Arch Bridges

by

Douglas A. Nettleton
Deputy Chief

Assisted by
John S. Torkelson
Structural Engineer

Bridge Division
Office of Engineering
Federal Highway Administration
U.S. Department of Transportation
Washington, D.C. 20590

PREFACE

Considering the extensive coverage in text books and technical literature of arch stress analysis and design, it might be thought that little of value could be added by a new publication on this subject. Emphasis in this paper is on aspects of arch design which are not covered in many text books, such as wind stress analysis and deflection, stress amplification due to deflection, consideration of rib shortening moments, plate stiffening, and calculations for preliminary design. In order for a designer to safely and economically design any structure, he must have a clear understanding of all aspects of the structural behavior. An unfortunate fact of most computer program usage is that the designer is much less cognizant of the basic action and assumptions. Chapter I covers steel arches and Chapter II covers concrete arches in all matters where concrete arches differ from steel. Chapter III covers arch construction.

Much of the material herein is from papers published by Engineering and Scientific Societies of the United States and other nations, and text books. Credit is given in the text to these sources. The basis for the equations developed by the author is given in the Appendix.

Every effort has been made to eliminate errors; but should errors be found, the author would appreciate notification from the readers.

ACKNOWLEDGEMENTS

The author would like to express thanks to those people who have helped in the preparation of this publication.

The following are from the Washington Office Bridge Division of FHWA: Mr. John S. Torkelson, Structural Engineer, made or checked the design calculations for steel and concrete arch examples; Mr. Emile G. Paulet, Staff Specialist, assisted by discussions with the author and by helping to find sources of material; Mrs. Willy Rudolph and Miss Karen Winters typed the drafts and final pages of the manual.

The many people and organizations which had a part in the design or construction of the steel and concrete arch bridges used as examples in this publication have given valuable indirect aid which is appreciated by the author.

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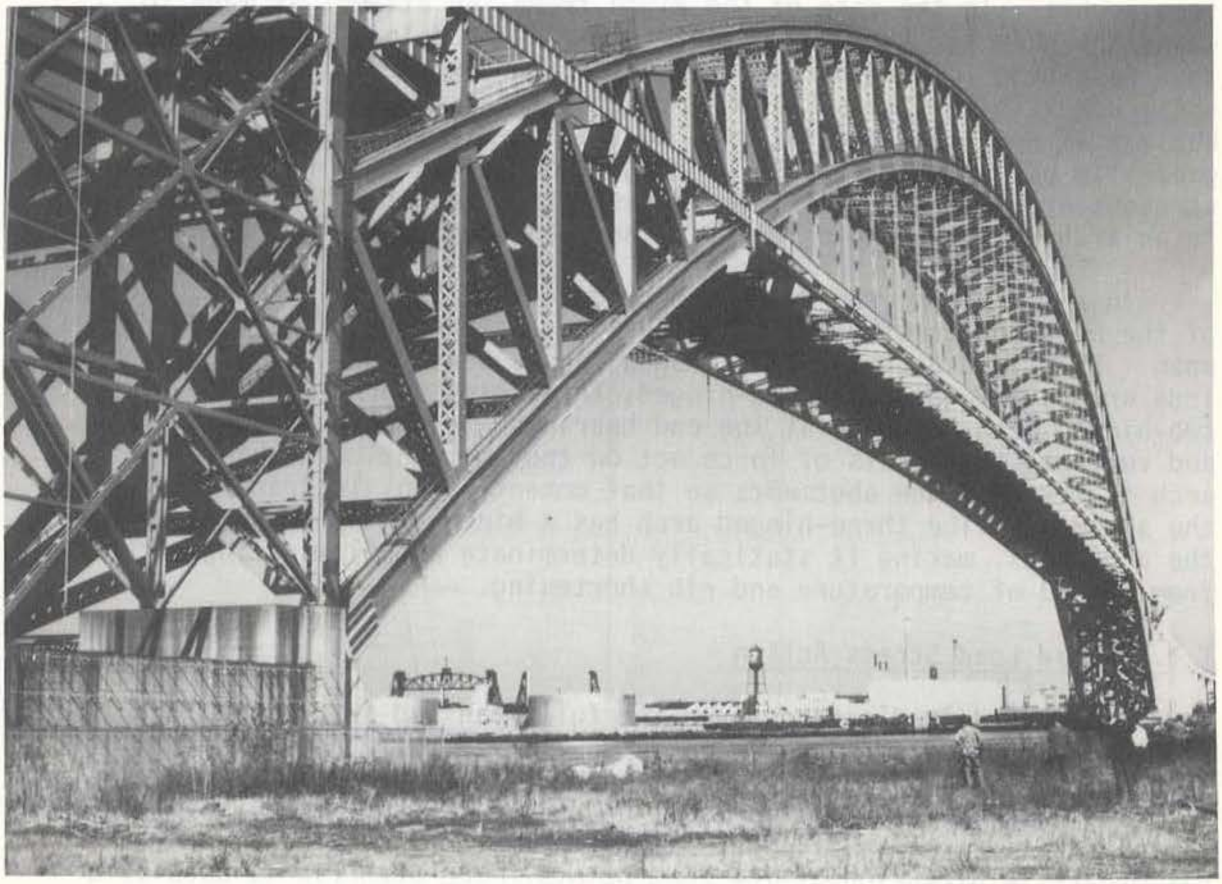
BAYONNE BRIDGE
over
THE KILL VAN KULL
New York - New Jersey
Span 1652 Feet

Built 1928-31



Designed By
The Port of New York Authority

Photograph By
Bruce Brakke - FHWA

CHAPTER I - STEEL ARCHES

1.1 Basic Arch Action

The distinguishing characteristics of an arch are the presence of horizontal reactions at the ends, and the considerable rise of the axis at the center of span, see Figure 1. Rigid frames and tied arches are closely related to the arch. However, both of these types have characteristics which cause them to act quite differently from true arches. In the case of the rigid frame, no attempt is made to shape the axis for the purpose of minimizing dead load bending moments, thus resulting in bending stresses which are considerably larger than axial compressive stress. In the case of the tied arch rib, the horizontal reactions are internal to the superstructure, the span generally having an expansion bearing at one end. As a result the stresses are different, in several respects, for a tied arch as compared to an arch with abutments receiving horizontal thrust.

In a true arch, the dead load produces mainly axial stress, and most of the bending stress comes from live load acting over a part of the span. Live load over the entire span causes very little bending moment. True arches are generally two-hinged, three-hinged or hingeless. The two-hinged arch has pins at the end bearings, so that only horizontal and vertical components of force act on the abutment. The hingeless arch is fixed at the abutments so that moment, also, is transmitted to the abutment. The three-hinged arch has a hinge at the crown as well as the abutments, making it statically determinate and eliminating stresses from change of temperature and rib shortening.

1.1.1 Dead Load Stress Action

Since dead load extends over the full span and is a fixed load, the arch axis should be shaped to an equilibrium polygon passing through the end bearings and the mid-depth of the rib at the crown, for dead load only. Since part of the dead load is generally applied to the rib as a series of concentrated loads, the equilibrium polygon has breaks in direction at the load points. The arch rib is usually a continuous curve in the case of a solid web rib, and this results in some dead load moments. Trussed ribs have breaks at each panel points and, if the dead loads are applied at every panel point, the chord stresses will not be affected by the moment effect mentioned for the solid web ribs on a continuous curve.

Usually a five-centered curve for the arch span can be fairly closely fitted to the dead load equilibrium polygon. Due to the greater dead load in the outer parts of the span, the radii should increase from the crown toward the springing, resulting in an axis lying between a parabola and a circular curve of constant radius.

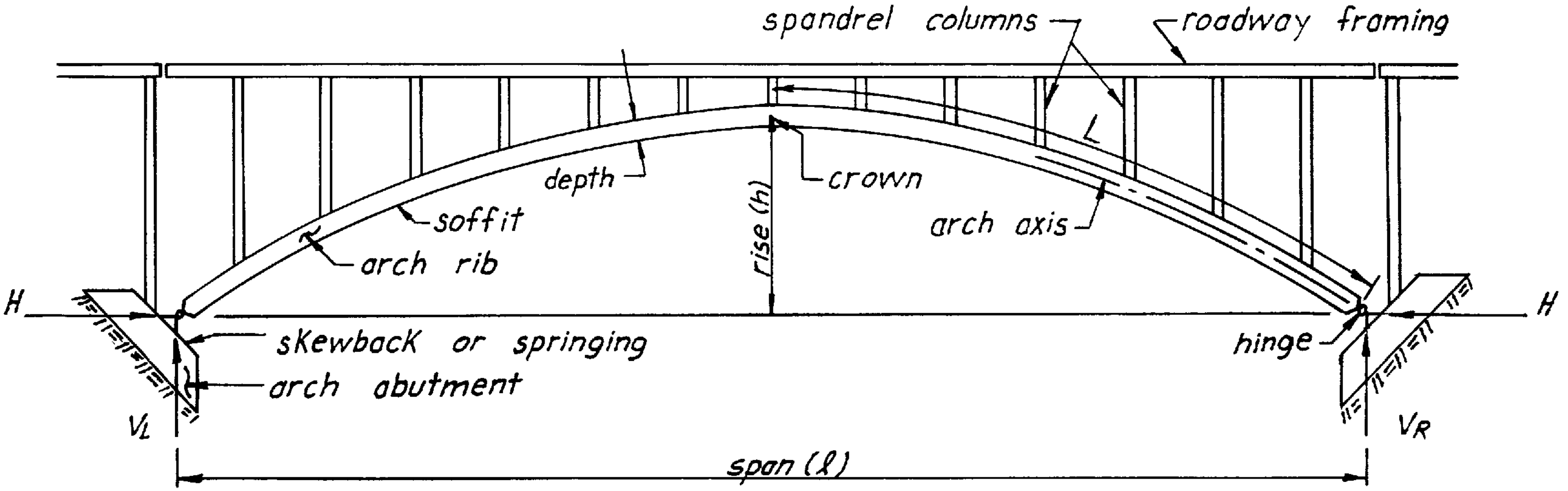


FIG. 1 - ARCH NOMENCLATURE

1.1.2 Live Load Stress Action

Although there is some live load axial stress, most of the live load stress is bending stress. The maximum live load bending stress at the quarter point of the span is produced by live load over approximately half the span, the moment being positive in the loaded half and negative in the unloaded half of the span. The maximum axial stress is produced by load over the full span. Thus the loaded length for maximum stress is different from the loaded length for the maximum moment alone. Under loading for maximum live load stress, the loaded half of the arch deflects downward and the unloaded half deflects upward. This deflection in combination with the axial thrust from dead and live load results in additional deflection and moment. This will be discussed at more length under Moment Magnification.

For "L" in the AASHTO Impact Formula use one half the span length for approximate calculations. For exact calculations use the loaded length as indicated by the influence line for the point in question, for either lane loading or truck loading.

With two arch ribs, the live load should be placed laterally, in accordance with AASHTO Specifications, to give the maximum load on one rib under the assumption of simple beam action between the two ribs. In the case of four or more ribs, the effect of live load eccentricity can be distributed in proportion to the squares of the distance from ribs to the center line. Where there are two lateral systems, a more exact method of distribution is explained under 2.1.1.

1.2 Buckling and Moment Magnification

An arch has a tendency to buckle in the plane of the arch due to the axial compression. This in-plane buckling tendency is usually greater than the lateral buckling because an arch bridge generally has two ribs braced together, and spaced apart a distance related to the width of the roadway. The usual in-plane buckling deflection is in the form of a reverse curve with part of the arch rib going down and the other part going up, as shown in Figure 2. Thus the buckling length for a two-hinged arch is approximately equal to L, defined as one-half the length of the rib, and a kL/r value of 80 or more is usual for most solid web arch ribs. This buckling tendency should be taken into account in the allowable axial stress, just as it is in other compression members. Note that L as used here is the half-length of the axis, and ℓ is the horizontal span.

In most arches live load deflection causes an increase in stress over that shown by a classical elastic analysis. This effect has been known for a long time, but it is often overlooked or else treated as a secondary stress for which an increased allowable stress is permitted. There is a similar effect in suspension bridges, but there is a vital difference in the deflection effect on the two types of structures. In the case of suspension bridges, deflection decreases stresses and, when taken into account, effects an economy. In the case of arches, deflection increases stresses, and reduces the safety factor if neglected in the analysis and design.

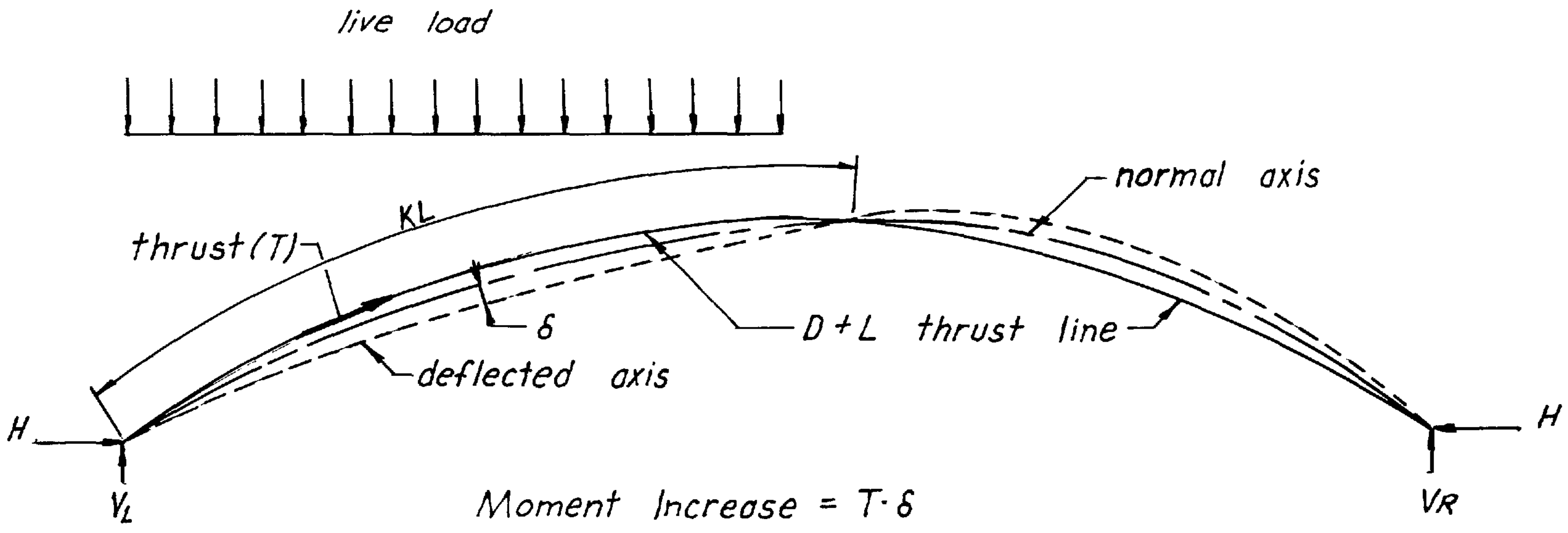


FIG. 2 – ARCH DEFLECTION

The live load moment in an arch is increased by the product of the total dead plus live arch thrust by the deflection from live load, as shown in Figure 2. The major component of the total arch thrust is dead load. Thus dead load interacts with live load in arch moment magnification. Maximum positive and negative moments are increased in about the same proportion.

Additional live load deflection is produced by the increase in moment, and this increase in deflection produces an additional increase in moment. This effect continues in a decreasing series. An approximate method of taking this effect into account is to use a moment magnification factor, A_F.

$$A_{Fs} = \frac{1}{1 - \frac{T}{AF_e}} \quad \text{(for deflection only)} \qquad \text{Eq. 1}$$

A_{Fs} = moment magnification factor for deflection under service load

T = arch rib thrust at the quarter point (approximately equal to H x secant of slope of line from springing to crown).

A = arch rib area at the quarter point

$$F_e = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} \quad \text{the Euler buckling stress}$$

L = half the length of the arch rib (approximately equal to $\ell/2$ x secant of slope of line from springing to crown)

r = radius of gyration at the quarter point

k is a factor varying from 0.7 to 1.16, depending on end restraint, see Figure 3.

Equation 1 gives the moment magnification factor at service load, and it should be used only for figuring service load deflection. Figure 3 can be used to get the value of this factor. For example, if T/A = 6 ksi and kL/r = 80, the factor is 1.155.

FIG. 3

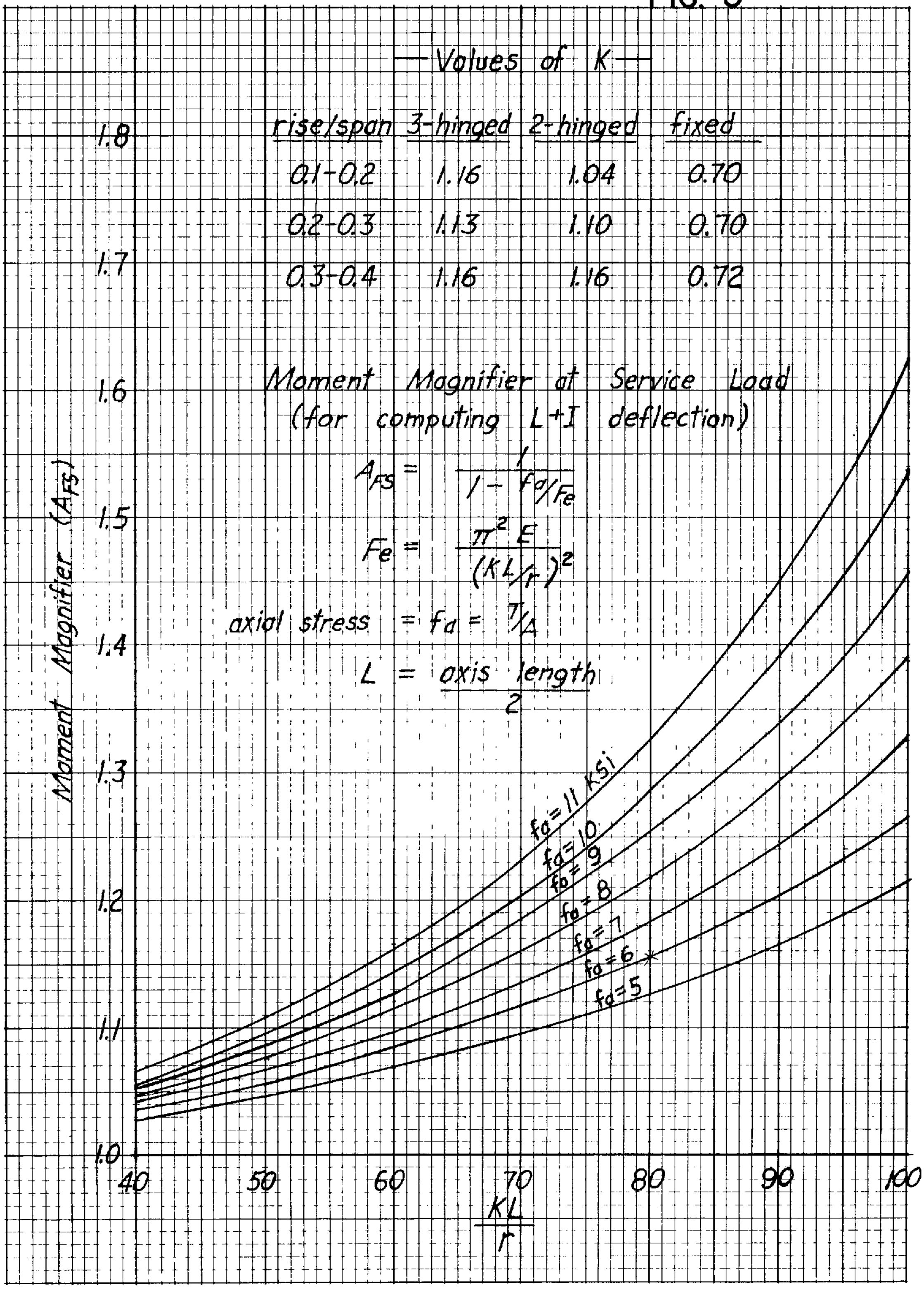
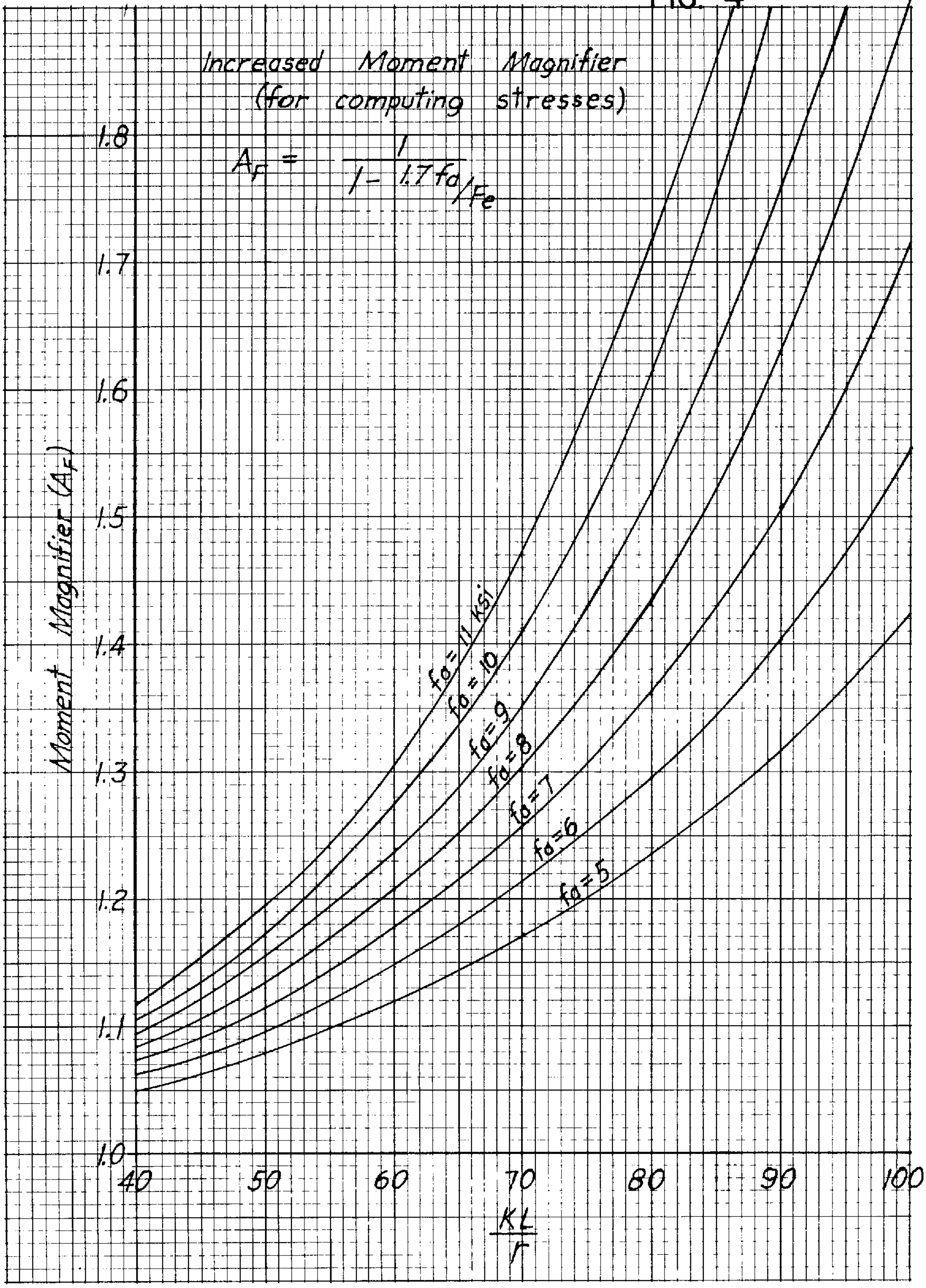

Values of K
rise/span 3-hinged 2-hinged fixed
0.1-0.2 1.16 1.04 0.70
0.2-0.3 1.13 1.10 0.70
0.3-0.4 1.16 1.16 0.72
Moment Magnifier at Service Load
(for computing L+I deflection)
$A_{FS} = \frac{1}{1 - f_a/F_e}$
$F_e = \frac{\pi^2 E}{(KL/r)^2}$
axial stress = f_a = T/A
$L = \frac{\text{axis length}}{2}$
Moment Magnifier (A_{FS})
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
40
50
60
70
80
90
100
$\frac{KL}{r}$
f_a = 11 KSi
f_a = 10
f_a = 9
f_a = 8
f_a = 7
f_a = 6
f_a = 5

FIG. 4



Increased Moment Magnifier
(for computing stresses)
$A_F = \frac{1}{1 - 1.7 f_a / F_e}$
Moment Magnifier (A_F)
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
40
50
60
70
80
90
100
$\frac{KL}{r}$
fa = 11 ksi
fa = 10
fa = 9
fa = 8
fa = 7
fa = 6
fa = 5

For an overload, the moment magnification factor does not remain the same but increases considerably. This is due to the fact that deflection is not proportional to load in the case of an arch. To take account of this fact, the following equation for A_F should be used for Service Load Design in order to maintain a desired safety factor:

$$A_F = \frac{1}{1 - \frac{1.7\,T}{AF_e}} \quad \text{(Service Load Design)} \qquad \text{Eq. 2}$$

Figure 4 can be used to get the value of this design factor. For $T/A = 6$ ksi and $kL/r = 80$, the design A_F is 1.295 as compared to 1.155 for service load deflection. The constant of 1.7, used in the A_F equation, is less than the corresponding numerical constant of 2.12 which is used for moment magnification in the AASHTO equations of Art. 1.7.17, for combined bending and axial stress in columns. The AASHTO value of 2.12 is the safety factor for compression. Since the thrust in an arch is mainly from dead load and the moment is mainly from live load, the increase in thrust due to overload is likely to be less than that which may occur in a column. Since the numerical factor in the A_F equation is to allow for non-linearity under overload, it is logical to use a smaller value in an arch formula than in a column formula. The numerical constant value of 1.7 has been derived from Load Factor Design safety factors which are smaller for dead load than for live load.

The AASHTO Load Factor Design safety factor for compression axial dead load is $1.3 \div 0.85 = 1.53$, and for compression axial live load is $5/3 \times 1.53 = 2.55$. Thus a weighted safety factor, for a case where live load thrust is equal to say 15% of total thrust, would be: $0.85 \times 1.53 + 0.15 \times 2.55 = 1.30 + 0.38 = 1.68$. A numerical factor of 1.7 is used in the equation. The numerical constant for Load Factor Design is $1 \div 0.85 = 1.18$ and the equation for A_F, Load Factor Design is:

$$A_F = \frac{1}{1 - \frac{1.18T}{AF_e}} \quad \text{(Load Factor Design)} \qquad \text{Eq. 3}$$

The effective length factors k, as given in Figure 3, are based on Guide to Stability Design Criteria for Metal Structures - Structural Stability Research Council, Chapter 16. (1)

1.2.1 Tied Arch Buckling and Moment Magnification

Moment magnification should not be used for tied arches. At any point in a tied arch the tie and the rib deflect practically the same amount. Thus the moment arm between the tension in the tie and the horizontal component of thrust in the rib remains constant for any section, regardless of deflection. In this respect the tied arch acts very much like a bow string truss, and there is no stress amplification due to deflection in either. In a true arch, however, the line of the horizontal component of the reaction is unchanged in position by deflection. Since the arch rib position does change due to deflection, the moment arm of "H" is changed by deflection in a true arch, and therefore the net moment, which is the difference between simple beam moment and the moment resisted by the effect of H, is increased by deflection in a true arch.

Buckling in the plane of a tied arch becomes a matter of buckling between suspenders rather than in a distance kL. Where wire ropes suspenders are used, the difference in stretch of the suspenders due to concentrated live load may be sufficient, due to the high allowable unit stress, to cause the buckling length to be somewhat longer than the distance between suspenders. (Ref. 5, page 14).

1.3 Ratio of Rib Depth to Span and Live Load Deflection

Moment magnification is quite sensitive to the ratio of rib depth to span. This is shown by Figure 4. At an axial stress of 8 ksi, an increase of kL/r from 80 to 100 increases AF from 1.44 to 1.91. A two-hinged design first studied for the 950 foot span Rainbow Arch at Niagara Falls (2) had an ℓ/d ratio of 66.5 and a kL/r ratio of 99. This preliminary design showed a very large moment magnification and was not used. A fixed arch with an ℓ/d of 78 and a kL/r of 75 was used. In this case, due to the long span and the use of silicon steel, the axial stress is 11,700 psi. As can be seen from Figure 4, the moment magnification is still large.

It can be seen from Figure 5 that deflection also is quite sensitive to the ratio of span to depth. For a magnified bending stress of 10,000 psi at service load, the magnified live load deflection is 1/800 of the span for $\ell/d = 75$, and 1/1200 of the span for $\ell/d = 50$.

AASHTO Specifications give a maximum value for live load deflection of 1/800 of the span for simple or continuous span. It is questionable whether such a high deflection in terms of spans should be permitted for an arch. Maximum deflection for an arch occurs for approximately half span loading and, under this loading, about one half the span goes down and the other part of the span goes up. It could be argued, therefore, that the maximum deflection for an arch should be 1/1600 of the span. Very few existing steel arch bridges would meet such a criteria. Some will barely meet the criteria of 1/800 of the span. We suggest a value of 1/1200 of the span. An equation $\ell/d = 44 + 0.6\sqrt{\ell}$, for two-hinged solid web ribs, is plotted in Figure 6. Use of this depth-to-span ratio should result in arch ribs which can meet the deflection criterion of 1/1200 of the span without loss of economy. Several existing arches were checked in relation to this curve. Most of them have a lesser depth than that shown by the curve. Two hinged trussed arches are generally about 25% deeper at the crown than solid-web arches, and increase in depth toward the springing. As a result, live load deflection is considerably smaller for trussed arches. Moment magnification is reduced

FIG. 5

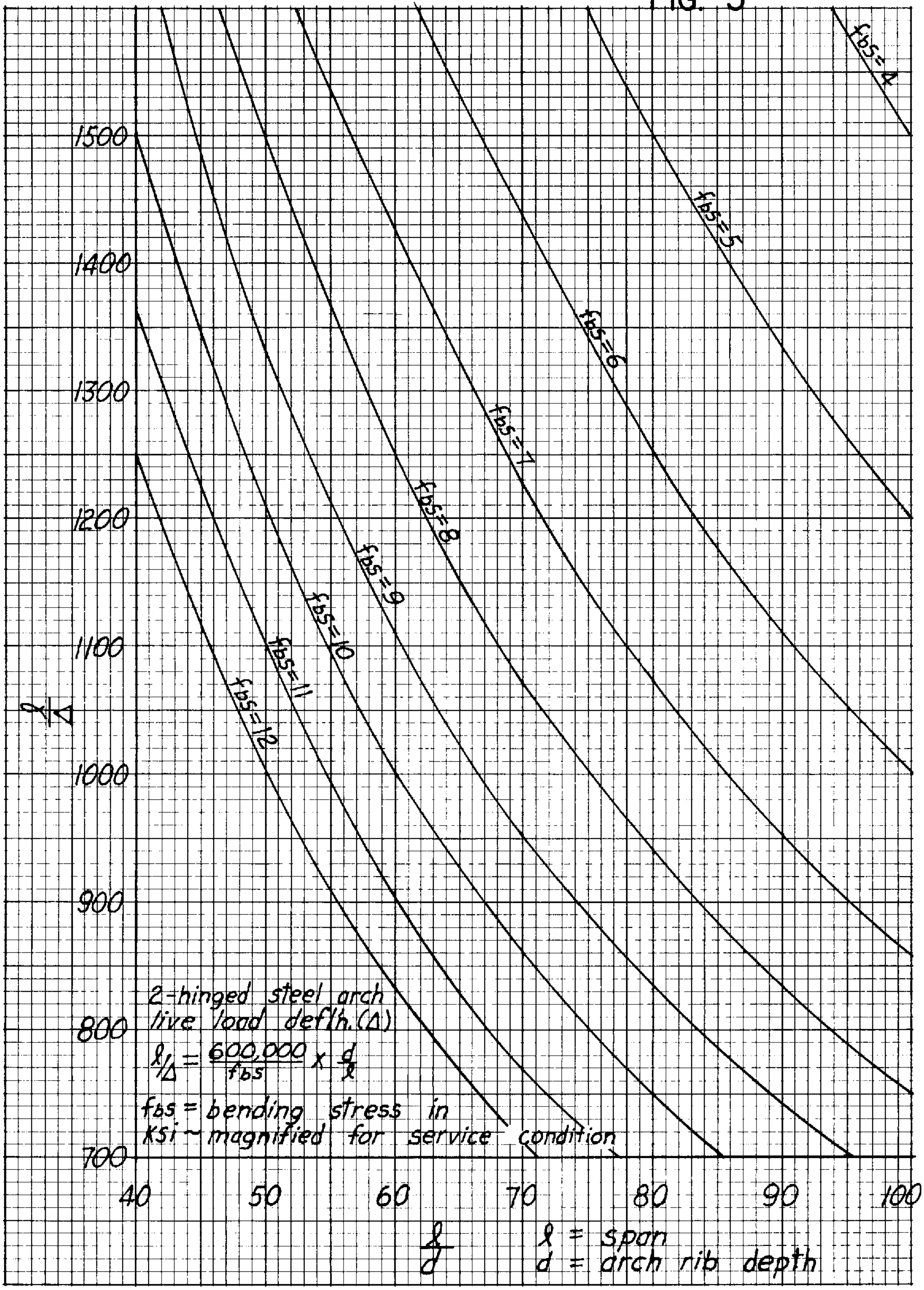
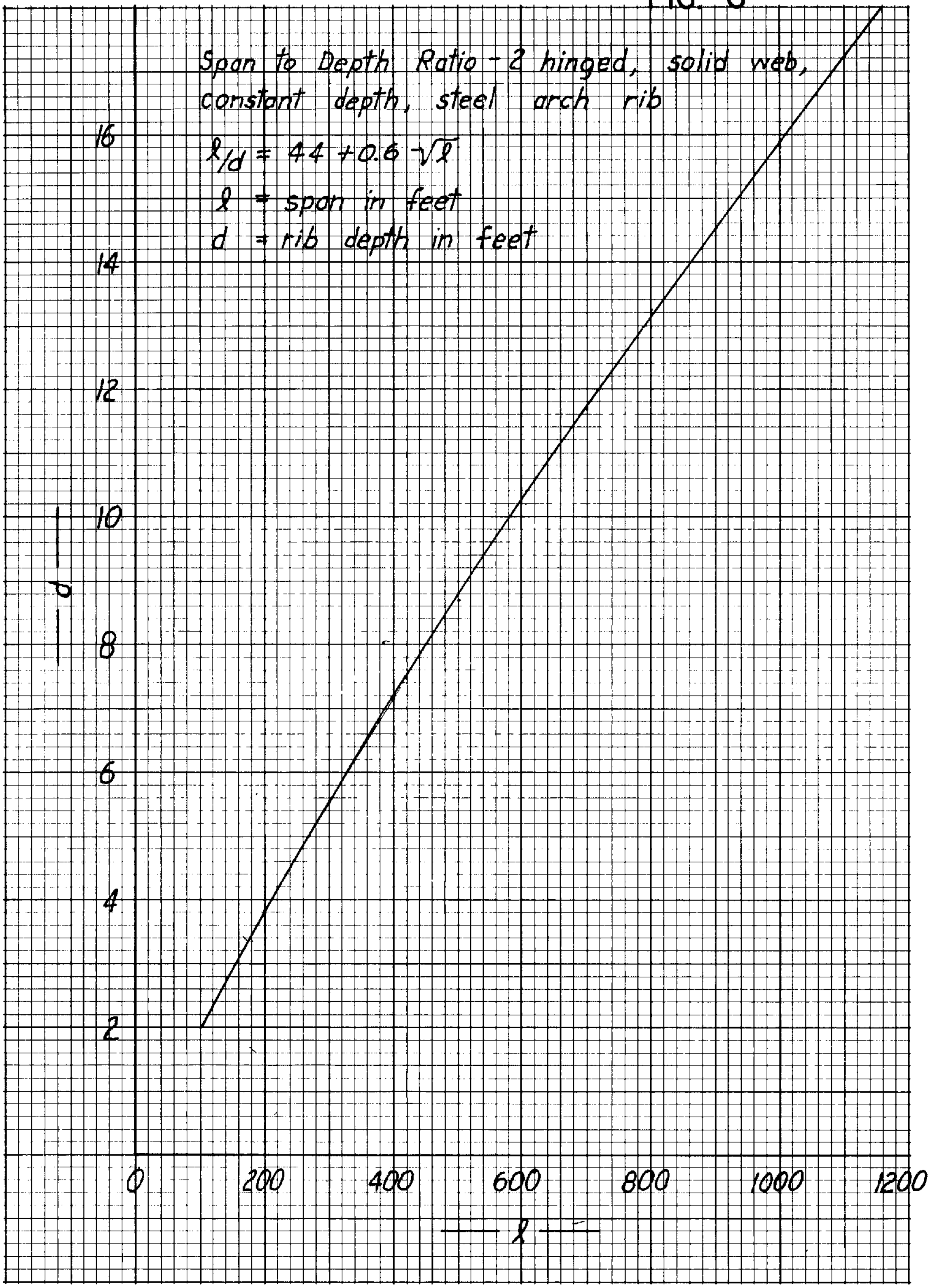

fbs=4
fbs=5
fbs=6
fbs=7
fbs=8
fbs=9
fbs=10
fbs=11
fbs=12
1500
1400
1300
1200
1100
1000
900
800
700
$\frac{\ell}{\Delta}$
2-hinged steel arch
live load defl'n. (Δ)
$\ell/\Delta = \frac{600,000}{f_{bs}} \times \frac{d}{\ell}$
f_{bs} = bending stress in
ksi ~ magnified for service condition
40
50
60
70
80
90
100
$\frac{\ell}{d}$
ℓ = span
d = arch rib depth

FIG. 6



Span to Depth Ratio - 2 hinged, solid web, constant depth, steel arch rib
$\ell/d = 44 + 0.6\sqrt{\ell}$
ℓ = span in feet
d = rib depth in feet
16
14
12
10
8
6
4
2
d
0
200
400
600
800
1000
1200
ℓ

in a trussed arch for the same reason, and also because the ratio of r to d is almost 0.5 for a trussed arch and about 0.4, or less, for a solid web arch.

Fixed arches may have about 0.8 the depth of two-hinged arches. With this depth ratio and the same bending stress, fixed arches will have about 2/3 the live load deflection of two-hinged arches.

See section 1.15 for approximate L.L. deflection equations.

1.3.1 Tied Arch-Rib and Tie Depths

Tied arches may be designed with a rib of sufficient depth to take almost all of the bending moment. In that case the tie may be made the minimum depth permitted by the Specifications (3) for a tension member with an unsupported length equal to the spacing of the suspenders. The tie in this case is designed mainly for the tension produced by the horizontal component of arch thrust. The tie will receive some moment due to arch rib deflection under partial live load. This effect can be approximately allowed for by dividing the total live load moment between the rib and tie in proportion to their moments of inertia.

Many tied arches, however, are designed with a deep tie and shallow rib. The tie then takes most of the bending from partial live load or other causes. The rib then serves principally as a compression member to take the arch thrust, and may be made as shallow as consideration of allowable compressive stress and maximum ℓ/r, based on supports at the suspender points, will economically allow. The rib depth will be such that bending stress as well as axial stress should be considered in its design.

An approximate method of analysis of a tied arch is to calculate the bending moment in the same manner as for a true arch, and then divide the moment, at any point, between the rib and tie in proportion to their moments of inertia. An exact method of analysis would be to treat each suspender force as an "unknown." The amount of work would then be such that a computer program would probably be needed.

1.4 Rise-to-Span Ratio

The rise-to-span ratio for arches varies widely. A range from 0.12 to 0.3 would include almost all bridge arches. Most are in the range from 0.16 to 0.2. The site along with navigation clearances or vehicular clearances and roadway grades may determine, or have a predominent effect on the rise to span ratio used. A tied arch with suspended roadway over a navigation channel is one case in which there is full freedom to vary the rise-to-span ratio to suit economy and appearance.

An increase of rise decreases arch thrust inversely with the rise-to-span ratio, reducing the axial stress from dead and live load and the bending stress from temperature change. The axial tension in a tie, if used, is also decreased in the same way. Offsetting these effects from the standpoint of economy is the increased length of the arch rib. This greater length increases the quantity of steel and the dead load. It also increases the buckling length in the plane of the arch and the moment magnification factor. The lengths of the suspenders are increased. The total length of lateral bracing between the ribs is increased, and the wind overturning and stresses are increased. Many existing tied arches have a rise to span ratio of about 0.2.

1.5 Stress from Change of Temperature

An arch responds to temperature change by increasing or decreasing its rise, instead of by increasing or decreasing the span. If the arch has a hinge at the crown and at the ends, no stresses are produced by a change of temperature. In the case of a two-hinged arch, positive moment is produced by a drop in temperature and negative moment by a rise in temperature. For a fixed arch, the moments reverse in the outer portions of the span. The greater the ratio of rise to span, the smaller is the temperature stress. A lesser depth of rib also results in smaller temperature stresses. The following approximate equations are based on an assumed uniform moment of inertia:

Temperature Rise
2-Hinged Arch

$$H_t = \frac{15EI\omega t}{8h^2} \qquad \text{Eq. 4a}$$

$$M_x = -H_t \cdot y \qquad \text{Eq. 4b}$$

Fixed Arch

$$H_t = \frac{90EI\omega t}{8h^2} \qquad \text{Eq. 4c}$$

$$M_s = \frac{3}{4} H_t \cdot h \qquad \text{Eq. 4d}$$

$$M_x = -H_t \left(y - \frac{3}{4} h\right) \qquad \text{Eq. 4e}$$

Where H_t = horizontal thrust from change of temperatures

M_x = moment at point x

y = ordinate to arch axis at point x

I = assumed uniform moment of inertia

ω = temperature expansion coefficient

t = change of temperature

h = arch rise

1.5.1 Tied Arch Stress from Change of Temperature

A tied arch will not be stressed by change of temperature. A difference in temperature between the rib and tie will produce stresses. In fact differences in temperature between parts of most bridge structures will produce stresses, but these are generally neglected. It may be that this effect in a tied arch should not be neglected due to the large distance between the rib and tie, and the fact that the tie may be protected from the direct sun by the roadway slab. It can be calculated by the equation for a two-hinged arch, using the sum of the I values for rib and tie and the difference in temperature. As for any tied arch, the calculated bending moment may then be divided between the rib and tie in proportion of their respective moments of inertia, as an approximation.

1.6 Rib Shortening, Camber and Arch Rib Closure

The shortening of the arch axis from axial thrust due to loads is called rib shortening. The stress produced by loads may be divided into two parts: that resulting from rib shortening, and that resulting from flexual deformation. The arch axis can be shaped so as to practically eliminate other dead load moments, but the shape of the axis does not affect the moment from dead load rib shortening. This dead load axial deformation produces negative horizontal reactions and positive moment throughout the span of a two-hinged arch. This positive moment results in a required larger top flange or chord area than that required for the bottom flange or chord. Using a larger area in the top flange than in the bottom flange will not balance the maximum unit stresses in the two flanges, because more of the axial thrust will then go to the top flange or chord.

Therefore the net result of dead load rib shortening is an understressed bottom flange or chord, which means that maximum economy is not reached. However, dead load rib shortening stress can be eliminated.

As an example of the effect of rib shortening on stress, the stress sheet for the 1700 foot arch span over the New River Gorge in West Virginia (4) shows that, in the central two-thirds of the span, the dead load upper chord stresses in kips are from 35 to 60 percent higher than the lower chord stresses. This large difference is partly due to rib shortening and partly due to larger upper chord areas. However, the design unit stresses in the upper chord are very close to the allowable, whereas the design unit stresses in the lower chord are about 15 percent below the allowable. This is a typical relationship for many arches, and is due to the effect of dead load rib shortening.

It is not necessary to have rib dead load shortening stresses in an arch, if certain methods of member camber and erection are used. If the lengths of the members are cambered for dead load axial stress, there will be no dead load rib shortening stresses. Steinman recognized this in 1936 in the design of the Henry Hudson Bridge (29). This is because the closure of the arch must be forced if the member lengths have been cambered for dead load axial stress but not for curvature due to moment from rib shortening. The effect of this forced closure, which can be done with jacks that are needed for erection in any case, is to produce equal and opposite flexure to that produced by rib shortening. The almost complete elimination of dead load moments in the Fremont Bridge (5) by the use of length camber, resulting in forced reverse moments under zero load is illustrated in Figure 15.

The method of analysis used, such as a computer program, may include rib shortening stresses. If so, they may be separately figured and then excluded by use of the following approximate equations for rib shortening. These equations are for a uniform depth. See equations 48a - 48h (Concrete Arches) for a varying depth of rib.

2-Hinged Solid Web

$$H_{rs} = \frac{-15}{8}(r/h)^2 H_{D.L.} \qquad \text{Eq. 4f}$$

$$M_x = -H_{rs}\, y \qquad \text{Eq. 4g}$$

Fixed-Solid Webs

$$H_{rs} = \frac{-90}{8}(r/h)^2 H_{D.L.} \qquad \text{Eq. 4h}$$

$$M_x = -H_{rs}(y - 0.67h) \qquad \text{Eq. 4i}$$

2-Hinged-Trussed Rib

$$H_{rs} = -0.47\ (d_c/h)^2 H_{D.L.} \qquad \text{Eq. 4k}$$

Where $H_{D.L.}$ = horizontal reaction from dead load

H_{rs} = horizonal reaction from rib shortening

r = radius of gyration of a solid web rib

d_c = depth center to center of chords at the crown

h = arch axis rise at crown

y = arch axis rise at point x, measured from springing

The method used for analyzing a trussed rib will most likely include rib shortening deformation and stress, and the above equations will be needed for exclusion of dead load rib shortening stress. The method for solid web ribs may or may not include rib shortening. This illustrates the importance of a designer knowing the basic assumptions of the method of analysis he is using.

A trussed arch may be closed on a temporary pin in either the upper or lower chord. After the arch becomes self-supporting, with all of the thrust going through the pin, the gap for the closing chord member is jacked open an amount sufficient for the length of that member. In order to insure that the stresses are as calculated, it is preferable to have the connection at one end of the closing member blank. Holes are drilled in the field to fit the opening produced by a precalculated jacking force. This force is equal to the calculated stress in the member from the dead load, including erection equipment, which is on the bridge at the time of jacking. In order to exclude all dead load rib shortening stress, the rib shortening stress from dead load to be placed after closure should be subtracted (algebraicly) from the calculated jacking force. Dead load rib shortening stress should be excluded from the design stress and camber in all members. The members are cambered, of course, for the stress from dead load thrust, and the stress from dead load moment due to causes other than rib shortening.

An arch may be designed as three-hinged, only for the dead load at time of closure. It would be closed on a pin at the crown, and then provided with a full depth moment connection. No jacking is required, and errors of surveying or fabrication would not affect the stresses. The remainder of the dead load would produce rib shortening stresses and these should be included in the design. If it is desired to eliminate these stresses, it could be done by light jacking at closure. If the pin is in the lower chord, jacks in the line of the upper chord would exert a pull in the upper chord line, and release after closure would leave a precalculated tension in the upper chord equal to the rib shortening compression form the additional dead load. For a pin at mid-depth of a solid web rib, simultaneous jacking would be required at both flanges to eliminate all rib shortening stress.

It is necessary to indicate the assumptions with regard to rib shortening on the plans, and the requirements at closure to insure the realization of such assumptions. The contractor should have freedom in choosing the method of erection, but he must be required to achieve the desired stress condition.

1.6.1 Tied Arch Rib Shortening and Tie Lengthening

The effect of rib shortening in a tied arch is accentuated by the effect of tie lengthening, due to tension, from load effects. The following equation can be used for calculating H:

$$H_{rt} = \frac{-15H_{DL}(I_t+I_r)}{8h^2}\left(\frac{1}{A_t} + \frac{1}{A_r}\right) \qquad \text{Eq. } 4\ell$$

Where I_t = moment of inertia of tie

I_r = moment of inertia of rib

A_t = area of tie

A_r = area of rib

Values of I and A at the crown may be used

$$M_{r+t} = H_{rt} \cdot y \qquad \text{Eq. 4m}$$

Where M_{r+t} = sum of moments in rib and tie at point x

M_{r+t} may be divided between rib and tie in proportion to their respective values of I. Camber will eliminate this moment and that from hanger stretch.

1.7 Effect of Location of End Pins with Respect to the Arch Axis

For trussed arch ribs the end pins are often located at the center of the lower chord instead of on the arch axis. This has the advantage of simpler framing for the arch truss and for the lateral system at the ends of the span. It may result in somewhat more steel in the arch chords.

Locating the pin in the lower chord, in effect, is equivalent to introducing a negative dead load and live load truss moment at the pin, instead of zero moment for a pin on the arch axis. For dead load, this negative moment reduces to zero in a little more than one quarter of the span length and a maximum positive moment is produced at the center of the span. As a result the lower chord has more compression than the upper chord in the outer parts of the span, and the less in the central part of the span. The lowered pin produces similar results for live load stresses at the ends and center of the span. The areas of the chord members cannot be fully adjusted to meet these large differences in maximum stress between upper and lower chords at the same point in the span. The true arch axis is changed by the change in chord areas. The result is similar to the effect of rib shortening stress. The designer will be forced to use reduced unit stresses in the upper chords at the ends of the span, and in the lower chords in the central part of the span. These effects may offset the saving in cost of framing details at the ends of the span for a pin located on the arch axis.

Appearance may also enter into the decision of end pin location. Tapering the end panel almost to a point from a fairly deep truss may give the appearance of weakness. When the pin is placed in the lower chord, the abutments are sometimes designed so as to hide the fact that the upper chord does not thrust against the abutment. The pin is always placed on the axis for a solid web arch.

1.8 Fixed Arches Versus Hinged Arches

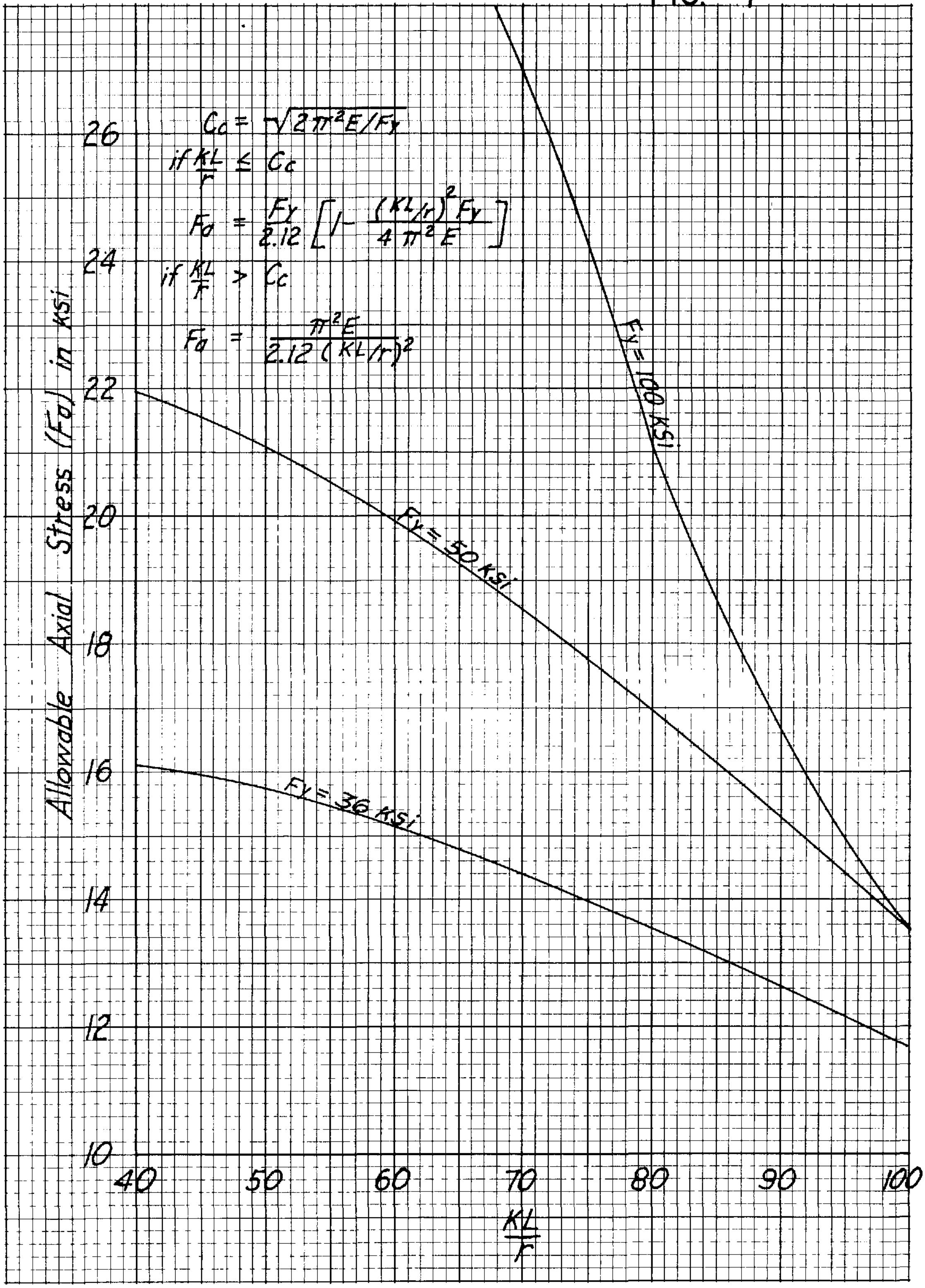
Unlike concrete arches, most steel arches are not fixed. The Rainbow Arch over Niagara Falls (2) is a notable exception. This arch was originally planned as a two-hinged arch and changed to a fixed arch to reduce deflection and moment magnification. Of course, reduced deflection and reduced moment magnification could have been obtained by use of a deeper two-hinged arch. The fixed arch was found to be lighter in weight, and was probably also favored because of its slender appearance.

Abutment costs are higher for a fixed arch because a large moment must be transmitted to the foundation by anchorage ties or by sufficient spread of foundation bearing to keep the edge pressure within the allowable. Also heavy anchorage details between the steel and concrete are required. Although the designers of the Rainbow Arch found economy in the use of a fixed arch, studies by the designers of the Bayonne and the Glen Canyon arches indicated the opposite.

Erection is more complicated for a fixed arch, particularly if it is by the tie-back method. Complex calculations were made for the erection of the Rainbow Arch.

The fixed arch is statically indeterminate to the third degree. Analysis is generally made by cutting the arch at the center of span, and taking the horizontal thrust, moment and shear at that point as the "unknowns." Simultaneous equations can be avoided by use of the "elastic center," a point below the crown. However, there may not be a net saving in work by use of this method.

FIG. 7



$C_c = \sqrt{2\pi^2 E/F_y}$
if $\frac{KL}{r} \le C_c$
$F_a = \frac{F_y}{2.12}\left[1 - \frac{(KL/r)^2 F_y}{4\pi^2 E}\right]$
if $\frac{KL}{r} > C_c$
$F_a = \frac{\pi^2 E}{2.12\,(KL/r)^2}$
F_y = 100 ksi
F_y = 50 ksi
F_y = 36 ksi
Allowable Axial Stress (F_a) in ksi
10
12
14
16
18
20
22
24
26
40
50
60
70
80
90
100
$\frac{KL}{r}$

1.9 Allowable Stress and Plate Buckling

The allowable axial stress, F_a, is given by the equation for F_a in Table 1.7.1 of AASHTO Specifications, as revised by 1974 Interim 7. This equation is:

$$F_a = \frac{F_y}{2.12}\left(1 - \frac{(KL/r)^2 F_y}{4\pi^2 E}\right)$$

L for a solid web arch rib is one-half the length of the arch axis. This equation has been plotted in Figure 7, and a table of k values for arch ribs is given in Figure 3. As in the AASHTO Specifications, when KL/r exceeds $(2\pi^2 E/F_y)^{1/2}$, the following equation for F_a should be used:

$$F_a = \frac{135 \times 10^6}{(KL/r)^2}$$

The allowable bending stress, F_a, for a solid web arch is $0.55F_y$. No reduction for lateral buckling is needed for a box section, or for two plate girders laced together, as in the case of the Henry Hudson Bridge.

The interaction equation, $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$, is used to check the assumed section.

The plates making up the cross-section must meet requirements for local buckling. Where the overall design is based on an interaction equation, as is the case of the arch rib, the equations for plate buckling cannot be based on F_y, or on F_a and F_b. They must be based on f_a and f_b with a safety factor included in the equation.

The axial stress f_a, from arch thrust, produces uniform compressive stress across the width of both the flange and the web plates. The bending stress f_b, from moment, produces uniform stress across the flange plate; but a varying stress from maximum compression at one edge to maximum tension at the other edge in the web plate. As a result, separate buckling equations are needed for the web and flange plates.

1.9.1 Web Buckling

For an unstiffened web, the effect of f_b on buckling is very small compared to the effect of f_a. For a stiffened web, f_b has a very appreciable effect on the buckling of the individual panels between stiffeners. Although f_b does not appear in the following equations, its effect has been taken into account in the numerical coefficients of the equations. These coefficients are based on a value of f_b at the edge of the web equal to about 1.75 f_a. It is very unlikely that any arch would have a higher ratio of f_b to f_a than 1.75.

The plate buckling equations, which are based on the AASHTO Specification equations, are quite conservative for the web. In the case of the web plate, particularly the unstiffened web, the axial stress has the major effect on local buckling, and this stress is generally in the range from 0.15 F_y to 0.3 F_y.

The AASHTO plate buckling equations, and the equations given here, cover the full range of unit stress in the plate. Reference to USS Steel Design Manual, pages 74 and 75, will show two equations for plate buckling, one to be used when the critical stress is below 0.5 F_y and the other for a critical stress above 0.5 F_y. It will be noticed, in Figure 4.1, that the curve, for critical buckling stress below 0.5 F_y, gives too high a stress above 0.5 F_y. In order to use one equation over the entire range, it is necessary to accept a reduced allowable stress for the lower part of the stress range. Since the axial stress in the web plate of a bridge arch rib will be in this lower range, the allowable D/t for the web will be on the conservative side.

The effect of this conservatism on economy is small, because a heavier web will permit lighter flanges.

Web Buckling Equations:

No longitudinal stiffener

$D/t = 5000/\sqrt{f_a}$, max. $D/t = 60$ Eq. 5

One stiffener at mid-depth

$D/t = 7500/\sqrt{f_a}$, max. $D/t = 90$ Eq. 6

$I_s = 0.75Dt^3$ Eq. 7

Two stiffeners at the 1/3 points

$D/t = 10000/\sqrt{f_a}$, max. $D/t = 120$ Eq. 8

$I_s = 2.2Dt^3$ Eq. 9

Outstanding element of stiffener

$b'/t' = 1625/\sqrt{f_a + f_b/3}$, max. $b'/t' = 12$ Eq. 10

Where D = web depth

t = web thickness

I_s = moment of inertia about an axis at the base of the stiffener

b' = width and t' = thickness of outstanding stiffener elements

Generally two stiffeners should be used in order to get the most economical section. For spans of 450 feet or more, longitudinal diaphragms across the width of a box section should probably be used. These would act as rigid lines of support for the webs. Such a diaphragm could be used at mid-depth of the box, and each panel formed thereby could be stiffened at its mid-depth. The b/t value for each panel would then be based on the average stress in the panel rather than on the axial stress for the whole rib. In the case of the Rainbow Arch (2), continuous longitudinal diaphragms were used at the mid and fourth points of the web depth. These longitudinal diaphragms were supported by radial diaphragms about 20 feet apart. The stress in a longitudinal diaphragm at mid-depth is f_a.

Longitudinal Diaphragm Buckling Equation

$b/t = 4500/\sqrt{f_a}$, max. $b/t = 54$ Eq. 10a

1.9.2 Flange Buckling

Flange Buckling Equations (Unstiffened)

Between webs

$b/t = 4250/\sqrt{f_a + f_b}$, max. $b/t = 47$ Eq. 11

Overhang

$b'/t = 1625/\sqrt{f_a + f_b}$, max. $b'/t = 12$ Eq. 12

Where b = distance between webs

b' = flange overhang outside web

t = flange thickness

Stiffeners are seldom used on arch rib flanges. A single stiffener at mid-width of the flange would permit a $b/t = 8500/\sqrt{f_a + f_b}$, and the required I_s for that value of b/t would be $4bt^3$. About the only case where flange stiffening might be used would be where the two ribs are not braced together, laterally.

1.9.3 Equation for Load Factor Design

All the above equations in this article are for Service Load Design. The corresponding equations for Load Factor Design Are:

Web Plates

No longitudinal stiffener

$D/t = 6750/\sqrt{f_a}$ Eq. 13

One longitudinal stiffener

$D/t = 10{,}150/\sqrt{f_a}$ Eq. 14

Two longitudinal stiffeners

$$D/t = 13{,}500/\sqrt{f_a} \qquad \text{Eq. 15}$$

$$b'/t' = 2200/\sqrt{f_a + f_b/3} \qquad \text{Eq. 16}$$

Flange Plates

Between webs

$$b/t = 5700/\sqrt{f_a + f_b} \text{ - unstiffened} \qquad \text{Eq. 17}$$

$$b/t = 11{,}500/\sqrt{f_a + f_b} \text{ - one stiffener} \qquad \text{Eq. 17a}$$

Overhang

$$b'/t' = 2200/\sqrt{f_a + f_b} \qquad \text{Eq. 18}$$

Otherwise the equations are the same.

1.9.4 Web Buckling from Shear

Transverse stiffeners on the web are not generally required for stress in either the rib or tie of an arch, because of the low unit shear stress. The AASHTO equation $t = \frac{D\sqrt{f_v}}{7500}$ (Article 1.7.71) (3), which permits the omission of transverse stiffeners, is almost certain to be met. The dead load produces very little shear since the thrust line follows the axis. Live load shear is small because the shear is only a component of the thrust and, at any point, is equal to the thrust multiplied by the sine of the angle between the direction of the thrust and the direction of the axis at that point.

In the case of the tie, the large axial tension more than nullifies any buckling tendency from shear. The Fremont Bridge (5) tie is 18 feet deep with 1/2-inch webs, a D/t ratio of 432. Transverse stiffeners are used on these webs midway between floorbeam diaphragms, thus giving web panels of 5 feet 7 inches. These stiffeners were probably used to prevent distortion from handling in shipping and erection.

Longitudinal stiffeners are not required in an arch tie because of the large tension.

Diaphragms are used for arch ribs and ties at points of loading. The Rainbow Arch has diaphragms at the columns and midway between, giving a 20-foot spacing for the 12-foot rib.

1.10 Wind Stress and Wind Deflection

For lateral forces such as wind, the arch is a curved member in the plane normal to the direction of loading. The central angle is large and, therefore, the torsional effects are very important in the wind stress analysis. The exact analysis of torsional effects is quite complicated because of the interaction of St. Venant torsion with warping torsion.

If the two arch ribs are connected by a single lateral system at the mid-depth of the ribs, the St. Venant torsion is of minor importance and can be neglected in the analysis. The torsional effects are resisted in this case by equal and opposite bending of the arch ribs in the vertical plane. With lateral systems at both the upper and lower flange or chord levels, St. Venant torsion becomes predominant because the overall structure is a closed torsional section.

1.10.1 Single Lateral System

The forces acting on the arch are applied at the connections of the laterals to the ribs, and they act tangentially to the arch curve. Since the laterals carry the transverse wind shear, the tangential forces acting on the arch ribs are equal to the transverse shear, in the lateral system panel, multiplied by the ratio of the panel length to the distance between ribs. For the symmetrical load case of wind over the full span, the shear is zero at the center of the span, and the shear in any panel can be found by summing the panel loads outward from the center of the span. This procedure is illustrated in Figure 8. These tangential forces act toward the crown of the windward rib and opposite on the leeward rib.

One method of determining the arch stresses produced by these loads is to resolve the tangential forces into vertical and horizontal components. The horizontal component of the reaction at the springing is then determined by assuming one end bearing to be on horizontal rollers, figuring the horizontal movement on the rollers, and then the horizontal force required to reduce the movement to zero. The windward arch rib will move downward in the outer portions of the span under positive moment and upward in the central portion of the span under negative moment. The leeward rib will do the opposite. In other words the ribs rotate clockwise for the outer parts of the span and counter-clockwise for the inner part of the span, as seen from the left springing with the wind coming from the right. The wind stress at any section is composed of two parts, the axial stress and the bending stress in the vertical plane. The stresses can also be obtained by taking the "unknown" as the crown moment instead of as the horizontal reaction

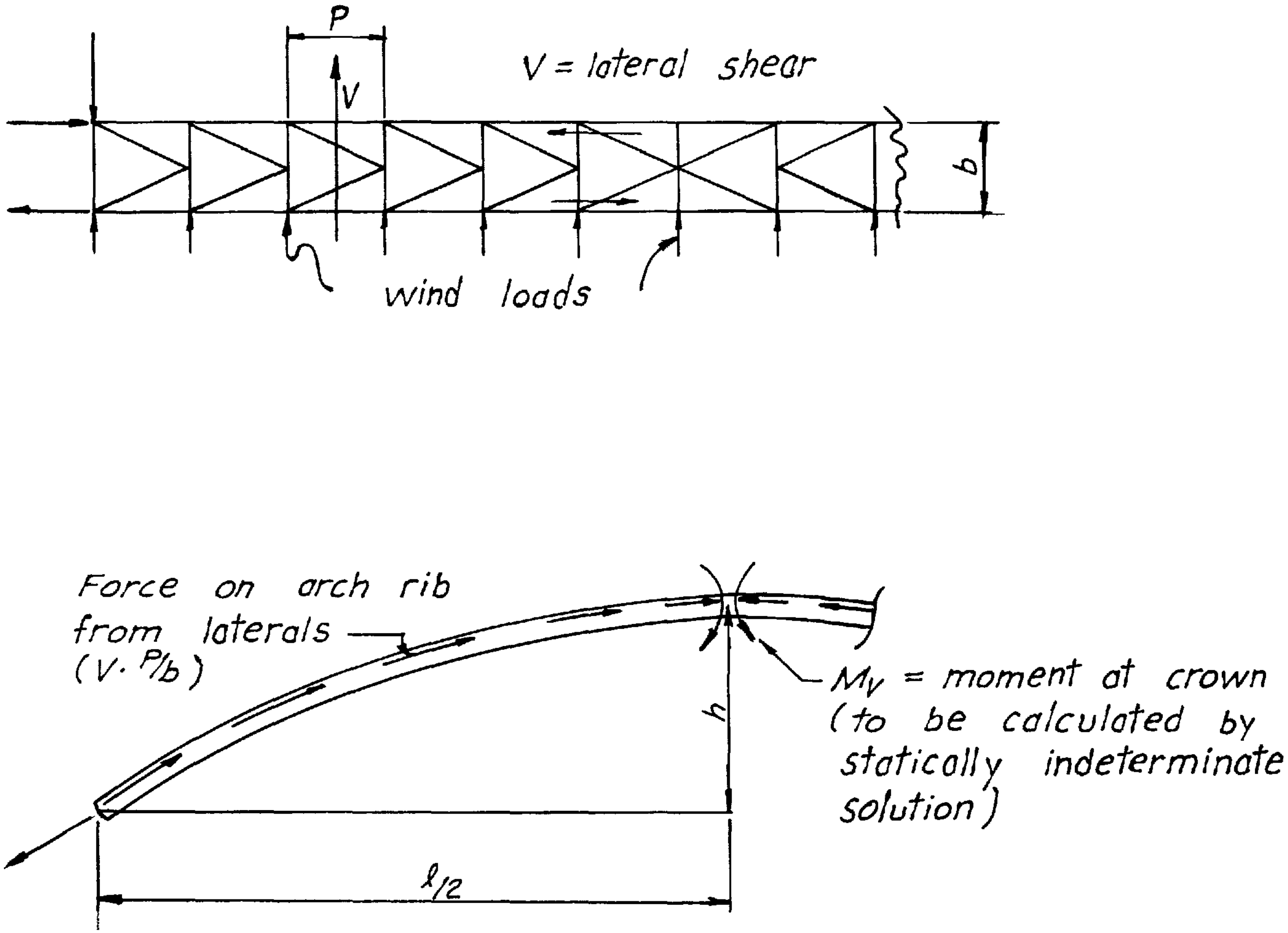


FIG. 8 – WIND ANALYSIS BY ARCH BENDING METHOD
(single lateral system at mid-depth of rib)

as indicated in Figure 8. The statically determinate structure is a three-hinged arch. The moment in the vertical plane, at the crown, is then solved for by making the rotation in the plane of the rib at the crown equal to zero, in the case of symmetrical loading. This method of analysis is illustrated by an example.

1.10.2 Two Lateral Systems

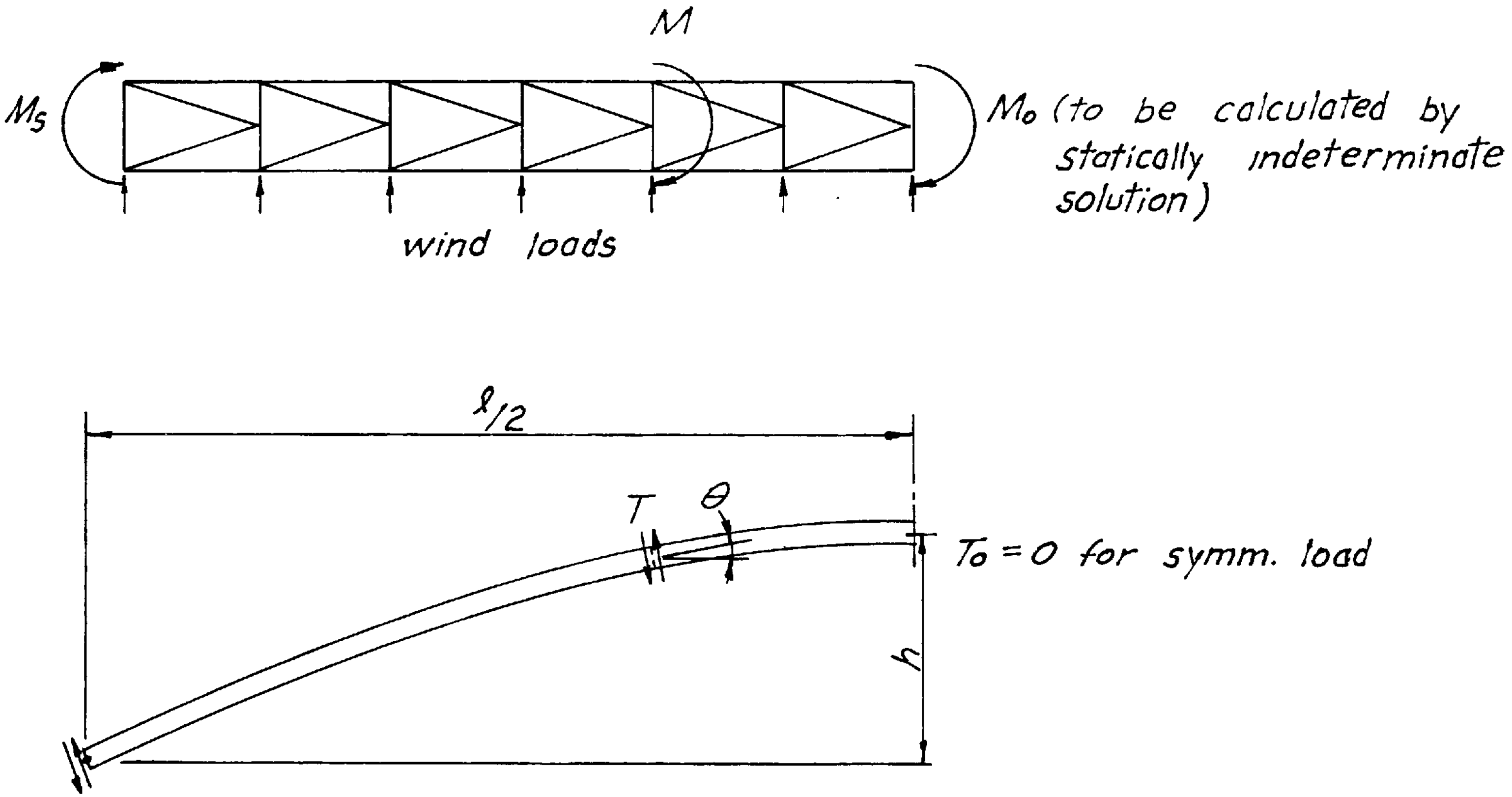
On long spans, two lateral systems are used between the arch ribs, at the levels of the upper and lower chords or flanges. With two lateral systems, the structure is stiff in St. Venant torsional action, because it is a closed torsional system with one dimension equal to the arch rib depth and the other dimension equal to the distance center to center of ribs. There will be some vertical arch rib bending action but the St. Venant torsional action will be dominant. For two lateral systems between the ribs, it is sufficiently accurate to neglect the arch rib bending stress and consider only the St. Venant torsional stress and the lateral bending stress. The St. Venant torsional action produces stresses in the web members of the rib and in the laterals. The lateral bending stress which accompanies the St. Venant torsional analysis produces axial stresses in the arch ribs which correspond to the axial stresses in the arch ribs found by the vertical arch bending method of analysis. An example will later be analyzed by the St. Venant torsional method and by the vertical arch bending method to give a comparison of results. The St. Venant torsional analysis involves cutting the arch at the crown and solving for the unknown lateral bending moment at the crown. Under symmetrical loading the torsional moment at the crown is equal to zero. Figure 9 illustrates the stress action. For varying moment of inertia and any shape of arch axis, the arch axis may be divided into several sections of equal length Δ_L, and the "unknown" quantity, the lateral moment M_O at the crown, is found by solving the following equation which was derived by equating the rotation in a horizontal plane at the crown to zero:

$$M_O = \frac{\Sigma[\cos\Theta(M_x\cos\Theta + M_y\sin\Theta) \div I] + \frac{E}{G}\Sigma[\sin\Theta(M_x\sin\Theta - M_y\cos\Theta) \div K]}{\Sigma[\cos^2\Theta \div I] + \frac{E}{G}\Sigma[\sin^2\Theta \div K]} \qquad \text{Eq. 19}$$

Where I = average moment of inertia for each section

K = average torsional constant for each section

G = shear modulus of elasticity



m_x = moment about vertical axis of wind loads between crown ℄ & any pt.
m_y = " " horizontal " " " " " " " " "

$$M = M_0 \cos\theta - (m_x \cos\theta + m_y \sin\theta)$$

$$T = M_0 \sin\theta - (m_x \sin\theta - m_y \cos\theta)$$

FIG. 9 - WIND ANALYSIS BY ST. VENANT TORSIONAL METHOD
(double lateral system - at top and bottom of ribs)

The second terms in the numerator and denominator come from torsional deformation. For most steel arches the second terms are so much larger than the first terms that the effect of the first terms may be neglected. This is not true for concrete arches. Neglecting the first terms and assuming constant cross section, the equation becomes:

$$M_o = \frac{\Sigma \sin\Theta(M_x \sin\Theta - M_y \cos\Theta)}{\Sigma \sin^2\Theta} \qquad \text{Eq. 20}$$

If we assume a single centered arch axis of radius R and constant cross section, the equation for M_o becomes:

$$M_o = WR^2 \left\{ \frac{[\sin\beta - \beta/2 - \frac{\sin 2\beta}{4}] + \frac{EI}{GK}[\sin\beta - \beta\cos\beta - 1/2\,(\beta - \sin\beta\cos\beta)]}{[\beta/2 + \frac{\sin 2\beta}{4}] + \frac{EI}{2GK}[\beta - \sin\beta\ \cos\beta]} \right\} \qquad \text{Eq. 21}$$

Neglecting the first terms in the numerator and denominator, Eq. 21 becomes:

$$M_o = WR^2 \left[\frac{2(\sin\beta - \beta\cos\beta)}{\beta - \sin\beta\cos\beta} - 1 \right] \qquad \text{Eq. 22}$$

W is the wind load per foot, R is the radius and β is one-half the central angle. R and β are obtained by passing a circular curve through the crown and springing of the actual arch axis. At any point, M and T are as follows:

$$M = M_o\cos\Theta - WR^2(1-\cos\Theta) \qquad \text{Eq. 22a}$$

$$T = M_o\sin\Theta - WR^2(\Theta-\sin\Theta) \qquad \text{Eq. 22b}$$

1.10.3 Lateral Deflection - Two Lateral Systems

The lateral deflection at any point may be found by applying a unit transverse force to the cantilevered half arch, and multiplying the lateral bending moments due to this unit load by the actual lateral bending moments. These products are summed up and divided by EI to give the lateral deflection at the point of application of the unit load. The torsional deformation is practically negligible insofar as any effect on lateral deflection. The lateral deflection at the crown, assuming a circular axis is:

$$\delta_c = \frac{(R\sin\beta)^2}{2EI} \left[-WR^2\left(1 - \frac{2}{1+\cos\beta}\right) - M_o \right] \qquad \text{Eq. 23}$$

A more approximate formula will give practically the same answer:

$$\delta_c = \frac{L^2}{2EI} \left[\frac{WL^2}{4} - M_o \right] \qquad \text{Eq. 23a}$$

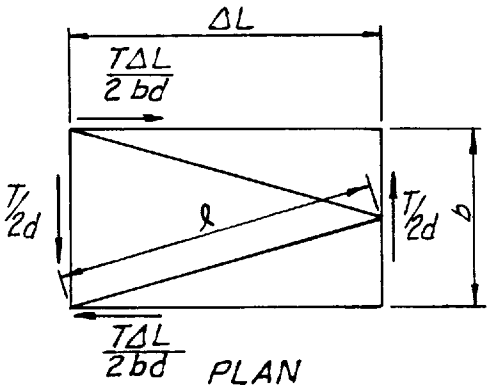
The value of M_o for an arch of average rise to span ratio is about six tenths of the center moment in a fixed-end beam of span equal to the rolled out length of the arch axis. Using $0.6 \times \frac{W\ell^2}{24}$ for M_o, the crown lateral deflection for the arch is 1.75 times that of a fixed-end beam of equivalent rolled out length. It will be shown further on that the lateral deflection of an arch with only a single lateral system is considerably greater. An approximate formula for the deflection at any point x, measured along the arch axis from the crown is:

$$\delta_x = \frac{1}{2EI} \left[\frac{W}{12} (x^4 - 4L^3x + 3L^4) - M_o (L - x)^2 \right] \qquad \text{Eq. 24}$$

Torsional moment acting on an arch with two lateral systems produces shearing forces on the laterals and on the web members of the ribs as shown in Figure 10. These shearing forces produce axial stresses in truss type laterals and rib webs, and shearing stresses in solid rib webs. For Equation 19, it is necessary to determine the torsional stiffness of the arch cross section, corresponding to the bending stiffness EI and the axial stress stiffness EA. The total torsional stiffness of the cross section is somewhat complicated by the fact that some of the members resisting torsion will have axial stress and others may have shearing stress. The torsional stiffness will be determined by the rotation in one panel length produced by a unit torque. Dividing the panel length by the rotation gives the torsional stiffness at that point on the arch axis. Referring again to Figure 10 and using the method described above, the following equation for the torsional factor K is obtained:

$$K = \frac{\Delta_L (bd)^2}{\frac{G}{E} \left[\frac{b^3}{8A_s} + \frac{\ell^3}{A_d} \right] + \frac{\Delta_L d}{t}} \qquad \text{Eq. 25}$$

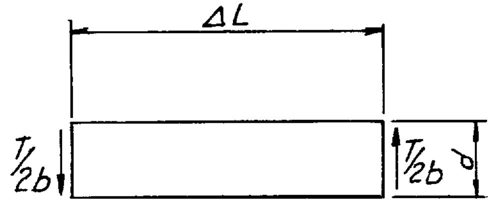
This equation applies only for a K lateral system with equal size members for the upper and lower systems and a solid web rib, with t representing the combined thickness of two web plates in the case of a box section. For other lateral system configurations and for a trussed rib instead of a solid web rib the equation for K will be different. The method of derivation described for the above equation can be used for any type of lateral system and for trussed ribs. There will generally be small stresses from torsional moments in the flanges or chords, but they are negligible in relation to the areas of the chords or flanges, from the standpoint of either stress or deformation.



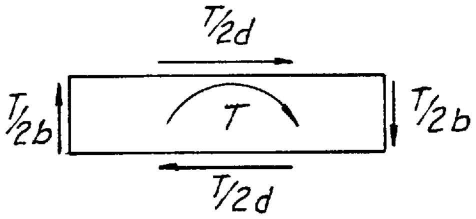
PLAN

A_d = diagonal area
A_s = strut area
t = combined thickness of webs of one rib

$$K = \frac{\Delta L\,(bd)^2}{\frac{G}{E}\left[\frac{b^3}{8A_s} + \frac{\ell^3}{A_d}\right] + \frac{\Delta L d}{t}}$$



ELEV.



CROSS-SECTION

stress in lateral strut = $T/4d$
" " " diagonal = $T\ell/2db$
shear in arch rib web = $T/2b$

FIG. 10

In addition to the lateral deflection from wind, there is also rotation in a plane normal to the arch axis. This rotation causes the leeward rib to move down and the windward rib to move up at the crown. This movement is of importance in distributing wind loads between the arch lateral system and a lateral system in the plane of the roadway above the arch. The rotation angle multiplied by the vertical distance from the arch axis to the roadway lateral system adds to the arch deflection to give the total transverse movement, from arch wind loading, at the roadway level. There is a slight reduction of the rotational effect due to the bending of any columns at the crown. However, these columns will be very short at the crown so that this bending effect is small. The rotation angle at the arch crown can be calculated from the following equation:

$$\text{Crown rotation} = \frac{R}{GK}\left[M_o x \frac{1}{2}\sin^2\beta + WR^2\left(\frac{1}{2}\sin^2\beta - \cos\beta - \beta\sin\beta + 1\right)\right] \qquad \text{Eq. 26}$$

This equation has been derived on the basis of a circular arch axis of uniform cross-section. Torsional deformation only has been included because the effect of lateral bending deformation is negligible for rotation. The effect of variable cross section and the actual shape of the arch axis can be taken into account by the summation method. By this method, a unit couple in the vertical plane is applied to the cantilevered half arch at the point where the rotation is desired. The products of the actual torsion by the torsion from the unit couple, divided by the torsional K at each point are summed to give the desired rotation at the point of the unit load.

1.10.4 Interaction Between Arch Rib and Roadway Lateral Systems

Distribution of wind forces between the roadway and arch rib laterals may be accomplished by a trial and error process of balancing lateral deflections and rotations. Lateral bending deflection of the columns is involved in this process unless transverse bracing is used between the columns. Use of such bracing is unusual and is not recommended because of the rotation of the arch.

Quite often lateral struts only are used between the ribs with no lateral diagonals. This type of lateral system can be solved for the thrust and moment in the vertical plane of the arch rib in the same manner as for a single lateral system. However, since there are no diagonals to take the lateral shear, lateral bending moments are produced in the arch ribs and in the struts. An approximate solution can be made for these moments by assuming points of contraflexure in the members midway between the panel points and midway in the length of the transverse struts. With this type of bracing, the lateral deflection from shear will be considerably greater than when diagonals are used. The deflection for one panel is:

$$\Delta = \frac{Vp^2}{24E}\left[\frac{p}{I_r} + \frac{2b}{I_s}\right] \qquad \text{Eq. 27}$$

Where V = panel shear

p = panel length

b = distance center to center of ribs

I_r = lateral moment of inertia of one rib

I_s = lateral moment of inertia of the strut

The total deflection from shear at any point is found by adding the panel deflections outward from the end of the span.

1.10.5 Unsymmetrical Wind Load

The discussion of wind stress up to this point has been based on symmetrical wind load only. Wind on the live load would be unsymmetrical for most cases. The same general methods of analysis, as have been explained for symmetrical wind load, can be used for unsymmetrical wind load. However, symmetry can no longer be used to eliminate "unknowns" in the solution. In the case of a double lateral system, there will be an unknown lateral shear and an unknown torque at the crown in addition to the unknown lateral moment. It will be necessary to set up three equations to be solved simultaneously, and the full length of the arch will have to be used in the summation method.

In the solution for unsymmetrical wind with a single lateral system, the tangential forces can no longer be determined from the fact of symmetry. An approximate way of handling this is to assume that the transverse lateral reactions at the ends of the span are the same as for a straight fixed end beam of uniform moment of inertia. In the part of the solution involving the determination of the thrust at the crown of a three-hinged arch, the vertical shear at the crown must be taken into consideration. This can be found by statics. The remainder of the solution is the same as for the symmetrical case, except that the full arch instead of the half arch must be included in the summation. After the determination of the arch thrust at the crown, the thrust can be multiplied by the distance between ribs to get M_0. The M_0 can be used to determine the lateral end reactions. If these differ appreciably from those assumed in the beginning, the whole process can be repeated.

Obviously the solution for unsymmetrical wind load involves considerable work, and there is a question as to whether it is really necessary. The main reason for such a solution would be for design of the laterals in the central portion of the arch. The unsymmetrical wind load stresses in these members can be approximated by assuming the length of the rib as a straight member and getting the end reactions on the basis of a fixed end beam of constant section.

1.10.6 Longitudinal Wind and Forces from Braking and Traction

An arch may have stresses produced by longitudinal forces caused by wind parallel to the roadway or wind at an angle, and by braking and traction from live load. AASHTO Specifications mention arches under wind and link arches with trusses for transverse wind force. Seventy-five pounds per square foot is specified for trusses and arches, and 50 pounds per square foot for girders and beams. The lower figure for girders and beams would appear to be based on the shielding effect of a solid web girder with respect to girders behind it, as compared to the shielding of a truss. If so, it would seem that the lower figure of 50 pounds per square foot should have been specified for a solid web arch rib.

AASHTO Specifications for "Substructure Design" give a table of longitudinal components of wind acting on the superstructure. However, this is intended to be used only for the purpose of obtaining forces on the substructure from the superstructure.

In the case of an arch, the effect of longitudinal wind is of more importance because of the total height of the structure. However, where the roadway passes over the crown of the arch, the arch itself would generally be shielded in the longitudinal direction by high ground at each end. Where the roadway is suspended below the arch, there would be no shielding above the roadway level.

For the design example in 1.16, the following analysis is made for a 60° wind acting on the deck. It is assumed that all of the longitudinal force acting on the deck, in the length of the arch span, is transmitted to the arch at the crown.

1.11 Lateral Buckling and Lateral Moment Magnification

An exact analysis of lateral buckling and lateral moment magnification is quite complicated. An approximate analysis is proposed here because buckling in the plane of the arch is almost always controlling, and therefore the effect of lateral buckling is minor in the overall design of most arch bridges.

Arches have overall fixity at the ends for lateral moment and for torsion. Due to the arch rise, resulting in a combination of torsion and flexure for lateral buckling, the buckling condition will always be more severe than that for a straight member with a length equal to the length of the arch axis. The third edition of "Guide to Stability Design Criteria for Metal Structures," pages 479 and 480, gives an equation (16.25) and a table (16.7) for calculating the critical vertical load for lateral arch buckling. The values for KL given below apply to a closed torsional system. In the case of steel arches, this would require two planes of lateral bracing between the ribs. Most concrete arches do have a closed torsional system. This equation and table have been used to work out the following lateral buckling lengths for use in the general equations involving KL/r, such as those for allowable unit stress and moment magnification. The following values apply to single arch ribs having a ratio of torsional stiffness to lateral bending stiffness between 0.5 and 1.5:

h/ℓ	KL
0.1	1.04L
0.2	1.17L
0.3	1.34L

Where L = the half length of the arch axis

The above values, in the case of a single rib, would be used to get F_e for use in Equations 2 and 3 for the lateral moment magnification factor, and in Figure 7 to get allowable stress based on lateral KL/r.

Braced Ribs with One Plane Diagonal Type Lateral System

h/ℓ	KL
0.1	1.36L
0.2	1.88L
0.3	2.30L

r = half the distance between two outer ribs

To allow for deformation of the laterals, multiply KL above by k from Eq. 28 (Bleich (28) page 169).

$$k = \sqrt{1 + \frac{\pi^2 I}{(KL)^2} \cdot \frac{1}{pb^2} \left(\frac{\ell_d^3}{A_d} + \frac{b^3}{A_b} \right)} \qquad \text{Eq. 28}$$

The above equation applies to a bracing system with one diagonal of length ℓ and area A_d, and one strut of length b and area A_b per panel. I is the lateral moment of inertia of the braced system and p is the panel length. Bleich's notation has been changed to correspond to the notation being used in this paper. Also KL has been used in place of ℓ, and E_t has been assumed equal to E. This last assumption is based on the fact that the bridge arch thrust would always be appreciably less than the maximum allowable because of the presence of bending stress due to partial live load. Therefore, it is reasonable to assume that, when loaded to ultimate strength, the stress from thrust would be within the proportional limit of the stress strain curve.

Braced Ribs with One Plane of Struts Only

The Guide to Stability Design refers to theoretical studies for the case of lateral bracing by transverse struts only, made by Ostlund (25) and Almeida (26); but goes on to state that these methods are more involved than is desirable for use in preliminary design. The Guide then mentions an approximate method by Wastlund (27) and others in which the arches are assumed to be straightened out so that the ribs and bracing members lie in a horizontal plane. The ASCE paper by Wastlund gives a simple equation for the buckling thrust. However, this equation involves a coefficient "C" which must be obtained from Ostlund's paper. The following method, based on Bleich (28), is similar to the method proposed for diagonal bracing. The following is a modification of Bleich's Equation (350) on page 178.

$$k = \sqrt{1 + \frac{\pi^2 I}{12\ (KL)^2} \left(\frac{p^2}{2I_1} + \frac{p_b}{I_b} \right)} \qquad \text{Eq. 28a}$$

Where I_1 = lateral moment of inertia of one rib

I_b = moment of inertia of the strut about an axis normal to the arch axis.

As with diagonal bracing, k is to be used as multiplier of $k\ell/r$ for use in equations for allowable stress and moment magnification. The moment magnification factor should be applied to both the axial forces in the ribs produced by lateral flexure of the arch as a whole and to the bending stresses in the individual ribs and struts produced by the shear accompanying lateral flexure. KL/r is the same as for braced ribs with one plane diagonal type lateral system.

Stress in Laterals Due to Arch Lateral Buckling

The buckling tendancy will produce lateral shears on the arch as a whole. This shear may be taken as 2 percent of the axial thrust. The maximum shear from buckling will occur at the lateral inflection point, near the arch quarter point. Since the arch is fixed laterally at the ends and is symmetrical, the buckling shear will be zero at the ends of the span and at the crown, and maximum at about the quarter point.

The lateral wind shear is a maximum at the ends of the span and, since it may be taken as a partial load, the maximum crown shear is about one fourth of the end shear. Thus the maximum lateral buckling shear and the maximum lateral wind shear do not add directly. The lateral wind shear at the quarter point may be assumed as 0.6 times the end shear. It is proposed that the laterals be of constant cross-section throughout the span, and be designed for either the full wind shear at the end of the span or for a shear equal to 2 percent of the total axial force at the quarter point plus 0.6 times the end wind shear, whichever is larger. The wind shear factor of 0.6 is very conservative because it assumes that the wind can blow at maximum velocity over three quarters of the span, with negligible velocity over the remainder of the span. For this reason, it is recommended that wind on the live load be neglected, because of its minor effect and the large amount of work in movable wind load analysis.

1.12 Wind Vibration

The hangers for suspended roadways and the columns for a roadway above an arch have had wind vibration difficulties in several arches. This vibration is due to vortex shedding. The wind stream is split by the member, causing turbulence and downstream vortexes with alternating transverse pressures on the member, resulting in vibration in a direction at right angles to the wind direction. This type of vibration was noticed in hangers of the Tacony-Palmyra bridge, a tied arch in Philadelphia, built before 1930. The hangers were H shaped members. Horizontal and diagonal bracing between the hangers, in the plane of the arch rib, was added to stop the vibration. This occurred about 1929, at the time the Bayonne bridge (6) was under design. H shaped hangers had been designed for this bridge, but they were changed to wire rope hangers, after the trouble at Tacony-Palmyra became known. The Bayonne bridge was probably the first arch bridge to use wire rope hangers. They have been used for a number of arches since. The wire rope hangers in the Fremont bridge (5) developed wind vibration during construction. Spreaders have been used between the four ropes at each panel point to change their vibration characteristics.

Several tied arches have developed vibration in their structural hangers. In one case, the H shaped sections were converted to box sections by welding plates on the open sides. The long columns at the ends of the Glen Canyon bridge developed vibration. Two long columns at the end of each rib were braced by horizontal struts to the rib.

This type of vibration is caused by a steady wind with a velocity such that the frequency of the vortex shedding is in resonance with the natural frequency of the member. The following equations may be used to determine this velocity for a given member.

$$f_v = C\sqrt{\frac{EI}{m\ell^4}} \qquad \text{Eq. 29}$$

and $V = \dfrac{f_v \cdot d}{S}$

Where f_v = natural frequency of the member in cycles per second

I = moment of inertia about the axis parallel to the wind

ℓ = length of member

m = mass per unit length of member

C = 1.57 for pinned ends and 3.57 for fixed ends

V = wind velocity

d = width of member at right angles to the wind direction

S = Strouhal number = 0.12 for H shape
0.15 for box shape

The longest column for the Glen Canyon bridge (7) has a length of 156 feet with an overall depth in the plane of the arch of 3' - 3 1/2" and at right angles to the plane of the arch of 2' - 7 1/4". The moment of inertia about the axis at right angles to the arch plane is 19,850 in.^{4} and the cross-sectional area is 82.7 square inches. Assuming average end conditions as half way between fixed and free, the frequency is:

$$f_v = 2.57\sqrt{\frac{32.2 \times 29 \times 10^6 \times 19{,}850}{281 \times 156^4 \times 144}} = 2.26 \text{ cycles per second}$$

$$V = \frac{2.26 \times 39.5}{0.15 \times 12} = 49.6 \text{ fps} = 34 \text{ mph}$$

The presence of axial load will modify the above figures by the following factor:

$$\sqrt{1 \pm \frac{f}{\pi^2 E} \left(\frac{Kl}{r}\right)^2}$$, where f is the axial unit stress. The plus sign is for tension and the minus is for compression.

Assuming f = 2500 psi and k = 0.75, the modifying factor is:

$$\sqrt{1 - \frac{2.5}{3.14^2 \times 29{,}000} \left(\frac{0.75 \times 156 \times 12}{15.5}\right)^2} = 0.96$$

and the critical wind velocity is 0.96 x 34 = 33 mph.

The bracing strut reduced the maximum column length, vibration in the plane of the arch, to about 100 feet. This would increase the critical wind velocity to:

$$\left(\frac{156}{100}\right)^2 \times 33 = 80 \text{ mph.}$$

The above equations are based on vibration transverse to the wind direction. Torsional vibration may occur for an H-shaped section but would be very unlikely in a box shape. Vibration can probably be avoided by using a box or double laced section, and designing for a critical wind velocity above that likely to occur for a steady wind at the structure site.

Since the suspended roadways of some suspension bridges have suffered severe vibration, it might be though that the same type of floor vibration could occur for a floor suspended from an arch. We know of no arch bridge which has been subjected to such vibration. The probable reason is that the rib or the tie of an arch is much stiffer for a given span than the stiffening truss of a suspension bridge. The stiffening member of an arch must be designed to carry the live load moment for a span approximately equal to one half the arch span. The stiffening member of a suspension bridge may be made as light as the designer determines to be adequate. The George Washington bridge, with a span of 3500 feet, had no stiffening truss in its initial single deck condition, which lasted for a period of approximately 25 years. During this period, no vibration of any seriousness occurred. This was due to the fact that the dead load cable tension supplied the necessary stiffening. The arch rib thrust does not supply similar stiffening because it is a compressive instead of a tensile force, and a compressive force amplifies change of shape, rather than resisting as does tension.

1.13 Interaction Between Rib and Roadway Framing

If the roadway longitudinal members are continuous across the columns or the suspenders they will participate in the arch bending moments, since they will take the same deflection curve as the arch rib. They will have, approximately, a participation bending stress equal to that in the arch rib multiplied by the ratio of the floor member depth to the depth of the rib.

The Hampton Road bridge (8) at Dallas, with a span of 192 feet, has 24 inch stringers and 58 inch deep ribs. With a maximum rib bending stress of about 10,000 psi, the participation stress in the stringers is roughly 24 x 10,000/58 = 4000 psi. This could be considered a secondary stress in the stringers. The total moment of inertia of the stringers is about one twelfth that of the ribs, so the reduction in moment on the ribs due to the action of the stringers would be about 8 percent. This was neglected in the design of the arch ribs.

The effect of deck participation is likely to be less as the arch span gets longer.

1.14 Welding and Other Connections

Welding is now being very extensively used for connecting the plates making up the cross-sections of box ribs and ties; and for making the shop splices of these members. The corner welds for these box sections carry the longitudinal shear between the flanges and webs of the rib and tie. Although full penetration welds are sometimes used at these corners, they are not needed for stress. By overlaping the web plate on the edge of the flange plate, a fillet weld, meeting the longitudinal shear requirement and the minimum size requirement as specified in AASHTO 1.7.26, may be used. The fillet weld costs less than the full penetration weld and is desirable from the standpoint of lesser shrinkage stresses due to its smaller volume.

Because the tie is a tension member, it is much more susceptible to possible cracking due to improper welding than is the compression rib. The corner welds receive the same stress as the tie, which is the full allowable tensile stress for the steel used. A defect in the weld may start a crack that can rapidly spread across the tie. Failure at any point in one tie of a bridge supported by only two arches is certain to cause complete collapse of the bridge. The tied arch is, in effect, a simple span, and one end rests on an expansion bearing. Even if the pier or abutments could take the horizontal arch thrust, which they are not designed for, the expansion bearing would prevent transfer to the pier of the horizontal force caused by failure of the tie.

Transverse and longitudinal stiffeners are not needed on the tie in the completed bridge. If the webs need stiffening for transport and erection, some form of temporary bracing, not welded, should be used. Use of unnecessary members and welding adds to the cost and increases the probability of occurance of a weld defect.

Due to the 100 percent certainty of complete collapse of the span following tie failure, the obtaining of perfect welds cannot be relied upon. A tougher steel that is capable of arresting crack growth is necessary in a tie that is fabricated by welding. The added cost of this tougher steel in the tie is a very small percentage of the total cost of the bridge, and is well justified on the basis of safety.

Suspender Connections

Where the roadway is suspended below the arch, the connections of the hangers should be designed to permit inspection with minimum trouble, and should avoid loss of cross-section in the arch rib. For a large truss, such as the Bayonne Arch, the hangers may be connected to vertical gusset plates extending below the lower chord. For solid web sections, the hangers are sometimes extended through large holes in the bottom flange of the rib. This gives a good appearance, but involves considerable loss of section or considerable reinforcement around the holes. Slots for gusset plates parallel to the arch rib result in a minimum loss of section. The upper connection, if inside the rib, can be inspected by a man having access through the inside of the box rib, manholes being provided at the diaphragms.

Wire rope hangers are more frequently used now than structural hangers. They have given some trouble, particularly on suspension bridges, by corrosion of the wires, due to holding damp dirt at the sockets. The connection of the ropes to the sockets at the lower end should be easily accessible and visible to a man on the deck.

Column Connections

Short stiff columns near the crown of the arch may result in considerable interaction between the deck and the rib. This can be minimized by using rocker type connections.

Splices

Since the arch ribs are principally in compression, 50 percent of the load in bearing at splices, as permitted by AASHTO Specifications, can be utilized for boltedsplices. In the case of the Hampton Road rib, both shop and field splices were welded. Welding of field splices is generally not desirable for an arch because of locked-in stresses from weld shrinkage and from the method of erection.

1.15 Equations and Curves for Design

The equations of this section and Figures 3 to 7 are intended for preliminary design use, to arrive at an arch rib section for more exact analysis by classical methods or by computer. Some of the curves, such as those for deflection and moment magnification may be sufficiently accurate for final design.

This section should be used with Section 1.16, a design example using the approximation of Section 1.15.

Steel Weight

The first requirement for preliminary design is an assumed dead load. The equations for weight of steel include not only that in the arch ribs and bracing, but also the roadway framing steel and the members connecting the roadway framing to the arch. The reason for lumping these different sections together is their interdependence with regard to steel weight. It is easily seen that a greater spacing of suspenders or columns will result in more weight of roadway framing steel and less weight of suspender or column steel. It is not so obvious, but nevertheless true that suspender or column spacing also affects the weight of arch rib steel. The equations for steel weight are:

2-Hinged Arch

Solid Web

No tie	$W_s = 0.18\,\ell + 20$	Eq. 30
With tie	$W_s = 0.23\,\ell + 20$	Eq. 31

Truss type rib

(no tie)	$W_s = 0.15\,\ell + 20$	Eq. 32

Fixed Arch

Solid Web	$W_s = 0.16\,\ell + 20$	Eq. 33
Truss Type	$W_s = 0.14\,\ell + 20$	Eq. 34

Where W_s = steel weight in pounds per sq. ft. of deck

ℓ = span in feet

The above equations are roughly applicable regardless of the grade of steel, assuming that A36 steel will be used in the shorter spans and higher strength steel in the arch rib main members of long spans.

This steel weight will not be uniformly distributed over the length of the span. By assuming it as uniform for preliminary design, the total dead load thrust will be overestimated, but probably by not more than 6 percent. If desired, the weight of steel only could be reduced by a factor of 0.9 in figuring the dead load thrust by the uniform load formula. For a partially suspended deck, such as the Bayonne Bridge type, the steel weight would be slightly less due to the use of suspenders instead of columns.

Thrust

The horizontal component of dead load thrust in an arch may be approximately calculated by the following:

$$H_{u.\ell.} = w\ell^2/8h \qquad \text{Eq. 35}$$

Where $H_{u.\ell.}$ = horizontal thrust from uniform load over the full span

w = load per ft.

ℓ = arch span

h = arch rise

This equation may also be used for uniformly distributed live load over the full span.

For a concentrated load, P, the horizontal thrust is approximately:

(P at the crown) $H = P\ell/5h$ for 2-hinged and
$P\ell/4h$ for fixed arch — Eq. 36

(P at the quarter point) $H = P\ell/7.3\,h$ for either 2-hinged or fixed — Eq. 37

The thrust at any point is approximately equal to the secant of the angle of slope of the axis times H.

Moment

Live load positive moment in the vicinity of the quarter point may be approximately calculated by using simple spans of the following lengths:

Fixed arch, equiv. simple span = $0.36(1-0.1I_s/I_c)\ell$ — Eq. 38

2-Hinged arch, equiv. simple span = 0.36ℓ — Eq. 39

where I_c = crown and I_s = springing moments of inertia

The above moments, which occur near the quarter point, are the maximum, except at a fixed end. The crown moments will be somewhat less than the quarter point moments. Negative moments in a 2-hinged arch will generally be smaller than positive moments. Positive moments at the springing of a fixed arch will be of the order of 2.5 times the maximum positive quarter point moment, and the maximum negative springing moment will generally be less than the positive springing moment.

The dead load moments in any arch will be quite small, provided the arch axis is closely fitted to the dead load equilibrium polygon. There will be rib shortening moments from dead load unless these are eliminated by the fabricating and erection procedure.

Effect of Curving Arch Axis

As previously mentioned, uniformly distributed loads applied through columns or suspenders will produce additional moments due to their application as concentrations on a continuously curving axis. A positive moment will be produced at the point of load and will have approximately the following value:

$$M_{conc.} = P\Delta_L/12 \qquad \text{Eq. 40}$$

Where M_{conc} = additional moment due to non-uniform application

ΔL = distance between columns and suspenders

P = load

The above moment, from the roadway D.L. and the uniform live load, will add to the maximum positive moments at the columns. The maximum negative moment, at points mid-way between columns, will be increased by one half of the above, and will be from roadway D.L. only, since live load will be placed on a different part of the span for negative moment.

Deflection

Live load deflection may be approximately calculated by the following equations:

2-Hinged Arch

$$\Delta = \frac{M\ell^2}{41EI} \qquad \text{Eq. 41}$$

Fixed Arch

$$\Delta = \frac{M\ell^2}{76\ EI} \qquad \text{Eq. 41a}$$

Where Δ = live load deflection (maximum)

M = live load moment (maximum)

ℓ = arch span

I = moment of inertia at quarter point

The above equations may be used for either steel or concrete arches, by using the appropriate value for E. The moment for computing deflection should be magnified for service loading only.

Dead load deflection at the crown may be calculated by the following approximate equations:

2-Hinged Arch

$$\frac{f_a}{E} \cdot \frac{\ell^2}{5h} + \frac{f_a}{E} \cdot h \qquad \text{Eq. 42}$$

Fixed Arch

$$\frac{f_a}{E} \cdot \frac{\ell^2}{4h} + \frac{f_a}{E} \cdot h \qquad \text{Eq. 42a}$$

Where f_a = dead load axial unit stress at the crown.

The above equations assume that the arch rib shape has been determined by the dead load equilibrium polygon. For a tied arch, f_a in the first term should be the sum of the dead load axial unit stress in the tie and in the rib at the center line of the span.

Temperature deflection at the crown may be calculated by the following approximate equations:

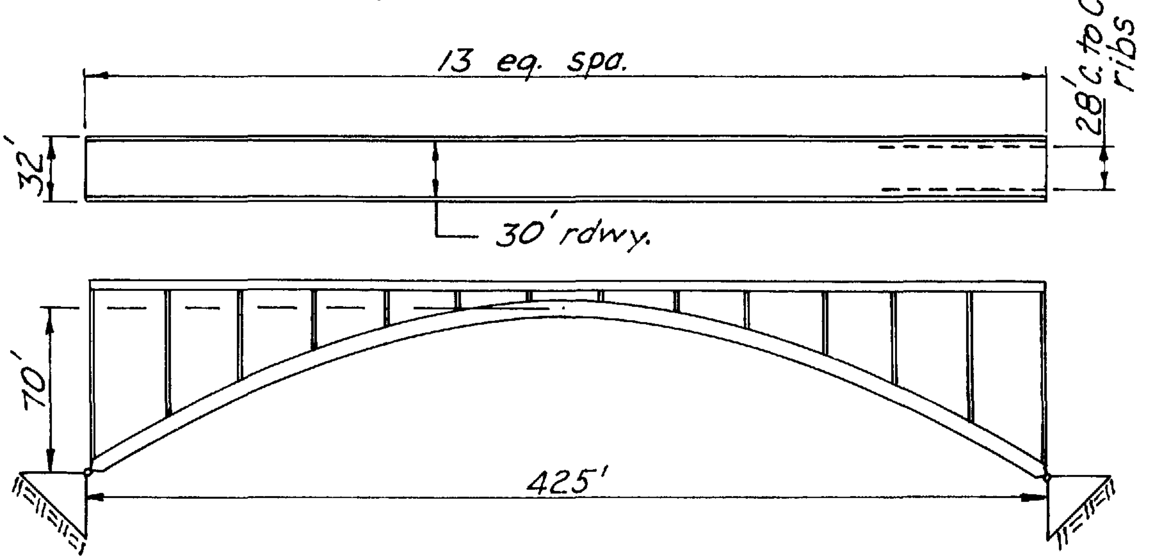
2-Hinged Arch

$$\omega t\left(\frac{\ell^2}{5h} + h\right) \qquad \text{Eq. 42b}$$

Fixed Arch

$$\omega t\left(\frac{\ell^2}{4h} + h\right) \qquad \text{Eq. 42c}$$

1.16 Design Example - Vertical Loads



Specifications

L.L. = 2 lanes HS20-44

maximum L.L + I deflection = $\ell/1200$

A36 steel

Dead Load (30' roadway)

slab + bituminous pavement ~ $\frac{10}{12} \times 150 \times 32$ = 4000 #/'

curbs + railing ~ 700 #/'

structural steel (eq. 30) ~ 32 [0.18(425)+20] = 3090 #/'

7790 #/'

reduction in steel wt. for equivalent uniform load ~ -0.1×3090 = − 309 #/'

7480 #/'

Live Load + Impact at quarter point

traffic lanes distributed to one rib = $\frac{2 \times 18}{28}$ = 1.286

equivalent simple span (eq. 39)

$0.36 \times 425 = 153'$

live load moment (A.A.S.H.T.O. Appendix A)

$1.286 \times 2560 = 3290^{\prime K}$

% impact = 50÷[0.5(425)+125] = 14.8 %

LL + I moment = $1.148 \times 3290 = 3780^{\prime K}$

dead load vertical reaction = $\frac{1}{4} \times 7.79 \times 425 = 828^K$

dead load horizontal reaction (eq. 35)

$$H = \tfrac{1}{2}(7.48)(425)^2 \div (8 \times 70) = 1210^K$$

Thrust at quarter point ~ secant of slope × H

dead load ~ 1.06×1210	=	1280
uniform lane load over half span (eq. 35)		
$1.06 \times 1.286 \times 0.64 \times 425^2 \div 16 \times 70$	=	141
concentrated lane load at quarter point (eq. 37)		
$1.06 \times 1.286 \times 18.0 \times 425 \div 7.3 \times 70$	=	20
impact ~ $0.148\,(141 + 20)$	=	24
		1465^K

From fig. #6 ~ $d = 7.5'$

$$\ell/d = 425/7.5 = 56.7$$

From fig #5 given $\ell/\Delta = 1200$ & $\ell/d = 56.7$

$$f_{bs} = 8.9 \text{ ksi}$$

$$L \approx \text{secant of slope at q.p.} \times \ell/2$$

$$\text{approximate } \frac{KL}{r} = \frac{1.04(1.06 \times 212.5)}{0.4 \times 7.5} = 78$$

assume $f_a = 6.0$ ksi

From fig. #3 & #4 ~ $f_b = 8.9 \times 1.28 \div 1.15 = 9.91$ ksi

From fig. #7 ~ $F_a = 13.7$ ksi

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{6.0}{13.7} + \frac{9.91}{20.0} = 0.44 + 0.50 = 0.94 < 1.0$$

Web size

required D/t for web with two stiffeners (eq. 8)

$$10{,}000/\sqrt{6000} = 129 \sim \text{use } 120 \text{ max.}$$

try web $90'' \times \frac{3}{4}''$ ~ $D/t = 90/0.75 = 120$

Stiffener size ~ stiffeners at $\frac{1}{3}$ points (eq. 9)

required $I_s = 2.2 \times 90 \times 0.75^3 = 83.5 \text{ in}^4$

use $7'' \times \frac{3}{4}''$ plates ~ $I_s = 0.75 \times 7^3/3 = 85.7 \text{ in}^4$

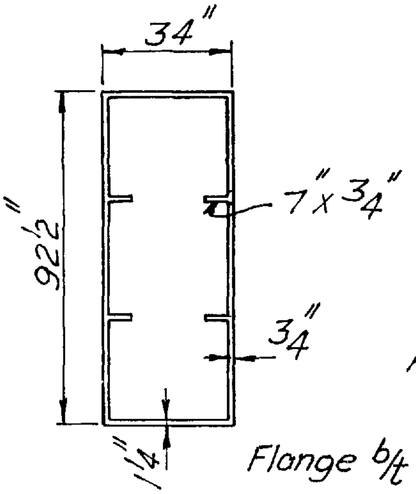
Flange size

approximate rib area = $1465/6.0 = 244^{\square''}$

approximate flange area = $244 - (2 \times 90 \times \frac{3}{4}) - (4 \times 7 \times \frac{3}{4}) = 88^{\square''}$

use $34'' \times 1\frac{1}{4}''$ plates $b/t = 34/1.25 = 2.72$

req'd. b/t (eq. 11) $= 4250/\sqrt{f_a + f_b} = 4250/\sqrt{6000 + 9910} = 33.7$



		Area	I
2 webs	90 x ¾	135	91,100
4 stiff.	7 x ¾	21	4,730
2 flanges	34 x 1¼	85	176,900
		241 □"	272,730 in.⁴

$r = \sqrt{272{,}700/241} = 33.6''$

$KL/r = 1.04 \times 1.06 \times 212.5 \times 12 / 33.6 = 83.7$

$S = 272{,}700 / 46.25 = 5896\ in^3$

Flange $b/t = 3.4/1.25 = 27.2$, Allow. $= 4250/\sqrt{6{,}080+10{,}310} = 33.2$

Web $b/t = 91/0.75 = 121.$, Allow. $= 10{,}000/\sqrt{6080} = 128.$

Combined stresses

$f_a = 1465/241 = 6.08$ ksi

from fig. #10 ~ $F_a = 13.2$ ksi

from fig. # 4 ~ $A_F = 1.34$

$f_b = 1.34 \times 3780 \times 12 / 5896 = 10.31$ ksi

$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{6.08}{13.2} + \frac{10.31}{20.0} = 0.461 + 0.516 = 0.977 < 1.0$

Service Load deflection

from fig. #3 ~ $A_{FS} = 1.18$

$f_{bS} = (1.18/1.34)\ 10.31 = 9.08$

$\ell/d = 425/7.71 = 55.1$

from fig. #5 ~ $\ell/\Delta = 1200$

Dead Load rib shortening stress (eq. 4f)

$H_{rs} = 1.875\,[33.6/70 \times 12]^2\ 1210 = 3.63^K$

rib shortening moment at quarter point

$3.63 \times 0.75 \times 70 = 191'^K$

rib shortening stress at quarter point

$191 \times 12 / 5896 = 0.39$ ksi

Additional stress from column concentration (eq. 40)

$D.L. = \frac{1}{2}\,[47 + (0.02 \times 32)]\ 32.7^2/12 = 238$

$L.L. + I. = 1.148 \times 1.286 \times 0.64 \times 32.5^2/12 = \underline{83}$

$321'^K$

$f_b = 321 \times 12 / 5896 = 0.65$ ksi

$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.977 + \frac{0.39 + 0.65}{20.0} = 1.029$

Alternate cross section with thinner web by using a horizontal diaphragm

Web size

ave. stress in one half of web $= 6.08 + \frac{1}{2}(10.31 + 0.39 + 0.65)$
$= 11.76$ ksi

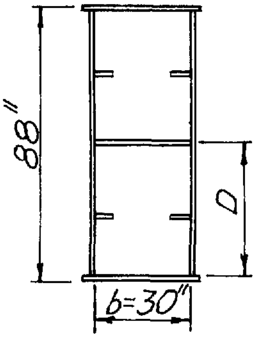
required D/t for web with one stiffener at mid-depth
(eq. 6) " " $7500/\sqrt{11760} = 69.2$
use $88'' \times \frac{5}{8}''$ web ; $D/t = 44/0.625 = 70.4$

Stiffener size (eq. 7)

required $I_s = 0.75 \times 44 \times 0.625^3 = 8.06 \text{ in}^4$
use $4'' \times \frac{3}{8}''$ plates $I_s = 0.375 \times 4^3/3 = 8.0 \text{ in}^4$

Diaphragm size (eq. 10a)

required $b/t = 4500/\sqrt{6080} = 57.7$
use $30'' \times \frac{1}{2}''$ plate ~ $b/t = 30/0.5 = 60$



	A	I
2 webs $88'' \times \frac{5}{8}''$	110	71000
1 diaphragm $30'' \times \frac{1}{2}''$	15	–
4 stiffeners $4'' \times \frac{3}{8}''$	6	2900
2 flanges $34'' \times 1\frac{1}{2}''$	102	204300
	233 $\square''$	278,200 in.^{4}

$r = \sqrt{278000/233} = 34.5''$
$KL/r = 1.04 \times 1.06 \times 212.5 \times 12/34.5 = 81.5$
$S = 278,000/45.5 = 6110 \text{ in.}^3$

Combined stresses

$f_a = 1465/233 = 6.29$ ksi
from fig. # 7 ~ $F_a = 13.45$ ksi
from fig. # 4 ~ $A_F = 1.335$
$f_b = 1.335 \times 3780 \times 12/6110 = 9.91$ ksi
$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{6.29}{13.45} + \frac{9.91}{20.0} = 0.468 + 0.496 = 0.964 < 1.0$

Service load deflection

from fig. # 3 ~ $A_{FS} = 1.173$
$f_{bS} = (1.173/1.335)\,9.91 = 8.71$
$\ell/d = 425/7.58 = 56.1$
from fig. # 5 ~ $\ell/\Delta = 1240$

Dead load rib shortening stress (eq. 4f)

$H_{rs} = 1.875 \left[34.5/70 \times 12 \right]^2 1210 = 3.83^{k}$

rib shortening moment at quarter point

$3.83 \times 0.75 \times 70 = 201^{\prime k}$

rib shortening stress at quarter point

$201 \times 12 / 6110 = 0.39$ ksi

Additional stress from column concentration (eq. 40)

$f_b = 321 \times 12 / 6110 = 0.63$ ksi

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.964 + \frac{0.39 + 0.63}{20.0} = 1.015$$

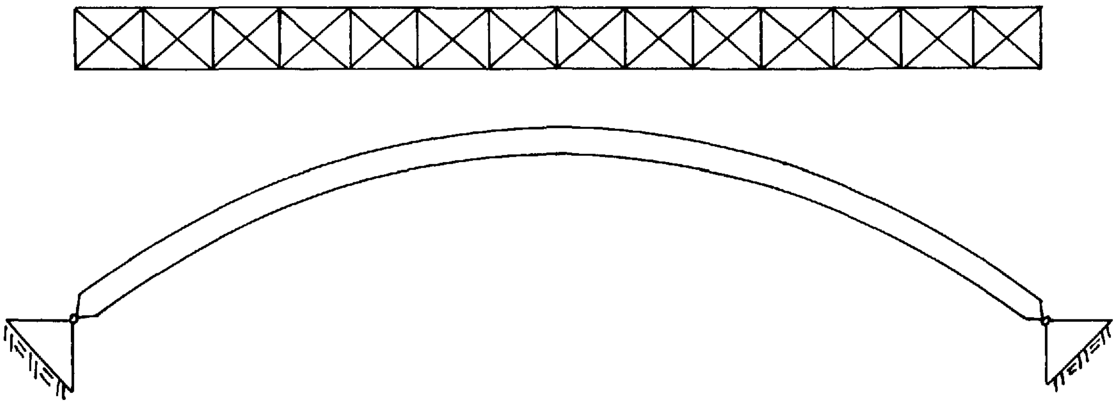
Temperature stress (eq.

assume ave. $I = 0.9\, I_{q.p.}$

$$H_t = \frac{15 \times 29 \times 10^6 \times 0.9 \times 278{,}000 \times 6.5 \times 10^{-6} \times 60}{8\,(70 \times 12)^2} = 7520^{\#}$$

stress at quarter point $= (7.52/3.83)\,0.39 = 0.766$ ksi

1.16.1 Wind Analysis — Double Lateral System



Wind load on rib $(W) = 7.5 \times 75 = 563\ \#/{\prime}$ use $0.6\ k/{\prime}$

The equation for a single centered circular curve and rib of constant cross-section will be used to calculate M_o. (See fig. #8)

$$\text{radius } (R) = \frac{\ell^2}{8h} + \frac{h}{2} = \frac{425^2}{8 \times 70} + \frac{70}{2} = 357.545'$$

β (radians) = 0.63643

Moment at crown (eq. #22)

$$M_o = 0.6 \times 357.5^2 \left[2\frac{(\sin\beta - \beta\cos\beta)}{(\beta - \sin\beta\cos\beta)} - 1 \right] = 3174^{\,\prime K}$$

rib stress at crown = $3174/28 \times 241$ = 0.47 ksi

moment at springing $= M_o \cos\beta - WR^2(1-\cos\beta) = 12{,}460^{\,\prime K}$

torsion at springing $= M_o \sin\beta - WR^2(\beta - \sin\beta) = -1350^{\,\prime K}$

torsion at quarter pt. $= M_o \sin\beta/2 - WR^2(\beta/2 - \sin\beta/2) = 583^{\,\prime K}$

rib stress at springing = $12460/28 \times 241$ = 1.85 ksi

length of half arch = $357.5\,\beta$ = 227.5'

Due to the fact that wind force is applied as concentrated loads at the panel points, stress in the laterals, using the approximate equation, should be based on the angle from the crown to the center of the panel in question.

It will be assumed that the arch is divided into 14 equal panels along the axis of 32.5' each.

<u>Maximum shear in end panel</u>

$\theta = (6.5/7.0)(0.63643) = 0.59097$

$\sin\theta = 0.55717$

torsion for end panel $= M_o \sin\theta - WR^2(\theta - \sin\theta)$

$3174 \times 0.55717 - 0.6 \times 357.5^2(0.59097 - 0.55717) = -824^{\,\prime K}$

torsional shear for laterals (see fig. #10)

$T/2d = -824/2 \times 7.5 = \pm 55^K$

bending shear in one lateral system

$\tfrac{1}{2} \times 0.6 \times 6.5 \times 32.5 = 63^K$

combined shear in upper lateral system = 8^K

" " " lower " " = 118^K

<u>Maximum shear at quarter point</u>

$\theta = (3.5/7.0)(0.63643) = 0.31822$

torsion at q.p. $= 3174 \times 0.31287 - 76{,}700(0.31822 - 0.31287)$

$= 583^{\,\prime K}$

torsional shear for laterals

$T/2d = 583 / 2 \times 7.5 = \pm 39^k$

bending shear in one lateral system

$\tfrac{1}{2} \times 0.6 \times 3.5 \times 32.5 = 34^k$

combined shear in upper lateral system $= 73^k$

" " " lower " " $= 5^k$

<u>Minimum shear for lacing</u> (A.A.S.H.T.O. 1.7.83)

$$V = \frac{2 \times 1465}{100}\left[\frac{100}{\frac{1.26 \times 227}{14}+10} + \frac{1.26 \times 227/14}{3300/36}\right] = \begin{matrix} 84^k \sim 2 \text{ systems} \\ 42^k \sim 1 \text{ system}\end{matrix}$$

Assuming an X-system with the diagonal designed to take tension only, the maximum diagonal stress is $1.53 \times 118 = 181^k$. Use a WT8X35.5 diagonal. The maximum strut stress is 118^k requiring a WT 6X22.5. The upper and lower struts will be braced together to act as cross-frames. In determining rotation at the crown by the approximate equation, assume an average lateral diagonal area of 9.4 in.^{2} and an average lateral strut area of 5.9 in.^{2} Assuming only one diagonal of the X-system in action, the equation for K is:

$$K = \frac{\Delta L\,(bd)^2}{\frac{G}{E}\left[\frac{b^3}{2A_s} + \frac{\ell^3}{2A_d}\right] + \frac{\Delta L d}{t}}$$

$$= \frac{32.5(28 \times 7.5)^2}{0.4 \times 144\left[\frac{28^3}{2 \times 5.9} + \frac{42.9^3}{2 \times 9.4}\right] + \frac{32.5 \times 7.5}{1.5/12}}$$

$$= 4.09\ ft^4$$

<u>Crown rotation</u> (eq. #26)

$$\frac{357.5}{0.4 \times 144 \times 29{,}000 \times 4.09}\left[3174 \times 0.1766 + 76{,}700 \times -0.00582\right] = .00597$$

lateral $I = 2(241/144)(14)^2 = 656 \text{ ft}^4$

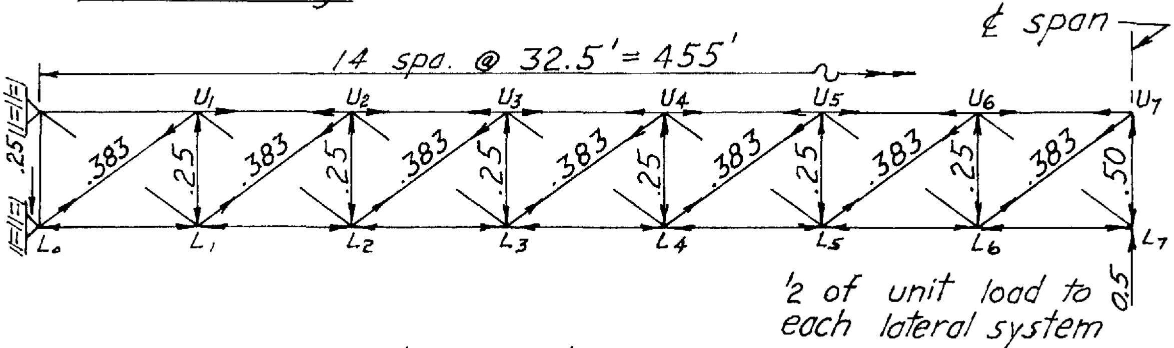
<u>Lateral deflection at crown from arch rib axial stress</u>

(eq. #23a)

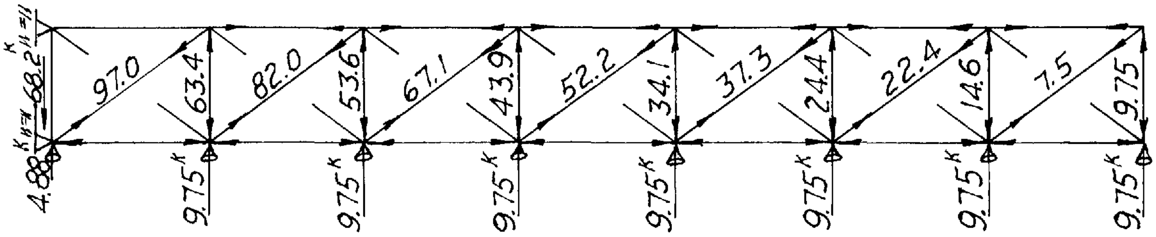
$$\Delta = \frac{227.5^2}{2 \times 29{,}000 \times 144 \times 656}\left[\frac{0.6 \times 227.5^2}{4} - 3174\right]$$

$$= 0.0434' \sim 0.52''$$

<u>Lateral deflection at crown from shear stress in wind bracing</u>



<u>Unit load stresses</u>



<u>Wind shear stresses</u>

$\Delta = \Sigma \Delta L(u)$

$\Delta L = PL/AE$

u = unit load stress

member	L (in)	A (in^2)	P (Kips)	ΔL	u	$\Delta L(u)$
$L_0 U_1$	515	9.4	97.0	.1832	.383	.0702
$L_1 U_2$	"	"	82.0	.1549	"	.0593
$L_2 U_3$	"	"	67.1	.1268	"	.0486
$L_3 U_4$	"	"	52.2	.0986	"	.0378
$L_4 U_5$	"	"	37.3	.0705	"	.0270
$L_5 U_6$	"	"	22.4	.0423	"	.0162
$L_6 U_7$	"	"	7.5	.0141	"	.0054
$L_1 U_1$	336	5.9	-63.4	-.1245	-.250	.0311
$L_2 U_2$	"	"	-53.6	-.1053	"	.0263
$L_3 U_3$	"	"	-43.9	-.0862	"	.0216
$L_4 U_4$	"	"	-34.1	-.0670	"	.0167
$L_5 U_5$	"	"	-24.4	-.0479	"	.0120
$L_6 U_6$	"	"	-14.6	-.0287	"	.0072
$L_7 U_7$	"	"	- 9.8	-.0191	"	.0048
						.3842"

Since we have considered only ½ of one lateral system: $\Delta = 4 \times 0.3842 = 1.54''$

This deflection may be determined by the following abbreviated method also:

diagonal stress from unit load $= \frac{1}{2} \times \frac{1}{2} \times 1.53 = .383$
strut " " " " $= \frac{1}{2} \times \frac{1}{2} = .250$
ave. shear from wind $= \frac{1}{2} \times 0.6 \times 227.5 = 68.2$
ave. diagonal wind stress $= 2 \times 68.2 \times .383 = 52.2$
ave. strut " " $= 2 \times 68.2 \times .25 = 34.1$
since there are 28 panels, $\Delta = 28(PL/AE)u$

$$\Delta = 28\left[\frac{52.2 \times 28 \times 1.53 \times .383}{9.4 \times 29{,}000} + \frac{34.1 \times 28 \times .25}{5.9 \times 29{,}000}\right]$$

$$= 1.53''$$

total deflection at crown $= 0.52 + 1.53 = 2.05''$

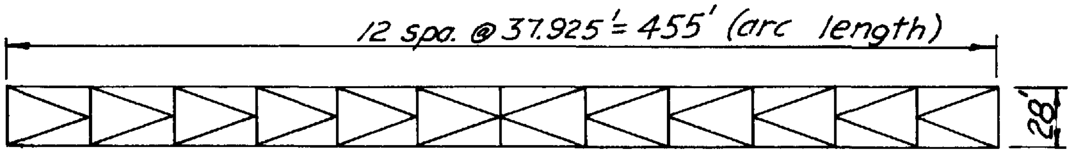
If the roadway lateral system is five feet above the center of the arch rib at the crown, total lateral deflection from wind on the arch rib is $2.05 + 5 \times 12 \times .00597 = 2.41''$ at the roadway lateral system level.

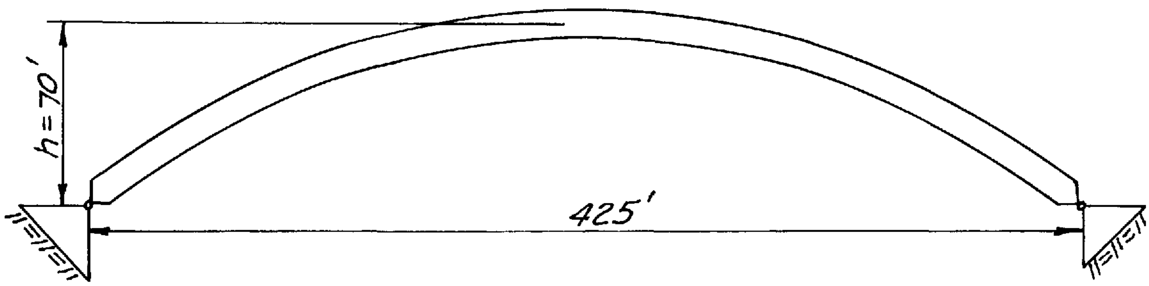
For a straight member with fixed ends and constant I, the lateral deflection at the center of span due to moment from a uniform wind load is $w\ell^4/384EI$. This would give:

$$0.6(455)^4/384 \times 29000 \times 144 \times 655 = 0.0245' \sim 0.30''$$

as compared to $0.52''$ for the arch.

1.16.2 Wind Analysis - Single Lateral System





For analysis, divide the half arch into six equal segments. Transverse wind shear in the lateral system may be calculated by summing the wind forces outward from the crown for symmetrical wind loading. Transverse shears result in tangential longitudinal forces ($\Delta LWR\theta/b$) acting on the arch rib at each point of connection of the diagonal to the rib. For simplification of analysis it will be assumed that tangential forces are applied at the center of the panels.

For this analysis, a constant cross-section is assumed. The arch rib moments will be determined for a 3-hinged condition & converted to 2-hinged

F_t = tangential force for one panel = $\frac{WR\theta}{b}(\Delta L)$

ΔL = length of panel

M_{tf} = moment at θ of tangential forces
$= \sum_0^\theta F_t[1-\cos(\theta-\theta_F)]R$

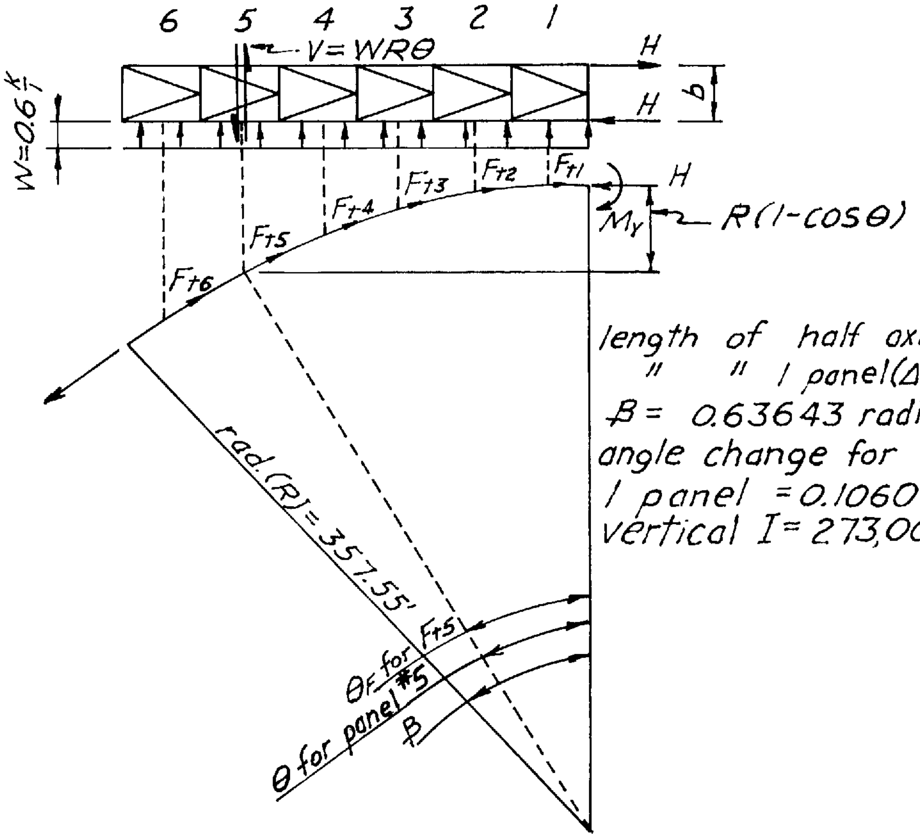
H_3 = thrust at crown for 3-hinged condition
$= \sum_0^\beta F_t[1-\cos(\beta-\theta_F)]R$

M = moment for 3-hinged condition
$= M_{tf} + H_3 R(1-\cos\theta)$

m = moment from unit crown moment = $1 - \frac{R(1-\cos\theta)}{h}$

M_V = 2-hinged crown moment = $-\Sigma Mm/\Sigma m^2$

H_2 = thrust at crown for 2-hinged condition
$= H_3 + M_V/h$



length of half axis = 227.553'
" " 1 panel(ΔL) = 37.925'
β = 0.63643 radians
angle change for 1 panel = 0.106073 rad.
vertical I = 273,000 $\frac{in^4}{rib}$

Determine H_3

1	2	3	4	5
panel	θ	F_t	$R[1-\cos\beta-\theta_3]$	3 X 4
1	.05304	15.4	59.1	910
2	.15911	46.2	40.0	1848
3	.26518	77.0	24.3	1871
4	.37125	107.9	12.5	1349
5	.47732	138.7	4.52	627
6	.58340	169.5	0.503	85
			140.9	6690

$H_3 = 6690/70 = 95.57^K \sim$ tension in leeward rib

Determine M_{tf} values – leeward rib

panel	2	3	4	5	6
$\Delta\theta = \theta - \theta_F$	.10607	.21214	.31822	.42429	.53036
$R[1-\cos\Delta\theta]$	2.01	8.02	17.95	31.70	49.12
F_t					
15.4	31.0	123.5	276	488	756
46.2		92.9	371	829	1465
77.0			155	618	1382
107.9				217	865
138.7					279
M_{tf}	31	216	802	2152	4747

Determine M_y

panel	1	2	3				
	M_{tf}	$1-\cos\theta$	$H_3R(1-\cos\theta)$	$M = 1+3$	m	$M \cdot m$	m^2
1	0	.001406	-48	-48	.993	-48	.985
2	31	.012631	-432	-401	.935	-375	.875
3	216	.034953	-1192	-975	.822	-801	.675
4	802	.068078	-2325	-1523	.653	-995	.427
5	2152	.111765	-3820	-1668	.429	-716	.184
6	4747	.165416	-5655	-909	.1543	-140	.023
springing	6690	.195783	-6694	0	0	0	0
						-3075	3.169

$M_y = 3075/3.169 = 970.3$ (leeward rib)

$H_2 = 95.6 + (970.3/70) = 109.5$ (tension in leeward rib)

$M_o = 109.5 \times 28 = 3070^{1K}$

Determine arch rib moment and thrust

panel	1	2	1+2	3	4	3+4
	M	$M_v \cdot m$	moment	$H_2 \cos\theta$	$\frac{WR^2}{b}(1-\cos\theta)$	thrust
1	- 48	963	915	109.4	-3.8	105.6
2	-401	907	506	108.2	-34.6	73.6
3	-975	797	-178	105.9	-95.7	10.2
4	-1523	633	-690	102.2	-186.7	-84.5
5	-1668	416	-1252	97.5	-306	-209
6	-909	50	-859	91.5	-453	-362
springing	0	0	0	88.1	-536	-448

stress at crown $= 109.5/241 \pm 970 \times 12/5896 = 2.42$ ksi
" " point 5 $= -209/241 \pm -1252 \times 12/5896 = -3.42$ ksi

Lateral deflection

The deflection may be considered as consisting of three parts:

Δ from arch rib axial stress
Δ from arch rib flexure in the vertical plane
Δ from shear stress in the laterals

The axial stress effect may be calculated by equation #23a as in the double lateral system.

$$\delta_c = \frac{227.5^2}{2 \times 29000 \times 144 \times 656}\left[\frac{0.6 \times 227.5^2}{4} - 3070\right] = 0.0443' \sim 0.53''$$

Determine deflection from arch rib flexure

1. Apply unit lateral load at crown and find tangential forces at each point
2. Calculate moments, M_u, in 3-hinged system from unit load.
3. Multiply the actual wind moments by the unit load moments. (It is a well known fact that deflection in a statically indeterminate system can be calculated by using the unit load moments in the determinate system.)
4. Deflection at crown $= \frac{\Delta L}{EI} \sum M \cdot M_u$

tangential forces from unit load = $0.5\Delta L/28 = 0.01786\Delta L$ (all points)

H from tangential forces $= 0.01786\Delta L\, R \sum_0^{\beta} \frac{1-\cos(\beta-\theta_F)}{70}$

$= 0.036\Delta L$

$$M_u/\Delta L = \left[0.01786\, R \sum_0^{\theta} 1-\cos(\theta-\theta_F)\right] - \frac{HR(1-\cos\theta)}{\Delta L}$$

panel	1	2	3	4	5	4X5
	$R\sum_0^{\theta} 1-\cos(\theta-\theta_F)$	0.01786X1	$\frac{HR(1-\cos\theta)}{\Delta L}$	$M_u/\Delta L = 2-3$	M	
1	0	0	0.0181	-0.0181	-915	16.6
2	2.01	0.0359	0.1626	-0.1267	-506	64.1
3	10.03	0.179	0.450	-0.271	+178	-48.2
4	27.98	0.500	0.878	-0.378	+690	-261
5	59.68	1.066	1.440	-0.374	+1252	-468
6	108.80	1.943	2.130	-0.187	+859	-161

$$\frac{\sum M M_u}{\Delta L} \doteq -857.5$$

$$\text{deflection at crown} = \frac{4 \times 37.9^2 \times 144 \times 858}{29{,}000 \times 273{,}000} = 0.0897' \sim 1.08''$$

The factor (4) accounts for the 4 half ribs in the system.

The lateral deflection from chord action is 1.08" + 0.53" = 1.61" which is $1.61 \div 0.3 = 5.37$ times that for a straight beam. The total lateral deflection = 1.61 + 1.53 = 3.14", assuming the same bending shear deflection as for the double lateral system. The ratio of lateral deflection for a single to that for a double lateral system = 3.14/2.05 = 1.5

To find the lateral deflection from vertical moment at other points in the span, apply unit lateral forces symmetrically at the points where deflection is desired and go through the same process as above. A factor of ½ must be applied in the final equation since two unit loads are applied.

The lateral deflection at any point from axial stress may be obtained from the approximate equation involving M_0 given for the double lateral system.

Vertical deflection

$\Delta = (\Delta L/EI) \Sigma M \cdot m$

1. Apply a unit vertical load to the 3-hinged arch at the crown
2. Multiply $\Delta L/EI$ by the sum of the products of the unit load moments (m) and actual wind moments (M) in the vertical plane to get the deflection.

H due to unit load = $\ell/4h$

m " " " " $= [(R\ell/4h)(1-\cos\theta)] - (R/2)\sin\theta$

panel	1	2	m = 1-2	M	M·m
	$\frac{R\ell}{4h}(1-\cos\theta)$	$\frac{R}{2}\sin\theta$			
1	0.76	9.47	- 8.71	+ 915	- 7970
2	6.84	28.3	- 21.5	+ 506	-10880
3	18.94	46.8	- 27.9	- 178	+ 4960
4	36.9	64.8	- 27.9	- 690	+19250
5	60.6	82.0	- 21.4	- 1252	+26800
6	88.5	98.5	- 10.0	- 859	+ 8590
					+40,750

deflection at crown $= \dfrac{2 \times 40750 \times 37.9 \times 144}{29000 \times 273{,}000} = 0.056' \sim 0.67''$

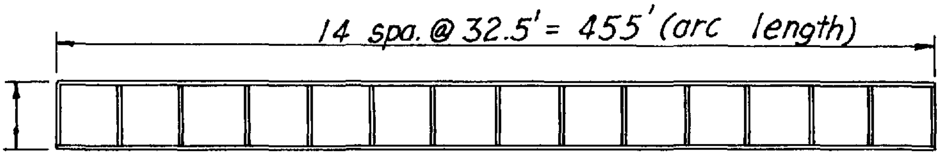
The factor (2) accounts for the 2 half ribs in the system.

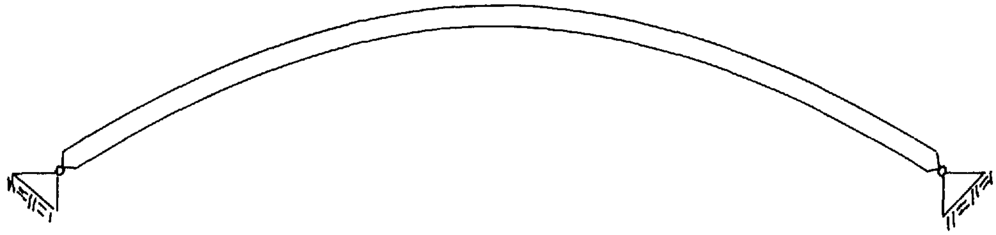
rotation angle $= 0.67/(12 \times 14) = 0.004$

The total lateral deflection at slab level from wind on the arch is $3.14 + (0.004 \times 60) = 3.38''$.

To find the vertical deflection at other points in the span, apply symmetrical unit loads at those points and go through the same procedure as above.

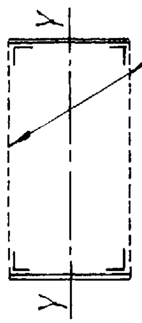
1.16.3 Wind analysis – Strut bracing between ribs





Assume 14 equal panels in the arch length. Transverse wind shear will be divided equally between the two ribs. Points of contraflexure will be assumed in the ribs midway between struts. Balancing longitudinal shear forces are then required in the struts at assumed points of contraflexure midway between the arch ribs.

Design wind shear	=	$0.6 \times 455/2$	$= 137^{K}$
Rib moment in end panel	=	$137 \times 32.5/2 \times 2$	$= 1110^{'K}$
Strut shear in end panel	=	$137 \times 32.5/2 \times 14$	$= 159^{K}$
Strut moment in end panel	=	159×14	$= 2230^{'K}$



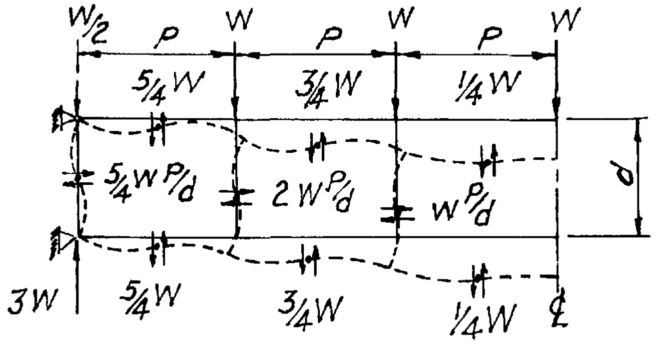
lacing

2 PLs $41'' \times \frac{7}{8}''$

4 ∠s $9 \times 4 \times 1$

lateral $I = I_{Y-Y} = 4 \times 12 \times 20^2 + 1.75 \times 41^3 \times \frac{1}{12} = 29{,}200\ in^4$

← strut cross section ~ strut depth equals rib depth



Wind shears

Bending stresses (f_b)

strut f_b = 2230 x 12 x 20.5 / 29,200 = 18.8 ksi

lateral I in rib = $156 \times 16^2 + 2.5 \times 34^3 \times 1/12$ = 48,100 in^4

rib f_b = 1110 x 12 x 17 / 48,100 = 4.7 ksi

Lateral deflection

As in the previous shear deflection calculations, assume an average $V = 68.2^k$. From eq. #27, the lateral shear deflection for one panel is:

$$\frac{68.2\,(32.5 \times 12)^2}{24 \times 29{,}200}\left[\frac{32.5 \times 12}{48{,}100} + \frac{2 \times 28 \times 12}{29{,}200}\right] = 0.461''$$

total shear deflection at crown = 7 x 0.461 = 3.23"

This compares with 1.53" shear deflection with diagonals. Total lateral deflection at crown equals 0.52 + 1.08 + 3.23 = 4.83"

Deflection at slab level = 4.83 + 0.004 x 60 = 5.07"

1.16.4 Lateral Buckling and Moment Magnification

For diagonal type lateral system: (single lateral)

assume diagonal area = 22 in.^{2}

" strut " = 15 in.^{2}

KL = 1.7 x 227.5 ≐ 387'

overall lateral $I = 2 \times 241 \times 14^2$ = 94,500 in^2 ft^2

from eq. #28

$$K = \sqrt{1 + \frac{\pi^2 \times 94{,}500}{273^2} \cdot \frac{1}{32.5 \times 28^2}\left[\frac{42.9^3}{22} + \frac{28^3}{15}\right]}$$

$= 1.87$

$K \times \frac{KL}{r} = 1.87 \times \frac{387}{14} = 51.7$

Vertical $\frac{KL}{r}$ governs for allowable compressive stress since it is 83.7.

For lateral moment magnification:

$F_e = \pi^2 \times 29{,}000 / 51.7^2 = 107$ ksi

lateral moment magnifier $= \dfrac{1}{1 - \dfrac{1.7 \times 2926}{482 \times 107}} \doteq 1.11$

For struts only between ribs:

$KL = 387'$

overall lateral $I = 94{,}500$ in² ft²

from eq. #28 a

$$K = \sqrt{1 + \frac{\pi^2 \times 94500}{12 \times 273^2}\left[\frac{32.5^2 \times 144}{2 \times 48{,}100} + \frac{32.5 \times 28 \times 144}{29{,}200}\right]}$$

$= 2.71$

$K \times KL/r = 2.71 \times \frac{387}{14} = 74.9$

$F_e = \pi^2 \times 29{,}000 / 74.9^2 = 51$ ksi

lateral moment magnifier $= \dfrac{1}{1 - \dfrac{1.7 \times 2926}{482 \times 51}} = 1.25$

Wind stresses in the rib for the two systems:

(stress at point 5)	strut bracing	diagonal type single lateral
vertical flexure	2.56	2.56
lateral flexure	1.25 x 0.87 = 1.09	1.11 x 0.87 = .97
local bending	1.25 x 4.6 = 5.75	0
	9.40	3.53

1.16.5 Distribution of Wind Load Between Arch Rib and Deck Lateral Systems

The wind stress analyses made in 1.16.1 and 1.16.4 are for the two arch ribs, braced together, and subjected to the lateral forces from wind blowing directly against the arch. There will also be wind lateral forces acting directly on the roadway deck. Since the roadway deck and the arch are not free to deflect laterally, independently of each other, there will be lateral force transfers between them through the columns or suspenders.

The roadway deck will probably have a lateral system of its own and the roadway slab may also act as a lateral system. If there is no diagonal bracing between the columns within the arch span, the main lateral force transfer would be through the short columns at or adjacent to the center of span. The bents at the abutments may have diagonal bracing between the columns, in which case they will act as vertical truss cantilevers to take the wind shear from the deck down to the arch abutments. A rough method of analyzing the lateral load transfer is to assume that the deck lateral system and the arch lateral system have the same lateral deflection at the center of the arch span, but are free to deflect independently at the other columns within the arch span. The following example will illustrate a method of solving for the value of the lateral force required at the center of the span, between the arch and deck, to equalize the deflection.

The arch single lateral system will be used in this example. For this case, the arch was calculated to have a lateral deflection at the center of span of 3.38 inches, for the wind directly against it. It will be assumed that the deck system, acting alone, has a deflection of 2.3 inches from the wind loads directly on it and that a unit lateral force of 1 kip at the center of span will deflect the deck 25×10^{-3} inches. These values, of course, depend on the deck lateral system, the roadway slab and the end support, and would have to be calculated.

The arch lateral deflection of 3.38 inches from wind loads acting directly on the arch, was calculated in Section 1.16.2. The lateral deflection of the arch for a 1 kip lateral load at the crown will be calculated in a similar manner. This lateral deflection from vertical arch bending is:

$$\delta_{vb} = \Sigma M_2^2 \, \Delta L/EI$$

where M_2 is the moment at any point, in the 2-hinged arch, produced by a unit horizontal load at the crown. M_2 is calculated by the following equation:

$$M_2 = M_u + M_v m$$

where M_u is the moment at any point, in the 3-hinged arch, produced by a unit horizontal load at the crown; m is the moment at any point due to a unit crown moment; and M_v is the 2-hinged crown moment, produced by a unit lateral load at the crown. M_v is found by the following equation:

$$M_v = \Sigma m M_u \div \Sigma m^2$$

To get the total lateral deflection, the lateral deflection from shear and from axial stress must be added to the lateral deflection from vertical bending.

The shear from a unit lateral load of 1 kip at the crown is 0.5 kips and the shear deflection is calculated in the example by multiplying the shear deflection from wind loads by the ratio of 0.5 to the average shear from the wind loads.

The axial stress deflection may be approximately calculated by multiplying the wind load deflection by the ratio of one to the total wind load on the span and by the ratio of the fixed end beam deflections constants of 1/192 and 1/384 for concentrated load at the center and uniformly distributed load, respectively.

An equation to solve for the lateral force transfer at the arch crown is set up in the example in terms of the independent deflections, the unit load deflections and the unknown force W_L. The following example illustrates the procedure described above.

Rib deflection for 1 Kip lateral load at the crown
(Single diagonal system)
Values for $m, \Sigma m^2,$ & $M_u/\Delta L$ are found in Sec. 1.16.2

	1	2	3	4	2+4=5	$5^2=6$
panel	m	$M_u/\Delta L$	$mM_u/\Delta L$	$M_v m/\Delta L$	$M_2/\Delta L$	$M_2^2/\Delta L^2$
1	.993	-.0181	-.0180	.249	.231	.0534
2	.935	-.127	-.119	.235	.108	.0117
3	.822	-.271	-.223	.206	-.065	.0042
4	.653	-.378	-.247	.164	-.214	.0458
5	.429	-.374	-.160	.108	-.266	.0708
6	.154	-.187	-.0289	.039	-.148	.0219
			-.796			.2078

$M_v = 0.796 \times \Delta L / 3.169 = 0.251 \Delta L$
For 1 Kip lateral load
lateral deflection from vertical bending

$$\delta_{vb} = \frac{0.208 \times 37.9^3 \times 4 \times 144}{29{,}000 \times 273{,}000} = 0.824 \times 10^{-3} \text{ ft} \sim 9.9 \times 10^{-3} \text{ in}$$

lateral deflection from shear (with diagonals)

$$\frac{0.5}{52.5} \times 1.53 = 14.6 \times 10^{-3} \text{ in.}$$

lateral deflection from axial stress (approximate)

$$\frac{1}{.6 \times 425} \times \frac{384}{192} \times 0.52 = 4.1 \times 10^{-3} \text{ in.}$$

total lateral deflection $= (9.9 + 14.6 + 4.1) \times 10^{-3} = 28.6 \times 10^{-3}$ in.

W_L = lateral force tranfer from arch to deck at center of span.

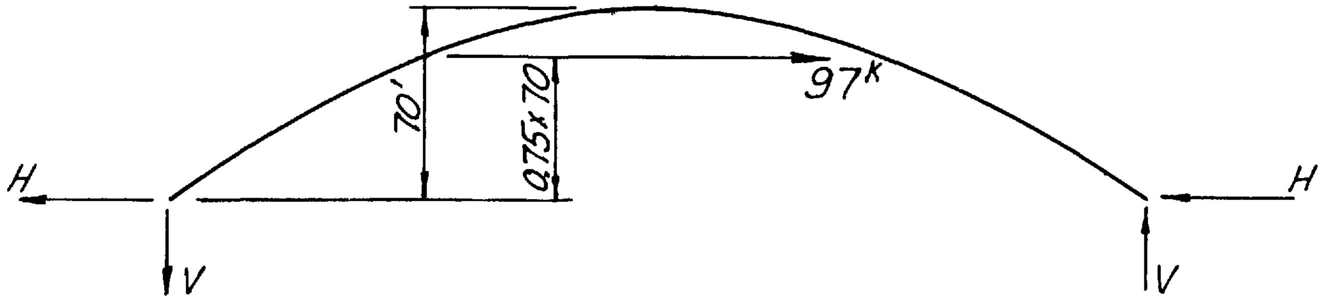
$$3.38 - 28.6 \times 10^{-3} W_L = 2.3 + 25 \times 10^{-3} W_L$$

$$W_L = \frac{3.38 - 2.3}{(28.6 + 25.0)10^{-3}} = 20.2^K$$

Consideration of Longitudinal Forces for Design Example in 1.16

Moments from 60° wind acting on the arch:

total longitudinal component $= 0.6 \times \frac{19}{50} \times 425 = 97^K$



$H = 97 \div 2 = 48.5^K$

$V = -97 \times 70 \times 0.75 \div 425 = -12^K$ (assuming parabolic axis)

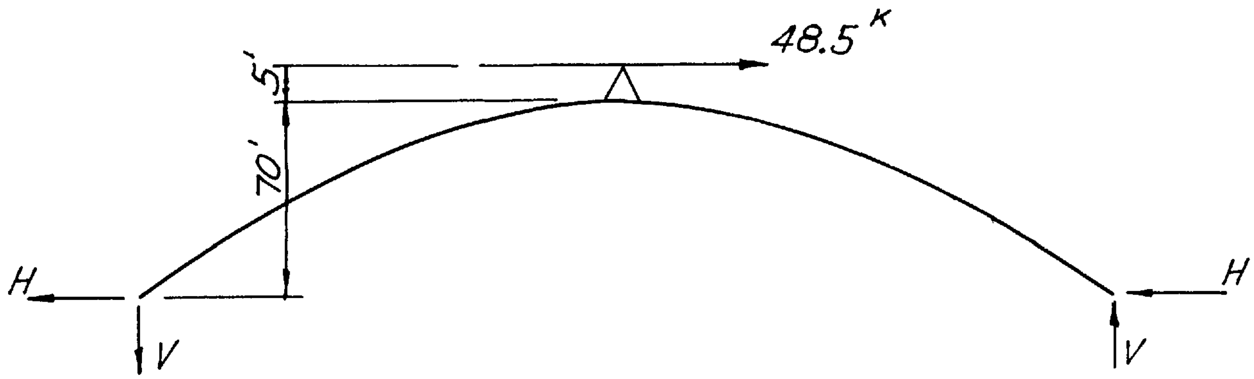
$$M \text{ at quarter point} = 48.5 \times 0.75 \times 70 - 12 \times 106 - \frac{97}{4} \times \frac{0.75 \times 70}{2}$$
$$= 2550 - 1270 - 640$$
$$= +640'^K$$

$$M \text{ at crown} = 48.5 \times 70 - 12 \times 212.5 - 48.5 \times 70 \times 0.25$$
$$= 3400 - 2550 - 850$$
$$= 0$$

Moments from 60° wind acting on the roadway:

assume longitudinal component $= 6 \times 0.019 \times 425 = 48.5^K$

assume this force to act 5 feet above the arch at the crown, and to be transmitted to the arch at the crown.



$H = \frac{1}{2} \times 48.5 = 24.25^{K}$
$V = -48.5 \times 75 \div 425 = -8.6^{K}$
M at quarter point $= 24.25 \times 0.75 \times 70 - 8.6 \times 425 \div 4$
$= 1270 - 910$
$= 360^{'K}$
M at crown $= \frac{1}{2} \times 48.5 \times 5$
$= 121^{'K}$

Total moment at quarter point from longitudinal wind
$M = \frac{1}{2}(640 + 360) = 500^{'K}$

Wind on live load (WL) moment at quarter pt. - half span loaded

$$M = \frac{0.038 \times 212}{48.5} \times \frac{360}{2} = 30^{'K} \text{ per rib}$$

Moment from L.L. longitudinal force
force $= 0.05 \times 2\ (0.64 \times 425 + 18) = 29^{K}$
quarter point moment $= \frac{29}{48.5} \times \frac{360}{2} = 107^{'K}$

For Group III loading with 60° wind
$M_{q.p.} = 107 + 0.3 \times 500 + 30 = 287^{'K}$
longitudinal $f_{q.p.} = 287 \times 12 \div 5896 = 0.584$ ksi
lateral f_5 (see page 57) $= 3.42 \times 0.3 \times \frac{17}{50} = 0.349$ ksi
0.933 ksi

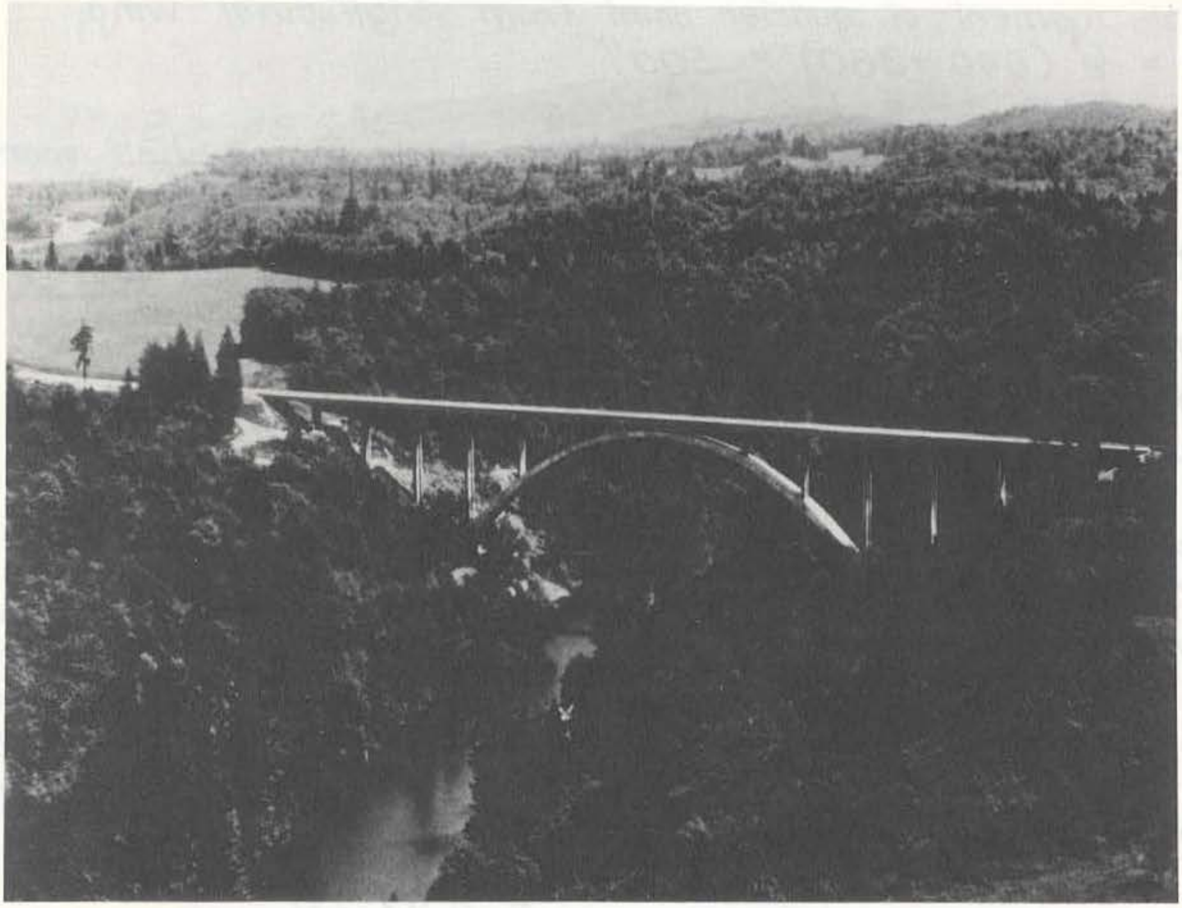
Total stress from 60° wind $= 3.42 \times \frac{17}{50} + \frac{(500+30)\,12}{5896}$

$= 2.24$ ksi
Total stress from 90° wind $= 3.42$ ksi

$\therefore$ 90° wind governs over 60° wind for this example.

COWLITZ RIVER BRIDGE
Lewis County - Washington
Span 520 feet

Built 1967-68



Designed By
Howard, Needles, Tammen and Bergendoff

CHAPTER II - CONCRETE ARCHES

2.1 Basic Arch Action

Much of what has been said in Chapter I with regard to steel arches applies to concrete arches also. It is the intent to discuss in this chapter those features in which the action and design of concrete arches differ from that of steel arches.

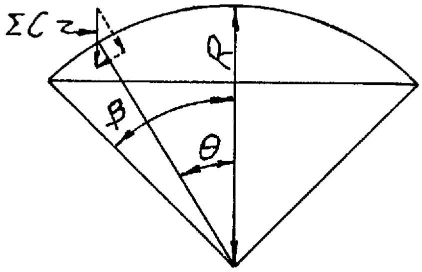
Concrete arches are generally fixed at the springing, and generally have a varying rather than a uniform depth. They may be of the barrel type, in which a single rib is used, with a width approximately equal to that of the deck, or they may have individual ribs braced together by lateral struts. In recent years a number of hollow box-section concrete arches have been built. These are generally a single or a multiple cell box with an overall width somewhat less than the width of the roadway. For short spans many spandrel-filled concrete arches have been built. These have retaining walls at the edges of the barrel to hold an earth fill on which the roadway slab is placed. Concrete arches are frequently built in multiple spans, so that the dead load horizontal reactions of adjacent spans balance each other, and the longitudinal force on the piers is from live load only. Thus high piers with the arch springing well above the foundation may be used.

Concrete arches are subjected to appreciable stresses from shrinkage and creep of the concrete under dead load stresses. Stress interaction between the rib and the deck is likely to be greater for concrete arches than for steel arches. This is due to the greater degree of monolithic construction used in concrete and the fact that the concrete columns and deck members are generally larger in relation to the arch rib than is the case for steel arches. The method used in the construction of concrete arches does not generally result in the elimination of dead load rib shortening stresses, as is the case for steel arches. Dead load is greater for a concrete arch than for a steel arch, resulting in a higher percentage of the stress being axial and, therefore, a greater predominance of compressive stress over tensile stress.

2.1.1 Dead and Live Load Action

The comments in Chapter I with regard to dead and live load action apply, in general, to concrete. There is, however, one fundamental difference in the case of many concrete arches. These are arches which have a single, wide rib of either the barrel or box type. Live load located laterally eccentric to the rib center line applies twisting moments as well as vertical loads to the rib, resulting in torsional shearing and lateral flexural stresses. This is quite different from arches with two or more ribs, where live load eccentricity is carried by an increase in vertical load to the ribs on the side of the eccentricity and a decrease to the other ribs.

The single rib or barrel arch under live load w with eccentricity e is subjected to a twisting moment, $w \cdot e$, in addition to vertical load. A stress analysis similar to that used for wind load may be used. The following equations apply to a uniform load w with eccentricity e over the full span, and a circular axis.



M_O = transverse bending moment at crown

$\Sigma C = w \cdot e \cdot R\sin\Theta$

$M_\Theta = M_O\cos\Theta - w \cdot e \cdot R\sin^2\Theta$ Eq. 43

$T_\Theta = M_O\sin\Theta + w \cdot e \cdot R\sin\Theta\cos\Theta$ Eq. 44

Assuming constant cross-section, and using the method of cutting the arch at the crown and solving for statical unknown, M_O · (T_O= zero, by symmetry).

$$M_O = \frac{\frac{1}{EI}\int_0^\beta MmRd\Theta + \frac{1}{GK}\int_0^\beta T \cdot t \cdot R \cdot d\Theta}{\frac{1}{EI}\int_0^\beta m^2 R \cdot d\Theta + \frac{1}{GK}\int_0^\beta t^2 \cdot R \cdot d\Theta}$$

Where M = moment due to external forces on cut structure

T = torsion due to external forces on cut structure

m = moment due to unit $M_O = \cos\Theta$

t = torsion due to unit $M_O = \sin\Theta$

G = shearing modulus of elasticity = E ÷ 2.3

$$M_o = \frac{\frac{w \cdot e \cdot R}{EI} \int_o^{\beta} -\sin^2\theta\cos\theta R d\theta + \frac{w \cdot e \cdot R}{GK} \int_o^{\beta} \sin^2\theta\cos\theta R d\theta}{\frac{1}{EI} \int_o^{\beta} \cos^2\theta R d\theta + \frac{1}{GK} \int_o^{\beta} \sin^2\theta R d\theta}$$

$$= \frac{2R\, w \cdot e \cdot \sin^3\beta \left(\frac{EI}{GK} - 1\right)}{3\left[(\beta + \sin\beta\cos\beta) + (\beta - \sin\beta\cos\beta)\frac{EI}{GK} \right]} \qquad \text{Eq. 45}$$

2.2 Buckling and Moment Magnification

A concrete arch will generally have a greater dead load thrust than a steel arch of the same span. However, at service load the stiffness, EI, of the concrete arch is generally greater than that of the steel arch. Although the dead load may be twice as great for the concrete arch, the stiffness at service load may be 1.5 times as great as that of the steel arch. In addition, since concrete arches are generally fixed and steel arches are generally two-hinged, the k factor for buckling length of the concrete arch may be about seven tenths that for steel arch. For a concrete arch, therefore, the ratio T/AF_e, used in magnification Equation 1, may be $2 \times 0.7^2 \div 1.5$ = (approximately) 0.65 that of a steel arch. Thus moment magnification is generally less at service load for a concrete arch. The combination of more stiffness at service load with the effect of fixity causes the fixed concrete and to have about 1/3 the live load deflection of a two-hinged steel arch.

At ultimate load, however, the picture changes completely and the concrete arch becomes much more flexible. This is due to the very large downward curve, from a straight line, of the concrete stress-strain curve as ultimate strength is approached. The AASHTO Specifications Article 1.5.34 give a value of concrete $EI = (E_cI_c/5) + E_sI_s$ for compression member magnification of live load moment. Since an arch may have as little as 0.5 percent reinforcing in each face, the above equation may reduce the EI of an arch section to 1/4 the value at service load. The net result is that a concrete arch will generally have considerably more moment magnification than a steel arch, for design purposes.

AASHTO Article 1.5.33, Compression Members With or Without Flexure, and Article 1.5.34, Slenderness Effects in Compression Members may be used for the design of the concrete arch cross-sections. These articles are under Load Factor Design. Service Load Design makes use of these Load Factor Design Articles by the application of certain factors.

Since ultimate strength, rather than working stress, is therefore the basis of both design methods, it is recommended that load factor design be used for arches. It is also recommended that a minimum of 1 percent total reinforcement be used for concrete arches. More reinforcement may be needed because of tensile requirements, or to reduce moment magnification.

There is a question of the value of ϕ to be used in AASHTO Equation (6-15). Although ϕ should be taken as 0.7 (except for $P < 0.1f_cA_g$) for determining the section resistance to combined moment and axial load as outlined in Article 1.5.33, it is recommended that ϕ be taken as 0.85 in Equation (6-15) for the determination of moment magnification. The reason for this recommendation is that moment magnification applies to flexural action only. This ϕ value is also consistent with the numerical constant of 1.18 used in Equation 3 for Load Factor Design of steel arches.

Moment magnification should not be applied to either stresses or deflection produced by rib shortening, shrinkage or temperature. The reason is that this deflection, unlike partial live load deflection, is in the same direction (either up or down) over the entire span. As a result of this type of deflection the horizontal reaction and the position of the thrust line can, and does, adjust to this deflection. The calculated amount of H is correct for the final arch position, because of camber.

2.3 Ratio of Rib Depth to Span

As explained in the previous section, live load deflection at service load is quite small for a fixed concrete arch and, therefore, does not govern the depth of rib to be used. Buckling in the plane of the arch and moment magnification are the important factors in determining the rib depth. Table I gives dimensions and data for existing concrete arches. Six of the eight have a ratio of span to rib depth at the crown between 70 and 80. A ratio of 75 is a good average figure for a fixed-concrete arch. The Sando Arch (9) in Sweden has a ratio of 99. Radious of gyration, rather than overall depth, is a measure of resistance to buckling and moment magnification. The radius of gyration of a solid section is about 3/10 of the depth, and about 4/10 of the depth for a box section. On this basis the box section might have 4/3 the span-to-depth ratio of a solid section. However, there would appear to be no advantage, from the standpoint of economy, in using the smaller depth for a box section. The width of rib and thickness of slabs and webs can be such that the area of cross-section can be made a minimum consistent with stress.

Excluding the Hokawazu Arch (10), which is two hinged, the other four box sections have a ratio of springing depth to crown depth varying from 1.55 to 1.72. A value of about 1.7 is suggested for the ratio. From the crown to the springing the depth may vary approximately linearly along the arch axis.

TABLE I - EXAMPLES OF CONCRETE ARCHES

	Gladesville Australia (12)	Sando Sweden (9)	Port Elizabeth South Africa (13)	Hokawazu Japan (10)	Cowlitz River Washington St. (14)	Mississippi Minnesota (11)	Memorial Wash. D.C. (11)	Arroyo Sec California (11)
Deck-Width(B_d)	84'	42'	85.3	33'	31'	57'	94'	32'
Span (l)	1000'	866'	656'	557'	520'	300'	180'	105'
Rise (h)	134	130'	145'	87'	148.6'	80'	27.4'	45'
Width Rib	4@20[1]	31'	47.8'	26.2'	27'	2@12'	86'	2@3.5'
Rib Depth-Crown	14'	8.75'	9.02'	7.87'	7'	4.0'	2.25'	2.0'
Rib Depth-Springing	23'	14.75'	14.0'	9.84'	12'	10.00'	5.86'	3.5'
No. of Cells	4	3	3	2	3	---	---	---
Slab b/t	14.4	9	19.6	11.3	12.6	---	---	---
Web b/t (max)	19.5	12.75	14.1	5.5	16	---	---	---
Rise/Span	0.134	0.15	0.22	0.156	0.286	0.267	0.139	0.43
Span/Crown Depth	71.5	99	72.8	70.8	74.3	75	80	53
Span/Rib Width	12.5[1]	28	13.7	21.2	20	25	1.92	21
f'c in psi	6000	5750	5700	5700	4000	2500?	3900	2500?
Spring Depth/Crown Depth	1.64	1.69	1.55	1.25	1.72	2.5	2.6	1.75
Rib Area-Sq. Ft (at crown)	292	89	100	72	51.5	96	193	14

All of the above are fixed except Hokawazu which is 2-hinged

[1] The 20-foot wide ribs are separated by 1-foot gaps, and connected by prestressed diaphragms at 50-foot intervals. The ribs are precast in lengths of about 10 feet and have no continuous reinforcement.

The span-to-crown depth ratio of 75 appears to be applicable to solid as well as box section. The lower value of 53, used for the Arroye Seco Bridge (11) may have been for architectural reasons, possibly to obtain a narrow rib in relation to depth in this short span, high rise arch. We see no reason for a larger ratio of springing to crown depth for a solid as compared to a box section and, therefore, recommend that 1.7 be used for both.

2.4 Rise-to-Span Ratio

Just as with steel arches, the rise-to-span ratios for concrete arches vary over a wide range. The ratios for the arches listed in Table I vary from 0.14 to 0.43. The site in combination with required clearances and roadway grades generally control the rise and minimum span. For a bridge over a deep valley either the span or the rise, or both, may be increased for reasons of economy. In a single span over a canyon the rise may be reduced and the span increased by raising the abutments, to effect economy.

A high rise-to-span ratio for a given span reduces the dead load thrust and the moments from rib shortening, shrinkage and temperature change, but the wind stresses are increased. The reduction in dead load thrust reduces the live load moment magnification. The resulting added length of rib, of course, adds material cost and construction cost.

2.5 Rib Shortening, Shrinkage, Temperature Effects and Camber

Rib shortening, as used here, refers to the shortening of the arch axis due to axial stress and the stresses produced by that shortening. Shrinkage refers to arch axis shortening and resultant stresses due to the drying out of the concrete after setting. Temperature involves the lengthening or shortening of the arch axis due to rise and fall of the average concrete temperature from the temperature at the time of closure. These effects are modified by the plastic quality of concrete resulting in creep and stress relaxation. Changes in length of the axis and the resultant deflections which determine camber will be discussed first.

2.5.1 Permanent Arch Deflections

Since dead load is a permanent load, plastic flow or creep increases the initial deflection over a long period of time, at a decreasing rate. The following equations can be used to calculate ultimate crown deflection, approximately:

Fixed Arch

Initial Δ_{rs}	$= f_a/E_c\ (h + \ell^2/4h)$	Eq. 46a
Creep Δ_{rs}	$= 2.4$ Initial Δ_{rs}(Additional)	Eq. 46b
Δ_s	$= 0.0003\ (h + \ell^2/4h)$	Eq. 46c
Δ_t	$= \omega t\ (h + \ell^2/4h)$	Eq. 46d

Two-Hinged Arch

Initial Δ_{rs}	$= f_a/E_c(h + \ell^2/5h)$	Eq. 47a
Creep Δ_{rs}	$= 2.4$ Initial Δ_{rs}(Additional)	Eq. 47b
Δ_s	$= 0.0003\ (h + \ell^2/5h)$	Eq. 47c
Δ_t	$= \omega t\ (h + \ell^2/5h)$	Eq. 47d

Where f_a = axial unit stress at the crown

Δ_{rs} = crown deflection from dead load rib shortening

Δ_s = final crown deflection from shrinkage

Δ_t = crown deflection from temperature change

ω = coefficient of expansion = 0.000006

t = temperature change in degrees Farenheit

2.5.2 Arch Stresses from Rib Shortening, Shrinkage and Temperature Change

Stress relaxation does not reduce deflection but has a considerable effect in reducing the stress effects of creep and shrinkage. The initial rib shortening deflection and stress are elastic effects of load. The effect of creep is to increase initial deflection and stress. At the same time that creep is increasing stress, relaxation is reducing stress but not deflection. There is a residual net stress effect from the combined action of creep and stress relaxation. This residual stress factor will be taken as 0.38. Therefore, the final stress from dead load rib shortening will be 1.38 times the initial elastic stress due to rib shortening. In other words, the initial elastic rib shortening stress is a load stress which in itself remains unchanged, but the deformation effects of creep and relaxation add 38 percent to this initial load stress, in a period of time.

Shrinkage is entirely a deformation effect, having no connection with any load. Shrinkage stress, therefore, is subject to the full effect of stress relaxation. The shrinkage factor of 0.0003 should, therefore, be multiplied by the relaxation residual factor of 0.38, giving a net shrinkage stress factor of 0.00012.

In line with the above discussion, the following equations should be used in calculation of reactions and stresses:

Fixed Arch

$$H_{rs} = \frac{-90}{8}\left[\frac{0.5\ (r_s + r_c)}{h}\right]^2 \times 1.38\ f_c A_c \qquad \text{Eq. 48a}$$

$$H_s = \frac{-90}{8}\left[\frac{0.5\ (r_s + r_c)}{h}\right]^2 \times 0.00012\ EA_c \qquad \text{Eq. 48b}$$

$$H_t\text{(drop)} = \frac{-90}{8}\left[\frac{0.5\ (r_s + r_c)}{h}\right]^2 \times ct\ EA_c \qquad \text{Eq. 48c}$$

$$M_x = -H\left[y - h(1 - 0.33\sqrt{d_c/d_s})\right] \qquad \text{Eq. 48d}$$

2-Hinged Arch

$$H_{rs} = \frac{-15}{8}\left[\frac{rc}{h}\right]^2 \times 1.38\ f_c A_c \qquad \text{Eq. 48e}$$

$$H_s = \frac{-15}{8}\left[\frac{rc}{h}\right]^2\ 0.00012\ EA_c \qquad \text{Eq. 48f}$$

$$H_t\text{(drop)} = \frac{-15}{8}\left[\frac{rc}{h}\right] \times ct\ EA_c \qquad \text{Eq. 48g}$$

$$M_x = -Hy \qquad \text{Eq. 48h}$$

r_s = radius of gyration at the springing

r_c = radius of gyration at the crown

y = vertical distance from springing to point x

f_c & A_c = unit stress and area of cross-section at the crown

M_c = crown moment

M_s = springing moment

2.5.3 Reduction of Rib Shortening and Shrinkage Stresses by Construction Methods

The majority of concrete arch designs allow for full rib shortening and shrinkage stresses, and only minor steps are taken in construction to reduce these stresses. One generally used method is to pour the arch rib in separated sections along the axis. Keyways are left between these sections and reinforcing is lapped in the keyways. The keyways are poured last. This will result in some reduction of shrinkage stress, due to some shrinkage having occurred in the time period between the start of concrete placement and the final pour. The reduction effect is greater in long span arches because of the greater length of time involved in concrete placement as compared to short span arches.

Stresses opposite to rib shortening and shrinkage stresses can be jacked into the rib before final rib closure. This is similar to the method in steel arches. Freyssinet developed one method. The two halves of the rib may be separated at the crown by an opening. Pairs of jacks are used at the introdos and extrados at the crown to further separate and raise the two halves of the rib. The jacking forces are precalculated to force initial stresses in the arch for the purpose of counteracting rib shortening and shrinkage stresses. The sum of the jacking forces must be very close to the crown thrust from the vertical load at that time. By making the lower jacking forces larger than the upper forces, negative moment (compression in the bottom of the rib)is forced into the arch. The jacking results in separating the two halves at the crown, and lifting the entire rib from the falsework, Concrete is then placed in the opening created by jacking, and the jacking forces are transferred to the crown key section after it has attained sufficient strength. The crown jacking produces rotation, so that the key section is wedge shaped. The calculated stresses to be jacked are opposite in sign and considerably larger in magnitude than the rib shortening and shrinkage stresses, because of stress relaxation. Using the previously mentioned residual factor of 0.38 for the combined effect of creep and stress relaxation, the calculated total, final rib shortening and shrinkage forces at the crown must be multiplied by $1 \div 0.38 = 2.6$ to get the jacking forces.

Other methods of producing counteracting stresses, just before the arch becomes self supporting, will be discussed under arch construction.

2.6 Buckling of Elements of Box Cross-Sections

The slabs and walls of concrete arch box cross-sections should be checked for buckling, just as are the flanges and webs of steel arch boxes. The slabs are more critical than the walls because the average stress in the slab, at any cross-section, is practically equal to the sum of the axial compression and the bending compression, whereas the average stress in the wall is equal to the axial compression.

The following equations are based on two independent sets of tests, one set by S. E. Swartz and V. H. Rosebraugh, reported in the October 1976 ASCE Proceedings; and the other set by G. C. Ernst, reported in the December 1952 Journal of the American Concrete Institute. The equations give results about midway between the two sources.

For slabs

$$b/t = 80\left(1 - 0.85\frac{f_a + f_b}{\text{allowable } f_c}\right), \text{ maximum} = 20 \qquad \text{Eq. 49}$$

For walls

$$b/t = 80\left(1 - 0.85\frac{f_a}{\text{allowable } f_c}\right), \text{ maximum} = 20 \qquad \text{Eq. 49a}$$

Where f_a = computed axial compressive stress

f_b = computed bending stress

allowable f_c = maximum allowable stress for the concrete, without consideration of buckling

b = clear width of slab or wall

t = thickness of slab or wall

The above equations will permit full stress in the slab, with no reduction for buckling, for b/t = 12 or less, and a maximum reduction of stress of 12 percent is required for b/t = 20.

The same is true for the walls, except that the rib bending stress is not considered, and only the rib axial stress is used. Generally b/t for the walls can be made 20, except in some cases where the wall may become too thin for proper placement of the concrete.

The slabs should have a minimum transverse reinforcement of 0.5 percent, and this reinforcement should be equally divided between the top and bottom. The transverse reinforcement should extend to the exterior faces of the outside walls and be anchored by standard 90 degree hooks. The webs, or walls, should have the same minimum percentage of reinforcement as the slabs.

The longitudinal reinforcement should be a minimum of 1 percent for both slabs and walls, and should be divided equally between the two faces. This minimum longitudinal reinforcement is desirable for reduction of dead load axial creep and of live load moment magnification.

Table I shows the ratio of b/t for the slabs of 5 box-section arches. The only one which considerably exceeds 12 is the Port Elizabeth Bridge in South Africa. The b/t ratio for the slabs of this cross-section is 19.6. This ratio, by equation 49, would require a stress reduction of 11 percent.

The proposed equation may be somewhat conservative. However, in most arches, the value of b/t = 12 can be easily met by changing the relationship of overall box width to slab thickness, with no loss of economy.

2.7 Wind Stress and Wind Deflection

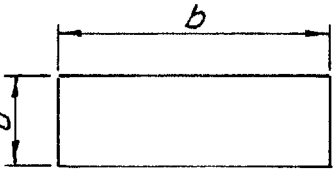
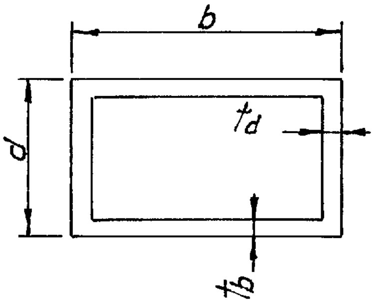
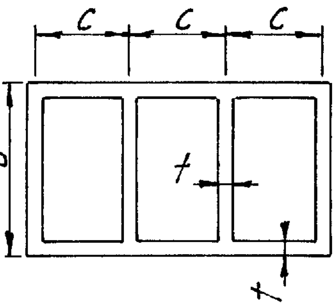
The equations and methods for wind stress analysis and wind deflection of steel arches apply to concrete arches as well. Concrete arches consisting of a single rib should be analyzed by the method of Article 1.13.2. Where two or more ribs are braced together by transverse struts, Article 1.13.1 and Design Example 1.22 should be used.

Table II, on the following page, gives formulas for the torsional stiffness constant K and for torsional shear stress.

2.8 Interaction Between Rib and Roadway Framing

Interaction between the roadway framing and the arch rib is likely to be greater for a concrete arch than for a steel arch. This is mainly due to the relatively deeper members used for concrete framing and to the greater rigidity at the intersections. A model arch was tested at the University of Illinois, and the results are described in the ASCE 1935 Transactions, page 1429, "Concrete and Reinforced Concrete Arches - Final Report of the Special Committee."

Table II ~ Equations for K and Torsional Stress

Section	K	Torsional Stress
 For $b/d \geq 10$	eq. 50 a $\frac{bd^3}{3.1}$	eq. 50 d $\frac{3.1\,T}{b d^2}$
	eq. 50 b $\frac{2b^2d^2}{b/t_b + d/t_d}$	eq. 50 e $\frac{T}{2bdt}$ Use t for side of desired stress
	eq. 50 c $\frac{4c^2d^2t(3c+5d)}{2(c+d)^2 - d^2}$	eq. 50 f $\frac{T(c+2d)}{2cdt(3c+5d)}$

The equations for a single cell box may be used for a three cell box with negligible error in K. The effect of the two interior walls is to increase the stress at the center of the long side by about 13%.

This test arch had a springing depth of twice the crown depth, with nine column spaces, and the deck framing was against the rib for for the length of the center space. The moment of inertia of the deck was approximately 1.5 times that of the rib at the crown, and the column moment of inertia was 0.4 that of the rib at the crown.

A horizontal force of 2540 pounds was required to reduce the span by 0.1 inch and this produced a moment at the springing of 131,300 inch pounds. Calculations, for the arch rib acting alone, show a horizontal force of 950 pounds and a springing moment of 59,000 inch pounds for a span reduction of 0.1 inch. Thus the effect of the interaction was to increase the springing moment due to shortening of the rib from shrinkage, axial stress, and temperature by a ratio of 2.2.

The total moment from these causes, at the mid-point of the span, is increased in a ratio of 4.1 by interaction. The springing moment would be carried by the arch rib alone, but the moment at the center line of span would be carried by the combined action of the deck and rib. This is just one example, of course, but it gives an idea of the effect of interaction on stresses from shrinkage, rib shortening, and temperature.

The effect of interaction on live load stress would be a reduction at the springing and at all points in the rib. The combination of rib, columns, and deck framing would act as a Vierendeel truss to resist live load moments. The net effect of interaction would be to reduce the total stress from all causes in all parts of the rib.

If the rib is designed to act alone under all loads, it should be adequate. Some stresses from arch action will be induced in the columns and deck framing, but these can be considered as being in the category of secondary stresses in a truss. Just as in a truss the secondary stresses will be minimized by using slender members, for the columns and deck framing.

Some arches, particularly in Europe, have very slender ribs, designed to take only axial load, and deep deck members are designed to take the moment. This is similar in action to a tied arch in which a deep tie is designed to take the moment, and a slender rib is used.

In some cases, expansion joints have been introduced in the deck, within the arch span, to reduce interaction. This is not desirable because expansion joints should be kept to a minimum and because of a possible high stress produced in the rib in the vicinity of the joints.

The 1938 ASCE Transactions, page 62, has a very good paper by Nathan Newmark, "Interaction Between Rib and Superstructure in Concrete Arch Bridges."

2.9 Lateral Buckling and Lateral Moment Magnification

Arches of the single barrel or multiple box type are appreciably stiffer laterally than vertically so that lateral buckling and lateral moment magnification are of minor consequence. When two or more individual ribs are used, lateral stiffness may play a more important part in the design. Referring to Table I, the Sando multiple box arch has a ℓ/b ratio of 28, and the Cowlitz River multiple box has a ℓ/b ratio of 20. The Minnesota multiple rib arch has an individual rib span to width ratio of 25, and the two ribs are braced together by several transverse struts. The Arroyo Seco multiple rib arch has two ribs with individual span-to-width ratios of 21, and the ribs are braced together at each column with a spacing of 10 feet 6 inches.

Thus the Sando Bridge is the most flexible laterally of this group of bridges. A multiple box section has a transverse radius of gyration approximately equal to 0.32b. Referring to Section 1.11, and interpolating in the table for KL; the Sando Arch, with a rise to span ratio of 0.15, has a $k\ell/r$ ratio of $1.11 \times 1.05 \times 433 \div 0.32 \times 28 = 56.4$. In the plane of the arch, the Sando Bridge has a ratio of crown depth to span of 1/99 which would give a $k\ell/r$ value of approximately $0.55 \times 99 \times 0.7 \div 0.4 = 86$. Thus, for this arch, lateral buckling is considerately less critical than in-plane buckling. The span-to-rib width ratio of 28 is entirely satisfactory.

The equations of AASHTO Article 1.5.34 may be used for Load Factor Design Moment Magnification. As with in-plane moment magnification, a ϕ factor of 0.85 may be used and C_m should be taken as 1. Use lateral

For moment magnification to determine service load unit stresses, the elastic value of EI should be used and ϕ should be taken as 1.

A concrete arch with narrow ribs braced together by lateral struts may be investigated for lateral buckling and moment magnification by the methods of Section 1.11.

2.10 Load Factor Versus Service Load Design

An arch rib is a slender compression member with flexure. AASHTO Specifications for Service Load Design of such members refer to the pertinent Articles under Load Factor Design, requiring a factor of 0.35 to be applied to capacity and a factor of 2.5 to be applied to the axial load for use in moment magnification. In effect, Ultimate Strength Design is factored for use in Service Load Design. It is more direct to simply use Load Factor Design. A desirable feature of Service Load Design, the calculation of unit stresses, is not used in AASHTO Specifications for compression members. A designer using Load Factor Design may wish to calculate unit stresses, as well as deflection, at service load to gain a better insight into the member action under service load.

Load Factor Design for compression members subjected to flexure involves the construction of an Interaction Diagram. Such diagrams are generally available for solid sections. For box sections the Interaction Diagram can be constructed according to AASHTO Article 1.5.31 - Design Assumptions. Load Factor Design for Reinforced Concrete Bridge Structures (29) by P.C.A. gives examples of calculations for such construction.

2.11 Minimum Reinforcing Steel Requirements and Other Details

A minimum of 1 percent of reinforcing steel should be used in a concrete arch rib. For a wide barrel type rib this reinforcement should be divided half and half between an upper and lower layer. For a box section this minimum should be distributed uniformly over the cross section. Two layers of steel should be used in both slabs and webs. The use of this steel reduces creep and gives a minimum tensile strength. This steel may be very highly stressed in compression due to creep. This is counted on for stiffness in moment magnification.

Additional reinforcement may be required because of tensile stress. As required in the AASHTO Specifications, a design dead load reduction by 0.75 should be used in investigating tension.

For box sections, diaphragms should be used at columns. Columns may be used in pairs, or a single wide column at each panel point may be used. In addition to the diaphragm at columns, diaphragms are frequently used at the mid-panel points between columns.

2.12 Design Example

As an example, a concrete arch for the same conditions as those assumed for the steel arch design example of Chapter 1 will be designed. The initial preliminary design for the concrete arch will not be given since it is similar to the method used for the steel arch. The more exact analysis is used, and it is revised to bring the assumed cross-section closer to the required cross-section. The following assumptions in regard to the cross-section to be analyzed are made:

Use cellular type box cross-sections

Rib depth at crown $= \frac{425}{75} = 5.67$, try 5'-9"

Rib depth at springing = 1.7 x 5.75 x 9.78, try 10'-0"

Rib width = 4.25 $\div$ 20 = 21.2, try 20'-0"

Table I gives examples of the dimensions of a wide variety of types and spans of concrete arches. The last line in the table gives the area of the rib cross-section at the crown. The Cowlitz River Bridge is the closest to the design example. It has a cross-sectional area at the crown of 51.5 square feet. The design example has about the same roadway width but a span about 8/10 that of Cowlitz River. A two-celled section with 10 inch webs and 8 inch slabs will be assumed. The crown area A_C is:

$$20 \times 5.75 - 17.5 \times 4.42 = 37.7 \text{ sq. ft.}$$

The ratio of b/t for the slabs = $\frac{105}{8} = 13.1$

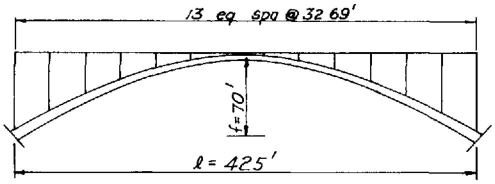
Assume 35 #6 bars, top and bottom, in slabs

$$p \text{ for slabs} = \frac{2 \times 35 \times .441}{240 \times 8} = 1.6\%$$

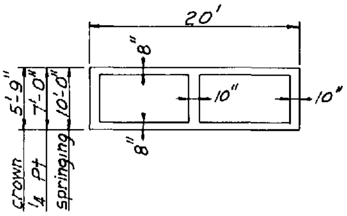
Assume 6 #6 bars, each face, in webs

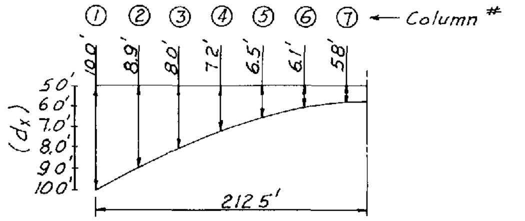
$$p \text{ for webs} = \frac{2 \times 6 \times .441}{53 \times 10} = 1.0\%$$

The weight of the rib can now be calculated. The deck weight will simply be assumed for the purpose of this example. Normally, of course, it would be designed first in order to more accurately get the superimposed dead load on the arch.



HS20-44
f'_c = 4.0 ksi
f_y = 60.0 ksi
rdwy. deck ~ 5 K/'
bent cap ~ 80K
column ~ 2 K/'
reinforcement:
webs ~ 12-#6's ea web
top and bott slabs ~ 2 layers-#6's @ 7"



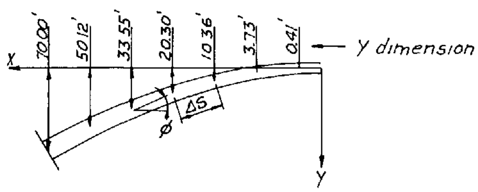


Arch Rib Depth (d_x)

Assume parabolic arch axis:

$$y = 4fx^2/\ell^2$$

$$\sec\phi = \sqrt{1+\left(\frac{8fx}{\ell^2}\right)^2}$$



Column length $\simeq y + \frac{d_{crown}}{2} - \frac{d_x \sec\phi}{2}$

Col #	length
1	66.89'
2	47.90'
3	32.03'
4	19.36'
5	9.88'
6	3.52'
7	0.38'

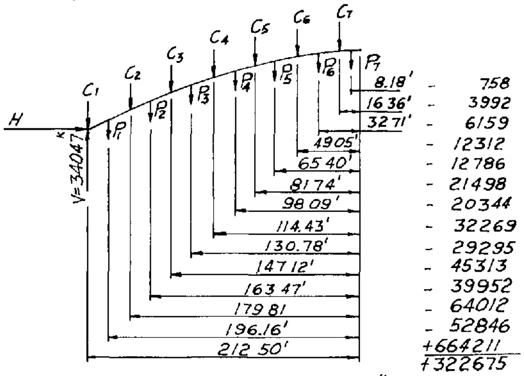
Concentrated Column Loads (C)

C_1 = 0.4(5.0×32.7) + 80.0 + (2.0×66.89) = 279 K
C_2 = 1.1(") + " + (2.0×47.90) = 356
C_3 = 1.0(") + " + (2.0×32.03) = 308
C_4 = " + " + (2.0×19.36) = 282
C_5 = " + " + (2.0×9.88) = 263
C_6 = " + " + (2.0×3.52) = 251
C_7 = " + " + (2.0×0.38) = 244
1983 K

Arch Rib Loads (P)*

load	sec. ϕ	ΔS	d_x	A_g	Kips
P_1	1.1704	38.26	9.45	46.95	269.4
P_2	1.1211	36.65	8.45	44.46	244.4
P_3	1.0791	35.28	7.60	42.33	224.0
P_4	1.0452	34.17	6.85	40.46	207.4
P_5	1.0203	33.35	6.30	39.08	195.5
P_6	1.0051	32.86	5.95	38.21	188.3
P_7	1.0003	16.36	5.77	37.76	92.7
					1421.7

* ϕ, d_x & A_g at segment center



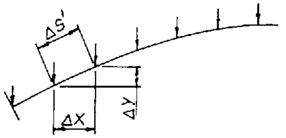
$$H = \frac{322675}{70} = 4609.6^K$$

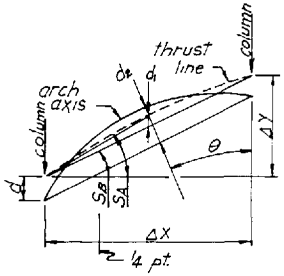
Determine Dead Load Equilibrium Polygon

$$\Delta Y = \text{shear}\ \Delta X / H$$

load	shear	ΔX	ΔY	ΣΔY
C_1				0.00
	3125.7	16.345	11.08	
P_1	-269.4			
	2856.3	"	10.13	
C_2	-356			21.21
	2500.3	"	8.87	
P_2	-244.4			
	2255.9	"	8.00	
C_3	-308			38.08
	1947.9	"	6.91	
P_3	-224.0			
	1723.9	"	6.11	
C_4	-282			51.10
	1441.9	"	5.11	
P_4	-207.4			
	1234.5	"	4.38	
C_5	-263			60.59
	971.5	"	3.44	
P_5	-195.5			
	776.0	"	2.75	
C_6	-251			66.78
	525.0	"	1.86	
P_6	-188.3			
	336.7	"	1.19	
C_7	-244			69.84
	92.7	8.18	0.16	
P_7	-92.7			
	0.0	"	0.00	
crown				70.00

Concentrated column loads applied to the arch produce an angular equilibrium polygon. The arch axis is a smooth curve. Keep dead load moments to a minimum by passing the axis a distance (d) below the thrust line at columns and half this distance ($d/2$) above the thrust line at midpoints between columns.





If a circular arch axis of constant radius (R) is assumed

$$R = \sqrt{(212.5)^2 + (R-70.0)^2} = 358'$$

$$S_A = \frac{V_{1/4\,pt}}{H} \qquad S_B = \frac{\Delta Y}{\Delta X}$$

$$d_1 = (S_A - S_B)\,\frac{\Delta X}{2}$$

$$\theta = \sin^{-1}(\tfrac{1}{2}\Delta S'/R)$$

$$d_2 = R\,(1-\cos\theta)$$

$$d = \frac{d_2 - d_1}{1.5}$$

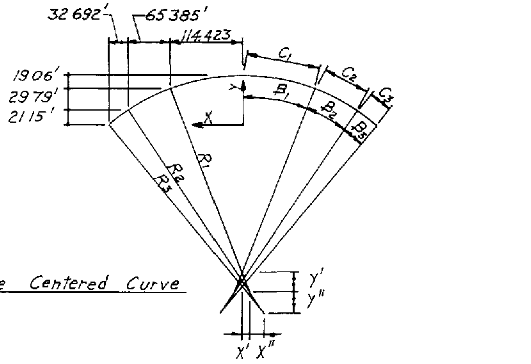
location	$V_{1/4\,pt}$	S_A	S_B	d_1 (ft)	θ degrees	d_2 (ft)	d (ft)
1	3058	.6634	6488	24	3 06	51	18
2	2439	.5291	.5161	21	2 93	.47	.17
3	1892	.4104	.3983	20	2 82	.43	.15
4	1390	.3015	.2903	.18	2 74	.41	.15
5	923	.2002	.1894	18	2 67	39	.14
6	478	.1037	0936	.17	2 63	.38	.14
7	46	.0100	.0000	.16	2.62	37	.14

Assume the thrust line will be a distance d (0.18') above the arch axis at the springing and a distance $d/2$ (0.07') below the arch axis at the crown. A new horizontal thrust results:

$$H = \frac{322675}{70.00 - 0.25} = 4626.2^{K}$$

Recalculate the thrust line coordinates and construct the arch axis to pass through those points.

load	shear	ΔX	ΔY	ΣΔY	ΣΔY−d
C_1				0 18	0 00
	3125.7	16 345	11 04		
P_1					
	2856 3	"	10 09		
C_2				21.32	21 15
	25003	"	8.83		
P_2					
	22559	"	7.97		
C_3				38 12	37.97
	19479	"	6.88		
P_3					
	17239	"	6.09		
C_4				51.09	50.94
	14419	"	5.09		
P_4					
	12345	"	4 36		
C_5				60.55	60.41
	971.5	"	3 43		
P_5					
	776 0	"	2 74		
C_6				66.72	66.58
	525 0	"	1.85		
P_6					
	336.7	"	1.19		
C_7				69.77	69.63
	92 7	8.18	0.16		
P_7					
	0.0	"	0.0	69.93	70.00



Five Centered Curve

$$R_1 = \sqrt{(114.423)^2 + (R_1 - 19.06)^2} = 352.988'$$

$$C_1 = \sqrt{(114.423)^2 + (19.06)^2} = 116.000'$$

$$\beta_1 = 2\sin^{-1}(58.000/352.988) = 18.9145^\circ$$

$$X' = (R_2 - R_1)\sin\beta_1 = 0.324157R_2 - 114.423'$$

$$Y' = (R_2 - R_1)\cos\beta_1 = 0.946003R_2 - 333.928'$$

$$R_2 = \sqrt{(179.808 + X')^2 + (304.138 + Y')^2} = 369.473'$$

$$X' = 5.344'$$

$$Y' = 15.595'$$

$$C_2 = \sqrt{(65.385)^2 + (29.79)^2} = 71.852'$$

$$\beta_2 = 2\sin^{-1}(35.926/369.473) = 11.1600^\circ$$

$$X'' = (R_3 - R_2)\sin(\beta_1 + \beta_2) = 0.501126R_3 - 185.152'$$

$$Y'' = (R_3 - R_2)\cos(\beta_1 + \beta_2) = 0.865375R_3 - 319.733'$$

$$R_3 = \sqrt{(212.50 + X' + X'')^2 + (282.988 + Y' + Y'')^2} = 394.918'$$

$$X'' = 12.752'$$

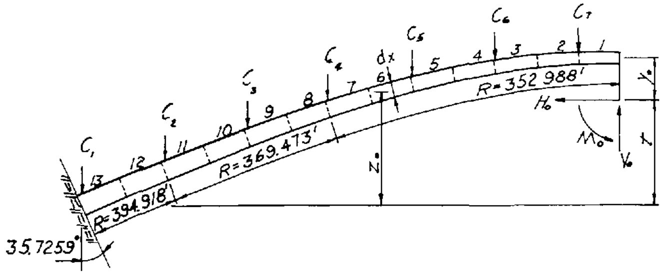
$$Y'' = 22.019'$$

$$C_3 = \sqrt{(32.692)^2 + (21.150)^2} = 38.937'$$

$$\beta_3 = 2\sin^{-1}(19.468/394.918) = 5.6514^\circ$$

$$\Sigma\beta = 35.7259^\circ$$

column	1	2	3	4	5	6	7
X	2125	179.81	147.12	11442	81 73	49.04	16.35
Y	0.0	21.15	37.97	50.94	60.41	66.58	69.62



$Z_o = R_1 \cos\beta - 282.988'$ (seg 1-7)
$Z_o = R_2 \cos\beta - 298\ 583'$ (" 8-11)
$Z_o = R_3 \cos\beta - 320.602'$ (" 12&13)
$Y = t - Z_o$
$A_g = 2\ 5 d_x + 23\ 333$
$A_T = A_g + (2n-1)(0538)$ for dead load
$A_T = A_g + (n-1)(0538)$ for live load
$n = 8$

$$I_c = \frac{20 d_x^3 - 175 (d_x - 1333)^3}{12}$$

$$I_s = (n-1)\left[0428 \left(\frac{d_x - 0\ 667}{2}\right)^2 + (000917)(d_x - 1.333)^2\right]$$

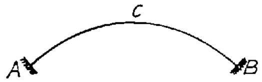
$X = R_1 \sin\beta$
$X = R_2 \sin\beta - 5\ 344'$
$X = R_3 \sin\beta - 18\ 096'$

$$t = \frac{\Sigma Z_o \Delta S/I}{\Sigma \Delta S/I} = 53.82'$$

Properties of Arch Rib

	radians β	ft Z_o	ft ΔS	ft d_x	I_c	I_s	ft⁴ I	$\Delta S/I$	$Z_o\Delta S/I$	ft X	ft Y	ft² A_g
1	0236	69 90	16 65	5 77	192 8	20 8	213 6	.0779	5448	8.32	- 16 08	37 76
2	0707	69 12	"	5 88	201 7	21 7	223 4	.0745	5150	24 95	-15 30	38 03
3	1179	67 55	"	6 03	214 3	23 0	237 3	.0702	4 739	41 52	-13.73	38 41
4	.1651	65.20	"	6 20	229.1	24 5	253 6	.0656	4.280	58 00	- 11 38	38.83
5	2122	62 08	"	6.40	247 2	26.3	273 5	0609	3 779	74 35	-8 26	39 33
6	2594	58 19	"	6 68	273 9	28 9	302 8	0550	3 199	90 54	- 4 37	40 03
7	3065	53 55	"	7.03	309 4	32.4	341 8	.0487	2.608	106.52	0.28	40.91
8	3545	47 92	17 99	7 40	349.7	36.3	386.0	0466	2 234	122.90	5 90	41.83
9	4032	41 27	"	7 80	396 5	40.8	437 3	0411	1698	139.61	12.55	42 83
10	4519	33 81	"	8 23	450.6	45.9	496 5	.0362	1225	155 98	20.01	43 91
11	5006	25 56	"	8 68	511.6	51 6	563 2	.0319	0817	171 97	28 26	45 03
12	.5496	16 17	19 48	9 18	584 7	58.2	642.9	.0303	0.490	188.17	37.65	46.28
13	.5989	5 59	"	9.73	671.8	66.0	737.8	.0264	0.148	204 53	48 23	47.66
								.6654	35.813			

	ft² A_T(DL)	ft² A_T(LL)	X^2	Y^2	$X\Delta S/I$	$Y\Delta S/I$	$X^2\Delta S/I$	$Y^2\Delta S/I$	ft-Kips M	$M\Delta S/I$	$M\Delta SX/I$	$M\Delta SY/I$
1	45 83	41 52	69	258 6	0.649	-1 253	5.4	20.16	- 0	0	0	0
2	46 10	41 80	621	234 0	1 859	-1 140	46 4	17.44	- 3667	- 273	- 6818	4180
3	46 48	42 17	1724	188 5	2 913	-0 963	120.9	13 22	- 10846	- 761	-31591	10446
4	46 90	42 60	3364	129.5	3.807	-0 747	220.8	8.50	- 21816	- 1432	-83056	16299
5	47 40	43 10	5528	68 2	4 526	-0 503	336.5	4.15	- 36159	- 2201	-163644	18182
6	48 10	43 80	8197	19.1	4 977	-0.240	450.6	1 05	-54259	-2983	-270066	13042
7	48 98	44 67	11346	0.1	5 188	0.013	552.6	0.00	-75651	-3685	-392470	-1013
8	49 90	45 60	15104	34 8	5 728	0275	704 0	1.62	-101632	-4737	-582159	-27948
9	50 90	46 60	19491	157.6	5 744	0.516	801.9	6.48	-132308	-5443	-759942	-68330
10	51 98	47 67	24330	400.4	5.652	0.725	881 6	14.51	-166978	-6051	-943787	-121079
11	53 10	48 80	29574	798 5	5.493	0.903	944.7	25.51	-204988	-6548	-1126083	-185037
12	54 35	50 05	35409	1417 8	5.701	1 141	1072.7	42.95	-248460	-7527	-1416357	-283416
13	55 73	51 42	41830	2326 3	5.399	1.273	1104.2	61.41	-297353	-7849	-1605363	-378584
							7242.3	217.01		-49490	-7381336	-1003258



Dead Load Reactions at Elastic Center Neglecting Rib Shortening.

$$M_o = \frac{-\sum_A^B \frac{M\Delta S}{I}}{\sum_A^B \frac{\Delta S}{I}} = \frac{-2(-49,490)}{2(0.6654)} = 74,374'^{k}$$

$$H_o = \frac{-\sum_A^B \frac{MY\Delta S}{I}}{\sum_A^B \frac{Y^2\Delta S}{I}} = \frac{-2(-1,003,258)}{2(217.01)} = 4623.0^{k}$$

$$V_o = \frac{-\left[\sum_A^C \frac{MX\Delta S}{I} - \sum_C^B \frac{MX\Delta S}{I}\right]}{\sum_A^B \frac{X^2\Delta S}{I}} = \frac{-\left[(-7,381,336)-(-7,381,336)\right]}{2(7242.3)} = 0^{k}$$

Dead Load Moment in Cantilevered Half-Arch (M)

Item	1	C7	2	3	C6	4	5	C5	6	7
Vertical load (Kips)	94.28	244 0	94.97	95.91	251 0	96.97	98.22	263.0	99.96	102.15
Σ " " "	94 28	338.28	433.25	529 16	780.16	877.13	975 35	1238.4	1338.3	1440.5
Moment arm (ft)	8 024	8 604	16.571	7 517	8.962	16.35	7.381	8.804	15.984	7.904
Moment -(ft-Kips)	0.0	756	3667	10846	14824	21816	36157	43356	54259	75651
Item	C4	8	9	C3	10	11	C2	12	13	C1
Vertical load	282.0	112 89	115 59	308.0	118 5	121 53	356 0	135 21	139 23	279 0
Σ " "	1722 5	1835.4	1950.9	2258.9	2377.4	2499.0	2855 0	2990 2	3129.4	3408.4
Moment arm	8.474	16.714	7 504	8.867	15.988	7.838	8.366	16.351	7.975	0.0
Moment	87036	101632	132308	146948	166978	204988	224575	248460	297353	322310

	$\sin\phi$	$\frac{\cos^2\phi\Delta S}{A_T(LL)}$	$\frac{\sin^2\phi\Delta S}{A_T(LL)}$	Unit load at C_2			Unit load at C_3			Unit load at C_4			Unit load at C_5			Unit load at C_6			Unit load at C_7		
				N	$\frac{N\cos\phi\Delta S}{A_T}$	$\frac{N\sin\phi\Delta S}{A_T}$	N	$\frac{N\cos\phi\Delta S}{A_T}$	$\frac{N\sin\phi\Delta S}{A_T}$	N	$\frac{N\cos\phi\Delta S}{A_T}$	$\frac{N\sin\phi\Delta S}{A_T}$	N	$\frac{N\cos\phi\Delta S}{A_T}$	$\frac{N\sin\phi\Delta S}{A_T}$	N	$\frac{N\cos\phi\Delta S}{A_T}$	$\frac{N\sin\phi\Delta S}{A_T}$	N	$\frac{N\cos\phi\Delta S}{A_T}$	$\frac{N\sin\phi\Delta S}{A_T}$
1	02358	.40068	.00023																		
2	07068	39627	.00199																.07068	02808	.00199
3	11763	.38926	00546																11763	.04611	.00546
4	.16431	.38023	.01055													16431	06334	.01055	.16431	.06334	.01055
5	.21063	36911	01714													21063	.07953	.01714	.21063	.07953	.01714
6	.25648	35508	02500										.25648	.09422	02500	25648	.09422	.02500	25648	.09422	.02500
7	.30176	33870	.03393										30176	.10721	.03393	30176	.10721	03393	.30176	.10721	.03393
8	.34709	.34702	.04753							.34709	.12843	.04753	.34709	12843	.04753	34709	12843	.04753	34709	.12843	.04753
9	.39233	.32666	.05943							39233	13932	05943	.39233	13932	.05943	.39233	.13932	05943	39233	.13932	05943
10	.43664	30543	.07195				.43664	.14824	.07195	.43664	.14824	.07195	.43664	.14824	.07195	.43664	.14824	.07195	.43664	14824	.07195
11	.47991	28377	.08491				.47991	.15523	.08491	.47991	.15523	.08491	.47991	.15523	.08491	47991	.15523	.08491	47991	.15523	08491
12	52231	.28298	.10616	.52231	.17332	10616	52231	.17332	.10616	.52231	.17332	.10616	.52231	.17332	.10616	.52231	.17332	.10616	.52231	.17332	.10616
13	.56371	.25838	.12035	56371	.17634	12035	.56371	.17634	.12035	.56371	17634	.12035	.56371	17634	.12035	.56371	.17634	.12035	.56371	.17634	12035
Σ		443356	.58463		.34967	.22651		65313	.38337		.92088	.49033		112231	.54926		126518	.57695		133937	.58440
H_o					.0918			.3345			.6669			1.0195			1.3150			1.4845	
V_o					.0125			.0505			.1140			.2021			.3113			.4353	

N = dead load shear $\times$ $\sin\phi$

	N (Kips)	$\frac{\Delta S}{A_T(DL)}$	$\cos\phi$	$\frac{N\cos\phi\Delta S}{A_T(DL)}$	$\frac{\cos^2\phi\Delta S}{A_T(DL)}$
1	0	.3633	.99972	0	.3630
2	23.9	3611	99750	8.61	.3593
3	51.0	.3582	.99306	18 13	.3532
4	128 2	3549	.98641	44 88	.3453
5	184.8	3512	.97757	63 42	.3356
6	317.6	.3461	96655	106.24	.3233
7	403.9	.3399	.95338	130.86	.3089
8	597.9	.3605	.93783	202 14	.3171
9	720.1	.3534	.91983	23409	.2990
10	986.3	.3461	.89964	307.14	.2801
11	1141.0	.3388	.87732	339.13	.2608
12	1491.2	.3583	.85276	455.65	.2606
13	1685 6	.3495	.82597	486 57	.2887
				2396 86	4.0447

$$H_o = \frac{-(-1{,}003{,}258 + 2396.86)}{217.01 + 4.045} = 4527.6^{K}$$

Reactions Including Rib Shortening (R)

$$H_o = \frac{-\left[\sum_A^B \frac{MY\Delta S}{I} + \sum_A^B \frac{N\cos\phi\Delta S}{A_T}\right]}{\sum_A^B \frac{Y^2\Delta S}{I} + \sum_A^B \frac{\cos^2\phi\Delta S}{A_T}}$$

$$V_o = \frac{-\left[\sum_A^C \frac{MX\Delta S}{I} - \sum_A^C \frac{N\sin\phi\Delta S}{A_T}\right]}{\sum_A^B \frac{X^2\Delta S}{I} + \sum_A^B \frac{\sin^2\phi\Delta S}{A_T}}$$

M_o is not affected

<u>Dead Load Rib Shortening Effect</u>

H_o neglecting R = 4623.0^K
H_o including R = 4527.6^K
Negative thrust from R = 95.4^K
V_o and M_o are not affected by a symmetrical loading.
Thrust(N) = $-95.4\cos\phi$
Moment(M) = $-95.4 \times$ distance vertically to elastic center

$N^7 = -953^K$ $\quad$ $N^4 = -902^K$
$M^7 = 1507'^K$ $\quad$ $M^4 = -270'^K$

$N^5 = -928^K$ $\quad$ $N' = -774^K$
$M^5 = 629.'^K$ $\quad$ $M' = -5134'^K$

Live Load Rib Shortening Effect

	Unit load at:					
	C_2	C_3	C_4	C_5	C_6	C_7
H_o neglecting R	.0945	.3428	.6826	1.0429	1.3447	1.5179
H_o including R	.0918	.3345	6669	1.0195	1.3150	1.4845
Negative thrust from R	-.0027	-.0083	-.0157	-.0234	-.0297	-.0334
N^7 = Neg thrust $\times\cos\phi$	-.0027	-.0083	-.0157	-.0234	-.0297	-.0334
M^7 = " " $\times (Y)$	.0427	.1311	.2481	.3697	.4693	.5277
N^5	-.0026	-.0081	-.0153	-.0228	-.0289	-.0325
M^5	.0178	.0547	.1035	.1542	.1957	.2201
N^4	-.0026	-.0079	-.0149	-.0221	-.0281	-.0316
M^4	-.0078	-.0239	-.0452	-.0674	-.0856	-.0962
N'	-.0022	-.0067	-.0127	-.0190	-.0241	-.0271
M'	-.1453	-.4467	-.8450	-1.2594	-1.5985	-1.7976

The rib shortening effect on V_o due to live load is so small that it is neglected in this case

	Unit load at C_2				Unit load at C_3				Unit load at C_4			
	M	$M\Delta S/I$	$MX\Delta S/I$	$MY\Delta S/I$	M	$M\Delta S/I$	$MX\Delta S/I$	$MY\Delta S/I$	M	$M\Delta S/I$	$MX\Delta S/I$	$MY\Delta S/I$
1												
2												
3												
4												
5												
6												
7												
8									- 8 47	- .3948	- 48 5	- 2.33
9									- 25.19	-10363	- 144 7	- 13 01
10					-8 87	-.3214	-50 1	-6 43	- 41.57	-15063	- 235 0	- 30 14
11					-24 85	-7938	-136.5	-22 43	- 57 55	-1.8384	-316 1	- 51 95
12	-8.37	- 2536	-47 7	- 9 55	-41 06	-12439	-234 1	-46 84	-73 75	-2 2342	-420 4	- 84 13
13	-2472	- 6525	-133.5	- 31.47	-57 41	-15155	-309 9	-73 09	-90 10	-2.3784	-486 4	-114 71
Σ		- 9061	-181 2	-41 02		-3 8745	-730 7	-148.79		-9 3884	-1651 2	-296 27
M_o	0.6808				2.9113				7 0545			
H_o	0.0945				0.3428				0 6826			
V_o	0 0125				0.0504				0 1140			

	Unit load at C_5				Unit load at C_6				Unit load at C_7			
	M	$M\Delta S/I$	$MX\Delta S/I$	$MY\Delta S/I$	M	$M\Delta S/I$	$MX\Delta S/I$	$MY\Delta S/I$	M	$M\Delta S/I$	$MX\Delta S/I$	$MY\Delta S/I$
1												
2									- 8 60	- .6409	- 16 0	10 31
3									- 25.18	-17664	- 73 3	24 25
4					- 8 96	- .5881	- 34 11	6 69	- 41.65	-2.7339	- 158 6	31 12
5					-25 31	-1 5406	-114 55	12 73	-58 00	-3.5309	- 262 5	29.17
6	- 8 81	- 4844	- 43 9	2 12	- 41 50	-2 2816	-206 6	9 97	-74 19	-4.0787	-369 3	17 83
7	- 24 79	-12074	-128 6	- 0 33	- 57 48	-2 7995	-298.2	- 0.77	-90 17	-4 3916	-467.8	- 1 21
8	- 41 17	-19189	-235 8	- 11 32	- 73 86	-3 4425	-423.1	-20.31	-106.55	-4 9662	-610 3	- 29 30
9	- 57 88	-2 3812	-332 4	- 29 89	-90.58	-3.7266	-520 3	-46 78	-123 27	-5 0715	-708.0	-63 66
10	- 74 25	-2 6905	-419 7	- 53 84	-106 94	-3.8751	-604 4	-77 54	-139.64	-5 0600	-789.3	-101 26
11	- 90 24	-2 8890	-496 8	- 81 64	-122 93	-3.9269	-675.3	-110.97	-155.62	-4 9711	-854 9	-140 47
12	-106 44	-3 2245	-606 8	-121 41	-139.14	-4 2151	-793.2	-158 72	-171.83	-5 2054	-979.5	-196 00
13	-122 79	-3 2413	-662.9	-156 33	-155 49	-4.1045	-839.5	-197 97	-188 18	-4.9674	-1016 0	-239 59
Σ		-18.0372	-2926.9	-452.65		-30.5004	-4509.2	-583.66		-47 3836	-6305.4	-658.82
M_o	13.5532				22.9181				35.6042			
H_o	1.0429				1.3447				1 5179			
V_o	0 2021				0.3113				0.4353			

$$M_o = \frac{-\sum_A^C M\Delta S/I}{\sum_A^B \Delta S/I}$$

$$H_o = \frac{-\sum_A^C MY\Delta S/I}{\sum_A^B Y^2\Delta S/I}$$

$$V_o = \frac{-\sum_A^C MX\Delta S/I}{\sum_A^B X^2\Delta S/I}$$

$M_C = M_o - Y_o H_o$

Moment $(M) = M_C + Y H_o + X V_o + m'$

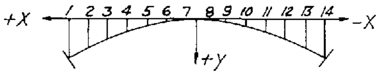
Thrust $(N)_{Lt} = H\cos\phi - V\sin\phi$

" $(N)_{Rt} = H\cos\phi + V\sin\phi$

m' = moment from unit load on cantilevered half-arch

$$f = N/A \pm MC/I$$

	Column #7		Column #5		Column #4		Column #1	
	A = 41 60, C = 34 8", I = 216 2		A = 43 35, C = 39 0", I = 283 7		A = 45 10, C = 43 2", I = 361 8		A = 52 10, C = 60 0", I = 787 3	
	f_u	f_l	f_u	f_l	f_u	f_l	f_u	f_l
C_2	- .0409	.0723	.1002	- 0716	.1778	- .1516	-.9832	1 1574
C_3	- 0998	.2134	.4312	- 3280	.7153	- 6205	-13582	1 5802
C_4	- 0610	2868	1 0467	- 8425	1 6183	-14309	-11829	14685
C_5	2077	.1369	2.0005	-16905	.6746	- .2912	-.6252	.9750
C_6	.8516	-.4080	.7766	- 3064	.0246	.4358	1100	.2882
C_7	1 9952	-14958	- 0771	.5921	-.3238	8224	8100	- 3936
C_8	6761	-.1631	- 5409	1 0463	-.4365	.9221	1 2917	- 8955
C_9	- 0918	5450	- 6847	11269	-.3986	.8214	1 4548	-11154
C_{10}	- 4047	7557	-.6126	9526	-.2918	.6158	1 3077	-1 0505
C_{11}	- 4064	.6358	- 4273	.6485	-.1729	.3831	9457	- .7803
C_{12}	-.2526	3676	-.2205	.3311	- 0767	1815	5107	- 4287
C_{13}	-.0788	0947	-.0615	.0767	- 0186	.0474	1484	- 1260



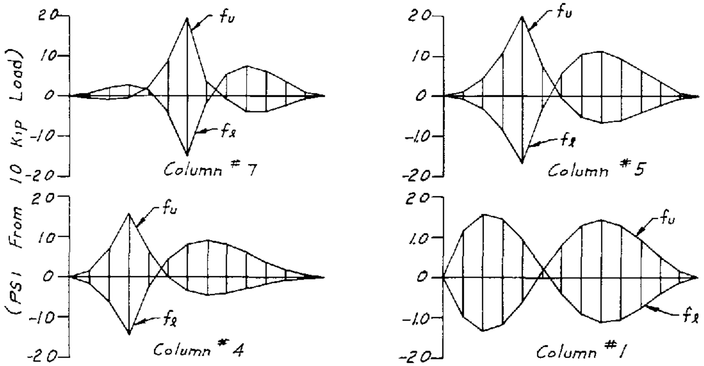
	Unit load At:					
	C_2	C_3	C_4	C_5	C_6	C_7
M_C	-.8482	-2 6352	-3 9900	-3 3209	1 1609	11.0446
M_7	- 6079	-16809	-18667	3797	6 7616	18 7386
M_8	-1 0167	-3 3290	-5.5945	-6 2289	-3 4179	4 5042
N_7	.0938	3401	6766	1 0324	1 3289	1 4961
N_8	0950	3447	6872	1 0512	1 3577	1 5365
M_5	1 0797	4.7714	11.8734	23 1981	6 8068	-4 2063
M_{10}	-.9636	-3 4669	-6 7611	-9 8371	-11 3860	-9 9758
N_5	.0890	.3218	6376	9678	1 4677	1 6075
N_{10}	0948	.3452	6904	1 0614	1 3803	1 5775
M_4	2 3832	9 6653	22 0642	6 9888	-2 9748	-8.2941
M_{11}	- 4773	-18682	-4 0235	-6 5675	-8 8281	-9 8313
N_4	.0853	.3080	.6087	1 2452	1 4953	1 6189
N_{11}	.0935	.3406	.6827	1 0521	1 3730	1 5770
M_1	-24 2693	-33 3138	-30 0599	-18 1409	-2 0204	+13 6451
M_{14}	3 1106	10 6508	19.5670	26.7359	29 1387	24 7964
N_1	.6533	.8328	1 0714	1 3125	1 4938	1 5620
N_{14}	.0840	.3077	6207	9646	1 2735	1 4865

$y^7 = y^8 = 0.38'$
$x^7 = -x^8 = 16\ 35'$
$\phi^7 = \phi^8 = 2.655°$

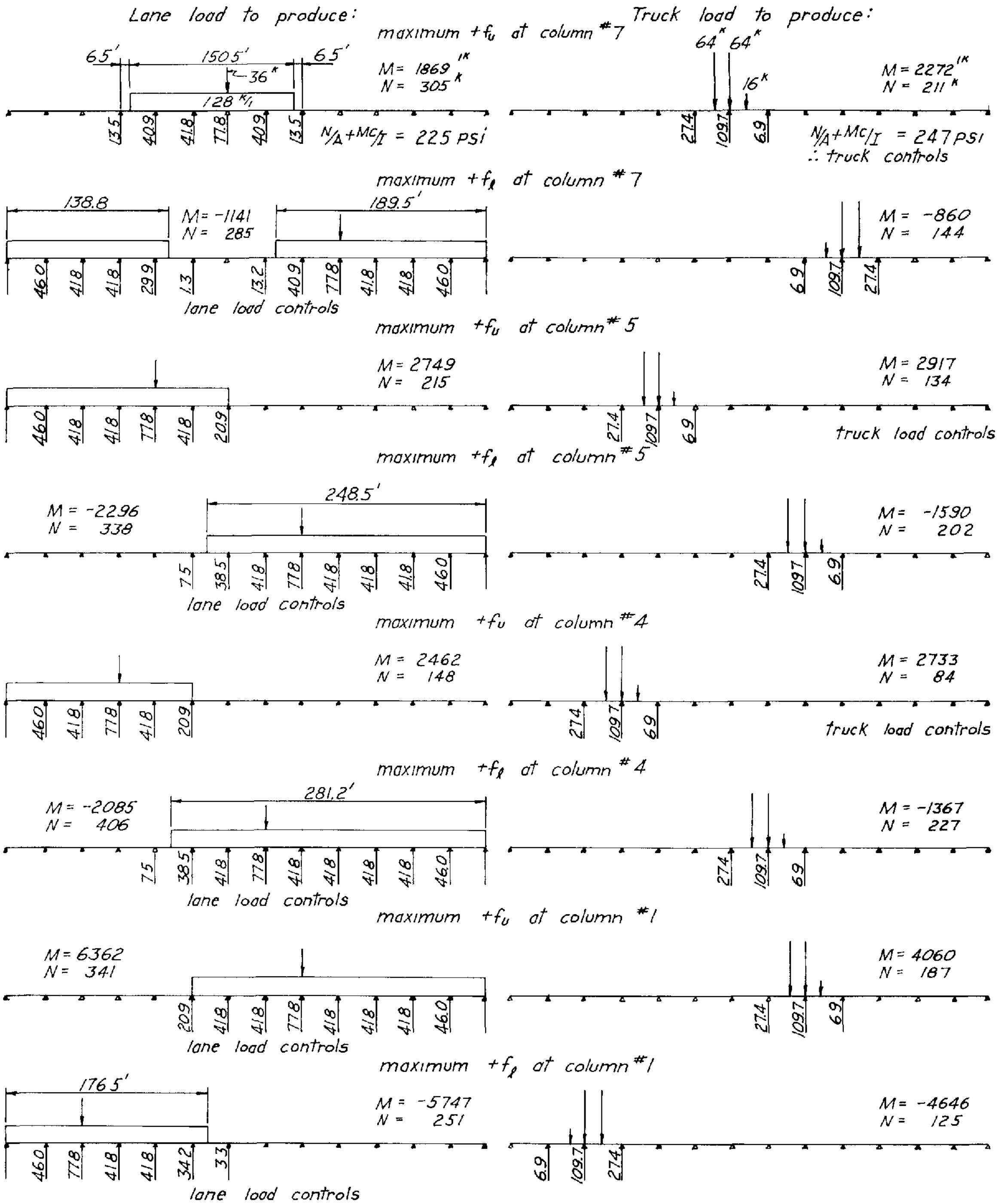
$y^5 = y^{10} = 9.59'$
$x^5 = -x^{10} = 81.73'$
$\phi^5 = \phi^{10} = 13.388°$

$y^4 = y^{11} = 19\ 06'$
$x^4 = -x^{11} = 114.42'$
$\phi^4 = \phi^{11} = 18.914°$

$y^1 = y^{14} = 70.00'$
$x^1 = -x^{14} = 212\ 50'$
$\phi^1 = \phi^{14} = 35\ 726°$



$+f_u$ = compressive stress in extreme upper bending fiber of arch rib
$+f_l$ = " " " " lower " " " " "



Dead Load Moment & Thrust

$M_D = M_o + H_o y +$ cantilever moment
$N_D = H_o \cos\phi + V \sin\phi$

Column	#7	#5	#4	#1
M_D ('K)	575	552	652	874
N_D (K)	4634	4784	4932	5743

Live Load Moment & Thrust

Moment Magnification Factor (δ)
(based on $E, I, \&\ N$ at col # 4)

$I_{gross} = 20 \times 7.0^3/12 - 17.5 \times 5.67^3/12 = 306\ ft^4$

$I_{reinf.} = 2 \times 31 \times 38^2 + 15.9 \times 68^2/12 = 95{,}700\ in^4 \sim 4.62\ ft^4$

A.A.S.H.T.O. 6-17 $\quad \beta_d \approx 0$

$EI = [3600 \times 306/5 + 29000 \times 4.62]\, 144 = 510 \times 10^6\ K\text{-}ft^2$

A.A.S.H.T.O. 6-16

$$P_c = \frac{3.14^2 \times 510 \times 10^6}{(0.7 \times 106 \times 212.5)^2} = 20250^K$$

$$P_u = 13\,[4932 + 5/3\,(84)] = 6594^K$$

$$\delta = \frac{1}{1 - \dfrac{6594}{0.85(20{,}250)}} = 1.62$$

Impact (I)

$$I = \frac{50}{L + 125}$$

L = length of lane loading

Col. #	%I for +fu	%I for +fl
7	18.1	11.0
5	15.6	13.4
4	17.3	12.3
1	12.9	16.6

$M_{L+I} = \delta I M$
$N_{L+I} = I N$

Col #		+fu	+fl
7	M_{L+I} =	4347	-2052
	N_{L+I} =	249	316
5	M_{L+I} =	5463	-4218
	N_{L+I} =	155	384
4	M_{L+I} =	5193	-3793
	N_{L+I} =	98	456
1	M_{L+I} =	11636	-10856
	N_{L+I} =	385	292

Rib Shortening Moment & Thrust

$M_R = M_R(DL) + I\, M_R(LL)$
$N_R = N_R(D.L.) + I\, N_R(LL)$

Col #		R_{LL} +fu	R_{LL} +fl
7	M_R(LL)	74	102
	N_R "	-5	-7
5	M_R "	21	48
	N_R "	-3	-7
4	M_R "	-6	-25
	N_R "	-2	-8
1	M_R "	-422	-190
	N_R "	-6	-3

Increase dead load rib shortening stress 38% to account for creep.

Col #		1.38 R_{DL} +fu	1.38 R_{DL} +fl	$R_{Total} = 1.38 R_{DL} + I \cdot R_{LL}$ +fu	$R_{Total} = 1.38 R_{DL} + I \cdot R_{LL}$ +fl
7	1.38 M_R(D.L.)	2080	2080	2167	2193
	" N_R "	-132	-132	-138	-138
5	" M_R "	868	868	892	946
	" N_R "	-128	-128	-131	-136
4	" M_R "	-373	-373	-380	-408
	" N_R "	-124	-124	-126	-133
1	" M_R "	-7085	-7085	-7561	-7306
	" N_R "	-107	-107	-114	-110

Temperature Change Moment & Thrust

$$H_o = \frac{\alpha t L}{\sum_A^B \dfrac{y^2 \Delta S}{E I} + \sum_A^B \dfrac{\cos^2\phi\, \Delta S}{A E}} \qquad \alpha = 6 \times 10^{-6}$$

$= -134.3^K$ for 45 °F drop
$= 104.5^K$ for 35 °F rise

$M_T = H_o y$
$N_T = H_o \cos\phi$

Col #	-45° M_T	-45° N_T	+35° M_T	+35° N_T
7	2122	-134	-1650	104
5	885	-131	-688	102
4	-387	-127	301	99
1	-7228	-109	5622	85

Shrinkage Moment & Thrust

$$H_o = \frac{-cL}{\sum_A^B \dfrac{y^2 \Delta S}{E I} + \sum_A^B \dfrac{\cos^2\phi\, \Delta S}{A E}} \qquad C = 0.00012$$

$= -59.8^K$

$M_S = H_o y$
$N_S = H_o \cos\phi$

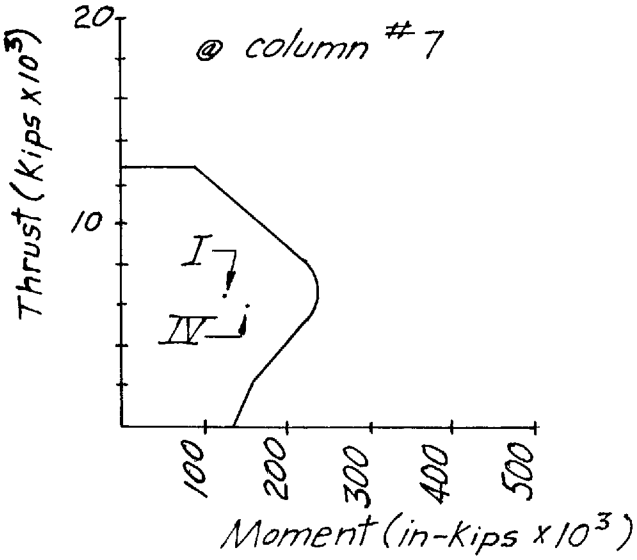
Col. #	M_S	N_S
7	945	-60
5	394	-58
4	-172	-57
1	-3218	-49

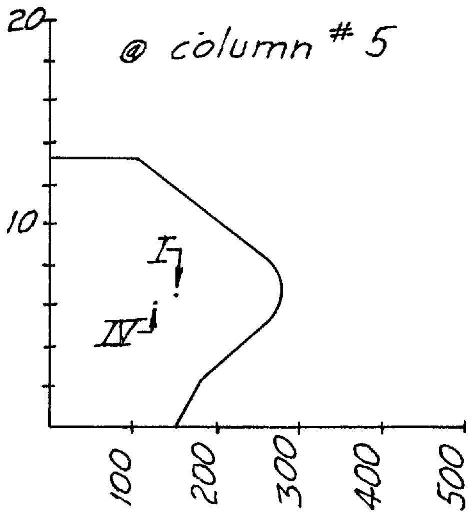
Design Moment & Thrust

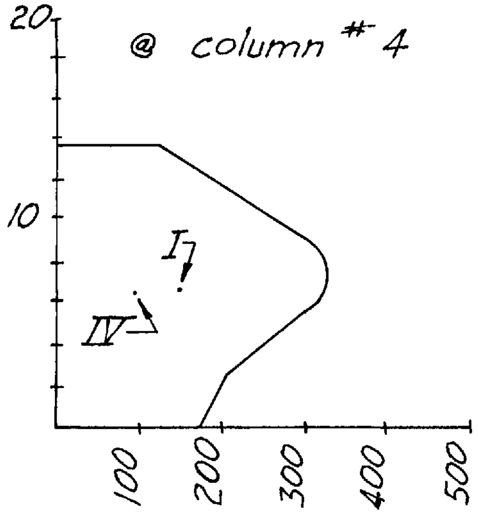
Group I $= 1.3\,[D + 5/3\,(L+I)]$
Group IV $= 13\,[D + (L+I) + R + S + T]$

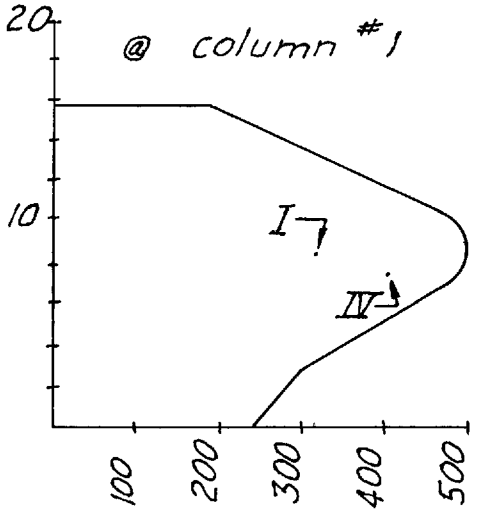
Col #	group	fu-ΔT M	fu-ΔT N	fu+ΔT M	fu+ΔT N	fl-ΔT M	fl-ΔT N	fl+ΔT M	fl+ΔT N
7	I	10168	6564	10168	6564	-3699	6709	-3699	6709
	IV	13203	5916	8299	6225	4918	6001	14	6310
5	I	12554	6555	12554	6555	-8422	7051	-8422	7051
	IV	10642	6005	8597	6308	-1873	6296	-3918	6599
4	I	12099	6624	12099	6624	-7371	7400	-7371	7400
	IV	6378	6136	7272	6430	-5340	6592	-4446	6886
1	I	26348	8300	26348	8300	-22386	8099	-22386	8099
	IV	-7146	7613	9559	7865	-36054	7497	-19349	7749

Interaction Diagrams









2.12.1 Revision of Cross-Section

The cross-section which has been analyzed is understressed. There are three possible ways of reducing the section, a reduction in overall width, a reduction in overall depth, and a reduction in thickness of the members of the box. Each of these will reduce the moment of inertia, resulting in an increase in the moment magnification factor with a resulting increase in live load moment. This will be partially offset by a decrease in dead load thrust due to reduced weight of the arch rib. A reduction in depth would have the biggest effect on the reduction in moment of inertia and the least effect on decrease of dead load. Therefore, depth reduction is ruled out, and the change will be confined to thinning the slab and reduction of overall width to give a slab b/t of 12 so as to prevent possible buckling control. The walls could be reduced in thickness, but this will not be done because of difficulty of concrete placement between vertical forms.

The ratio of force to section strength is about 0.85 at the springing. A 7 inch slab with a 17 foot overall width would give a b/t ratio for the slab of $87 \div 7 = 12.4$, and roughly a reduction of section moment of inertia of about 20 percent. The live load moment will increase but the thrust will decrease, resulting in only a slight change in force on the cross-section. The spacing of reinforcing bars in the slabs will be increased from 7 to 8 inches to keep the same percentage of roughly 1.5 percent in the slabs.

Since the variation of moment of inertia and area along the arch axis will remain practically the same, a new analysis is not necessary. The thrusts and moments can be modified by simple calculations, and then compared with a new interaction diagram.

The span-to-width ratio will be $425 \div 17 = 25$. Wind stresses, live load lateral eccentricity and lateral buckling and lateral moment magnification will be investigated.

$A_{q.p.} = 204 \times 84 - 174 \times 70 = 4950 \text{ in}^2$
$I_{q.p.} = (204 \times 84^3 - 174 \times 70^3) \div 12 = 5.10 \times 10^6 \text{ in}^4$

Previous $A_{q.p.} = 240 \times 84 - 210 \times 68 = 5880 \text{ in}^2$
Previous $I_{q.p.} = (240 \times 84^3 - 210 \times 68^3) \div 12 = 6.35 \times 10^6$

Obtain the previous thrust at the quarter point for Group I loading by interpolation from the design moment and thrust tabulation.

Previous $N_{q.p.} = 6607^k$

$$\text{Decrease in } N_{q.p.} = \frac{1.06(5880-4950)(0.150)(425)^2}{144\ (8 \times 70)}$$
$$= 331^k$$
$$N_{q.p.} = 6607 - 1.3 \times 331 = 6177^k$$

$$\text{Ratio change in rib shortening moment} = \frac{6607 - 331 \times 1.3}{6607}$$
$$= 0.935$$

$$\text{Ratio change in shrinkage \& temp. moment} = \frac{4950}{5880}$$
$$= 0.842$$

$$I \text{ of reinforcement} = 2 \times 22.9(38.5)^2 + 15.9(70)^2 \div 12$$
$$= 74{,}400 \text{ in}^4$$

$$EI = (3600 \times 5.10 \times 10^6 \div 5) + (29000 \times 74400)$$
$$= 5830 \times 10^6$$

$$P_c = \frac{\pi^2 \times 5830 \times 10^6}{(0.7 \times 1.06 \times 212.5)^2\, 144} = 16{,}072$$

$$\delta = \frac{1}{1 - \frac{6177}{0.85 \times 16{,}072}} = 1.83$$

Revised section at springing:

Group IV loading

new thrust $= 7497 - 1.3 \times \frac{1.2}{1.06} \times 331$

$= 7497 - 487$

$= 7010^{k}$

new moment $= 1.3[874 - \frac{1.83}{1.62} \times 10{,}856 - 0.935 \times 7306$

$- 0.842 \times 10{,}446$

$= 35{,}120'^{k}$

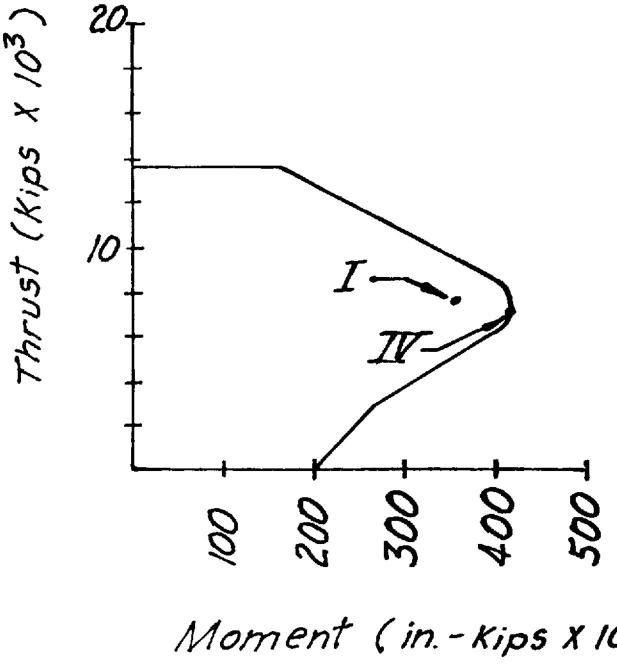
Group I loading

new thrust $= 8300 - 487$

$= 7813^{k}$

new moment $= 1.3[874 + 5_3(\frac{1.83}{1.62} \times 11636)]$

$= 29{,}620'^{k}$



Revised Interaction Diagram at Springing

2.12.2 Vertical Crown Deflection
(dead load, creep, & shrinkage)

Dead load rib shortening deflection

transformed area at crown (A_c)

$$A_c = 204 \times 69 - 174 \times 55 + 7 \times 61.6 = 4940 \text{ in}^2$$
$$H_{D.L.} = 4623 - 331 = 4292^K$$
$$f_{c\,D.L.} = 4292/4950 = 0.867 \text{ ksi}$$
$$\Delta_c = \frac{0.867}{3600}\left(70 + \frac{425^2}{4 \times 70}\right)$$
$$= \frac{0.867}{3600}(715)$$
$$= 0.172' = 2.06'' \text{ initially}$$

Creep

$$\Delta_c = 2.4 \times 2.06'' = 4.94''$$

Shrinkage

$$\Delta_c = 0.0003\ (715) = 0.215' = 2.58''$$

Total deflection for camber at crown ($\Sigma\Delta_c$)

$$\Sigma\Delta_c = 2.06 + 4.94 + 2.58 = 9.58''$$

Deflection from change in temperature (45°F drop)

Crown deflection $= 6 \times 10^{-6} \times 45\ (715) = 0.193' = 2.32''$

Live load + Impact crown deflection

Moment magnification at service load

$$I_{g.p.} = 5.10 \times 10^6 + 7 \times 74{,}400 = 5.62 \times 10^6 \text{ in}^4$$
$$N_{g.p.} = 6177 \div 1.3 = 4752^k$$
$$P_c = \frac{\pi^2 \times 3600 \times 5.62 \times 10^6}{(0.7 \times 1.06 \times 212.5)^2\ 144} = 55{,}800^k$$
$$\delta = \frac{1}{1 - \frac{4752}{55{,}800}} = 1.09$$
$$\Delta_c = \frac{M\ell^2}{76EI} = \frac{4347 \times 1.09 \times 425^2 \times 1728}{76 \times 3600 \times (5.75/7.0)^2 \times 5.62 \times 10^6} = 1.43'' \ (\ell/3600)$$

2.12.3 Wind Analysis

use wind load $= 7 \times 75 = 525$ #/ft. of arch axis

lateral $I_{q.p.} = (7 \times 17^3 - 5.83 \times 15.33^3) \div 12 = 1116 \text{ ft}^4$

$$K_{q.p.} = \frac{2 \times 16.17^2 \times 6.42^2}{\frac{16.17}{0.583} + \frac{6.42}{0.833}} = 608 \text{ ft}^4$$

$$E/G = 2.3$$

$$\frac{IE}{KG} = \frac{1116 \times 2.3}{608} = 4.22$$

transverse moment at crown (M_o), using equation 21:

$$M_o = (WR^2)\,\frac{\sin\beta - \frac{\beta}{2} - \frac{\sin 2\beta}{4} + \frac{IE}{KG}\left[\sin\beta - \beta\cos\beta - \frac{\beta - \sin\beta\cos\beta}{2}\right]}{\frac{\beta}{2} + \frac{\sin 2\beta}{4} + \frac{IE}{KG}\left[\frac{\beta - \sin\beta\cos\beta}{2}\right]}$$

$\beta = 0.63643$ radians $\qquad R = 357.545'$

$$M_o = (67100)\,\frac{.59432 - .31822 - .23899 + 4.22[.00326]}{.31822 + .23899 + 4.22[.07923]} = 3828'^{K}$$

torsion at springing (T_S)

$$T_S = M_o \sin\beta - WR^2(\beta - \sin\beta)$$
$$= 3828 \times .59432 - 67100 \times .04211$$
$$= -551'^{K}$$

transformed lateral I at crown (I_c)

$$I_c = \frac{1.167 \times 17^3 \times (1 + 7 \times .0160)}{12} + 1.667 \times 4.58 \times 8.08^2 \times (1 + 7 \times .0096)$$
$$= 1063 \text{ ft.}^4$$

lateral bending stress at crown (f_c)

$$f_c = \frac{3828 \times 8.5 \times 1000}{1063 \times 144} = 213 \text{ psi}$$

lateral shear stress at springing (f_s)

lateral reaction $= 0.525 \times 1.06 \times 212.5 = 118^{K}$

bending shear $= 118{,}000 / 204 \times 14 = 41$ psi

torsional shear $= \dfrac{551 \times 12000}{2 \times 113 \times 194 \times 7} = \underline{22 \text{ psi}}$

63 psi

lateral deflection from wind (δ_c), from equation 23a:

$$\delta_c = \frac{L^2}{2EI}\left[\frac{WL^2}{4} - M_o\right]$$

$$= \frac{225^2}{2 \times 3600 \times 1063 \times 144}\left[\frac{.525 \times 225^2}{4} - 3828\right]$$

$$= 0.129'$$

$$= 1.55''$$

This compares with a lateral deflection of 2.41" for the steel arch with two planes of lateral bracing, 3.38" for the steel arch with one plane of lateral bracing, and 2.23" for the deck.

The concrete arch is less flexible laterally than the roadway floor. This results in a transfer of wind load from the roadway slab to the arch. The opposite is true for the steel arch.

2.12.4 Effect of live load lateral eccentricity

Assume 2 - 12 foot lanes at one edge of the 30 foot roadway with vehicles shifted 1 foot in the lanes. Resulting eccentricity (e) is 4 feet.

from equation 45:

$$M_o = \frac{2 \times 357.5 \times 1.28 \times 4 \times .1953\,[4.22 - 1]}{3\,[1.1144 + .15846 \times 4.22]} = 430^{\prime K}$$

$$T_s = -430 \times .59432 + 357.5 \times 1.28 \times 4 \times .47797 + 2 \times 26 \times 4$$

$$= 827^{\prime K}$$

Bending stress at crown (f_c)
$f_c = 430 \times 8.5 \times 1000 / 1063 \times 144 = 24$ psi

lateral shear stress at springing
shear stress from eccentric LL $= \dfrac{827 \times 12000}{2 \times 113 \times 194 \times 7} = 32$ psi

shear stress from wind $= \underline{63 \text{ psi}}$
Total shear stress in transverse slabs $= 95$ psi

Due to the large axial compressive stress, diagonal tension is not involved with this shear stress. The minimum requirement for transverse reinforcement in the slabs must still be met.

2.12.5 Lateral buckling and moment magnification

At service load:
lateral $r = \sqrt{I/A} = \sqrt{1116/34.4} = 5.7'$
lateral $KL/r = 1.13 \times 1.06 \times 212.5 / 5.7 = 44.7$
This compares with a vertical $KL/r = 0.7 \times 1.06 \times 212.5 / \sqrt{246/34.4}$
$= 59.0$
lateral $P_c = \dfrac{\pi^2 \times 3600 \times 114 \times 1116}{(1.13 \times 1.06 \times 212.5)^2} = 88{,}200^K$

lateral moment magnifier $= \dfrac{1}{1 - \dfrac{4752}{88200}} = 1.06$

vertical moment magnifier $= 1.09$ (for comparison)

At ultimate load:
$EI_{g.p.} = (1116 \times 3600 \div 5 + .0124 \times 1116 \times 29000)144 = 173.5 \times 10^6$

lateral $P_c = \dfrac{\pi^2 \times 173.5 \times 10^6}{(1.13 \times 1.06 \times 212.5)^2} = 26{,}400^K$

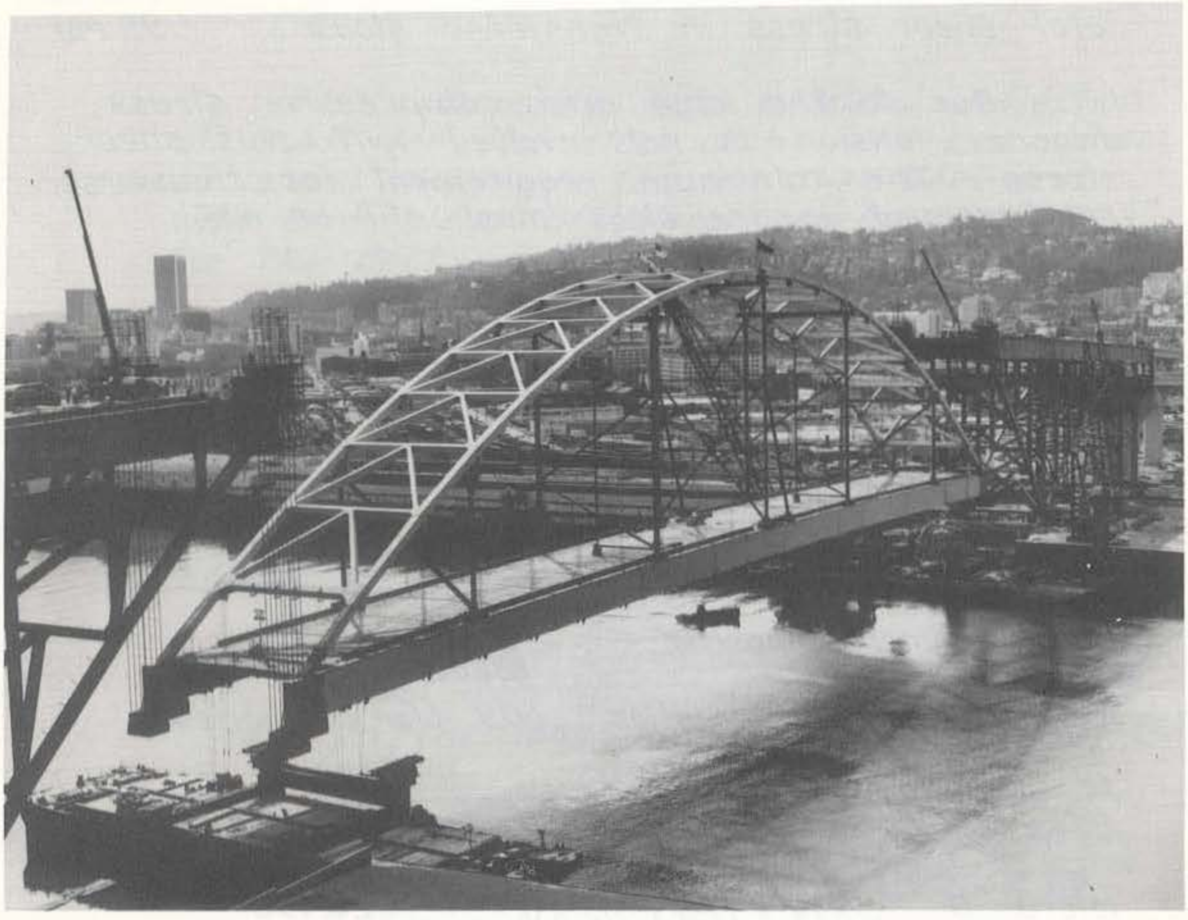
lateral moment magnifier $= \dfrac{1}{1 - \dfrac{1.3 \times 4752}{0.85 \times 26{,}400}} = 1.38$

vertical moment magnifier $= 1.83$ (for comparison)

Fremont Bridge
Over The
Willamette River
Portland, Oregon

Center Span - 1255 Feet

Built 1969-1971



Designed By
Parsons, Brinckerhoff, Quade and Douglas

Photograph - Courtesy of
Oregon Department of Transportation

CHAPTER III - ARCH CONSTRUCTION

3.1 Steel Arches

It is important for the designer to consider the probable method of construction of the arch. This may have an effect on the choice of type and on the design. The principal methods of construction are: full falsework support; falsework bents with increasing cantilevers to each succeeding bent; cantilevering from each abutment by the use of tiebacks; and off-site construction for tied arches, followed by flotation to the site and vertical lifting of the full tied arch by means of cables.

As pointed out in earlier chapters, erection procedure may be used to eliminate stress from rib shortening, shrinkage and creep.

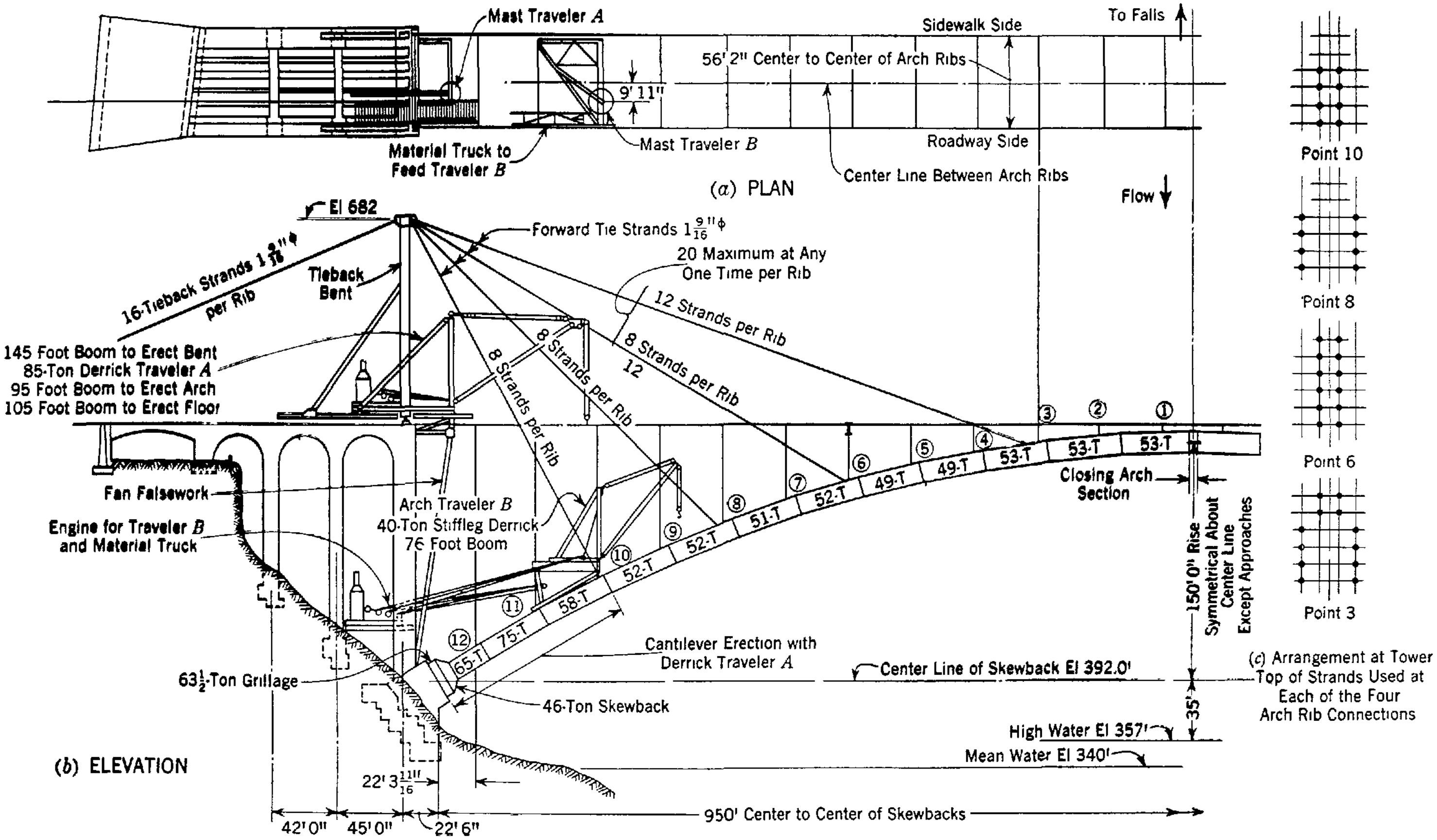
As is the case for all long span bridges, the designer should check and follow the erection procedure. This is necessary to insure that the actual structure is equivalent to the one assumed in design; and, in the case of arches, to see that the necessary closure and jacking procedure is followed.

3.1.1 Cantilevering from the Abutments by Tie-Backs

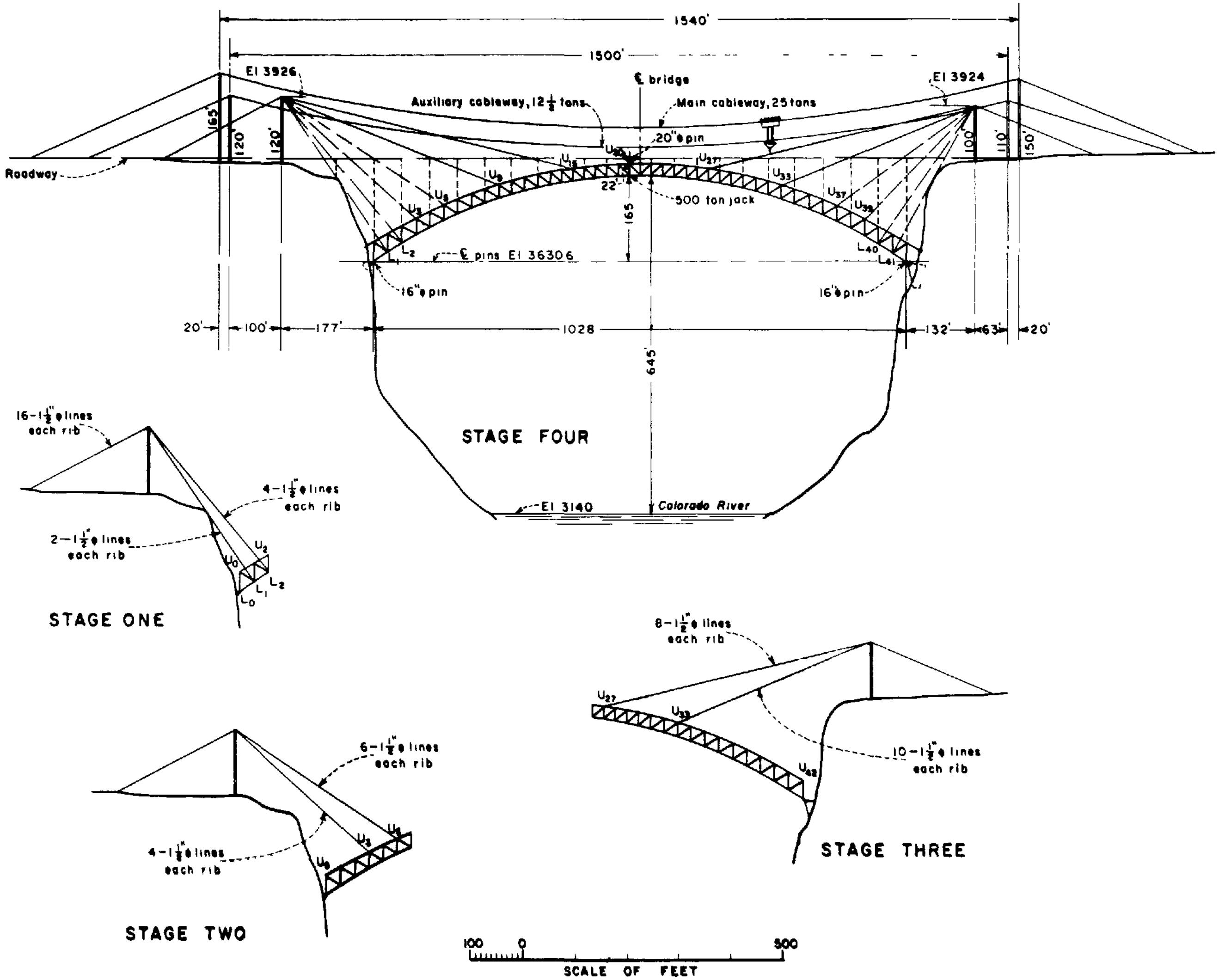
The Eads Bridge over the Mississippi at St. Louis started in 1868, was the first to make use of this method of erection. Temporary wooden towers were built on the piers to support steel tie bars. Since this is a multiple span bridge, balanced cantilever erection, avoiding the use of anchorages, could be used for all of the bridge except one-half of each end span.

No adequate method was provided for adjustment at closure. As a result, difficulty was encountered in the insertion of the closing sections. The arch chords are steel tubes and an improvised closing section, made adjustable by screw threads at the ends, was used.

Since then many long span steel arches have been erected by the cantilever tieback method. Some of the notable ones are the Hell Gate Bridge over the East River in New York City, built in 1916; the Rainbow Bridge at Niagara Falls, Figure 11, completed in 1941; the Glen Canyon Bridge, Figure 12, over the Colorado in 1958; the Lewiston-Queenston Bridge over the Niagara River; the Cold Spring Canyon Bridge in California; the Snake River Bridge in Idaho; and the New River Gorge Bridge in West Virginia, see cover.



FROM: "Rainbow Arch Bridge"
Erection of Steel Superstructure, by E. L. Durkee
ASCE Proceedings, October 1943, page 1240
Fig. 11



GLEN CANYON BRIDGE - ERECTION
From "Technical Record of Design and Construction," Nov. 1959
U.S. Dept. of the Interior - Bureau of Reclamation - Fig. 12

For the tiebacks of the Rainbow Bridge, prestrained wire bridge strands were used. These were provided with adjustable links adjacent to their connections to the arch rib. It was necessary to adjust these strands to keep the arch profile close to the theoretical. Jacks were provided on both flanges at the crown to permit insertion of the closing key section of the arch rib, 11 inches long and wedge shaped. This key section was actually fabricated 13 inches long before milling. If adjustment were found necessary to get the design crown moment, it would have been possible to mill the section to a length lesser or greater than the theoretical 11 inches and to change the angle of wedge.

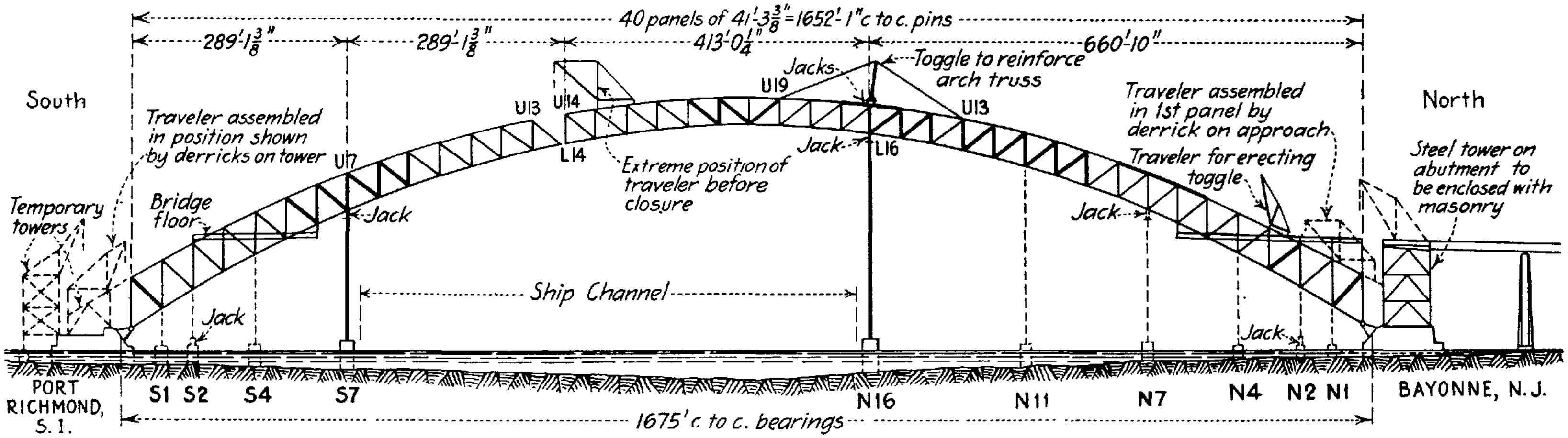
After the arch was self-supporting, the jacking forces were measured. Making allowance for the dead load and erection load on the arch at that time, it was decided to use the theoretical size of the key section. The moment and thrust did not check out exactly, but the difference was considered to be within the limits of possible accuracy of steel weight calculations.

The Glen Canyon Arch, see Figure 12, a truss type rib, was closed on a pin in the upper chord. Jacks inside the lower chord were used to give a calculated force of 525 kips at that stage. This force had been calculated on the basis of the loads, including erection equipment, on the arch at that time and on the temperature at that time. The chord was then shimmed in that position, and the holes match-marked in the blind connection. The holes were drilled and the jacks were then used to remove the shims and allow the placement of drift pins and bolts. The connection at this point, of course, did not provide any bearing and the rivets were designed to take the full maximum stress. Unlike the Rainbow Arch where an erection traveler on the top flanges was used for erection, a cableway was used at Glen Canyon to deliver the steel members to their final position.

The horizontal part of the tie backs for the New River Gorge Bridge consisted of hollow oil well drill stems. Two of these failed during construction. Fortunately, the remaining intact ties were strong enough to take the additional load and impact.

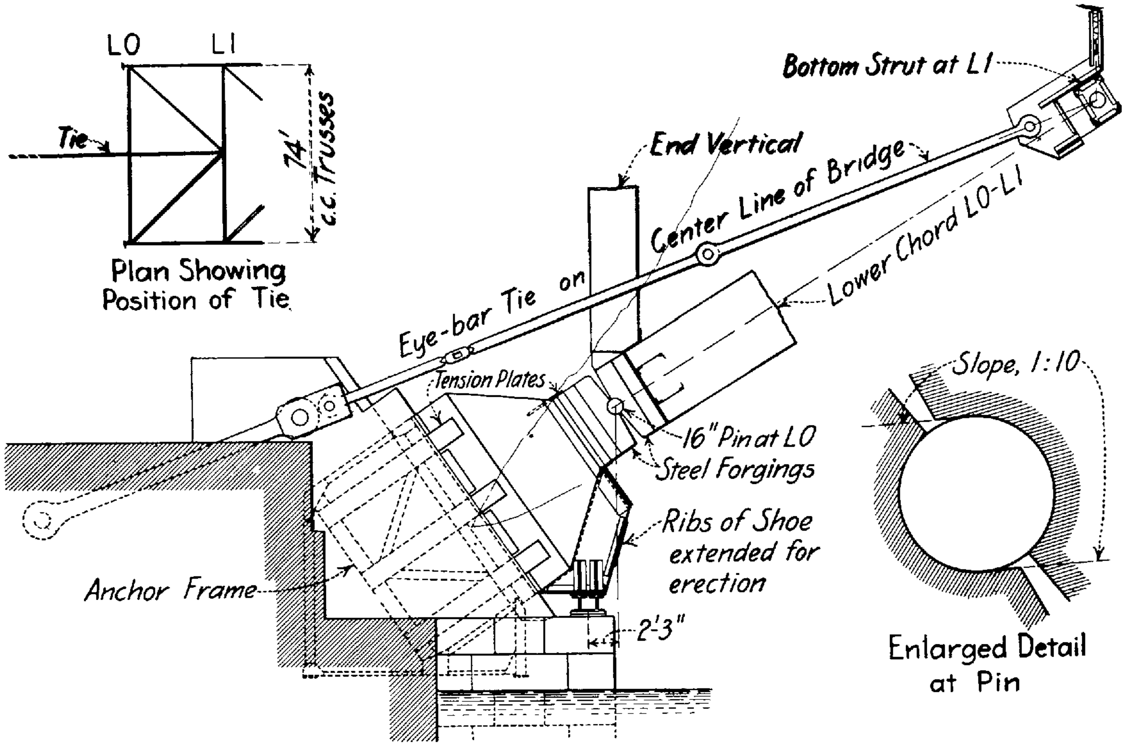
3.1.2 Cantilevering over Falsework Bents

The Bayonne Bridge with a span of 1652 feet over the Kill Van Kull was completed in 1931. As shown in Figure 13 this arch was built by cantilevering over falsework bents. Rock foundation for the bents was at a shallow depth and plate girders for the approaches were used in the bents. The arch was designed to act as 3-hinged for its own weight on the basis of closure on a pin in the lower chord at mid-span. The arch was to then be converted to 2-hinged for the remainder of the dead load and live load by connecting the top chord over the pin in a stressless



BAYONNE BRIDGE OVER THE KILL VAN KULL - ERECTION
Courtesy American Bridge Division of United States Steel Corporation

Fig. 13



South Shoe During Erection.

BAYONNE BRIDGE - SHOE ERECTION
Courtesy American Bridge Division of United States Steel Corporation

Fig. 14

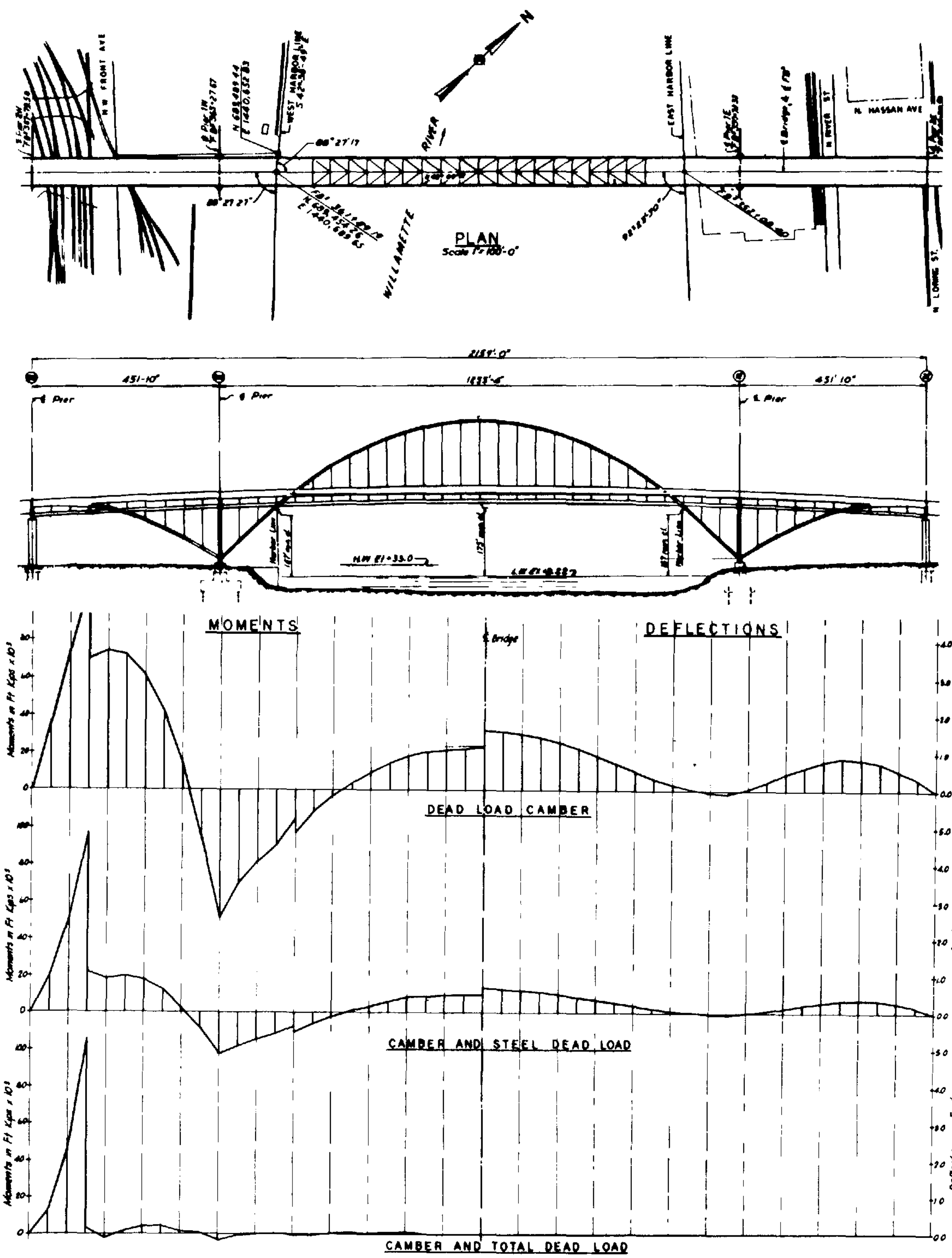
condition. However, the off-center location of the ship channel would have required a very long cantilever on one side. It was, therefore, decided to close the arch at a point about 250 feet from the center of the span with a cantilever overhang, from the temporary bent, of 413 feet. This required a temporary toggle arrangement over the bent to reinforce the arch for the large cantilever moment,

Since the arch was closed on a lower chord pin 250 feet from the center of the span, calculated jacking at closure was required. The stress in the closing top chord member at this point was calculated on the basis of a pin at the center of the span and the dead load at time of closure. The vertical force, acting at the closest bent, required to produce that stress in the 2-hinge was calculated. This jacking force was applied at the bent to the 3-hinged arch. The closing top chord member was then drilled at the blank connection to fit the opening under this load. The connection was riveted and the jacks released. This produced the required compression in the closing member and zero stress in the top chord member at the crown as the arch was designed.

Figure 14 shows temporary supports of the arch shoe required during erection. Since the reaction on the pin was vertical until closure, temporary connections to the shoe were provided to prevent overturning. These consisted of a temporary extension of the shoe in front to provide bearing at the edge of the abutment and tension connections at the back of the shoe to the anchor frame in the concrete. Another temporary connection was required because of wind. Due to the extremely long cantilevered arch rib, a wind blowing at the right angles to the span would tend to move the windward rib away from the pin, due to negative wind moment. It was decided to relieve this condition by allowing the pin to act as a roller, as shown in the enlarged detail at the pin, Figure 14. This, however, introduced another problem. There was a possibility of an off-shore wind rolling the pin up the 10 percent slopes and off the shoes. To offset this possibility, an eyebar tie on the center line of bridge was anchored to the abutment as shown in Figure 14. This eyebar chain was strong enough to take the off-shore wind, but flexible enough to allow the transverse wind negative moment rotation without overstress of the eyebar chain.

3.1.3 Off-Site Construction

Since tied arches form a self-contained structural system, requiring only vertical support, they can be assembled at a convenient shore point, and floated to the site for bridges over navigable water. They are then raised to their final position. The center section of the main span of the Fremont Bridge in Portland, Oregon, is, in effect, a tied arch with a span of 902 feet. This section, weighing 6,000 tons was raised by means of jacks and rods mounted on the cantilevered sections of the span, Figure 15. This is quite similar to the erection of the suspended span of cantilever bridges. Two 535 ft.

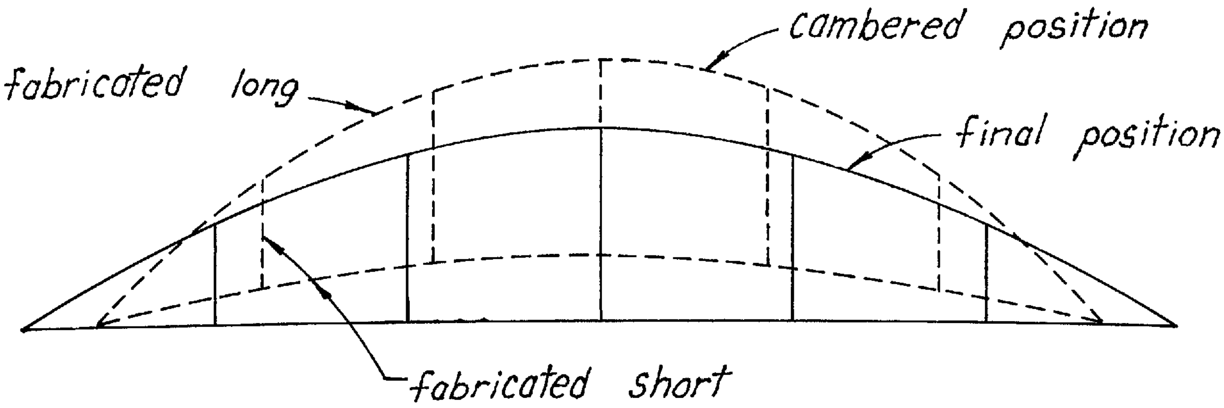


FREMONT BRIDGE - ELIMINATION OF DEAD LOAD MOMENTS BY CAMBER
From ASCE April 1970 Portland, Oregon Meeting Preprint 1210
by A. Hedefine and L. G. Silano

Fig. 15

tied arch spans over the Tennessee River near Paducah, Kentucky, were floated to the site on barges and raised immediately adjacent to the line of the bridge on falsework towers by means of jacks and rods. The spans of the parallel twin bridges were then skidded sideways onto the permanent concrete piers. Falsework in the span opening would probably have been less expensive, but could not be used because of navigation requirements.

3.1.4 Camber for Tied Arches



Camber of the rib and tie sections is for length only. Each section is fabricated to the radius of curvature that the rib or tie is intended to have under final dead load. Both rib and tie will be forced to assume sharper curvature while supported on falsework, in order to connect the stressless, cambered members. This forcing induces flexural stresses in the rib and tie equal and opposite to rib shortening stresses. The arch is erected by use of blocking under the tie and struts between the tie and rib. After the members are connected and the blocking and struts are removed, so that only end support is provided, the temporary, forced flexural stresses are counteracted by rib shortening so that zero rib shortening stress, under full dead, results. The vertical and horizontal position of the points of falsework support can be precalculated, and this will give a check on the accuracy of the fabrication and erection. In the case of the Fremont Bridge, all the design moments for the tie girder included an allowance of ± 5,000 ft. kips, to allow for possible inaccuracies in fabrication and erection.

3.2 Concrete Arches

Dating back to the early stone arches and continuing to the present time, the most used method of building masonry or concrete arches has been on timber falsework. The Cowlitz River Bridge (14) in the State of Washington, Figure 16, is an example of the use of full falsework. This arch has a span of 520 feet and the crown is 220 feet above the ground. In order to avoid flexural stresses from uneven support during removal of falsework, some means of uniform lowering of the centering over the full span should be used. This may be done by the use of jacks, sand boxes, or even wedges in the case of short spans.

In setting the elevation of the soffit forms, the deformation of the timber supports, including the local deformations due to post bearing normal to the grain of the wood, must be taken into account. The actual movement during concrete placement should be monitored and adjustments made by the jacks if needed.

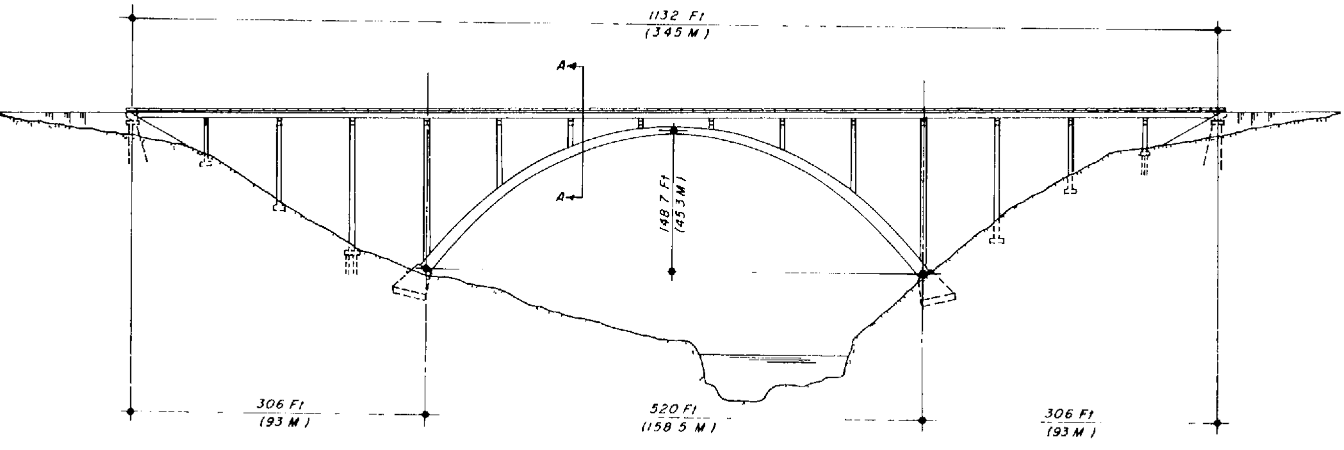
Where openings for traffic, stream flow or navigation are required, steel falsework or a combination of steel and timber falsework may be used.

The Sando Arch overthe Angerman River in Sweden with a span of 866 feet, completed in 1943, was built on multiple timber falsework in the river. The first attempt at building this arch resulted in collapse in 1939. This support was a lattice type timber arch of the same span as the final arch. Collapse occurred after only a small amount of concrete had been placed. The rib was jacked at the crown with a force of 6,700 tons.

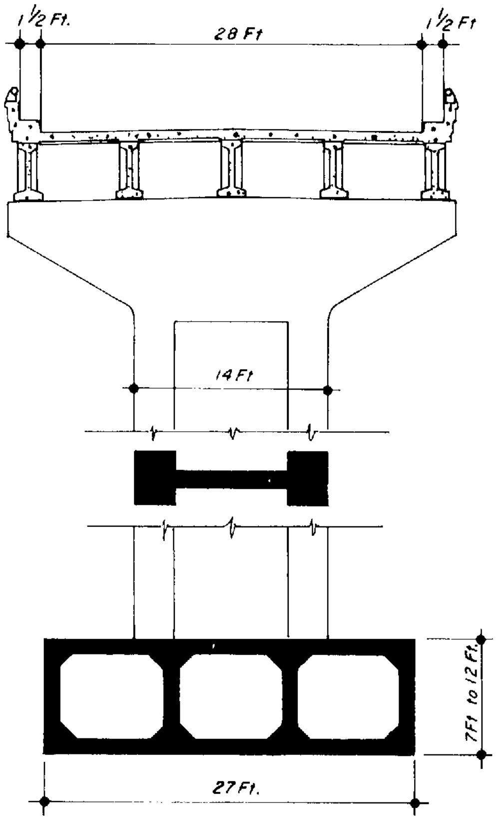
Truss type steel arch centering of the same span as the final concrete arch has been frequently used. A step from this was the use of a structural steel arch as reinforcing inside the concrete. The forms were suspended from the reinforcing. An example of this type of construction is an arch bridge over the Connecticut River at Springfield, built in the early twenties.

3.2.1 Freyssinet and Mènager Hinge Construction

Stresses from rib shortening, creep and shrinkage in concrete arches are much more important than the rib shortening stresses in steel arches. They can be practically eliminated by the method of construction.



Elevation



Section A–A

COWLITZ RIVER BRIDGE - WASHINGTON STATE
Reproduced from IABSE Bulletin 25 - 1969, pg. 54

Fig. 16

One of these methods discussed in 2.5.3 is that developed by Freyssinet before 1930. This method involves jacking at the crown, or the quarter point. The effect of jacking also raises the arch rib off the centering. This eliminates the need for a number of vertical jacks for lowering the centering. Spans as great as 1,000 feet have been built by the Freyssinet Method. The necessary camber is produced by the jacking.

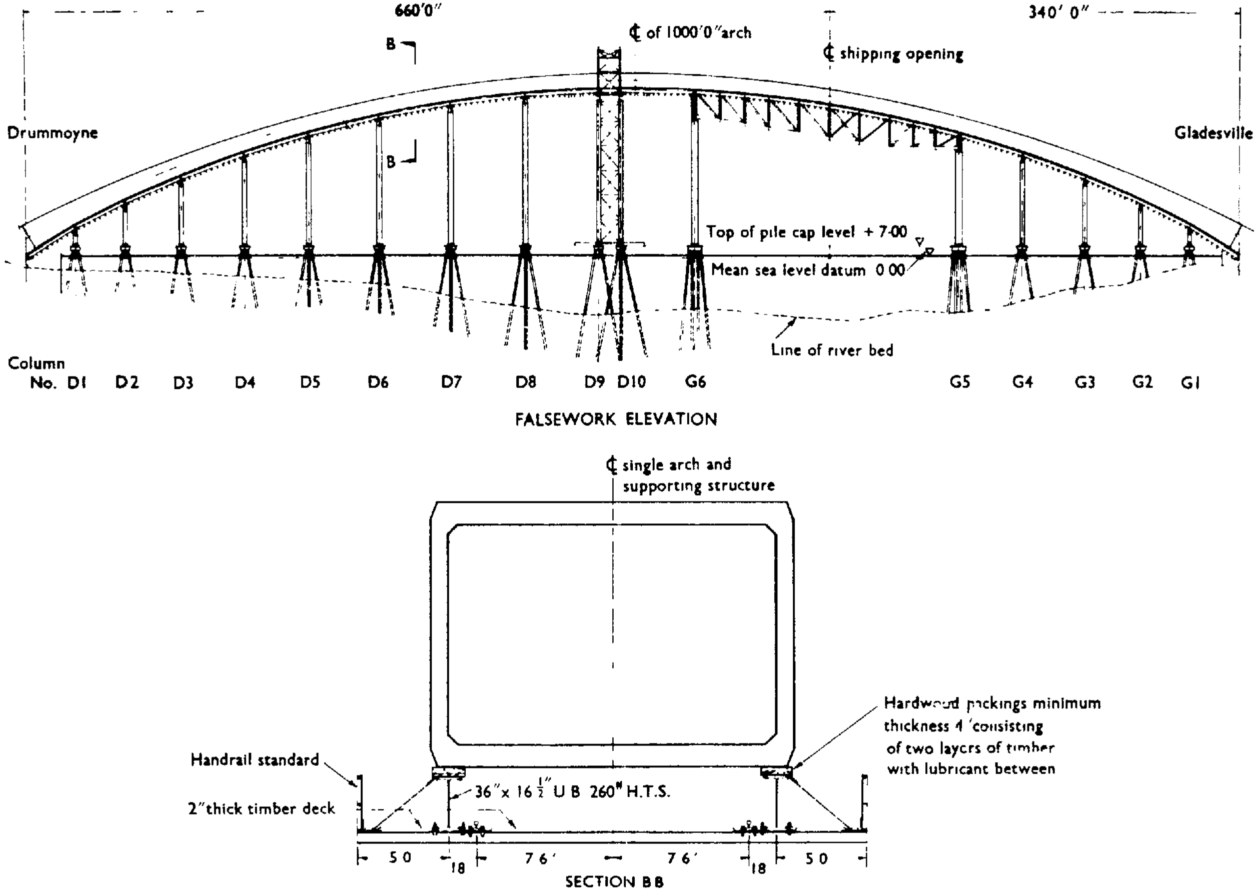
Another method for eliminating "parasitical stresses" is by the use of temporary hinges which cause the arch to act as 3-hinged for a specified length of time. The 3-hinged arch, being statically determinate, does not develop these stresses. The Menager hinge is a well known type of hinge for this purpose. It consists of reinforcing bars bent in the vertical plane so as to cross each other at an angle, at mid-depth of the rib. These bars have sufficient strength to take the stress from arch thrust, and the gap in the concrete is just wide enough to permit rotation of the concrete. Jacks to produce quick rotation can also be used in conjunction with this hinge for elimination of stress.

The Gladesville Bridge at Sydney, Australia, Figure 17, with the world's longest concrete arch span of 1,000 feet, was built on steel falsework with columns at varying intervals. The arch consists of four parallel ribs, each made up of precast single-cell unreinforced units 20 feet wide by approximately 10 feet long. The units were raised to the top of the falsework at the center of the span, and then moved down the falsework. Three inch joints between the units were poured in place. Freyssinet flat jacks were placed between the units at the quarter points of the span.

After closure at the crown, the jacks were used to open the ribs 3 1/2 inches at each quarter point. This was for the purpose of lifting the rib from the falsework and for reducing stresses from rib shortening, creep and shrinkage.

The ribs were set with a 1 foot wide opening between them transversely, and diaphragms inside the ribs and between the ribs were used at 50 foot intervals. These diaphragms were post-tensioned transverse to the ribs. The 1 foot space between the ribs was continuously filled with concrete of a thickness equal to that of the top slab of the cells.

Use of jacks at the quarter points instead of the crown permitted jacking for axial force only, since the quarter point is practically on the line of the elastic center of the rib. Jacks at the crown would have been required to jack in a large moment as well as an axial force.



DETAILS OF FALSEWORK

GLADESVILLE BRIDGE, AUSTRALIA - CONSTRUCTION
From March 1965 Proceedings - The Institution of Civil Engineers, page 489
by J.W. Baxter, A.F. Gee and H.B. James

Fig. 17

The use of unreinforced units required a larger cross-sectional area; and the resulting increased dead load on such a long span, in turn, further increased the area. Reinforced units with projecting bars and sufficient space between the units for lap of the bars in the poured-in-place concrete might be more economical.

3.2.2 Tieback Construction

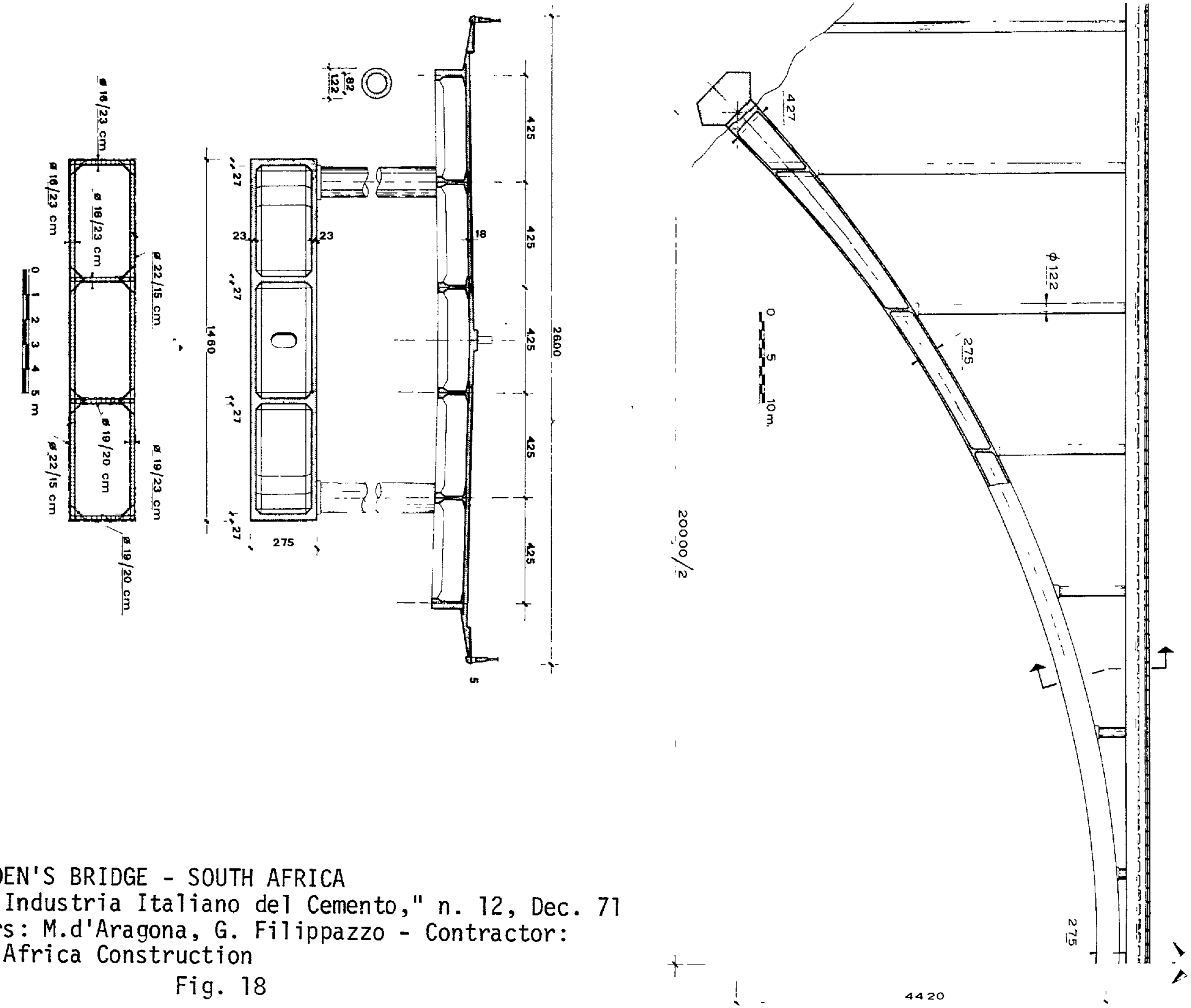
Construction of concrete arches by means of anchored tie-backs is becoming increasingly popular. An arch bridge with a span of 656 feet was built near Port Elizabeth in South Africa about 1970, Figure 18. The Hokawazu Creek Bridge in Japan, Figure 19, with a span of 560 feet was built about 1973; and a 315 foot span was recently built in Austria. All of these bridges have hollow box cross-sections and the concrete was poured in place in about 10 to 20 foot lengths on movable forms cantilevered from the completed sections.

The South African bridge used cable tie-backs passing over a temporary tower built on top of the land pier adjacent to the arch abutment, and anchored in the rock slope. Anchorage jacks were used for adjustment of these ties.

The Hokawazu Bridge used 76 prestressing rods for tie-backs. See Figure 20. These rods were anchored in the permanent bridge abutments, designed for this purpose. The horizontal rods were laid on the roadway deck which was constructed along with the arch rib. Temporary diagonal rods from the deck to the arch rib were used between the columns. The arch is two-hinged and the columns are pinned at both ends. There are also joints in the deck over the crown and at the bridge abutments. The deck is a voided slab with a depth of only 26 inches. The arch rib has a depth of about 8 feet at the crown and 10 feet at the springing. The arch rib is flared laterally at the springing for earthquake resistance. The column hinges, shallow deck and joints minimize deck participation with the arch rib. The columns are heavier than normally used, because they must take the vertical component of the temporary diagonals. The temporary rod diagonals are protected from the sunshine by tubes of vesicated styrene. The heat would effect tension in the bars. The horizontal bars on the deck are protected by thick wood planks.

3.2.3 Elimination of Rib Shortening, Creep and Shrinkage Stress

Bridges constructed by the tie-back method also permit the reduction or elimination of stresses from rib shortening, creep and shrinkage. The tension in the several ties can be adjusted by jacks so that their release, after closure, results in stresses opposite in sign to the "parasitical stresses." Due to the effect of stress relaxation,



VAN STADEN'S BRIDGE - SOUTH AFRICA
From "L'Industria Italiano del Cemento," n. 12, Dec. 71
Designers: M.d'Aragona, G. Filippazzo - Contractor:
Dipenta Africa Construction

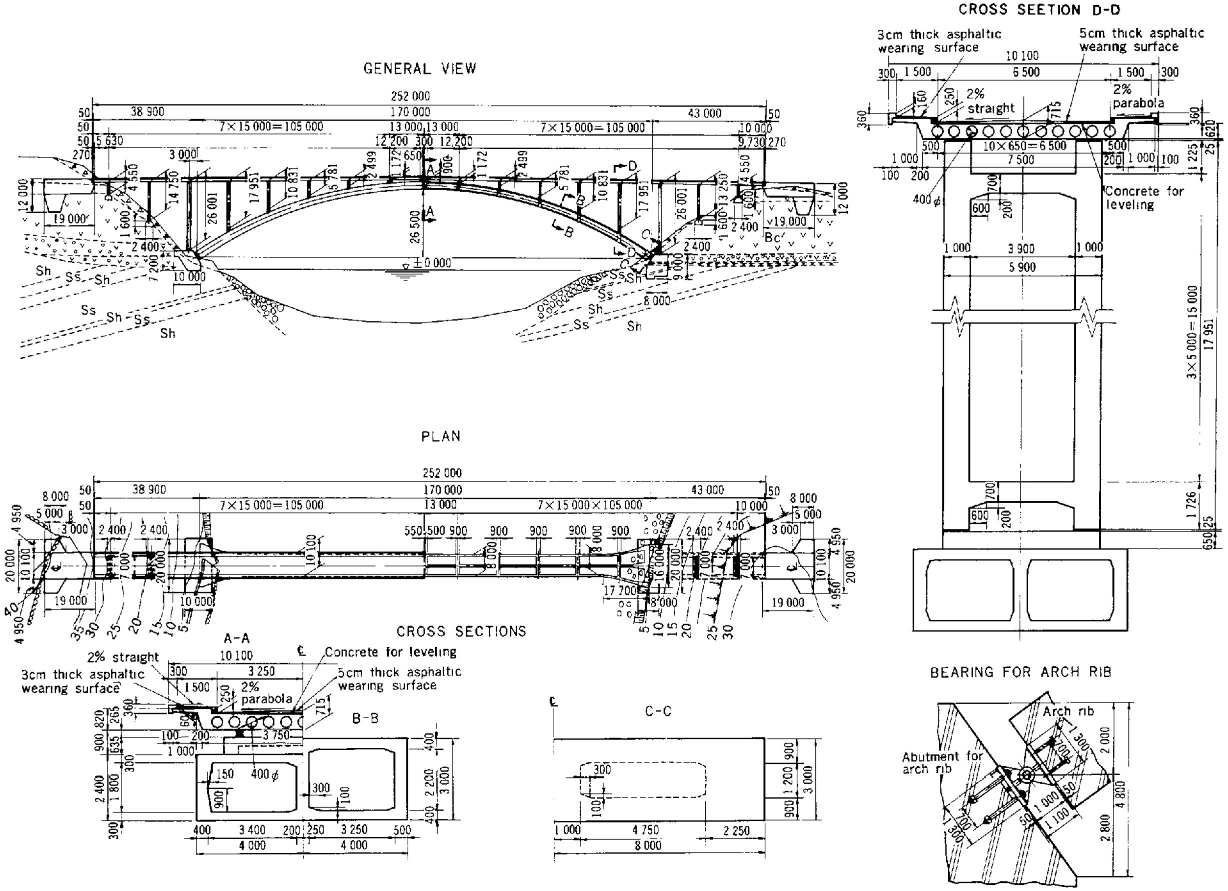
Fig. 18

larger reverse moments are required than the moments actually produced by rib shortening, creep and shrinkage, see 2.5.3.

An article on the Hokawazu Bridge, in "Annual Report of Roads, 1974" of the Japan Road Association, gives a table of stresses. An analysis of these stresses indicate that reverse stresses were introduced in this bridge. The effect of creep and shrinkage results in a large change in dead load stress after the time at which the full dead load is in place. In the case of the Hokawazu Bridge, the immediate maximum dead load stress is 1,700 psi, which compares to a maximum dead load stress after ending of creep and shrinkage of 1,230 psi. The initial stresses fall off rapidly in the beginning, so this initial condition is only for a short time.

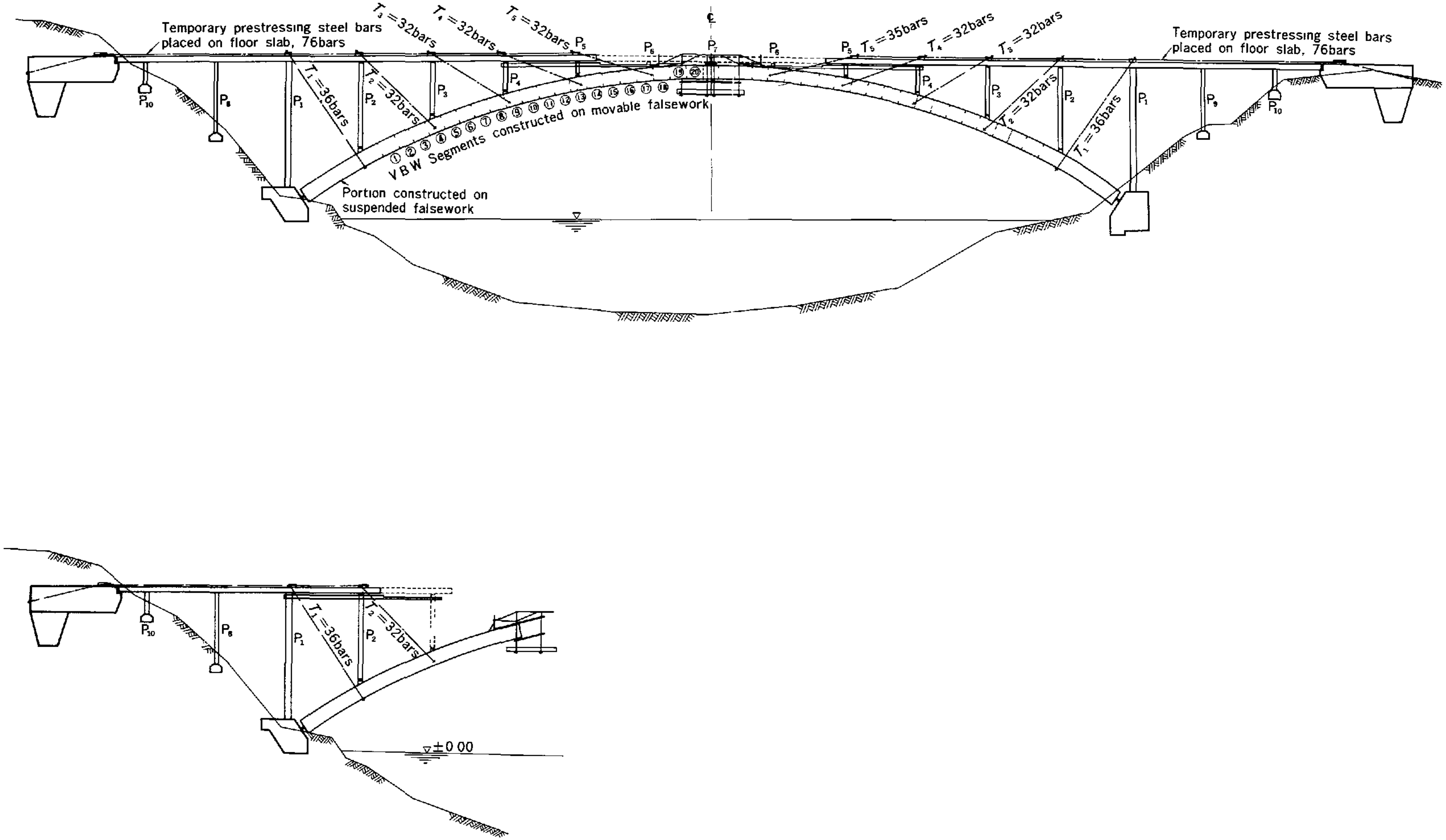
The arch rib should be constructed high to allow for the subsequent deflection, after closure, from dead load rib shortening, creep and shrinkage. This is true regardless of whether or not measures are taken in construction to eliminate stresses from rib shortening, creep and shrinkage. The same ultimate deflection will still occur, because this deflection is due to the shortening of the arch axis from dead load thrust and shrinkage.

If the transfer of load is from a rib supported by tie backs to a self-supporting rib after placing the key, there is a considerable change of stress and position of the rib at release of the tie-backs. The maximum stress in the Hokawazu Arch rib during erection was about 2,200 psi near column P3, Figure 19, even though the section area used at this point was about 1.5 times the section area used at the crown. The maximum stress in the arch rib under design load is about 1,400 psi at the same point. However, this stress is at the intrados, whereas the 2,200 psi erection stressis at the extrados. The effect of change in stress and position of the rib in going from the cantilevered condition to the arch condition must be taken into account in determining the position of the two cantilevers at the time of making the closing pour at the crown.



General view of the Hokawazu Bridge

From Japan Road Association
Annual Report of Roads, 1974, page 14
"Design Construction of the Hokawazu Bridge" by Y. Inaue, Y. Miyazaki
Fig. 19



HOKAWAZU BRIDGE
From Japan Road Association
Annual Report of Roads, 1974, page 14
"Design Construction of the Hokawazu Bridge" by Y. Inaue, Y. Miyazaki
Fig. 20

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APPENDIX - DERIVATION AND ORIGIN OF EQUATIONS

Eq. 1, 2 & 3

Equations 2 and 3 are the same as given in the 1977 Interim AASHTO Specification for Bridge, #20, replacing Article 1.7.90, and explained in the Commentary.

Equation 1 is for use in calculating live load deflection under service load, and is therefore the same as equations 2 and 3 with the numerical overload factor omitted.

K values, Fig. 3, are from Third Edition, Col. Research Council Guide, Chapter 16.

Fig. 5 and Eq. 41 &41a

$$\frac{\ell}{\Delta} = \frac{600{,}000}{f_{bs}} \times \frac{d}{\ell} \quad \text{for 2-hinged steel arch}$$

Where ℓ = arch span

d = depth of arch rib

Δ = live load deflection

f_{bs} = live load bending stress for service condition

Maximum live load deflection for a 2-hinged arch occurs approximately at the quarter point under a load over half the span. The loaded half of the span deflects downward and the unloaded half moves upward with a point of zero deflection at approximately the center of the span.

The approximate equation for live load deflection is based on the assumption of a simple span equal to one half the arch span and a parabolic shape to the moment curve over this simple span. Using the well known method for deflection whereby the moment curve is treated as a load and the moment due to this "load", divided by EI, is equal to the deflection:

$$\Delta = \frac{M\ell}{6} \times \frac{5\ell}{32} \times \frac{1}{EI} = \frac{5M\ell^2}{192EI}$$

The bending stress, f_{bs}, is equal to $\frac{Md}{2I}$, where d = rib depth; therefore:

$$\frac{M}{I} = \frac{2f_{bs}}{d}$$

$$\text{and} \quad \Delta = \frac{5\ell^2}{192 \times 29{,}000} \bullet \frac{2f_{bs}}{d} = \frac{\ell^2 f_{bs}}{557{,}000d}$$

rounding off:

$$\Delta = \frac{f_{bs}\ell^2}{600{,}000d}$$

and $$\frac{\ell}{\Delta} = \frac{600{,}000}{f_{bs}} \cdot \frac{d}{\ell}$$

substituting $\frac{Md}{2I}$ for f_{bs} and transposing, gives Eq. 41:

$$\Delta = \frac{M\ell^2}{1{,}200{,}000I} = \frac{M\ell^2}{41EI}$$

Maximum live load deflection for a fixed arch occurs at the crown. Under a load over the central part of the span, the crown moves downward, and the outer parts of the span move upward. The points of zero deflection are assumed 0.35ℓ apart. This is based on the K value of 0.7L for buckling of a fixed arch. Equation 41a was derived in a similar manner to equation 41, but using the span as 0.35ℓ.

Fig.6 $$\frac{\ell}{d} = 44 + 0.6\sqrt{\ell}$$

This equation for the ℓ/d ratio is based partly on the ratios used for existing arches, and partly on the ratio required to meet a live load deflection-to-span ratio of 1/1200, as determined by the equation of Figure 5. The ℓ/d curve of Figure 6 will meet this requirement for f_{bs} = 9 ksi up to a span of 400 feet, and for f_{bs} = 8 ksi up to a span of 900 feet. As the span gets longer the dead load axial stress becomes larger, resulting in less allowable stress available for bending. The use of higher strength steels for longer spans will counteract this lowering of the available allowable stress for bending. The net effect is usage of a somewhat smaller ratio of depth to span as the span gets longer.

Eq. 4a-4e These equations for the horizontal reactions due to change of temperature were obtained by integrating the equations given in many text books for arches, assuming constant I and a parabolic arch axis.

Eq. 4f-4g These equations are based on the same assumptions as for Eq. 4a - 4e, and the assumption that the change in span length from dead load axial stress, if unrestrained, would be:

$H_{DL}\ell \div A_cE$, where A_c = rib area at the crown

and $r^2 = \frac{I_c}{A_c}$

For the trussed rib, $\frac{d_c}{2}$ is substituted for r.

Eq. 4ℓ &4m These equations are obtained in a similar manner to 4f and 4g, using the sum of the axial change in length of the rib and tie = $H_{DL}/A_r + H_{DL}/A_t$ and by adding I_r to I_t in the equation for H_{rt}.

Eq. 5 to 18 These are as adopted in the 1977 AASHTO Interim Specifications and explained in the Commentary.

Eq. 19 to 27 The derivation of these equations is explained in the text.

Eq. 28 & 28a The origin of these equations is explained in the text.

Eq. 29 This equation and the equation on the following page are given in various forms and rotation in text books on vibration. "Engineering Vibrations" by Jacobsen and Ayre is one.

Eq. 30-34 These equations are based on weights in actual bridges. Since they include both the roadway framing and the arch steel, different strength steels may be included in a single bridge. In a general way, the "ℓ" term includes both the effect of more steel per foot in a longer span and the use of higher strength steels as the spans get longer.

Eq. 35 H = simple beam moment from uniform load divided by h = $W\ell^2/8h$. This assumes that the arch axis is so shaped as to eliminate dead load bending moment.

Eq. 36 & 37 These are based on typical influence lines for arches.

Eq. 38 & 39 These equations are based partly on typical influence lines for arches. If I_s is placed equal to zero in equation 38, this equation becomes the same as equation 39. Placing I_s equal to zero might be considered equivalent to hinged ends. Equation 38 goes from 0.324ℓ for constant I to 0.222ℓ for $I_s = 4I_c$.

The following is proof of equation 38 for use in preliminary design. Assume a parabolic axis and uniform live load.

Ordinate to arch axis at quarter point = 0.75h

Full span loaded: $H = 1/2\ W\ell^2/8h$ and $V = W\ell \div 2$

Left half of span loaded: $H = 1/2\ W\ell^2/8h = W\ell^2/16h$

$$V_L = 3W\ell/8$$

$$M_{q.p.} = \frac{3W\ell}{8} \times \frac{\ell}{4} - \frac{W\ell}{2}\left(\frac{\ell}{4}\right)^2 - \frac{W\ell^2}{16h} \times \frac{3h}{4}$$

$$= \left(\frac{3}{32} - \frac{1}{32} - \frac{3}{32}\right)W\ell^2 = \frac{1}{64}\ W\ell^2$$

$$\frac{W\ell_e^2}{8} = \frac{1}{64}W\ell^2, \quad \text{where } \ell_e = \text{equiv. simple span}$$

$$\ell_e^2 = \frac{1}{8}\ \ell^2$$

$$\ell_e \approx 0.354\ell$$

therefore, use equivalent simple span = 0.36ℓ

A similar approach for a concentrated load at the quarter point gives approximately the same equivalent simple span.

Eq. 40 The effect of applying uniform load through uniformly spaced columns or hangers to the arch rib is to add moment to the moment which would occur for directly applied uniform load. This added moment is equivalent to the moment occurring in a uniformly loaded continuous beam on equally spaced supports, but opposite in sign. Thus, for a column spacing of Δ_L, the added rib moment is $+W\Delta_L^2/12 = +P\Delta_L/12$ under the columns, and $-P\Delta_L/24$ midway between columns.

Eq. 41 & 41a See under derivation for Figure 5.

Eq. 42, 42a, 42b, 42c These equations assume no dead load bending moment in the arch except dead load rib shortening moment, and uniform dead load axial stress equal to f_a. The second term of each equation is the downward deflection at the crown due to shortening of the arch axis from dead load axial stress, assuming the arch to be unrestrained horizontally at one abutment. The second term represents the effect of the bending moment in the rib produced by the horizontal restraint at the abutments. The second term can be arrived at from the horizontal reactions produced at the abutments by a unit vertical load at the crown. The equations assume constant f_a and constant temperature along the arch axis.

Eq. 43, 44 &45 These equations are the effect of lateral eccentricity of the live load on a wide arch barrel. Their derivation is explained in the text.

Eq. 46a-47d These equations are similar to equations 42 - 42c for steel arches. The effect of creep has been assumed as 2.4 times the initial rib shortening. The shrinkage coefficient has been taken as 0.0003.

Eq. 48a-48h These equations differ from the corresponding ones for steel arches to allow for a non-uniform depth of rib.

Eq. 49a & 49b	The basis for these equations is explained in the text.
Table II Eq. 50a to 50f	Equations 50a and 50b, with variable numerical constants, are given in a number of texts on torsion. The numerical constants given here are close enough within the limit of $b/d \geq 10$.
	Equations 50b to 50f were derived by a method given in "Torsion in Structures," by C. F. Koelbrunner and K. Basler.