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A Dynamic Model of Asymmetric Price Negotiation

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A Dynamic Model of Asymmetric Price Negotiation

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Dissertation

Presented to the Faculty of the Graduate School of

the University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

May 2005

Acknowledgements

I am grateful for the numerous insights and suggestions of the following people: Dr. Alex Chien, Dr. Won-Yong Kim, Professor Dan Slesnick, Dr. Elsie Echeverri-Carroll, Professor Leigh McAlister, Joe Aarons, and Xiuli He. In addition, I would like to thank my committee members, Frenkel ter Hofstede, Rajashri Srinivasan, Jay Koehler and Dale Stahl for their strong support including. Finally, I am very grateful for the advice and counsel from my dissertation chair Professor Robert A. Peterson, whose patience and meticulous attention to detail were instrumental to my successful completion of this dissertation.

A Dynamic Model of Asymmetric Price Negotiation

Publication No. ___________

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The University of Texas at Austin, 2005

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Many buyer-seller interactions are characterized by *asymmetric negotiation* in which the seller makes explicit offers that the buyer either accepts or rejects. Asymmetric negotiations are prevalent in situations where the buyer is either reluctant or unable to make an explicit counteroffer. A common constraint that buyers face during negotiation is a purchase deadline, which causes the buyer's opportunity cost of delay (negotiation cost) to increase over time. In turn, an increasing negotiation cost causes the buyer's minimum purchase threshold (reservation value) to vary over time. This dissertation proposes a new model of asymmetric price negotiation that allows buyers' reservation values to change over time. The model reflects buyer decisions with respect to negotiation costs, the seller's offer rate, a discount rate, and time.

A dynamic structural model of asymmetric price negotiation is derived from the economic theory of search behavior that integrates findings from the behavioral literature. In particular, buyers are assumed to maximize their net present expected utility, but do so myopically over a short time horizon. Buyers evaluate a seller's *relative offer* or the difference between the seller's current offer price and a reference price. The model implies that the purchase hazard rate increases with time and negotiation cost, but decreases with offer rate and average relative offer.

Model properties are empirically tested using a competing-risks proportional hazard model derived from the structural model. The empirical model is estimated on a sample of actual negotiations over the rental of a durable product; the results confirm the properties of the structural model. The empirical model is used to explore alternative specifications of the buyer's reference price. It is shown that buyers tend to rely on the most recent offer price when evaluating the seller's offer in the sample of negotiations.

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1 Introduction

Negotiations are an important form of conflict resolution pervading a wide variety of social interactions. However, despite calls for more research on the negotiation process (e.g., Binmore, Osborne, and Rubinstein 1992; Gale 1986), few negotiation process models have been proposed (Balakrishnan and Eliashberg 1995). The extant negotiation literature is dominated by two perspectives. The behavioral perspective emphasizes the role of social perception regarding how negotiators selectively categorize, interpret, and infer information (Thompson 1990). This perspective identifies the antecedents of negotiation and the factors affecting the overall negotiation process. Alternatively, the economic perspective applies utility theory and game theory to predict equilibria for specific types of negotiations (Sutton 1986) and focuses on predicting negotiation outcomes. Unfortunately, neither perspective provides a unified theory of negotiation that incorporates important antecedents into a model predicting negotiation outcomes. Moreover, there is limited empirical support for economic models based on game theory (Ochs and Roth 1989; Rapoport, Erev, and Zwick 1995; Srivastava, Chakravarti, and Rapoport 2000). The model proposed here offers important insights into the limitations of outcome-oriented economics models while clarifying the role of different reference prices identified in the behavioral literature.

The proposed model addresses negotiations between a single buyer and a seller. Before I describe the type of situations I will model, some terminology is required. A *negotiation* is a process by which parties exchange information with the goal of reaching a jointly desirable agreement that prescribes future actions, behavior, or responsibilities. The *process* involves a series of offers and counter-offers between parties. An *offer* is a suggested agreement that at least one party finds desirable. A *party* to a negotiation is an individual or group that seeks a common outcome in accord with its preferences. The information exchanged may include provisional offers, stated preferences, or attempts to persuade the other party to change an offer or preference. A *settlement* is a set of required actions that the parties mutually agree to undertake upon completion of negotiation. Although the settlement is jointly desirable, each party has the option of exiting the negotiation at any time.

Negotiations can be characterized several ways. A negotiation is *monolithic* when the parties represent their own interests and do not require ratification from a third party (Raiffa 1982). A negotiation is *distributive* when the settlement represents a division of some fixed available payoff whereby one party benefits to the detriment of the other party. A negotiation is *integrative* when the settlement that benefits one party is not made at the cost of the other party (Walton and McKersie 1965). Although there is no agreed upon terminology for the number of issues being negotiated, negotiations involving one issue will be called *simple* and those involving more than one issue will be called *complex*. When the outcome of one negotiation affects the outcome of another negotiation between the same parties, there is said to be *linkage* across the negotiations (Raiffa 1982). These negotiation characteristics are not necessarily exclusive, since more than one characteristic may be present during the same negotiation.

The negotiation characteristics are illustrated by the following examples. When a husband and wife negotiate over where to go to dinner, the individuals typically represent their own interests; this represents a *monolithic* negotiation. This situation contrasts with a labor contract negotiation involving multiple parties in which representatives must seek ratification to reach a settlement. The negotiation between a husband and wife is often *integrative*, since a settlement that is advantageous to the husband need not be disadvantageous to the wife and vice versa. Again, this situation contrasts with labor contract negotiations which are often *distributive* since an outcome that is advantageous for labor (higher wages) usually represents a loss for management (lower profits). The husband and wife negotiation over where to dine is *simple*, whereas labor negotiations are usually *complex*, since they involve multiple issues such as wages, benefits, and work schedules. Since husbands and wives and labor and management have a continuing relationship with each other, there is often *linkage* across both types of negotiations.

Negotiations between a buyer and a seller over the purchase price of a product or service exemplify many of these negotiation characteristics. The negotiations are *simple*, since they often involve only the purchase price, and are *monolithic* since the buyer and seller represent their own interests.[1] Buyer-seller negotiations are *distributive* since a gain for the buyer (lower price) results in a loss to the seller (lower profit). Depending on the relationship between the buyer and seller, there may or may not exist

[1] Although spouses and families often shop together and make joint purchase decisions, for the purposes of this dissertation they will be treated as a single party to the negotiation.

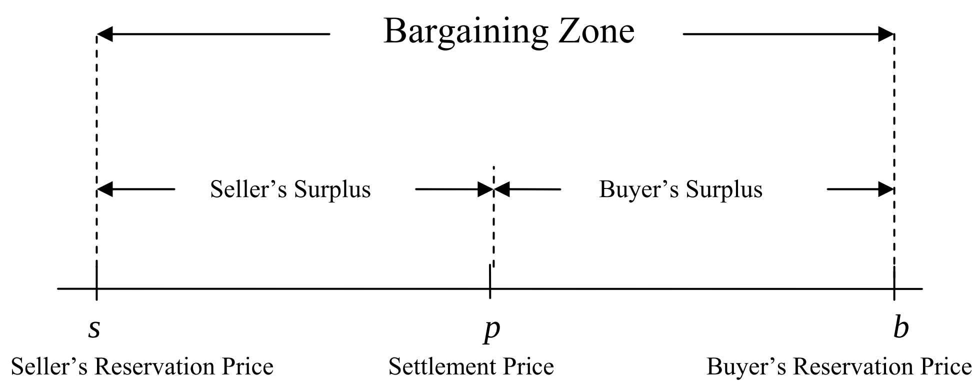
linkages across negotiations. Hence, buyer-seller negotiations are characterized as *monolithic, distributive, simple,* and with or without *linkages.*

1.1 A Simple Conceptual Model

One useful way to conceptualize the process of buyer-seller price negotiation is the Bargaining Zone model (Raiffa 1982). In this model, a buyer is interested in purchasing a single product from a seller. The buyer and the seller each possess some *reservation price* for the product. For the buyer, the reservation price *b* is the maximum price that the buyer is willing to pay for the product. Any price higher than *b* represents a situation at which the buyer is worse off than not settling. For the seller, the reservation price *s* is the minimum price at which the seller is willing to sell the product. Any price less than *s* represents a situation in which the seller is worse off than not selling. There are two possible situations. If *b* is less than *s*, the buyer is not willing to purchase at a price high enough for the seller and the seller is not willing to sell at a price low enough for the buyer. Consequently, there is no room for negotiation and no purchase takes place. If *s* is less than *b*, there is a region -- the Bargaining Zone -- within which buyer and seller reservation prices overlap. In this case, the parties are better off settling at some negotiated final price *p* between *s* and *b* than choosing not to settle. The difference *p-s* is the seller's *surplus* and the difference *b-p* is the buyer's *surplus*. Since the total available surplus, *b-s*, does not depend upon the final price *p*, this model represents a distributive negotiation. It is often assumed that during negotiation both parties seek to maximize their share of the available surplus.

Figure 1

SIMPLE CONCEPTUAL MODEL OF BUYER-SELLER NEGOTIATION



The Bargaining Zone model presented in Figure 1 provides a concise illustration of the problem that buyers and sellers face when negotiating. This conceptual model forms the basis for many economic theories of negotiation (cf. Nash 1950, Rubinstein 1982). However, the simplicity of this model belies an important characteristic of actual negotiations; the buyer's and seller's reservation prices may change over the duration of negotiation. Real negotiations involve dynamic changes in how the parties perceive the situation. In particular, a party's reservation price may change over time as a function of pertinent factors such as the cost of negotiation or the rate at which offers are made. The proposed model in this dissertation is an attempt to rectify this deficiency in the static Bargaining Zone model.

1.2 An Illustrative Example

By their nature, negotiations involve a dynamic process in which pertinent factors change over time. The dynamic nature of the negotiation process introduces several difficulties in specifying a model that reflects buyer behavior.[2] This issue is particularly acute when a buyer faces a purchase deadline. For example, the buyer may face a deadline when purchasing airplane tickets, appliances for a new home, or items for a vacation. In this situation, the time spent negotiating represents foregone opportunities to purchase. As the deadline nears, the need to reach a settlement becomes increasingly urgent. This urgency is reflected in an increasing opportunity cost of delay. In this sense, the buyer faces an increasing cost of negotiation even if the transaction costs are fixed over time. In turn, the increasing negotiation costs will affect the minimum threshold or *reservation value* that the buyer uses as a basis for a purchase decision. In the face of an impending deadline, a buyer may adjust his/her reservation value to ensure that a suitable offer arrives in time. The complexity that a time-varying reservation value has on a buyer's decisions during negotiation is illustrated by the following stylized example.

John has just finished a business trip and has a few hours before his plane leaves. As an avid reader, he is happy to find a used book store near the airport that specializes in 19th century American literature. When perusing the books, he

[2] In order to simplify the exposition of the proposed model, the terms "buyer" and "potential buyer" will be used interchangeably.

comes across the collected works of his favorite author, Henry James. The collection is priced at $140, but John thinks this is too expensive. John wants at least $30 off this price. Because John knows from prior experience that book dealers are often willing to negotiate price, he indicates to the dealer that he'd love to buy the Henry James collection, but $140 seems a little high. The dealer offers a new price of $130. John is excited about the $10 discount, but the new price doesn't meet his minimum threshold of $30 off the initial price. John decides he wants to think over this decision for a little while so he thanks the dealer and walks across the street to get a cup of coffee. His plane is leaving in two hours, and he realizes that he needs to make up his mind soon if he wants to get the collection. Given enough time, John thinks he could get the dealer to agree to a lower price, but he realizes that he should adjust his minimum threshold to $20 off to ensure that he has enough time to make his plane if the dealer meets the new threshold. He walks back into the store and politely asks the dealer if he would consider a lower price. The dealer makes a new offer of $120 or $20 less than the original price. Since the new offer meets John's new threshold, he buys the book collection and makes it to his plane just before takeoff.

This stylized example illustrates the dynamic nature of price negotiation when buyers face a purchase deadline. In this case, John only had a few hours to spend negotiating or he would miss his flight. Although John's instantaneous effort during negotiation remained fixed, he faced an increasing opportunity cost of delay as his deadline loomed closer. This increasing negotiation cost influenced John's decision rule for making a purchase. In particular, as John approached his flight time, he realized that he must adjust his minimum threshold or reservation value. Specifically, the time constraint forced John to adjust his minimum discount from $30 to $20 to increase the probability that the dealer would meet the reservation value within the remaining time. This example also illustrates the critical role of negotiation duration. Since the buyer's reservation value depends on time, the buyer's probability of choice will be time-dependent. The goal of the model proposed in this dissertation is to capture the dynamics of a changing reservation value on a buyer's decision making during a negotiation.

1.3 Overview of Proposed Model

The proposed model focuses on the same type of negotiation that John faced in the stylized example. John engaged in an asymmetric price negotiation where only the seller made explicit price offers that John either accepted or rejected. This type of negotiation characterizes many buyer-seller interactions in which the buyer merely indicates a willingness to consider a lower price, but only the seller makes explicit offers. Asymmetric negotiations are prevalent in situations where the buyer is either reluctant or unable to offer an alternative price because of comfort level or lack of requisite expertise to generate an explicit counteroffer. The proposed model reflects the buyer's decision making over time in reaction to a sequence of offers by a seller. In the stylized example, the model explains John's time-varying minimum threshold for making a purchase with respect to his increasing negotiation cost, the seller's offer rate, and time. In addition, although John's negotiation occurred over a short duration, the model incorporates a discount rate that a buyer may use to value future offers.

The proposed model integrates findings from the behavioral and economics literatures on negotiation into a structural model that captures the tradeoff between increasing negotiation costs and the expected benefit from negotiation. This structural model is derived from the theory of search (Flinn and Heckman 1982), but is augmented by important characteristics of actual negotiations (Blount, Thomas-Hunt, and Neale 1996) and decision making over time (Prelec and Loewenstein 1991). In this model, buyers exist in several states and transition according to choices that maximize utility during negotiation at each point in time. Buyers are assumed to be forward looking, but act myopically by only considering potential outcomes within a short time horizon. Furthermore, buyers are assumed to consider *relative offers* or the difference between the most recent offer and a reference price. Although these assumptions impose time-varying restrictions on the buyer's choices, they enable the model to more accurately reflect the dynamics of actual negotiations than if these assumptions are not made.

1.4 Research Questions

The proposed model addresses three broad research questions: How does time, the buyer's negotiation cost, the seller's offer rate, and the average relative offer affect the negotiation duration? How does the negotiation duration affect the buyer's decision to purchase or exit negotiation? What reference price(s) do buyers rely on in evaluating a seller's relative offer? These questions will be empirically addressed by estimating a reduced-form specification of the proposed model on a database of actual negotiations over the rental of a durable product.[3] Since this database contains actual negotiations, it permits an evaluation of the external validity of the proposed model. This contrasts with prior research on negotiation that relies exclusively on simulated negotiations or purely theoretical models.

The dissertation is organized as follows. This chapter provided a brief introduction to the research problem. In Chapter 2, the existing negotiation literature is reviewed, with an emphasis on work conducted in the behavioral, economics, and marketing literatures and on identifying the underlying approaches and constructs that are important in understanding the negotiation process. Chapter 3 introduces a static negotiation model based on search theory, and several extensions and modifications to the static negotiation model are introduced. The proposed model is presented in Chapter 4. This chapter opens with a conceptual model of asymmetric price negotiation that integrates several findings from the economics and behavioral literatures as formal assumptions of the model. These assumptions are used to derive a structural model of the buyer's decision making during negotiation. In Chapter 5, a competing-risks proportional hazard model is derived from the properties of the structural model. The competing-risks model is used to empirically validate several properties of the proposed

[3] Due to the proprietary nature of the data, the product type, attribute names, and the company from which the database originated cannot be specified here.

model. This chapter provides a description of the database used to estimate the competing-risks model as well as the empirical results and tests of the propositions implied by the structural model. Chapter 6 presents a discussion of the implications of the model for the seller investigated, the limitations of the model, and extensions to the model. A technical appendix provides several derivations and proofs.

2 Literature Review

The negotiation literature is dominated by two research perspectives. The behavioral perspective emphasizes the role of social perception regarding how negotiators selectively categorize, interpret, and infer information related to negotiation (Thompson 1990). This perspective is descriptive, since it addresses the motivations and behavior of actual negotiations. Behavioral measures of negotiation include people's judgments of the bargaining process, evaluation or liking of the other party, expectations of fairness, bargaining skill, and perceptions of negotiation goals. The primary focus of the behavioral perspective is on identifying the antecedents of negotiation and the factors affecting the overall negotiation process.

The economic perspective applies utility theory and game theory to identify equilibria for specific types of negotiations (Sutton 1986). Utility theory assumes people maximize a function representing their preferences and risk attitudes for choice alternatives (von Neumann and Morgenstern 1944). Game theory formalizes situations in which each party's welfare depends on the party's own preferences and actions as well as on those of the other party (Luce and Raiffa 1957). *Equilibria* are outcomes with the property that neither party would be better off altering the terms of the outcome with respect to their own utility function. Hence, economic theory is normative in that if a person's preferences satisfy certain axioms, the theory identifies choices a person *should* make. The primary focus of the economic perspective is on identifying optimal negotiation outcomes.

Most of the marketing literature on negotiation focuses on applications of economic negotiation theory. For example, there are several articles on applications of game theory to the study of channel negotiations between suppliers and distributors (e.g., Banks, Hutchinson, and Myer 2002; Iyer and Villas-Boas 2003; Srivastava, Chakravarti, and Rapoport 2000). A smaller stream of marketing negotiation literature tests the implications of game theory models for the outcome of buyer-seller negotiations (e.g., Curry, Menasco, and Van Ark 1991; Menasco and Roy 1997; Neslin and Greenhalgh 1983, 1986). However, with the exception of Balakrishnan and Eliashberg (1995) and Chen, Yang, and Zhao (2004), the marketing negotiation literature does not offer any new structural models of the negotiation process. One stream of marketing literature relevant to the proposed model reports research on reference prices. Several articles assess alternative specifications of a buyer's reference price on their subsequent purchase probability (e.g., Briesch, Krishnamurthi, Mazumdar, and Raj 1997; Hardie, Johnson, and Fader 1993; Kalwani, Yim, Rinne, and Sugita 1990; Mayhew and Winer 1992; Winer 1986).

Below, I focus on articles most representative of each research perspective and those offering important insights for modeling the negotiation process. I emphasize research identifying important factors that affect behavior and present models that reflect the social interactions characterizing the negotiation process. I conclude with a brief overview of the negotiation literature, highlighting strengths and weaknesses, and identify several contributions of the proposed model in light of the extant literature.

2.1 Behavioral Literature

The behavioral perspective on negotiation contains a number of distinct theories related to a negotiating party's social perception of the situation. These theories can be roughly classified into three groups (Thompson 1990). The first group includes theories about *individual differences* or stable characteristics of people that predictably affect negotiation behavior. Green, Gross, and Robinson (1967) exemplify this group research. They find that when a party with a high level of rigidity negotiates with a party of low rigidity, the former extracts a higher proportion of the available payoffs. Additional individual differences identified in this group include egocentricity (Corfman and Lehman 1993), cognitive ability (Pruitt and Lewis 1975), Machiavellianism (Huber and Neal 1986), perspective-taking (Neale and Bazerman 1983), and cultural norms (Graham, Kim, Lin, and Robinson 1988). Many of these individual differences require specialized measurement instruments and thus would be difficult to operationalize outside of a laboratory setting.

The second group of behavioral theories includes research identifying *motivational* factors, such as a party's aspirations, goals, and expectations. For example, several authors show that expectations of fairness affect negotiation behavior. Maxwell, Nye, and Maxwell (1999) find that buyers primed with an expectation of fairness settle more often and more quickly than unprimed buyers. Luft and Libby (1997) find high expectations of fairness yield equitable settlements. Negotiator goals can also affect the negotiation process. In channel negotiations, McAlister, Bazerman, and Fader (1986) show that the effect of moderately high profit goals on negotiator performance depends on the power relationship between the parties. In particular, a party with a moderately high profit goal acquires a higher profit only when the party exhibits more power compared to the other party.

The third group of behavioral theories includes *cognitive* theories of how parties perceive the negotiation situation, process the available information, and make judgments. An important issue in this stream of research is how the parties frame the negotiation process and the possible settlements. For example, Thomas and Hastie (1990) show that parties sometimes assume there is conflict when none exists (*incompatibility error*). This error can lead to settlements in which both parties are worse off than if they negotiated without the assumption of conflict. Neale and Bazerman (1983) find that one party sometimes assumes the other party has the same preferences and prioritizes the outcomes equivalently (*fixed-pie error*). This error can lead to a distributive settlement even when an integrative outcome is feasible. Several authors identify the effect of perceived negotiator roles. For example, a party with the perceived role as "buyer" often acquires a greater payoff than the "seller" (Eliashberg, LaTour, Rangaswamy, and Stern 1986; Huber and Neale 1986; McAlister et al. 1986; Neale and Northcraft 1987). Finally, offers often are correlated with a reference price. For example, White and Neale (1994) show that the settlement price is highly correlated with the price a party would like to pay (*aspiration price*).

2.1.1 Prospect Theory

An important behavioral theory used in the proposed model draws on Prospect Theory. Although Prospect Theory is not a theory of negotiation *per se,* several facets of this theory are relevant to the study of negotiation (and are used in the articles cited above.) Prospect Theory explains how people perceive gains and losses with respect to individual reference points (Kahnemann and Tversky 1979). This theory explains certain observed systematic departures from the normative utility theory of economics. For example, consumers often value the purchase price of a product with respect to a reference price, evaluating the price as a “gain” if it is priced less than their reference price and a “loss” if it is priced greater than their reference price (Winer 1986). Reference price effects have been identified in consumer price negotiations for the purchase of a house (Northcraft and Neale 1987).

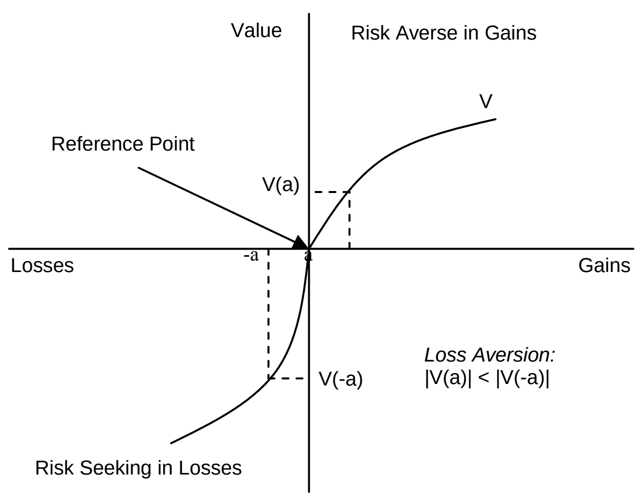
Prospect Theory rests on three key assumptions. First, there exists a reference point from which an individual’s choices are perceived by that individual as either a gain or a loss. The reference point may be different across individuals. This suggests that the same choice can be perceived as a gain for one person, and perceived as a loss by someone else. For example, a buyer may perceive a purchase as a loss since the buyer pays (“loses”) money , whereas a seller may perceive the same choice as a gain, because the seller profits from the sale (Neale et al. 1987). Second, Prospect Theory assumes that the marginal value of gains is decreasing and the marginal value of losses is increasing. This assumption implies that people are risk-averse in the choices between gains, but are risk-seeking when choosing between losses. Third, the theory assumes that the perceived change in value between gains is less than the perceived change between losses, an effect known as *loss aversion*. These three assumptions are represented by a *value function* that specifies preferences and risk attitudes in a way similar to a utility function. Figure 2 below illustrates an idealized value function that displays the characteristics implied by the three assumptions of prospect theory.

The importance of Prospect Theory to the study of negotiation is evidenced by several articles that show how a buyer's reference price affects the negotiation settlement. For example, Northcraft and Neale (1987) find that buyers often use the initial list price as a reference price in negotiating the purchase price of a house. White and Neale (1996) show that the average selling price of a brand affects the final negotiated price of that brand. In the context of dyadic negotiation over the purchase of a home, White, Valley, Bazerman, Neale, and Peck (1994) show that the average selling price of other homes significantly affects the purchase price. These examples demonstrate the importance of considering the effect of reference price on a buyer's decision making during negotiation. Reference prices will play a crucial role in the proposed model. I will return to this topic in the discussion of the marketing literature.

Figure 2 shows the effect of gains and losses on a person's *value function,* denoted by V. The reference point is depicted by the origin, with losses on the negative side and gains on the positive side of the origin. Risk-aversion is illustrated by the concave downward shape of the graph in the gains region. Risk-seeking is illustrated by the concave upward shape of the graph in the loss region. Loss aversion is shown by the greater slope in the loss region compared to the gain region of the graph. Consequently, the absolute value of V at “a” is less than the absolute value of V at “-a”, or $|V(a)| < |V(-a)|$. Hence, a person will value a loss greater than a gain of the same amount. For example, if the value function is defined over prices, then a person will perceive a larger change in value for a decrease in price of $\$a$ compared to an increase in price of $a, with respect to their reference price. This effect of *loss aversion* has important implications for how buyers value offers made during negotiation. In particular, a buyer may react more strongly to an offer that is a decrease with respect to the buyer’s references compared to an offer that is an increase of the same amount.

Figure 2

PROSPECT THEORY VALUE FUNCTION



2.2 Economics Literature

The economic perspective applies utility theory and game theory to identify equilibria for specific types of negotiations (Sutton 1986). Utility theory assumes that people maximize a function representing their preferences and risk attitudes for choice alternatives (von Neumann and Morgenstern 1944). Game theory formalizes situations in which each party's welfare depends on the party's own preferences and actions as well as those of the other party (Mas-Colell, Whinston, and Green 1995). An *equilibrium* is an outcome with the property that neither negotiating party would be better off altering the terms of the outcome with respect to its own utility function. Hence, economic theory is normative in that if a negotiating party's preferences satisfy certain axioms, the theory identifies the choices that the party *should* make. The primary focus of the economic perspective is to identify negotiation outcomes that are equilibria for particular types of negotiations.

Historically, the economics negotiation literature contains two broad perspectives on how a party maximizes utility. Early process theories of negotiation assume a party only maximizes utility with respect to that party's preferences. In contrast, game theory allows a party to maximize utility with respect to its own preferences as well as its beliefs about the preferences of the other party involved in the negotiation. In game theory, a party's strategy during negotiation is a best response to beliefs about the other party's actions. Although the early process theories have fallen out of favor, they are relevant to the model proposal in this dissertation because they focus on explaining the negotiation process rather than the outcome.

In the earlier process theories, the primary focus was to understand why a party would make a concession from an initial offer, since a concession would imply taking a position of lesser utility. Consequently, early process theories tended to focus on the sequence of offers and counter-offers (i.e., concessions) made between negotiation parties. This approach proved problematic, because many of the process-oriented models could not predict the value of the final settlement (or if a settlement was even possible!). Game theories overcome this deficiency by formalizing the negotiation process through a set of axioms that specify the allowable negotiation behavior. In this way, by an appropriate choice of axioms, game theory guarantees that a settlement is feasible and predicts its value. Game theories also allow parties to engage in more realistic decision making, since utility maximization is conditional on both parties preferences and their beliefs about each other's preferences.

In formalizing the negotiation process, game theories have largely abandoned the initial interest in studying the negotiation process. Consequently, the game theory perspective is fundamentally limited in what it can explain and predict about the negotiation process because the process is "fixed" by a set of axioms or stipulations.[4] Despite this shortcoming, I will provide a brief overview of the main theories and features of both the early process models and several game theories with the goal of emphasizing aspects of each that provide insights on how best to specify a dynamic model the negotiation process.

[4] In their defense, some game theorists maintain that formalization of the negotiation process is necessary in order for their models to be relevant to a broad range of possible processes (Sutton 1986). Unfortunately, the practice of game theory has at times introduced formalizations of the process that have proved unrealistic in empirical tests (Ochs and Roth 1989).

2.2.1 Early Process Models

A negotiation can be characterized by a series of concessions culminating in a settlement representing a compromise among the offers. If a party's utility decreases with each concession, an important issue to explain is why the parties would ever concede. Several models address this issue by introducing the notion of *bargaining power* or the advantage of accepting or rejecting an offer. Bargaining power restores the basic utility maximization framework for explaining negotiation behavior because parties are assumed to maximize utility weighted by bargaining power. The research presented below is also noteworthy because it identifies several important characteristics of the negotiation process that will prove important in my subsequent model. Zuethen (1930) provides the earliest model to explain concession making in terms of bargaining power.[5] This model is developed using the following example. Suppose two parties are involved in a negotiation over two possible settlements, A_1 and A_2. The utility functions for Party 1 and Party 2's are respectively denoted by $U_1(\cdot)$ and $U_2(\cdot)$. Party 1 prefers A_1 over A_2 and Party 2 prefers A_2 over A_1. Suppose Party 2 offers Party 1 the settlement A_2. Zeuthen proposes that the decision by Party 1 to counter-offer with A_1 depends on Party 1's assessment of the probability, p_2, that Party 2 would reject this offer and choose not to settle. Party 1 faces the choice of obtaining $U_1(A_1)$ with certainty or obtaining $U_1(A_2)$ with probability $(1-p_2)$. Since the parties maximize utility, Party 1 will accept A_2 if:

[5] The explanation of Zuethen's model presented here is due to Harsanyi (1956).

(1) $$\frac{U_1(A_1) - U_1(A_2)}{U_1(A_1)} < p_2 .$$

Zuethen argues that the maximum value of the left-hand term in equation (1) represents the greatest "risk" that Party 1 would face in order to achieve A_1. Party 2 faces a similar situation. The maximum risk each party is willing to face to achieve its most preferred outcome represents each party's *bargaining power*.[6] Therefore, Zeuthen argues that a party will make a concession when the other party's bargaining power is stronger.[7]

There are several noteworthy features of Zuethen's concession model. First, the choice to concede and the amount of bargaining power are determined solely by each party's assessment of its utility. Second, although the model provides conditions for a concession, it provides no mechanism for predicting the concession value. For example, in negotiations over price, the model does not provide a way to predict the sequence of offer prices made by the parties. Third, the model assumes that each party knows the other party's utility (i.e., there exists *perfect information*) and that the utilities are comparable (i.e., cardinal utility functions). These assumptions limit the applicability of Zuethen's model. In particular, it may unreasonable to assume that in a buyer-seller negotiation both parties know each other's preferences.

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[6] This terminology is due to Harsanyi (1956)
[7] Harsayni (1956) shows that this process will eventually lead to a settlement at the midpoint between the original offers.

Despite these shortcomings, Zeuthen's model provides a key insight into the negotiation process. Namely, each party considers the probability the other party will accept an offer. In choosing an offer, a party maximizes utility conditional upon the likelihood the other party accepts the offer. Since this probability depends on the other party's utility, Zuethen's model implicitly assumes that a party takes into consideration beliefs about its opponent's likely response to an offer. This feature will prove useful in my proposed model.

Cheng (1968) offers a model of concession that predicts the concession values and offers further insight on how bargaining power impacts the choice to concede. The model closely resembles that of Zeuthen (1930). To illustrate Cheng's approach, again consider the example in which Party 2 offers A_2 to Party 1 and Party 1 considers the counter-offer A_1. Cheng argues that if bargaining power is defined as in Zuethen's model, then a concession by one party may reduce both parties' bargaining power. Hence, it is unclear when a party would concede. Cheng proposes that bargaining power should be defined in relative terms, or that bargaining power should be standardized with respect to the total amount of power of both parties. This modification provides Cheng with a justification for interpreting bargaining power as the probability that a party's offer will be accepted.

Cheng models the concession as the maximum of a party's expected utility with respect to its bargaining power. For example, if $P_1(x_1 \mid A_2)$ is the relative bargaining power of Party 1 for an offer x_1 with respect to Party 2's outstanding offer A_2, then

Party 1 chooses an offer x that maximizes expected utility or solves $\max_x P_1(x \mid A_2) * U_1(x)$. Party 2 maximizes a similar expression, given Party 1's current offer of A_1. This process continues until the expected utility of each party's optimal current offer equals the certain utility of accepting the other party's optimal offer.

The primary contribution of Cheng's model is its ability to predict the concession value by interpreting bargaining power as the probability of acceptance. However, the model suffers from many of the same limitations as Zeuthen's model, such as the need for perfect information and the existence of interpersonal utility comparisons. Also, it is not clear that bargaining power is equivalent to the probability of acceptance or that the proposed process converges to a settlement.[8] Nonetheless, the model provides a useful conceptualization of the negotiation process in terms of utility maximization with respective to bargaining power and the probability of acceptance.

Cross (1965) provides a further improvement over the models of Zuethen and Cheng. He argues that concession making is motivated by a desire to minimize utility loss that occurs as the value of settlement erodes over time. For example, in buyer-seller negotiations, the utility of the good being negotiated may decrease over time due to a limited shelf-life, competitive entry, or changing preferences. Cross argues that time has a threefold impact on the negotiation process. First, future outcomes are discounted for time. The longer the negotiation process takes, the longer it will take to realize a gain. The present value of a distant gain is less than a current gain. Second, utility changes

[8] In fact, Cheng notes that a settlement is guaranteed only when the utility comparison is expressed as a linear function.

over time. This implies that the party's reservation values will change over the negotiation duration. Finally, there are negotiation costs that accrue over time. These costs include the effort involved to negotiate and the opportunity costs associated with not pursuing alternative settlements.

Since the present value of a future outcome is a function of the time it takes to realize the outcome, Cross argues that the parties estimate the time to settlement as part of their utility maximization. However, Cross does not assume that the parties have perfect information or that they can make interpersonal utility comparisons. Instead, he argues that each party makes an initial estimate of the other's party's rate of concession and updates this estimate with the actual rate of concession observed during negotiation. Hence, Cross's model only depends on the ability of each party to learn the other party's concession rate.

Cross proposes the following model to capture the sequence of concessions made during negotiation. Suppose two parties negotiate over a divisible good whose total quantity is *M*. Let q_1 and q_2 denote the quantity demanded by each party, respectively. Negotiation involves dividing the difference $(q_1 + q_2) - M$. Let $U_1(\cdot)$ represent Party 1's utility and r_2 denote Party 1's estimate of Party 2's concession rate. Let T denote the total time to settle which depends on r_2 through the equality $T = {[(q_1 + q_2) - M]}/{r_2}$. Let C_1 denote Party 1's fixed cost of negotiation during each time period and let a_1 denote the discount rate for $U_1(\cdot)$ and C_1. Cross assumes that discounting is exponential, so that the discounted utility at time T for q_1 is given by $U_1(q_1)e^{-aT}$, and the total cost of negotiation is given by $C_1(T) = \frac{C_1}{a}(1 - e^{-aT})$. Consequently, Party 1 chooses q_1 that maximizes the net present value of utility:

$$U_1^{net}(q_1) = U_1(q_1)e^{-aT} - \frac{C_1}{a}(1 - e^{-aT}). \tag{2}$$

Party 2 faces a similar utility maximization problem. Cross proposes that a settlement occurs when the party's demands equal the available surplus or when $q_1 + q_2 = M$.

Cross's model offers several contributions to an understanding of negotiation. It is the first model to incorporate the effect of time discounting on the negotiation process. If there is no cost associated with negotiation, then the parties would continually change their bargaining power *ad infinitum* without ever reaching a settlement. Second, Cross's model provides a mechanism for estimating each party's sequence of offers in terms of the estimated rate of concessions of both parties. Note that in estimating the other party's concession rate, Cross does not assume that a party has knowledge of the other party's utility function.

Unfortunately, Cross leaves open exactly how learning takes place, providing only first order conditions that a learning function should satisfy. For example, Cross argues that if Party 2's actual concession rate is faster than Party 1's expectation of this rate, then Party 1 will increase the value of its expectation, and the amount of increase will depend on the size of the difference between the actual and expected concession rates. Cross does not provide an exact specification for how the updating occurs. Before I explain some possible remedies to Cross's model, I turn next to a brief description of more recent theories of negotiation.

2.2.2 Game Theory Models

Negotiation models subsequent to Cross's model are grounded in game theory. A "game" is any situation in which each party's welfare depends on the party's own preferences and actions as well as those of the other party (Luce and Raiffa 1957). A game theory model of negotiation ("bargaining problem") specifies a set of axioms governing conduct, the payoffs to each party, and possible strategies. The set of strategies that are best responses to each other are called a *Nash equilibrium*. A *solution* to a bargaining problem is a rule specifying the Nash equilibrium of the game. A central focus of game theory is to derive conditions that guarantee certain properties of a solution (e.g., existence of a unique Nash equilibrium). Consequently, game theory focuses on the negotiation outcome(s), given specific types of negotiation processes. Fundamentally, a game theory model takes the process as given and seeks outcomes that result from the specified process.

Two types of game theory models have been applied to negotiations. In *cooperative* models, negotiation is assumed to satisfy several *a priori* axioms. Cooperative models usually have a unique equilibrium expressed as the maximum of some function of the individual's utilities. In *non-cooperative* models, the negotiation process is assumed to follow a specific sequence and the parties act in response to the information available to them at a point in time. Non-cooperative games may have more than one equilibrium solution, depending on the information each party is assumed to possess at the start of negotiation (e.g., information about the other party's preferences). Although the approaches to cooperative and non-cooperative games differ in many ways, the two perspectives can be viewed as complementary with respect to their explanation of negotiation outcomes (Sutton 1986).

Most game theory models of bargaining share a common abstract representation of the bargaining process (Thomson 1994). Two or more parties have access to a set of alternatives called the *feasible set*, denoted by S. The parties have different preferences for the alternatives in S. If the parties come to an agreement, they both will receive a settlement in S. If the parties do not agree, they each obtain a pre-specified alternative called the conflict point, denoted by c (which is also in S). The parties' preferences for the alternatives and the conflict point are captured in their utility functions. The bargaining solution is a prediction of the settlement, given a set of axioms dictating the behavior of the parties. The axioms may be interpreted as a set of normative objectives of fairness that the parties each obey during the negotiation process. Hence, different sets of axioms embody alternative rules of conduct that the parties are thought to operate under during negotiation. The principle models and results from both cooperative and non-cooperative game theory traditions are presented next.

2.2.2.1 Cooperative Games

The first cooperative game theory model of negotiation is Nash's Bargaining Model (Nash 1950). This model is notable both in its simplicity and in its widespread use as a benchmark in subsequent research. An important factor influencing a settlement in Nash's model is the existence of a *conflict point* or an outcome arising from no settlement. The parties use the conflict point to value possible outcomes. Nash's model is based on the following axioms thought to characterize bargaining. In the statement of these axioms, *S* is the feasible set, and *F(S)* is the settlement of *S*.

(Axiom 1) *Cardinal Utilities*: Each party's utility is unique up to an affine transformation.

(Axiom 2) *Symmetry*: The settlement is valued the same for all parties.

(Axiom 3) *Independence of Irrelevant Alternative (IIA):* If $S' \subseteq S$ and $F(S) \in S'$, then $F(S') = F(S)$.

(Axiom 4) *Pareto-Optimality:* $F(S) \in PO(S) = \{s \in S \mid s \geq s', \forall s' \in S\}$.[9]

Nash shows that if negotiation satisfies these assumptions, then there is a unique equilibrium. This outcome is the solution maximizing the product of each party's utility with respect to its conflict point. In particular, if the *i-th* party's utility is U_i and the utility of no settlement is c_i, then the Nash Bargaining Solution is the maximum of $(U_1 - c_1)(U_2 - c_2)$. In the case where the parties negotiate solely over price and each party's utility for the conflict point is zero, then the settlement is the outcome that gives

[9] Note that Axiom 4 implies every negotiation ends in a settlement other than the conflict point. The fact that many actual negotiations often end with no settlement suggests that Axiom 4 may not be characteristic of actual negotiations.

each party the same profit. Hence, the Nash Bargaining Solution can be interpreted as emphasizing the party's concern for equity in the outcome (Corfman and Gupta 1993).

Several alternatives to Nash's static bargaining model have been proposed. Each alternative results from "relaxing" or altering one or more of the four axioms in Nash's model. Similar to the Nash model, these alternatives derive an equilibrium solution based on a particular negotiation process. Harsanyi (1955) replaces Axiom1 with the assumption that utilities are ordinal, and the parties are able to make interpersonal utility comparisons. This seemingly innocuous alteration changes the bargaining solution to the maximum of $U_1 + U_2$. Kalai-Smordinsky (1975) replaces Axiom 3 with the assumption of *Individual Monotonicity*.[10] The solution to the Kalai-Smordinsky bargaining problem is the maximum of $k_1U_1 + k_2U_2$ subject to the constraint that $\frac{k_1}{k_2} = \frac{U_1 - c_1}{U_2 - c_2}$. Geometrically, the Kalai-Smordinsky solution is the Pareto-optimal point that lines on the line connecting the conflict to the ideal point (or pair of maximal utilities). Finally, Gupta and Livne (1988) replace the conflict point with any *reference point* that is Pareto-superior to the conflict point, but not to the settlement. Gupta and Livne's solution is represented geometrically as the Pareto-optimal point on the line connecting the reference point to the ideal point. Figure 3 provides a geometric interpretation of these cooperative game theory models of negotiation.

[10] *Individual Monotonicty* is the assumption that if the feasible set S is enlarged to S' such that for every payoff to Party 1, the range of feasible payoffs to Party 2 is increased, then Party 2's final payoff in S' should be at least as large as Party 2's payoff in S (Corfman and Gupta 1993)

Although the cooperative bargaining models abstract away from the negotiation process, these models do provide important insights. First, since the axioms define situations for which the models are applicable, they may be interpreted as conditions under which the proposed bargaining solutions might occur. For example, the Nash Bargaining Solution provides for an equitable settlement when there are cardinal symmetric utility functions satisfying the IIA property. Hence, actual settlements that are not equitable may result from negotiations that fail one of the assumed properties (e.g., symmetry). Second, the unique solutions provide testable benchmarks for evaluating an empirical model of negotiation.[11] Cross (1965) shows that when both parties have the same discount rate and conflict point, the outcome of his model is equivalent to the Nash Bargaining Solution.

2.2.2.2 Non-Cooperative Games

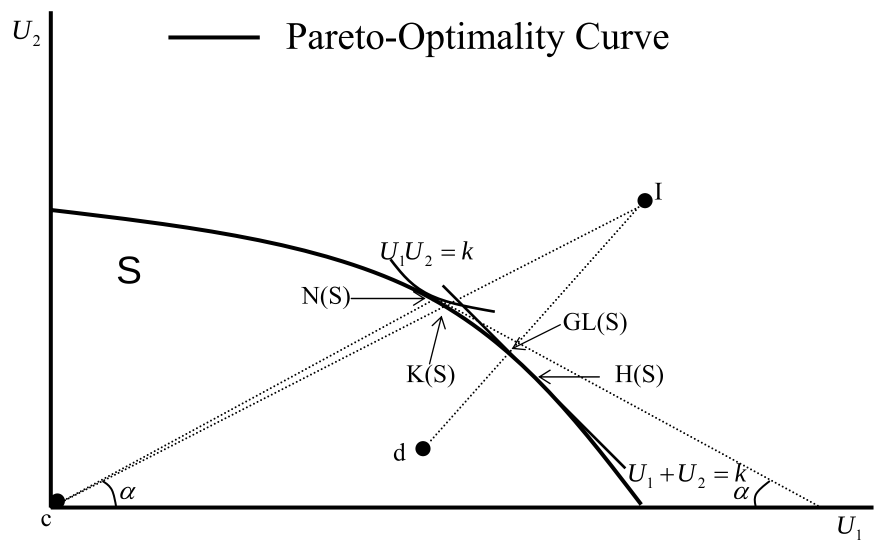
Non-cooperative games attempt to incorporate characteristics of the negotiation process into the axioms that prescribe allowable behavior by the parties. However, non-cooperative games often do not have a unique Nash equilibrium. Instead, sequential or subgame perfect equilibria are sought that represent a subset of *credible* Nash equilibria. Within non-cooperative games, there is a distinction between games that assume complete information and those that assume incomplete information (e.g., only

[11] In fact, several authors have shown that cooperative game theory models can predict negotiation outcomes in certain situations (Curry, Menasco, and Van Ark 1991; Eliashberg, LaTour, Rangaswamy, and Stern 1986; Menasco and Roy 1997). However, since these models leave the negotiation process unspecified, they are unable to fully explain why models fail. This "black box" approach to specifying the negotiation process of the earlier model provides motivation for including the negotiation process as part of the proposed model.

one party knows all the relevant information). I will briefly describe a prominent example of a game theory that is representative of each type of non-cooperative model in order to identify constructs that will be important in modeling the negotiation process.

Figure 3

GEOMETRIC REPRESENTATION OF BARGAINING SOLUTIONS



Note: S is the feasible set, c is the conflict point, d is a status quo point, and I is the ideal point. N(S) is the Nash solution, K(S) is the Kalai-Smordinsky solution, GL(S) is the Gupta-Livne Solution, and H(S) is the Harsanyi solution.

Rubinstein (1982) presents a non-cooperative game that assumes complete information. In this game, two parties seek to divide a fixed quantity *M*. Negotiation is characterized by the parties making alternating offers to receive a portion of *M*, giving the remaining portion to the other party. Each party is assumed to face some cost of delay that motivates them to settle early. This cost of delay is given by discount factors δ_1 and δ_2 for Party 1 and Party 2, respectively. Time is divided into periods, and in the odd-numbered periods Party1 makes an offer to Party 2 of some division $(x, M - x)$. If Party 2 accepts, then the game ends with Party 1 receiving the payoff $\delta_1^{t-1}x$ and Party 2 receiving the payoff $\delta_2^{t-1}(M - x)$. If Party 2 rejects, and period *t* is not the end of the game, then Party 2 makes a counter-offer to Party 1 and the roles of the parties are reversed.

Since any division of *M* is a Nash equilibrium, Rubinstein sought *subgame-perfect equalibria* or the set of credible Nash equilibria. Subgame-perfect equilibria rule out non-credible strategies such as a threat by Party 1 to walk away if that party does not receive 70% of *M*. This threat is not credible because if Party 2 were to offer taking only 10% of *M*, it would not be in Party 1's best interest to walk away with nothing (i.e., if Party 1 accepted, that party would get 90% of *M,* which is even better than the threat). Rubinstein showed that this game has a unique subgame-perfect equilibrium where a settlement is immediate and Party 1 receives the share $\frac{1-\delta_2}{1-\delta_1\delta_2}$ and Party 2 receives the share $\frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}$.

Rubinstein's solution has several important properties relevant to modeling the negotiation process. First, the model provides a useful way to conceptualize two-party negotiations as a sequence of alternating offers over the division of a fixed amount *M*. Second, the model identifies the important role that time discounting plays in how the parties value the settlement. Third, the solution has the appealing property that the more impatient a party is (i.e., the party with the greater discount factor), the less payoff that party will receive. This intuitive result provides some face validity for the model, since one would expect this type of outcome in actual negotiations.

The *Tunisian Bazaar* exemplifies non-cooperative models with *incomplete* information (Fudenberg, Levine, and Ruud 1986). In this model, a single seller negotiates with a single buyer for the price of a good. The value of the good to the seller is noted by *s*, and is common knowledge. The value of the good to the buyer is denoted by *v*, and it is only known through some probability distribution *Pr(v)* to the seller (the probability distribution is common knowledge). The negotiation process follows a similar offer structure as in Rubinstein's model. However, the seller is the only party that makes offers. The buyer either rejects or accepts the offer. If the price is rejected by the buyer, then the seller makes another offer. There is a discount factor δ that affects the payoffs. The seller's payoff is δp and the buyer's payoff is $\delta(v - p)$. Under suitable conditions, this game has a unique sequential equilibrium.[12] However, as with non-cooperative games, several empirical tests of negotiations that follow the rules of the Tunisian Bazaar show that actual behavior is not consistent with the sequential

[12] The conditions for a unique sequential equilibrium include the support for the distribution of the buyer's value strictly exceeds the seller's value *s*, the seller cannot bargain with anyone else, and the seller's prior beliefs about the buyer's valuation are uniform on (0,1].

equilibrium predicted by theory (Rapoport, Erev, and Zwik 1995; Srivastava, Chakravarti and Rapoport 2000).

Although lacking as a descriptive theory of actual negotiation behavior, the Tunisian Bazaar offers several clues on how to represent the negotiation process. First, many negotiations are characterized by parties that do not have complete information about each other's utility functions, but possess some probability distribution for each other's reservation price. Second, negotiations need not entail explicit offers from both parties. In consumer negotiations, the retailer is often the only party making explicit offers and the consumer either accepts or rejects each offer. Finally, the seller acquires information about the buyer's reservation price through the sequence of offers. Each rejection by the buyer provides information about the likelihood of reservation prices.

2.3 Marketing Literature

There are two main bodies of relevant marketing literature for the proposed model. The first body is comprised of the articles by Balakrishnan and Eliashberg (1995) and Chen, Yang and Zhao (2004), both of which offer structural models of the negotiation process. These models will provide a backdrop against which to explicate the contributions of the proposed model. The second body includes the rich stream of research in marketing on reference prices that will prove useful when specifying the proposed model. Several alternative reference price definitions are identified in this literature stream.

2.3.1 Process Models

Balakrishnan and Eliashberg (1995) draw on the economics and behavioral literatures to develop a model of the negotiation process based on the demand and concession model of Pruitt (1981) in which the parties' behavior during negotiation represents a tradeoff between a "resistance force" and a "concession force." The forces are specified as functions of each party's current offer price, and Balakrishnan and Eliashberg's model predicts the sequence of offers and counter-offers as a function of these opposing forces and each party's aspiration level, reservation price, subjective relative bargaining power, and time pressure. An agreement is reached when the offers exceed a party's concession point or the point at which the concession force vanishes. Balakrishnan and Eliashberg derive several propositions corresponding to the overall pattern of offers and a lower bound on negotiation duration. They test their model using a mail survey of industry experts and find that most experts agree with the predictions of the model. However, they provide no direct empirical evidence supporting their model.

Chen, Yang, and Zhao (2004) present a model of choice behavior when prices are negotiable. They derive a utility-based model that draws on the applied game theory literature (Iyer and Villas-Boas 2003) and incorporate each party's bargaining power, reservation price, and individual differences (e.g., race, gender, income, occupation). Chen, Yang, and Zhao define a party's bargaining power as the difference between a party's reservation price and the purchase price. A unique contribution of their approach is the ability to infer a party's bargaining power and reservation price from data containing only the final offer price and brand choice information regarding the purchase of a new automobile. They find that individual differences such as gender affect a party's bargaining power and the negotiated price. However, they show that heterogeneity in the price coefficient is not significant. This implies that it is sufficient to model an overall effect for price on negotiation outcomes.

2.3.2 Reference Prices

The marketing literature has long recognized that consumers use some reference point in evaluating the price of a product (Monroe 1973). This literature contains several conceptualizations of reference price formation. For many researchers reference prices are based on a consumer's memory of past prices of that brand or others in the same product category (Bucklin and Lattin 1989; Mayhew and Winer 1992). Others include contextual factors such as how often the brand is promoted, store characteristics, and price trends (Kalwani et al. 1990; Winer 1986). Another group of researchers argues that since consumers have poor memory of past prices (Dickson and Sawyer 1990), they are more likely to use current prices of certain brands (Hardie, Johnson and Fader 1993; Rajendran and Tellis 1994).

Reference price is often operationalized as an ordinary least squares regression model of price on relevant independent variables (e.g., past prices, contextual factors, etc.). A common justification for this specification is provided by the Rational Expectations Hypothesis (Muth 1961). Muth proposed that consumer price expectations are essentially the same as those predicted by economic theory. In particular, consumers learn over time the decision rules that retailers use to set price and form expectations about these pricing rules. Consumer expectations are drawn from a price distribution with mean zero, which implies that the price expectation is an unbiased estimate of the actual price. Consequently, consumer price expectations can be operationalized as a simple regression whose error distribution is serially uncorrelated across time and has mean zero (i.e., using an ordinary least squares equation).

Several empirical tests confirm the effect of reference price by including terms in the consumer's utility specification to represent perceived price gains and price losses (Kalwani et al. 1990; Mayhew and Winer 1992; Winer 1986). A perceived gain occurs when the list price is less than a consumer's reference price, whereas a perceived loss occurs when the list price is above the consumer's reference price. The empirical results are consistent with the *loss aversion* implication of Prospect Theory. In particular, results show that perceived losses have a greater impact on a consumer's choice than perceived gains. Briesch, Krishnamurthi, Mazumdar, and Raj (1997) tested a variety of reference price models to determine the best fit, prediction, and parsimony. They found that the best model of consumer's reference price was one that used the past history of a brand's prices. Furthermore, for many products, using only the most recent past price was sufficient to capture the effect of a consumer's reference price on brand choice.

Although the marketing literature on reference prices is limited to exogenous prices, a similar process may occur for negotiated prices. Buyers might form expectations about the final settlement price given their memory of past prices and the seller's current offer price. Several reference price specifications from the marketing literature are particularly relevant to the study of negotiation. For example, several researchers have found that consumers often use past prices of a brand as a reference price during future purchase occasions (e.g., Briesch et al. 1997; Bucklin and Lattin 1989; Hardie, Johnson, and Fader 1993; Kalwani, Yim, Rinne, and Sugita 1990; Kalyanaram and Winer 1995; Mayhew and Winer 1992). This suggests that negotiators may use the most recent own-brand offer as their reference price. Jacobson and Obermiller (1990) explore several reference prices based on the rational expectations theory of Muth (1961). Reference price is modeled as a regression of current price onto past purchase prices. This suggests that negotiators may use a weighted-average of past offers as a reference price. Similar definitions of competitor-brand reference prices have been used in the literature. The initial and most recent competitor-brand prices have been used in several articles (e.g., Briesch et al. 1997). Jacobson and Obermiller (1990) also consider a rational expectations model using competitor-brand prices, which suggests using a similar definition here. Several definitions of reference price from the marketing literatures are summarized in Table 1.

2.4 Literature Summary

Although economic models provide important insights into negotiation, several researchers have failed to find empirical support for game theory models of negotiation. For example, Ochs and Roth (1989) show Rubinstein's sub-game perfect equilibrium is not consistent with actual behavior. They found disagreements occurred more often than predicted, counter-offers were less advantageous than initial offers, and observed outcomes were more consistent with an equitable division of the available payoff compared to the normative predictions. Srivastava, Chakravarti, and Rapoport (2000) show that actual negotiations often last longer than the duration predicted by the noncooperative game theory of Fudenberg, Levine, and Ruud (1985). Also, Rapoport, Erev, and Zwik (1995) show that there is little empirical support for non-cooperative games with incomplete information. Collectively this research suggests that game theory models may prove too restrictive to explain actual negotiation behavior.

Table 1

REFERENCE PRICES IN MARKETING AND BEHAVIORAL LITERATURES[a]

Reference Price	Definition
Initial price (Northcraft and Neale 1987)	$Price(t=0)$
Most recent price (Mayhew and Winer 1992)	$Price(t-1)$
Regression of past prices (Jacobson and Obermiller 1990)	$\sum \hat{\beta}_j Price(t_j < t)$
Average past prices (White and Neale 1996)	$Ave.Price(t_j < t)$
Initial competitor price (Briesch et al. 1997)	$CompPrice(0)$
Most recent competitor price (Briesch et al. 1997)	$CompPrice(t-1)$
Regression of past competitor prices (Jacobson and Obermiller 1990)	$\sum \hat{\gamma}_j CompPrice(t_j < t)$
Average past competitor prices (Blount et al. 1994)	$Ave.CompPrice(t_j < t)$

[a] *Price* is the own-brand price; *CompPrice* is the competitor-brand price; $\hat{\beta}_j, \hat{\gamma}_j$ are estimated coefficients from the OLS regression of *Price(t)* onto past prices.

Although the models introduced by Balakrishnan and Eliashberg (1995) and Chen, Yang, and Zhao (2004) overcome some of the limitations of prior research, they each suffer from limitations. For example, the Balakrishnan and Eliashberg (1995) model requires that at least two offers are observed from the buyer in order to estimate model parameters. This obviously limits the applicability of the model. The model of Chen, Yang, and Zhao (2004) is limited to negotiations that eventually end in a purchase. However, many negotiations result in the parties failing to reach agreement. Finally, both models assume that the structural parameters (e.g., reservation price and bargaining power) are fixed over time. However, these parameters are likely to vary over time in response to changing situational variables, such as rising negotiation costs due to an impending deadline.

2.4.1 Research Contributions

There are five main contributions of the proposed model. First, the proposed model captures how a buyer's reservation value changes over time. Prior negotiation models assume buyers maintain a fixed reservation value throughout negotiation. The proposed model predicts a buyer's reservation value as a function of time, negotiation costs, a discount rate, and the seller's offer rate. The model quantifies the intuition that when buyers face a purchase deadline, the instantaneous probability of purchase (i.e., the purchase hazard rate) should decrease over time and as negotiation costs increase, but increase with the seller's offer rate and average relative offer. Second, the proposed model easily leads to an empirically estimable model. Prior negotiation models (e.g., those based on game theory) are often not estimable, making it difficult to test their implications using actual negotiation data. Consequently, it is relatively easy to empirically validate the proposed model. Third, the proposed model is empirically validated using a sample of actual negotiations over the rental of a durable product. Prior empirical negotiation research relies almost exclusively on simulated negotiations within a laboratory setting. Consequently, the present results provide managerially relevant insights into real negotiation processes between a buyer and seller. Fourth, the model does not assume that a buyer always reaches agreement with a seller. Consequently, the proposed model can differentiate between how the relevant parameters affect the choice to purchase versus the choice to exit the negotiation. Fifth, the proposed model clarifies the effect of a buyer's reference price on the negotiation outcome. The prior negotiation literature offers conflicting findings on which reference price is most important. In the proposed model, the effect of alternative reference prices can be precisely estimated. In summary, the proposed model overcomes the limitations of prior models of negotiation while providing both theoretically and managerially relevant insights on actual negotiations.

3 Search Theory

The choices characterizing asymmetric negotiations are very similar to the choices that an unemployed individual makes when searching for a new job (Merlo 1997). Remarkably, many of the constructs and relationships in job-search models are analogous to those characterizing asymmetric negotiations. The economic theory of job-search describes the dynamics of labor force participation in terms of a job-seeker's decision making under uncertainty (Flinn and Heckman 1982). The process of job search is characterized by a series of choices to either accept or reject wage offers made by prospective employers. Job-seekers are assumed to maximize utility by seeking the highest paying jobs (largest wage offers). In the simplest models, the possible choices are to either accept a current wage offer or to continue searching. The choice is made by comparing the value of a current wage offer with the expected present value of searching over some short period in the future. Job-seekers are assumed to posses a reservation wage, such that wage offers above the reservation wage are accepted, and wage offers below the reservation wage are rejected and job search continues. The entire process is represented by the job-seeker existing in either a state of search or a state of employment. Job-search models predict the transitions between these states, given the optimality constraints imposed by the properties of the reservation wage, utility maximization and the choices characterizing job search. Additionally, these models identify and measure the effect of factors impacting the transition probabilities.

The analogy between job-search and asymmetric negotiation is straight-forward. Buyer-negotiators are like job-seekers searching for a new job. Buyers maximize utility by seeking the lowest offer price. Buyers either accept a current offer price or continue negotiating. The choice is made by comparing the utility of the current offer with the expected present value of negotiation over a short period in the future. Buyers are assumed to posses a reservation price, such that offer prices below the reservation price are accepted and offers above the reservation price are rejected and negotiation continues. The entire process is represented by a series of state-spaces in which the consumer occupies only a single state at any particular time. A model of this process can be used to explain and predict the transitions between states given a set optimality constraints.

3.1 A Static Negotiation Model

In this section, I describe a static job search model in terms of its analogous constructs in an asymmetric negotiation over the purchase of a product. The structural model is similar to the two-state, structural job-search model of Lancaster (1990) in that the negotiation process consists of two states: negotiation (n) and purchase (p). Buyers start out negotiating and eventually transition to the purchase state, where they permanently reside. The negotiation transpires over a finite time horizon. There is no learning and all parameters and distributions are known to the buyer. The buyer's instantaneous utilities at time *t* in the purchase and negotiation states are $u_p(t) = r(t)$ and $u_n(t) = -c$, respectively. The *relative offer r(t)* is a realization of the random variable $R(t) = \rho(t) - P(t)$, where $\rho(t)$ is the buyer's reference price and *P(t)* is the seller's offer price at time t, respectively.[13] Relative offers are assumed to be positive valued. In the subsequent discussion, the terms "relative offers" and "offers" are used synonymously. The density function for *R(t)* is denoted by *f* and the distribution function is *F*. The offer distribution *f* is assumed to be independent of time. The offers arrive in a Poisson process at the rate λ per unit of time. The value -*c* is the negotiation cost, which is fixed over time. Future utility is discounted at the constant rate δ .

The buyer is assumed to be forward looking but myopic in assessing future outcomes. This means that at each time *t*, the buyer's choice of what state to occupy is based on the buyer's consideration of the net present utility of being in each state over a short horizon *h*. The buyer chooses the state with the highest expected net present utility over *h*. The buyer incurs a negotiation cost, discounted for the time that transpires over the short horizon. The horizon is chosen small enough to ensure that if an offer arrives, one and only one offer arrives within the horizon. If a new offer arrives, the buyer evaluates the expected net present utility of the new offer with respect to the reservation value. This expectation will depend on the relative offer distribution along with the discounted value of purchasing at the relative offer value. If no new offer arrives, the buyer will continue to receive the net present utility of negotiation. The total net present utility of negotiation is the sum of the discounted negotiation costs, the discounted utility of purchase given that a new offer arrives, and the discounted utility of continued negotiation given that no new offer arrives.

[13] In the subsequent analysis, several reference price specifications are considered. Some of these specifications allow the reference price to change over time.

The buyer's tradeoffs during negotiation can be formalized in the following way. The present utility of purchase at *t* is the instantaneous utility of purchase discounted over an infinite time horizon, or $V_p\left(r(t),-c\right)=\frac{r(t)}{\delta}$.[14] The present value of negotiation over *h* is the discounted cost of negotiation plus the expected present utility of continued negotiation. The negotiation cost is given by $-ch$. Since offers arrive in a Poisson process with rate λ, the probability of receiving a new offer within the short horizon *h* is λh, and the probability of not receiving a new offer during the short horizon is $(1-\lambda h)$. If no new offer arrives, the buyer continues to obtain the present utility of negotiation V_n. However, if a new offer *r(t)* arrives, the buyer receives either the present utility of purchase, $V_p\left(r(t),-c\right)$, or the present value of negotiation, whichever is larger. Therefore, the expected present utility of negotiation at time *t* is:

$$(3) \qquad V_n=\frac{-ch}{\left(1+\delta h\right)}+\frac{(1-\lambda h)}{(1+\delta h)}V_n+\frac{\lambda h}{(1+\delta h)}E_R\left\{\max\left(V_n,V_p\left(r(t)\right)\right)\right\}$$

In equation (3), $(1+\delta h)^{-1}$ is the discount factor over the short period *(t, t+h)*. The expectation E_R is taken with respect to *f* and can be simplified by the following:

$$(4) \qquad E_R\left\{\max(V_n,V_p\left(r(t)\right)\right\}=V_n+\frac{1}{\delta}\int_{\xi}^{\infty}(r-\xi)dF\ .$$

[14] This assumes risk-neutral utility and that the buyer will enjoy the product from *t* to ∞. Hence, the present value of consumption is the discounted instantaneous utility over an infinite time horizon.

The term $\xi = \delta V_n$ is the offer that equates $V_p(r(t))$ to V_n. Hence, ξ is the reservation offer that determines when the consumer purchases. Once the reservation offer is calculated, the optimal decision rule is to purchase when the offer is greater than ξ. The value of ξ is found by substituting (4) into (3), multiplying by $h^{-1}(1+\delta h)$, taking the limit as $h \to \infty$, and substituting $\xi = \delta V_n$. This yields the following implicit function for ξ :

$$\xi = -c + \frac{\lambda}{\delta}\int_{\xi}^{\infty}(r-\xi)dF \,. \tag{5}$$

Equation (5) has a unique solution and acts as a constraint on the consumer's optimal decision path.[15] Equation (5) is the structural model for the buyer's static reservation value. This describes the reservation value ξ as a function of the discount rate δ, the negotiation cost *c*, the offer rate λ, and the expected relative offer value. Note that it is possible for $V_n < 0$, in which case the optimal decision would be to stop negotiating and to not purchase. This possibility requires a second exit state of no-purchase.

Several probability distributions can be deduced from the above analysis. Some of these are shown below and will be referred to in the subsequent analysis. In these expressions, the following variables are used: *t* denotes the negotiation duration; *a*

[15] See the Appendix for details on this and other derivations.

denotes the accepted offer; *n* denotes the number of rejected offers; and $r_1,...,r_n$ denotes the rejected offers. These distributions assume that data exist for each variable in the distribution (e.g., no variable is censored or unobserved)

The probability distribution of the negotiation duration, denoted by $g(t)$, is given by equation (6). This is found by first considering the hazard function or the probability that a buyer purchases in the short interval *(t,t+h)*. This equals the probability the buyer receives an offer, λh, times the probability that the offer exceeds the reservation offer, $\overline{F}(\xi) = 1 - F(\xi)$. Therefore, the hazard function associated with entering the purchase state is $\theta(t) = \lambda\overline{F}(\xi)$. The hazard is independent of time since the parameters are stationary by assumption. Note that the hazard for ending negotiation is expressed in terms of the offer distribution. This relationship characterizes all structural search models and is an important feature that a formal hazard specification would have to satisfy in order to reflect optimal decision making.

The duration probability distribution (6) is found by applying properties of hazard functions. Equation (7) is the probability distribution of accepted offers. Since the probability of acceptance is independent of time, the joint distribution (8) of duration and accepted offers is the product of (6) and (7). Finally, (9) is the joint distribution of the negotiation duration (*t*); the accepted offer (*a*); the number of rejected offers (*k*), and the rejected offers ($r_1,...,r_n$). A derivation of (9) is given in the Appendix.

(6) $$g(t) = \lambda\overline{F}(\xi)e^{-\lambda\overline{F}(\xi)t}\text{, for } t \geq 0$$

(7) $$g(a) = \frac{f(a)}{\overline{F}(\xi)}, \quad \text{for a} > \xi$$

(8) $$g(a,t) = \lambda f(a) e^{-\lambda \overline{F}(\xi)t}, \quad \text{for a} > \xi; t \geq 0$$

(9) $$g(r,a,k,t) = \lambda^{k+1} e^{-\lambda t} f(a) \prod_{i=1}^{k} \frac{f(r_i)}{k!},$$

$$\text{for } r_1,\ldots,r_k < \xi;\ k = 1,2,\ldots;\ a \geq \xi;\ t \geq 0$$

Note that in (9), the joint density is expressed only in terms of the parameters of the observed variables. In particular, the reservation price ξ appears only in the constraint. This fact will play an important role in model estimation. However, before I discuss how to estimate this model, I turn next to several extensions of the basic model that are needed in order to capture key aspects of the negotiation process.

3.2 Static Model Extensions

The static model makes several restrictive assumptions. In particular, the parameters do not depend upon time. This implies that: (a) the probability of purchase is independent of time and (b) the negotiation duration does not depend on the exit state.[16] Hence, the static model only reflects the situation when the purchase probability depends on a negotiation having taken place (i.e., irrespective of its duration). The static model does not capture the effect of time-varying parameters, e.g., $u_n(t) = -ct$. In the paragraphs below, I present several extensions to the static model that address the

[16] Implication (b) results from properties of mixed duration models (e.g., competing risks) when the state-specific "hazards" are constant over time (Lancaster 1990).

assumption of time-invariant parameters and other limitations. Some of these extensions will be employed in my proposed model.

3.2.1 Multiple States and Transitions

Although the description of the static model only has two-states, it can accommodate additional exit states. However, these states must be defined with respect to the same utility function (e.g., "no purchase" when $V_n < 0$). Sometimes the transitions to separate exit states depend on decision rules derived from different utility functions. For example, if a consumer simultaneously negotiates with several sellers over the same product, then the choice to not purchase from one seller depends on the expected present value of utility obtained from another seller. In this case, several reservation offer values are needed to specify the model (e.g., $\xi_1,...,\xi_K$), with each one corresponding to a different utility function associated with purchasing from another seller. For example, in the case of two sellers, the utility for negotiation (n), purchase (b), and no purchase (b') would be: $u_n(t) = -c$, $u_b(t) = d(t)$, and $u_{b'}(t) = e(t)$. In this case, the expected present utility for negotiation includes additional terms corresponding to the optimal choice between negotiation and no purchase. Also, there is an additional implicit function similar to equation (5) specifying the constraint on the second reservation offer (cf., Flinn and Heckmann 1982). Unfortunately, the additional negotiations are often not observed, limiting applications of this approach to reduced form specifications of additional utility functions.[17]

3.2.2 Time-varying Parameters

The most severe limitation of the static model is the assumption that the offer distribution *f*, the offer rate λ, the discount rate δ, and the costs *c* are independent of time. However, some of these parameters are likely to change over time in an actual negotiation. For example, a buyer's knowledge of the offer distribution may increase over time as the buyer acquires more information through the sequence of offers made by the seller. In this case, *f* would depend on time. Also, it is reasonable to assume costs increase over time. In fact, increasing costs are a primary motivation to achieve an early settlement (Cross 1965). Unfortunately, allowing these parameters to vary over time substantially complicates the basic model. For example, if costs depend on time, then the optimal reservation offer will also depend on time (i.e., $\xi = \xi(t)$). I will return to this issue in the description of the proposed model.

[17] Van Den Berg (1990) notes that one rarely observes returns to non-participation (i.e., the equivalent of the no-purchase state) and therefore assumes the transition to the out-of-market state is a Poisson process with transition rate ς.

4 Proposed Model

The behavioral and economics literatures provide complementary theories of negotiation behavior. The proposed model of negotiation behavior is based on assumptions drawn from both literatures. Several properties are adopted from the two literatures as “structural” features of the proposed model. Although formally these properties are treated as model assumptions, they are justified by empirical and theoretical results in the two literatures. However, since the best specification of reference price is likely to depend on the particular application, the issue of how to specify the reference price is left as an empirical question to be explored by the reduced-form model.

4.1 Model Assumptions

Buyer preferences in the proposed model are represented by a von-Neumann and Morgenstern (1944) utility function and observed choices result from maximizing a buyer’s indirect utility function (Mas-Colell, Whinston, and Green 1995). Following Koopmans (1960), Cross (1965), and others, the present utility of consumption over time is assumed to be a discounted function of the utility in each time period. In particular, there exists a discount rate δ such that the present utility (V) of future consumption equals the discounted utility at the time of consumption. For example, if x_T represents consumption at some future time T, then the present utility of x_T is $V(x_T) = \delta_t u(x_T)$. Sequences of consumption over time are similarly discounted.

Therefore, the present utility of the consumption sequence $x = (x_0, x_1, ..., x_T)$ is given by $V(x) = \sum_{t=0}^{T} \delta_t u(x_t)$. If consumption lasts indefinitely, the present utility is the infinite sum of discounted utilities over time. The proposed model assumes that buyers maximize the net present value of utility, where costs are treated as consumption with negative utility.

Several researchers have shown that when evaluating a sequence of future events, more attention is given to outcomes occurring in the near future (e.g., Prelec and Lowenstein 1991), so that utility maximization effectively occurs only over a short time horizon (Strotz 1955). This suggests that buyers myopically maximize utility during negotiation by only considering outcomes occurring over a short time horizon. Also, Prospect Theory (Kahneman and Tversky 1979) implies that buyers value offers with respect to a reference point, typically a reference price. In the context of brand choice, the marketing and behavioral negotiation literatures have identified several types of reference prices (Kalyanaram and Winer 1995; Northcraft and Neale 1987; White and Neale 1994). Since the negotiation literature provides equivocal support as to which reference price dominates during negotiation, the precise specification of the reservation price is undefined in the structural model. Instead, buyers are assumed to consider *relative offers*, defined as the difference between the current offer price $P(t)$ and the reference price $RP(t)$. Finally, the model assumes there is a minimum threshold or reservation value ξ with the property that all offers above ξ are accepted and all offers below ξ are rejected (Raiffa 1982). Buyers are indifferent between accepting and rejecting the reservation value. The reservation value ξ will be allowed to vary with time.

The model incorporates the same abstract representation of an asymmetric negotiation process as commonly found in the game theory literature (cf. Fudenberg, Levine, and Ruud 1985). The representation is augmented by modifications regarding how buyers value an offer and myopically consider future prospective utility streams. Only the seller makes explicit offers and the buyer either accepts or rejects the seller's offers. If an offer is rejected, the buyer can either continue negotiating or exit negotiation. If negotiation continues, the seller makes a new offer.[18] Once an offer is rejected, it is assumed that the buyer cannot go back to the previous offer at a later date. If an offer is accepted, the buyer purchases at the most recent offer price. Hence, the negotiation process is characterized by a sequence of offers by the seller and decision by the buyer to accept or reject each of the seller's offers.

4.2 Conceptual Model

As previously mentioned, The choices characterizing asymmetric price negotiations are very similar to the choices made when searching for a new job (Merlo 1997; Flinn and Heckman 1982) or a new automobile (Ratchford and Srinivasan 1993). Buyers maximize utility by seeking the highest relative offer less the negotiation costs.

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[18] Note that since negotiations may proceed over a long duration, the seller's offers may fluctuate due to changing market conditions that may affect its costs (e.g., inventory fluctuations).

The seller makes offers to buyers, who accept, continue negotiating, or exit negotiation. A choice is determined by comparing the utility of the current relative offer with the expected present value of negotiation over a short period in the future. The events in the negotiation process are represented by a set of state-spaces. The buyer occupies a single state at a particular time. Transitions between states correspond to choices during negotiation. An illustration of this conceptual model of the negotiation process in terms of state-space transitions is depicted in Figure 5.

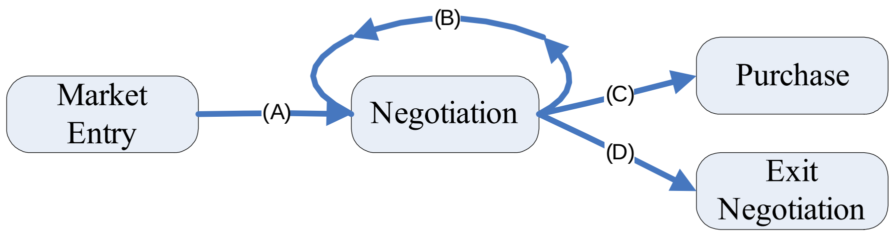
The state transitions in Figure 5 correspond to choices during the negotiation process. The buyer starts in the market entry state. If the buyer chooses to negotiate, the buyer transitions along the path (A) to the negotiation state.[19] After transitioning to the negotiation state, the buyer has several options. The buyer can continue negotiating along path (B), purchase along path (C), or exit negotiation along path (D). If a buyer enters the purchase or exit negotiation states, the buyer permanently resides there. The transition from negotiation state to negotiation state represents a choice to continue negotiating, and the entire process may entail multiple loops within the negotiation state.

Note that the model can be generalized to accommodate multiple purchases or simultaneous negotiations with more than one seller with the addition of more state-spaces (not pictured here). Also, since the negotiation occurs over time, the probability of transitioning to the purchase state is not necessarily the complement of the

[19] In the subsequent structural model, buyers are assumed to start in the negotiation state.

probability of transitioning to the exit negotiation state. This situation contrasts with a static choice model, where choices do not depend on time, and the probability of purchasing equals one minus the probability of not purchasing.

Figure 4
STATE TRANSITIONS IN PROPOSED MODEL



4.3 Structural Model

This section describes the proposed structural model for asymmetric price negotiations. Several properties of this model are derived when relative offers are assumed to be exponentially distributed and negotiation cost increases over time. The instantaneous probability of transitioning to the purchase state is shown to be a proportional hazard function. This proportional hazard will facilitate estimation and interpretation of an empirical model that is used to test several properties of the model. First, a description of the two-state model is given in which buyers exist in either a negotiation or a purchase state. Later this model is generalized to include an exit negotiation state.

4.3.1 Structural Model of Time-varying Reservation Value

The proposed structural model is similar to the two-state, static structural job-search model of Lancaster (1990) described in Section 3.1. The negotiation process consists of two states: negotiation (n) and purchase (p). Buyers start out negotiating and eventually transition to the purchase state, where they permanently reside. The negotiation transpires over a finite time horizon. There is no learning and all parameters and distributions are known to the buyer. The buyer's instantaneous utilities at time *t* in the purchase and negotiation states are $u_p(t) = r(t)$ and $u_n(t) = -c(t)$, respectively. The *relative offer r(t)* is a realization of the random variable $R(t) = \rho(t) - P(t)$, where $\rho(t)$ is the buyer's reference price and *P(t)* is the seller's offer price at time t, respectively.[20] Relative offers are assumed to be positive valued. In the subsequent discussion, the terms "relative offers" and "offers" are used synonymously. The density function for *R(t)* is denoted by *f* and the distribution function is *F*. The offer distribution *f* is assumed to be independent of time. The offers arrive in a Poisson process at the rate λ per unit of time. The function *c(t)* is the negotiation cost, which increases over time. Future utility is discounted at the constant rate δ.

[20] In the subsequent analysis, several reference price specifications are considered. Some of these specifications allow the reference price to change over time.

The structure of this model is the same as the static model presented in Section 3.1. The buyer is assumed to be forward looking but myopic in assessing future outcomes. The buyer chooses the state with the highest expected net present utility over *h*. The buyer incurs a negotiation cost, discounted for the time that transpires over the short horizon. The horizon is chosen small enough to ensure that if an offer arrives, one and only one offer arrives within the horizon. If a new offer arrives, the buyer evaluates the expected net present utility of the new offer with respect to the reservation value. If no new offer arrives, the buyer will continue to receive the net present utility of negotiation. The total net present utility of negotiation is the sum of the discounted negotiation costs, the discounted utility of purchase given that a new offer arrives, and the discounted utility of continued negotiation given that no new offer arrives.

The buyer's tradeoffs during negotiation can be formalized in the following way. The present utility of purchase at *t* is the instantaneous utility of purchase discounted over an infinite time horizon, or $V_p\left(r(t),-c(t)\right)=\frac{r(t)}{\delta}$.[21] The present value of negotiation over *h* is the discounted cost of negotiation plus the expected present utility of continued negotiation. The negotiation cost is given by $-c(t+h)$. Since offers arrive in a Poisson process with rate λ, the probability of receiving a new offer within the short horizon *h* is λh, and the probability of not receiving a new offer during the short horizon is $(1-\lambda h)$. If no new offer arrives, the buyer continues to obtain the present

[21] This assumes risk-neutral utility and that the buyer will enjoy the product from *t* to ∞. Hence, the present value of consumption is the discounted instantaneous utility over an infinite time horizon.

utility of negotiation V_n. However, if a new offer *r(t)* arrives, the buyer receives either the present utility of purchase, $V_p\left(r(t),-c(t)\right)$, or the present value of negotiation, whichever is larger. Therefore, the expected present utility of negotiation at time *t* is:

$$(13) \qquad V_n(t) = \frac{-c(t+h) + (1-\lambda h)V_n(t+h) + (\lambda h)E_R \max\left(V_n(t+h), V_p\left(r(t)\right)\right)}{\left(1+\delta h\right)}.$$

In equation (13), $(1+\delta h)^{-1}$ is the discount factor over the period *(t, t+h)*. The expectation E_R is taken with respect to the offer distribution *f*. Multiplying equation (13) on both sides by $h^{-1}(1+\delta h)$ yields:

$$(14) \quad \frac{V_n(t) - V_n(t+h)}{h} + \delta V_n(t) = -c(t+h) - \lambda V_n(t+h) + \lambda E_R\left(\max(V_n(t+h), V_p\left(r(t)\right)\right).$$

Taking the limit $h \to 0$ gives:

$$(15) \qquad -\frac{d}{dt}V_n(t) + \delta V_n(t) = -c(t) - \lambda V_n(t) + \lambda E_R\left(\max(V_n(t), V_p\left(r(t)\right)\right).$$

The expectation can be written (after taking the limit as $h \to 0$) as:

$$(16) \qquad E_R\left(\max(V_n, V_p(r)\right) = V_n \int_0^{\delta V_n} dF + \int_{\delta V_n}^{\infty} \frac{r(t)}{\delta} dF .$$

The term $\xi(t) = \delta V_n(t)$ is the reservation value that equates the net present utility of purchase, $V_p(r(t))$, to the net present utility of continued negotiation, $V_n(t)$. Recall that F is the cumulative distribution function for the relative offers. The reservation value $\xi(t)$ provides a rule for determining if the buyer purchases at each time t. The decision rule is to purchase when the relative offer *r(t)* is greater than $\xi(t)$. Substituting $\xi(t) = \delta V_n(t)$ into (16), and adding and subtracting terms yields:

$$(17) \qquad E_r\left(\max(V_n(t), V_p(r(t))\right) = V_n(t)\int_0^{\xi(t)} dF + V_n(t)\int_{\xi(t)}^{\infty} dF - V_n(t)\int_{\xi(t)}^{\infty} dF + \int_{\xi(t)}^{\infty} \frac{r(t)}{\delta} dF$$

$$= V_n(t) + \frac{1}{\delta}\int_{\xi(t)}^{\infty} (r(t) - \xi(t))dF \ .$$

Substituting (17) into (15) yields the following implicit function for $\xi(t)$:

$$(18) \qquad \frac{d}{dt}\xi(t) = \delta\xi(t) + \delta c(t) - \lambda \int_{\xi(t)}^{\infty} \left(r(t) - \xi(t)\right) dF \ .$$

Thus, equation (18) is the proposed structural model for the buyer's time-varying reservation value. Note that equation (18) is a nonlinear differential equation of $\xi(t)$ with respect to time. This equation describes how the reservation value $\xi(t)$ changes over time as a function of the discount rate δ, the negotiation cost *c(t)*, the offer rate λ, and the expected relative offer value. Equation (18) acts as a constraint on the buyer's transition path between the state of negotiation and the purchase state. This equation is used to derive several interesting properties that govern asymmetric price negotiations.

Equation (18) has a unique solution as long as the negotiation cost is finite. This condition is guaranteed when the buyer faces a purchase deadline. If the purchase deadline occurs at some point in time T, then the negotiation cost is fixed after T or $c(t) = c_T$ for $t \geq T$. However, since the negotiation cost is assumed to be fixed after the deadline, this implies that the reservation value will be fixed for $t \geq T$ or $\xi(t) = \xi_T$. A similar derivation to the one above shows that for $t > T$, the fixed reservation value ξ_T is implicitly defined by the following equation:[22]

(19) $$\xi_T = -c_T + \frac{\lambda}{\delta} \int_{\xi_T}^{\infty} (r - \xi_T) dF .$$

Equation (19) provides a terminal condition that can be used to solve differential equation (18) for the time-varying reservation value $\xi(t)$ when $t \leq T$. Equation (19) is also useful to formally prove the properties of the structural model.

4.3.2 Structural Model Implications

There are several interesting implications of the proposed structural model of asymmetric price negotiation. First, in order to ensure that a buyer's decision rule for

[22] See Section 3.1.

purchasing will be met within the time left, the buyer must lower his/her reservation value $\xi(t)$ over time. Hence, the reservation value will decrease over time. Second, since the negotiation cost is increasing, equation (13) implies that the buyer faces a decreasing expected net present utility from continued negotiation. Consequently, the buyer's reservation value $\xi(t)$ will decrease as negotiation costs increase. If the relative offers are exponentially distributed, then the instantaneous probability of purchase or purchase hazard rate is given by the expression $\theta(t) = \lambda \exp(-\lambda\xi(t))$. Hence, the hazard rate $\theta(t)$ increases if and only if the reservation value $\xi(t)$ decreases. Consequently, the hazard rate will increase with increases in both time and negotiation cost. These properties are summarized in the following proposition, whose formal proof is given in the Appendix:

> Proposition 1: The purchase hazard rate $\theta(t)$ increases as time and negotiation cost increase.

Although the relative offer distribution and offer rate are assumed fixed over time, how the buyer might react to changes in these parameters is interesting because the offer distribution and offer rate are decision variables for the seller. Consider the effect of an increase in the average relative offer on the buyer's reservation value, which is defined as the value that equates the discounted net present utility of purchase to the discounted net present utility of continued negotiation. Since an increase in the average relative offer will increase the expected net present utility of purchase, this will increase the buyer's reservation value in kind. Consequently, increasing the average relative offer will increase the buyer's reservation value. Second, consider the effect of an increase in the seller's offer rate. If the buyer faces a high offer rate, then the buyer knows that new offers will arrive shortly. Consequently, the buyer can afford to set a high reservation value, since the buyer can assume that a viable offer (i.e., one that exceeds the buyer's reservation value) will arrive within a short period of time. Therefore, the purchase hazard rate will decrease with an increase in the average relative offer and an increase in the offer rate. These properties are summarized in the following proposition, whose proof is given in the Appendix.

> *Proposition 2*: The purchase hazard rate $\theta(t)$ decreases as the offer rate and the average relative offer increase.

5 Empirical Model and Validation

In general, it is difficult to solve equation (18) to derive a closed-form expression for the reservation value $\xi(t)$. This difficulty arises since the integral in equation (18) does not have a closed form solution for many distributions. Nonetheless, the time-dependent reservation value $\xi(t)$ can be used to derive the probability distribution of the negotiation duration. Consider the probability that a buyer negotiates up to time t, and then purchases within a short interval *(t,t+h)*. This probability equals the probability that the buyer receives an offer (λh) times the probability that the offer exceeds the reservation value or $1 - F(\xi(t))$. If the limit is taken of the ratio of this probability to the length of the time interval as the short time interval shrinks to zero, the purchase hazard rate results. Using the notation $S(\xi(t)) = 1 - F(\xi(t))$, the hazard rate is $\theta(t) = \lambda S(\xi(t))$. Properties of the hazard rate imply that the duration probability is given by $g(t)$ in equation (20).

(20) $$g(t) = \lambda S(\xi(t)) \exp\left(-\int_0^t \lambda S(\xi(s)) ds \right), \text{ for } t \geq 0 .$$

Thus, equation (20) is the proposed structural model expressed in terms of negotiation duration. Note that equation (20) is complicated by the potentially non-trivial integral inside the exponential function. This model can be estimated by first solving for the optimality constraint represented by equation (18), inserting this estimate into equation (20), and then estimating the structural parameters (e.g., by using maximum likelihood procedures). In applications, the structural parameters may depend on individual differences or product characteristics.

5.1 Empirical Model Parameter Estimation

Although all of the parameters can be estimated using a likelihood function derived from the structural model (i.e., using equations (18) and (20)), it is often the case that some of the structural parameters (e.g., the discount rate δ) are not observed. Inclusion of an additional transition state (exit negotiation) introduces additional unidentifiable parameters (e.g., the offer rate for exiting the market). However, a reduced-form specification can be found that is consistent with the structural model. If the relative offers are distributed according to an exponential distribution and the negotiation cost is linear, then differential equation (18) can be solved and used in equation (20) to a derive closed-form expression for the hazard rate $\theta(t)$. In the Appendix, the closed-form expression is shown to be equivalent to a proportional hazard model in which $\theta(t,x) = \kappa_1(x)\kappa_2(t)$, where $\kappa_1(x)$ is a buyer-specific term and $\kappa_2(t)$ is common to all the buyers considered (Lancaster 1990). The proportional hazard model parameters are easy to interpret, which facilitates testing Propositions 1 and 2 and exploring alternative specifications of the reference price.

The model described above is for two-state transitions between the negotiation state and the purchase state. However, buyers also face the option of not making a purchase. This is incorporated into the proposed model by adding a second outcome state, exit negotiation, which has an associated utility V_E and reservation value $\xi_E(t)$. Although the "offers" corresponding to the exit negotiation state are not identified, if entrants into the exit negotiation state are observed and if the utility of no purchase is conceptualized as the *disutility* of purchase, then it is reasonable to assume that transitions to exiting negotiation are modeled using the same functional form as for transitions to purchase. This assumption is made in estimating the model below.

5.2 Competing-risks Specification

The empirical specification is motivated by the properties of the structural model (i.e., a proportional hazard model). Since the goal is to accommodate multiple terminal states (purchase and exit negotiation), a competing-risks hazard model is considered. This model differs from a standard hazard model in that the transition to each terminal state *j* is governed by a separate instantaneous probability function $\theta_j(t,x)$, known as the *transition rate*. The function $\theta_j(t,x)$ is the probability of exiting in the short interval *(t,t+h)* to the *j-th* state. The overall hazard function $\theta(t,x)$ is the probability of exiting in the short interval *(t,t+h)*, regardless of the terminal state, which is the sum of the transition rates $\theta(t,x) = \sum_{j=1}^{J} \theta_j(t,x)$. Identification of the parameters in a competing-risks model requires information on which terminal state the buyer transitions into after negotiation.

The following specification is used to estimate the effects of time (*t*), negotiation cost (*cost*), offer rate (*rate*), and relative offer value (*value*) on the transition rates to the purchase and exit negotiation states:

$$(21) \qquad \theta_{ij}\left(t_i, X(t_i)\right) = \exp\left(\beta_{0j} + \beta_{1j} t_i\right) \exp\left(\beta_{2j} cost(t) + \beta_{3j} rate + \beta_{4j} value\right).$$

In equation (21), the subscript *i* denotes the buyer and the subscript *j* denotes the terminal state. A value for *j* equal to 1 corresponds to purchasing and a value equal to 2 corresponds to exiting negotiation. The symbol X_i denotes the value of the covariates (*cost(t)*, *rate*, and *value*) for the *i-th* buyer. Equation (21) is a proportional hazard model since it is expressed as the product of a buyer-specific function and a function of time that is common to all the buyers in the dataset.[23] Note that this property is consistent with the structural model presented above. The buyer-specific covariates include negotiation costs, the offer rate, and the relative offer value. The relative offer value is the difference between the offer price and a reference price. The variable *cost(t)* is a time-varying covariate whose complete time path is known *a priori* to the customer.[24]

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[23] Technically, equation (21) is not a hazard function because it depends on the terminal state. Nonetheless, expressions like equation (21) are almost always called hazard functions within a competing-risks framework (Lancaster 1990), and this terminology is used here to emphasize that the functional form in equation (21) is the same proportional function as in the structural model.

[24] This assumption is required to justify using the proposed likelihood expression given below (Lancaster 1990, pp. 24-25).

The discount rate is not included since there is no suitable available proxy. The competing-risks model for two terminal states is estimated using maximum likelihood, where the log-likelihood function is given by equation (22):

$$\ell(t,d)=\sum_{i=1}^{N}\sum_{j=1}^{2}\left[d_{ij}\log\theta_{ij}\left(t_i, X_i(t_i)\right)-\int_{0}^{t_i}\theta_{ij}\left(s_i, X_i(s_i)\right)ds\right]. \quad (22)$$

The variable d_{ij} appearing in the log-likelihood function in equation (22) is a dummy variable indicating whether person *i* transitioned to state *j*.

5.3 Reference Price Alternatives

A final consideration is the operationalization of the relative offer. In the development of the structural model, a relative offer is the difference between the current offer price and a reference price. The specification of reference price was left undefined. The competing-risks hazard model is estimated using several reference price specifications, and it is left as an empirical question as to which specification yields the best fit to the data. The specifications employed are drawn from the behavioral negotiation and marketing literatures. For example, Northcraft and Neale (1987) find that buyers often employ the initial list price as their reference price when negotiating the purchase price of a house. This suggests specifying the reference price as the list price. Several researchers have found that consumers often use past prices of a brand as a reference price during future purchase occasions (e.g., Briesch, Krishnamurthi, Mazumdar, and Raj 1997; Mayhew and Winer 1992). This suggests specifying the reference price as the most recent offer price. Finally, White and Neale (1996) show that the average selling price is used as the reference price when determining the final negotiated price. This suggests specifying the reference price as the average of previous offer prices.

In summary, three specifications of reference price are used to model the effect of relative offers on negotiation duration. These specifications are given in Table 2. All relative offers are defined as the difference between a reference price and the offer price at the point of transition (i.e., to either the purchase or exit negotiation state).

Table 2

REFERENCE PRICES USED IN ESTIMATION

Label	Reference Price	Specification[a]
InitialPrice	Initial price (Northcraft and Neale 1987)	$Price(0)$
RecentPrice	Most recent price (Mayhew and Winer 1992)	$Price(t-1)$
AveragePrice	Average past price (White and Neale 1996)	$Ave.Price(0,...,t-1)$

[a] The value appearing in parentheses in the specification of each reference price is the period during which the price is evaluated, and t is the period corresponding to the total negotiation duration T. Hence, t-1 corresponds to one period prior to the last period of negotiation.

5.4 Data Description

The purpose of estimating the competing-risks model is to validate Propositions 1 and 2 and to assess the best reference price specification. The model was estimated using a sample of actual customer negotiations over the rental of a durable product. The propriety transaction database contains information about telephone call transactions between potential buyers and the seller. Buyers call to request information about the products and to rent a product. The buyer has the option to negotiate a lower price, but not everyone chooses to negotiate. Some of the non-negotiators purchase at the list price. If a negotiation occurs, only the seller makes explicit offers that the buyer either accepts or rejects. Hence, the dataset contains only asymmetric price negotiations.

Negotiators are explicitly identified by an indicator variable. Approximately 14 percent of the buyers in the sample negotiated over the purchase price. For transactions involving a negotiation, only the seller's offer prices are recorded. A purchase indicates acceptance of the current offer price, whereas a non-purchase indicates rejection of the current offer price. A series of transactions with the same buyer over the same product in which negotiation occurs is considered to be an ongoing negotiation. Since the model assumes parties start in the negotiation state, only those individuals who negotiate are included in the model. The analysis sample of negotiators includes 3846 potential buyers, of which 235 eventually rent the durable product.[25]

[25] In the subsequent discussion, buyers who rent are considered to have entered the purchase state.

Although a single transaction that ends in a non-rental is clearly identified, an entire negotiation that *eventually* ends in a non-rental is inferred using additional information in the database. The difficulty in identifying eventual transitions to the exit negotiation state arises from the possibility that negotiations are censored. Fortunately, a date-time "stamp" is associated with each transaction. The buyer's desired rental date is also identified and is treated here as the buyer's deadline. If the desired rental date occurs before the last date on file, then the negotiation has ended. If no rental has occurred for these negotiations, it is inferred that the buyer has entered the exit negotiation state. For some negotiations, the desired rental date occurs after the last date on file. If no purchase occurs for these negotiations, it is inferred that the negotiation duration is censored. Fortunately, censoring is not a major concern in this dataset, since it affects only 3% of the negotiations.[26]

5.5 Offer Rate and Negotiation Cost

Offer rate and negotiation cost are not directly observed. Instead, proxies are used. Since at least one offer is made during each telephone call, the actual number of offers is at least as large as the number of telephone calls. Also, the seller may set a higher offer rate for negotiations in which buyers face a shorter deadline. Consequently, the proxy for the offer rate is inversely proportional to the number of days until the buyer's deadline. Therefore, the *offer rate* proxy is defined as the ratio of telephone

[26] In order to facilitate estimation and minimize the adverse effect of outliers, observations were removed that contained the top and bottom 1% from each variable's distribution. Coincidently, no censored observations remained in the analysis sample after removing the outliers.

calls (*OFS)* to the number of days until the buyer's deadline (*TDL*), or $\lambda = \dfrac{OFS}{TDL}$. Note that for a given number of offers, the offer rate is large for a close deadline and small for a distant deadline. Moreover, since every day that passes without a rental represents an increasing opportunity cost to the buyer, negotiation cost increases for the buyer at a rate that is inversely and linearly proportional to the initial number of days until the buyer's deadline (*ITDL*). Therefore, the negotiation cost proxy is defined as $cost(t) = \dfrac{t+1}{ITDL}$, where *t* is the current duration (i.e., the number of days since the start of negotiation).[27] Note that this proxy sets the initial negotiation cost (i.e., when t=0) to be small for distant initial deadlines and large for closer initial deadlines. Also, both proxies accommodate buyer heterogeneity because the deadline will depend on individual differences across buyers. Summary statistics for the variables used to estimate the empirical model are given in Table 3.[28]

The table of summary statistics reveals several interesting characteristics of the dataset. On average, negotiation durations that end in a purchase (Mean = 5.33 days) are signficantly shorter ($p<.001$) than those that end in the potential buyer exiting negotiation without a purchase (Mean = 8.76 days). Shorter negotiation durations for purchasers may be a result of their lower reservation value or closer deadlines. The longer average offer rate ($p<.001$) for purchasers (Mean = .281) compared to those

[27] Note that *cost(t)* is less than or equal to one since the negotiation duration must end prior to the desired delivery date. This characteristic of the negotiation cost ensures that the differential equation (18) has a unique solution (Edwards and Penny 1989).
[28] The summary statistics appearing in Table 3 reflect the values after removal of the outliers.

exiting negotiation (Mean = .198) is also consistent with the proposed model. Sellers who make more offers per unit of time to a potential buyer have more opportunities to meet the buyer's reservation value. Consequently, buyers who purchase would be expected to have received offers at a higher rate than buyers who exit negotiation.

Table 3

SUMMARY STATISTICS FOR ATTRIBUTES USED IN MODEL ESTIMATION

	Overall (N=3846)		Buyers Who Purchase (N=235)		Buyers Who Exit Negotiation (N=3611)	
	Mean	Std.Dev	Mean	Std.Dev	Mean	Std.Dev
Duration[a]	8.55	7.83	5.33***	4.83	8.76	7.94
Cost(t)[b]	.372	.221	.363	.236	.372	.220
Offer Rate	.203	.255	.281***	.363	.198	.246
InitialPrice[c]	$205.90	$192.56	$220.73	$157.41	$204.93	$194.61
RecentPrice	$256.74	$251.52	$181.11***	$163.93	$261.66	$255.43
AveragePrice	$147.97	$130.02	$152.66	$114.78	$147.67	$130.96

[a] Duration is measured in days.
[b] Negotiation cost is evaluated at the final negotiation duration.
[c] The prices listed are relative offers.
*** These differences between Buyers Who Purchase and Buyers Who Exit Negotiation are statistically significant at the p<.001 level

Finally, consider the relative offers, i.e., the difference between the most recent offer price and a references price. When the reference price is RecentPrice, there is a significantly higher relative offer (p<.001) for buyers who exit negotiation (Mean = $261.66) compared to buyers who purchase (Mean = $181.11). This is consistent with the second part of Proposition 2, since those who exit have longer negotiation durations. All other differences (i.e., for Cost(t), InitialPrice, and AveragePrice) are not statistically significant at the p=.05 level.

5.6 Empirical Results

A competing-risks proportional hazard model is used to validate Propositions 1 and 2 and to assess the best reference price specification. Although no specific hypotheses were given for transitions to the exit negotiation state, separate estimates for time, negotiation cost, offer rate, and offer value are included because many potential buyers (the majority) are observed to exit negotiation without a purchase. Furthermore, additional validation of the proposed model is provided if these effects are shown to depend on the terminal state. The competing-risks model is also used to explore alternative reference price specifications found in the negotiation literature. Also, these alternatives provide an informal test of the model's robustness to alternative specifications of the relative offer. In particular, the generalizability of the model is supported if the propositions remain valid under alternative reference price specifications.

5.7 Estimating the Marginal Effects

Since Propositions 1 and 2 express how the hazard changes with respect to a change in the structural parameters (e.g., negotiation cost), they are tested by considering the marginal effect of each structural parameter on the purchase hazard rate.

For example, Proposition 1 states that the marginal effect of negotiation cost and time on the purchase hazard rate is positive, whereas Proposition 2 states that the marginal effect of the offer rate and the average relative offer on the purchase hazard rate is negative. Note that the marginal effect of time involves more than the simple effect of time, since negotiation cost is a function of time. The marginal effect of time on the purchase hazard rate is given by equation (23):

(23) $$\frac{d\hat{\theta}_{ij}(t, X_i)}{dt} = \left(\hat{\beta}_{1j} + \hat{\beta}_{2j}\frac{1}{ITP}\right)\hat{\theta}_{ij}(t, X_i),$$

where $\hat{\theta}_{ij}(t, X_i)$ is given by equation (21) evaluated at the estimated coefficients and average attribute values. There is a statistically significant marginal effect of time if $H_0 : \{\beta_{10} = 0, \beta_{20} = 0\}$ is rejected and the sign of equation (21) is positive when evaluated at the average value of $\frac{1}{ITP}$ and the other variables (Greene 2000). There is a statistically significant marginal effect for the negotiation cost if the hypotheses $H_0 : \beta_{20} = 0$ can be rejected in equation (7). There is a statistically significant marginal effect for the offer rate if the hypothesis $H_0 : \beta_{30} = 0$ can be rejected in equation (7). Unfortunately, the second part of Proposition 2 (i.e., the purchase hazard rate decreases with an increasing average relative offer) cannot be tested using the competing-risks model. This is because the parameter estimates are the same across all the buyers and the proposition would require a separate parameter estimate for each buyer. The dataset used here contains insufficient information to estimate separate parameter estimates for each buyer.

The dataset was randomly partitioned into an estimation sample (80 percent) and holdout sample (20 percent). The estimation sample was used to estimate model parameters and test the model properties in Propositions 1 and 2. The holdout sample was used to compare the reference price specifications by assessing the best model fit. The estimates for the effects of time, negotiation cost, offer rate, and relative offer are reported for the estimation sample and grouped according to the transition state (buyers who purchase and buyers who exit negotiation). Three separate models are estimated corresponding to each of three alternative reference prices. Model performance is assessed using the Log-likelihood (LL) and the Bayesian Information Criterion (BIC) on the holdout sample. The parameter estimates for the three models and two transition states for the estimation sample are presented in Table 4. The corresponding marginal effects evaluated at the average values are presented in Table 5. The labels correspond to the reference prices defined in Table 2.

The results support Propositions 1 and the tested part of Proposition 2. As seen in Table 5, for every specification of reference price, the marginal effects of time and negotiation cost are positive and statistically significant ($p<.001$), as implied by Proposition 1. For every specification of reference price, the effect of the offer rate is negative and statistically significant ($p<.001$), as implied by Proposition 2. Therefore, both Proposition 1 and the tested part of Proposition 2 are completely supported by the empirical results. Although no specific hypotheses are given regarding the effect of the relative offers, the statistical significance of these effects validates the underlying assumption of the structural model that buyers value *relative offers*.

Table 4

PARAMETER ESTIMATES FOR COMPETING RISKS MODEL

	Buyer Who Purchase[a]			**Buyer Who Exit Negotiation**		
Parameter	**InitialPrice**	**RecentPrice**	**AveragePrice**	**InitialPrice**	**RecentPrice**	**AveragePrice**
Constant	-5.35 (<.001)	-4.86 (<.001)	-5.30 (<.001)	-2.81 (<.001)	-2.73 (<.001)	-2.74 (<.001)
Time	-0.34 (<.001)	-0.33 (<.001)	-0.33 (<.001)	-0.17 (<.001)	-0.16 (<.001)	-0.17 (<.001)
Neg.Costs	11.22 (<.001)	11.13 (<.001)	11.11 (<.001)	9.68 (<.001)	9.65 (<.001)	9.65 (<.001)
Offer Rate	-4.24 (<.001)	-4.29 (<.001)	-4.17 (<.001)	-5.64 (<.001)	-5.63 (<.001)	-5.62 (<.001)
Relative Offer	0.0006 (.031)	-0.0016 (<.001)	0.0005 (.288)	0.0001 (.255)	-0.0002 (<.001)	-0.0003 (.0193)

[a] P-values appear in parentheses. Each column corresponds to a model estimated using one of the three specifications of reference price (i.e., InitialPrice uses the list price as the buyer's reference price, RecentPrice uses the most recent offer price as the buyer's reference price, and AveragePrice uses the average of prior offer prices as the buyer's reference price.)

Table 5

MARGINAL EFFECTS FOR COMPETING RISKS MODEL

	Buyers Who Purchase[a]			Buyers Who Exit Negotiation		
	InitialPrice	**RecentPrice**	**AveragePrice**	**InitialPrice**	**RecentPrice**	**AveragePrice**
Time	0.000114	0.000134	0.000123	0.015161	0.015401	0.014896
Neg.Costs	0.049185	0.051166	0.051077	0.986421	0.997215	0.966313
Offer Rate	-0.018596	-0.019705	-0.019183	-0.574655	-0.581434	-0.562903
Relative Offer	0.000003	-0.000007	0.000002	0.000010	-0.000022	-0.000033

[a] The values correspond to the marginal effect of the independent variable (e.g., Time) on the transition rate $\hat{\theta}_{ij}$ evaluated at the parameter estimates and attribute averages. For example, for the marginal effect of Time on the purchase transition rate when the reference price in InitialPrice is given by .000114.

5.8 Comparing Reference Price Alternatives

A comparison of model fits using the three reference price alternatives appears in Table 6. Based on the results from the holdout sample, the model using the most recent price performs best (BIC = -4291.5), the model using the initial price performs second best (BIC = -4301.0), and the model using the average past price performs worst (BIC = -4302.9). This result suggests that the buyers in this dataset rely more on the most recent offer price in evaluating the relative offers than on the list price or average past offer price. This inference is further supported by the fact that the relative offer is not statistically significant when the reference price is defined as the AveragePrice (p=.288) and is marginally significant when the reference price is defined as the InitialPrice (p=.031).

The results presented in Table 6 are consistent with the findings in the marketing literature on reference price (e.g., Briesch, Krishnamurthi, Mazumdar, and Raj 1997; Mayhew and Winer 1992), but are not consistent with some of the behavioral negotiation literature (e.g., Northcraft and Neale 1987; White and Neale 1996). One reason for this discrepancy may be that the results from the marketing literature are based on actual choice information whereas the results from the behavioral negotiation literature are based on simulated negotiations. In particular, laboratory simulations often either measure the subjects' responses at the end of the negotiation or ask the subject to

Table 6

MODEL COMPARISONS USING ALTERNATIVE REFERENCE PRICES

Estimation (N=3077)	**InitialPrice[a]**	**RecentPrice**	**AveragePrice**
Log-Likelihood	-9005.3	*-8994.2*	-9004.4
BIC	-17975.7	*-17953.4*	-17973.9
Holdout (N=769)	**InitialPrice**	**RecentPrice**	**AveragePrice**
Log-Likelihood	-2164.9	*-2160.2*	-2165.9
BIC	-4301.0	*-4291.5*	-4302.9

[a] Each column corresponds to the competing-risks proportional hazard model estimated using the reference price indicated in the column heading (i.e., InitialPrice uses the list price as the buyer's reference price, RecentPrice uses the most recent offer price as the buyer's reference price, and AveragePrice uses the average of prior offer prices as the buyer's reference price).

make an evaluation after the negotiation has terminated. However, in actual negotiations, the parties evaluate offers as they arrive and may terminate at any point during the negotiation. According to Hogarth and Einhorn (1992) this distinction is critical with respect to which reference point is used in evaluating the final offer. Hogarth and Einhorn (1992) show that when people are asked to evaluate a sequence of events at the end of the sequence, they tend to use the first event as a reference point.

Hence, the results of Hogarth and Einhorn (1992) imply that in simulated negotiations, parties should use the list price as their reference price in evaluating successive offers. This was the result shown by Northcraft and Neale (1987) and White and Neale (1996). Alternatively, Hogarth and Einhorn (1992) show that when the evaluation is done after each event in the sequence, a person tends to use the most recent event as the reference point. Hence, in actual negotiations, parties should use the most recent offer price as their reference price in evaluating successive offers. This result is shown in the present study. Similar results appear in the behavioral pricing literature that shows buyers often adapt their reference price as a product's price changes over time (e.g., Monroe 1979).

6 Discussion

This dissertation proposes a new model of asymmetric price negotiation between a buyer and a seller. The proposed model combines insights from several research streams to model a buyer's decisions during negotiation. Several properties were derived from the proposed model. Analytical and empirical evidence indicates that as negotiation cost and time increase, the purchase hazard rate increases, but as the seller's offer rate increases, the purchase hazard rate decreases. An analytic proof is given that the purchase hazard rate for a buyer decreases as the average relative offer for that buyer increases. Empirical validation of the proposed model was made by estimating a competing-risks proportional hazard model on a sample of actual negotiations over the rental of a durable good. The proposed model was shown to be robust to alternative specifications of reference price, although buyers were shown to rely more on the most recent offer price as a basis for evaluating relative offers.

The proposed model offers several contributions to the negotiation literature. First, the proposed model extends prior marketing literature on negotiation by allowing the negotiation cost and reservation value to vary over time. Since prior research in marketing on negotiation assumes that these parameters are fixed over time, the proposed model subsumes previous models and is more realistic. Second, since the structural model easily leads to an empirically estimable model, the actual effect of a seller's marketing decisions on negotiation outcomes can be precisely estimated. Prior negotiation research tends to rely on either experiments or game theory models, neither of which provides an estimable model to measure the precise effects of negotiation cost, time, offer rate, or the relative offer on the negotiation process. Third, the proposed model clarifies the role that the buyer's reference price has on the negotiation duration in the current dataset. Prior research has found conflicting evidence regarding which reference price dominates negotiation. Finally, the proposed model shows that a seller might decrease negotiation duration by decreasing the offer rate and average relative offer to potential buyers. Prior models do not predict negotiation duration.

6.1 Managerial Implications

The empirical results possess several managerial implications for the present study. The model shows that when negotiating over the rental price, a potential buyer will value an offer with respect to a reference price. In particular, buyers rely on the most recent offer price in evaluating a subsequent offer price. Consequently, in this instance, the seller should emphasize the savings from purchasing the product at the new offer price compared to the most recent offer price. Also, the model implies that the seller will have a better chance at reaching an agreement on the first day of negotiation for buyers who face an impending deadline. Buyers with close deadlines face a high initial negotiation cost that increases very rapidly. Consequently, the seller can leverage a buyer's rapidly increasing negotiation cost by being less aggressive in lowering the offer price when the buyer is facing a close deadline. Finally, the structural model implies that a large average relative offer with a particular buyer will increase the negotiation duration with that buyer. A larger average relative offer means buyers face a large discount with respect to their reference price. Over time, a larger average offer will increase a buyer's reservation value, making it more difficult for the seller to satisfy the buyer's purchase criterion. Consequently, the seller should be cautious in offering steep discounts over the duration of negotiation. In summary, the proposed model provides several guidelines for sellers that engage in asymmetric price negotiation.

6.2 Limitations and Future Research

The current study contains several potential limitations. Although the relative offer varies over time, the offer distribution is assumed to be stationary. This assumption was made to simplify the derivation of the structural model, but it might not always be realistic. For example, the offer distribution may change over time, reflecting the seller's reactions to changing market conditions or inventory levels. Also, buyers might update their expectations about likely offers in response to changes in the information received during negotiation according to Bayes's rule. In particular, the buyer may have a prior belief about likely offers and update these beliefs as the negotiation proceeds.

The model assumes a buyer is risk-neutral so that utility is a linear function of its parameters. In general, a buyer may exhibit risk-seeking or risk-averse behavior. For example, Prospect Theory implies that a buyer will be risk-seeking or risk-adverse depending on how the buyer frames the negotiation with respect to a reference point. Suppose the buyer uses the average offer of other sellers $\bar{d}$ as the reference point. The buyer may frame an offer $d < \bar{d}$ as a "loss" and exhibit risk-seeking behavior. Offers satisfying $d > \bar{d}$ may be perceived as a "gain", inducing risk-adverse behavior. Furthermore, *loss aversion* implies that a perceived loss (i.e., $d < \bar{d}$) will have a greater effect on utility than a perceived gain (i.e., $d > \bar{d}$). One functional form for utility satisfying these assumptions is given by:

$$(24) \qquad u_p(t) = \frac{g(t)^{\alpha} - 1}{\alpha} + \frac{\gamma[l(t)^{-\beta} - 1)]}{\beta}; \alpha, \beta > 0, \gamma > 1,$$

In equation (24), $g(t) = \begin{cases} d, d > \bar{d} \\ 0, \ \mathrm{d} \leq \bar{\mathrm{d}} \end{cases}$ and $l(t) = \begin{cases} 0, d > \bar{d} \\ d, \ \mathrm{d} \leq \bar{\mathrm{d}} \end{cases}$. The parameter γ captures the effect of loss aversion and the α, β parameters measure the level of risk-aversion and risk-seeking, respectively.

The use of a general utility function changes the optimality constraint for the reservation offer. For example, in the stationary two-state model, the implicit function defining the reservation offer changes to:

$$(25) \qquad \xi = -c + \frac{\lambda}{\delta} \int_{\xi}^{\infty} (u_p - \xi) u'_p dF,$$

where u'_p is the derivative of the purchase utility with respective to the offer (Van Den Berg 1990).

Another potential limitation is the assumption that the offer rate (λ) is fixed over time. This too may not be realistic. For example, the seller may increase the offer rate as the buyer approaches his/her deadline in an attempt to entice the buyer to settle. Also, the model assumes everyone shares the same parameter values, e.g., λ is the same for all buyers. However, some parameters may vary across individuals. For example, the offer rate may depend on either observed (e.g., gender) or unobserved individual differences (e.g., bargaining power). If observed differences do not depend on time, they are easily incorporated into the basic model, e.g., $\lambda = \lambda(X'\beta)$.

Lancaster (1979) shows one way of modifying the static model to incorporate unobserved individual differences in the offer rate. If the hazard function has a "proportional form" $\theta(t) = \lambda(X)\psi_2(t)$, then Lancaster suggests $\lambda(X) = \upsilon_i \exp(X'\beta)$ for observed *X* and unobserved υ_i. If $\upsilon_i \sim Gamma(1,\sigma^2)$, then the survival function conditional on *X* is given by:

(26) $$S(t \mid X) = \int_0^\infty \overbrace{e^{-\upsilon\mu I(t)}}^{S(t|x,\upsilon)} \overbrace{\upsilon^{\sigma^{-2}-1} e^{-\upsilon\sigma^{-2}}}^{\upsilon\sim Gamma(1,\sigma^2)} d\upsilon,$$

where $\mu = e^{(X'\beta)}$ and $I(t) = \int_0^t \psi_2(s)ds$. Hence, for an appropriate specification of $\psi_2(t)$, the static model adequately accounts for some types of unobserved heterogeneity in the offer rate. An alternative approach to modeling unobserved heterogeneity is to apply a hierarchical Bayes formulation.

The optimality constraint (5) introduces several problems for estimating the negotiation model. First, since $\xi(t)$ is a minimum value, the corresponding test statistics for $\xi(t)$ are different from those used to test parameters in a standard maximum likelihood problem (cf. Flinn and Heckman 1982; Lancaster 1990). Most researchers sidestep these issues by assuming a specific functional form for $\xi(t)$ (e.g., Lancaster and Chesher 1984; Van Den Berg 1990). A complementary approach is to apply Bayesian methods such as MCMC to sample from the posterior distribution of the parameters (Gelman, Carlin, Stern, and Rubin 1995). In the static model, a Bayesian analysis is relatively straightforward due to the special form of the conditional distributions. In the Appendix, I sketch the method proposed by Lancaster (1997) to estimate the parameters of the static model using a Bayesian approach.

The model presented in this study can be extended. The offer rate can be modeled as a source of unobserved heterogeneity since unforeseen individual differences (e.g., bargaining skill and experience) may alter the seller's offer rate for particular buyers. The structural model could be augmented by placing a prior distribution on the offer rate. For example, since the offers are assumed to arrive in a Poisson process, a gamma prior distribution could be used to capture unobserved heterogeneity in the offer rate (Wagner and Decker 2000). Incorporating a prior distribution on the offer rate would help the model capture more of the dynamics of actual negotiations. Furthermore, incorporating unobserved heterogeneity might offer a way to validate the negative effect of the average relative offer on the purchase hazard rate (i.e., the second part of Proposition 2).

Finally, additional empirical studies should be conducted in order to further validate the structural model. The empirical model should be estimated using additional product categories and research conducted over different lengths of time. Also, data containing a more precise measure of the negotiation duration might overcome a potential limitation of this dataset, the fact that negotiations were initiated by a buyer's telephone call to the seller. In the current dataset, suitable proxies existed for the negotiation cost and offer rate. However, the discount rate and its effect on negotiation duration are completely unobserved. Also, negotiation duration was an estimated value based on the time between the phone calls. Furthermore, the setting within which negotiations transpired was not observed and potential confounding affects were not controlled. For example, the buyer may have conversed with a friend or family member between the phone calls, influencing the buyer's responses to the seller's counter-offers. Although the dataset and model estimated thus far provide preliminary support for the proposed model and its properties, additional research is needed to validate these results.

The deficiencies in the preliminary study can be overcome by manipulating the structural parameters and observing their effect on negotiation behavior in a controlled laboratory setting. An experiment can be conducted using human subjects who negotiate over the purchase of a product. In order to directly test the affect of negotiation costs, offer rate, and discount rate, subjects would negotiate in scenarios in which these factors are directly manipulated. Also, the negotiation duration could be precisely measured since the participants negotiate in a controlled setting.

Appendix

Stationary Model Distributions

Derivations for several of the probability distributions and optimality constraints are given below. First, consider the probability distribution for the number of rejected offers *k*, given the duration *t* and the accepted offer *a*. This is found by the observation that since the offers arrive in a Poisson process at the rate λ, the accepted and rejected offers arrive in independent Poisson processes with rates $\lambda F(\xi)$ and $\lambda \overline{F}(\xi)$, respectively. Therefore, if the first acceptable offer arrives at time *t*, then the number of rejected offers that arrive before *t* is also a Poisson process with rate $\lambda F(\xi)$:

(A1) $$g(k \mid a,t) = \frac{[\lambda F(\xi)t]^k e^{-\lambda F(\xi)t}}{k!}, \text{ k} = 1,2,\ldots.$$

Next, consider the joint distribution for the number of rejected offers and duration, which is just the product $g(k \mid t)g(t)$:

(A2) $$g(k,t) = \frac{\lambda^{k+1}\overline{F}(\xi)[F(\xi)t]^k e^{-\lambda t}}{k!}, \text{ k} = 1,2,\ldots.$$

Next, consider the distribution of rejected offers conditional on *k*, *a* and *t*. Denote the rejected offer by $\{r_1,\ldots,r_k\}$ and note that $\Pr(r_i \mid k,a,t) = \dfrac{\Pr(r_i)}{\Pr(r_i < \xi)}$. Therefore:

(A3) $$g(r \mid k,a,t) = \prod_{i=1}^{k} \frac{f(r_i)}{F(\xi)}, \text{ for } r_1,\ldots,r_k < \xi .$$

The product of equation (A3) and equation (A1) is the joint distribution of rejection number and rejected offers:

(A4) $$g(r,k \mid a,t) = \prod_{i=1}^{k} \frac{f(r_i)(\lambda t)^k e^{-\lambda F(\xi)t}}{k!}, \text{ for } r_1,\ldots,r_k < \xi; \text{ k} = 1,2,\ldots .$$

Finally, the joint distribution of the accepted offer (*a*), rejected offers ($r_1,\ldots,r_k$), the number of rejections (*k*), and the duration (*t*) is given by the product of equation (A4) and equation (8):

(A5) $$g(r,a,k,t) = \lambda^{k+1} t^k e^{-\lambda t} f(a) \prod_{i=1}^{k} \frac{f(r_i)}{k!},$$

$$\text{for } r_1,\ldots,r_k < \xi; \text{ k} = 1,2,\ldots; \text{ a} \geq \xi; \text{ t} \geq 0 .$$

Derivations from the Proposed Model

It is shown that for exponential offers $f(r) \sim \gamma \exp(\gamma r)$, and linear negotiation cost *c(t)=ct*, the purchase hazard rate $\theta(t) = \kappa_1(t)\kappa_2(x)$, where $\kappa_1(t)$ is a function common to all buyers and $\kappa_2(x)$ is buyer-specific (i.e., $\theta(t)$ is a proportional hazard function). Also, the purchase hazard $\theta(t)$ is shown to increase with time and negotiation cost (Proposition 1), but decrease with the offer rate and the average relative offer (Proposition 2).

Derivation of Purchase Hazard Rate

The differential equation describing how the reservation value $\xi(t)$ changes over time is given by the following expression:

(A6) $$\frac{d}{dt}\xi(t) = \delta\xi(t) + \delta ct - \lambda \int_{\xi(t)}^{\infty} (x - \xi(t))\gamma \exp(-\gamma x)dx\,.$$

The integral in expression (A6) can be expanded as:

$$\int_{\xi(t)}^{\infty} (x - \xi(t))\gamma \exp(-\gamma x)dx = \int_{\xi(t)}^{\infty} \gamma x \exp(-\gamma x)dx - \int_{\xi(t)}^{\infty} \gamma\xi(t)\exp(-\gamma x)dx\,.$$

The first integral can be solved using integration by parts, and since the second integral is trivial,

$$\int_{\xi(t)}^{\infty} \gamma x \exp(-\gamma x)dx = \xi(t)\exp(-\gamma\xi(t)) + \frac{\exp(-\gamma\xi(t))}{\gamma}$$

$$\Leftrightarrow \int_{\xi(t)}^{\infty} \gamma\xi(t)\exp(-\gamma x)dx = \xi(t)\exp(-\gamma\xi(t)).$$

Therefore, $\int_{\xi(t)}^{\infty}(x-\xi(t))\gamma\exp(-\gamma x)dx = \dfrac{\exp(-\gamma\xi(t))}{\gamma}$, and equation (A14) becomes:

(A7) $$\frac{d}{dt}\xi(t) = \delta\xi(t) + \delta ct - \frac{\lambda\exp(-\gamma\xi(t))}{\gamma}.$$

Solving equation (A7) is made difficult by the exponential function. However, a power series expansion of the exponential function implies that $\exp(-\gamma\xi(t))$ is approximated by $1-\gamma\xi(t)$. Therefore, equation (A7) is approximated by the first-order linear differential equation given by (A8):

(A8) $$\frac{d}{dt}\xi(t) - (\delta+\lambda)\xi(t) = \delta ct - \frac{\lambda}{\gamma}.$$

This differential equation is solved using the integration factor $\rho(t) = \exp\big(-(\delta+\lambda)t\big)$. In particular, multiplying both sides of expression (A8) by the integration factor $\rho(t)$ yields:

(A9) $$\frac{d}{dt}\left[\xi(t)\exp\left(-(\delta+\lambda)t\right)\right]=\left(\delta ct-\frac{\lambda}{\gamma}\right)\exp\left(-(\delta+\lambda)t\right).$$

Integrating both sides of expression (A9) and rearranging terms yields:

(A10) $$\xi(t)=\exp\left((\delta+\lambda)t\right)\left(\int \delta ct\exp\left(-(\delta+\lambda)t\right)dt-\int\frac{\lambda}{\gamma}\exp\left(-(\delta+\lambda)t\right)dt\right).$$

Solving the integrals inside the right-hand parentheses yields:

$$\int \delta ct\exp\left(-(\delta+\lambda)t\right)dt=\frac{-\delta ct\exp\left(-(\delta+\lambda)t\right)}{(\delta+\lambda)}-\frac{\delta c\exp\left(-(\delta+\lambda)t\right)}{(\delta+\lambda)^2}+K'\text{, and}$$

$$\int\frac{\lambda}{\gamma}\exp\left(-(\delta+\lambda)t\right)dt=\frac{-\lambda\exp\left(-(\delta+\lambda)t\right)}{\gamma(\delta+\lambda)}+K''.$$

Therefore, expression (A10) reduces to the following expression:

(A11) $$\xi(t)=K\exp\left((\delta+\lambda)t\right)-\left(\frac{\delta ct}{(\delta+\lambda)}-\frac{\lambda}{\gamma(\delta+\lambda)}+\frac{\delta c}{(\delta+\lambda)^2}\right).$$

[Note: An explicit solution is provided by solving for the integration constant K using the terminal condition $\xi(T) = -cT + \frac{\exp(-\gamma\xi_T)}{\gamma}$.] The hazard rate is given by $\theta(t) = \lambda S(\xi(t))$, which in light of equation (A11) can be written as:

$$\text{(A12)} \quad \theta(t) = \lambda \exp\left(-\gamma K \exp((\delta+\lambda)t) + \frac{\gamma\delta c t}{(\delta+\lambda)}\right)\exp\left(\frac{\gamma\delta c}{(\delta+\lambda)^2} - \frac{\lambda}{(\delta+\lambda)}\right).$$

Expression (A12) is in the form of a proportional hazard, which completes the proof.

Proof of Proposition 1

Since the purchase hazard rate is given by $\theta(t) = \lambda S(\xi(t))$, $\theta(t)$ increases whenever $\xi(t)$ decreases. Also, since negotiation cost is increasing in time, $\xi(t)$ decreases in negotiation cost whenever $\xi(t)$ decreases in time. Therefore, in order to prove that $\theta(t)$ increases with time and negotiation cost, it suffices to show that $\xi(t)$ decreases with time.

The proof that $\xi(t)$ decreases with time follows by first showing that $\xi(t)$ is monotonic and then showing that $\xi(t)$ is decreasing at a specific point, namely the deadline T. Recall, for exponential offers and linearly increasing negotiation cost, the reservation value $\xi(t)$ is the solution to the following differential equation (where $\xi'(t) = \frac{d\xi(t)}{dt}$):

(A13) $$\xi'(t) = \delta\xi(t) + \delta ct - \frac{\lambda \exp(-\gamma\xi(t))}{\gamma}.$$

Differentiating expression (A13) with respect to time *t* yields:

(A14) $$\xi''(t) = \delta\xi'(t) + \delta c + \lambda\xi'(t)\exp\left(-\gamma\xi(t)\right).$$

Expression (A14) reduces to the following:

(A15) $$\frac{\xi''(t) - \delta c}{\xi'(t)} = \delta + \lambda \exp\left(-\gamma\xi(t)\right) > 0 .$$

The right-hand side is greater than zero since by assumption the discount and offer rate are great than zero (i.e., $\delta, \lambda > 0$). Also, since $c > 0$, it follows that $\frac{\xi''(t)}{\xi'(t)} > 0$. This implies that $\xi(t)$ is either monotonically increasing or monotonically decreasing in time. Therefore, it is only necessary to consider the value of $\xi(t)$ at a specific point in time to determine whether it is increasing. Consider some point $\tilde{t}$ that is close to the deadline T. The terminal condition is given by $\xi(T) = -cT + \frac{\exp\left(-\gamma\xi_T\right)}{\gamma}$, for $t \geq T$. Subtracting $\xi(T)$ from $\xi(\tilde{t})$ yields:

(A16) $$\xi'(\tilde{t}) = \delta\xi(\tilde{t}) + \delta c\tilde{t} - \frac{\lambda \exp\left(-\gamma\xi(\tilde{t})\right)}{\gamma} - \xi(T) - cT + \frac{\lambda \exp\left(-\gamma\xi(T)\right)}{\delta\gamma} .$$

Suppose that $\xi'(t) > 0$, which implies $\xi(\tilde{t}) - \xi(T) < 0$ and $-\gamma\xi(\tilde{t}) > -\gamma\xi(T)$. Since the exponential function is monotonically increasing, this implies that $\lambda \exp\left(-\gamma\xi(T)\right) - \lambda \exp\left(-\gamma\xi(\tilde{t})\right) < 0$. Furthermore, by definition negotiation cost is increasing and $\delta < 1$, which implies that $\delta c\tilde{t} - cT < 0$. Hence, the right-hand side of expression (A16) is negative, which contradicts the assumption that $\xi'(t) > 0$. A similar argument shows that $\xi(t) \neq 0$. Therefore, it follows that $\xi(t) < 0$, which completes the proof. Note that although this proof utilizes the assumption of exponential offers and linearly increasing costs, it can be easily generalized to any offer distribution and increasing cost function.

Proof of Proposition 2

As in the proof to Proposition 1, the fact that the purchase hazard rate $\theta(t)$ decreases whenever $\xi(t)$ increases is utilized. To show that $\xi(t)$ increases with the offer rate λ, consider the derivative of $\xi(t)$ with respect to λ:

(A17) $$\frac{d\xi(t)}{d\lambda} = Kt \exp\left((\delta + \lambda)t\right) + \frac{\delta ct}{(\delta + \lambda)^2} - \frac{\lambda}{\gamma(\delta + \lambda)^2} + \frac{2\delta c}{(\delta + \lambda)^3} .$$

Taking the derivative of (A17) with respect to time yields:

(A18) $$\frac{d^2\xi(t)}{dtd\lambda} = K\exp\left((\delta+\lambda)t\right)\left[1+(\delta+\lambda)t\right] + \frac{\delta c}{(\delta+\lambda)^2}\,.$$

Now, taking the derivative of equation (A18) with respect to λ yields:

(A19) $$\frac{d^2\xi(t)}{d\lambda dt} = \delta\frac{d\xi(t)}{d\lambda} - \frac{\exp(-\gamma\xi(t))}{\gamma} + \lambda\frac{d\xi(t)}{d\lambda}\exp(-\gamma\xi(t))$$

or $$\frac{d^2\xi(t)}{d\lambda dt} = \left[\delta + \lambda\exp(-\gamma\xi(t))\right]\frac{d\xi(t)}{d\lambda} - \frac{\exp(-\gamma\xi(t))}{\gamma}\,.$$

However, since $\frac{d^2\xi(t)}{dtd\lambda} = \frac{d^2\xi(t)}{d\lambda dt}$, (A18) and (A19) can be equated and solved for $\frac{d\xi(t)}{d\lambda}$:

(A20) $$\frac{d\xi(t)}{d\lambda} = \frac{K\exp\left((\delta+\lambda)t\right)\left[1+(\delta+\lambda)t\right] + \frac{\delta c}{(\delta+\lambda)^2} + \frac{\exp(-\gamma\xi(t))}{\gamma}}{\left[\delta+\lambda\exp(-\gamma\xi(t))\right]}\,.$$

Since all of the parameters are positive (i.e., $K, \delta, \lambda, c > 0$) and the exponential function is always positive, this implies that both the numerator and denominator of (A20) are positive. Therefore, $\frac{d\xi(t)}{d\lambda} > 0$, so that $\xi(t)$ is increasing in the offer rate λ, which completes the proof for the case of an increasing offer rate λ.

Finally, the fact that $\xi(t)$ increases with the average relative offer follows from the properties of the exponential distribution and the expression for $\xi(t)$ given by equation (A11):

$$\xi(t) = K\exp\big((\delta+\lambda)t\big) - \left(\frac{\delta ct}{(\delta+\lambda)} - \frac{\lambda}{\gamma(\delta+\lambda)} + \frac{\delta c}{(\delta+\lambda)^2}\right).$$

In particular, when relative offers are exponentially distributed, the average relative offer is given by $\frac{1}{\gamma}$. However, equation (A11) is obviously increasing with respect to $\frac{1}{\gamma}$, which completes the proof for increases in the average relative offer.

Bayesian Estimation of Static Model

Lancaster (1997) proposes an MCMC method for sampling from the posterior of the parameters in the basic model (stationary parameters). The joint density for an individual observation is given by equation (A21):

(A21) $$g(r,a,k,t) = \lambda^{k+1}e^{-\lambda t} f(a)\prod_{i=1}^{k}\frac{f(r_i)}{k!},$$

for $\mathrm{r}_1,\ldots,r_k < \xi;\ \mathrm{k}=1,2,\ldots;\ \mathrm{a} \geq \xi;\ \mathrm{t} \geq 0$.

Using the notation from the previous section, the likelihood is comprised of the product of terms like (A21), and is given by:

(A22) $$\ell(\lambda,\delta,\gamma,\xi \mid r,k,a,t) \propto \begin{Bmatrix} e^{-\lambda T}\lambda^{A+K}\prod_{i=1}^{A} f(a_i)\prod_{j=1}^{K} f(r_{ij}) & ,r<\xi<a \\ 0 & ,.otherwise \end{Bmatrix}$$

or $$\ell(\lambda,\delta,\gamma,\xi \mid r,k,a,t) \propto \begin{Bmatrix} e^{-\lambda T}\lambda^{n}\prod_{i=1}^{n} f(v_i) & ,r<\xi<a \\ 0 & ,.otherwise \end{Bmatrix}.$$

In the second part of (A22), v_i is an offer (accepted or rejected). A buyer chooses ξ by solving the optimality constraint:

(A23) $$\xi + c - \frac{\lambda}{\delta}\int_{\xi}^{\infty}(d-\xi)dF = 0 .$$

Let the distribution of offers have the lognormal distribution, or $\log v_i \sim n(\mu,\sigma^2)$, so that in (A22), $\gamma = (\mu,\sigma)$. Note that the model parameters $\{\lambda,\mu,\sigma,\delta)$ are independent of each other. Lancaster chooses the improper prior $\pi(\lambda,\mu,\sigma,\delta) \propto \frac{1}{\lambda\sigma}\pi(\delta)$, so that the posterior density is given by:

(A24) $$g(\lambda,\mu,\sigma,\delta)\propto\begin{cases}\lambda^{n-1}e^{-\lambda T}\dfrac{\exp\left(-(\dfrac{n}{2\sigma^2})[s^2+(\mu-\overline{v})^2]\right)}{\sigma^{n+1}}\pi(\delta) & r<\xi<a\\ 0 & ,otherwise\end{cases}.$$

Expression (A23) suggests a way to transform the constraint involving ξ into a constraint involving δ. In particular, rearranging terms in (A23) and using properties of $\log v_i \sim n(\mu,\sigma)$ implies:

(A25) $$\frac{\lambda}{\delta}=\frac{\xi+c}{\int_{\xi}^{\infty}\overline{\Phi}\left(\dfrac{\log(x)-\mu}{\sigma}\right)dx}=\kappa(\xi,\mu,\sigma).$$

The function $\kappa(\xi,\mu,\sigma)$ is monotonically increasing in its argument. Applying $\kappa(\xi,\mu,\sigma)$ to the constraint $r<\xi<a$ yields the equivalent:

(A26) $$\delta_a(\lambda,\mu,\sigma)=\frac{\lambda}{\kappa(a,\mu,\sigma)}<\delta<\frac{\lambda}{\kappa(r,\mu,\sigma)}=\delta_r(\lambda,\mu,\sigma).$$

Lancaster samples from equation (A24) using the transformed constraint (A25) via the SIR (sampling/importance sampling) algorithm of Gelfand and Smith (1990). In particular, δ is integrated out of (A24) to get the marginal posterior distribution of $\theta=(\lambda,\mu,\sigma)$:

(A27) $$g(\theta) \propto r(\theta)\left[\pi(\delta_r(\theta)) - \pi(\delta_a(\theta))\right]$$

where, $$r(\theta) = \lambda^{n-1} e^{-\lambda T} \frac{\exp\left(-(\frac{n}{2\sigma^2})[s^2 + (\mu - \overline{v})^2]\right)}{\sigma^{n+1}}.$$

After drawing from the marginal of $\theta = (\lambda, \mu, \sigma)$, draws from the corresponding conditional posterior for δ are taken from: $g(\delta \mid \theta) \propto \pi(\delta),\ \ \delta_a < \delta < \delta_r$ .This completes drawing samples from the joint posterior (A24). Note that the SIR procedure requires calculations of δ_a, δ_r which are defined in terms of (A25). The integral in equation (A25) is estimated using the equality:

(A28) $$\int_{\xi}^{\infty} \overline{\Phi}\left(\frac{\log x - \mu}{\sigma}\right) dx = \exp\left(\frac{\mu + \sigma^2}{2}\right) \overline{\Phi}\left(\frac{\log \xi - \mu}{\sigma} - \sigma\right) - \xi \overline{\Phi}\left(\frac{\log \xi - \mu}{\sigma}\right).$$

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Vita

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