu, \text{ if } z = z^h \text{ and } z' = z^h \\
R^2 &= 0.9974,\ F = 2915907,\ p = 0
\end{aligned} \tag{3.21}$$

Parameter values reported in (3.21) are the values that have converged after several iterations of the same functional form, and they are all significant at 5% level of significance. All three measures of fitness reported underneath each regression equation indicate that the simple linear functional form is a good way of describing how μ evolves[18].

[18]Another measure of "goodness of fit" can be the discrepancy between the μ's implied by $H(\mu, z, z')$ and actual ones. It turns out that the maximum discrepancy in a period was 0.00087. Furthermore, this difference was always less than 0.0007 in all but 9 periods (out of 15000).

Other studies that modeled on-the-job search either assumed simplifying assumptions to get rid of the endogenous effects of heterogeneity through meeting rates or simply restricted the analysis to non-stochastic equilibrium. Examples of the first approach are Mortensen and Nagypal (2005) and Krause and Lubik (2004). They abstract from match specific productivity changes, which shuts off the channel through which the employment distribution becomes a state variable. As this chapter argues, this channel is indeed very significant. Studies taking the latter approach fails to show the full picture. They only focus on comparing different steady states or analyzing only the transitional dynamics (Barlevy (2002), Nagypal (2005), Shimer (2003)). However, this chapter provides a solution for the stochastic equilibrium allowing us to conduct a thorough business cycle analysis. These results are discussed in the next section.

3.7 Results

To understand the role of on-the-job search in generating labor market amplification, both the benchmark model and the model with on-the-job search are simulated. Table 4. presents the results from simulations of the model with on-the-job search. As it is evident from the reported standard deviations, the presence of on-the-job search creates significant variations in our key labor market variables. Although the underlying aggregate productivity process is assumed to be the same, a comparison of Table 3.5 and either Table 3.2 or 3.3 shows that the model amplifies the effects of productivity shocks to a large extent. In standard search models, unemployment, vacancies and market tightness are almost as variable as the exoge-

Simulations with On-the-Job Search							
	u	v	v/u	u-e	e-u	e-e	z
Std	0.19	0.13	0.31	0.14	0.29	0.14	0.02
Auto	0.88	0.73	0.84	0.80	0.67	0.81	0.81
Cross Correlations							
u		-0.89	-0.98	-0.94	0.86	-0.96	-0.97
v			0.96	0.99	-0.97	0.98	0.98
v/u				0.99	-0.92	0.99	0.99
u-e					-0.96	0.99	0.99
e-u						-0.95	-0.95
e-e							0.99

Table 3.5: Simulations with On-the-Job Search

nous forcing process. However, same process generates quite encouraging results in the model with on-the-job search. For instance, the standard deviation of unemployment and the Beveridge curve relationship are easily matched (See Table 3.1). It also implies significantly large variations in vacancies and market tightness, even though, they are a little far off from the data.

One might argue that, a direct comparison between Table 3.2, Table 3.3 and Table 3.5 is not reasonable. This is a legitimate concern, because the implied improvements in performance might be due to one (or more) of the several extensions inherent in the model presented in Section 3.3 In order to distinguish the effect of on-the-job search, I compare the results from simulations of the benchmark model, which is exactly identical except the on-the-job search aspect.

Simulation results for the benchmark model are presented in Table 3.6[19].

[19]The difference between Table 3.6 and Table 3.3 is only because of the different functional

Benchmark Model (No OJS)						
	u	*v*	*v/u*	u-e	*e-u*	*z*
Std	0.13	0.05	0.08	0.07	0.16	0.02
Auto	0.76	0.67	0.81	0.81	0.36	0.89
Cross Correlations						
u		0.96	−0.98	−0.98	0.76	−0.98
v			-0.90	0.90	0.78	−0.90
v/u				1	−0.71	0.99
u-e					−0.71	0.99
e-u						−0.70

Table 3.6: Simulations of the Benchmark Model with no On-the-Job Search

There are at least two important differences between the benchmark model and the model with on-the-job search. First of all, the standard deviations of the cyclical variations show significant decline, especially for vacancies and vacancy-unemployment ratio. This proves that the relevant mechanism for the amplification is on-the-job search. Another fact that stands out in Table 3.6 is related to the Beveridge curve. The correlation between unemployment and vacancies implied by the benchmark model is 0.96. This counterfactually positive relationship is a result of the countercyclical vacancies in the benchmark model.

Without on-the-job search, equilibrium of this economy will only be a function of the level of aggregate productivity. This follows from the constant returns to scale matching function and the fact that the search input is proxied only by unemployment. Since all unemployed workers are ex-ante identical and there are

forms used for matching function. The model that generates the simulations in Table 3.3 assumes a standard Cobb-Douglas form.

no meetings between vacancies and employed workers, the probability of forming a match for a firm only depends on the v/u ratio. For two different unemployment levels, as long as the aggregate state stays the same, free entry of the firms will ensure the same equilibrium level of v/u through variations in v[20]. Then v/u becomes a sufficient statistic that determines vacant firms' problem. Hence, fluctuations in aggregate productivity is almost perfectly correlated with variations in v/u. In this case, market tightness, v/u, is an increasing function of z.

In order to understand the counterfactually positive correlation, consider staying in the same aggregate state for a few periods. Since z is constant over periods, there will be no discrete changes in the profitability of an ongoing match. In other words, separations do not fluctuate a lot due to changes in $\bar{\gamma}(z)$. Thus, with the same level of v/u, unemployment will start shrinking over this episode. But, a constant v/u implies that v should also decline over the same episode. Thus, there will be a positive correlation between unemployment and vacancies over this particular episode. This seems to be a common feature of the standard model too. So, why do I get positive correlation here instead of the negative correlation implied by the standard model (with only exogenous separations) ? The answer is intimately related to the behavior of separations. The absence of endogenous separations forces the economy to adjust to new aggregate shocks via changing the hiring behavior. If this is the only channel, then changes in unemployment is induced only through the job finding probability, $f(v/u)$. However, with endogenous separations due to low

[20] This is a fairly standard observation made about the labor market search and matching models.

productivity, unemployment is also affected by countercyclical separations. This dampens the magnitude of variations required in v in response to negative productivity shock. Since such a negative shock induces a sharp decline in u, the lower equilibrium value for v/u could be attained even with small changes (and in Table 3.5 with large declines) in v[21].

On the other hand, the model with on-the-job search implies a much more realistic picture in this regard. In contrast to the benchmark model, the relevant equilibrium object is not v/u but only v. Hence, aggregate productivity fluctuations are accommodated through changes in the number of vacancies posted. But as argued in the rest of the discussion here, firms' incentive to create vacancies respond to the behavior of expected job-to-job transitions. This is the channel which reverse the counterfactual implications of the benchmark model.

How does job-to-job transition create amplification and help to reconcile endogenous separations with the Beveridge curve? The answer to this question lies at the heart of the model. First, it is important to understand how the match quality distribution evolves in response to aggregate productivity shocks. In order to illustrate this, it might be useful to analyze what match quality distributions would look like in steady states with different productivity levels. These two different match quality distributions are shown in Figure (3.7). It is clear from the figure that, in the high productivity steady state there are some employed workers in low quality matches, which would have been unprofitable otherwise. It also happens to be the

[21]Shimer (2005a) considers separation shocks as a possible driving force for unemployment fluctuations and discusses the point made here further.

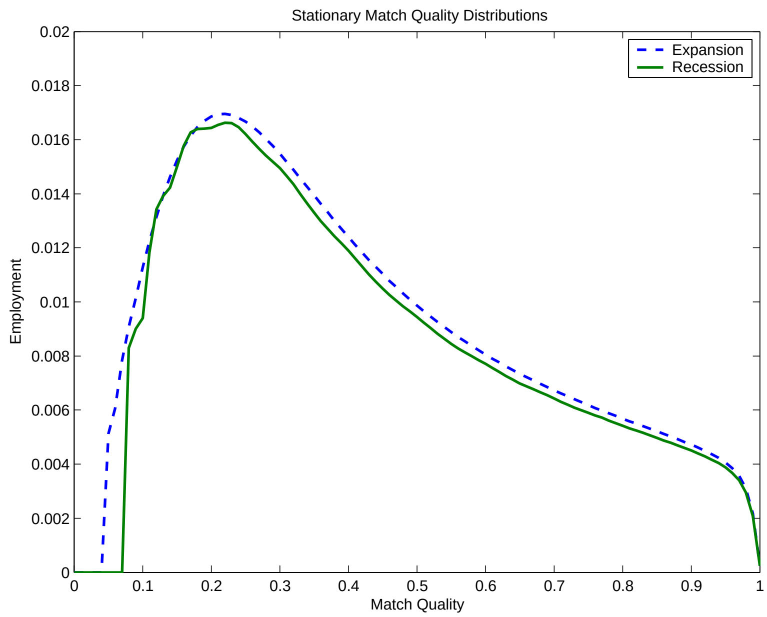

Figure 3.7: Stationary Match Quality Distribution

case that, since employment is higher in high productivity steady state, there are uniformly more workers employed in each match quality level. Because, workers are better of by switching to new jobs with higher quality, the odds of quitting and changing a job would be higher in the high aggregate productivity state. This is crucial for the firms that are considering to post vacancies. Remember that one critical object in the model was the value for μ, which summarized this probability. Two distributions pictured in Figure (3.7) clearly show why one should expect more workers to be willing to accept job offers from vacancies. Hence, vacant firms might expect to meet with workers that are more willing to change jobs and accept their offers in expansions.

In the standard search model, when firms intend to create vacancies due to high aggregate productivity, unemployment pool starts to shrink. Since new matches are formed only between vacancies and unemployed workers, as the high productivity prevails, firms loose incentive to create any more vacancies due to smaller pool of potential matches. Here, however, high productivity state serves as a good opportunity for workers to reallocate themselves for better quality matches. This improved reallocation across matches also gives further incentive to firms to create vacancies when aggregate productivity is high. Thus, the model implies a significantly procyclical labor market reallocation through procyclical job-to-job transitions.

This model's implications for labor market flows are also instructive in this regard. First of all, by having the possibility of job-to-job transitions, the model has richer implications than the standard model. As Table 3.5 indicates, e-e flows are significantly procyclical and as variable as u-e flows. On the other hand, the flows from employment to unemployment is countercyclical as expected but has very large variation. The cyclicality of job-to-job transitions is in line with quits (Nagypal 2004a). However, we need to be cautious when interpreting u-e flows. In theory, these separations happen with mutual consent, so it is neither layoff nor quit. If both e-u and e-e flows are considered to be "separations", the enormous volatility in e-u does not carry over to all separations. This is due to the negative relationship between both flows constituting "separation," i.e the procyclicality of e-e dampens the effect of e-u.

3.8 Conclusion

The Mortensen-Pissarides labor market search model has been recently criticized because of the model's quantitative implications for business cycles. In particular, researchers have pointed out the discrepancy between the implied level of variation in unemployment, vacancies and market tightness and the observed variation in these variables in the United States. This chapter extends the baseline labor market search model to include on-the-job search and match specific heterogeneity to generate the missing amplification. The mechanism works through the effects of aggregate productivity shocks on the entire employment distribution.

There is incomplete information about the quality of the employee-firm match which provides persistence in employment relationships and the rationale for on-the-job search. Amplification arises because productivity changes not only affect firms' probability of contacting unemployed workers but also of contacting already employed workers. Since the measure of workers in low quality matches is greater, this probability is higher during expansions. This introduces strongly procyclical labor market reallocation through procyclical job-to-job transitions, which has been a generally ignored feature in the literature. Hence, the model provides a possible channel which does not require changing the wage determination process or the information structure to a large extent to create more variation. In particular, simulations with a plausible forcing process show that the standard deviations for unemployment, vacancies and vacancy-unemployment ratio (market tightness) match the U.S. data. The model also reconciles the presence of endogenous separation with the negative correlation of unemployment and vacancies over business cycle frequencies.

This chapter also has a methodological contribution. On-the-job search with match heterogeneity requires to take the entire employment distribution into account as part of the state space. It is well known in the literature that this complicates the numerical solution of the equilibrium. I adapt the algorithm used by Krusell and Smith (1998) to the problem described in this chapter. The computational exercise suggests that approximating the worker's acceptance probability of a firm's job offer suffices to characterize the equilibrium. Other studies that have modeled on-the-job search either assumed simplifying assumptions to get rid of the endogenous effects of heterogeneity or simply restricted the analysis to non-stochastic equilibrium.

Several possible variations of the model has been ignored in this chapter. One key feature that should be considered is the effect of strategic bargaining at the wage determination stage. Incorporating such an additional feature might be a natural extension of the model to get wage rigidity and therefore might provide a comparison to the mechanism advocated here. I have also focused on the mechanism itself without much discussion on the implied magnitudes of the flows. This is mostly the case, because the underlying calibration of the model does not target job-to-job flow statistics from recent studies. These possible extensions are left for future research.

Chapter 4

Screening Costs, Hiring Behavior and Volatility

This chapter proposes yet another mechanism to increase the response of key aggregate labor market variables to productivity shocks. Instead of focusing on the pool of searchers among the unemployed, we try to understand the recruitment and hiring behavior of employers and its potential contribution to the amplification mechanism through costly screening. The proposed mechanism incorporates match specific symmetric incomplete information and the availability of a screening technology into otherwise standard MP model. In the model, due to incomplete information about the actual quality of the match, both parties in a match (a worker and an employer) potentially learn about the quality of the match by engaging in a productive relationship. Although it is assumed that, when they meet, agents do not know exactly what quality match they are going to form, they have some prior information suggesting how good a match it is going to be. This information relates to how good a match is likely to be formed and summarizes an underlying efficiency in the matching technology. One of the key ingredients of the model, the availability of another screening technology, is modelled as the availability of another distribution of potential matches which promises a higher expected average match quality. In this framework, allowing employers to choose this costly screening technology leads to more intensive recruiting when aggregate productivity is low. Intuitively, employers choose to utilize this costly technology when opportunity cost of not producing is relatively low and when the cost of a low quality worker is high. This in turn leads to higher volatility in the number of vacancies created. Simulations show that, reasonable costs of screening in this model might explain more than half of the observed variability in key labor market variables.

Although we lack empirical studies about the intensity of recruitment or screening over the business cycle, there is some evidence regarding the importance of this margin and the costs associated with it. Best example of a recruitment process modeled here comes from Dutch labor markets. Van Ours and Ridder (1992) report that vacancy durations in the Dutch labor market are mainly selection periods but not search periods for applicants. This means employer search in this market is not sequential. Employers advertise vacancies and then form a pool of applicants. This is consistent with the endogenous decision of whether to form a match upon meeting in the model. The costs associated with this screening/recruitment process might also be significant. Based on empirical evidence provided by Barron et. al. (1997) and Senesky (2003) from Employment Opportunity Pilot Project (EOPP), Silva and Toledo (2006) reports that average cost of recruitment/screening is around 4% of the quarterly wage. Incorporating costs like advertising, travel costs for applicants and agency fees this cost rises up to 20% of employee compensation according to an estimate by Saratoga Institute (2004).

One key assumption in the chapter is that information about the productivity on the match is incomplete and it is revealed through output realizations over the job. Nagypal (2004) focuses on the importance of this feature of employment relationships and argues that learning about match quality is an important element determining the worker's turnover behavior especially after the first 6 months of tenure. This feature of the model also provides persistence in employment relationships and a rationale for potential gains from better information about prospective match quality. There is also some empirical evidence which suggests that employers more frequently use informal recruitment channels when the vacant position requires higher education, skills and training or when the cost of recruitment is high (Barron et. al. (1985) and Gorther et. al. (1997)). We also interpret this empirical evidence as suggestive of the presence of information incompleteness about the prospective match and the potential benefits from getting better screening.

Several recent studies offered models to improve the lack of propagation in the standard MP model (Hagedorn and Manovskii (2005), Krause and Lubik (2004), Nagypal (2005) and Silva and Toledo (2005)). Hagedorn and Manovskii (2005) use an unrealistically high value of leisure to generate amplification. Krause and Lubik (2004) and Nagypal (2005) both model on-the job search aspect. In general on-the-job search introduces the heterogeneity of job seekers into the picture. Coupled with the aggregate uncertainty, this complicates the problem to a great extent. This might be the reason why Krause and Lubik (2004) assumes a segregated market for different kind of jobs to simplify the potential complexity of the model, whereas Nagypal (2005) only restricts the analysis to non-stochastic equilibrium. Both of these problems are addressed in the previous chapter. The study which is probably the closest to the current paper is by Silva and Toledo (2005). They also model the role of turnover costs in the standard MP model. However, they impose these costs on employers in every state of the world, whereas we model the costly screening such that employers are free to incur these costs. This indeed, leads to higher volatility endogenously.

The next section describes the economic environment and lays out the dynamic optimization problem of agents. Section 4.2 characterizes the equilibrium of the economy. Section 4.3 describes the computational procedure, calibration of the parameters and the simulation results. Finally, Section 4.4 concludes.

4.1 The Economic Environment

The model presented here is a simpler version of the model in the previous chapter), without on-the-job search and with a costly screening technology. The following discussion closely follows the presentation of the model in Chapter 3.

4.1.1 Learning, Production and Screening Technology

Employers are endowed with a production technology that produces $y_t \in Y = \{y^h, y^l | y^h > y^l\} \subset \mathbb{R}_{++}$ when matched with a worker. Hence, when a worker and a firm form a productive match, they produce $z_t y_t$, which depends on the inherent match quality and aggregate state, $z_t \in Z \subset \mathbb{R}_{++}$. Aggregate productivity is governed by a Markov process, $\Psi(z_{t+1}|z_t)$ and is independent of the idiosyncratic component. Even though both workers and firms observe the match specific component of the output, y_t, and the aggregate state, they do not observe their actual match quality, q, which can be *good* or *bad*. Match specific output is determined by the following relationship:

$$\begin{aligned}\Pr(y_t &= y^h|q=g) = \Pi_g > \Pr(y_t = y^l|q=g) = 1-\Pi_g \\ \Pr(y_t &= y^l|q=b) = \Pi_b > \Pr(y_t = y^h|q=b) = 1-\Pi_b\end{aligned} \tag{4.1}$$

Though q is unobservable, agents receive an initial signal $\gamma_0 \in [0,1]$ that corresponds to the probability that the match will be good if formed. It is same for both the worker and the firm.

The major feature of the model that leads to substantial variability in vacancies, unemployment and market tightness is the availability of another technology for firms. This technology will be called a *screening technology* and can be utilized by the firms upon paying a fixed cost, $\kappa > 0$. When a firm pays this cost, the initial signal γ_0 will be drawn from a distribution that has a higher mean. Let the distribution of initial signals be denoted by $\Gamma(\eta, \sigma)$ in the absence of this better screening. Then, paying κ entitles the firm to get an initial draw from another distribution $\Omega(\tilde{\eta}, \sigma)$, where $\tilde{\eta} > \eta$. Since agents are all risk neutral, a higher expected output might be desired when κ is low relative to the expected gains from a higher match quality prospect.

After the initial period, both parties start learning about their match quality based on output realizations. Since there is no asymmetric information and the output is observed by both, they will have the same posterior belief about the match quality. Let $\Pr(q = g|y_{t-1}) = \gamma$ denote this probability that the current match is a good match conditional on the past output realization on the match, y_{t-1}. Agents need to infer $\Pr(q = g|y_t)$ and $\Pr(q = b|y_t)$ for $y_t \in \{y^h, y^l\}$. As in Chapter 3, updating rule follows from simple Bayesian inference.

$$\Pr(q=g|y_t=y)=\frac{\Pr(q=g|y_{t-1})\Pr(y_t=y|q=g)}{\Pr(q=g|y_{t-1})\Pr(y_t=y|q=g)+\Pr(q=b|y_{t-1})\Pr(y_t=Y-y|q=b)}$$

After some algebra, I arrive at the following posteriors implied by prior belief, γ, and time t output realization.

$$\begin{aligned}\Pr(q &= g|y_t=y^h)=\gamma^h=\frac{\gamma\Pi_g}{\gamma\Pi_g+(1-\gamma)(1-\Pi_b)} \\ \Pr(q &= g|y_t=y^l)=\gamma^l=\frac{\gamma(1-\Pi_g)}{\gamma(1-\Pi_g)+(1-\gamma)\Pi_b}\end{aligned} \quad (4.2)$$

The posterior is updated to γ^h after observing a high output and to γ^l after low output. Intuitively, γ^h is expected to be higher than the current state γ. More formally, the current state is related to the future state in the following way under (4.1).

Remark 4.1.1. If (4.1) holds, $\gamma^h(\gamma)\geq\gamma\geq\gamma^l(\gamma)$ and both $\gamma^h(\gamma)$ and $\gamma^l(\gamma)$ are increasing in γ. In addition, $\gamma^h(\gamma)$ is concave whereas $\gamma^l(\gamma)$ is convex[1].

Finally for simplicity, I refer to the unconditional probability of observing a high output (low output) at each period as α $(1-\alpha)$ in the rest of the chapter. They are simply defined as linear functions of γ: $\alpha(\gamma)=\gamma\Pi_g+(1-\gamma)(1-\Pi_b)$, $(1-\alpha(\gamma))=\gamma(1-\Pi_g)+(1-\gamma)\Pi_b$.

The presence of match specific productivity implies that all matches are indexed by their quality (γ) at any point in time. This requires a way of describing

[1] The fact that these two mappings are increasing in γ could be established by the positive sign of the first derivatives. Also, comparing γ^h,γ^l and γ, one can easily establish that $\gamma^h(\gamma)\geq\gamma\geq\gamma^l(\gamma)$ is true. Concavity of $\gamma^h(\gamma)$ and convexity of $\gamma^l(\gamma)$ follow from these arguments and the fact that both $\gamma^h$and γ^l maps $[0,1]$ to itself.

the heterogeneity at any point in time. A mapping that gives the measure of the employment at any subset of the match quality state space is a natural way of describing this heterogeneity. Let $n_t(\gamma) \in [0, 1]$ be the total number of matches that are believed to be good quality with probability γ. Since, each match employs one worker, this also gives the total number of employed workers with match quality index γ. I also assume that agents are rational in their expectations. In other words, γ fraction of all matches among $n_t(\gamma)$ are actually good quality. How this distribution evolves over time depends both on the endogenous decisions made by agents and the equation of motion for aggregate productivity. I assume that this is summarized by a function G such that:

$$n_{t+1} = G(n_t, z_t, z_{t+1}) \tag{4.3}$$

Employment distribution is not going to be a state variable in the dynamic problem outlined in this model, in contrast to Chapter 3. The reason for this simplification is discussed in the following section.

4.1.2 Matching Technology and Wage Determination

The meeting process is facilitated by an aggregate matching function, which maps the number of searchers on both sides of the market into meetings. There will be no search effort and only unemployed workers search.

The matching technology is summarized by a constant returns to scale matching function, $M(v_t, u_t)$, that takes the amount of job seekers, u_t, and vacancies, v_t as its arguments[2]. This implies that the rate at which workers meet a job opportunity is $f(\theta_t) = M(v_t, u_t)/u_t = M(\theta_t, 1)$, where $\theta_t = v_t/u_t$, or market tightness. Similarly, a vacancy will meet a worker at the rate $h(\theta_t) = M(v_t, u_t)/u_t$. The meeting rate is not equal to matching rate in this model, because not all meetings end as successful matches. Meetings between an unemployed worker and a vacant firm turns into productive matches if their common beliefs about the match quality are above a certain threshold, which is to be determined endogenously in the equilibrium. For simplicity, I assume that there is no recall of past job offers.

The firm pays a wage that is determined by Nash bargaining over the match surplus which is common in the literature. Nash bargaining leads to a linear sharing rule, which is standard in labor market search literature. The sharing rule is such that workers keep $\phi \in (0, 1)$ fraction of the match surplus whereas firms get $(1 - \phi)$ of it. Wages are renegotiated each period by splitting the surplus with the same rule. This does not preclude persistence in wages because inherent match quality, γ, and aggregate productivity, z, are both persistent.

Finally, the alternative to a match for a firm is posting a vacancy , which costs $c > 0$ units of consumption per period and generates a possibility of a new match in the next period. Firms have incentive to post vacancy as long as the value of posting one is positive. This is ensured by the free entry of firms and implies that equilibrium value of vacancy is driven to zero. For workers, the outside option is to be unemployed and to consume $b > 0$, which could be interpreted as unemployment

[2] Since not all meetings result in a match, the term "matching" is used for meeting, in the context of this chapter.

benefits and/or value of leisure. This implies that ongoing matches are destroyed endogenously when the match surplus becomes negative. Because of the particular sharing rule I use, such a decision does not create any disagreements, i.e. both parties agree to end the match jointly. In the equilibrium, this implies a reservation prior, $\bar{\gamma}$. below which the match ceases to be productive and dissolves. Matches are also subject to an exogenous shock in each period that renders the match unproductive. This probability is denoted by λ[3].

4.1.3 Timing of Events

It would be instructive to describe the timing of events within a period to understand agents' information set at each point in time. Events with a time period follows the sequence below:

- Matches that were productive in the last period start the period t with the information, γ_t, z_t . Unemployed workers and vacant firms start the period with z_t.

- Workers and firms within a match decide whether to stay or exit the match. Because of the surplus sharing rule, there is no disagreement between two parties.

  – If the decision is to stay, production occurs, y_t is realized. Workers consume wages, firm consumes net output. Match quality is updated to

[3] This exogenous probability ensures that in the nonstochastic steady state of this economy we have a non-degenerate employment distribution over match quality space.

γ_{t+1}.

  - If the decision is to exit, worker becomes an unemployed searcher and consumes b. Firm becomes idle.

- Firms decide to post vacancy at the cost of c until the value of a vacant position is driven to zero. This pins down the market clearing market tightness, θ_t.

- If a firm decides to post a vacancy, then it also has to choose which screening technology to be used.

- Meetings occur according to $M(\theta_t, 1)$, and initial signals are drawn from either Γ or Ω. Unemployed worker who meets a vacancy decides whether to form a match or stay unemployed.

- New matches are formed, which will be productive in $t+1$. Existing matches are subject to exogenous destruction with probability λ.

4.1.4 Bellman Equations

In order to define the equilibrium of this economy, I start with the Bellman equations that determine values of being in different labor market states. State variables for agents form a list $\{\gamma, z\}$, where time subscripts are dropped for convenience. Aggregate state variable z is not correlated with the law of motion for the individual state variable γ, since the learning process is independent of the aggregate state[4]. Equation of motion for γ is given by posteriors defined in (4.2), whereas that

[4]The matches that will survive in different aggregate states will be a function of the aggregate state in equilibrium. What is meant here is that conditional on survival of the match in the next

of z is governed by the Markov process.

Let $J_u(z)$ and $J_e(\gamma, z)$ be values of having a vacant job and being in a match for firms respectively.

$$J_u(z) = -c + \beta(1 - h(\theta)) E_{z'|z} J_u(z') + \max \left\{ \begin{array}{c} \beta h(\theta) E_{z'|z} \int J_e(\gamma', z') d\Gamma(\gamma'), \\ -\kappa + \beta h(\theta) E_{z'|z} \int J_e(\gamma', z') d\Omega(\gamma') \end{array} \right\}, \text{ for all } z. \tag{4.4}$$

This definition of $J_u(z)$ leads to a decision rule for the choice of screening technology. In other words, a firm should decide whether to draw the initial signals from a higher mean (Ω) at the expense of paying out of pocket costs, κ. Let μ_s express this decision, assuming that $\mu_s = 1$ when firms decide to choose Ω over Γ. Then it takes a simple form.

$$\mu_s(z) = \left\{ \begin{array}{l} 1 \text{ if } z \in Z_s \subseteq Z \\ 0 \text{ if } z \in Z - Z_s \end{array} \right. \tag{4.5}$$

The interpretation of $J_u(z)$ is straightforward. A firm has to pay c in order to post a vacancy, which will remain vacant with probability $1 - h(\theta)$ next period. Depending on whether the firm decides to utilize the screening technology, firms will meet with workers who are different with respect to their γ's with probability $h(\theta)$. As (4.5) puts it, in some aggregate states firms will choose to have this costly screening. Expectation operator takes $\Psi(z'|z)$ into account.

Similarly, the value of a match for a firm is defined as follows.

period posterior is only determined by the exogenous learning process.

$$J_e(\gamma, z) = \max \left\{ \begin{array}{c} z(\alpha(\gamma)y^h + (1-\alpha(\gamma))y^l) - w(\gamma, z) \\ +\beta(1-\lambda)E_{z'|z}\left[\alpha(\gamma)J_e(\gamma^h, z') + (1-\alpha(\gamma))J_e(\gamma^l, z')\right] + \beta\lambda E_{z'|z}J_u(z') \\ , J_u(z) \end{array} \right\} \tag{4.6}$$

A firm that starts the current period with a match that is of good quality with probability γ, has to decide whether to go on with this arrangement and pay $w(\gamma, z)$ to the worker or destroy the match (or not start the match at all). In the latter case, payoff to the firm is simply the value of being a vacant job. Current return from the match to the firm is the expected net output which is defined as $z(\alpha(\gamma)y^h + (1-\alpha(\gamma))y^l) - w(\gamma, z)$. Future return from current match depends on how beliefs and the aggregate state change. If the match survives exogenous separation (with probability $1-\lambda$), posterior for the next period will either be γ^h or γ^l, with probabilities $\alpha(\gamma)$ and $(1-\alpha(\gamma)$ respectively.

Let $V_u(z)$ be the value of being unemployed for a worker when aggregate productivity is z, then it takes the following form.

$$V_u(z) = b + \beta E_{z'|z}\left\{f(\theta)\left[\begin{array}{c} \mu_s(z)\int V_e(\gamma', z')d\Omega(\gamma') \\ +(1-\mu_s(z))\int V_e(\gamma', z')d\Gamma(\gamma') \end{array}\right] + (1-f(\theta))\,V_u(z')\right\} \tag{4.7}$$

An unemployed worker consumes b in this period and expects to come up with a possible match with probability $f(\theta)$, in which case, she gets the value of having a match, denoted by $V_e(\gamma', z')$. The value of γ', however, might be drawn either from Ω or Γ. Alternatively, she will stay unemployed with probability $1-f(\theta)$.

The value of having a match which is of good quality with probability γ is slightly more complicated.

$$V_e(\gamma, z) = \max \left\{ \begin{array}{c} w(\gamma, z) \\ +\beta(1-\lambda)E_{z'|z} \left[\begin{array}{c} \alpha(\gamma)V_e(\gamma^h, z') \\ +(1-\alpha(\gamma))V_e(\gamma^l, z') \end{array} \right] + \beta\lambda E_{z'|z} V_u(z') \\ , V_u(z, n) \end{array} \right\} \tag{4.8}$$

The worker compares the returns on retaining the match and not accepting or dissolving it. Current return from the match equals wage payments, $w(\gamma, z)$. Depending on the current output realization, she will update her belief about the quality of the match according to (4.2). Since a high output is realized with probability $\alpha(\gamma)$, the expected future value of the match becomes $V_e(\gamma^h, z')$. Alternatively, a low output realization leads to a lower posterior and a corresponding expected future value of being in a match, $V_e(\gamma^l, z')$.

4.2 Equilibrium

There are three endogenous equilibrium objects to be determined in this economy: Workers' and firms' decision as to when to destroy an existing match, equilibrium market tightness and the wage to be paid.

Let $\chi(\gamma, z)$ denote the optimal decision rule on match formation (and destruction) and $\theta(z)$ denote equilibrium market tightness as a function of the aggregate state. Then the equilibrium of this economy can be easily defined.

Definition 4.2.1. The **equilibrium** of this economy is a list $w(\gamma, z)$, $\theta(z)$, $\chi(\gamma, z)$,

$\mu_s(z)$, $J_e(\gamma, z)$, $J_u(z)$, $V_e(\gamma, z)$, and $V_u(z)$ such that;

1. Given $w(\gamma, z)$, $\theta(z)$, $\mu_s(z)$ and $\chi(\gamma, z)$, value functions satisfy (4.4)-(4.8)

2. Given $w(\gamma, z)$, $\theta(z)$, $\mu_s(z)$ and value functions, $\chi(\gamma, z)$ is optimal.

3. Given $w(\gamma, z)$, $\theta(z)$, $\chi(\gamma, z)$ and value functions, $\mu_s(z)$ is optimal.

4. (Free entry of firms) Given $w(\gamma, z)$, $\chi(\gamma, z)$, $\mu_s(z)$ and $\theta(z)$, each firm posts a vacancy as long as $J_u(z) > 0$. Hence, aggregate $\theta(z)$ makes the value of posting a vacancy zero, i.e. $J_u(z) = 0, \forall z$.

5. (Nash Bargaining) Each period: $V_e(\gamma, z) - V_u(z) = \phi \left[J_e(\gamma, z) - J_u(z) + V_e(\gamma, z) - V_u(z) \right]$ and $J_e(\gamma, z) - J_u(z) = (1 - \phi) \left[J_e(\gamma, z) - J_u(z) + V_e(\gamma, z) - V_u(z) \right]$.

The specific Nash Bargaining rule used in this chapter implies that both workers and firms agree to leave when the surplus of the match falls below zero[5]. Surplus of the match is defined as the quantity, $J_e(\gamma, z) - J_u(z) + V_e(\gamma, z) - V_u(z)$. When the match surplus is negative, the share each party gets become negative simultaneously. Hence, in order to describe the decision rule $\chi(\gamma, z)$, it is essential to write down the surplus function. Subtracting outside options from $J_e(\gamma, z)$ and $V_e(\gamma, z)$, and adding them up leads to an expression defining the value of the surplus from a match with quality γ, in aggregate state z. Details of the derivation is presented in the appendix. Let this value be denoted by $S(\gamma, z)$. The appendix shows that this surplus function has the following recursive form.

[5]This is quite standard in labor market search literature with match specific heterogeneity. See for instance, Mortensen and Pissarides (1994).

$$S(\gamma,z)=\max\left\{\begin{array}{c} z(\alpha(\gamma)y^h+(1-\alpha(\gamma))y^l)-b \\ +\beta(1-\lambda)E_{z'|z}\left[\alpha(\gamma)S(\gamma^h,z')+(1-\alpha(\gamma))S(\gamma^l,z')\right] \\ -\beta f(\theta)\phi E_{z'|z}\left[\begin{array}{c}\mu_s(z)\int S(\gamma',z')d\Omega(\gamma') \\ +(1-\mu_s(z))\int S(\gamma',z')d\Gamma(\gamma')\end{array}\right] \end{array}, \quad 0\right\}$$

This equation is one of the key equations characterizing the equilibrium. For any $\theta > 0$, this equation describes when an existing match should be destroyed bilaterally. Since the right part of the expression within the outermost bracket is constant and the left is increasing in γ, it is easy to show that the decision rule $\chi(\gamma, z)$ takes the following form for a given θ.

$$\chi(\gamma,z)=\left\{\begin{array}{l} 1 \text{ if } \gamma \geq \bar{\gamma}(z) \\ 0 \text{ if } \gamma < \bar{\gamma}(z) \end{array}\right. \tag{4.9}$$

The reservation threshold, $\bar{\gamma}(z)$, determines whether a match should survive. It also summarizes the hiring decision. In other words, a meeting will turn into a match if the reservation threshold is reached and an ongoing match is terminated if the match quality falls below this threshold. In equilibrium, it turns out that $\bar{\gamma}(z)$ is a decreasing function of z. Intuitively, agents become less willing in recessions to undertake matches that are less probable to be good quality. Since all matches are less productive the threshold for a match to survive is higher in recessions.

The second important equation determining the equilibrium of this economy comes from the free entry condition and pins down the equilibrium number of vacancies posted. As it is shown in the appendix, the value of vacancy can be written as a function of the surplus function. It takes a simple form:

$$J_u(z) = -c + \beta E_{z'|z} J_u(z') + \beta h(\theta)(1-\phi) E_{z'|z} \left[\begin{array}{c} (1-\mu_s(z)) \int S(\gamma', z') d\Gamma(\gamma') \\ +\mu_s(z) \left(-\kappa + \int S(\gamma', z') d\Omega(\gamma')\right) \end{array} \right] \tag{4.10}$$

However, free entry of firms imply that $J_u(z) = 0$ for all z in equilibrium. Substituting this in (4.10) gives the second key condition describing the equilibrium.

$$\frac{c}{\beta h(\theta)(1-\phi)} = E_{z'|z} \left[(1-\mu_s(z)) \int S(\gamma', z') d\Gamma(\gamma') + \mu_s(z) \left(-\kappa + \int S(\gamma', z') d\Omega(\gamma') \right) \right] \tag{4.11}$$

Finally, $\mu_s(z)$ will be pinned down by equilibrium condition 3 as follows.

$$\mu_s(z) = \begin{cases} 1 \text{ if } z \in Z_s \subseteq Z | -\kappa + \int S(\gamma, z) d\Omega(\gamma) > \int S(\gamma, z) d\Gamma(\gamma) \\ 0 \text{ if } z \in Z - Z_s | -\kappa + \int S(\gamma, z) d\Omega(\gamma) \leq \int S(\gamma, z) d\Gamma(\gamma) \end{cases} \tag{4.12}$$

Equations (3.11), (4.11) and (4.12) jointly determine the equilibrium values for $\theta(z), \mu_s(z)$ and $\bar{\gamma}(z)$. First two equilibrium conditions are standard in models of labor market search with endogenous job destruction (Mortensen and Pissarides, 1994). Availability of a costly screening technology, however, changes the equilibrium value of market tightness slightly. Since, workers need to anticipate whether firms will choose to use this technology, we need the last equation (4.12) to complete the picture.

For the purpose of finding out equilibrium of the dynamic problem for the individuals laid out in the previous section, it is important to note that employment distribution, n, never appears as a state variable. First of all, note that n only appears in the matching function indirectly through u (e.g. $u = (1 - \int_0^1 n(\gamma')d\gamma')$). The reason n is not a state variable is intimately related to the particular form of the matching function and the fact that aggregate number of vacancies is a jump variable. Because the matching function is constant returns to scale, a sufficient statistic to summarize the meeting rates is the v/u ratio, rather than the actual levels of u or v. In addition to this, we have assumed (as the rest of the literature) that vacancies are posted up to the level such that the value of posting an additional one is zero. This guarantees that v, the number of aggregate vacancies, will instantaneously adjust to the equilibrium level of θ regardless of the value of u. However, when simulating the model, it is important to assign an initial value to unemployment to track how it evolves over time. Once, $\theta(z), \mu_s(z)$ and $\chi(\gamma, z)$ are known, the evolution of the employment (match quality) distribution will be straightforward.

Let n_{t-1} be the match quality distribution at the end of time period $t-1$. In other words, $n_{t-1}(\gamma)$ gives the number of workers employed with match quality index γ at time $t-1$. I assume that agents, both workers and firms, are rational in their expectations about match quality. In other words, among the matches that are currently believed to be good with probability γ, fraction of the good matches are indeed γ. Then the measure of workers in next period will be easily determined.

$$n_t(\gamma) = \chi(\gamma, z_t) \left\{ \begin{array}{c} (1-\lambda)\left[(1-\alpha(\gamma^1))n_{t-1}(\gamma^1) + \alpha(\gamma^2)n_{t-1}(\gamma^2)\right] | \\ +f\left(\theta(z_t)\right)\left(1 - \int_0^1 n_{t-1}(\gamma')d\gamma'\right)\left[\mu_s(z_t)d\Omega(\gamma) + (1-\mu_s(z_t))d\Gamma(\gamma)\right] \end{array} \right\}, \forall \gamma \in [0,1] \tag{4.13}$$

where and γ_1 and γ_2 are defined as;

$$\gamma = \frac{\gamma^1 \Pi_g}{\gamma_1 \Pi_g + (1-\gamma_1)(1-\Pi_b)} \text{ and } \gamma = \frac{\gamma_2(1-\Pi_g)}{\gamma_2(1-\Pi_g) + (1-\gamma_2)\Pi_b} \tag{4.14}$$

Then the definition of u follows:

$$u_t = (1 - \int_0^1 n_t(\gamma')d\gamma') \tag{4.15}$$

The set of equations determining the equilibrium conditions and the equation of motion for match quality distribution presented in this section forms the foundation of the computational algorithm to solve the equilibrium. We outline the computational strategy and the calibration in the next section along with the simulation results.

4.3 Calibration and Simulation Results

This model demands a much simpler algorithm than Chapter 3. Given the parameter values, it basically consists of three steps, each involving solving a fixed point problem over $\theta(z), \mu_s(z)$ and $\chi(\gamma, z)$. Equations (3.11), (4.11) and (4.12) are used simultaneously to solve for these three objects. Then the model is simulated based on (4.13) and an initial condition. The precise algorithm for the computation of the equilibrium involves following steps:

1. Select T-point grid on z and M-point grid on γ.

2. Start with a Tx1 vector of initial guess on $\mu_s(z)$.

3. Guess an initial Tx1 vector of $\theta(z)$.

4. Given $\theta(z)$ and $\mu_s(z)$ solve the decision rule $\chi(\gamma, z)$ by Iterating over the surplus function defined in recursive equation (3.11) until convergence. Obtain the value of $S(\gamma, z)$.

5. Given the surplus function, $S(\gamma, z)$, and $\mu_s(z)$, check whether (4.11) is satisfied. If it is satisfied, then $\theta(z)$ is potentially an equilibrium and continue to Step 6. Otherwise, update your guess for $\theta(z)$ and go back to Step 4 with this new guess.

6. Given $\theta(z)$ and $S(\gamma, z)$, evaluate whether (4.12) is satisfied. If it is satisfied, $\theta(z), \mu_s(z)$ and $\chi(\gamma, z)$ constitute the equilibrium. Otherwise update $\mu_s(z)$ with a new guess and go back to Step 3 with this guess. Repeat this until equilibrium is reached.

4.3.1 Calibration

In order to solve the model with this algorithm we need to assign parameter values for the primitives. The parameter values are assigned to match some key long-run features of the U.S. labor markets. The time period is one month and β is calibrated to match 4% annual interest rate. This implies that $\beta = 0.9967$.

First step is to calibrate a productivity process. This is achieved by estimating a three state Markov-Chain approximation for the AR(1) process for the real output per worker in the non-farm business sector. A three state approximation is chosen in order to simplify the computation process and to have sufficient dispersion in productivity. It is important to have enough number of aggregate states to see how different technologies might be chosen by firms in response to these shocks. This productivity data has a standard deviation of 0.02 and a first order autocorrelation of 0.89. Since the standard deviation of this process will be affecting the volatility of other variables directly, matching the exact standard deviation would be desired. Thus I find the following Markov process as the best approximation, which implies a standard deviation of 0.02 and a first order autocorrelation of 0.81.

$$\Psi(z_{t+1}|z_t) = \begin{bmatrix} 0.9564 & 0.0435 & 0.0001 \\ 0.0435 & 0.9129 & 0.0436 \\ 0.0001 & 0.0436 & 0.9682 \end{bmatrix}, \ Z = \{0.9672, 1, 1.0339\}$$

Several parameters are taken from other studies. The share of the surplus taken by worker, ϕ, is chosen to be 0.36 (Shimer, 2005). The mean of the truncated normal density Γ is set to 0, i.e. $\eta = 0$. This is from Pries and Rogerson (2005). I also normalize the value of the match specific output when it is low to 1, i.e. $y^l = 1$.

Matching function takes the following simple Cobb-Douglas form:

$$M(v_t, u_t) = m u_t^{1-\delta} v_t^{\delta} \tag{4.16}$$

Efficiency requires that $\delta = \phi$ (Hosios, 1990) and m is normalized to 1.

In order to calibrate the rest of the parameters, I target three moments from the model; steady state unemployment rate of 5.68%, steady state job finding prob-

Calibrated Parameters					
Parameter	Value		Parameter	Value	
β	0.9967	4% interest	Π_g	0.58	Match
ϕ	0.36	Shimer (2005)	Π_b	0.53	Match
y^l	1	Normalization	c	2.69	Match
z	-	U.S. Avg.	b	4	Restriction
δ	0.36	Hosios	y^h	8.73	Match
η	0	PR (2005)	λ	0.0041	DHS (1996)
σ	0.235	PR (2005)	z^{ss}	1	Normalization

Table 4.1: Calibration with Screening Technology

ability of 45% and steady state probability of accepting a match conditional on meeting of 50%. The first two statistics are from the data. The latter statistic equals $1 - \Gamma(\bar{\gamma}^{ss})$ and chosen to be in line with Pries and Rogerson (2005). Note that I implicitly assume that firms will choose not to use the screening technology in the steady state. Hence, in the steady state, Γ is preferred[6]. These targets imply a steady state monthly separation probability of 2.71%. This is slightly lower than the average separation probability in the U.S. data, which is 3.4%. However, we cannot simultaneously match an average job finding probability of 45% and an average separation probability of 3.4%. The exogenous job destruction probability, λ, is then calibrated to match the fraction of shutdowns among all job destruction. Davis et al. (1996) estimates that 11% of all job destruction is accounted for by shutdowns. Since all separations in the benchmark model's steady state are either exogenous (due to λ) or due to learning about the quality of the match, a slightly higher value for this fraction is targeted. Specifically, it is assumed that 15% of all

[6]This indeed is true in the simulations.

separations are exogenous. This pins down λ, which is set to 0.0041.

Since job finding probability is $f(\theta^{ss})\,(1-\Gamma(\bar{\gamma}^{ss})\ )$ in the steady state equilibrium of this economy, the target for $1-\Gamma(\bar{\gamma}^{ss})$ implies that $\theta^{ss}=0.672$. Given these targets, learning process can be calibrated to match them. For instance, the standard deviation of the distribution for prior signals, σ, implies an equilibrium value for $\bar{\gamma}^{ss}$ if $(1-\Gamma(\bar{\gamma}^{ss})\ )=0.5$. The value of $\bar{\gamma}^{ss}$ does not have any intrinsic value for the purpose of this chapter. Hence, I target a value of 0.15 for this equilibrium value. This target and $(1-\Gamma(\bar{\gamma}^{ss})\ )=0.5$, implies that $\sigma=0.235$. With the targets for $\bar{\gamma}^{ss}$ and θ^{ss} and values for λ,σ and η, the model can be easily simulated to generate a stationary match quality distribution. Given these parameters, only determinants of this equilibrium distribution are learning parameters Π_g and Π_b. Recall that these two parameters determine the pace of learning and the attrition over the job. They are calibrated to match the steady state unemployment rate and the U.S. tenure distribution. This implies that $\Pi_b=0.53$ and $\Pi_g=0.58$. It turns out that the model generates a tenure distribution such that 24.76% of the employed are with 1 year of tenure, 21.23% of the employed are with 3-4 years of tenure and 23.82% of the employed are with 5-9 years of tenure. The corresponding values from BLS's Employment Tenure Summary are 20.7%, 19% and 20.7% respectively[7].

Calibrating a value for b is not straightforward. Recall that $y^l=1$. In this benchmark model, finding an interior solution for $\bar{\gamma}$, requires that the value of surplus is at least 0 for $\gamma=0$ and strictly positive for $\gamma=1$. In general, this implies

[7]The data could be found in Table 3 of the Summary at http://www.bls.gov/news.release/tenure.t03.htm.

that b should be somewhere between y^h and y^l. Taking this restriction into account and the targets for $\bar{\gamma}^{ss}$ and v^{ss}, I pick the values for b, y^h and c. Implied values for these parameters are 4, 8.73 and 2.69 respectively. This completes the calibration of the model except κ and $\tilde{\eta}$. Simulation results for various values of κ and $\tilde{\eta}$ are presented in the following subsection.

4.3.2 Simulations

Employers' response to productivity shocks will vary with respect to the size of κ and the potential gains in terms of higher $\tilde{\eta}$. The cost of screening and recruiting from better quality workers is κ. However, choosing to draw initial signals from Ω improves the odds of finding a better quality worker on average. Therefore, the magnitude of the response employers give to the productivity shocks will crucially depend on the balance between these two parameters. Hence, simulation results will depend on the assigned parameter values for κ and $\tilde{\eta}$.

Unfortunately, there is not a large body of literature which provides us with robust estimates of screening costs as interpreted by κ. Although there is relatively a well established literature on search behavior of workers, we lack economy wide estimates of the costs incurred by employers while they search. This could be attributed to the fact that we lack reliable data on employers' side of the market. There is, however, still some evidence about the extent of these costs. Based on empirical evidence provided by Barron, et. al. (1997) and Senesky (2003) from Employment Opportunity Pilot Project (EOPP), Silva and Toledo (2006) reports that average cost of recruitment/screening is around 4% of the quarterly wage. Incorporating

U.S. DATA	*(Quarterly, 1951Q1-2003Q4)*					
	u	v	v/u	u-e	e-u	z
Std	0.19	0.20	0.38	0.12	0.07	0.02
Auto	0.94	0.95	0.95	0.91	0.73	0.89
Cross Correlations						
u		−0.89	−0.97	−0.95	0.71	−0.42
v			0.97	0.90	−0.69	0.37
v/u				0.95	−0.72	0.40
u-e					−0.58	0.41
e-u						−0.52

Table 4.2: Quarterly U.S. Labor Market Data

costs like advertising, travel costs for applicants and agency fees this cost rises up to 20% of employee compensation according to an estimate by Saratoga Institute (2004). Therefore, we simulate the model for various different values of κ such that the ratio of κ to average wages vary between 4% to 33%.

In order to have a benchmark to compare, let's first look at the key business cycle features of the U.S. labor market data. Table 4.2 summarizes he variables of interest: *u; unemployment, v; vacancies, v/u; vacancy to unemployment ratio or market tightness, u-e; transition probability from unemployment to employment, e-u; transition probability from employment to unemployment and z; average labor productivity.* It reports standard deviations, one period autocorrelations and contemporaneous cross correlations of the detrended variables in the post WWII period. The details of the data are discussed in Chapter 3 extensively. For brevity, only major facts about this table will be discussed here.

As Table 4.2 indicates, unemployment, vacancies and market tightness are all

MP Model with Constant Separation						
	u	v	v/u	u-e	e-u	z
Std	0.01	0.02	0.03	0.01	0	0.02
Auto	0.85	0.74	0.81	0.81	1	0.81
Cross Correlations						
u		-0.87	-0.94	-0.99	0	-0.94
v			0.99	0.92	0	0.99
v/u				1	0	1
u-e					0	1
e-u						0

Table 4.3: MP Model with Constant Separations

much more variable than the underlying productivity shocks. Since this fact provides the motivation for this chapter, we will focus on the performance of the model on this dimension mainly. Another major fact about the U.S. labor markets is the significantly negative correlation between unemployment and vacancies (Beveridge Curve). The observed high volatility in key variables of labor markets led to a large literature focusing on the inability of standard Mortensen-Pissairdes model in this aspect[8]. This puzzling performance of the standard MP model can be seen from Table 4.3. Table 4.3 presents simulated moments from a calibrated version of the standard MP model. The table is replicated from Chapter 3 and is consistent with Shimer (2005). As with the reported U.S. data, simulated data is detrended by HP filter with smoothing parameter 10^5.

As Table 4.3 clearly indicates, unemployment, vacancies and market tightness

[8]For an excellent survey of this literature, see Mortensen and Nagypal (2005).

Simulations with $\kappa = 0.18$ and $\tilde{\eta} = 0.0563$						
	u	v	v/u	u-e	e-u	z
Std	0.11	0.03	0.10	0.06	0.10	0.02
Auto	0.78	0.64	0.85	0.85	0.72	0.85
Cross Correlations						
u		0.52	-0.97	-0.97	0.98	-0.97
v			-0.33	-0.34	0.64	-0.35
v/u				0.99	-0.93	1
u-e					-0.91	0.99
e-u						-0.93

Table 4.4: Simulations with $\kappa = 0.18$ and $\tilde{\eta} = 0.0563$

show almost as much variability as the average labor productivity in the standard MP model. This implies the lack of a proper amplification mechanism in the standard model. A comparison between these two tables reveals that there is not enough variation in vacancy creation in the standard model over the cycle. The screening technology, which is modeled as a higher mean expected match quality draw has the potential to change the number of posted vacancies by imposing disproportinate changes due to the use of screening technology. In all the simulation results to follow, firms choose to use screening technology (choose initial priors from Ω) always but the highest aggregate productivity level. Since it is costly to draw from Ω, equilibrium market tightness increases dramatically in the highest productivity state, when the cost κ is not incurred. To see this relatively high volatility generated in the model, consider Table 4.4 below.

Table 4.4 reports the results from the simulation of the model presented in this chapter. The cost of screening, κ, is calibrated to be 0.18 which is approximately

4% of the average wage in the model. Since, we look for an equilibrium in which firms only choose to utilize this screening technology when the aggregate state is highest, we calibrate the value of $\tilde{\eta}$ such that it is the lowest value that implies this equilibrium. For instance, in the simulations presented in Table 4.4, a lower $\tilde{\eta}$ will imply not paying the cost κ for all z.

Standard deviations of key aggregate labor market variables clearly imply relatively high volatility with the added features of the model. This implies that even a low screening cost, with a slightly better prospect of better quality match will increase the desired volatility measures to some extent. The major pitfall of the model is about the implied Beveridge Curve relationship. As Table 4.4 indicates, correlation between unemployment and vacancies is implied to be significantly positive. This is a counterfactual result. However, it is not unique to this particular model. As Shimer (2005) and Chapter 3 argue, models of labor market search with endogenous separations through match specific heterogeneity imply a counterfactually positive correlation.

Since the model has more than one additional component relative to standard MP model, it is instructive to disentangle the contribution of learning about match quality from costly screening. To this end, we also solve the model with parameter values $(\kappa, \tilde{\eta}) = (0, 0)$. This will give the extent of variability caused by just learning due to incomplete information about the quality of the match. As Table 4.5 indicates, most of the variation with low κ, is due to the endogenous separations and the learning about match quality feature of the model. However, this is the case when κ is relatively low.

Simulations with $\kappa = 0$ and $\tilde{\eta} = 0$						
	u	v	v/u	u-e	e-u	z
Std	0.09	0.02	0.08	0.06	0.08	0.02
Auto	0.81	0.64	0.85	0.85	0.78	0.85
Cross Correlations						
u		0.79	-0.99	-0.99	0.99	-0.97
v			-0.70	-0.71	0.83	-0.69
v/u				0.99	-0.97	1
u-e					-0.96	0.99
e-u						-0.97

Table 4.5: Simulations without Costly Screening

Variatins in κ and $\tilde{\eta}$				
	Standard Deviations			*Ratio*
(κ,$\tilde{\eta}$)	u	v	v/u	$\kappa/avg(w)$
$(0.4, 0.1023)$	0.103	0.03	0.10	0.09
$(0.6, 0.132)$	0.112	0.05	0.136	0.13
$(0.8, 0.155)$	0.116	0.065	0.15	0.18
$(1.5, 0.208)$	0.15	0.11	0.21	0.33

Table 4.6: Variations in Screening Technology Parameters

In order to shed some more light on the performance of the model in the volatility dimension, we report the results of the simulations for four more values of κ. In each case, the corresponding $\tilde{\eta}$ value is determined such that it is the lowest value that implies an equilibrium in which firms choose to screen intensively in all but highest aggregate productivity state.

Table 4.6 supports the claim that volatility of labor market variables increase with costly screening technology in an environment with match specific heterogeneity.

The extent of variation is increasing in the level of screening costs, κ. However, in order to explain all of the variation, the cost of drawing from a higher mean distribution should be unreasonably high relative to average wage.

4.4 Conclusion

The Mortensen-Pissarides labor market search model has been recently criticized because of the model's quantitative implications for business cycles. In particular, researchers have pointed out the discrepancy between the implied level of variation in unemployment, vacancies and market tightness and the observed variation in these variables in the United States. This chapter extends the baseline labor market search model of Mortensen and Pisasrides (1994) to include costly screening and match specific heterogeneity to generate the missing amplification. The mechanism works through the endogenous choice of costly screening when the average labor productivity is low enough. Firms choose to invest in costly screening when the opportunity cost of not producing is low enough. It happens during aggregate state in which firms want to hire cautiously and create less vacancies. Hence, employers become pickier during recessions. Countercyclical changes in screening investment improves the cyclical behavior of unemployment, vacancies and market tightness. The result holds even stronger if the cost of screening increases relative to the average wage. One major drawback of the model is that it implies counterfactually positive correlations between unemployment and vacancies.

Further analysis of recruitment behavior seems to be a promising avenue of research to understand the business cycle features of key labor market variables.

This chapter was a first step to understand how the intensity of employer search might change over the cycle. Understanding the costs associated with screening and recruitment will further enlighten our thinking about labor market policies.

Appendices

Appendix A

Data and Estimation in Chapter 2

A.1 The Data

Unemployment, U, is the unemployment rate (unemployed persons per member of the labor force) constructed as a quarterly average of the seasonally adjusted monthly series from the Current Population Survey (CPS) of the Bureau of Labor Statistics (BLS); series downloaded from the CPS home page *http://www.bls.gov/cps*.

Employment, N, is computed as the identity $N = 1 - U$.

The vacancies series, V, represents vacancies per member of the labor force and is constructed by multiplying two seasonally adjusted monthly series – the ratio of help-wanted advertising to unemployed compiled by the Conference Board (downloaded as variable LHELX from the DRI Basic database), and the unemployment rate U (defined above) – and averaging the monthly values to obtain the quarterly series. The commonly reported help-wanted advertising index is a scalar transformation of this series.

Consumption, C, is proxied by aggregate output per member of the labor force. The output variable is real gross domestic product (billions of chained 2000 dollars, seasonally adjusted annual rate) downloaded from the Federal Reserve Bank of St. Louis FRED II database at *http://research.stlouisfed.org/fred2/series/GDPC1.*

We divide this series by the seasonally-adjusted civilian labor force (averaged from monthly to quarterly), appropriately scaled, to express the variable in year 2000 chained dollars per person. The civilian labor force measure is constructed by the Bureau of Labor Statistics (BLS) as part of the Current Population Survey (CPS) and is downloaded from *http://www.bls.gov/cps.*

A.2 Estimation of VAR(1) Shock Process

As explained in section 4, solving the model to ultimately recover the histories of the exogenous forcing processes, requires that we complement the log-linearized versions of the efficiency conditions (2.6)–(2.8) with the VAR(1) system (2.12) summarizing the probability characteristics of the shocks:

$$\widetilde{e}_{t+1} = A\widetilde{e}_t + \varepsilon_{t+1}$$

where $\widetilde{e}_t = (z_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)'$, A is a 3×3 matrix of constants, and $\varepsilon_t = (\varepsilon_{zt}, \varepsilon_{\widetilde{\chi}t}, \varepsilon_{\widetilde{\sigma}t})'$ is trivariate normal with $\mathrm{E}\varepsilon_t = 0$ and $\mathrm{E}[\varepsilon_t \varepsilon_t'] = \Sigma$. Given that we have no information, *a priori*, regarding the 15 distributional parameters comprising the vector θ – the 9 parameters of the VAR(1) coefficient matrix A and the 6 independent parameters of the variance-covariance matrix Σ – we determine them using a simulated method of moments procedure. Thus, the vector θ is chosen to minimize the objective function

$$Q(\theta) = [\mathbf{m} - \mathbf{m}(\theta)]' [\mathbf{m} - \mathbf{m}(\theta)]$$

where $\mathbf{m}(\theta)$ is a 15×1 vector of simulated theoretical moments that are implied by the model, and $\mathbf{m}$ is the corresponding 15×1 vector of data moments. The data moments are simple sample averages:

$$\mathbf{m} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{m}_t, \quad i = 1, \ldots, 15,$$

where $T = 224$ is the number of time series observations in our sample of observed exogenous variables.

Given technology and preference parameters, the estimation procedure is initiated with three additional pieces of information: 1) an initial guess of the unknown parameters θ_0, 2) a $3 \times nT$ draw from a univariate standard normal distribution forming the random matrix $\mathbf{u}'$, where $T = 224$, and $n = 10$, and 3) the initial state-vector $(n_1, \widetilde{e}_1)$. By fixing the 9 coefficients of matrix A, the initial guess θ_0 determines the exact log-linear decision rules (2.13); it also determines the 6 independent values of the variance-covariance matrix Σ. Given $\mathbf{u}$ and Σ, we generate the $3 \times nT$ random matrix ε', a draw from the trivariate normal distribution $N(\mathbf{0}, \Sigma)$, as follows. Let R be the Cholesky decomposition of Σ so that $R'R = \Sigma$ where R is an upper triangular 3×3 matrix. The random draw of innovations from the trivariate normal density is then constructed as $\varepsilon = \mathbf{u}R$, where ε is the realization $\varepsilon_t' = (\varepsilon_{zt}, \varepsilon_{\widetilde{\chi}t}, \varepsilon_{\widetilde{\sigma}t})_{t=1}^{nT}$.

Next, we create a model simulation nT periods in length beginning from an initial state equal to the steady state: $(n_0, \widetilde{e}_0) = (0, \mathbf{0})$. Given the initial draw of innovations $\{\varepsilon_t\}_{t=1}^{nT}$, we solve (2.13) forward producing a simulation nT periods in length, discarding the first 10 percent of simulated data observations to remove potential bias due to initial conditions. The simulated theoretical moments are then computed using the remaining $T^* = (0.9)(nT) = 2016$ simulated data observations:

$$\mathbf{m}(\theta) = \frac{1}{T}\sum_{i=1}^{T^*} \mathbf{m}_i(\theta), \quad i = 1, \ldots, 15.$$

Given these simulated moments, we locate the parameter vector θ that minimizes the expression $Q(\theta)$. The initial θ is subsequently replaced with $\theta = \arg\min Q(\theta)$, and the next iteration begins. The algorithm continues in this fashion until $Q(\theta) < \delta$, where δ is sufficiently close to a machine zero to deem the most recent computed θ the solution. The estimation requires that the same set of innovations be used at each function evaluation ensuring that any reduction in $Q(\theta)$ is due to a better θ, and not to a different set of innovations. This clearly poses a challenge to identify the true vector of structural parameters that minimize $Q(\theta)$ in our problem because Σ is the parameter that determines the process which generates the random innovations necessary for the simulations. This problem is solved by exploiting the fact that a joint normal distribution of three random variables could be generated from standard normal distribution given the parameters defining means and variance of the joint process. Hence, at each function evaluation we use the same $3 \times T$ random sample, ε, from a standard normal distribution. This implies that the set of innovations (z, χ and σ) are generated using this ε and the updated Σ at each step.

In practice, the minimization problem includes two additional constraints. First, for the exogenous shock process to be stationary, the VAR(1) coefficient matrix A needs to possess eigenvalues that lie within the unit circle. Second, the variance-covariance matrix Σ is naturally positive semi-definite. Since it is impossible to impose these two constraints explicitly in the minimization procedure, our algorithm assigns an arbitrarily large number (10^{30}) to the value of the objective function whenever either of these two constraints is violated. Given an initial guess for θ, this ensures that the algorithm attains a minimum that satisfies these restrictions.

We determine the moments to be matched with three considerations in mind. First, and most obviously, the distributional parameters that we estimate do not affect the first moments of the endogenous variables implied by the non-stochastic steady state. Hence, we choose dynamic covariances between endogenous variables. Second, given that the dynamic behavior of employment is central to our analysis, the second moments relating employment to aggregate output at various leads and lags are included. Finally, it is straightforward to choose moments that show considerable variation with the parameter vector θ. Given that θ primarily determines the dynamic behavior of the exogenous forcing variables z, χ, and σ, second and higher order moments of these will generate the most substantial variation. Unfortunately, these are not observed. The model property allowing labor productivity to be inferred directly from the social planner's feasibility constraint ameliorates this problem somewhat, and thus we apply a considerable number of moments that relate to z. With these considerations in mind, we match the following 15 moments: $\text{cov}(n_t,z_{t-2})$, $\text{cov}(n_t,z_{t-1})$, $\text{cov}(n_t,z_t)$, $\text{cov}(n_t,z_{t+1})$, $\text{cov}(n_t,z_{t+2})$, $\text{cov}(n_t,c_{t-2})$, $\text{cov}(n_t,c_{t-1})$, $\text{cov}(n_t,c_t)$, $\text{cov}(n_t, c_{t+1})$, $\text{cov}(n_t,c_{t+2})$, $\text{cov}(z_t,z_{t-2})$, $\text{cov}(z_t,z_{t-1})$, $\text{var}(z_t)$, $\text{cov}(z_t,z_{t+1})$, and $\text{cov}(z_t,z_{t+2})$.

Appendix B

Standard Mortensen Pissarides Model and Derivation of Equilibrium Conditions in Chapter 3

B.1 Standard Mortensen-Pissarides Model

In order to facilitate comparison the notation for describing the standard Mortensen-Pissarides model will be similar to the notation used in the model presented in section 4, whenever possible.

There is a continuum of risk neutral workers and employers who discount the future at the rate $\beta \in (0, 1)$. The measure of workers is normalized to 1 and they are either unemployed looking for a job or employed and producing $z_t > 0$ each period. All matches are identical. Match output, z_t, is stochastic and governed by a Markov process, $\Psi(z_{t+1}|z_t)$. There is no on-the-job search. Hence, only unemployed workers are searching for a job. Active firms could be either producing by employing a worker, or waiting for a possible match after posting a vacancy. There is free entry of firms which guarantees that as long as the value of posting a vacancy is positive, there will be active firms posting vacancies. Posting a vacacny costs $c > 0$ per period and enables vacant firms to meet an unemployed worker through a matching function, $M(v_t, u_t)$, where v_t is the aggregate number of vacancies and u_t is the aggregate number of unemployed workers. $M(v, u)$ is constant returns to scale, which implies the following probabilities of finding a job (for unemployed workers) and filling a vacancy (for vacant positions).

$$f(\theta_t) = M(v_t, u_t)/u_t \;\text{ and }\; h(\theta_t) = M(v_t, u_t)/v_t \text{ where } M(v_t, u_t) = M u_t^{1-\eta} v_t^{\eta}$$

Here $\theta_t = v_t/u_t$ is usually referred to as market tightness. When unemployed, workers consume $b > 0$. Continuing matches are subject to exogenous destruction with probability λ each period. Wage, $w(z_t)$, is determined each period via Nash bargaining between worker and firm taking the threat points as value of unemployment and value of being a vacant job respectively. Let the value of being unemployed be $V_u(z_t)$ and the value of a vacancy be $J_u(z_t)$. Similarly, the value of being employed for a worker and the value of being in a match for a firm are denoted by $V_e(z)$ and $J_e(z)$ respectively. These value functions are summarized in four Bellman equations:

$$V_u(z) = b + \beta E_{z'|z} \left\{ f(\theta(z)) V_e(z') + (1 - f(\theta(z))) V_u(z') \right\} \tag{B.1}$$

$$V_e(z) = w(z) + \beta E_{z'|z} \left\{ (1-\lambda) V_e(z') + \lambda V_u(z') \right\} \tag{B.2}$$

$$J_u(z) = -c + \beta E_{z'|z} \left\{ h(\theta(z)) J_e(z') + (1 - h(\theta(z))) J_u(z') \right\} \tag{B.3}$$

$$J_e(z) = z - w(z) + \beta E_{z'|z} \left\{ (1-\lambda) J_e(z') + \lambda J_u(z') \right\} \tag{B.4}$$

where time subscripts are dropped for convenience.

Consequently, the equilibrium of this economy satisfies the following conditions:

1. (Optimization) Given, $\theta(z)$ and $w(z)$, value functions $V_u(z)$, $V_e(z)$, $J_u(z)$ and $J_e(z)$ satisfy (B.1) - (B.4).

2. (Free entry) Given $\theta(z)$ and $w(z)$ a firm is willing to post a vacany as long as $J_u(z) > 0$. Therefore in equilibrium, due to free entry $J_u(z) = 0$.

3. (Nash Bargaining) $V_e(z) - V_u(z) = \phi\left[V_e(z) - V_u(z) + J_e(z) - J_u(z)\right]$, where $\phi \in [0,1]$ is the worker's bargaining strength.

4. (Equation of motion for unemployment) Given an initial unemployment u_0 decisions should be consistent with the evolution of unemployment.

$$u' = (1-u)\lambda + u(1 - f(\theta)) \tag{B.5}$$

The simulations of the standard Mortensen-Pissarides model in Chapter 3 use the following calibration.

B.2 Surplus Function and Equilibrium Value of Vacancy

First, write down the values of $J_e(\gamma, z, n) - J_u(z, n)$ and $V_e(\gamma, z, n) - V_u(z, n)$ by subtracting (4.4) and (4.7) from (4.6) and (4.8) respectively.

Calibration for MP Model		
Parameter	Value	
β	0.9967	4% interest
ϕ	0.36	Shimer (2005)
η	0.36	Shimer (2005)
y^l	1	Normalization
z^h	1.0259	U.S. Avg.
z^l	0.9748	Output p/w
z^{ss}	1	Normalization
λ	0.0339	Shimer (2005)
b	0.4	Shimer (2005)
c	0.24	Match $u^{ss} = 0.0568$
M	0.35	Match $f(\theta^{ss}) = 0.45$

Table B.1: Calibration for the Standard Mortensen-Pissarides Model

$$V_e(\gamma, z, n) - V_u(z, n) =$$

$$\max \left\{ \begin{array}{c} w(\gamma, z, \mu) \\ +\beta(1-\lambda)(1-f(v))E_{z'|z} \left[\begin{array}{c} \alpha(\gamma)\left(V_e(\gamma^h, z', \mu') - V_u(z', \mu')\right) \\ +(1-\alpha(\gamma))\left(V_e(\gamma^l, z', \mu') - V_u(z', \mu')\right) \end{array} \right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z} \left[\begin{array}{c} \Gamma(\gamma^h)\left(V_e(\gamma^h, z', \mu') - V_u(z', \mu')\right) \\ +\int_{\gamma^h} \left(V_e(\gamma', z', \mu') - V_u(z', \mu')\right) d\Gamma(\gamma') \end{array} \right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z} \left[\begin{array}{c} \Gamma(\gamma^l)\left(V_e(\gamma^l, z', \mu') - V_u(z', \mu')\right) \\ +\int_{\gamma^l} \left(V_e(\gamma', z', \mu') - V_u(z', \mu')\right) d\Gamma(\gamma') \end{array} \right] \\ -V_u(z, \mu) + \beta\lambda E_{z'|z} V_u(z', \mu') + \beta(1-\lambda)(1-f(v))E_{z'|z} V_u(z', \mu') \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z} V_u(z', \mu') \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z} V_u(z', \mu'), 0 \end{array} \right\} \tag{B.6}$$

$$J_e(\gamma,z,n)-J_u(z,n)=$$

$$\max\left\{\begin{array}{c} z(\alpha(\gamma)y^h+(1-\alpha(\gamma))y^l)-w(\gamma,z,n) \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\begin{array}{c}\alpha(\gamma)\left(J_e(\gamma^h,z',n')-J_u(z',n')\right)\\ +(1-\alpha(\gamma))\left(J_e(\gamma^l,z',n')-J_u(z',n')\right)\end{array}\right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^h)\left(J_e(\gamma^h,z',n')-J_u(z',n')\right)\\ +\left(1-\Gamma(\gamma^h)\right)J_u(z',n')\end{array}\right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^l)\left(J_e(\gamma^l,z',n')-J_u(z',n')\right)\\ +\left(1-\Gamma(\gamma^l)\right)J_u(z',n')\end{array}\right]-J_u(z,n) \\ +\beta\lambda E_{z'|z}J_u(z',n')+\beta(1-\lambda)(1-f(v))E_{z'|z}J_u(z',n') \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\Gamma(\gamma^h)J_u(z',n') \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\Gamma(\gamma^l)J_u(z',n'),0 \end{array}\right\} \tag{B.7}$$

Imposing the free entry condition, $J_u(z,n)=0\ \forall z,n$, and some simplification, yield the following two equations.

$$V_e(\gamma,z,n)-V_u(z,n)=$$

$$\max\left\{\begin{array}{c} w(\gamma,z,n) \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\begin{array}{c}\alpha(\gamma)\left(V_e(\gamma^h,z',n')-V_u(z',n')\right)\\ +(1-\alpha(\gamma))\left(V_e(\gamma^l,z',n')-V_u(z',n')\right)\end{array}\right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^h)\left(V_e(\gamma^h,z',n')-V_u(z',n')\right)\\ +\int_{\gamma^h}\left(V_e(\gamma',z',n')-V_u(z',n')\right)d\Gamma(\gamma')\end{array}\right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^l)\left(V_e(\gamma^l,z',n')-V_u(z',n')\right)\\ +\int_{\gamma^l}\left(V_e(\gamma',z',n')-V_u(z',n')\right)d\Gamma(\gamma')\end{array}\right] \\ -V_u(z,n)+\beta E_{z'|z}V_u(z',n'),0 \end{array}\right\} \tag{B.8}$$

$$J_e(\gamma,z,n) - J_u(z,n) =$$
$$\max\left\{\begin{array}{c} z(\alpha(\gamma)y^h + (1-\alpha(\gamma))y^l) - w(\gamma,z,n) \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\begin{array}{c}\alpha(\gamma)\left(J_e(\gamma^h,z',n') - J_u(z',n')\right) \\ +(1-\alpha(\gamma))\left(J_e(\gamma^l,z',n') - J_u(z',n')\right)\end{array}\right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^h)\left(J_e(\gamma^h,z',n') - J_u(z',n')\right) \\ +\left(1-\Gamma(\gamma^h)\right)J_u(z',n')\end{array}\right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^l)\left(J_e(\gamma^l,z',n') - J_u(z',n')\right) \\ +\left(1-\Gamma(\gamma^l)\right)J_u(z',n')\end{array}\right], 0\end{array}\right\} \tag{B.9}$$

Let $S(\gamma,z,n) = J_e(\gamma,z,n) - J_u(z,n) + V_e(\gamma,z,n) - V_u(z,n)$ denote the match surplus. Adding (B.8) and (B.9) provides the following expression for the match surplus.

$$S(\gamma,z,n) = \max\left\{\begin{array}{c} z(\alpha(\gamma)y^h + (1-\alpha(\gamma))y^l) \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\alpha(\gamma)S(\gamma^h,z',n') + (1-\alpha(\gamma))S(\gamma^l,z',n')\right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^h)S(\gamma^h,z',n') \\ +\int_{\gamma^h}\left(V_e(\gamma',z',n') - V_u(z',n')\right)d\Gamma(\gamma')\end{array}\right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\begin{array}{c}\Gamma(\gamma^l)S(\gamma^l,z',n') \\ +\int_{\gamma^l}\left(V_e(\gamma',z',n') - V_u(z',n')\right)d\Gamma(\gamma')\end{array}\right] \\ -V_u(z,n) + \beta E_{z'|z}V_u(z',n'), 0\end{array}\right\} \tag{B.10}$$

The surplus sharing rule implies that $V_e(\gamma,z,n) - V_u(z,n) = \phi S(\gamma,z,n)$ for all γ,z,n. This leads to following equalities:

$$\int_{\gamma^h}\left(V_e(\gamma',z',n') - V_u(z',n')\right)d\Gamma(\gamma') = \phi\int_{\gamma^h}S(\gamma',z',n')d\Gamma(\gamma')$$
$$\int_{\gamma^l}\left(V_e(\gamma',z',n') - V_u(z',n')\right)d\Gamma(\gamma') = \phi\int_{\gamma^l}S(\gamma',z',n')d\Gamma(\gamma')$$

Then the surplus function reduces to

$$S(\gamma, z, n) = \max \left\{ \begin{array}{c} z(\alpha(\gamma)y^h + (1-\alpha(\gamma))y^l) \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\alpha(\gamma)S(\gamma^h, z', n') + (1-\alpha(\gamma))S(\gamma^l, z', n')\right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\Gamma(\gamma^h)S(\gamma^h, z', n') + \phi\int_{\gamma^h} S(\gamma', z', n')d\Gamma(\gamma')\right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\Gamma(\gamma^l)S(\gamma^l, z', n') + \phi\int_{\gamma^l} S(\gamma', z', n')d\Gamma(\gamma')\right] \\ -V_u(z,n) + \beta E_{z'|z}V_u(z', n'), 0 \end{array} \right\} \tag{B.11}$$

Now I need to use (4.7) to pin down the value of $-V_u(z,n) + \beta E_{z'|z}V_u(z', n')$ in terms of surplus function. It is possible by substituting $V_u(z,n) + \phi S(\gamma, z, n)$ for $V_e(\gamma, z, n)$, using the surplus sharing rule.

$$V_u(z,n) = b + \beta E_{z'|z}\left\{ f(v)\int\left[V_u(z', n') + \phi S(\gamma', z', n')\right]d\Gamma(\gamma') + (1-f(v))\,V_u(z', n')\right\} \tag{B.12}$$

And further simplification of (C.6) yields the desired expression.

$$V_u(z,n) - \beta E_{z'|z}V_u(z', n') = b + \beta E_{z'|z}f(v)\phi\int S(\gamma', z', n')d\Gamma(\gamma') \tag{B.13}$$

Substituting (C.7) in (B.11) reduces (B.11) to a recursive functional equation in $S(\gamma, z, n)$.

$$S(\gamma, z, n) = \max \left\{ \begin{array}{c} z(\alpha(\gamma)y^h + (1-\alpha(\gamma))y^l) - b \\ +\beta(1-\lambda)(1-f(v))E_{z'|z}\left[\alpha(\gamma)S(\gamma^h, z', n') + (1-\alpha(\gamma))S(\gamma^l, z', n')\right] \\ +\beta(1-\lambda)f(v)\alpha(\gamma)E_{z'|z}\left[\Gamma(\gamma^h)S(\gamma^h, z', n') + \phi\int_{\gamma^h} S(\gamma', z', n')d\Gamma(\gamma')\right] \\ +\beta(1-\lambda)f(v)(1-\alpha(\gamma))E_{z'|z}\left[\Gamma(\gamma^l)S(\gamma^l, z', n') + \phi\int_{\gamma^l} S(\gamma', z', n')d\Gamma(\gamma')\right] \\ -\beta E_{z'|z}f(v)\phi\int S(\gamma', z', n')d\Gamma(\gamma'), 0 \end{array} \right\} \tag{B.14}$$

On the other hand, one can write down the equilibrium value of vacancy as a function of the surplus function. This follows from the definition of (4.4) and surplus sharing rule, $J_e(\gamma, z, n) - J_u(z, n) = (1-\phi)\, S(\gamma, z, n)$ for all γ, z, n. Using these two conditions I arrive at the following condition expressed in the text as :

$$J_u(z) = -c + \beta E_{z'|z}J_u(z') + \beta h(\theta)\,(1-\phi)\, E_{z'|z}\int S(\gamma', z', n')d\Gamma(\gamma') \tag{B.15}$$

Appendix C

Derivation of Equilibrium Conditions in Chapter 4

First, write down the values of $J_e(\gamma, n) - J_u(z)$ and $V_e(\gamma, z) - V_u(z)$ by subtracting (4.4) and (4.7) from (4.6) and (4.8) respectively.

$$V_e(\gamma, z) - V_u(z) = \max \left\{ \begin{array}{c} w(\gamma, z) \\ +\beta(1-\lambda)E_{z'|z} \left[\begin{array}{c} \alpha(\gamma)\left(V_e(\gamma^h, z') - V_u(z')\right) \\ +(1-\alpha(\gamma))\left(V_e(\gamma^l, z') - V_u(z')\right) \end{array} \right] + \beta\lambda E_{z'|z} V_u(z') \\ +\beta(1-\lambda)E_{z'|z} V_u(z') - V_u(z), 0 \end{array} \right\} \tag{C.1}$$

$$J_e(\gamma, z) - J_u(z) = \max \left\{ \begin{array}{c} z(\alpha(\gamma)y^h + (1-\alpha(\gamma))y^l) - w(\gamma, z) \\ +\beta(1-\lambda)E_{z'|z} \left[\begin{array}{c} \alpha(\gamma)\left(J_e(\gamma^h, z') - J_u(z')\right) \\ +(1-\alpha(\gamma))\left(J_e(\gamma^l, z') - J_u(z')\right) \end{array} \right] \\ +\beta\lambda E_{z'|z} J_u(z') \\ +\beta(1-\lambda)E_{z'|z} J_u(z') - J_u(z), 0 \end{array} \right\} \tag{C.2}$$

Imposing the free entry condition, $J_u(z) = 0 \; \forall z$ and some simplification, yield the following two equations.

$$V_e(\gamma,z)-V_u(z)=\max\left\{\begin{array}{c} w(\gamma,z) \\ +\beta(1-\lambda)E_{z'|z}\left[\begin{array}{c}\alpha(\gamma)\left(V_e(\gamma^h,z')-V_u(z')\right)\\ +(1-\alpha(\gamma))\left(V_e(\gamma^l,z')-V_u(z')\right)\end{array}\right] \\ +\beta E_{z'|z}V_u(z')-V_u(z),0\end{array}\right\} \tag{C.3}$$

$$J_e(\gamma,z)-J_u(z)=\max\left\{\begin{array}{c} z(\alpha(\gamma)y^h+(1-\alpha(\gamma))y^l)-w(\gamma,z) \\ +\beta(1-\lambda)E_{z'|z}\left[\begin{array}{c}\alpha(\gamma)\left(J_e(\gamma^h,z')-J_u(z')\right)\\ +(1-\alpha(\gamma))\left(J_e(\gamma^l,z')-J_u(z')\right)\end{array}\right],0\end{array}\right\} \tag{C.4}$$

Let $S(\gamma,z)=J_e(\gamma,z)-J_u(z)+V_e(\gamma,z)-V_u(z)$ denote the match surplus. Adding (C.3) and (C.4) provides the following expression for the match surplus.

$$S(\gamma,z)=\max\left\{\begin{array}{c} z(\alpha(\gamma)y^h+(1-\alpha(\gamma))y^l) \\ +\beta(1-\lambda)E_{z'|z}\left[\alpha(\gamma)S(\gamma^h,z)+(1-\alpha(\gamma))S(\gamma^l,z)\right] \\ +\beta E_{z'|z}V_u(z')-V_u(z),0\end{array}\right\} \tag{C.5}$$

The surplus sharing rule implied by the Nash bargaining problem leads to $V_e(\gamma,z)-V_u(z)=\phi S(\gamma,z)$ for all γ,z. This leads to following equalities:

$$\int_{\gamma^h}\left(V_e(\gamma',z')-V_u(z')\right)d\Gamma(\gamma') = \phi\int_{\gamma^h}S(\gamma',z')d\Gamma(\gamma')$$

$$\int_{\gamma^l}\left(V_e(\gamma',z')-V_u(z')\right)d\Gamma(\gamma') = \phi\int_{\gamma^l}S(\gamma',z')d\Gamma(\gamma')$$

$$\int_{\gamma^h}\left(V_e(\gamma',z')-V_u(z')\right)d\Omega(\gamma') = \phi\int_{\gamma^h}S(\gamma',z')d\Omega(\gamma')$$

$$\int_{\gamma^l}\left(V_e(\gamma',z')-V_u(z')\right)d\Omega(\gamma') = \phi\int_{\gamma^l}S(\gamma',z')d\Omega(\gamma')$$

Now I need to use (4.7) to pin down the value of $-V_u(z) + \beta E_{z'|z} V_u(z')$ in terms of surplus function. It is possible by substituting $V_u(z)+\phi S(\gamma, z)$ for $V_e(\gamma, z)$, using the surplus sharing rule.

$$V_u(z) = b + \beta E_{z'|z} \left\{ f(\theta) \left[\begin{array}{c} \mu_s(z) \int (V_u(z') + \phi S(\gamma', z'))\, d\Omega(\gamma') \\ + (1-\mu_s(z)) \int (V_u(z') + \phi S(\gamma', z'))\, d\Gamma(\gamma') \end{array} \right] + (1 - f(\theta))\, V_u(z') \right\} \tag{C.6}$$

And further simplification of (C.6) yields the desired expression.

$$V_u(z) - \beta E_{z'|z} V_u(z') = b + \beta E_{z'|z} f(\theta) \phi \left[\mu_s(z) \int S(\gamma', z') d\Omega(\gamma') + (1-\mu_s(z)) \int S(\gamma', z') d\Gamma(\gamma') \right] \tag{C.7}$$

Substituting (C.7) in (C.5) reduces (C.5) to a recursive functional equation in $S(\gamma, z)$.

$$S(\gamma, z) = \max \left\{ \begin{array}{c} z(\alpha(\gamma) y^h + (1-\alpha(\gamma)) y^l) - b \\ +\beta(1-\lambda)(1-f(v)) E_{z'|z} \left[\alpha(\gamma) S(\gamma^h, z', n') + (1-\alpha(\gamma)) S(\gamma^l, z', n') \right] \\ -\beta E_{z'|z} f(\theta) \phi \left[\mu_s(z) \int S(\gamma', z') d\Omega(\gamma') + (1-\mu_s(z)) \int S(\gamma', z') d\Gamma(\gamma') \right], 0 \end{array} \right\} \tag{C.8}$$

On the other hand, one can write down the equilibrium value of vacancy as a function of the surplus function. This follows from the definition of (4.4) and surplus sharing rule, $J_e(\gamma, z) - J_u(z) = (1-\phi)\, S(\gamma, z)$ for all γ, z. Using these two conditions I arrive at the following condition:

$$J_u(z,n) = -c + \beta E_{z'|z} J_u(z') + \beta h(\theta)(1-\phi) E_{z'|z} \left[\begin{array}{c} (1-\mu_s(z)) \int S(\gamma', z') d\Gamma(\gamma') \\ +\mu_s(z) \left(-\kappa + \int S(\gamma', z') d\Omega(\gamma')\right) \end{array} \right] \tag{C.9}$$

Finally, (4.12) can be easily obtained by solving the maximization problem in (4.4).

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Vita

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