Properties of White Dwarfs

Teacher Guide

Introduction

Astronomers determine the properties of white dwarfs based on observations using a telescope and light sensitive instruments. Students will follow many of the same steps astronomers do to find the basic properties of a white dwarf. During their investigation, students will draw on mathematics skills (algebra) and geometry concepts.

Materials

For each group of two students	Classroom equipment
1/4 inch grid graph paper Thick cardboard sheet or foam core as large as the graph paper 3 push pins 2 lengths of cotton string/twine (1 mm thick) 1 Ruler 1 Calculator Pencil 2 Student guides(one for each student) *Optional: graphing calculator or Microsoft Excel*	Straw and candle (birthday cake candle) Computer running PowerPoint Video projector and screen to project the PowerPoint; or a set of computers running PowerPoint for student groups to use.

Objectives

1. Make a model of the Sirius binary system.
2. Interpret the observed motion of Sirius A as a gravitational interaction with a white dwarf, Sirius B.
3. Calculate the size, mass, and density of Sirius B using astronomical data.
4. Compare the properties of Sirius B, a white dwarf, to our Sun.

NSES

Grades 9 – 12 Physical Science: motions and forces; structure and properties of matter; interactions of energy and matter.

TEKS

IPC	Physics	Astronomy
(2) Scientific Inquiry: B, C, and D **(3) Critical Thinking & Problem Solving:** E **(4) Force and Motion**: B **(6) Energy transformations:** A	**(2) Scientific Inquiry:** B, C, D, and E **(3) Critical Thinking & Problem Solving:** E **(4) Motion**: B **(5) Conservation of Energy and Momentum**: D. **(6) Forces:** A	**(2) Scientific Inquiry:** B, C, and D

Preparation (30 minutes)

Straw balance:
> Slide a small cake candle into one end of a straw to conceal it from view.
> Practice balancing the straw on your finger. Use both hands with outstretched index fingers to support the straw, and then slowly draw your fingers inward. The balance point will be toward the heavy end of the straw.

PowerPoint of Sirius observations:
> Prepare your computer and video projector to present the "SiriusA.ppt" and "SiriusA+B.ppt" PowerPoints. Preview the two PowerPoint files to familiarize yourself with the content and timing. If you prefer, you may print the slides onto overhead transparency film to display on an overhead projector.

Binary star model:
> Student will draw two ellipses on graph paper using cotton string loops, push pins, and a pencil. For each group, cut two 40 cm lengths of cotton string. Let each student group make the loops (5 to 10 minutes).

Sirius B loop: 33 centimeters	Sirius A loop: 15 centimeters
1. On a piece of cardboard, press a push pin into the cardboard.	1. On a piece of cardboard, press a push pin into the cardboard.
2. Measure **16.5 centimeters** away from the push pin, and mark the spot.	2. Measure **7.6 centimeters** away from the push pin, and mark the spot.
3. Insert another push pin into that spot.	3. Insert another push pin into that spot.
4. Make a **tight** loop around the push pins, and tie off the string to complete the loop.	4. Make a **tight** loop around the push pins, and tie off the string to complete the loop.
5. Check that the total length of your loop is 33 centimeters.	5. Check that the total length of your loop is 15 centimeters

Making the Sirius B loop

16.5 cm
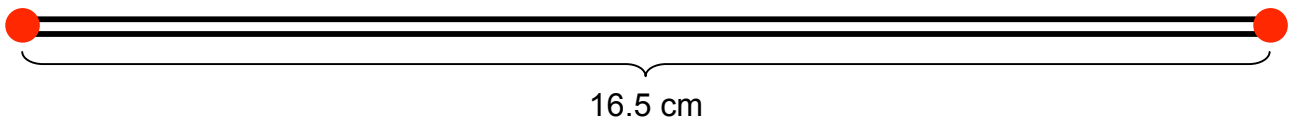
Making the Sirius A loop

7.6 cm
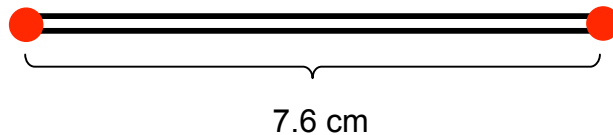
Activity Time Table

Engage	15 minutes
Explore: Part I	45 minutes
Explore: Part II	20 minutes
Explain	20 minutes
Elaborate	15 minutes
Total	**115 minutes (2 hours)**

Engage (15 minutes)

Balanced Straw

Hold up the "loaded" straw for the whole class to see. Ask them where they think the balance point is on the straw. Use both hands with outstretched index fingers to support the straw, and then slowly draw your fingers inward. The balance point will be toward the heavy end of the straw.

Once you have the straw balanced on one finger, hold it up and ask how this could happen? Accept all answers.

Sirius Plotting

1. Show the first frame to display a star map of stars in the constellations Canis Major and Orion. Tell students a little about Sirius and where it is:

★ Point out Sirius, <u>the brightest *visible-light* star</u> in the sky <u>(besides our Sun).</u>
★ The stars grouped into Canis Major and Orion are best viewed during winter months in the Northern Hemisphere.
★ Sirius is also <u>a nearby neighbor: only 9 light-years away</u>. Our solar system is only a few *light-hours* in diameter. The Sun is about 8 *light-minutes* away.
★ Sirius's photosphere (the layer we see) is a little bit hotter than our Sun: 9,910 Kelvin vs. 5,770 Kelvin for our Sun.
★ The mass of a star determines its luminosity (power), photosphere temperature, radius, and life cycle. Sirius is about 2.3 times the mass of the Sun (2.3 solar masses), or 4.6 x 10^{30} kilograms.

2. Go to the next frame. The PowerPoint shows the motion of Sirius over the course of 100 years in five year "snapshots".

★ Display the snapshot.
★ <u>Students plot the position of Sirius on their graph.</u>
★ <u>Students label the position with the observation date.</u>
★ Go to the next snapshot
★ Stop when the graph is full (1955)

The last frame shows a close up of Sirius's motion.

3. When students complete their plots, guide them through answering the following questions:

A. What kind of motion does Sirius show? Justify your answer.

Sirius shows a periodic motion over a long time. It follows a pattern.

Some background: Sirius is a nearby star – only 9 light-years away. All stars in our Galaxy orbit the Galactic barycenter, so they are all moving. Astronomers detect and track the motion of stars relative to our Sun. There are two motions that can be measured: radial velocity (in the line of sight) measured through spectroscopy and tangential velocity (perpendicular to the line of sight). Tangential velocity is obtained from knowing the star's distance and "proper motion" (motion across the sky versus relatively stationary backgroud stars) that is measured in angular units. The motion of Sirius in these plots is from changes in proper motion over time. So, as Sirius orbits the Galaxy, it also appears to orbit something else every 50 years. For instance, the Moon and the Earth orbit the barycenter of the Solar System (very close to the Sun), and they orbit the Earth-Moon barycenter (inside the Earth).

B. What variables can you identify to describe this motion?

Period and amplitude. They may see acceleration, since Sirius appears to slow down and speed up at the same places in its periodic motion. If they identify the period, ask students to measure it as precisely as their data allows. Its period is 50 years.

C. What do you think is causing Sirius's motion?

Sirius has an unseen companion – Sirius B. Together, these two masses form a system with a center of mass or barycenter. Each member orbits the barycenter with the same period. Sirius B is the white dwarf that was too dim for Bessel to see back in the early 19th century. But by the motion of Sirius A, he suggested the existence of Sirius B. Let students work with Newton's laws of force and motion in order to explain how they think a star like Sirius could be tugged around in a periodic motion.

Motion through the galaxy →
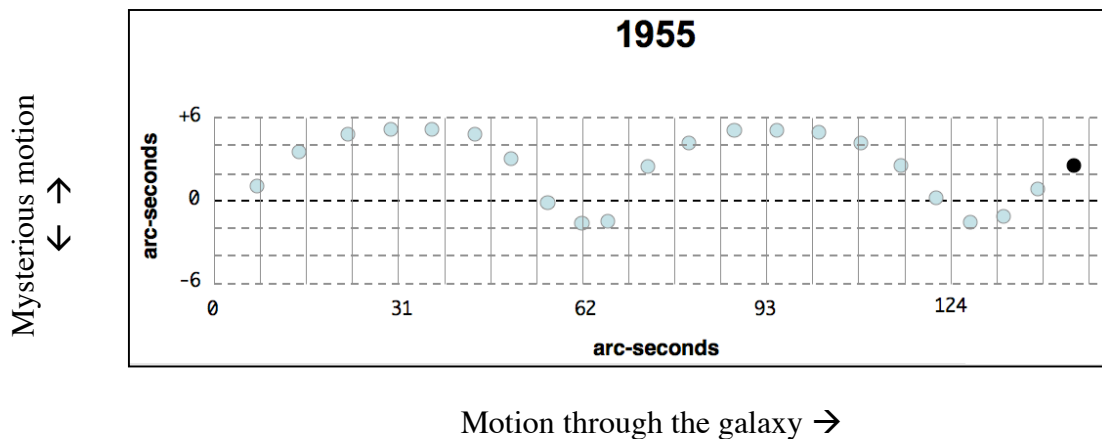
Explore Part I: Calculating the Mass of a White Dwarf, Sirius B (45 minutes)

Divide students into groups of two. Pass out materials to each group:
* Graph paper, 1/4 inch squares
* Cardboard sheet or foam core
* 3 pushpins
* Two 40 centimeter lengths of cotton string to make loops to draw Sirius A and Sirius B orbits.

1. Make the string loops (5 to 10 minutes)
Let each student group make the two string loops. The student's directions show illustrations to scale.
They can use the illustrations as templates to place their push pins and make string loops.

2. Set up the graph paper (5 minutes)
Turn the graph paper lengthwise. Find the center of the graph paper to the nearest grid cross.
Mark the center, and label it (0, 0) as an origin.. Mark these points: (-16, 0) (-7, 0), and (12, 0).
(-7, 0) represents the center of mass for the Sirius A and B binary star system.
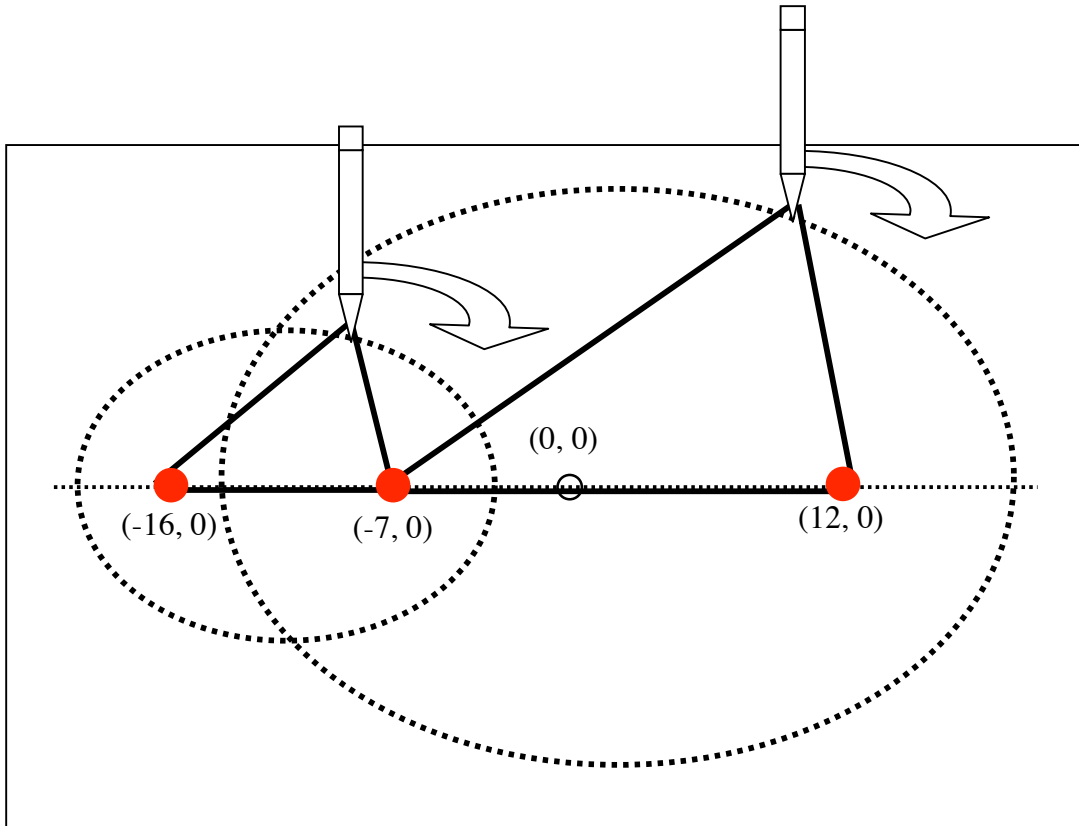
Place the graph paper on top of the cardboard sheet. At each of these points (-16, 0), (-7, 0), and (12, 0) place a push pin.

3. Draw the orbits of Sirius A and B (5 minutes)
Wrap the short loop around both pushpins at (-16, 0) and (-7, 0). Let the loop guide your pencil as you draw a curve. This represents the orbit of Sirius A, the brightest star in the sky.
For the orbit of Sirius B, wrap the long loop around the pushpins at (-7, 0) and (12, 0) and draw the curve with the loop guiding your pencil as you draw.
    Note: check that student follow these directions. Some may draw circles instead of ellipses.

Ellipse geometry and relationships
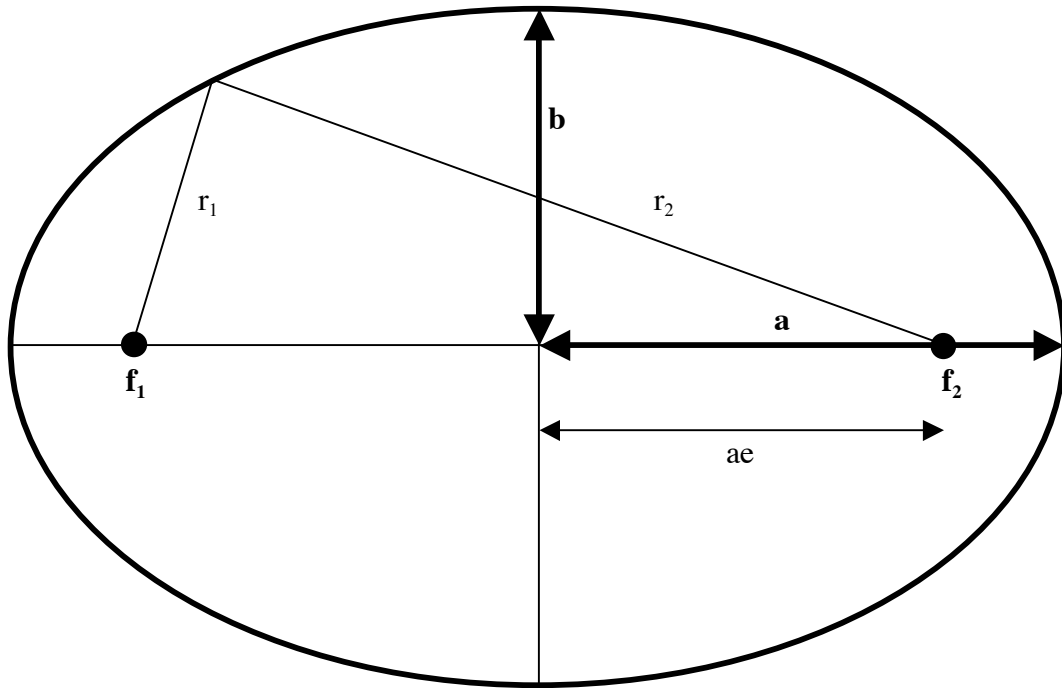
Semimajor axis: a Semiminor axis: b

For each point on the ellipse, $r_1 + r_2 = 2a$.

The loop of string represents the relationship between the two radii and the semimajor axis: $r_1 + r_2 = 2a$

$$\text{eccentricity} = e = \sqrt{1 - \frac{b^2}{a^2}}$$ or the distance between the foci divided by "a".

4. What is the mass of Sirius B? (10 to 15 minutes)

Pick a point on the orbit of Sirius A. Put a pushpin at that point. Put another push pin at (-7,0). With a ruler, line up the two push pins. Place the third push pin where the ruler crosses the orbit of Sirius B, on the *other side* of the push pin at (-7, 0).

> *Note: Some students may get confused about where to place the Sirius B push pin. Their ruler will intersect Sirius B's orbit twice: once between Sirius A and the center of mass, and once on the opposite side of the center of mass. Students should mark Sirius B opposite of Sirius A, with the center of mass between Sirius A and Sirius B: Sirius A ----- center of mass ----- Sirius B*

Sirius A   (-7, 0)   Sirius B

Draw a line segment that connects the pushpins.
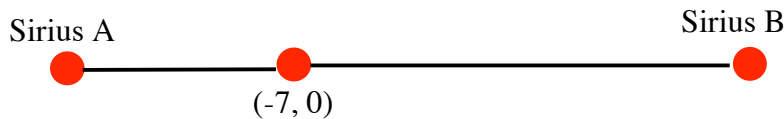
Sirius A   (-7, 0)   Sirius B

Measure the distance between the center of mass (middle push pin) and each push pin. This is the distance between Sirius A or Sirius B and the barycenter.

Make a two column table:

Sirius A distance	Sirius B distance

Students are testing the relationship

$$M_A r_A = M_B r_B \quad \text{or} \quad \frac{M_A}{M_B} = \frac{r_B}{r_A}$$

They should discover that the ratio of the radii is constant.

Record your first set of data points, then repeat for six more points. Pick points evenly around the orbit of Sirius A.

Plot your data on a graph Sirius B vs. Sirius A.

> *Students may want to use Excel or a graphing calculator to input their data and make plots.*
> *Note: remind students to use as much of the graphing area as possible to make their plots. They will see the relationship between the variables more clearly.*

What kind of relationship does the data show between the two distances?

> *A linear relationship. The points should tightly stager around a line. The slope represents the ratio of the distances of Sirius A and B from the center of mass:*
>
> $$slope = \frac{y}{x} = \frac{r_B}{r_A}$$
>
> *Since Sirius B is 1.1 solar masses, and Sirius A is 2.3 solar masses, the slope should be about 2.*
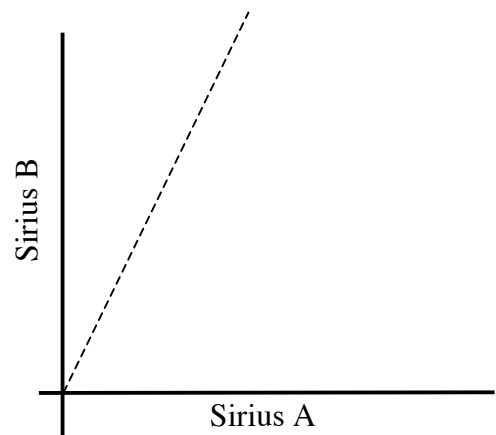
History: Freidrick Whilhelm Bessel

Freidrick Bessel observed the motion of Sirius with respect to the background stars between 1831 and 1844. In that time, he saw part of this pattern in Sirius's motion. The dark spots represent Sirius A, the brightest star in the sky. Sirius is bright not only because it is intrinsically bright (about 2.5 times the luminosity of our Sun) but also because it is very close to us (only about 9 light-years away).

Based on your previous activity, you know that Sirius has a companion. But where is it? Bessel did not see it. It was too faint and too close to Sirius. But Bessel had enough information to predict where to look.
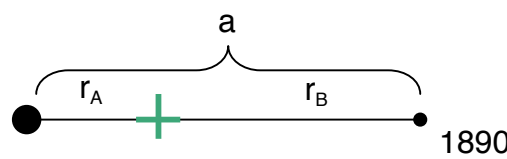
Kepler's Law:

$$P^2 = \frac{4\pi^2 a^3}{G(m_A + m_B)}$$

P: orbital period in seconds
a: semimajor axis in meters ($r_A + r_B$)
G: gravitational constant 6.67×10^{-11} N•m^2/kg^2
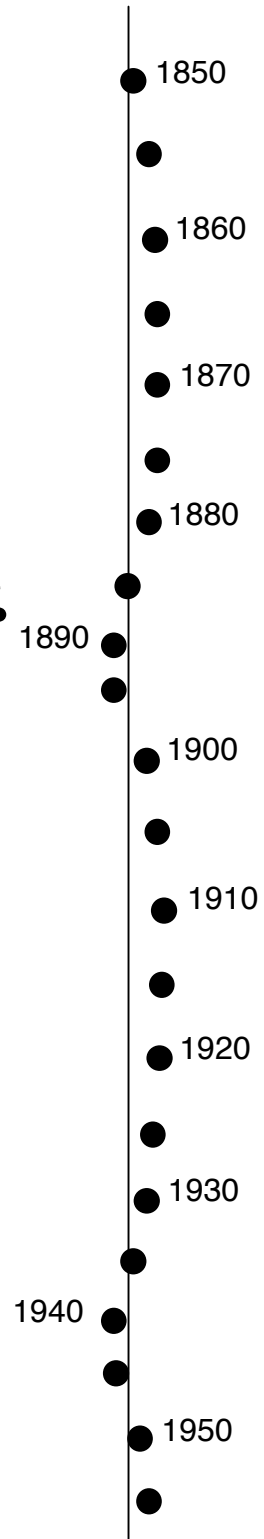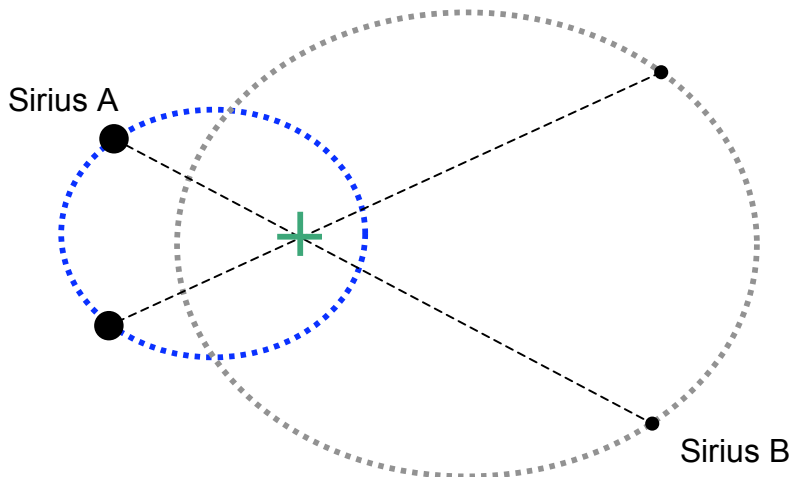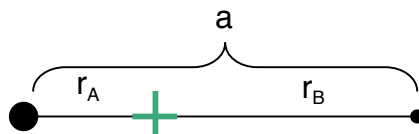m: mass in kilograms.

A simpler way to express this equation is to use units of the mass in terms of the Sun (solar mass), time in years, and distances in Astronomical Units (the semimajor axis of the Earth's orbit).

$$P^2 = \frac{a^3}{(m_A + m_B)}$$

P: orbital period in years
a: semimajor axis in Astronomical Units
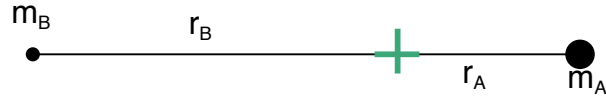m: ratio of the star's mass to the sun's mass (solar masses)

Sirius A

Sirius B

For the Sirius binary system:

If a = 20.21 AU and P = 50 years
What is the total mass? $(m_A + m_B)$?

m_B r_B r_A m_A

$(m_A + m_B) = 3.3$ solar masses
"a" is the mean separation between Sirius A and B for the whole 50 year period.

Calculating the mass of Sirius B (10 to 15 minutes)

Now that you know the sum of the masses in the binary star system, you can calculate the mass of Sirius B. Applying conservation of angular momentum, the product of the mass and distance (r) for Sirius A and B must balance.

$$m_A r_A = m_B r_B$$

$$\frac{r_A}{r_B} = \frac{m_B}{m_A}$$
            (mass of Sirius A = 2.2 solar masses)

From your earlier plot, you know the ratio of the distances so you also know the ratio of the masses. Combining the mass ratio with the sum of the masses (in solar mass units), you can calculate the mass of Sirius B.

Equation 1 $$m_A = m_B \frac{r_B}{r_A}$$

Equation 2 $$m_A + m_B = 3.3$$

Isolate m_B and calculate the mass for Sirius B.

The current value for the mass of Sirius B is 1.1 solar masses. Students may not compute this exact number due to experimental errors in tracing orbits and measuring the distances. The point is that they go through a process of deriving the distances and masses from observations, and see how many steps are involved in finding the mass of a star using basic physics and mathematics.

$m_A = m_B \dfrac{r_B}{r_A}$ Equation 1

$m_A + m_B = 3.3$ Equation 2

$m_B \dfrac{r_B}{r_A} + m_B = 3.3$ Substitute equation 1 expression for m_A into equation 2.

$m_B (\dfrac{r_B}{r_A} + 1) = 3.3$ Isolate m_B, then plug in the value for r_B/r_A.

$m_B = \dfrac{3.3}{(\dfrac{r_B}{r_A} + 1)}$

Once students finish these problems, run the PowerPoint file "Sirius A+B.ppt" *which shows the orbits of Sirius A and B with the observation data. Here are few properties to point out while students view the PowerPoint:*

1. *Ask students to look at the distances between Sirius A, Sirius B, and the barycenter. Point out the ratio of the distances (Sirius A : Sirius B) for a few of the frames.*
2. *Ask students what changes between the frames. The distances from the barycenter changes, but the ratio of the distances r_A:r_B remains constant.*
3. *Also, Sirius A and B are always on opposite sides of the barycenter.*

Explore Part II What is the size and luminosity of Sirius B? (20 minutes)

Size of a White Dwarf

The name dwarf indicates that this may be a small object. If you know the white dwarf's temperature and luminosity (amount of energy it radiates per second), then you can calculate its radius.

$$L = 4\pi R^2 \times \sigma T^4$$

L: luminosity in watts

T: temperature of the surface in Kelvin (K)

R: radius of the white dwarf in meters

σ: Stephan-Boltzmann constant = 5.67 x10^{-8} (W/[m^{2}K^4])

π: Pi, the ratio of a circle's circumference to its diameter

> **Astronomical Magnitudes**
> Astronomers measure brightness in units of magnitudes: the fainter the object, the larger the magnitude. For instance, our Sun's apparent magnitude is –26, while the faintest stars you can see in the sky without optical aid is +6. For a difference of 5 magnitudes, the flux changes by a factor of 100. But to your eye and brain, an object 1 magnitude lower than another looks about twice as bright, rather than $\sqrt[5]{100} = 2.5$ times as bright.

A spectroscopic analysis of Sirius B shows that the surface temperature is about 27,000 Kelvin. Compare that to our Sun at 5,770 Kelvin, and you understand why these objects when first discovered were called "white". They are very hot. But what about "dwarf"? Based on Bessel's calculations for Sirius's companion, astronomers were able to find Sirius B and measure its apparent brightness (magnitude), which turned out to be very dim: +8 compared to Sirius at –1.4. That difference in brightness magnitudes means that Sirius A radiates 5,864 times more light than Sirius B in the *visible region* of the electromagnetic spectrum.

Reflect

Before you calculate Sirius B's luminosity, think about what the astronomers had discovered. Sirius A and B orbit a common center of mass (barycenter) with "A" more luminous than our Sun, and "B" thousands of times fainter than "A". Sirius B is so faint that it barely shows up in a photograph. Sirius B certainly was an odd object, unlike any star previously known at the time. It was about the same mass as our Sun, yet extremely dim. In the picture, Sirius B is the tiny speck next to Sirius A. In order to reveal Sirius B, Sirius A is overexposed so the size difference in this picture is not real.
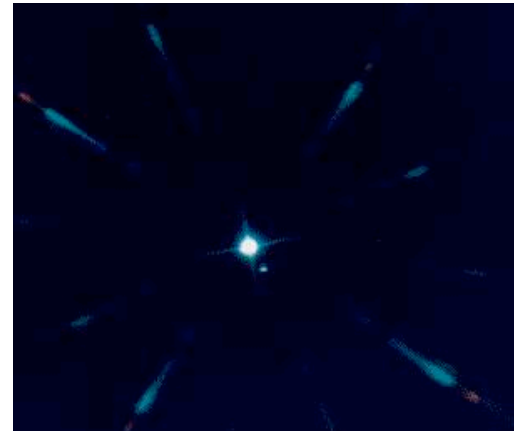
McDonald Observatory photo

Calculate the diameter of Sirius B

Since they also knew the distance to Sirius B, astronomers calculated Sirius B's luminosity.
For L = 1.15 x 10^{25} Watts and T = 27,000 K, calculate the radius of the star in meters. For comparison, the Sun is 1.4 x 10^9 meters in diameter and the Earth is about 1.28 x 10^7 meters in diameter. Express Sirius B's diameter as a ratio: Sirius B:Sun and Sirius B:Earth.

> *Sirius B radius: 5.5 x 10^6 meters, diameter: 1.1 x 10^7 meters, or 0.008 solar diameters, or 0.86 Earth diameters.*

Density

Based on your calculation for Sirius B's radius, **calculate its average density.** The volume of a sphere is:

$$V = \frac{4}{3}\pi R^3$$

Density is the total mass divided by the volume.

> *(Note: the Sun's mass is 2.0 x 10^{30} kg)*

> *Sirius B's density is 3.0 x 10^6 grams per cubic cm, or 3.0 x 10^9 kg per cubic meter.*

For comparison, water is 1 gram per cubic centimeter, or 10^3 kg per cubic meter. The Sun's average density is about 1.4 grams per cubic centimeter, or 1.4 x 10^3 kg per cubic meter.

Explain (20 minutes)

1. How does Sirius B compare to the Sun in terms of:

Property	Sun	Sirius B	Sirius B / Sun
Luminosity	1 (3.26 x 10^{26} Watts)	0.03 (1.15 x 10^{25} Watts)	0.03
Diameter	1 (1.4 x 10^6 kilometers)	0.008 (1.12 x 10^4 kilometers)	0.008
Average Density	1 (1.4 grams / cm^3)	(2.1 x 10^6) 3.0 x 10^6 grams / cm^3	2.1 x 10^6

2. Imagine that you are an astronomer in the early 1900's, and were just learning about the strange properties of Sirius B. Why do you think your fellow astronomers might have difficulty accepting the proposed properties of Sirius B?

This was such a strange result because Sirius B is unusual compared to the stars we normally see in the night sky. Sirius B is tiny (the size the Earth) yet more massive than the Sun (1.1 solar masses). It is also very hot (27,000 Kelvin, with the Sun at 5,770 K), but very dim (0.03 solar luminosities). In fact, its physical characteristics challenged the limits of physics of the day (late 19th and early 20th century). If the observations are reliable, Sirius B's density was at the time unexplainable – how could such an object exist at that density? A new understanding of physics at the atomic level was required to understand the physical state of a white dwarf.

3. Summarize the properties of a white dwarf in one sentence, as if you were making up a newspaper headline.
Dense stuff like an elephant in a marble!

Elaborate (15 minutes)

Since the early 20ᵗʰ century, many white dwarfs have been discovered. Since they are so small and dim, they are hard to find. But today at the University of Texas at Austin, a group of astronomers is not only looking for white dwarfs, they are trying to use them as a clock to determine the age of our Galaxy.

Read the following StarDate scripts about white dwarfs.
> *Please note that these scripts are dated. Any references about when to view an object may be inaccurate. See the underlined sentences in the scripts.*

Dog and Pup January 20, 2003

The dog star Sirius trots across the southern sky on winter nights. It's the brightest star in the entire night sky. Only the Moon and a couple of planets regularly outshine it. <u>Tonight, Sirius rises in the southeast about the time darkness falls, and climbs into good view by an hour later. It stands highest in the sky after midnight.</u>

Sirius appears so bright in part because it really IS a fairly bright star, but mostly because it's one of our closest neighbors -- less than nine light-years away.

That closeness made Sirius a favorite target for astronomers who were building new telescopes and new techniques in the 19th century. In 1844, for example, German astronomer Friedrich Bessel discovered that something was tugging Sirius back and forth a little bit -- an unseen star. Alvan Clark first saw this star in 1862. It's small and faint, so it's hard to find beside Sirius. Since it's a little companion to the DOG star, astronomers called it the Pup.

A half-century later, astronomers found that the Pup was unlike any other star they'd ever seen. Its surface is extremely hot, yet the star produces so little light that it must be tiny -- no bigger than Earth. But it's about as massive as the Sun, which has a diameter a hundred times GREATER than Earth's.

The Pup was the first of a previously unseen type of star called a white dwarf -- the crushed corpse of a once normal star.

White Dwarfs January 21, 2003

As our Sun ages, it'll undergo dramatic changes. It'll get hotter and brighter in its core, then puff up like a balloon. Eventually, it'll shed its outer layers, surrounding itself with a colorful but ephemeral bubble of gas. As the gas dissipates, only the Sun's collapsed core will remain -- a cosmic cinder slowly fading from sight.

This dead remnant is called a white dwarf. Billions of white dwarfs probably inhabit the Milky Way galaxy. But they're so faint that they're hard to see. In fact, not a single white dwarf is visible to the unaided eye. The closest one is a companion to Sirius, the brightest star in the night sky, which wheels across the south on winter nights. It's less than nine light-years away. But the white dwarf is so faint compared to Sirius that it wasn't discovered until the 1860s.

A white dwarf is the inevitable fate of a certain class of stars -- stars that are no more than about 8 times as massive as the Sun. As these stars age, they "burn up" the hydrogen fuel in their cores. They then burn heavier elements, until they can't generate the temperatures needed to keep going. Their cores collapse, forming ultradense, hot balls about as big as Earth. Over many billions of years, they'll cool and fade from sight.

Evolution to a white dwarf is a long-term project, though. The Sun won't reach that final stage of life for another five or six billion years.

An animation of a white dwarf is available at:
http://hubblesite.org/newscenter/newsdesk/archive/releases/2002/10/video/b
and at:
http://www.whitedwarf.org/index.html?education/&0

Setting a Date July 17, 2002

   After decades of debate, astronomers are reaching a rough consensus on the age of the Universe. And recent observations by Hubble Space Telescope not only strengthen this estimate, they may help astronomers narrow the range even more.

   Several years ago, astronomers used the orbiting telescope to measure the distances to remote galaxies. These measurements reveal how fast the universe is expanding now, and how the rate of expansion has changed over time. From these measurements, the astronomers estimated that the Big Bang took place 13 billion to 14 billion years ago.

   More recently, astronomers used Hubble to measure the ages of the oldest stars yet discovered -- DEAD stars called white dwarfs. These stars no longer produce energy in their cores. They shine simply by radiating their intense heat into space. The older a white dwarf, the cooler its surface, so measuring its temperature provides a good estimate of its age.

   Astronomers used Hubble to study white dwarfs in a star cluster called M4. The telescope watched M4 for several days, and found some of the coolest white dwarfs ever discovered -- and therefore, some of the oldest. From the temperatures of these stars, astronomers estimate that they're 12 billion to 13 billion years old. Since the stars probably formed soon after the Big Bang, their ages agree well with the earlier estimates of the age of the Universe.

This M4 image is at the Hubble WWW site:
http://hubblesite.org/newscenter/newsdesk/archive/releases/2002/10/image/a

To make an Excel spreadsheet model of a white dwarf, visit:
http://www.unisi.it/fisica/luciana/articoli/starsimu/homepage.htm

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