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Modeling Equilibria in Integrated Transportation-Land Use Models

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Dedication

This work is dedicated to my father, an experienced transportation engineer

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Modeling Equilibria in Integrated Transportation-Land Use Models

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This research focuses on equilibrium modeling of integrated transportation-land use models (ITLUMs) based on spatial input-output (SIO) theory. First, it analyzes equilibria in the SIO model by studying sales prices' and trade volumes' solution existence and uniqueness. A fixed-point formulation is proposed for the uncongestible, random-utility-based multiregional input-output (RUBMRIO) model, which consists of a set of model equations. Under *weak* conditions regarding sales prices, the set of price solutions is shown to exist. And these are unique under sufficiently small dispersion parameters. Price solutions uniqueness is also discussed under more general conditions which permit much larger dispersion parameter values. Once prices are known, commodity flows are found to be unique. The fixed-point formulation established here verifies that the common/original RUBMRIO iterative algorithm converges *almost surely*, regardless of the initial values. However, a modified algorithm is demonstrated to be more efficient. Second, the dissertation examines two methods to approach the *overall* equilibria of a full, congestible ITLUM based on the RUBMRIO model. First, a combined model is constructed which synthesizes all the ITLUM components including location choices, travel frequencies choices, mode choices, and route choices. The optimization conditions are derived, and they can be assembled to show the equivalence of the combined model to these component choice models. The uniqueness of equilibrium solution is also discussed. Evans' algorithm is proposed to solve this combined model. Second, a "linked" model is assembled using two feedback strategies: the first uses a single loop; the second implements an additional internal loop. A numerical example for the Dallas-Fort Worth network suggests that both feedback methods converge to the unique solution; but the second one, with double loops, converges more efficiently. In summary, the two methods developed here are of theoretical interest and practical application.

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Chapter 1 Introduction

1.1 BACKGROUND

A transportation-land use equilibrium describes a stable state in which both transportation's effects on land use and land use's effects on transportation achieve balance in urban or regional systems. This equilibrium is not yet well understood.

Urban development and activities take place spatially. Land use models describe demographic and economic transitions in the spatial allocation of activities across regions. Most land use models deal with the demand for residential, commercial, and other activities at different sites. The demand is constrained by the physical limits of land space, and the artificial, planned utilization rates (Oryani and Harris, 1996). Transportation systems are fundamental to urban development. Transportation models describe commodity and human travel patterns on the transportation infrastructure network, which connects the sites. Most travel demand models describe travel frequency, destination choice, mode choice, and route choice, given the allocation of land-based activities. The travel demand is constrained by the transportation infrastructure and system supply.

The connection and interaction between land use and transportation were widely recognized half a century ago (Mitchell and Rapkin, 1954). The location of land use and economic activities creates the demand for transportation to move goods and people between places. The availability and efficiency of transportation facilities influence people's location choices.

However, the understanding of the interactions between land use and transportation, and, in turn, the empirical modeling of these interactions, is still limited. It is difficult to answer questions such as how transportation projects, which often intend to accommodate economic development, in fact induce more urban expansion and congestion, and how inadequate transportation facilities become obstacles to economic growth.

There is a growing appreciation of the importance of understanding and modeling the interactions between land use and transportation. First, direct and indirect legislation encourages the development of land use-transportation modeling as a decision support tool for policies designed to mitigate urban development problems. In the United Sates, the legislation includes the 1990 Clear Air Act Amendments (CAAA), the 1991 Intermodal Surface Transportation Efficiency Act (ISTEA), and ISTEA's successor, the 1997 Transportation Equity Act for the Twenty First Century (TEA-21). Additionally, recent lawsuits undertaken in the San Francisco Bay Area and Chicago (NRDC vs. USDOT and Caltrans, 1992; S. Club vs. Illinois DOT, 1996; S. Club vs. Wasatch Front Regional Council, 2000; details can be found in Johnston, 2002) have intensified such pressure on metropolitan planning organizations (MPOs) and other agencies to integrate land use and transportation in their models.

Second, the sustainability of our cities is a crucial social and economic issue, which drives a great deal of effort to model urban systems in order to improve the forecasts of future urban patterns. For example, highway pollution has been a critical issue for urban sustainability. In the U.S., mobile source emissions from the highway transportation sector alone are estimated to account for about 70% of the carbon monoxide generated, 39% of nitrogen dioxide, 30% volatile organic compounds (VOCs), and 28% of small particulate matter (PM-10) (Curran et al., 1992). Furthermore, highway transportation accounts for about 22% of the nation's annual energy consumption (Davis, 1994).

Finally, an increasing number of MPOs, state departments of transportation (DOTs), and other local and regional authorities rely on land use-transportation models in the management of urban systems to facilitate long-range planning and to simulate the potential outcomes (especially congestions) of decisions affecting the cities. A better understanding and representation of land use-transportation interactions is essential to urban policy-making.

Many frameworks have been proposed over the past three decades to model the ways in which transportation improvements influence land uses and vice versa. Recent efforts attempt to couple the two in an integrated fashion for operational models. Integrated transportation-land use models (ITLUMs) simulate the decision hierarchy both in land use including location choice and in transportation, including trip generation, destination choice, mode choice, and route choice.

In modeling the interaction between transportation and land use, most research has relied on an equilibrium hypothesis (e.g., Putman’s ITLUP model, 1983; Martínez’s “5-LUT” model, 1996). The transportation-land use equilibrium assumes that spatial land use allocation is consistent with transportation “prices” (travel times), which reflects traffic congestion as a result of the allocation.

One simple but widely used approach under this assumption is that land use and transportation are modeled separately, and a feedback loop is built between their two systems in such a way that, after several cycles, an equilibrium situation is reached in the integrated system (e.g., Putman’s ITLUP model, 1983; Martínez’s “5-LUT” model, 1996). These models are often referred to as “linked” models (de la Barra, 1994). Another approach tries to capture the equilibria between and within land use and transportation systems simultaneously (e.g., Kim’s model, 1989). This method synthesizes location and transportation choices into a unified mathematical framework. Models using this approach are commonly called “combined” models (e.g., Boyce, 1977).

Equilibrium mechanisms in integrated transportation-land use systems are not well understood or modeled. For example, in the “linked”-type approach described above, it is not clear if the converged results represent the equilibrium and why the system converges through the feedback iterations. Also, it is not clear that sub-system equilibria are realized, when the entire system converges. Moreover, land use changes exhibit some time lag in response to transportation improvements. This dynamic relationship cannot be easily formulated in static-equilibrium model systems.

This study investigates the equilibrium existence and mechanisms underlying one popular group of integrated transportation-land use models based on spatial input-output (SIO) theory. The study examines the existence and uniqueness of the equilibrium solutions to random-utility-based multi-regional input-output (RUBMRIO) models to clarify if the subsystem achieves equilibrium. Then it models location choices and travel demand decisions under an equilibrium framework, while recognizing many stochastic and dynamic properties of such systems. The equilibrium conditions of the model characterize economic activities and travel behaviors simultaneously.

The solution existence and uniqueness of RUBMRIO models provide a theoretical base to link or combine such models into integrated models and obtain a solution for the overall model system. The proposed combined model presents a starting point for other possible models in integrating the RUBMRIO model with a transportation model. This comprehensive model system can serve as a policy tool for transportation engineers and planners.

1.2 Equilibrium Concepts In Land Use-Transportation Interaction

Equilibrium analysis is very common in economic studies. Economic analyses of markets often proceed by defining demand and supply models for a set of goods or services, and then searching for an equilibrium consistent with both. In general, the equilibrium is represented by specific prices. In the context of an urban system, equilibrium describes a stable state in which both transportation's effects on land use and land use's effects on transportation achieve balance. This general equilibrium approach also implies equilibria between supply and demand in both land and transportation markets.

Transportation demand is usually considered a derived demand: it is normally undertaken to allow participation in a spatially varied set of activities such as work, shopping, and recreation. That leads to the demands behind the travel, the residential and job location choices, which are the core of land use demand. On the other hand, the transportation supply is always constrained by infrastructure capacities. So, from this macro-level perspective, transportation networks can be roughly viewed as the supply side of land use-transportation systems, with land use driving the demand for travel. Then, the system equilibrium characterizes the balance between transportation supply and land use demand, although such macro-balance is not explicitly in equilibrium formulations.

While a complex urban system in the real world may never actually be in a state of equilibrium (thanks to constant shocks, growth, and decline), it is assumed that the system is at least close to its equilibrium state and, given enough time, would approach that equilibrium (Bell and Iida, 1997). So equilibrium is indeed a static hypothesis, which may characterize the most probable state of the system. The most probable state, as suggested in Wilson's entropy maximization theory (Wilson, 1969), is the statistical phenomenon where the simple, individual probability choice hypothesis leads to very regular patterns of spatial interaction behavior.

At the static equilibrium, the decision makers (e.g., the commuters and land developers) have no motivation to change their travel behaviors or location choices. However, in reality, due to lack of information and the stochastic nature of human behavior and the environment, decision makers may keep adjusting their choices to maximize their utilities or benefits (see, McFadden, 1978; Sheffi, 1985). In other words, the system is dynamic in many ways. Then, any observed, actual state of the system is a disequilibrium state of the system. These disequilibrium states build up the trajectories to the equilibrium state (Nagurney, 1999). These disequilibrium states may oscillate around the equilibrium states. From a long-term point of view, the equilibrium state describes the average state of the disequilibrium trajectories. Therefore, a good understanding of land use-transportation equilibrium can help policy-makers and planners to develop sustainable land use-transportation policies and plans.

In summary, the concept of equilibrium in complicated land use and transportation systems is a conventional research hypothesis, which has been used to describe the most possible static state of the system.

The advantage of equilibrium modeling for land use-transportation systems derives from three aspects. First, despite the fact that disequilibrium descriptions may be more realistic, they are difficult to model in mathematical form. Only recently, Nagurney and Zhang (1998) proposed a projected dynamic system to simulate disequilibria and Waddell's (1999) UrbanSim model provided possible ways to imitate the urban disequilibria. In contrast, equilibrium analysis is basically time independent: there is no need for dynamic details. Moreover, successful equilibrium modeling is fundamental to successful modeling of disequilibrium (for example, Nagurney and Zhang's [1998] definition of system disequilibrium is based on their equilibrium assumptions). Second, equilibrium modeling provides access to existing operational algorithms, which are essential for solving large, real problems. Finally, there are a variety of applicable equilibrium approaches, both in land use and transportation models, which are successful in describing a rational system status and formulating in mathematical forms to obtain solutions. For example, user equilibrium and stochastic user equilibrium are common in network analysis. On the land use model side, there are spatial market (or price) equilibrium and economic activity equilibrium (EAE) (see Nagurney, 1999; Bertuglia and Leonardi, 1980).

The limitations of equilibrium modeling are also obvious. First, in concept, equilibrium only models the ***cleared market*** situation; in other words, supply equals demand at the prevailing prices. There are various situations where the market (or system) does not clear, and disequilibrium exists. Second, due to the complexity and variability of urban systems, continuous fluctuations mean that equilibrium states are not observable, and thus, cannot be validated.

Despite such limitations, the theoretical soundness of, convenient formulations for, and operational algorithms for large problems make equilibrium modeling very attractive for integrated transportation-land use models. These motivate this study.

1.3 RESEARCH OBJECTIVES

This work investigates equilibrium modeling approaches for integrated transportation and land use models. The primary objectives of this research are to:

- State, formulate, and analyze the equilibria between and within transportation and land use systems based on spatial input-output models in a way that explicitly takes the stochastic nature of decision makers' behavior into account,

- Seek and apply efficient mathematical methods and algorithms to find the equilibrium solutions in the problems; and compare different feedback strategies for the existing linked land use-transportation models.

Related to these objectives, two main assumptions are made. First, one can empirically formulate, in mathematical equations, the equilibrium between transportation and land use; and the existence of a mathematical solution implies that the equilibrium exists. The concept of such an equilibrium is stated in the previous section. The second assumption is that solution uniqueness leads to equilibrium uniqueness. A unique equilibrium is highly desired since the equilibrium hypothesis describes the most probable static state of the system; this hypothesis will be much less attractive if there are multiple probable solutions.

In order to achieve the research objectives, this dissertation undertakes the following key tasks:

Task 1. Equilibrium modeling of spatial input-output models

A number of operational land use-transportation models make use of interzonal input-output (IO) models, including de la Barra's TRANUS (1994), Echenique and colleagues' MEPLAN (Hunt, 1993), and Kim's model (1989). Such spatial models can directly calculate transportation demand via interzonal interactions, obviating the need for trip generation and distribution steps (Hunt and Simmonds, 1993).

The solutions to a set of equations provide a spatial equilibrium solution for trade volumes. However, this complex specification requires searching for the equilibrium via iterative calculations. This work constructs a general fixed-point formulation of RUBMRIO models and provides a modified, efficient algorithm to achieve the equilibrium solution. We also provide the conditions for a unique solution and the stability of any equilibrium solution.

Task 2. Equilibrium modeling of RUBMRIO models with congestible networks

Although spatial input-output (SIO) models describe many aspects of transportation and land use, they generally consider transportation costs as constant. In order to make the transportation cost adjustable according to the changes in location choices, an external traffic assignment model is usually required and the travel costs are fed back to the SIO model.

This research synthesizes the SIO and trip assignment model into one general formulation to ensure the simultaneous determination of the equilibrium solutions. It then identifies efficient algorithms to solve such equilibrium problems.

Given that most current ITLUMs rely on feedbacks to link sub-models, this study also explores different feedback techniques to identify the most appropriate and efficient. A number of feedback strategies can be implemented in practice; some may imply different sub-system equilibrium assumptions, and others mimic the temporal equilibria of quasi-dynamic models. For example (Figure 1.3.1), by linking a location choice model to a four-step travel demand model, one can build a single feedback loop from traffic assignment to the land use model. Another strategy is to construct a feedback loop within the four-step transportation model, and assemble a second feedback loop between the land use and transportation models. By composing two-level feedback loops, the second strategy assures that equilibria exist between and within land use and transportation systems. Most existing ITLUMs make use of the first feedback method for equilibrium solutions (e.g., Martínez's "5-LUT" model (1996)). However, the second method appears to be more efficient at achieving such equilibria.

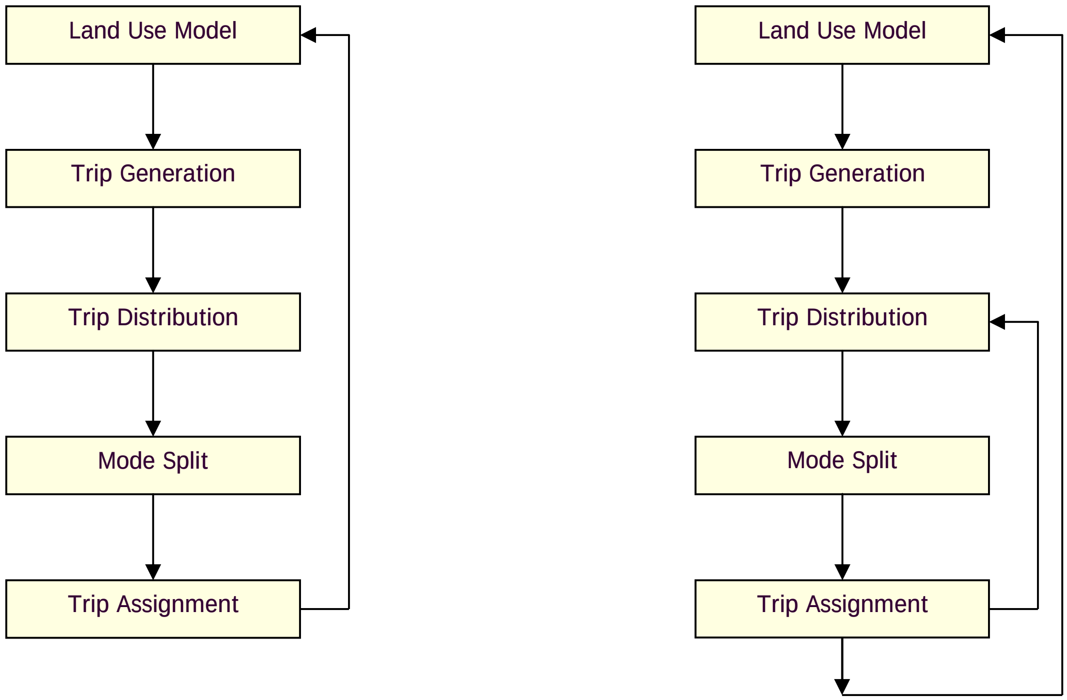

Figure 1.3.1 Feedback loops examples

Boyce, Lupa, and Zhang (1994) have examined the feedback approaches in four-step travel demand models and shown that iterated solutions eventually converge to the equilibrium solution. This study extends their work to integrated transport-land use models. By making use of the RUBMRIO model as the land use model, this research constructs a typical travel demand model. It then builds two types of feedback loops and examines their efficiencies to equilibrium solutions. It also examines whether or not the sub-system equilibria realize at convergence.

1.4 DISSERTATION ORGANIZATION

The dissertation is organized in the following manner: Chapter 2 presents a review of related work in the literature and covers other necessary background information. Chapter 3 introduces the conceptual and theoretical framework of equilibrium formulation of RUBMRIO models. Chapter 4 provides the general formulation of ITLUMs based on the RUBMRIO model and examines feedback strategies for linked-type ITLUMs. Chapter 5 summaries the results of this dissertation, provides conclusions, and makes recommendations for future research.

Chapter 2 Literature Review

2.1 INTRODUCTION

In view of the enormous interest generated by rapid urban development after World War II, it is not surprising that a host of different theoretical approaches towards more comprehensively regional systems have been proposed. This chapter is not an exhaustive listing of all approaches. Rather, it aims to illustrate the main types of theories and modeling methods, and to emphasize their use of equilibrium concepts, solution calculation methods, and solution existence and uniqueness. As shown in this review, little research has been concerned with the equilibrium solution existence and uniqueness in integrated land use-transportation (LU-T) models. And this research attempts to fulfill this neglected topic and presents one step towards a better understanding of LU-T equilibria.

The rest of this chapter is organized in the following manner. Section 2.2 begins with a brief discussion of equilibrium methods in urban systems modeling, with the focus on what is primarily referred to as land use models in integrated studies. Section 2.3 introduces user equilibrium (UE) and stochastic user equilibrium (SUE) concepts, methods, and extensions for network and travel analysis. Section 2.4 reviews of the literature on the major theoretical and methodological underpinnings of integrated land use- transportation (LU-T) models that are most relevant to this research. The review summarizes the strengths and limitations of the existing, operational models, especially with respect to LU-T equilibria.

2.2 LAND USE MODELS AND SPATIAL EQUILIBRIA

Land use models describe demographic and economic transitions in the spatial allocation of activities across regions. Most operational land use models can be classified into three main categories (see Betugilia et al, 1987; Southworth, 1995; and Oryani and Harris, 1996): the Lowry gravity model, normative optimization models, and economic approaches. This last category can be further divided into two groups: microeconomic approaches, such as the bid-choice theory used in the MUSSA model (Martínez, 1992), and macroeconomic approaches, such as the spatial input-output (SIO) theory used in MEPLAN (Hunter, 1993). It should be mentioned that the classification is not exclusive; a number of land use models are the results of combining various theories and methods. For example, Wilson *et al.* (1981) extended the Lowry model under the SIO framework and formulated it into the entropy-maximization problem.

2.2.1 Lowry-Type Models

Many current urban land use models derive from Lowry's model (1964). The original Lowry model incorporates the spatial distribution of population, employment, retailing, and land use by (1) allocating workers to zones based on exogenous basic employment levels, (2) allocating dependent families of these workers, and (3) allocating service ("non-basic") employment to serve these workers and their families. The latter two allocations are calculated iteratively to then bring the resulting residential and non-basic employment into balance. However, Lowry's model did not rely on economic theory (Mill, 1972). A number of contributions have been made to modify and develop Lowry's model and merge it with urban economics (see e.g., Alonso, 1964; Mills, 1972; and Goldner, 1983). Today the most-used Lowry-type models are the Disaggregate Residential Allocation Model (DRAM) and the Employment Allocation Model (EMPAL) (Putman, 1974, 1983, 1991).

DRAM and EMPAL

DRAM and EMPAL are the most used successors to Lowry's model in the United States. Putman (1983) developed DRAM in allocating residents to a zone, which has the following form:

$$\hat{N}_{in} = \sum_{jm} r_{mn} E_{jm} \left[\frac{W_{in} (d_{ji})^{a_n} e^{-b_n d_{ji}}}{\sum_k W_{kn} (d_{jk})^{a_n} e^{-b_n d_{jk}}} \right] \qquad (2.2.1)$$

where $\hat{N}_{in}$ is the estimated number of type *n* of residents of zone i, E_{jm} is the employment in sector *m* in zone j, W_{in} is a composite attractiveness measure for zone i to employees from residential group *n*, d_{ji} is the travel time between zones j and i, r_{mn} is the region-wide ratio of type *n* residents per employee type *m*, and a_n and b_n are empirically derived parameters. Equation (2.2.1) is a gravity-type model to assign total residents $r_{mn}E_{jm}$ to zone *i* based on that zone's attractiveness and impedance measure, which is a Gamma function of travel time, i.e.: $f(d_{ij}) = 1 \cdot d_{ij}^{a} e^{-b \cdot d_{ij}}$.

The attractiveness of a zone is assumed to be a function of residential land, vacant land (potential residential land), and existing residents (characterized by income groups). It may be calculated as follows:

$$W_{in} = (L_i^v)^{q_{1n}} (1 + X_i)^{q_{2n}} (L_i^r)^{q_{3n}} \prod_q \left(1 + \frac{N_{iq}}{\sum_q N_{iq}} \right)^{g_{nq}} \quad (2.2.2)$$

where L_i^v is the area of vacant, developable land in zone i, X_i is the proportion of developable land in zone i that has already been developed, L_i^r is the area of residential land in zone i, N_{iq} is defined as the number of residents in zone i who are in the q^{th} income quartile, and q_{1n}, q_{2n}, q_{3n}, and g_{nq} are parameters to be estimated.

EMPAL allocates employment in each industry sector to a zone based on a similar gravity-type model, with consideration (via a weight parameter) of the prior period's employment level(s) (Putman, 1983). The relationship is as follows:

$$E_{j,t}^m = f \sum_i P_{i,t-1} \left[\frac{W_{j,t-1}^m (c_{ji,t})^{a_m} e^{-b_m c_{ji,t}}}{\sum_k W_{k,t-1}^m (c_{ki,t})^{a_m} e^{-b_m c_{ki,t}}} \right] + (1-f) E_{j,t-1}^m \quad (2.2.3)$$

where $E_{j,t}^m$ is the employment in sector *m* in zone j at time t, $P_{i,t-1}$ is the number of total population in zone i at prior time $t-1$, $W_{j,t-1}^m$ is the attractiveness function of zone j for sector *m* at time $t-1$, f, a_m, and b_m are empirically derived parameters. The value of f is also constrained to lie between 0 and 1.

Similarly, the attractiveness of a zone is assumed a function of prior time employments and total land area and may be calculated as follows:

$$W_{j,t-1}^m = (E_{j,t-1})^{d_1} (L_j)^{d_2} \quad (2.2.4)$$

where $E_{j,t-1}$ is the total employment in zone *j* in prior time *t-1*, L_j is the total land area of zone j, and d_1 and d_2 are parameters to be estimated.

As one can see, in EMPAL, the time lag effect has been empirically considered. Also, as Putman (1991) suggested, the possible lagged variables could be implemented in DRAM.

Both DRAM and EMPAL are deterministic mathematical models in the sense of calculating solutions; in other words, once the parameters and the exogenous variables (including the lagged variables from last time period) are known, both models yield unique solutions. DRAM and EMPAL require relatively less data. However, they allocate residents and employments instantly into zones and ignore land market clearing process. The approach implicitly assumes the market equilibrium between demand and supply at the prevailing price in short run, but clearly the land market needs multiyear, long- run process to clear. Moreover, the space constraints should be considered. For example, some vacant, developable land may be restricted to land use types other than residential; thus, using these land areas as attractiveness variables is inappropriate.

Essentially, Lowry-type models use iterative calculations to seek a convergent solution and, hence, achieve a static stable point of the land use system, given the basic employment.

2.2.2 Optimization Models

Another common approach makes use of optimization, though the system solutions are generally considered normative, rather than positive. The normative approach reflects a long held interest within the planning profession for best possible solutions or designs. For example, Wilson and his colleagues (1981) formulated the spatial interaction and activity allocation as entropy-based convex mathematical programs subject to a set of linear planning constraints. The objective function is to maximize the allocation entropy.

The primary advantage of optimization models is their use of generally simpler mathematical forms that are tied to theories of cost minimization, entropy (or utility) maximization, or net gain. Additionally, advances in operations research provide efficient algorithms allowing application of the optimization approach to large scale, practical problems.

There are a number of similar approaches, which pursue alternative formulations of interrelated facility location problems (see, for example, Herbert and Stephen, 1960; Wilson, 1981, 1987; Bertuglia and Leonardi, 1980; and Boyce and Lundqvist, 1987). Other, more comprehensive models, which combine residential, employment, and travel choice decisions within single jointly optimized modeling framework, will be discussed later (see Boyce and Southworth, 1979; Boyce *et al.*, 1983; Kim, 1989; Kim, Ham, and Boyce, 2002).

Optimization-based models usually form convex objective functions with linear constraints, so their unique equilibrium solutions are certain (Bertsimas and Tsitsiklis, 1997).

2.2.3 Economic Models

A third general approach involves economic interpretations of spatial interactions. For a microeconomic basis, one representative approach is the MUSSA model (Martínez, 1992). And one of the most applied *macroeconomic* approaches is the spatial input-output (SIO) model (see, e.g., Issad *et al*, 1960;

Leontief and Stuart, 1963; Wilson, 1970a; Anas, 1984; Hunt, 1993; and de la Barra, 19945).

The MUSSA model

MUSSA (Modelo del Uso de Suelo de Santiago) is based on "bid-choice" theory for competitive urban land markets (Martínez, 1992). The theory describes an equivalency between the bid approach (Alonso, 1964) and the maximum utility approach (McFadden, 1978).

Demand for a dwelling or a building (by households or firms) depends on the consumers' willingness to pay (W). Consumers attempt to maximize their surplus ($C = W\text{-}P$, where P is price actually paid), while sellers attempt to maximize price paid. So, among S dwelling alternatives, household h will chose the one which satisfies:

$$\max_{s\in S} \; C_{hs} = \max_{s\in S} \; (W_{hs} - P_s) \qquad \forall h \tag{2.2.5}$$

While the developer for this dwelling will accept the maximum price bid, the successful consumer must be the maximum bidder (i.e., the one with the highest willingness to pay for it). Thus, among N bidders for this property, the final price P_s is given by:

$$P_s = \max_{n} \; W_{ns} \qquad \forall s \tag{2.2.6}$$

Then, the bid-choice model presents an equilibrium if, when replacing (2.2.6) in (2.2.5), one has the following:

$$\max_{s} \; C_{hs} = \max_{s} \; [W_{hs} - (\max_{n} \; W_{ns})] \qquad \forall h \tag{2.2.7}$$

W is taken to be a function of dwelling characteristics, household income, and a household's utility level of each choice. Since the property's characteristics (e.g., accessibility) and household income are exogenous, the decision variable in (2.2.7) is the utility level, interpreted as "quality of life". Herbert and Stephens (1960) suggested that if a consumer is the highest bidder in a given group of bidders, then that is his/her optimal location.

Assume that the W function includes a random error term following an i.i.d. Gumbel distribution. Then, the probability that household h makes the highest bid on a property s, is given by the multinomial logit model:

$$\Pr(h \mid s) = \frac{\exp(mW_{hs})}{\sum_n \exp(mW_{ns})} \tag{2.2.8}$$

where m is the Gumbel's scale parameter, which is also called dispersion parameter.

The equilibrium conditions for MUSSA can be described as the following: (1) every household and firm should find a location, (2) land availability constraints is recognized by introducing linear constraints, and (3) dwelling supply must follow historical tendencies, consistent with developers' behavior. This tendency is captured by a time-series supply function estimated using previous data.

In summary, the MUSSA model maximizes the consumer surplus in (2.1.7), subject to the linear land-availability constraints. This is a linear programming problem once the expected land prices are evaluated by $P_s = \frac{1}{m}\ln\left[\sum_n \exp\left(mW_{ns}\right)\right]$; thus, there is a unique equilibrium solution for dwelling prices. Usually, location accessibilities in the MUSSA model are determined by a separate transportation model, which will be discussed later.

Macroeconomic Models

Another line of methods derive their inspiration from macroeconomic theories, especially from an intersectoral input-output model introduced by Leontief (1967).

The classic input-output (IO) model describes inter-industry production and relates to urban and regional trade where development depends mainly on exports demand (Leontief, 1967). The spatial disaggregations and interactions of the manufacturing and other basic industrial activities create land use and transportation demands. Unlike Lowry-based models using exogenous employment information, IO models determine productions, prices, and jobs endogenously based on the inter-industry interaction. The interregional development of an IO model provides a general framework for urban system analysis. Such intersectoral/interzonal models have a number of variations with embedding entropy-maximizing, utility-maximizing, and spatial surplus mathematical programming formulations (MacGill and Wilson, 1979; Wilson, 1981; and Hunt, 1993). Mills (1972) developed a linear programming model based on Leontieff's constant-coefficient technology describing the relationships between various goods and services. The amount of land allocated to roads is determined endogenously for each area by minimizing the total cost of the urban area including transportation congestion. The households in Mill's model are assumed to be cost minimizers, rather than utility maximizers. Mill's model centers activity around a central location, from where the basic goods are exported. Hartwick and Hartwick (1974) included intermediate goods and services, and investigated the relative location of different activities.

Spatially disaggregate IO models have many applications in economic analysis and have been used extensively (e.g., Kim's model [1989], Echenique and colleagues' MEPLAN [Hunt, 1993], and de la Barra's TRANUS [1994]).

There has been an increasing interest in, and substantial efforts devoted toward the spatial input-output (SIO) model, due to its theoretical soundness and wide applications. However, the equilibria problem of SIO models remains untouched. For example, to date there is no research on existence and uniqueness of equilibrium SIO solutions. Additionally, it is unclear to integrate SIO models with transportation models to represent the integrated transportation-land use equilibria. The study of equilibrium solutions to SIO models constructs one of the tasks addressed in the research.

2.3 Transportation Model Equilibria

Transportation models describe commodity and human travel patterns on a network. Given the allocation of land-based activities, most travel demand models specify four key components of the travel patterns: travel frequency (how many trips are made and from where), destination choice (where the trips are going to), mode choice (which available modes are used), and route choice (which paths are used). This modeling system is often referred to as the "four-step" process (see Meyer and Miller, 1984). Although recent efforts have tended to more disaggregate, activity-based modeling approaches (see Ben-Akiva and Lerman, 1985 and Bhat, 1996, 2001), the four-step process is by far the most common model in practice (Boyce, 1998), and is usually treated as the standard "state-of-the-practice" approach (Miller, 1997).

Unfortunately, the sequential or four-step approach suffers from many limitations. One of the most severe problems is the inconsistency among the steps (Boyce, Lupa, and Zhang, 1994), e.g., trip generation studies daily personal trips, while network assignment focuses on peak hour vehicle trips. Another problem is that uncertainties (or prediction errors) are compounded over the four steps of the travel demand model. Mispredictions at early stages of the multi-stage model (e.g., trip generation) may be amplified at later stages (Zhao and Kockelman, 2002).

One attempt to improve the model's application introduces travel time feedback loops among various steps. Considering that the components of travel demand models are highly interrelated, the feedback approach involves iterative calculations through the model sequence to reach an equilibrium state. However, many applications of travel demand forecasting lack such feedbacks (Barton-Aschman Associates, 1981; Boyce, 1998), and those that do incorporate feedbacks almost always limit these to two or fewer steps upstream in the submodel sequence. This is a significant weakness (Martin *et al.*, 1998).

Another viable alternative involves embedding the travel demand submodels components in the four-step approach within market equilibrium (Miller, 2001). Equilibrium analysis of transportation systems requires a system-oriented view of transportation and creates consistency among the components. This view is well recognized for network traffic assignment, but it can entail other transportation modeling, such as combined mode choice, trip distribution, and traffic assignment models. It is discussed in the following section.

2.3.1 Network Equilibrium

In transportation, network flow equilibria represent an interaction between congestion and travel decisions. The situation can be analyzed using two functions: (1) a performance function that describes how travel times rise with increasing demand; and (2) a demand function (e.g., a combination of trip-making, mode choice, destination choice, route choice, and departure-time decisions) expressing how travel demand adjusts as travel times increase (Sheffi, 1985).

In the example of a traffic assignment problem, a stable condition was defined by Wardrop in 1952 as “no traveler can improve his (/her) travel time by unilaterally changing routes” (Sheffi, 1985, p. 19). This is known as Wardrop’s first principle, and this network flow equilibrium is called a User Equilibrium (UE). Wardrop’s second principle can be described as a situation where overall network travel time is minimized. It is known as a System Optimum (SO). However, under an SO situation, a traveler may be able to decrease his/her own travel time by changing routes. Thus, such a situation is not behaviorally stable and unlikely to describe actual route choices; however, it can be helpful in deriving a mathematical formulation for a UE (as shown later).

Since it is unlikely that all travelers have full information about minimum travel times on every possible route and always make user-optimal route choices, another equilibrium condition is described as one where no traveler can improve his/her *perceived* travel time by unilaterally changing route and is known as a Stochastic User Equilibrium (SUE) (Danganzo and Sheffi, 1977).

Although UE and SUE concepts are intuitive and logical, some questions arise as to their existence, uniqueness, and stability. The issue of mathematically formulating and solving the equilibrium problems are shown to be essential, if not critical, to such concerns.

UE and SO Assignment

The key to studying network equilibria (and their existence, uniqueness, and stability) is tied to formulating the network problems as equivalent mathematical problems. For example, the SO problem is fundamentally described as a total travel time minimization problem subject to the constraint that path flows distribute the O-D demand.

Beckmann *et al.* (1956) suggested that the UE problem is equivalent to an artificial minimization problem. To illustrate this transformation, Sheffi's (1985) notation is used here. Given a network $G(N, A)$ with N nodes and A links, with a positive monotonically increasing link performance (travel time) function $t_a(x_a)$ of flow x_a on link $a \in A$, the UE trip assignment distributes the fixed demand $\vec{q}$ such that no individual can improve his/her route choice. Thus, the minimization problem is as follows:

$$\min_{\vec{x}} \quad Z(\vec{x}) = \sum_a \int_0^{x_a} t_a(w)dw \tag{2.3.1a}$$

subject to:

$$x_a = \sum_{rs} \sum_k f_k^{rs} d_{a,k}^{rs} \tag{2.3.1b}$$

$$\sum_{k \in K^{rs}} f_k^{rs} = q_{rs}, \quad \forall r, s \tag{2.3.1c}$$

$$f_k^{rs} \geq 0 \tag{2.3.1d}$$

where K^{rs} is the set of paths between O-D pairs *r* and *s*; f_k^{rs} is the flow on path *k* between O-D pairs *r* and *s*; $d_{a,k}^{rs} = 1$ if the path flow f_k^{rs} uses link *a*, and 0 otherwise. The objective function is to minimize the cumulative system travel time, which is measured by the sum of the integral $\int_0^{x_a} t_a(w)dw$. Condition (2.3.1b) describes the connection between link flows and path flows, condition (2.3.1c) ensures that all demands are distributed on the network, and condition (2.3.1d) represents the non-negativity constraints.

To show the equivalency, first build the Lagrange function of the minimization problem:

$$L\left(\vec{x}(\vec{f}), \vec{l}\right) = Z\left(\vec{x}(\vec{f})\right) + \sum_{rs} l_{rs}\left(q_{rs} - \sum_k f_k^{rs}\right), or$$
$$L\left(\vec{f}, \vec{l}\right) = Z\left(\sum_{rs}\sum_k f_k^{rs} d_{a,k}^{rs}\right) + \sum_{rs} l_{rs}\left(q_{rs} - \sum_k f_k^{rs}\right) \tag{2.3.2}$$

where l_{rs} is the Lagrangian multiplier. The Lagrange function has to be minimized with respect to nonnegative path flows, f_k^{rs}. Notice that the condition (2.3.1b) is used to replace the decision variable from x_a to f_k^{rs}. Then, the Karush-Kuhn-Tucker[1] (KKT) conditions (see Bazaraa and Shetty, 1979) for the minimization problem are the following:

[1] The Karush-Kuhn-Tucker optimality conditions in this case are also considered as the first-order conditions for a minimization problem with nonnegativity constrains. For example, the solution of the program $\min_{\vec{x}} \vec{z}(\vec{x})$ subject to: $x_i \geq 0, i = 1, 2, ..., I$ can occur either for a positive $\vec{x}$ (i.e., $\nabla\vec{z}(\vec{x}^*) = 0$), or it can be on the boundary of the feasible region, where some $x_i^* = 0$. So, the optimal conditions can be written as:

$$x_i^* \frac{dz(\vec{x}^*)}{dx_i} = 0, \text{ and } \frac{dz(\vec{x}^*)}{dx_i} \geq 0, \ \forall i$$

$$\frac{\partial L(\vec{x}(\vec{f}), \vec{l})}{\partial f_l^{rs}} \geq 0 \tag{2.3.3a}$$

$$f_l^{rs} \frac{\partial L(\vec{x}(\vec{f}), \vec{l})}{\partial f_l^{rs}} = 0 \tag{2.3.3b}$$

$$\frac{\partial L(\vec{x}(\vec{f}), \vec{l})}{\partial l_{rs}} = 0 \tag{2.3.3c}$$

Solving conditions (2.3.3a~c):

$$\frac{\partial L(\vec{x}(\vec{f}), \vec{l})}{\partial f_l^{rs}} = \frac{\partial Z(\vec{x}(\vec{f}))}{\partial f_l^{rs}} + \frac{\partial l_{rs} \sum_{rs} (q_{rs} - \sum_k f_k^{rs})}{\partial f_l^{rs}} = \frac{\partial Z}{\partial f_l^{rs}} - l_{rs} \geq 0 \tag{2.3.4a}$$

$$f_l^{rs} \left(\frac{\partial Z}{\partial f_l^{rs}} - l_{rs}\right) = 0 \tag{2.3.4b}$$

$$q_{rs} = \sum_k f_k^{rs} \tag{2.3.4c}$$

Notice that (2.3.4c) is identical to (2.3.1c), and

$$\frac{\partial Z}{\partial f_l^{rs}} = \frac{\partial}{\partial f_l^{rs}} \sum_a \int_0^{x_a} t_a(w) dw = \sum \frac{\partial \int_0^{x_a} t_a(w) dw}{\partial x_a} \cdot \frac{\partial x_a}{\partial f_l^{rs}} = \sum_a t_a(x_a^*) d_{a,l}^{rs} = C_l^{rs} \tag{2.3.5}$$

where C_l^{rs} is the total travel time along path *l* for O-D pair *r s*.

Then conditions (2.3.4a) and (2.3.4b) become the following:

$$l_{rs} \leq C_l^{rs} \tag{2.3.6a}$$

$$f_l^{rs} (C_l^{rs} - l_{rs}) = 0 \tag{2.3.6b}$$

Inequality (2.3.6a) suggests that l_{rs} is actually the minimum travel time between *r* and *s*, and (2.3.6b) shows that the path flow equals zero if the associated path travel time exceeds the minimum travel time and that the associated path travel time is equal to the minimum travel time, if the path flow is not zero. This precisely describes a UE situation. So, the first-order conditions for the minimization program are identical to the UE conditions.

The existence and uniqueness of user equilibria arise because the linear constraints set (i.e., [2.3.1b] and [2.3.1c]) is convex and the objective function *Z* is strictly convex on x_a (since $t_a(x_a)$ and is a positive, monotonically increasing function, so its integral is convex). However, one should recognize that the strict convexity (and thus uniqueness) of the UE problem is only on link flows, not on path flows (Sheffi, 1985).

Smith (1981) developed conditions that ensure the existence of an equilibrium solution to the assignment problem with both capacity constraints and junction interactions (i.e., link flows restricted by their capacities and one link's flow influenced by other flows [e.g., at intersection]). His sufficiency conditions are that link travel cost is a continuous function of flows and that costs go to infinity as flows near capacity.

Another approach to identifying conditions for the existence of a UE uses game theory. Dafermos and Sparrow (1969) raised the question of the relationship between network UE and Nash equilibria for noncooperative games. It was shown later that the two concepts are, indeed, equivalent (Rosenthal, 1973; Devarajan, 1980). The proof also uses the equivalent mathematical formulation, so this approach is not truly distinct.

An SO problem can be formulated similar to the above minimization problem by changing the objective function to the following:

$$\min_{\vec{x}} \quad Z(\vec{x}) = \sum_a x_a \cdot t_a(x_a) \qquad (2.3.7)$$

subject to conditions (2.3.1b~d).

This SO problem has a unique solution if all link performance functions are convex.

Heuristic equilibration techniques were used extensively before UE solution algorithms were developed. For example, capacity-restraint methods tried to capture the equilibrium nature of traffic assignment via iteration, but they did not guarantee convergence (Sheffi and Daganzo, 1978) because flows may "flip-flop" between some links and never be assigned to other links. Another heuristic method, incremental assignment, assigns a portion of the O-D demand matrix at each iteration. This method may not converge to a UE (Ferland *et al.*, 1975); but, as the number of increments increases, the assignment may generate a flow pattern close to UE.

The famous Frank-Wolfe (Frank and Wolfe, 1956) algorithm (also called a convex combination algorithm) is proven to be suitable to solve the UE problem (Murchland, 1969, LeBlanc *et al.*, 1975). In this Newton-Raphson-type algorithm, a minimization problem is solved by first deriving a descent direction $\vec{d}$ due to changing the decision variables $\vec{x}$, and then choosing a step size a . The decision variables in the next iteration are a convex combination of the current solution and the descent direction. To find the descent direction $\vec{d}^{\,n}$ at iteration *n*, one will actually find a feasible auxiliary solution $\vec{y}^{\,n}$, and $\vec{d}^{\,n} = \vec{y}^{\,n} - \vec{x}^{\,n}$ supposing $\vec{x}^{\,n}$ is given.

To solve $\min Z(\vec{x})$ with linear constraints like (2.3.1), a lower bound on $Z(\vec{x})$ is derived from a first-order Taylor series (i.e., linear) approximation:

$$Z_L^n(\vec{y}) = Z(\vec{x}^n) + \nabla Z(\vec{x}^n)(\vec{y} - \vec{x}^n)^T \quad (2.3.8)$$

Since $Z(\vec{x}^n) - \nabla Z(\vec{x}^n)\vec{x}^n$ is a constant (given $\vec{x}^n$), a bound for improving $\vec{y}$ can be found by minimizing $\nabla Z(\vec{x}^n)\vec{y}^T$. Denote the lower bound as $\vec{y}^n$. Then $\vec{d}^n = \vec{y}^n - \vec{x}^n$. And the solution to the step size a^n is derived by minimizing:

$$Z\left(\vec{x}^n + a^n(\vec{y}^n - \vec{x}^n)\right) \quad (2.3.9)$$

The optimal step size is determined by using the Bolzamo line search method (i.e., the bisection line search, see [Sheffi, 1985]). And the decision variables are updated as follows:

$$\vec{x}^{n+1} = \vec{x}^n + a^n(\vec{y}^n - \vec{x}^n) = (1 - a^n)\vec{x}^n + a^n\vec{y}^n \quad (2.3.10)$$

Thus, equation (2.3.10) is the convex combination of $\vec{x}^n$ and $\vec{y}^n$.

When the Frank-Wolfe algorithm is applied for UE solutions, (2.3.8) is derived as follows:

$$\nabla Z(\vec{x}^n)\vec{y}^T = \sum_a t_a^n y_a \quad (2.3.11)$$

Equation (2.3.11) is the total travel time assuming link travel times t_a^n. Solving the minimization of total travel time is simplified by applying an "all-or-nothing" assignment (i.e., assigning all demands between each r and s to their current shortest paths).

This Frank-Wolfe algorithm is similar to the capacity restraint method, except that the latter does not calculate an optimal step size. Use of a fixed step size or a successive average ($a = \frac{1}{n+1}$) will easily improve the standard capacity restraint method in achieving the UE solution.

Nguyen (1974) suggested an even faster convex simplex algorithm. Florain and Nguyen (1974) also proposed a reduced-gradient method and a modified reduced-gradient method for the UE problem. Both the Frank-Wolfe algorithm and the convex simplex method converge very fast when networks are uncongested. As congestion increases, the algorithms take a longer time to converge (Sheffi, 1985). If convergence is achieved, the unique solution to the equilibrium is approached, since the objective function is strictly convex. Thus, the existence and uniqueness of these algorithm solutions are guaranteed.

Since the UE concept has been widely accepted and applied to trip assignment models, it is nature to compare a real world situation with the UE solutions (and it is a rather common question in all equilibrium modeling processes). Bovy and Jansen (1981) showed the flows resulting from a UE assignment solution are very close to the actual traffic counts in Eindhoven, Netherlands. However, as suggested by Horowitz (1984), it is not clear that this kind of evidence can be interpreted to mean that the UE assumption is valid or violated because the approximations underlying the computations (i.e., network representations, performance functions, demand data, and models) may be inadequate. This leads to our previous interpretation on equilibrium concepts in LU-T models that the equilibrium is just a hypothesis and it could be never observed in real world.

SUE Assignment

Sheffi and Powell (1982) proved that the SUE problem based on a logit (or Probit) model of path choice also has an equivalent minimization formulation. This can be written as follows:

$$\min_{\vec{x}} \; Z(\vec{x}) = -\sum_{rs} q_{rs} S_{rs}[C^{rs}(\vec{x})] + \sum_{a} t_a(x_a)x_a - \sum_{a}\int_0^{x_a} t_a(w)dw \qquad (2.3.12)$$

subject to conditions (2.3.1b~d).

where $S_{rs}[C^{rs}(\vec{x})] = E[\min_{k\in K^{rs}}\{C_k^{rs}\} \mid C^{rs}(\vec{x})]$ (2.3.13)

Equation (2.3.13) is the expected perceived minimum travel time function. It is concave with respect to $C^{rs}(\vec{x})$ and its first derivative is the following:

$$\frac{\partial S_{rs}(\vec{C}^{rs})}{\partial C_k^{rs}} = P_k^{rs} \qquad (2.3.14)$$

where P_k^{rs} is the probability of choosing path k between r and s.

As shown by Sheffi (1985), (2.3.12) is strictly convex in the vicinity of the minimum, but not necessarily convex elsewhere. Nevertheless, its minimum is a global one. Thus, the existence and uniqueness of the SUE solution are certain.

One efficient algorithm for solving SUE problems is the method of successive averages (MSA). Again, the step size of each iteration is $a = \frac{1}{n+1}$ (Sheffi and Powell, 1981). In practice, an MSA solution usually converges (Sheffi, 1985).

Besides the equivalent optimization problem approach, alternative methods to show the solution existence and uniqueness of UE/SUE problems include variational inequalities and the fixed-point theory (Patrickson, 1994). Dafermos (1980) first proposed a fixed-point model for equilibrium assignment on road networks with fixed demand; her work also provided results for convergence analysis. More recently, the fixed-point approach has been adopted as a general framework to define UE and SUE problems and develop solution algorithms (see, Cantarella and Cascetta, 1995; Cantarella, 1997).

Virtually all algorithms for UE or SUE applications adopt an initial state as a starting point (to the equilibrium solution process), which usually assumes free-flow speeds (minimum travel times). However, the initial state may affect convergence. Beckmann *et al.* (1956) suggested that the UE would not necessarily be reached from an arbitrary initial state, even if equilibrium exists and is unique.

Using a two-link network, Horowitz (1984) investigated the stability of UE and SUE. He suggested that the stability of network equilibria depends on the relation between past network performance and current route choice decisions. In other words, it depends on how much the travelers know about the network's travel time. Thus, only under relatively restrictive assumptions concerning the route choice decision-making process (e.g., full information and user optimization), will the link flows converge to their equilibrium values. Moreover, even when convergence to equilibrium is assured, this convergence may take place too slowly to be of practical consequence.

2.3.2 Extensions of Equilibrium Modeling in Transportation

Based on the UE and SUE equilibrium principles for network flow equilibrium, a number of model specifications have been proposed for the overall transportation market equilibrium to include the remaining travel demand components (i.e., trip frequencies, destination choices, and mode choices). These remaining components are usually considered to be "higher-level" demand patterns which are linked tightly to the network equilibrium (Fermandez and Friesz, 1983). At an overall equilibrium, the travel pattern should exhibit stability that simultaneously encompasses all of the travel demand components. For example, at a UE-type equilibrium, no traveler should be able to unilaterally change his/her trip tendency, destination choice, mode choice, nor route choice without incurring higher "costs" (i.e., lower satisfaction).

Evans (1970) is one of the pioneers who extended the UE model to include a trip distribution model. She combined the "gravity"-type distribution model with a UE assignment model and formulated them into one mathematical problem as the following:

$$\min_{\vec{q},\vec{x}} \quad Z(\vec{q},\vec{x}) = \frac{1}{b}\sum_r \sum_s \left(q_{rs}\ln q_{rs} - q_{rs}\right) + \sum_a \int_0^{x_a} t_a(w)dw \tag{2.3.15a}$$

subject to:

$$x_a = \sum_{rs}\sum_k f_k^{rs} d_{a,k}^{rs} \tag{2.3.15b}$$

$$\sum_{k\in K^{rs}} f_k^{rs} = q_{rs}, \quad \forall r,s \tag{2.3.15c}$$

$$\sum_s q_{rs} = O_{r,} \qquad \forall r \tag{2.3.15d}$$

$$\sum_r q_{rs} = D_{s,} \qquad \forall s \tag{2.3.15e}$$

$$f_k^{rs}, q_{rs} \geq 0 \tag{2.3.15d}$$

where O_r and D_s are fixed, exogenous trip productions and attractions.

The objective function (2.3.15a) consists of two components. The first term corresponds to Wilson's (1967) "entropy maximizing," doubly-constrained spatial interaction model, and the second is the same to the UE model in (2.3.1). The optimality conditions of (2.3.15) imply the double constraint trip distribution model as the following:

$$q_{rs} = A_r B_s O_r D_s \exp(-bu_{rs}) \tag{2.3.16}$$

where $$A_r = \frac{1}{\sum_s B_s D_s \exp(-bu_{rs})} \qquad \forall r \tag{2.3.17a}$$

$$B_s = \frac{1}{\sum_r A_r O_r \exp(-bu_{rs})} \qquad \forall s \tag{2.3.17b}$$

and u_{rs} is the Lagrange multiplier associated with the constraint (2.3.15c), and b is an impedance parameter characterizing travelers' sensitivity to distance.

The objective function (2.3.15a) is strictly convex, since both terms are strictly convex functions. Therefore, there is a unique equilibrium solution.

Evans (1976) proposed a very efficient algorithm to solve her combined trip distribution and network assignment model. This technique is related to the Frank-Wolfe algorithm but only constructs a partial linearization of the objective function in finding a search direction. Friesz (1985) pointed out that Evans' method is only appropriate when the travel demands are consistent with a doubly-constrained spatial interaction model.

Florian and Nguyen (1977) further extended Evan's model to include mode split. Their model combined trip distribution, mode split (among automobile and bus), and UE assignment models as the following:

$$\min_{\vec{q}^{auto}, \vec{q}^{bus}, \vec{x}} Z(\vec{q}^{auto}, \vec{q}^{bus}, \vec{x}) = b \sum_r \sum_s (q_{rs}^{auto} \ln q_{rs}^{auto}) + \sum_r \sum_s q_{rs}^{bus} (\ln q_{rs}^{bus} + C_{rs}^{bus}) + \sum_a \int_0^{x_a} t_a(w)dw \tag{2.3.18a}$$

subject to:

$$x_a = \sum_{rs} \sum_k f_k^{rs,auto} d_{a,k}^{rs} + f_a^{bus} \tag{2.3.18b}$$

$$\sum_{k \in K^{rs}} f_k^{rs,auto} = q_{rs}^{auto}, \quad \forall r,s \tag{2.3.18c}$$

$$\sum_s (q_{rs}^{auto} + q_{rs}^{bus}) = O_{r,} \qquad \forall r \tag{2.3.18d}$$

$$\sum_{r} (q_{rs}^{auto} + q_{rs}^{bus}) = D_{s,} \qquad \forall s \tag{2.3.18e}$$

$$f_k^{rs}, q_{rs}^{auto}, q_{rs}^{bus} \geq 0 \tag{2.3.18d}$$

where q_{rs}^{auto} and q_{rs}^{bus} are automobile and public transit trip demands between *r* and *s*, and C_{rs}^{bus} is the fixed public transit cost between *r* and *s*.

The first and second terms measure the entropy of automobile and bus flows, which are consistent with the mode split and trip distribution models through the optimization conditions. The drawbacks of this model are the limitation to two modes and the fixed bus costs, which do not reflect the network congestion effects on bus if the public transit network is not separable.

Florian and Nguyen (1977) formulated a modified Frank-Wolfe algorithm to solve their model, where the direction-finding step is a Hitchcock transportation problem (i.e., a linear programming problem distributing flows using fix link costs, see [Sheffi, 1985]).

Safwat and Magnanti (1988) developed an overall transportation system equilibrium model, which encompasses all four travel demand components. Their simultaneous transportation equilibrium model (STEM) is based on the UE framework, and the objective function includes two entropy components, for the trip generation and trip distribution models. STEM's mode split component is incorporated by using separate sub-networks (for each mode). Trips are generated at origins based on zonal accessibility (defined as the expected maximum utility of trips and a composite variable that accounts for non-transportation factors, such as residential density and employment numbers. Trips are distributed according to a logit-type random utility function, based on the minimum trip costs between origins and destinations and the attractiveness of destinations.

Safwat and Walton (1988) proposed two solution algorithms for STEM. The first is an extension of the Frank-Wolfe algorithm, which determines a feasible direction at each iteration through a local linearization of the objective function. The second extends the Evans' algorithm and utilizes a partial linearization technique.

A more general transportation equilibrium model was suggested by Dafermos (1982) in the form of variational inequality (VI). This model assumes a non-separable cost function[2] whose Jacobian matrix is positive definite and a non-separable disutility function whose Jacobian matrix is positive definite. The cost function is as follows:

$$c_a(\vec{f}) = \vec{G} \cdot \vec{f} + \vec{b} \qquad (2.3.19)$$

where $\vec{G}$ is a matrix capturing interactions among network links, and $\vec{b}$ is a vector reflecting fixed, base costs for each link.

The "disutility function" is taken to be the inverse function of travel demand, and has the following form:

$$d_{rs}^{m}(q_{rs}^{m}) = \vec{M} \cdot q_{rs}^{m} + \vec{s} \qquad (2.3.20)$$

where $\vec{M}$ is a matrix capturing travel disutility interactions among O-D flows, and $\vec{s}$ is a vector containing fixed, base disutilities between O-D pairs.

[2] Separable cost functions require that a link's travel time or cost depends only on its own. A non-separable cost function relaxes this restriction and recognizes interactions between link flows.

Then, a network flow and travel demand pattern $(\vec{f}, \vec{q})$ is an overall transportation market equilibrium with combined trip generation, trip distribution, mode split, and traffic assignment if it satisfies the following VI problem:

$$\vec{c}(\vec{f}^*)^T(\vec{f}^* - \vec{f}) - \vec{d}(\vec{q}^*)^T(\vec{q}^* - \vec{q}) \geq 0 \quad \forall(\vec{f}, \vec{q}) \in K \tag{2.3.21}$$

where K is the set of feasible flow and demand patterns which is constructed by the constraints similar to those in (2.3.18).

Defermos (1980) constructed an algorithm to solve the VI problem (2.3.21) with a simple, linear link flow cost function for a single-mode network. The complexity of the algorithm comes from the calculation of the eigenvalues of the two interaction matrices, $\vec{G}$ and $\vec{M}$. This can be problematic when applied to large-scale networks. However, other standard VI algorithms may be suitable for larger problems, (e.g., the projection method and the relaxation method, see Nagurney, 1999).

Another general approach to describe an overall network equilibrium follows classic microeconomic consumer demand theory. Oppenheim's (1995) "trip consumers" (TC) approach maintains consistency between travelers' decisions and aggregate network equilibrium conditions by linking the utility of individuals to aggregate-level optimization problems. Oppenheim showed that the TC approach is particularly general and several other models, including UE models, Evans' model, and STEM, can be considered special cases of his approach.

The TC approach formulates the travel demand problem as a consumer utility maximization problem with to a nested logit (NL) structure to represent interrelationships among the travel demand components (see Wrigley, 1985, and

Ben-Akiva and Lerman, 1985). Oppenheim suggested that Evans' algorithm could be applied to solve the TC demand equilibrium problem, once the parameters are obtained.

In summary, a number of model approaches extended the UE and SUE models and incorporate other, traditionally separated travel demand components. From a mathematical point of view, the extensions usually involve formulating all travel demand components into an optimization problem, or a VI problem. The optimization formulations often rely on logit-type behavioral models and restate the "high-level" travel demand components as entropy-maximizing terms. Several general optimization solution algorithms including the Frank-Wolfe algorithm are shown to be efficient in solving these optimization problems.

2.4 Equilibria in Integrated Transportation-Land Use Models

Integrated transportation-land use models (ITLUMs) aim to describe behavioral interactions even further upstream, between land use and transportation systems. Generally, this interaction is modeled by means of feedback mechanisms or joint determination of system components. There are many styles of such models, including those based on spatial interaction, location choice, and economic activities, e.g., Putman's Integrated Transportation Land Use Package (ITLUP) (1991), Martínez's five stage land use-transportation model "5-LUT" (1996), de la Barra's TRANUS (1994), Echenique and colleagues' MEPLAN (Hunt, 1993), and Mackett's MASTER (1990).

2.4.1 Integrated Models with Feedback Loops

The most common method of LU-T integrations relies on a feedback mechanism, linking network performance with location choices through iterative calculations. The generalized travel costs or local accessibilities resulting from the transportation models can be fed back into the residential and employment activity location models, where they are used to allocate the area's residents and workers to specific urban zones within the land use model. This allows transportation system changes to affect land utilization, which in turn feeds back its effects in the form of new levels and locations of traffic generation. Local accessibility measures play a central role in most operational models. As an integral component of such accessibility, travel cost changes become part of the mechanism used to reallocate labor, residents, retail and service activities, and, when modeled, freight flows between separated land uses (Southwourth, 1995).

The feedback mechanism is rather straightforward and particularly easy to implement with existing four-step travel demand models. The method is so widely used in practice that it is sometime regarded as the standard of the "integrated" models (Southworth, 1995; Oryani and Harris, 1996). Most of the operational land use models reviewed in Section 2.2 can incorporate such feedback mechanisms, to link to transportation models. However, this feedback method just simply ***links*** land use models with transportation models and does not truly and simultaneously ***integrate*** the two systems.

For example, ITLUP (see Putman, 1983, 1991) provides a feedback mechanism between DRAM, EMPAL, and the mode split and traffic assignment components of the travel demand models described in Section 2.3. First, EMPAL allocates employment across analysis zones in the forecast time period (period ***t***) using prior period (***t-1***) accessibility, population, and employment totals. The time interval is typically 5 years. DRAM next forecasts the future allocation of households using prior period (***t-1***) local accessibilities but also using the forecast period's distribution of zonal employment. DRAM converts the housing allocation probabilities into vehicle trips using region-specific vehicle utilization rates, which correspond to a trip generation and distribution models, eliminating the need for these two transportation submodels. Typically three trip matrices are produced based on DRAM's trip purposes. Then these are split by a binomial logit model to private and public travel modes, and private vehicle trips are allocated to the highway network using a capacity-constrained traffic assignment (see Putman, 1983, 1991). The accessibility measures of this time period are re-calculated and fed back to the DRAM model. Thus, the ITLUP system effectively contains a mode split model and a traffic assignment model. However, DRAM and EMPAL can be easily linked to standard travel demand modeling software (Watterson, 1993).

The MUSSA model is also designed to interact with a four-stage transportation model called ESTRAUS (Martínez, 1996). MUSSA feeds the transportation model land use and density information for each zone. ESTRAUS uses these data to estimate trip generation, mode split, trip distribution, and traffic assignment. Zonal accessibilities are calculated and fed back to MUSSA.

In the MEPLAN model, resulting trade flows (of labor, materials, and services) by mode are translated into (mode-specific) trip matrices and loaded on to the network(s). The travel cost results can be fed back to the SIO model for updates on trade patterns. MEPLAN also can be linked with standard travel demand modeling packages (Hunt, 1993).

Rather clearly, simply feedback loops between time periods do not guarantee an instantaneous *system* equilibrium for single time period. It also is not quite clear what should be chosen as the equilibrium criteria or whether LU-T systems are in or even near equilibrium. In practice, most attention has been paid to the transportation network's performance for evidence of equilibrium. Of course, such a network criterion does not necessarily imply a system equilibrium between land use and transportation.

2.4.2 Integrated Models with Combined Formulations

Another integration method involves the joint modeling of land use and travel choices, including simultaneous model applications. These combined models adopt constrained-optimization approaches from operations research to describe the process of urban evolution, which depends on transportation costs and activity establishment costs (e.g., Putman, 1974; and Williams and Senior, 1978).

For example, Kim (1979) extended Mill's (1972) model to a multi-modal system to show a general urban equilibrium that combines location choice, IO models of product interactions, transportation, and land rent in a linear programming structure. Kim's normative model assumes that urban land is divided into zones, and the objective is to minimize total cost associated with product flows while meeting consumption and export demands. The objective function and all constraints (related to exports, commodity flows, transportation demand and supply, land, and transit provision) are linear. Thus, a unique equilibrium solution is certain (when feasible solutions exist). Moore (1986) later extended Kim's linear model to incorporate a dynamic structure. These linear models usually describe the urban area as a grid or other regular geometry, which makes Moore's model difficult to use in practice (Abraham, 1998).

An important step forward came with the recognition that spatial interaction models could also be written as convex programming problems which could themselves be embedded within activity-allocation models (see, e.g., Wilson, 1969, 1971b; Wilson *et al.*, 1981; Erlander, 1977.) The resulting urban "allocation models" usually take the form of convex mathematical programs subject to a set of linear planning constraints. These and related discoveries led researchers to use the mathematical programming approach to pursue alternative formulations of interrelated facility location-allocation problems (see, e.g., MacGill and Wilson, 1979; Wilson *et al.*, 1981; Bertuglia and Leonardi, 1980; Boyce and Lundqvist, 1987; Los, 1979).

Kim's Model

Kim (1986) proposed an integrated urban system model with a ***nonlinear*** structure by combining Wilson's (1969) entropy-based commodity model, Mill's (1972) cost-minimizing urban activity model, Boyce *et al.*'s (1983) notions of combined transportation-facility location models, and Beckman *et al.*'s (1956) concept of equilibrated demand and supply over networks. Kim's model specifies interactions between transportation and land use intensities (but assumes no interactions between alternative modes) and is closely related to SIO model.

Specifically, Kim's combined model of "land use and density, shipment route and mode choice with network congestion" solves the following variables (1989, p. 88): E_i^r, total export of commodity *r* from zone *i*; x_i^r, production of commodity *r* in zone *i* ; x_i^{rs}, production of commodity *r* with *s*-intensity of land input in zone *i*; x_{ij}^r, the units of *r* shipped from zone *i* to zone *j*; x_{ijp}^{rk} , the units of *r* shipped from *i* to *j* by path *p* on travel mode *k*; and $C_a^k(x)$, the generalized cost of travel (shipment) by mode *k* on link *a* at flow volume of *x*. $\sum_i x_{ij}^r$ represents the total amount of commodity *r* shipped to zone *i* from all other origins, and $\sum_j x_{ij}^r$ represents the total amount of commodity *r* shipped from *i* to all other destinations.

The exogenously variables include: E_r, the total export of commodity *r* from the urban area as a whole; d_i^r, the unit cost of exporting commodity *r* from each zone *i*, if *i* belongs to the set of export zones I^e; g^r, the passenger car equivalent of road space occupancy required for shipping commodity *r* ; d_{ijp}^{rk}, the incident matrix which equals 1, if route *p* from zone *i* to *j* by mode *k* includes link *a* for shipping *r* and equals 0, otherwise; S_r, the minimum level of spatial interaction (entropy) in the system for commodity *r*; *L,* the opportunity cost of land at urban periphery (it is assumed that as much land as needed can be rented by expanding the regions); *R,* the rental rate of a unit of capital (it is assumed that unlimited amounts of capital can be acquired at this rental rate); and a_{qrs}, the amount of input *q* required per unit output *r* with the *s* production technique when production takes place in an area at *s*-intensity of land use (e.g., an *s*-story building). Here *q* ranges from 1 to *r* + 2, in which the range 1 to *r*+1 represents input of produced goods), *r* represents labor input, *r* + 1 represents land inputs, and

r + 2 represents capital inputs. The range r = 1 to r+1 can specify typical production sectors, such as service, retail, and manufacturing. Sector r is the household sector, and each of households consumes some of each good produced plus housing. (Goods imported into the urban area for use by households are not in the model).

Kim's model takes the form of the following nonlinear optimization problem:

$$\min_{x,f} \quad Z = \sum_{k,a} \int_o^{f_a^k} t_a^k(w)dw + \sum_{i \in I^e, r} d_i^r E_i^r + \sum_{i,r,s} \left[L(a_{r+1,r,s} x_i^{rs}) + R(a_{r+2,r,s} x_i^{rs}) \right] \qquad (2.4.1a)$$

subject to:

$$f_a^k = \sum_r g^r \sum_{i,j,p} x_{ijp}^{rk} d_{aijp}^{rk} \quad \forall a, k \qquad (2.4.1b)$$

$$\sum_i E_i^r \geq E_r \quad \forall r \qquad (2.4.1c)$$

$$\sum_j x_{ji}^r + x_i^r \geq \sum_j x_{ij}^r + \sum_{q,s} a_{qrs} x_i^{qs} + E_i^r \quad \forall r, i \qquad (2.4.1d)$$

$$-\sum_{i,j,k} (\sum_p x_{ijp}^{rk}) \ln(\sum_p x_{ijp}^{rk}) \geq S_r \quad \forall r \qquad (2.4.1e)$$

The objective function *joins* a UE flow assignment objection function, the total costs of exporting commodities out of the region; and the total land plus capital costs summed over all zones, commodities, and production techniques used in the system. Constraint (2.4.1b) ensures that the model-assigned link traffic volumes equal the volumes assigned to all origin-to-destination paths using that link, and (2.4.1c) constrains zonal exports of each commodity r to match given totals. (2.4.1d) ensures that the total amount of commodity r produced in zone i plus any imports from other zones is at least equal to the amount sent to other zones, used in other sectors, and exported from the zone (via a typical IO relationship. (2.4.1e) ensures that a minimum level of entropy (spatial dispersion) in destination and mode choices takes place. These can be solved as a nested logit model. Note that these entropy terms appear as a set of constraints, rather than as part of the objective function (unlike Wilson's [1987] and Oppenheim's [1995] models).

Solution of the nonlinear problem yields a combined supply-demand balance supported by an allocation of activity levels to zones which ensure that the marginal cost of producing *r* at location *i* plus the equilibrium unit shipment cost from *i* to *j* by mode *k* should equal the marginal cost of producing *r* at location *j*. Also at equilibrium, commodity *r* in zone *i* will be produced at intensity level *s* as long as the net benefit associated with doing so is at least equal to the capital (*R*) plus land (*L*) costs of producing a unit of *r* in *i* at that intensity level. The feasible region is not convex since (2.4.1e) is nonlinear; thus, the whole formulation is not a convex programming problem and the uniqueness of the equilibrium solutions is uncertain.

In Kim's model, at equilibrium conditions, the route choice UE is achieved, along with the production and spatial equilibria, the export zone-choice equilibrium, and the interzonal shipment equilibria. Kim (1989) calibrated a version of this model for the Chicago area, at a rather aggregate spatial level, using various data sources. The model's extensive data requirement, its sophisticated formulation, and its substantial requisite computations limit its usage. Nevertheless, the approach demonstrates the possibility of bringing important aspects of urban economic theory into intersectoral, spatial-interaction-based discrete choice models in order to move towards more comprehensive urban modeling frameworks. The model does not contain a procedure for translating its activity allocations into actual land use arrangements within zones. However, it does operate directly upon detailed representations of modal (highway and rail transit) transportation networks. Moreover, Kim's model is often regarded as a normative approach rather than a (preferred) positive approach (Southworth, 1995).

Considering the interaction between public and private sectors, Boyce and Kim (1987) defined a bi-level programming problem where the upper problem is a welfare maximization problem and the lower problem is a UE problem. Bard and Falk (1982) suggested that such bi-level problems are not convex; and, therefore, there may be multiple local optimal solutions.

The POLIS Model

Among ITLUMs relying on a single mathematical formulation, Prastaco's POLIS model (Prastacos, 1986; Caindec and Prastacos, 1995) is one now used in actual planning practice (within the San Francisco Bay Area). POLIS incorporates a number of the theoretical developments introduced throughout this review. The model can be stated as a single mathematical program which seeks to maximize jointly the locational surplus associated with multimodal travel to work, retail, and local service sectors, as well as the agglomeration benefits accruing to basic-sector employers (Prastacos, 1986).

The joint objective function incorporates two spatial entropy terms, two travel cost terms (both for work and service-sector trips, respectively), and a term which adjusts the zonal distribution of basic employment within the region The objective function is maximized subject to a significant number of linear constraints. These include the usual non-negativity constraints on all flow and stock variables as well as constraints to ensure consistency between the flows (work trips, dollars of retail and service expenditures) generated by the model and the number of workers and households in each zone. They also include a set of linear planning constraints which both ensure consistency between the amount of residential and industrial land available in each zone and the additional amount of new housing and new employment assigned to those zones by the model. Finally, zonal totals for households and jobs are reconciled with county-wide sectoral as well as spatial totals in a manner that reflects the spatial agglomeration economies of basic sector activity at this more macro-spatial level. The joint objective function appears to be strict convex, in addition, all the constraints are linear, which ensure the solution is unique.

One major weakness of the POLIS model is that the network route choice model is based on a normative SO solution (rather than a UE) and does not represent the congestion-sensitive transportation network assignment.

Network Equilibrium Extensions

Kim's model and many other, similar models (e.g., Boyce et al, 1983; Boyce and Lundqvist, 1987; and Chu, 1999) can be considered as extensions of the network UE equivalent mathematical problem. These extensions can combine various type of land use models under a general network equilibrium framework. For example, Chu's (1999) model is based on a UE framework plus a transformed

EMPAL model to jointly determine employment location and travel choices. An equilibrium is achieved if no user can improve his/her utility by unilaterally changing his/her employment location, travel mode or trip routing. Shen (1995) derived two network equilibrium frameworks to combine travel and residential location choices: one is based on UE and the other is on SUE. His models combine the network equilibrium models with the DRAM model and are formulated into convex programming problems. The solutions to these models are proved to be unique. And the general network equilibrium assignment algorithms, Frank-Wolfe method and Evans' method are presented to be efficient in solving the problems.

Recent attempts by Ham, Kim, and Boyce (2000), and Kim, Ham, and Boyce (2002) combined a SIO model with a travel demand models based on the UE framework. The objective function involves minimizing system costs, which are computed as the sum of origin-destination flow distribution costs, and origin-destination flow distribution costs by mode. The function involves a sense of entropy terms and is consistent with the logit models of mode split and trip distribution. The model determines commodity flows on links by mode, and routes by mode, for each sector as a result of the dispersion of commodity flows among regions and choices of minimum-cost travel routes. Evans' algorithm was applied to solve the problem. The authors did not discuss the uniqueness of model solution. However, given the solution results from UE assignment and the entropy-maximizing approaches (Wilson, 1970b), one can easily establish the uniqueness of their equilibrium solutions.

To a certain extent, most of the LU-T optimization models may be viewed as normative in that they tend to describe how an urban system ought to behave. There are a number of reasons that such definitions of system equilibria may be unrealistic in practice. One of the conceptual concerns is the ambitious use of optimization frameworks, which seek to jointly solve for both travel activity patterns and urban activity allocations. The issue revolves around the validation and rationale of deriving a jointly optimal solution as the target for simulation. There is a general instability inherent in any human system as it evolves. Another issue is the extensive computational time and more sophisticated optimization routines required to achieve combined optimized solutions. The conceptual and computational issues create a substantial gap between state-of-the-art and state-of-the-practice models (see Boyce and Daksin, 1997).

On the other hand, such optimization mathematical programming developments open the door for analysts to understand the meaning of different model structures (Southworth, 1995). They also provide an effective mechanism to address planning restrictions (e.g., limited land) by explicitly incorporating them as constraints in the optimization problem.

Urban Dynamics

Although land market optima might explain long-run behavior, it may be inappropriate to consider long-term location and development choices under the same static system equilibrium as relatively short-term travel choices. Moreover, the temporal nature of travel prices (costs) raises the question of how and to what extension they influence urban form.

Even if an integrated system is at equilibrium, it is not necessarily at the global optimal state; it may be at one of the multiple local optimal states and hence difficult to justify (Nwosu, 1983) because most solution algorithms just search for local optima. Furthermore, transportation and land use systems involve substantial uncertainty in virtually all their components. Thus, any integrated system equilibrium should be considered in a dynamic, stochastic fashion.

Some current models (e.g., ITLUP and MEPLAN) simulate such urban dynamics by iterating urban systems through a series of discrete time intervals, and assume that at each time interval there is a quasi-equilibrium point and the system moves from one quasi-equilibrium to another by prudent use of lagged effects between some variables (Southworth, 1995). These models are referred to as quasi-dynamic models. However, such quasi-equilibria assumptions may be invalid. And it is also not clear how the interval size (i.e., time step) should be chosen.

A more elaborate modeling approach makes use of microsimulation to simulate the evolution of urban systems (see Wilson, 1981; Bertuglia *et al.*, 1981; and Allen, Engelen, and Sanglier, 1986). Although this approach is still in the early stage of application (e.g., UrbanSIM [Waddell, 2000] and TRANSIMS [Smith, 2000]), it seems rather appropriate to use such explicitly dynamic equations to microsimulate individual travel behaviors and location choices, and thus understand overall system evolution.

2.5 SUMMARY

This chapter provided a review of material related to the integrated modeling of transportation and land use. Land use models categorized as Lowry-type models, optimization models, and microeconomic and macroeconomic models. Example operational models of each of the four categories were described here. Section 2.2 reviewed literature on travel demand modeling with emphasis on network equilibrium models. The UE and SUE problems can be formulated as equivalent convex minimization problems with unique solutions. Furthermore, many extensions of the UE (or SUE) problems to incorporate other travel demand components in the traditional four-step models have a unique solution. This is true if it can be transformed into an equivalent convex mathematical problem (or an appropriate VI problem, a fixed-point problem). Wilson's entropy-maximizing theory for spatial allocation plays an important role in these formulations. The Frank-Wolfe algorithm and Evans' method are shown to be suitable for many such problems, and generally they converge to the equilibrium solutions. This convergence, however, is somewhat dependent on the initial state of the algorithm (i.e., the initial, assumed starting conditions).

Section 2.3's review focused on the integrated modeling of transportation and land use. There are two main methods for integration: one is the feedback mechanism in which individual model modules are linked by several key variables and iterative calculations are required to feed back (and forward) these variables to achieve the convergence. The second method absorbs all model components into one mathematical programming problem and simultaneously solves for the system equilibrium solutions. The latter method is often normative, but may be in either linear or nonlinear. Both may have unique solutions. When the model system extends to bi-level (or multi-level) programming, however, the global optimum cannot be easily achieved.

To summarize, existing ITLUMs exhibit the following drawbacks, among others, in their equilibrium approaches:

- Lack of system equilibrium evidence: The joint model approach involves local optimal solutions of the integrated LU-T system. However, the existence and uniqueness of the equilibrium solutions still remain uncertain, especially for the widely-used SIO models.
- Lack of validation of feedback strategies: Most models use feedback approaches which are thought to be efficient. Few studies compare alternative feedback strategies and explain the relationship between feedback mechanisms and equilibrium approaches (see Boyce, Lupa, and Zhang, 1994).

Recognizing the limitations of previous research, this study investigates the equilibrium in the widely used SIO model, its existence and uniqueness. It also generalizes the model formulation to provide more efficient solution algorithms. It provides a general formulation of the integrated SIO and transportation models as a single mathematical programming model and discusses the solution algorithms. Finally, it examines different feedback strategies for the existing linked land use-transportation models.

Chapter 3 Modeling the Equilibria in Spatial Input-Output Models

A number of operational land use-transportation models make use of spatial (or interregional, interzonal) input-output (SIO) models, including Echenique and colleagues' MEPLAN (Hunt, 1993), de la Barra's TRANUS (1994), and Kim's model (1989). MEPLAN and TRANUS are random-utility-based, and thus may be referred to as random-utility-based multiregional input-output (RUBMRIO) model. These combine traditional SIO models with a multinomial logit (MNL) model for trade and travel choices to represent the distributed nature of commodity flow patterns. The RUBMRIO model is usually solved by iteratively applying a set of equations (Hunt, 1993; de la Barra, 1994). Each equation describes a key model variable. This chapter examines the iterative solution's representation of the spatial equilibrium, its uniqueness and its stability.

3.1 Description of Spatial Input-output Models

Originally proposed by Leontief (1941), input-output (IO) analysis is a macroeconomic approach focusing on a single region's interactions via business expenditure patterns. The analysis is driven by exogenous demand for regional goods (e.g., exports). In contrast to Lowry-type models, the IO approach endogenously determines interactions of basic and non-basic industrial activities. SIO analysis extends the classical IO model to include spatial disaggregations (see Issad *et al*, 1960; Leontief and Stuart, 1963). Entropy concepts were then proposed, to establish a connection between SIO models, entropy-maximizing theory, and random utility theory (see Wilson, 1970a; Anas, 1984).

In this section, a brief description of single region IO models is presented, followed by the introduction of multiple-region SIO models and the operational transportation-land use packages based on the SIO approach. The discussion then focuses on one version of the SIO models, the RUBMIO model.

3.1.1 Single Regional IO Models

IO models characterize the interactions between various market actors (typically producers of commodities and services). The actors are usually aggregated into sectors. If one has M industry sectors, the basic IO model identifies the flow of commodities and services x^{mn} between sectors of within a single-region economy, where m is a producing sector and n is a purchasing sector (and $m, n = 1, 2, ..., M$). x^{mn} is the dollar value of sector m's output that is purchased by sector n.

The total output of any given sector of the single region economy, X^m, is given by

$$X^m = \sum_n x^{mn} + Y^m \qquad \forall m \tag{3.1.1}$$

where Y^m is the final demand for (or export of) sector m's output.

The direct purchase can be expressed as:

$$x^{mn} = a^{mn} X^n \qquad \forall m, n \tag{3.1.2}$$

where a^{mn} is the technical coefficient, representing the amount of sector m product required to produce one dollar of sector n product. So equation (3.1.1) can be rewritten as:

$$X^m = \sum_n a^{mn} X^n + Y^m \qquad \forall m \tag{3.1.3}$$

The single regional IO model assumes that equilibrium between total supply and total demand occurs, but substitution across inputs to production does not (i.e., one cannot substitute one input for another in producing any output; they are used in fixed ratios). The production technology does not change rapidly and is assumed constant over the period of model application.

In matrix notation, the IO model is as follows:

$$\mathbf{X} = \mathbf{AX} + \mathbf{Y}$$
$$\text{or} \quad \mathbf{(I - A)X} = \mathbf{Y} \tag{3.1.4}$$

where **Y** represents a vector of final demand, **X** a vector of outputs, **A** the technical coefficient matrix $\{a^{mn}\}$, and **I** the identity matrix.

Assuming that the **(I-A)** matrix is nonsingular, it is possible to solve for production levels given final demand:

$$\mathbf{X} = \mathbf{(I - A)^{-1} Y} \tag{3.1.5}$$

Thus, the equilibrium solution for **X** is deterministic in equation (3.1.5) given **Y** and **A**. The only necessary condition is that **(I-A)** matrix is nonsingular. Then x^{mn} is solved by equation (3.1.2).

3.1.2 Multiregional Input-output Models

The extension of the IO model to multiple regions was first proposed by Isard *et al* (1960), who introduced a spatial dimension into the intersectoral flow tables:

$$X_i^m = \sum_j q_{ij}^m \qquad \forall i, m \tag{3.1.6}$$

where q_{ij}^m is the flow of sector m from region i to region j, and $i, j = 1, 2, ..., J$. Equation (3.1.3) therefore becomes:

$$\sum_j q_{ji}^m = \sum_n a_i^{mn} \sum_j q_{ij}^n + Y_i^m \quad \forall i, m \tag{3.1.7}$$

where $\{a_i^{mn}\}$ is the set of technical coefficients for production processes in region ***i***.

Equation (3.1.7) describes the commodity balance condition, which requires that the flow of sector ***m***'s goods into region ***j*** equals the use of that sector's goods for producing goods of other sectors (intermediate demand) plus any final demand. Of course, a region can acquire many or even all of its inputs locally (i.e., from itself), but this is not required.

One should notice that equation (3.1.5) still holds for **X,** therefore, there is a unique solution for **X**. However, the solution for the interregional flow pattern, q_{ij}^n, is not uniquely defined by (3.1.7). Wilson (1970b) introduced the gravity model to describe the interregional relationship in SIO models. He developed an entropy-maximizing problem as follows:

$$\max_{\{q_{ij}^m\}} \quad Z = -\sum_{ijm} q_{ij}^m \ln q_{ij}^m \tag{3.1.8a}$$

subject to:

$$\sum_{ij} q_{ij}^m d_{ij}^m = D^m \quad \forall m \tag{3.1.8b}$$

$$q_{ij}^m \geq 0 \quad \forall i, j, m \tag{3.1.8c}$$

$$\sum_j q_{ji}^m = \sum_n a_i^{mn} \sum_j q_{ij}^n + Y_i^m \quad \forall i, m \tag{3.1.7}$$

where d_{ij}^m is the cost of transporting a unit of sector ***m*** from ***i*** to ***j***; D^m is the total transportation cost, which is known ***a priori***. However, this assumption is quite dubious since the total transportation cost is usually not observable in practice.

The Lagrangian is formed as the following:

$$L = -\sum_{ijm} q_{ij}^m \ln q_{ij}^m + \sum_{im} h_i^m \left(\sum_n a_i^{mn} \sum_j q_{ij}^n + Y_i^m - \sum_j q_{ji}^m\right) + \sum_m m^m \left(D^m - \sum_{ij} q_{ij}^m d_{ij}^m\right) \tag{3.1.9}$$

where h 's and m 's are the Lagrange multipliers associated with constraint (3.1.7) and (3.1.8b), respectively. Then, the Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial q_{ij}^m} = -\ln q_{ij}^m - 1 + \sum_n h_i^n a_i^{nm} - h_j^m - m^m d_{ij}^m = 0 \tag{3.1.10}$$

Equation (3.1.10) gives:

$$q_{ij}^m = \exp\left(\sum_n h_i^n a_i^{nm} - h_j^m - m^m d_{ij}^m - 1\right) \tag{3.1.11}$$

From (3.1.11), we have:

$$\frac{q_{ij}^m}{\sum_k q_{kj}^m} = \frac{\exp\left(\sum_n h_i^n a_i^{nm} - h_j^m - m^m d_{ij}^m - 1\right)}{\sum_k \exp\left(\sum_n h_k^n a_k^{nm} - h_j^m - m^m d_{kj}^m - 1\right)} = \frac{\exp\left(\sum_n h_i^n a_i^{nm} - m^m d_{ij}^m\right)}{\sum_k \exp\left(\sum_n h_k^n a_k^{nm} - m^m d_{kj}^m\right)} \quad \forall j \tag{3.1.12}$$

Note that $\sum_k q_{kj}^m$ is the total consumption of sector ***m*** in region ***i***. So equation (3.1.12) indicates that the commodity flow from ***j*** to ***i***, q_{ji}^m, equals the total consumption of ***i*** times the probability of choosing ***j*** as the flow origin. The choice probability is in the logit form and according to the shadow prices of the commodity balance and total travel cost constraints.

Similarly, from (3.1.11) we have:

$$\frac{q_{ij}^m}{\sum_k q_{ik}^m} = \frac{\exp\left(\sum_n h_i^n a_i^{nm} - h_j^m - m^m d_{ij}^m - 1\right)}{\sum_k \exp\left(\sum_n h_i^n a_i^{nm} - h_k^m - m^m d_{ik}^m - 1\right)} = \frac{\exp(-h_j^m - m^m d_{ij}^m)}{\sum_k \exp(-h_k^m - m^m d_{ik}^m)} \tag{3.1.13}$$

Equation (3.1.13) states that total production, $\sum_k q_{ik}^m = X_i^m$, is distributed according to a logit form with the shadow prices of the commodity balance and total travel costs constraints.

Although Wilson did not discuss solution uniqueness in his work, the entropy maximizing problem defined by (3.1.9) has a unique solution since the objective function is strictly concave and all the constraints are linear (providing a convex feasible region). The objective function is strictly concave because the Hessian matrix is negative definite (n.d.), i.e.,

$$\frac{\partial^2 Z}{\partial q_{ij}^m \partial q_{kl}^n} = \begin{cases} -\frac{1}{q_{ij}^m}, & i = k, j = l, m = n \\ 0, & otherwise \end{cases} \tag{3.1.14}$$

is a diagonal matrix with all diagonal elements strictly less than zero (since q_{ij}^m is assumed nonnegative[3]).

As described above, one major drawback of this Wilson's entropy-maximizing formulation is that the total transportation cost (*D*) is not usually known or observed. However, one can simply remove the total transportation cost constraint (3.1.8b) from the optimization problem (3.1.8) and absorb the total transportation cost into the objective function. Then the objective function is to maximize the entropy and to minimize the total transportation cost as the following:

$$\max_{\{q_{ij}^m\}} \quad Z = -\sum_{ijm} q_{ij}^m \ln q_{ij}^m - \sum_{ijm} q_{ij}^m d_{ij}^m \tag{3.1.15}$$

subject to (3.1.7) and (3.1.8c).

Or, changing the signs, the objective function (3.1.15) can be rewritten equivalently as follows:

$$\min_{\{q_{ij}^m\}} \quad Z = \sum_{ijm} q_{ij}^m \ln q_{ij}^m + \sum_{ijm} q_{ij}^m d_{ij}^m \tag{3.1.16}$$

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[3] Clearly, if q_{ij}^m equals zero, $-\frac{1}{q_{ij}^m}$ goes to negative infinite, the Hessian is still negative definite.

The uniqueness of the solution to (3.1.15) is also certain since the objective function is strictly concave and all the constraints are linear. The issue then arises as to whether (3.1.15) has became somewhat of a normative problem since it intends to minimize the total (system) transportation cost.

One should notice that the optimization conditions (3.1.12) and (3.1.13) have logit forms. This eventually leads the model to the random-utility-based approach as described in the following section.

3.1.3 The Random-Utility-Based Multiregional Input-Output Model

Comparing equations (3.1.13) and (3.1.14), random utility can then be adopted to describe how "industries" (including households) choose where to acquire their inputs, according to utility-maximizing or cost-minimizing principles, subject to certain constraints. For example, MEPLAN and TRANUS determine trade volumes essentially based on the following disutility function[4]:

$$-u_{ij}^{n} = p_{i}^{n} + d_{ij}^{n} + e_{ij}^{n} \qquad \forall i, j, n \tag{3.1.17}$$

where u_{ij}^{n} is the utility of purchasing one unit (one dollar) of sector ***n***'s goods from region ***i*** for use as inputs in region ***j***[5], p_{i}^{n} is the price of producing a unit of ***n*** in region ***i***, d_{ij}^{n} is the price of transporting a unit of ***n*** from ***i*** to ***j*** (which may be a

[4] In practice, there are other items in the disutility function, such as the "excess profit" made when producing a dollar of a commodity in a region, the region-specific (constant) disutility associated with producing a dollar of a commodity in a region, and the "size term" (proportional to the log of the number of sites available to a commodity in a region) (see Hunt, 1993). Additionally, the disutility function can include a "logsum" to construct a nested logit model to determine the shipping mode choice (see Jin, Kockelman, and Zhao, 2002). The introduction of these exogenously determined components does not affect the findings in this chapter.

[5] Note that this specification is independent of the user/consumer of the good. With better data (for example, a Commodity Flow Survey that specifies producing and consuming sectors, for each commodity shipped), one could make these equations user-dependent. In existing models (such as MEPLAN and TRANUS), however, they are independent.

logsum term, from lower-order mode choice, time-of-day choice, and/or transport choices within a nested logit model framework [Ben-Akiva and Lerman, 1979]), and e_{ij}^n is a random error term. If e_{ij}^n follows an i.i.d. Gumbel distribution (McFadden, 1974), then the trade volume of sector ***n*** from ***i*** to ***j*** is given by:

$$q_{ij}^n = C_j^n \frac{\exp(l^n v_{ij}^n)}{\sum_k \exp(l^n v_{kj}^n)} \quad \forall i, j, n \tag{3.1.18}$$

where l^n is a dispersion parameter (inversely related to the standard deviation of the Gumbel error terms), $v_{ij}^n = -(p_i^n + d_{ij}^n)$, the systematic utility, and C_j^n is the total consumption of commodity ***n*** in region ***j***, given by:

$$C_j^m = \sum_n a_j^{mn} X_j^n + Y_j^m = \sum_i q_{ij}^m \quad \forall j, m \tag{3.1.19}$$

where X_j^n is defined as in equation (3.1.6).

Equation (3.1.18) implies that

$$\sum_i q_{ij}^n = C_j^n \quad \forall j, n \tag{3.1.20}$$

Here we assume the final demand of each zone, Y_j^m, is known, which may be hard to determine and ideally should be endogenous (especially over the long run). One possible improvement for this model is to define a certain number of export zones (ports, airports, etc.), whose export amounts are observable, and use another logit model to distribute the export demands across production zones (Jin, Kockelman, and Zhao, 2002). The utility function is similar to (3.1.17). In fact, an equivalent way to implement export zones is to regard the expert zones as the same as the other, regular zones and impose infinite (or very large number in practice) intrazonal transportation costs to the expert zones (as well as the interzonal transportation cost among the expert zones) to prevent them producing any products. Then the model formulation and solution existence and uniqueness remain the same, as we will discuss later.

Since Leontief technology is linear, the average cost of sector *n* in zone *j* is taken to be the weighted average total consumption costs (including purchase prices at origin and transportation prices to region *j*).

$$c_j^n = \frac{\sum_i q_{ij}^n (p_i^n + d_{ij}^n)}{\sum_i q_{ij}^n} \quad \forall j, n \tag{3.1.21}$$

The average costs can be considered as the same as the prices offered by purchasers since we assume total supply equals total demand and the market clears. The sales price of sector *n* in region *j*, p_j^n, is assumed equal to its manufacture cost, which is given by the following:

$$p_j^n = \sum_m a_j^{mn} \times c_j^m \quad \forall j, n \tag{3.1.22}$$

The assumption here is that

$$\sum_m a_j^{mn} < 1 \forall j, n \tag{3.1.23}$$

which suggests that the production process is assumed efficient that to produce one dollar output, one needs less than one dollar input if profits are not considered in the accounting procedure. On the other hand, when calculating prices in (3.1.22), if one assumes that

$$\sum_m a_j^{mn} = 1 \forall j, n \tag{3.2.24}$$

then profits are considered for the balance of money flows, i.e., the sum of input expenditure equals the sum of output sales. Here we assume $\sum_m a_j^{mn} < 1 \forall j, n$.

The transportation costs, d_{ij}^n , are assumed to be fixed, implying a non-congestible network. Endogenously determined transportation costs, taking congestion into count, will be discussed in the next chapter. The dispersion parameters, l^n , are generally estimated ***a priori***, based on trade observations (for example, from the Commodity Flow Survey [BTS, 2001]).

Simultaneously solving (3.1.17) ~ (3.1.22) will provide a spatial equilibrium solution for trade volumes. An equilibrium is characterized here as a situation that satisfies all equations. In general, solving this complex set of equations requires iterative calculations. The standard algorithm, as suggested by Hunt (1993) and de la Barra (1994), can be summarized as follows:

Original RUBMRIO Algorithm, as Applied in Practice:

Given Y_i^m, a_j^{mn}, d_{ij}^n, and l^n, solve for q_{ij}^n, p_j^n, and c_i^n, for all i, j, m, n.

Step 0: Initialization. Set all x_{ij}^n, p_i^n, and c_i^n to initial values, usually zeros.

Step 1: Calculate all utilities u_{ij}^n from equation (3.1.17); calculate production levels X_i^m from (3.1.6), and consumption levels C_j^n from equation (3.1.21).

Step 2: Update all q_{ij}^n using equation (3.1.18).

Step 3: Update all c_i^n using equation (3.1.19), and p_i^n using equation (3.1.22).

Step 4: Convergence test. Check the predefined convergence criterion: for example, $\max(\left|q_{ij}^{n(t)} - q_{ij}^{n(t-1)}\right|) < 0.01 q_{ij}^{n(t-1)}, \forall i, j, n$, where t is the iteration number. If the convergence criterion is met, then stop and the current solution { q_{ij}^n } is taken to be the equilibrium solution; otherwise, go to step 1.

This iterative process is not clearly convergent. And it does not indicate whether its solution is unique (or whether it even exists). If the solution is not unique, a number of issues arise, such as which solution best represents the equilibrium of spatial interaction and how the initial values should be chosen to obtain the equilibrium solution. The following section formulates the RUBMRIO model as a fixed-point problem. The fixed-point formulation reveals that prices are based on the exogenous transportation prices and other parameters. In addition, the fixed-point formulation suggests that there is a unique solution for prices when transportation prices and other parameters are known and satisfied the weak conditions. Therefore, commodity flows are unique, once prices are determined. Such information is crucial to successful implementation of such models, since non-existence and/or non-uniqueness present serious problems for applications and predictions.

3.2 A Fixed-Point Approach to the RUBMRIO Model

The fixed-point approach is a primary mathematical tool for numerical analysis. It has been extensively used in showing the existence and uniqueness of solutions in game theory and economics (Border, 1985). Within the discipline of travel demand modeling, a number of studies make use of fixed-point formulations for trip assignment to networks. Dafermos (1980) first proposed a fixed-point model for equilibrium assignment on road networks with fixed demand; his work also provided results for convergence analysis. More recently, the fixed-point approach has been adopted as a general framework to define user equilibrium (UE) and stochastic user equilibrium (SUE) problems and develop solution algorithms (see, Cantarella and Cascetta, 1995; Cantarella, 1997).

Among other techniques (e.g., optimization formulation, variational inequality, and complementary formulation) in studying model solution existence and uniqueness, fixed-point methods is found to be successful. After investigating a variety of mathematical approaches and techniques to assess model solution existence and uniqueness, (e.g., optimization, variational inequality, and nonlinear complementarity formulations) the fixed-point method was found to be successful[6].

The fixed-point formulation of the RUBMRIO model has two objectives. First, it exposes summative relationships among key variables to improve model understanding. Second, it allows us to determine solution existence and uniqueness, as well as specify convergent algorithms, suitable for large-size problems.

3.2.1 The Fixed-Point RUBMRIO Formulation

Defining P_{ij}^{m} as the probability that region *j* purchases input *m* from region *i*:

$$P_{ij}^{m} = \frac{\exp(l^{m} v_{ij}^{m})}{\sum_{k} \exp(l^{m} v_{kj}^{m})} = \frac{\exp[-l^{m}(p_{i}^{m} + d_{ij}^{m})]}{\sum_{k} \exp[-l^{m}(p_{k}^{m} + d_{kj}^{m})]} \qquad (3.2.1)$$

Then, equation (3.1.21) can be rewritten as follows:

[6] The widely-used Newton's method is closely related to the fixed-point method. Both need the "contractive mapping theorem" for deriving the solution existence and uniqueness. And each can be derived via the other. The Newton's method solves *f(x)=0*, and the fixed-point solves *x=g(x)*. If we assume that *f(x)=g(x)-x*, then the Newton's problem and approach is the same as the fixed-point set-up. If one can show the contractiveness of the mapping *f()* or *g()*, either method can be used to prove the solution uniqueness.

$$p_j^n = \sum_m a_j^{mn} c_j^m = \sum_m a_j^{mn} \frac{\sum_i q_{ij}^m (p_i^m + d_{ij}^m)}{\sum_i q_{ij}^m}$$

$$= \sum_m a_j^{mn} \sum_i P_{ij}^m \cdot (p_i^m + d_{ij}^m) \tag{3.2.2}$$

$$= \sum_m a_j^{mn} \sum_i \frac{\exp[-l^m (p_i^m + d_{ij}^m)]}{\sum_k \exp[-l^m (p_k^m + d_{kj}^m)]} (p_i^m + d_{ij}^m)$$

In equation (3.2.2), the prices $\{p_i^n\}$ are clearly defined as a function of themselves, if the transportation prices $\{d_{ij}^m\}$, dispersion parameters $\{l^m\}$, and technical coefficients $\{a_j^{mn}\}$ are given exogenously. One should notice that the prices $\{p_i^n\}$ are *not* a function of the commodity flows $\{q_{ij}^n\}$ or consumption levels when written in this way. This suggests perfectly elastic supply, thanks to constant–rate IO technologies and an implicit lack of resource constraints.

Denote $\vec{p} = \{p_j^n\}$, and let

$$P_{ij}^m(\vec{p}) = \frac{\exp[-l^m (p_i^m + d_{ij}^m)]}{\sum_k \exp[-l^m (p_k^m + d_{kj}^m)]} \tag{3.2.3}$$

And, from equation (3.2.2), let

$$f_j^n(\vec{p}) = \sum_m a_j^{mn} \sum_i P_{ij}^m(\vec{p}) \cdot (p_i^m + d_{ij}^m) \tag{3.2.4}$$

So, we have a fixed-point problem as follows:

$$\vec{p} = \vec{f}(\vec{p}) \tag{3.2.5}$$

The elements of the function $\vec{f}$ are defined by (3.2.4). First, we impose a rather weak condition on the feasible set to ensure existence of a solution. Let $K_p = \left\{ p_{ij}^n \mid 0 \le p_{ij}^n \le p_{ij}^{n*}, \forall i, j, n \right\}$, where $\left\{ p_{ij}^{n*} \right\}$ are upper bounds which we assume can be determined a priori (in practice, one can usually choose very large numbers as upper bounds). Then K_p is a bounded and closed convex subset (therefore, a compact set) on the space R^{MJJ}. One can easily observe that, if the prices are bounded, the function $\vec{f}$ also can be considered bounded, since it is a convex combination of prices (plus transportation costs) across space (i.e., $\sum_i P_{ij}^m = 1, \forall m$) and economic sectors (i.e., $\sum_m a_j^{mn} \le 1,, \forall n, j$). If one assumes that $\vec{f}$'s upper bounds are also $\{p_{ij}^{n*}\}$, one essentially assumes that the upper bounds are large enough to accommodate the transportation prices' contributions to $\vec{f}$. Then, $\vec{f}$ is a mapping $K_p \to K_p$, and it is continuous. Following Brouwer's theorem (see Khamsi and Kirk, 2001), we then have the following condition:

Condition 1: Existence Condition for Price Solution

The fixed-point problem (3.2.5) provides at least one solution if and only if there exist positive constants $\{p_{ij}^{n*}\}$ such that the fixed-point problem (3.2.5) provides at least one feasible solution in the space K_b.

Sufficient conditions for the uniqueness of the solution of a fixed-point problem are given by Banach's theorem (see Border, 1985) which requires that the function be contractive over a complete set, or the function be quasi-contractive (implying monotonicity) over a compact set. We consider K_p is in a normed space, with norm of $\vec{p}$ denoted by $\|\vec{p}\|$. Here norm is a distance measure in the space. For purposes of definition, a function $\vec{f}$ provides contractive mapping of $\vec{p}$ if the following holds:

$$\|\vec{f}(\vec{p}) - \vec{f}(\vec{p}')\| < j \, \|\vec{p} - \vec{p}'\|, \vec{p} \ne \vec{p}', 0 < j \le 1 \tag{3.2.6}$$

Due to the mean-value theorem (see Khamsi and Kirk, 2001),

$$\vec{f}(\vec{p}) - \vec{f}(\vec{p}') = \nabla\vec{f}(\vec{d})(\vec{p} - \vec{p}') \tag{3.2.7}$$

where $\vec{d}$ is a convex combination of $\vec{p}$ and $\vec{p}'$. So one only needs to study the norm of the Jacobian matrix[7]; in other words, if $\left\|\nabla \vec{f}(\vec{p})\right\| < 1$, then the fixed-point problem has a unique solution and the sequence $\vec{p}^{(t+1)} = \vec{f}(\vec{p}^{(t)})$ converges on the unique solution $\vec{p} = \vec{f}(\vec{p})$, if $\vec{p}^{(o)} \in K_p$. This property is illustrated for four general cases in Figure 3.1.1.

[7] Matrix norm is a distance measure of the matrix space (see Border, 1985).

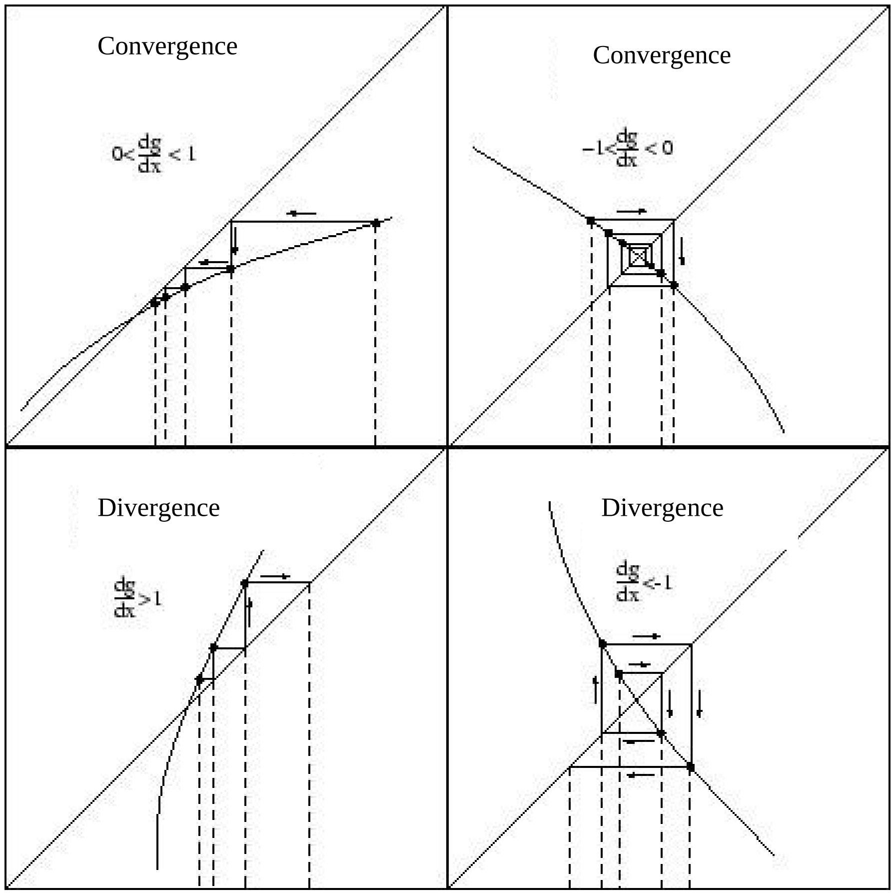

Figure 3.1.1 Examples of fixed-point problem convergence to unique solutions

Note: Fixed-point iteration for a general function $g(x)$ for four cases of interest. Positive-slope cases are shown on the left. Negative-slope cases are shown on the right. The solution sequences converge to the fixed-point solution only when the slopes lie between -1 and 1.

This is exactly what we provide in each of the following proofs for uniqueness. We first consider a simplified case where the probabilities $\{P_{ij}^{m}\}$ are fixed (i.e., they are not a function of prices). The Jacobian matrix is:

$$\nabla \vec{f}(\vec{p}) = \begin{bmatrix} \frac{\partial f_1^1(\vec{p})}{\partial b_1^1} & \frac{\partial f_1^1(\vec{p})}{\partial b_2^1} & \cdots & \frac{\partial f_1^1(\vec{p})}{\partial b_J^M} \\ \frac{\partial f_2^1(\vec{p})}{\partial b_1^1} & \frac{\partial f_2^1(\vec{p})}{\partial b_2^1} & \cdots & \frac{\partial f_2^1(\vec{p})}{\partial b_J^M} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_J^M(\vec{p})}{\partial b_1^1} & \frac{\partial f_J^M(\vec{p})}{\partial b_2^1} & \cdots & \frac{\partial f_J^M(\vec{p})}{\partial b_J^M} \end{bmatrix} = \begin{bmatrix} a_1^{11} P_{11}^1 & a_1^{11} P_{21}^1 & \cdots & a_1^{M1} P_{J1}^M \\ a_2^{11} P_{12}^1 & a_2^{11} P_{22}^1 & \cdots & a_2^{M1} P_{J2}^M \\ \vdots & \vdots & \vdots & \vdots \\ a_J^{1M} P_{1J}^1 & a_J^{1M} P_{2J}^1 & \cdots & a_J^{MM} P_{JJ}^M \end{bmatrix}$$

(3.2.8)

There are two properties of this Jacobian matrix: first, it is a positive matrix (i.e., all elements of it are strictly positive); and second, the row sums are the following:

$$\sum_{i,m} \frac{\partial f_j^n(\vec{p})}{\partial p_i^m} = \sum_i \sum_m a_j^{mn} P_{ij}^m = \sum_m a_j^{mn} (\sum_i P_{ij}^m) = \sum_m a_j^{mn}, \quad \forall j, n \qquad (3.2.9)$$

Therefore, we calculate the following the norm[8] of the Jacobian:

$$\left\| \nabla \vec{f}(\vec{p}) \right\| = \max_{\substack{1 \le j \le J \\ 1 \le n \le M}} \left| \sum_{i,m} \frac{\partial f_j^n(\vec{p})}{\partial p_i^m} \right| = \max_{\substack{1 \le j \le J \\ 1 \le n \le M}} (\sum_m a_j^{mn}) \qquad (3.2.10)$$

We also note that the technical coefficients have the following property:

$$\sum_m a_j^{mn} < 1 \forall j, n, \qquad (3.2.11)$$

Since, generally, the total value ($) of inputs required to produce one dollar of sector *n* product should be less than one dollar, if one does not wish to consider the recycling of profits (or wages) in the markets. However, if these are left endogenous and are re-spent in the region, then one could consider technical coefficients summing to 1. However, if this is done for consumption computations,

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8 According to the Norm-Equivalence Theorem (see Ortege and Rheinboldt, 1970), all norms on R^n are equivalent, here we use the l_∞ -norm (and later l_1 -norm) to obtain the sufficient results.

final demand effects will multiply infinitely through an IO model thanks to any column that sums to one (and the matrix **I-A** will not be invertible). In practice, equation (3.2.11)'s constraint is met through import or profit leakages[9], since labor is generally endogenous and represents an "industry" that can absorb all profits. Thus, $\left\|\nabla \vec{f}(\vec{p})\right\| < 1$, $\vec{f}$ is contractive on $\vec{p}$, and there exists a unique solution for the simplified, fixed-probability problem.

For purposes of price calculations, however, one needs to recognize the costs of those leakages. Those inputs are purchases outside the region, and they carry costs. So for purposes of sales price calculation, they should sum to 1.0 on the columns. In other words, one should have full-cost accounting in the price computations, but not in the consumption or flow calculations. In practice, modelers have not yet addressed this issue of leakages impacting the RUBMRIO technical coefficients (e.g., de la Barra [1994] suggest that the technical coefficients sum to 1.0 on the columns, but somehow avoid the explosion issue).

Following the same process, we now study the general situation where the probabilities are determined by relative disutilities which depend on prices.

$$\frac{\partial f_j^n(\vec{p})}{\partial p_i^m} = \frac{\partial}{\partial p_i^m}[\sum_m a_j^{mn} \sum_k P_{kj}^m(\vec{p}) \cdot (p_k^m + d_{kj}^m)] = a_j^{mn} \frac{\partial}{\partial p_i^m}\left[\sum_k P_{kj}^m(\vec{p}) \cdot (p_k^m + d_{kj}^m)\right] \tag{3.2.12}$$

[9] Money is spent outside the region, so no multiplying effects are locally/regionally generated by those expenditures.

where

$$\frac{\partial}{\partial p_i^m}\left[\sum_k P_{kj}^m(\vec{p})\cdot(p_k^m+d_{kj}^m)\right]$$

$$=\sum_{k\neq i}\frac{\partial}{\partial p_i^m}\left[P_{kj}^m(\vec{p})\cdot(p_k^m+d_{kj}^m)\right]+\frac{\partial}{\partial p_i^m}\left[P_{ij}^m(\vec{p})\cdot(p_i^m+d_{ij}^m)\right]$$

$$=\sum_{k\neq i}(p_k^m+d_{kj}^m)\frac{\partial}{\partial u_{ij}^m}P_{kj}^m(u_{ij}^m)\cdot\frac{\partial}{\partial p_i^m}(u_{ij}^m)+(p_i^m+d_{ij}^m)\frac{\partial}{\partial u_{ij}^m}P_{ij}^m(u_{ij}^m)\cdot\frac{\partial}{\partial p_i^m}(u_{ij}^m)+P_{ij}^m$$

$$=\sum_{k\neq i}(p_k^m+d_{kj}^m)P_{kj}^m\cdot(-P_{ij}^m)\cdot(-l^m)+(p_i^m+d_{ij}^m)P_{ij}^m\cdot(1-P_{ij}^m)(-l^m)+P_{ij}^m$$

$$=P_{ij}^m\cdot\{1-l^m[(p_i^m+d_{ij}^m)(1-P_{ij}^m)-\sum_{k\neq i}P_{kj}^m\cdot(p_k^m+d_{kj}^m)]\}$$

$$=P_{ij}^m\cdot\{1-l^m[(p_i^m+d_{ij}^m)-\sum_k P_{kj}^m\cdot(p_k^m+d_{kj}^m)]\}$$

$$=P_{ij}^m\cdot\{1-l^m[(p_i^m+d_{ij}^m)-c_j^m]\}$$

(3.2.13)

Then, equation (3.2.12) becomes:

$$\frac{\partial f_j^n(\vec{p})}{\partial p_i^m}=a_j^{mn}P_{ij}^m[1-l^m(p_i^m+d_{ij}^m-c_j^m)] \quad (3.2.14)$$

Notice that the second property of the Jacobian matrix in the simplified situation still holds here as well:

$$\begin{aligned}\sum_{i,m}\frac{\partial f_j^n(\vec{p})}{\partial p_i^m}&=\sum_i\sum_m a_j^{mn}P_{ij}^m\cdot[1-l^m(p_i^m+d_{ij}^m-c_j^m)]\\&=\sum_m a_j^{mn}\{\sum_i P_{ij}^m-l^m[\sum_i P_{ij}^m\cdot(p_i^m+d_{ij}^m)-c_j^m\sum_i P_{ij}^m]\}\\&=\sum_m a_j^{mn}[1-l^m(c_j^m-c_j^m)]\\&=\sum_m a_j^{mn},\quad\forall j,n\end{aligned} \quad (3.2.15)$$

If we want to apply the finding from the simplified situation (i.e., with fixed probabilities), we need to check whether all Jacobian matrix elements are positive[10]. Since the technical coefficients and the probabilities are all positive, we focus on the term $1 - l^m(p_i^m + d_{ij}^m - c_j^m)$. In general, the dispersion parameter, l^m, is positive. So we first need to discuss the values of commodity prices, transportation prices, and average costs. We examine three specific situations, which can be characterized as the following:

(i) $p_i^m + d_{ij}^m < c_j^m \quad \forall i, j, m$. The economic meaning of this condition is that the sales price at region *i* plus the transportation price to region *j* is less than the average input cost for that good *m* in region *j*. Under this condition, region *j* will purchase a positive amount of sector m from region *i*, so the derivative in equation (3.2.14) is positive (for any solution that satisfies this inequality).

(ii) $p_i^m + d_{ij}^m = c_j^m \quad \forall i, j, m$. This is a "spatial price equilibrium" situation (without randomness) (Samuelson, 1952), where the sales price at region *i* plus the transportation price to region *j* equals the average cost of input *m* in region *j* (see Nagurney, 1999). Then, equation (3.2.14)'s derivative (for any solution that satisfies this condition) is positive.

(iii) $p_i^m + d_{ij}^m > c_j^m \quad \forall i, j, m$. The economic interpretation of this condition is that the sales price at region *i* plus the transportation price to region *j* exceeds the average cost of input m in region *j*. If the prices satisfy "spatial price equilibrium", then there is no purchase from region *i* (Nagurney, 1999). However, in the RUBMRIO model, the commodity flow distribution is based on random utility theory, so there is a certain (small) amount of any commodity that will be purchased from any origin region whose sales price plus transportation cost

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[10] In fact, if the Jacobin matrix is a positive matrix, one can apply the Perron-Frobenius theorem (see, Golub and Van Loan, 1989) to study the matrix eigenvalues for the function's contractiveness property, which is equivalent to studying the norm.

exceeds the average input cost at the destination region (Abraham, 1998). Under this scenario, the condition to ensure that the Jacobian matrix elements are all positive is the following[11]:

$$l^m < \frac{1}{\max\limits_{1 \le i, j \le J} (p_i^m + d_{ij}^m - c_j^m)} \quad \forall m \tag{3.2.16}$$

Inequality (3.2.16) describes a sufficient condition wherein the dispersion parameters $\{l^m\}$ are sufficiently small: i.e., the commodity purchases are reasonably well spread over all regions. To summarize the above three conditions, we have a second condition:

Condition 2: (Restrictive) Uniqueness Condition for Price Solution

The fixed-point problem (3.2.5) results in at most one equilibrium price solution if the dispersion parameters $\{l^m\}$ *are sufficiently small such that the inequality (3.2.16) holds.*

This condition is rather restrictive for the dispersion parameters. In practice, it is common to have relatively large dispersion parameters (see Jin, Kockelman, and Zhao, 2002). We next discuss relaxed conditions under which dispersion parameters are relatively large. If the $\{l^m\}$ are very large, then the commodity flows become local and concentrated (i.e., the origin regions offering minimum total cost [sales price plus transportation price] will dominate the flow to the destination region). The flows (or the probabilities) from all other regions to this destination will be close to zero. The average cost then tends to be very close to the dominant, minimum (total) price. This satisfies the above condition (ii). Therefore,

[11] Here we impose a rather restrictive condition to ensure all the Jacobian matrix elements are positive. It is possible that the Jacobian remains p.d. even if some of its elements are negative, we will discuss that situation later.

the Jacobian matrix will have rows where the only positive elements tie to the dominant regions for each sector. Then $\vec{f}$ is contractive on $\vec{p}$, and the fixed-point problem (3.2.5) provides a unique equilibrium price solution. The contractiveness of $\vec{f}$ is important in the fixed-point sequence $\vec{p}^{(t+1)} = \vec{f}(\vec{p}^{(t)})$ calculation. This can be illustrated by a simple example. If one starts the iterative calculations from the upper bounds of *p*'s, the contractive mapping will redistribute the *p*'s values. And even given relatively large transportation prices *d*'s, the redistribution of *p*'s with a contractive function $\vec{f}$ will pull all *p*'s inside the bounds, eventually. However, this does not imply that during the iterative calculations the *p*'s will stay inside the bounds all the time; they can go out and back. But the solution will be in bounds, thanks to the contractive $\vec{f}$, due to the assumption (3.2.11).

Since the problem (3.2.5) has a unique price solution under the conditions that the $\{l^m\}$ are either sufficiently small or sufficiently large, it is natural to suspect that the uniqueness property holds with other, moderate $\{l^m\}$ values (which are also common in practice, see Jin, Kockelman, and Zhao, 2002). Suppose we sort the origin index as the following:

$$p_1^m + d_{1j}^m \le p_2^m + d_{2j}^m \le \ldots \le p_{i*}^m + d_{i*j}^m \le c_j^m \le p_{i*+1}^m + d_{i*+1,j}^m \le \ldots \le p_J^m + d_{Jj}^m \quad \forall j, m \tag{3.2.17}$$

i.e., small origin index indicates small origin sales price plus transport price. And the equal signs in (3.2.17) do not all hold in general, otherwise we have fixed probabilities as described in the simplified case above (and there is a unique solution). The probability in (3.2.1) can be rewritten as:

$$P_{ij}^m = \frac{\exp[-l^m(p_i^m + d_{ij}^m - p_1^m - d_{1j}^m)]}{\sum_k \exp[-l^m(p_k^m + d_{kj}^m - p_1^m - d_{1j}^m)]} \tag{3.2.18}$$

It is obvious that the largest amount commodity will be purchased from the region with the lowest sales price plus transportation price, and the purchase probabilities from other regions are dependent on the differences between their sales prices plus transportation prices and the lowest cost. If the difference is large enough, then the probability is close to zero. For example, if $l^m = 10$ and the difference is 2, then

$$P_{ij}^m = \frac{\exp(-20)}{1 + \sum_{k \neq 1} \exp[-l^m (p_k^m + d_{kj}^m - p_1^m - d_{1j}^m)]} < \exp(-20) = 2.06 \times 10^{-9} \approx 0$$

(3.2.19)

Now assume that for those regions under condition (iii), i.e., their sales prices plus transportation prices are larger than the average cost (and therefore larger than the lowest price), their price differences from the lowest price plus transportation price are so big that:

$$P_{ij}^m \approx 0, \; for \;\; i > i^*, \forall j, m \qquad (3.2.20)$$

i.e., the purchase probabilities can be ignored. Then under this assumption (and the rest regions satisfying situation [i] or [ii]), it is easy to obtain that:

$$\sum_{i,m} \left| \frac{\partial f_j^n(\vec{p})}{\partial p_i^m} \right| < 1 \qquad (3.2.21)$$

Then there is a unique price solution for the RUBRIO model for all dispersion parameters if the approximation (3.2.20) is practically acceptable[12]. In additionally, it is *almost certain* that the fixed-point sequence $\vec{p}^{(t+1)} = \vec{f}(\vec{p}^{(t)})$ converges.

---

[12] In fact, if the calculation precision is less than 10^{-8}, then (3.2.19) must hold.

Note that we can ignore the small purchase probabilities from regions with sales prices; plus transportation prices exceed the average cost in destination region *only* in the uniqueness analysis of the prices' fixed-point problem to obtain conditions (3.2.21). These small probabilities cannot be ignored for the following commodity flow calculation since the products of very small probabilities and very large total commodity consumptions can still result in nonzero flows. For example, if the total consumption $C_j^m = 5\times10^{11}$ ($), following (3.1.18) and (3.2.19), we have:

$$q_{ij}^m = C_j^m P_{ij}^m \approx 5\times10^{11}\times2.06\times10^{-9} = 1030 \qquad (3.2.22)$$

To summarize, the price fixed-point problem for prices allows one to develop the weak existence condition (condition 1) and sufficient uniqueness condition (condition 2). Additionally, the price solution uniqueness is discussed for more general situations and the fixed-point sequence's convergence is *almost certain*.

Once $\vec{p}$ is known, the probability vector $\vec{P}$ can be computed easily. From equation (3.2.2), we know that $\vec{p}$ can be written independent of commodity flows; in addition, from equation (3.2.3), it is clear that $\vec{P}$ is not implicitly a function of commodity flows.

From equations (3.1.9) and (3.1.10), we have:

$$q_{ij}^m = P_{ij}^m \sum_i q_{ij}^m = P_{ij}^m \cdot (\sum_n a_j^{mn} \sum_k q_{jk}^n + Y_j^m),\ \forall i, j, m \qquad (3.2.23)$$

Denoting:

$$g_{ij}^m(\vec{q}) = P_{ij}^m \cdot (\sum_n a_j^{mn} \sum_k q_{jk}^n + Y_j^m) \qquad (3.2.24)$$

produces another fixed-point problem:

$$\check{q} = \check{g}(\check{q}) \tag{3.2.25}$$

Similar to our prior discussions, we first impose a weak condition on the feasible set to guarantee existence of a solution. Let $K_q = \left\{ q_{ij}^m \mid 0 \le q_{ij}^m \le q_{ij}^{m*}, \forall i, j, m \right\}$, where $\left\{ q_{ij}^{m*} \right\}$ are upper bounds. Then, K_q is a bounded, closed, and convex subset of R^{MJJ}. Also, assume that $\check{g}$ maps $K_q \to K_q$ and is continuous. Following Brouwer's theorem (Khamsi and Kirk, 2001), we then have the following condition:

Conditions 3: Existence Conditions for Flow Solution

The fixed-point problem (3.2.25) permits at least one solution if and only if there exist positive constants $\left\{ q_{ij}^{m*} \right\}$*, such that the fixed-point problem (3.2.25) permits a flow solution in* K_q.

Again, we study the contractiveness of $\check{g}$ over K_q to obtain the sufficient conditions for the uniqueness of the flow solution. The elements of the Jacobian matrix of $\check{g}$ are:

$$\frac{\partial}{\partial q_{kl}^n} g_{ij}^m(\check{q}) = \begin{cases} P_{ij}^m a_j^{mn} & if\ k = j \\ 0 & otherwise \end{cases} \quad \forall i, j, k, l, m, n \tag{3.2.26}$$

The Jacobian matrix of $\check{g}$ is:

$$\nabla\vec{g}(\vec{q}) = \begin{bmatrix} \frac{\partial g_{11}^{1}(\vec{q})}{\partial q_{11}^{1}} & \frac{\partial g_{11}^{1}(\vec{q})}{\partial q_{21}^{1}} & \cdots & \frac{\partial g_{11}^{1}(\vec{q})}{\partial q_{JJ}^{M}} \\ \frac{\partial g_{21}^{1}(\vec{q})}{\partial q_{11}^{1}} & \frac{\partial g_{21}^{1}(\vec{q})}{\partial q_{21}^{1}} & \cdots & \frac{\partial g_{21}^{1}(\vec{q})}{\partial q_{JJ}^{M}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial g_{JJ}^{M}(\vec{q})}{\partial q_{11}^{1}} & \frac{\partial g_{JJ}^{M}(\vec{q})}{\partial q_{21}^{1}} & \cdots & \frac{\partial g_{JJ}^{M}(\vec{q})}{\partial q_{JJ}^{M}} \end{bmatrix} = \begin{bmatrix} P_{11}^{1}a_{1}^{11} & 0 & \ldots & 0 \\ P_{21}^{1}a_{1}^{11} & 0 & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & P_{12}^{1}a_{2}^{11} & \ldots & 0 \\ 0 & P_{22}^{1}a_{2}^{11} & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ P_{J1}^{M}a_{1}^{MM} & 0 & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & P_{J2}^{M}a_{2}^{MM} & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & P_{J-1,J}^{M}a_{J}^{MM} \\ 0 & 0 & \ldots & P_{JJ}^{M}a_{J}^{MM} \end{bmatrix} \tag{3.2.27}$$

There are two properties of this Jacobian matrix: first, it is a nonnegative matrix (i.e., all elements are equal to or larger than zero); second, the column sums are the following:

$$\sum_{i,j,m}\frac{\partial g_{ij}^{m}(\vec{q})}{\partial q_{kl}^{n}} = \sum_{i}\sum_{m}P_{ij}^{m}a_{j}^{mn} = \sum_{m}a_{j}^{mn}(\sum_{i}P_{ij}^{m}) = \sum_{m}a_{j}^{mn}, \quad \forall j,n \tag{3.2.28}$$

Therefore, we compute the following Jacobian matrix norm (Golub and Van Loan, 1989):

$$\|\nabla\vec{g}(\vec{q})\| = \max_{\substack{1\le k,l\le J \\ 1\le n\le M}}\left|\sum_{i,j,m}\frac{\partial g_{ij}^{m}(\vec{q})}{\partial q_{kl}^{n}}\right| = \max_{\substack{1\le k,l\le J \\ 1\le n\le M}}(\sum_{m}a_{j}^{mn}) < 1 \tag{3.2.29}$$

This implies that $\vec{g}$ is contractive on $\vec{q}$, and so there exists a unique solution for the fixed-probability problem (3.2.25), producing the following condition:

Condition 4: Uniqueness Condition for Flow Solution

The fixed-point problem (3.2.25) results in at most, one equilibrium price solution if any solution exists. Thus, the existence and uniqueness of solution flows are very general, once prices and probabilities are known.

One should notice that this condition is more relaxed than the restrictive uniqueness condition 2. Condition 4 only requires inequality (3.2.11) holds, without assumptions on other exogenous variables such as Y_j^m.

However, one assumption should be emphasized here on the technical coefficients, i.e. (3.2.23) $\sum_m a_j^{mn} < 1 \forall j, n$. If the original assumption (3.2.24) ($\sum_m a_j^{mn} = 1 \forall j, n$) is used, the above discussion is no longer valid. In practice, (3.2.23) is a rather applicable assumption due to the consideration of production efficiency, if one is not endogenizing labor and profits. However, here we do endogenize these inputs, and permit them to cycle through the productive process. Ideally, our technical coefficient rows should sum to 1 for price computations, and less than 1 for consumption flow/trade computations.

Another limitation of the production assumptions used here is that the technical coefficients are fixed. In reality, economic actors respond to price differences by shifting away from expensive inputs. Moreover, in the long-run, technologies will change (or improve), driven by innovations. Allowing technical coefficients to adjust based on commodity prices should be more realistic; but this will mean more complicated proofs of solution existence and uniqueness analysis.

In summary, through a general fixed-point approach, one can easily find an interesting relationship in the RUBMRIO model such that prices are independent of flows. Moreover, both price and flow solutions exist and are unique under the sufficient conditions described above. In the next section, we make use of the fixed-point approach to verify the convergence of the original RUBMRIO model, and we propose a modified solution algorithm, which efficiently applies the fixed-point formulation's properties.

3.2.2 A MODIFIED RUBMRIO ALGORITHM

Assuming that the conditions for the existence and uniqueness of the price solution hold, the sequence generated by the iterative function $\vec{p}^{(t+1)} = \vec{f}(\vec{p}^{(t)})$ converges on the unique solution $\vec{p} = \vec{f}(\vec{p})$, if $\vec{p}^{(o)} \in K_p$ (see Khamsi and Kirk, 2001). This price convergence does require that prices are bounded. But one can easily construct lower bounds of zero (since prices should be non-negative) and upper bounds as very large numbers. Additionally, the discussion about the dispersion parameters' values suggests that the fixed-point sequence converges *almost sure* in practice. Once the unique solution for prices is obtained (through the fixed-point sequence), a similar sequence for computation of flows can be generated as:

$$\vec{q}^{(t+1)} = \vec{g}(\vec{q}^{(t)}), \text{ if } \vec{q}^{(o)} \in K_q \qquad (3.2.30)$$

Convergence of the fixed-point sequence also suggests that the original RUBMRIO is indeed convergent, since it constructs a sequence of iterative price vectors, which are similar to the fixed-point sequence. The coincidence is not surprising, because nearly all iterative solution algorithms impose the fixed-point approach. However, only when the existence and uniqueness of solutions are ensured, can this sequence be guaranteed convergence at the correct solution.

Otherwise, different initial values could lead to different results, or the iterative process could never converge.

The original RUBMRIO algorithm calculates both prices and flows at each iteration. However, we have shown prices to be independent of the flows; so there is no need to calculate flows before the prices (or to compute prices after their convergence) in order to achieve the unique solution for prices and flows. Based on this consideration, we have developed a modified algorithm presented as follows, which efficiently applies the fixed-point approach for calculation of prices and flows.

<u>The Modified RUBMRIO Algorithm</u>

Step 0: Initialization. Set $\vec{q}^{(o)} \in K_q$ and $\vec{p}^{(0)} \in K_p$; let $t = 1$.

Step 1: Computation of prices. Calculate the prices $\{p_j^n\}$ using the following fixed-point equation:

$$\vec{p}^{(t+1)} = \vec{f}(\vec{p}^{(t)}) \qquad (3.2.31)$$

where $\vec{f}(\cdot)$ is defined in (3.2.4).

Step 2: Verification of price convergence If $\max(\left| p_j^{n(t)} - p_j^{n(t-1)} \right|) < t, \forall j, n$, with a pre-specified tolerance $t > 0$, then go to step 3; else, set $t = t + 1$, and go to step 1.

Step 3: Computation of probabilities. Compute probabilities using equation (3.2.1).

Step 4: Computation of flows. Set $t = 1$, and calculate flows using the following fixed-point equation:

$$\vec{q}^{(t+1)} = \vec{g}(\vec{q}^{(t)}) \qquad (3.2.32)$$

where $\vec{g}(\cdot)$ is defined in (3.2.24).

Step 5: Convergence test of flows. If $\max(\left|q_{ij}^{n(t)} - q_{ij}^{n(t-1)}\right|) < t$, $\forall i, j, n$, then stop, and the current solution, { q_{ij}^{n} } is the set of equilibrium flows; otherwise, set $t = t + 1$, and go to step 4.

3.2.3 Numerical Example

In this section a numerical example is given to demonstrate solution existence and uniqueness in the RUBMRIO model, and to compare the original and modified algorithms. We consider a simple case with only two regions and two commodity sectors; the exogenous variables' values are shown in Table 3.2.1. The dispersion parameters are set to $l^{1} = 15$ and $l^{2} = 0.2$ in order to represent different parameter values. The larger dispersion parameter implies less distributed flows and the smaller parameter entails wider distributed flows. These two values are arbitrarily chosen for the test. A more complex example, for the state of Texas, will be discussed in the next chapter (in which the network is congestible).

Convergence to the Unique Solution

Given the above example, we first checked the convergence patterns for prices and flows using the original algorithm. Our example convergence criterion requires that the absolute values of prices and flows between two successive iterations differ by no more than 0.00001. We tested three scenarios with different initial values: the first started with zero values (which are common start points) and the second and the third used some arbitrary, larger numbers, which were generated randomly.

The original algorithm converged after 138 iterations for the first scenario, 111 iterations for the second, and 120 iterations for the third. All converged to the same solution, as depicted in Figure 3.2.1 and Figure 3.2.2. The second and third scenario used non-zero initial values and converged faster, which suggests the traditional start values (zeros) probably are not the best choice. Finally, similar runs of all scenarios were made using the modified algorithm's fixed-point sequence. These are discussed here now.

Table 3.2.1 Exogenous values for numerical example

Variable	Value	Variable	Value	Variable	Value
Transportation Prices ($)		Technical Coef.		Final Demand ($)	
d_{11}	2	a^{11}	0.2	Y_1^1	100
d_{12}	10	a^{12}	0.8	Y_2^1	200
d_{21}	10	a^{21}	0.7	Y_1^2	20
d_{22}	1	a^{22}	0.1	Y_2^2	50

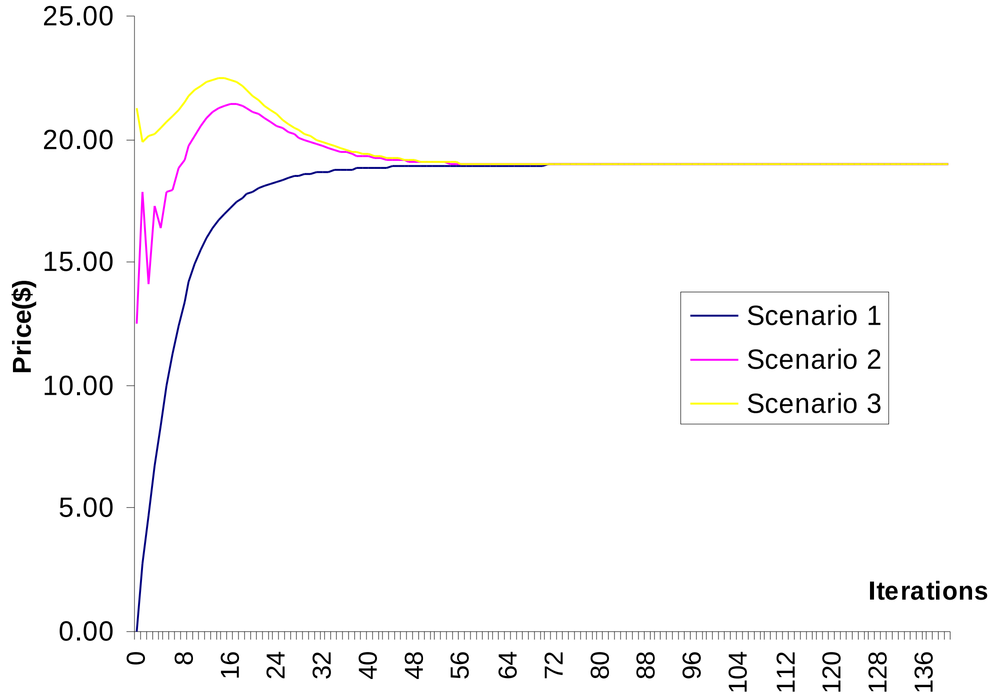

Figure 3.2.1 Convergence of price of sector 1 in region 1

Note here only one price is shown, other prices have the same convergence patterns.

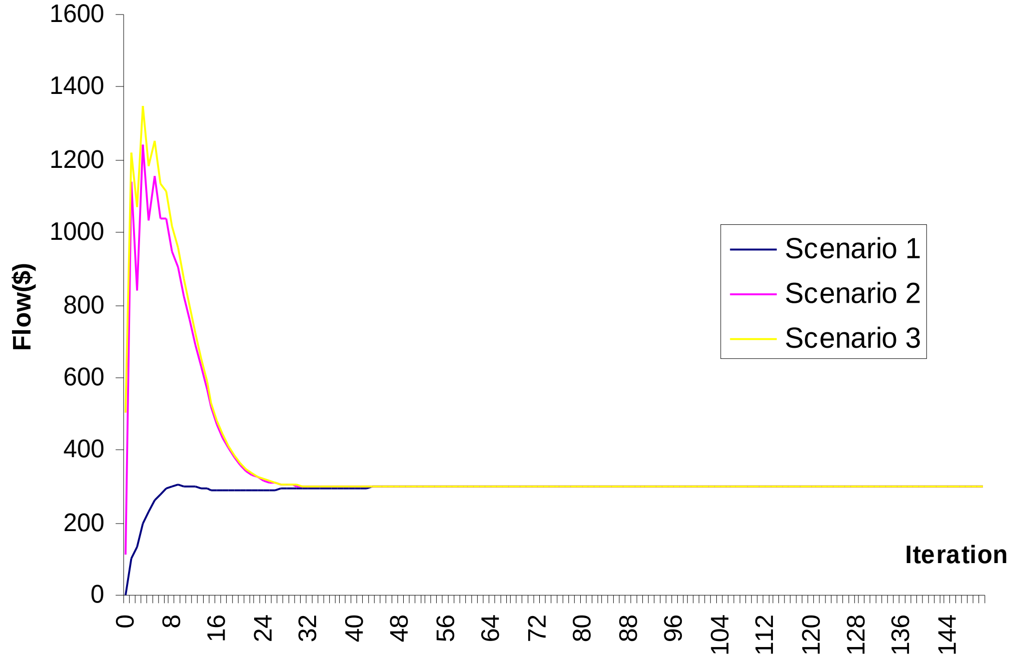

Figure 3.2.2 Convergence of flow of sector 1 from region 1 to 2

Note here only one flow is shown, other prices have the same convergence patterns.

Comparison of Algorithms

It is rather natural and efficient to adopt the modified algorithm to eliminate unnecessary computations in the original algorithm. Table 3.2.2 compares the original and modified algorithms for the RUBMRIO model. Both algorithms converged to the same, unique solutions. But the modified algorithm saved computational effort, especially in the computation of prices. Since the example is rather small, the computer calculation time difference is marginal. With the problem size increasing, one can expect the calculation time saving is significant.

Table 3.2.2 Comparison of the original and modified RUBMRIO algorithms

Scenario	Variables	Computation Effort	
		Original Algorithm	Modified Algorithm
1 (zero initial values)	Prices	138 iterations	86 iterations
	Flows	138 iterations	137 iterations
2 (arbitrary initial values)	Prices	111 iterations	95 iterations
	Flows	111 iterations	110 iterations
3 (arbitrary initial values)	Prices	120 iterations	94 iterations
	Flows	120 iterations	112 iterations

3.3 DERIVING THE OPTIMIZATION PROBLEM FOR THE RUBMRIO MODEL

For consistency with the entropy/optimization approach described in section 3.1, here we construct a similar optimization problem for the RUBMRIO model. We already discussed the uniqueness conditions for the RUBMRIO model using the fixed-point approach; here we just show the similarity of the model and the optimization of a specific objective function.

The approach to building an optimization problem may follow the process of developing equivalent problems for user equilibrium network assignment in travel demand modeling (see Sheffi, 1985).

Before proceeding, however, we need to simplify the equation system of RUMBRIO model. Equation (3.1.21) and (3.1.22) define the following relation between b_j^n and c_i^n:

$$c_j^n \sum_i q_{ij}^n = \sum_i \left[q_{ij}^n \left(\sum_m a_i^{mn} c_i^m + d_{ij}^n \right) \right] \quad \forall j, n \tag{3.3.1}$$

The original, full spatial I-O model aims to iteratively solve the equation system (3.1.18)~(3.1.22), with the results yielding solutions $\check{q}$ and $\check{c}$. Then, the optimization problem is:

$$\min_{\{q_{ij}^n, c_j^n\}} \quad Z = \sum_{ijn} \frac{1}{l^n} \left(q_{ij}^n \ln q_{ij}^n - q_{ij}^n \right) \tag{3.3.2a}$$

subject to:

$$\sum_j q_{ji}^m - \sum_n a_i^{mn} \sum_j q_{ij}^n = Y_i^m \quad \forall i, m \tag{3.3.2b}$$

$$c_j^n, \ q_{ij}^n \geq 0 \quad \forall i, j, n \tag{3.2.2c}$$

$$c_j^n \sum_i q_{ij}^n = \sum_i \left[q_{ij}^n \left(\sum_m a_i^{mn} c_i^m + d_{ij}^n \right) \right] \quad \forall j, n \tag{3.3.1}$$

To show the similarity of the optimization problem (3.2.2) and the original problem (3.1.18)~(3.1.22), we first construct the Lagrangian function, and then derive the first order conditions.

$$
\begin{aligned}
L(\vec{q}, \vec{p}, \vec{a}, \vec{g}) = & \sum_{ijn} \frac{1}{l^{n}} \left(q_{ij}^{n} \ln q_{ij}^{n} - q_{ij}^{n} \right) \\
& + \sum_{i} \sum_{m} a_{i}^{m} \left(Y_{i}^{m} - \sum_{j} q_{ji}^{m} + \sum_{n} a_{i}^{mn} \sum_{j} q_{ij}^{n} \right) \\
& + \sum_{j} \sum_{n} g_{j}^{n} \left[\sum_{i} q_{ij}^{n} \left(\sum_{m} a_{i}^{mn} c_{i}^{m} + d_{ij}^{n} \right) - c_{j}^{n} \sum_{i} q_{ij}^{n} \right]
\end{aligned}
\tag{3.3.3}
$$

The first-order derivative with respect to q_{ij}^{n} is:

$$
\frac{\partial L}{\partial q_{ij}^{n}} = \frac{1}{l^{n}} \ln q_{ij}^{n} - a_{j}^{n} + \sum_{l} a_{j}^{l} a_{j}^{lm} + g_{j}^{n} \left(\sum_{m} a_{i}^{mn} c_{i}^{m} + d_{ij}^{n} - c_{j}^{n} \right) \quad \forall i, j, n \tag{3.3.4}
$$

The first-order derivative with respect to c_{j}^{n} is:

$$
\frac{\partial L}{\partial c_{j}^{n}} = g_{j}^{n} \left(q_{jj}^{n} a_{j}^{nn} - \sum_{i} q_{ij}^{n} \right) \quad \forall j, n \tag{3.3.5}
$$

At the optimum, equation (3.3.4) equals zero and can be expressed as the following:

$$
\begin{aligned}
q_{ij}^{n} & = \exp\left\{ -l^{n} \left[-a_{j}^{n} + \sum_{l} a_{j}^{l} a_{j}^{lm} + g_{j}^{n} \left(\sum_{m} a_{i}^{mn} c_{i}^{m} + d_{ij}^{n} - c_{j}^{n} \right) \right] \right\} \\
& = \exp\left[l^{n} \left(a_{j}^{n} - \sum_{l} a_{j}^{l} a_{j}^{lm} + g_{j}^{n} c_{j}^{n} \right) \right] \exp\left[-l^{n} g_{j}^{n} \left(\sum_{m} a_{i}^{mn} c_{i}^{m} + d_{ij}^{n} \right) \right] \quad \forall i, j, n
\end{aligned}
\tag{3.3.6}
$$

Recalling equation (3.1.20), we have:

$$\frac{q_{ij}^{n}}{\sum_{k} q_{kj}^{n}} = \frac{\exp\left[l^{n}\left(a_{j}^{n} - \sum_{l} a_{j}^{l} a_{j}^{lm} + g_{j}^{n} c_{j}^{n} \right)\right] \exp\left[-l^{n} g_{j}^{n}\left(\sum_{m} a_{i}^{mn} c_{i}^{m} + d_{ij}^{n} \right)\right]}{\sum_{k} \exp\left[l^{n}\left(a_{j}^{n} - \sum_{l} a_{j}^{l} a_{j}^{lm} + g_{j}^{n} c_{j}^{n} \right)\right] \exp\left[-l^{n} g_{j}^{n}\left(\sum_{m} a_{k}^{mn} c_{k}^{m} + d_{kj}^{n} \right)\right]}$$

or

$$q_{ij}^{n} = C_{j}^{n} \frac{\exp\left[-l^{n} g_{j}^{n}\left(\sum_{m} a_{i}^{mn} c_{i}^{m} + d_{ij}^{n} \right)\right]}{\sum_{k} \exp\left[-l^{n} g_{j}^{n}\left(\sum_{m} a_{k}^{mn} c_{k}^{m} + d_{kj}^{n} \right)\right]} \qquad \forall i, j, n$$

(3.3.7)

If one assumes that $l^{n} = l^{n} g_{j}^{n}$ (i.e., $g_{j}^{n} = 1, \forall n, j$), equation (3.3.7) is the same as equation (3.1.18). Furthermore, one can easily find that the first-order conditions with respect to the Lagrange parameters a 's and g 's yield equations (3.1.19) and (3.3.1), respectively. Equation (3.3.1) resembles equations (3.2.21) and (3.2.22). So, for the simplified model, the optimization problem (3.3.2) is equivalent to the equation system (3.1.18~3.1.22). Solving the optimization problem for the minimum point will yield the same results as solving the equation system iteratively.

However, if the assumption, $l^{n} = l^{n} g_{j}^{n}$, does not hold, equation (3.3.7) is not exact the same as equation (3.1.18).Then the optimization problem cannot represent the original RUBMRIO model system and just a close approximation.

3.4 SUMMARY

This chapter described the equilibrium modeling of SIO models and, in particular, the RUBMRIO models. After a brief description of SIO model, the uniqueness of the entropy form of a standard SIO model has been shown. Then, a fixed-point formulation of the RUBMRIO model was constructed for solutions to many integrated land use-transportation models. This formulation and the problem properties allowed us to develop existence and uniqueness conditions for the RUBMRIO model solutions. In addition, the similar optimization problem of RUBMRIO model was developed.

Under weak conditions regarding sales prices, the set of price solutions was shown to be unique. Given prices and spatial purchase probabilities, commodity flows also were found to be unique. The fixed-point formulation established here verifies that the common/original RUBMRIO iterative algorithm does converge. However, a modified algorithm was demonstrated to be more efficient.

In this chapter we assumed that transportation prices were fixed. When integrating the RUBMRIO model with a congestible transportation network, one needs to treat transportation costs as a function of commodity flows, rather than as exogenous variables. One practical way is to link the RUBMRIO model with a UE or SUE assignment model. We discuss this in the next chapter.

It has been shown that there exists a unique solution to UE and SUE problems (see Sheffi, 1985), and we demonstrated here that the RUBMRIO has a unique solution. Thus, the only gap is a theoretical analysis of the uniqueness of the overall, congestible, integrated system solution. However, we fully expect that this exists, based on Cantarella's proofs [1997] of congestible travel demand model solution uniqueness using fixed point approach.

Chapter 4 Equilibrium Modeling of Integrated Transportation-Land Use Models

Chapter 3's RUBMRIO model calculates interregional trade flows, each region's commodity productions, and prices. These results essentially illustrate a regional system's economy, land use, and transportation patterns. However, the calculation is based on fixed transportation costs without empirically take into account network congestions. So it can be regarded as a simplified, integrated transportation-land use model. To reflect congestions on transportation networks, more sophisticated models are proposed and discussed in this chapter.

As described in Chapter 2's literature review, there are two major methods to integrate transportation and land use models: separated models with feedback strategies and combined models. The first method (or "feedback method") tends to solve a system of models, each of which describes one component of the transportation-land use system. Feedback techniques link individual models through key variables. The second method usually depends on a single mathematical programming problem which synthesizes all system components. It is relatively easy to build up a set of equations to represent relationships among the components in the complex system, especially if some of the submodels already exist. Thus, the feedback method is more common in practice. However, the combined model draws the full system picture and facilitates the theoretical study of solution existence, uniqueness, and stability. Additionally, the mathematical programming formulation can lead to a number of existing solution algorithms. In this chapter, we examine both methods of integrating transportation and land use models based on results from Chapter 3 for spatial input-output (SIO) models. This chapter begins with the formulation of an interregional flow model embedded in a transportation network equilibrium model. The optimization conditions and the characteristics of the solutions are discussed, and a solution algorithm, the adapted Evans' algorithm, is presented. The study then investigates two methods, both of which are "separated models" with feedback strategies. And it tests the efficiency of the two feedback strategies.

4.1 COMBINED ITLUM BASED ON RUBMRIO

We formulate a combined model for interregional trade and development by incorporating SIO relationships and the corresponding network flows. The model unifies the land use forecasts (i.e., the commodity production and flow patterns) and the conventional travel demand model steps (trip generation, mode choice, trip distribution, and route choice). This formulation is highly related to the research of Ham, Kim, and Boyce (2000) and Kim, Ham, and Boyce (2002). Their models combine commodity production and flow distribution, mode choices and route assignment but they are not a random-utility-based SIO model. They neglect prices and greatly rely on entropy-maximizing. In this chapter, we empirically show that the optimization condition of the combined model is equivalent to a modified RUBMRIO model. The extension provides a starting point for other possible models that integrate RUBMRIO and transportation models.

In this study, a research area is divided into regions (zones). We assume that final demands (exports) by commodity for each zone are given. As mentioned in Chapter 3, zonal final demands may be hard to determine and ideally should be endogenous. One possible improvement for this model is to define a certain number of export zones (ports, airports, etc.), whose export amounts are observable, and use a logit model to distribute the export demands across production zones (Jin, Kockelman, and Zhao, 2002).

The model determines each zone's commodity productions, distributes flows between zones, and allocates the flows to the congestible transportation network (i.e., the transportation costs reflect the flows on the network links). The sectors in this application include industries (e.g., agriculture and manufacture), services (e.g., education), government, and households (both as labor providers and commodity consumers). The period of analysis is one day, which differs from the traditional input-output model's application period of one year.

The flow consists persons and goods. The human flow can be transformed into personal trips, and the goods flow into truck trips. For example, the demand by other sectors for household labor produces work trips. The combined model considers alternative transportation modes through separate networks, or what Sheffi (1985) called "supernetworks". This neglects that the interactions between alternative transportation modes can be ignored and is a limitation of this research.

The modes operating on each network transport both humans and goods. If one mode is not available for a particular movement, the transportation costs are assumed to be infinite (or a very large number).

Let q_{ij}^{mw} denote the flow of sector m from zone i to zone j per day by mode w, and h_{ijr}^{mw} be the flow of sector m from i to j by route r and mode w. The sum of route flows equals total interzonal flows; i.e.,

$$\sum_r h_{ijr}^{mw} = q^m q_{ij}^{mw} \quad \forall m, w, i, j \tag{4.1.1}$$

where q^m converts flow units from dollars to vehicles on the network.

Let f_a^w denote the total flow on link a of the w mode network, which experiences a cost of $d_a^w(f_a^w)$. The link cost function $d_a^w(f_a^w)$ is assumed to be non-decreasing on link flows and it is assumed to be separable, which implies neglecting interactions among (adjacent) link flows.

Link flows are the sums of the route flows:

$$\sum_m \sum_{ijr} h_{ijr}^{mw} d_{ijr}^a = f_a^w \qquad \forall a, w \tag{4.1.2}$$

where $d_{ijr}^a = 1$ if route *r* from *i* to *j* uses link *a*, and zero otherwise.

4.1.1 The Combined Model Formulation

The model is formulated into a mathematical programming problem in which the objective function combines the entropy form of Chapter 3's SIO model the user equilibrium's (UE) objective function. A number of ITLUMs have taken such mathematical formulations to simultaneously determine key variables (see, e.g., Kim, 1989; Opperheinm, 1995; Kim, Ham, and Boyce 2002).

The model is to allocate an equilibrium SIO flow pattern consistent with a UE traffic flow pattern. An overall equilibrium between land use and transportation is then attained.

$$\min_{h,q} Z = \sum_{aw} \int_0^{f_a^w} d_a^w(w)dw \; + \sum_{ijmw} a^m q^m q_{ij}^{mw} \ln \frac{q_{ij}^{mw}}{q_{ij}^m} + \sum_{ijm} \frac{1}{l^n}\left(q_{ij}^m \ln q_{ij}^m - q_{ij}^m\right) \tag{4.1.3a}$$

subject to:

$$\sum_i q_{ij}^m = \sum_n a^{mn} \sum_k q_{jk}^n \; + \; Y_j^m \qquad \forall m, j \tag{4.1.3b}$$

$$c_j^m \sum_i q_{ij}^m = \sum_i \left[q_{ij}^m \left(\sum_n a_i^{nm} c_i^n + d_{ij} \right) \right] \quad \forall m, j \tag{4.1.3c}$$

$$\sum_w q_{ij}^{mw} = q_{ij}^m \quad \forall m, i, j \tag{4.1.3d}$$

$$\sum_r h_{ijr}^{mw} = q^m q_{ij}^{mw} \quad \forall m, i, j, w \tag{4.1.3e}$$

$$h_{ijr}^{mw} \geq 0, \ \ q_{ij}^{mw} \geq 0, q_{ij}^m \geq 0, \ \ c_j^m \geq 0 \quad \forall i, j, r, m, w \tag{4.1.3f}$$

where, $d_a^w(f_a^w)$ is the cost function of total flow on link a by mode *w;* $d_{ij} = E[\min(\sum_w d_a^w d_{ijr}^a)]$, the expected minimum travel cost between i and j; and a^m is the cost sensitivity parameter for sector *m*. The cost function is symmetric, i.e., there is no interaction between links. As suggested by Smith (1979) and Sheffi (1985), introducing asymmetric cost functions makes network equilibrium models' more realistic and sophistic, and the uniqueness properties of equilibrium models no longer hold. Here, instead, a symmetric, non-decreasing cost function (such as the BPR function, see Zhao and Kockelman, 2001) is used.

The interzonal flow distribution costs are constituted with the entropy terms with respect to distribution costs across modes and across zones. The model determines each sector's interzonal flows of each mode, together with link flows and route flows as a result of distributing commodity flows across zones through minimum travel-cost routes. This complex spatial distribution combines industries' and households' location choices, commodity flows' origin choices (purchasers' choices of suppliers), flow shipments' mode choices, and route choices. Thus, it integrates land use and transportation decisions simultaneously.

The Lagrange function is constructed as follows:

$$L(\vec{h},\vec{q},\vec{b},\vec{g},\vec{l},\vec{m}) = \sum_{aw}\int_0^{f_a^w} d_a^w(w)dw \quad + \sum_{ijmw} a^m q^m q_{ij}^{mw} \ln\frac{q_{ij}^{mw}}{q_{ij}^m}$$
$$+\sum_{ijm}\frac{1}{l^m}\left(q_{ij}^m \ln q_{ij}^m - q_{ij}^m\right) + \sum_{jm} b_j^m \left(Y_j^m - \sum_i q_{ij}^m + \sum_n a_j^{mn}\sum_k q_{jk}^n\right)$$
$$+\sum_{jm} g_j^m \left\{\sum_i \left[q_{ij}^m\left(\sum_m a_i^{nm}c_i^n + d_{ij}\right)\right] - c_j^m \sum_i q_{ij}^m\right\} \qquad (4.1.4)$$
$$+\sum_{mij} j_{ij}^m \left(q_{ij}^m - \sum_w q_{ij}^{mw}\right) + \sum_{mijw} m_{ij}^{mw}\left(q^m q_{ij}^{mw} - \sum_r h_{ijr}^{mw}\right)$$

where, $\vec{b},\vec{g},\vec{j},\vec{m}$ are Lagrange multipliers. The Lagrange function has to be minimized with respects to nonnegative constraints of h_{ijr}^{mw}, q_{ij}^{mw}, q_{ij}^m.

Then the Karush-Kuhn-Tucker (KKT) optimality conditions (see Hillier and Lieberman, 1995) are the following[13]:

$$\frac{\partial L}{\partial h_{ijr}^{mw}} = \sum_a d_a\left(f_a^w\right)d_{ijr}^{aw} - m_{ij}^{mw} \geq 0, \forall \boldsymbol{ijrmw} \qquad (4.1.5a)$$

$$h_{ijr}^{mw}\frac{\partial L}{\partial h_{ijr}^{mw}} = h_{ijr}^{mw}\left[\sum_a d_a\left(f_a^w\right)d_{ijr}^{aw} - m_{ij}^{mw}\right] = 0, \forall \boldsymbol{ijrmw} \qquad (4.1.5b)$$

$$\frac{\partial L}{\partial q_{ij}^{mw}} = a^m q^m\left(\ln\frac{q_{ij}^{mw}}{q_{ij}^m} + 1.0\right) - j_{ij}^m + m_{ij}^{mw}q^m \geq 0,\ \forall \boldsymbol{ijmw} \qquad (4.1.6a)$$

$$q_{ij}^{mw}\frac{\partial L}{\partial q_{ij}^{mw}} = q_{ij}^{mw}\left[a^m q^m\left(\ln\frac{q_{ij}^{mw}}{q_{ij}^m} + 1.0\right) - j_{ij}^m + m_{ij}^{mw}q^m\right] = 0,\ \forall \boldsymbol{ijmw} \qquad (4.1.6b)$$

$$\frac{\partial L}{\partial q_{ij}^m} = a^m q^m \sum_w \frac{q_{ij}^{mw}}{q_{ij}^m} + \frac{1}{l^m}\ln q_{ij}^m - b_j^m + \sum_l b_j^l a_j^{lm}$$
$$+ g_j^m\left(\sum_m a_i^{nm}c_i^n + d_{ij} - c_j^n\right) - j_{ij}^m \geq 0, \forall \boldsymbol{ijm} \qquad (4.1.7a)$$

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[13] The KKT conditions are obtained by deriving the first-order conditions for the Lagrange function with respect to each of the nonnegative variables and the Lagrangian multipliers. The first-order conditions with respect to the nonnegative variables are written in pair and referred to as the complementary slackness conditions (Bazaraa and Shetty, 1979).

$$q_{ij}^{m}\frac{\partial L}{\partial q_{ij}^{m}} = q_{ij}^{m}[a^{m}q^{m}\sum_{w}\frac{q_{ij}^{mw}}{q_{ij}^{m}} + \frac{1}{l^{m}}\ln q_{ij}^{m} - b_{j}^{m} + \sum_{l} b_{j}^{l} a_{j}^{lm}$$
$$+ g_{j}^{m}(\sum_{m} a_{i}^{nm} c_{i}^{n} + d_{ij} - c_{i}^{m}) - j_{ij}^{m}] = 0, \forall \boldsymbol{ijm} \qquad (4.1.7b)$$

and all the constraints (4.1.3b~f).

UE conditions for the route flows can be derived from the complementary slackness condition (4.1.5) as follows;

$$\text{If } h_{ijr}^{mw} > 0, m_{ij}^{mw} = \sum_{a} d_{a}^{w}\left(f_{a}^{w}\right) d_{ijr}^{aw}, \qquad (4.1.8)$$

$$\text{If } h_{ijr}^{mw} = 0,\ m_{ij}^{mw} \leq \sum_{a} d_{a}^{w}\left(f_{a}^{w}\right) d_{ijr}^{aw} \qquad (4.1.9)$$

The above conditions indicate that if the route flow of sector ***m*** by mode *w* is nonnegative, the route cost of route ***r*** equals the minimum cost (m_{ij}^{mw}) between ***i*** and *j* by mode *w*; otherwise, there is no flow on that route. This extends the UE problem to multiple modes and multiple users (sectors).

When the interzonal flows by mode are positive (recognizing the nonnegative constraint [4.1.3f]), condition (4.1.6a) can be taken at the equal sign as follows:

$$a^{m}q^{m}\left(\ln\frac{q_{ij}^{mw}}{q_{ij}^{m}} + 1.0\right) - j_{ij}^{m} + m_{ij}^{mw}q^{m} = 0 \qquad (4.1.10)$$

So,

$$q_{ij}^{mw} = q_{ij}^{m}\exp\left(\frac{j_{ij}^{m} - m_{ij}^{mw}q^{m}}{a^{m}q^{m}} - 1.0\right) = q_{ij}^{m}\exp\left(\frac{j_{ij}^{m}}{a^{m}q^{m}} - 1.0\right)\exp\left(\frac{-m_{ij}^{mw}}{a^{m}}\right)$$

(4.1.11)

From condition (4.1.3d), the sum of interzonal flows across all modes equals the interzonal flows; then,

$$\sum_w q_{ij}^{mw} = q_{ij}^m = \sum_w q_{ij}^m \exp\left(\frac{j_{ij}^{m}}{a^m q^m} - 1.0\right)\exp\left(\frac{-m_{ij}^{mw}}{a^m}\right)$$
$$= q_{ij}^m \exp\left(\frac{j_{ij}^{m}}{a^m q^m} - 1.0\right)\sum_w \exp\left(\frac{-m_{ij}^{mw}}{a^m}\right) \qquad (4.1.12)$$

Therefore:

$$\exp\left(\frac{j_{ij}^{m}}{a^m q^m} - 1.0\right) = \frac{1}{\sum_w \exp\left(-m_{ij}^{mw} / a^m\right)} \qquad (4.1.13)$$

By substituting (4.1.13) into (4.1.11), one has:

$$q_{ij}^{mw} = q_{ij}^m \frac{\exp\left(-m_{ij}^{mw} / a^m\right)}{\sum_k \exp\left(-m_{ij}^{mk} / a^m\right)} \qquad (4.1.14)$$

Recall that a^m is the cost sensitivity parameter for sector ***m***. Thus, equation (4.1.14) reveals a logit model of mode choice, where the disutility is the minimum cost m_{ij}^{mw} of mode *w* divided by sector ***m***'s cost sensitivity parameter.

The commodity flows are nonnegative due to the condition (4.1.3f), and the complementary condition (4.1.7) can be rearranged as follows if the flows are positive:

$$a^m q^m + \frac{1}{l^m}\ln q_{ij}^m - b_j^m + \sum_l b_j^l a_j^{lm} + g_j^m\left(\sum_m a_i^{nm} c_i^n + d_{ij} - c_j^n\right) - j_{ij}^m = 0$$

(4.1.15)

Rearranging it still further, one has:

$$q_{ij}^m = \exp\left[l^m\left(-a^m q^m + g_j^m c_j^n + b_j^m - \sum_l b_j^l a_j^{lm}\right)\right]$$
$$\mathrm{g}\exp\left\{l^m\left[-g_j^m\left(\sum_m a_i^{nm} c_i^n + d_{ij}\right) + j_{ij}^m\right]\right\} \qquad (4.1.16)$$

Therefore:

$$\frac{q_{ij}^m}{\sum_k q_{kj}^m} = \frac{\exp\left\{ l^m \left[-g_j^m \left(\sum_m a_i^{nm} c_i^n + d_{ij} \right) + j_{ij}^m \right] \right\}}{\sum_k \exp\left\{ l^m \left[-g_j^m \left(\sum_m a_k^{nm} c_k^n + d_{kj} \right) + j_{kj}^m \right] \right\}} \qquad (4.1.17)$$

Equation (4.1.17) presents a logit mode for interzonal commodity flows, which differs slightly from the form in the RUBMRIO model. Letting $l_j^m = l^m g_j^m$, the first part in the exponential function is the same of equation (3.3.7) in Chapter 3. The second part, j_{ij}^m, is actually a function of the "log-sum" (see, Ben-Akiva and Lerman, 1985) of transportation costs by mode. From equation (4.1.13), one has:

$$j_{ij}^m = a^m q^m \left[1 - \ln \sum_w \exp\left(-m_{ij}^{mw} / a^m \right) \right] \qquad (4.1.18)$$

By substituting (4.1.18) into (4.1.17), one has:

$$\frac{q_{ij}^m}{\sum_k q_{kj}^m} = \frac{\exp\left[-l_j^m (\sum_m a_i^{nm} c_i^n + d_{ij}) - l^m a^m q^m \ln \sum_w \exp\left(-m_{ij}^{mw} / a^m \right) \right]}{\sum_k \exp\left[-l_j^m (\sum_m a_k^{nm} c_k^n + d_{kj}) - l^m a^m q^m \ln \sum_w \exp\left(-m_{kj}^{mw} / a^m \right) \right]}$$

(4.1.19)

Equation (4.1.19) represents a nested logit model: the lower level is the transportation mode choice and the higher level is the origin choice (i.e., where to purchase the commodity). This is not a surprising finding since the model combines both choice types.

Comparing (4.1.19) to the random-utility-based flow distribution part of Chapter 3's modified RUBMRIO model (i.e., equation [3.3.7]), the utility function has changed to incorporate the lower level mode choices. But they are highly consistent. With constraints (4.1.3b~c), the combined model (4.1.3) synthesizes this modified RUBMRIO model.

The objective function (4.1.3a) is a convex function with respect to link flows, $f_a^w = f(\vec{h}) = \sum_m \sum_{ijr} h_{ijr}^{mw} d_{ijr}^a$, as one can see this from Chapter 2's UE formulation. It should be noted that this convexity is for link flows (f_a^w), not for path flows (h_{ijr}^{mw}) (see Sheffi, 1985). Moreover, it is a convex function with respect to interzonal flows (q_{ij}^m) and interzonal flows by mode (q_{ij}^{mw}). To illustrate this, it is sufficient to check whether the Hessian matrix of the objective function is positive semidefinite. The Hessian is calculated by using a representative term and taking its first-order and second-order derivatives with respect to $\vec{f}$ and $\vec{q}$.

$$\frac{\partial Z}{\partial f_a^w} = d_a\left(f_a^w\right) \ \forall aw \tag{4.1.20}$$

$$\frac{\partial Z}{\partial q_{ij}^m} = a^m q^m \sum_w \frac{q_{ij}^{mw}}{q_{ij}^m} + \frac{1}{l^m} \ln q_{ij}^m = a^m q^m + \frac{1}{l^m} \ln q_{ij}^m \qquad \forall ijm \tag{4.1.21}$$

$$\frac{\partial Z}{\partial q_{ij}^{mw}} = a^m q^m \left(\ln \frac{q_{ij}^{mw}}{q_{ij}^m} + 1.0 \right) \qquad \forall ijmw \tag{4.1.21}$$

where Z is the objective function in (4.1.3). The second-order derivatives are thus the following:

$$\frac{\partial^2 Z}{\partial f_a^w \partial f_b^v} = \frac{\partial}{\partial f_b^w} d_a\left(f_a^w\right) = \begin{cases} \dfrac{d\left[d_a\left(f_a^w\right)\right]}{df_b^w} & \forall a = b, w = v \\ 0 & otherwise \end{cases} \tag{4.1.22}$$

$$\frac{\partial^2 Z}{\partial q_{ij}^m \partial q_{lk}^n} = \frac{\partial}{\partial q_{lk}^n} \left(a^m q^m + \frac{1}{l^m} \ln q_{ij}^m \right) = \begin{cases} \dfrac{1}{l^m q_{ij}^m} & \forall m = n, l = i, k = j \\ 0 & otherwise \end{cases} \tag{4.1.22}$$

$$\frac{\partial^2 Z}{\partial q_{ij}^{mw} \partial q_{lk}^{nv}} = \frac{\partial}{\partial q_{lk}^n} a^m q^m \left(\ln \frac{q_{ij}^{mw}}{q_{ij}^m} + 1.0 \right) = \begin{cases} \dfrac{a^m q^m}{q_{ij}^{mv}} & \forall m = n, l = i, k = j \\ 0 & otherwise \end{cases} \tag{4.1.23}$$

$$\frac{\partial^2 Z}{\partial f_a^w \partial q_{ij}^m} = \frac{\partial^2 Z}{\partial q_{ij}^m \partial f_a^w} = 0 \ \forall ijamw \tag{4.1.24}$$

$$\frac{\partial^2 Z}{\partial f_a^w \partial q_{ij}^{mw}} = \frac{\partial^2 Z}{\partial q_{ij}^{mw} \partial f_a^w} = 0 \ \forall ijamw \tag{4.1.25}$$

$$\frac{\partial^2 Z}{\partial q_{ij}^{m} \partial q_{lk}^{nw}} = 0 \ \forall ijlkmnw \tag{4.1.26}$$

$$\frac{\partial^2 Z}{\partial q_{ij}^{mw} \partial q_{lk}^{n}} = \frac{\partial}{\partial q_{lk}^{n}} a^m q^m \left(\ln \frac{q_{ij}^{mw}}{q_{ij}^{m}} + 1.0 \right) = \begin{cases} -\frac{a^m q^m}{q_{ij}^{m}} & \forall m = n, l = i, k = j \\ 0 & otherwise \end{cases} \tag{4.1.27}$$

Since $\sum_w q_{ij}^{mw} = q_{ij}^{m}$, one has $q_{ij}^{mw} \le q_{ij}^{m}$, or $\frac{1}{q_{ij}^{mw}} \ge \frac{1}{q_{ij}^{m}}$. Then in the Hessian matrix, rows (which are defined with respect to q_{ij}^{mw}) have positive diagonal elements and negative off-diagonal elements, and the diagonal "dominates" (i.e., $\left| \frac{a^m q^m}{q_{ij}^{mv}} \right| \ge \left| -\frac{a^m q^m}{q_{ij}^{m}} \right|$ for each row) . It follows from Gershgorim's circle theorem (e.g., [Golub and Van Loan, 1983: Theorem 7.2.1]), that the Hessian has only nonnegative eigenvalues. Thus, the objective function is a convex function.

The feasible region defined by constraints (4.1.3b~f) are all linear except (4.1.3c), which is a fixed-point formulation for the intermediate price variables, as discussed in Chapter 3. In fact, it does not affect the feasible region since once the transportation costs are obtained, the average costs are uniquely determined. Then the feasible region is *almost* certain to be strictly convex. Overall, the uniqueness of solutions is *almost* certain.

4.1.2 Parameter Calibration

The above model consists of a large number of parameters and exogenous variables. In order to apply and solve the model, the parameters must be calibrated and the information about exogenous variables needs to be collected. Then the solution algorithms can be applied to obtain empirical results.

A convenient way to calibrate the model parameters is to estimate the submodels using observed data. For example, the mode choice model (4.1.14) can be used to estimate the parameters $\{a^m\}$ if the minimum transportation costs of each mode are known. In practice, the minimum transportation costs between any given pair of zones can be obtained by applying a shortest-path skim on the networks of different modes. Then the nested origin and mode choice model (4.1.20) can be used to estimate $\{l^m\}$ and $\{g_j^m\}$. The optimization problem (4.1.3) can be solved by a general nonlinear programming algorithm. In addition, recognizing the congestive characteristics of the network assignment embedded in the larger model, Evans' (1976) algorithm becomes very useful and so is discussed in the following section.

4.1.3 Solution Algorithms

Implementation of the combined model requires an algorithm for obtain solutions to the flows. Because the combined model is a convex programming problems, it can be solved efficiently by either the Evans' or the Frank-Wolf algorithm. Evans' algorithm is rather suitable and efficient for large-size problems and is preferred here since it requires fewer iterations than the Frank-Wolfe algorithm (see, e.g., Frank, 1978; Leblanc and Farhangian, 1981; and Boyce and Lundqvist, 1987). This partial linearization algorithm was originally proposed by Evans (1979) to solve the combined trip distribution and network assignment problem and it has been shown vastly useful in solving other complicated optimization problems (Patriksson, 1994). It combines Wilson's (1970a) iterative balancing method and the Frank-Wolfe algorithm (Sheffi, 1985). The adapted Evans' algorithm consists of the following steps:

Step 1 (Initialization): $t = 1$. Find an initial feasible link flows and commodity flows. Employ shortest path skims to obtain initial interzonal travel costs by mode, $d_{ij}^{w,0}$, and compute the weighted costs $d_{ij}^{0} = \sum_{w} \frac{\sum_{m} q_{ij}^{mw}}{\sum_{m} q_{ij}^{m}} d_{ij}^{w,0} \;\; \forall ij$.

Step 2 (Application of the RUBMRIO model): Carry out the RUBMRIO model using the original algorithm described in Chapter 3 to obtain "auxiliary" commodity distributions, $\{\tilde{q}_{ij}^{m}\}$. Then obtain $\{\tilde{q}_{ij}^{mw}\}$ from equation (4.1.14).

Step 3 (Network assignment): Apply an UE assignment from Step 2 results to calculate auxiliary link flows $\tilde{f}_a^{w}$.

Step 4 (Step size search): Set

$\{f_a^{w,r}, q_{ij}^{m,r}, q_{ij}^{mw,r}\} = (1-r)\{f_a^{w}, q_{ij}^{m}, q_{ij}^{mw}\} + \{\tilde{f}_a^{w}, \tilde{q}_{ij}^{m}, \tilde{q}_{ij}^{mw}\}$ to solve for optimal r, where $r \in [0,1]$, and minimizes the following:

$$\min Z(r) = \sum_{aw} \int_0^{f_a^{w,r}} d_a^{w}(w)dw \;\; + \sum_{ijmw} a^{m} q^{m} \left(q_{ij}^{mw,r} \ln \frac{q_{ij}^{mw,r}}{q_{ij}^{m,r}} \right)$$
$$+ \sum_{ijm} \frac{1}{l^{n}} (q_{ij}^{m,r} \ln q_{ij}^{m,r} - q_{ij}^{m,r})$$

Step 5 (Convergence tests): *t=t+1*, Set

$\{f_a^{w,(t)}, q_{ij}^{m,(t)}, q_{ij}^{mw,(t)}\} = \{f_a^{w,r}, q_{ij}^{m,r}, q_{ij}^{mw,r}\}$, If $\left\| f_a^{w,(t)} - f^{w,(t-1)} \right\| \le e \;\; (e > 0)$, then stop; else, go to Step 2.

As one can see, Step 2 and Step 3 solve for the auxiliary commodity distribution and the auxiliary link flows using the RUBMRIO model and UE assignment. The one-dimensional line search method for r considers commodity flows and link flows simultaneously, to provide an optimal step length at every iteration.

Evans (1976) proved that the solution to her algorithm is equivalent to the original optimization problem (in her case, the combined trip distribution and assignment problem). She also showed the algorithm converges to the unique solution to her problem regardless of initial conditions. These superb properties of Evans' algorithm have made it a general solving algorithm for many combined transportation-land use models (see, e.g., Chu, 1999; Kim, Ham, Boyce, 2002). So it is suggested in this study to obtain solutions for the combined model (4.1.3).

4.2 LINKED ITLUMS AND FEEDBACK METHODS

Since most current ITLUMs rely on feedbacks to link submodels, this study constructs a "linked" model based on the RUBMRIO model developed in Chapter 3. This section demonstrates the implementation of this linked model for the Dallas-Fort Worth (DFW), Texas region. It explores the convergence of the feedback method and compares different feedback techniques to identify the most appropriate and efficient one.

As described in Section 4.1.3's algorithm of the combined model, several steps are necessary to solve subproblems iteratively. Feedback methods also are solution techniques which iteratively solve subproblems (e.g., a land use model, trip distribution and mode split models, and trip assignment) without explicitly solving for moving directions and step sizes. So feedback methods can be regarded as simplified solution algorithms for the “fully integrated” (Kim, 1989), combined model.

However, a simple feedback method lacks the theoretical evidence that (1) it will converge, and (2) it will converge to the correct solution regardless of initial state. When one links the RUBMRIO model with a UE or SUE assignment model, Cantarella’s (1997) work becomes very valuable. He showed that there exists a unique solution to UE and SUE problems with multiple modes (i.e., with a combined mode split model; but the interaction among modes is not considered), and we demonstrated in Chapter 3 that the RUBMRIO has a unique solution. Thus, we only need to discuss the uniqueness of the overall, congestible, integrated system solution. Essentially, one efficient, equivalent way to construct a fixed-point problem is to assemble a pair of functions in which one function’s dependent variables are independent variables for another. This is called a “paired” fixed-point problem (Cascetta , 2001). The condition for the solution uniqueness of paired fixed-point problems is that one function is non-increasing and the other is decreasing (Cantarella, 1997). In our linked model case, the RUMBRIO model calculates commodity flows given transportation costs, and the transportation model (e.g., UE with/without mode split) computes transportation costs given the commodity flows. It is clear that flows are calculated from a set of non-increasing “function” of transportation costs (i.e., as the cost between a given origin-destination [OD] pair increases, the flow between the OD will not increase). Moreover, travel costs monotonically rise as OD demands (commodity flows) increase. Basically, the linked model calculation process with feedback of key variables (costs and flows) satisfies the unique solution condition of a paired fixed-point problem. Thus, we are ***almost*** certain there is a unique solution for the linked model and the computations converge to the unique solution if the initial costs and flows are feasible. In general, the initial state includes a shortest-path scan for initial transportation costs, which is certainly in the feasible region for costs. A formal proof using the paired fixed-point approach for the linked model is out of the range of this study. However, it is highly expected such proofs are practicable.

A number of feedback strategies can be implemented in practice; some may imply different sub-system equilibrium assumptions, while others may mimic the temporal equilibria of quasi-dynamic models. For example, by linking a location choice model to a multi-step travel demand model, one can build a single feedback loop from traffic assignment to the land use model. Another strategy is to construct a feedback loop within the four-step transportation model, and assemble a second feedback loop between the land use and transportation models. By using two distinct feedback loops, the second strategy assures that equilibria exist between and within the land use and transportation systems.

Figure 4.2.1 illustrates two possible feedback methods to link the RUBMRIO model with transportation models. Since RUBMRIO results include a spatial distribution of commodity flows, the trip generation and distribution steps of four-step approaches can be eliminated. Moreover, the RUBMRIO model is capable of incorporating mode choice (e.g., between highway and railway). However, we keep the mode split step in the following application of a transportation model because we test a second feedback loop. Also, it is common in existing ITLUMs to separated mode split and UE assignment modules and to rely on the first feedback method in Figure 4.2.1 (e.g., MEPLAN [Hunt, 1993]). We expect Figure 4.2.1's Method 2 more efficient in searching for an overall system equilibria than Method 1, since it explicitly obtains equilibria solutions in transportation.

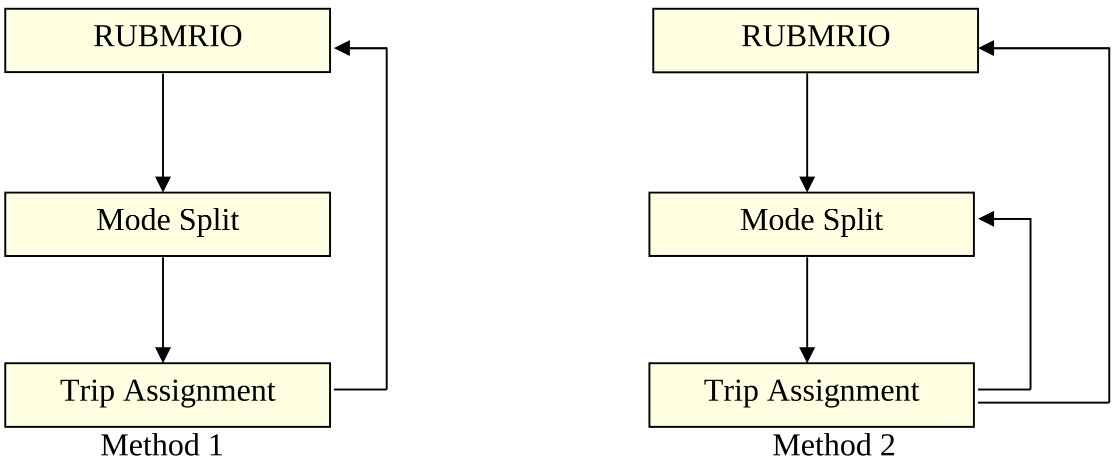

Figure 4.2.1: Sample feedback strategies for LU-T equilibria

Boyce, Lupa, and Zhang (1994) have examined the feedback approaches in four-step travel demand models and shown that iterated solutions will eventually converge to the equilibrium solution. This study extends their work to integrated transport-land use models. Using the RUBMRIO model as the land use model, this research links it to travel demand models of mode split and UE assignment. It then builds two types of feedback loops as shown in Figure 4.2.1. There are two objectives in this investigation for linked models with feedback. First, we examine whether the feedback methods converge to the equilibrium solution. Second, if they converge, we determine which is more efficient.

4.2.1 MODEL SETUP

It is illuminating to implement the RUBMRIO model in an urban area. The model allocates "industries" (including households) and production levels to zones and distributes commodity flows (including commuters) between zones. The various productive sectors in this study include manufactures, services (e.g., education), transportation, government, and households.

There are two types of flows: persons and goods. For our network loading, we focus on person flows and ignore goods flows. Person flows can be transformed into personal trips (and eventually vehicle trips). For example, other sectors' demands for the household sector are considered labor or employment, and flows from households to industries are work trips and to education sectors are school trips. Person flow results from the RUBMRIO model imply the location choices of households. They also determine trip frequency and distribution choices.

To "drive the model", we assume that final demands are known by zone and goods type, are proportional to household incomes, and are the triggers for the regional economy. These final demands may be commodity exports, destined for locations outside the study area. However, due to the lack of local export data, we assume that the total final demand is simply equal to the sum of household incomes. One should be aware that this assumption does not suggest that household consumption drives production and economic interactions. In that case, the household column and/or raw in the technical coefficient matrix should be excluded to avoid overestimating labor expenditures. One possible improvement for this model is to define a certain number of export zones (ports, airports, etc.), whose export amounts are observable, and use another logit model to distribute the export demands across production zones (see Jin, Kockelman, and Zhao, 2002).

The distribution of person flows is of great interest for the network loading. We first transform these flows of dollars person trips. Then a traditional binomial mode choice model splits the person trips into vehicle trips and public transit trips.

Mode Split Model

In practice, multinomial logit (MNL) and nested logit (NL) models are very common models of mode choice (see Ben-Akiva and Lerman, 1987). Using a MNL model, the proportion of trips made by mode *w* between zones *i* and *j* is the following:

$$\Pr_{w|ij} = \frac{e^{v_{w|ij}}}{\sum_{l} e^{v_{l|ij}}} \quad (4.2.1)$$

where $v_{w|ij}$ is the systematic utility of mode *w* given origin *i* and destination *j*. The utility function is specified to be a function of trip time, cost, and other variables. For example, a simple linear function can be specified as follows:

$$u_w = v_w + e_w = q_w + d t_w + e_w \quad (4.2.2)$$

where t_w is total travel time by mode *w*, e_w represents unobserved heterogeneity (assumed to be i.i.d. Gumbel, and q_w and d are model parameters.

So the total number of person trips by mode *w* from zone *i* to zone *j*, T_{ijw}, is the following:

$$T_{ijw} = T_{ij} \Pr_{w|ij} \quad (4.2.3)$$

where T_{ij} is the transformed personal trip between ***i*** and ***j***.

Here we focus on the personal vehicle trips and disregard the transit trips since vehicle trips are the main part of road traffic in the U.S..

UE Assignment

Once personal vehicle trips are known (from the mode split model described above), a vehicle occupancy factor is applied to transfer personal vehicle trips into vehicle trips, which can then be assigned to the road network. An all-or-nothing method assigns all traffic flows between an origin-destination (O-D) pair to the shortest path. Capacity-restrained assignments attempt to approximate an equilibrium solution by iterating from all-or-nothing traffic loading and link travel times based on link capacity functions. UE methods utilize an iterative process to achieve a convergent solution ("equilibrium") in which no traveler can improve his/her travel time by shifting routes. UE algorithms incorporate link capacity functions in their search for convergence to an equilibrium state. The main difference between capacity-restrained and UE assignments is that UE incorporates steps to determine search directions and step sizes, which ensures UE to approach the equilibrium solutions. Capacity-restrained assignments just simply repeat the iterations, so they may take a considerable long time to reach the solutions, or even fail (Sheffi, 1985).

A common link performance function used in UE assignments, developed by the Bureau of Public Roads (see Martin *et al.*, 1998), is the following:

$$t = t_f \left[1 + a_0 \left(\frac{f}{f_{\max}} \right)^{b_0} \right] \qquad (4.2.4)$$

where t is the impedance of a given link at flow *f*, t_f is free-flow (uncongested) impedance of the link, $f_{\max}$ is link "capacity", and a_0 and b_0 are volume/delay coefficients. The traditional BPR values for a_0 and b_0 are 0.15 and 4.0, respectively, but these are based on using a $f_{\max}$ for level of service C. For a $f_{\max}$ corresponding to true capacity (*i.e.*, maximum flow under level of service E), *NCHRP Report 365* (Martin *et al.*, 1998) recommends larger values, of 0.84 and 5.5, respectively. These larger values are applied here.

System Equilibration

The proposed model system links submodels by the UE assignment results for travel costs[14]. These feed back to the RUBMRIO model to calculate new commodity flows. Adopting proper feedback mechanisms may result in a general equilibrium for the whole system, as represented by convergent solutions for the UE assignment and the RUBMRIO model.

We examine the two feedback methods as shown in Figure 4.2.1. Method 1 consists of one feedback loop, which feeds the interzonal travel times back to the RUBMRIO model to recalculate interzonal flows. Method 2 includes two feedback loops: an internal one feeds interzonal vehicle travel times back to the mode split model and recalculates the share between car and public transit. After this two-step transportation model converges, the external loop feeds back interzonal travel times to the RUBMRIO model to compute the interzonal flows again. We set the internal feedback loop runs no more than five times because interzonal travel times

[14] Transportation costs can be transformed from time units into dollars using a time-value factor. However, because the unit of transportation costs do not affect the calculations of relative utilities (and distribution probabilities), there is unnecessary to make the transformation.

converges very fast when applying UE in transportation model. An overall LU-T equilibrium was obtained when the feedback results converged.

4.2.2 Data Acquisition and Parameter Calibration

A large amount data are required to implement the linked model. First, behavioral data such as observed commodity flows are used to estimate the model parameters. And then exogenous inputs are needed to run the model. Primary data sets include IMPLAN's industry transaction tables for Dallas county (Minnesota IMPLAN Group, Inc, 1997) and the 1993 Commodity Flow Survey (CFS) data set (BTS, 1995).

Our study area is the DFW metropolis, with over 4.4 million persons, 1.5 million households, and 2.7 million jobs in 1995 (NCTCOG, 1999). The DFW area is divided into 919 transportation analysis zones (TAZs). Figure 4.2.2 is a map of the 919 zones, as well as the coded DFW highway network, which consists of 45,112 links.

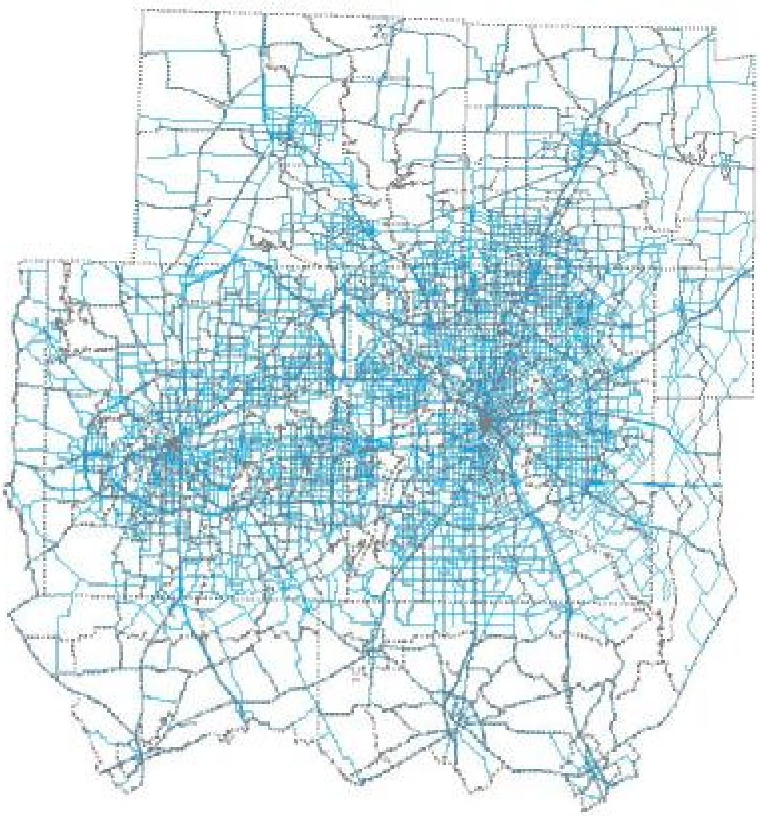

Figure 4.2.2 Dallas-Fort Worth metropolis with its 919 TAZs and 45,112 coded network links

Technical Coefficients

The key parameters of the RUBMRIO model are the technical coefficients a_i^{mn}, which reflect productive dependencies across zones. We assume that the technical coefficients are stable in the short run and therefore are exogenous to the model.

IMPLAN's industry-by-industry transaction tables at the county level were used to generate the technical coefficients. The transaction tables derive from U.S.

inter-industry accounts and estimate the values of purchases at relatively fine levels of resolution. The original industry transaction tables include 528 industry sectors, which are bridged to the Standard Industry Classification (SIC) codes. In this study, an aggregated sector system is used to represent the whole economy in the DFW area. The 528 industry sectors were categorized into 12 aggregate industry sectors and 2 other economic sectors (Government and Households) according to the SIC codes, on the basis of Min *et al'*s classification (2001). To do this, all transaction table values were summed up by 12 aggregate classifications, to create aggregate transaction values. Then, the total transaction amount corresponding to each productive sector (i.e., the sum of each aggregate column's transactions) was computed. The aggregate transaction values were divided by their respective column sums, creating the set of aggregate technical coefficients used here. Household and Government sectors were constructed from the value-added tables and final-demand tables generated by IMPLAN. Table 4.2.1 shows the 14 sectors and their corresponding IMPLAN and SIC sector codes.

Table 4.2.1 Description of economic sectors in the RUBMZIO application to DFW area

Sectors	Description	IMPLAN Code	SIC Code (2-digit)
1	Agriculture, Forestry, and Fisheries	1~27	01~09
2	Mining	28~47, 57	10~14
3	Construction	48~56	15~17
4	Food and Kindred Products	58~103	20
5	Chemicals and Allied Products	186~209	28
6	Primary Metals Industries	254~272	33
7	Fabricated Metal Products	273~306	34
8	Industrial Machinery and Equipment	307~354	35
9	Electronic and Electric Equipment	355~383	36
10	Transportation Equipment	384~399	37
11	Other Durable and Non-Durable Manufacturing	104~185, 210~253, 400~432	24~27, 29~32, 38~39
12	Utilities, Trade and Services	433~509	40~87
13	Households	From value-added tables	
14	Government	From final-demand tables	

Utility Function Parameters

Parameters in the random utility function (3.1.17) of the RUBMRIO model were estimated based on 1997 CFS data, which provides commodity flow estimates among the 49 continental states. We estimated the parameters for transportation costs at the state level using a multinomial logit model. The transportation costs recognize both highway and railway choices. Highway and railway distances between each state-to-state pair were generated based on the shortest path over the highway and railway network between geographic centroids. For within state trades, $\sqrt{\frac{Zone\ Area}{p}}$ was used as an estimate of distance traveled. For each needed input *m*, buyers can choose the providers based on a random cost minimization. Their sensitivity to distance is reflected by the dispersion parameters l^m. The results of these models show in Table 4.2.2. Household's and Government's parameters are assumed to be the average of all others. Then, the utility function and probabilities were calculated using equation (3.1.5).

Table 4.2.2 Origin choice model results

Sector	*l*	SE	T-Stat	P-Value
1	11.289	1.295	8.714	0.000
2	650.57	0.623	1043.256	0.000
3	11.35	1.299	8.733	0.000
4	11.435	1.306	8.754	0.000
5	10.678	1.250	8.537	0.000
6	8.536	1.107	7.71	0.000
7	7.123	1.008	7.066	0.000
8	5.372	0.893	6.015	0.000
9	3.387	0.771	4.393	0.000
10	6.546	0.969	6.75	0.000
11	6.580	0.972	6.769	0.000
12	5.141	0.964	5.333	0.000

Log-likelihood function: -1765.16
Log-likelihood function with (no coefficients): -2288.39
#Observations: 588

The final demand is assumed to be household incomes per zone. Census data for DFW provides the average household income for each TAZ, and we multiplied these by the number of TAZ households in order obtain zonal final demands. Then, each sector's final demand per zone was calculated using the technical coefficients.

The RUBMRIO model is coded in Compaq Visual FORTRAN 6.5 (Compaq, 2000). The convergence criterion is that the maximum absolute difference between two consecutive iterations' flows be less than 1%; i.e.,

$$\max(\left|q_{ij}^{n(t)} - q_{ij}^{n(t-1)}\right|) < 0.01 q_{ij}^{n(t-1)}, \forall i, j, n \, . \tag{4.2.5}$$

Transportation Model Parameters

For transportation model parameters, this study uses values from the DFW travel model description report (NCTCOG, 1999). Necessary simplifications and modifications have been made based on *NCHRP Reports 187* (Sosslau *et al.*, 1978) and *365* (Martin *et al.*, 1998). Table 4.2.3 presents the parameter values for the mode split (in equation [4.2.2]) and trip assignment models (in equation [4.2.4]).

Table 4.2.3 Transportation model parameters

Model	**Parameter**	**Value**	**Source**
Model Split	$\theta_{transit}$	-0.549	NCTCOG
	d	-0.0297	
Traffic Assignment	α_0	0.84	NCHRP Report *365*
	β_0	5.50	

TransCAD (Caliper Co., 2001) was used for mode split and trip assignment sub-models in order to apply its commercialized, embedded logit calculation and UE algorithm. The convergence of a UE assignment was assumed when the maximum absolute change in all link flows between consecutive iterations was less than 5 vehicles per hour. The daily vehicle trips were transformed into morning peak hour trips for the UE assignment, using a peak-hour factor of 12.5%.

4.2.3 Feedback Results

The linked models with feedbacks produced estimates of location choices, flow distributions, and link flows (by vehicle). The results of greatest interest often are the link flows. At each iteration, the sequence of linked sub-models produced a set of link-flow results. Then, link travel costs, which are directly associated with link flows, are fed back to the RUBMRIO model, to recalculate the demand, (i.e., interzonal flows for the household sector). This application ran the linked model sequence for about 100 iterations, and collected the link flows from each iteration. Most of the ratios of link flow versus capacity were relatively low (e.g., 85% of them were less than 0.72 and the mean was 0.33), indicating that the assignment equilibrium was not heavily congested. In fact, the result was a portion of a general assignment; it only included morning peak hour home-based work auto trip assignment.

We compared the efficiency of two alternative feedback strategies. In practice, it is not clear how many iteration runs are needed to obtain accurate, converged results[15]. In this study, our first objective is to examine if feedback methods converge. Thus, we ran the model for a relatively large number iterations.

[15] For a recent discussion of this, one may see the Transportation Model Improve Program mailing list (TMIP, 2001).

Convergence Pattern

To determine whether the two feedback methods converge, we first defined convergence. In general, an algorithm compares two consecutive iterations' results. If the difference is less than a pre-defined, small number, then "convergence" is achieved. For example, a convergence criterion can be set as:

$$\max_k \{ \left| T_k^{(t)} - T_k^{(t-1)} \right| \} < 0.01. \qquad (4.2.6)$$

where T_k is the kth element among the results of interest and t is the iteration number.

However, our results of interest consist of a larger number of elements. There are 22,557 positive link flows out of the 45,112 links (the rest unused links are mainly in suburban and consist of a large number of small link fragments). If we take (4.2.6)'s criterion, it may take too long to converge. Instead, we defined the following "average shift error" (ASE) as our convergence criterion:

$$ASE = \sqrt{\sum_k \frac{(T_k^{(t)} - T_k^{(t-1)})^2}{N}} \qquad (4.2.7)$$

where N is the total number of links with positive flow volumes. ASE is a measure of "average" shift in solution values between two consecutive iterations. Here, if the ASE of link flows between two consecutive iterations is less than 5, then the sequence converged.

Figure 4.2.3 illustrates the convergence patterns of the two methods. Both methods managed to converge at the 5.0 ASE level. Method 1 converged at the 97th iteration, while Method 2 succeeded at the 27th iteration. It took about 15.1 minutes of computation time for each iteration of Method 1, and 25.3 minutes for Method 2, on a 1.0 GHz Pentium III computer. So the convergence of Method 1 required

1465 minutes, while Method 2 required 683 minutes. We noticed that there are some oscillations for Method 1’s ASE values in the first 25 iterations. Then, it monotonically fell, through relatively slowly. In contrast, Method 2 converged rather fast. Comparing the convergent hourly volumes on links from both methods, they are almost identical (see Figure 4.2.5), with an ASE value of 1.2 (comparing each other) and a correlation coefficient value of 0.98.

To check whether the convergence patterns are consistent in the RUMBRIO model, we compared the trade flow results, which are the land use model’s outputs and the transportation model’s inputs. We calculated the ASE value for each iteration’s trade flow results (see Figure 4.2.4). Overall, the trade flows converged faster than the link flows: Method 1 needed 72 iterations to reduce the ASE value to less than 5.0, while Method 2 required only 21 iterations. Comparing the convergent daily trade flows from both methods, the results are rather close, with an ASE value of 0.14 and a correlation coefficient value of 0.99 (Figure 4.2.6).

This suggests that these two solution methods yield unique equilibrium results, as evidence from both the land use model results and the transportation model results. These solutions are regarded as the “true” solutions for the cross comparison in the next section.

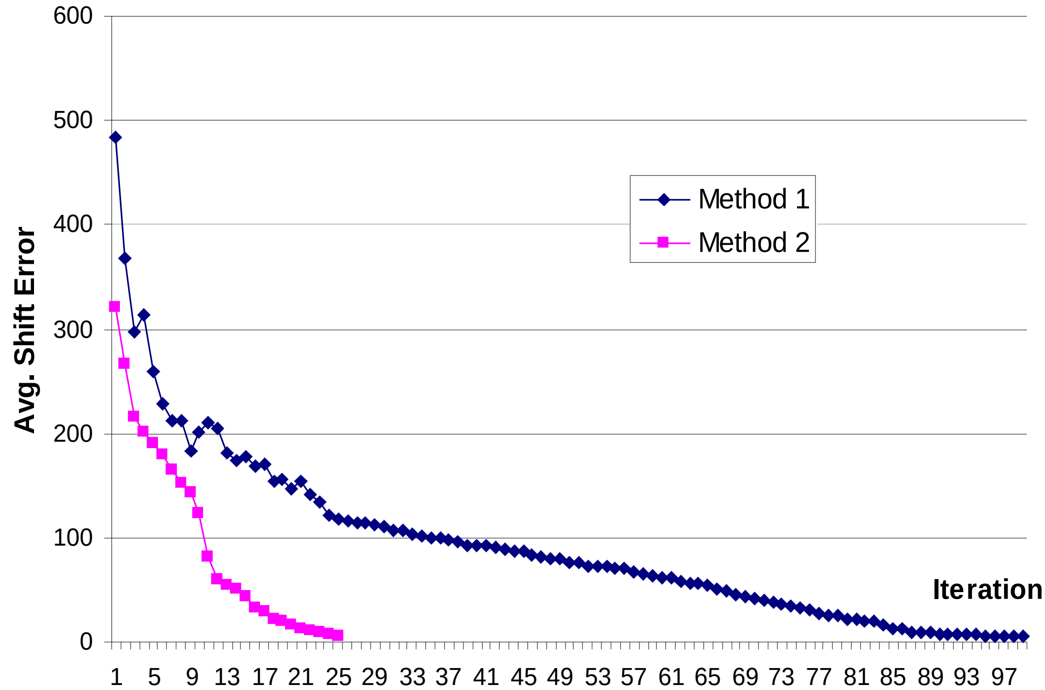

Figure 4.2.3 Convergence patterns of the feedback methods for hourly link flows

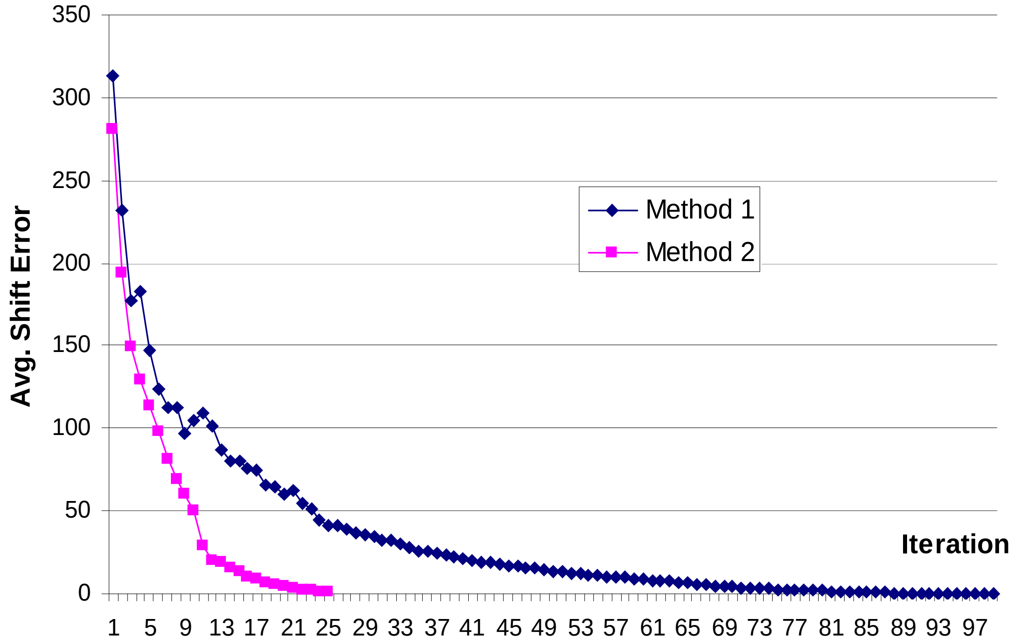

Figure 4.2.4 Convergence patterns of the feedback methods for trade flows

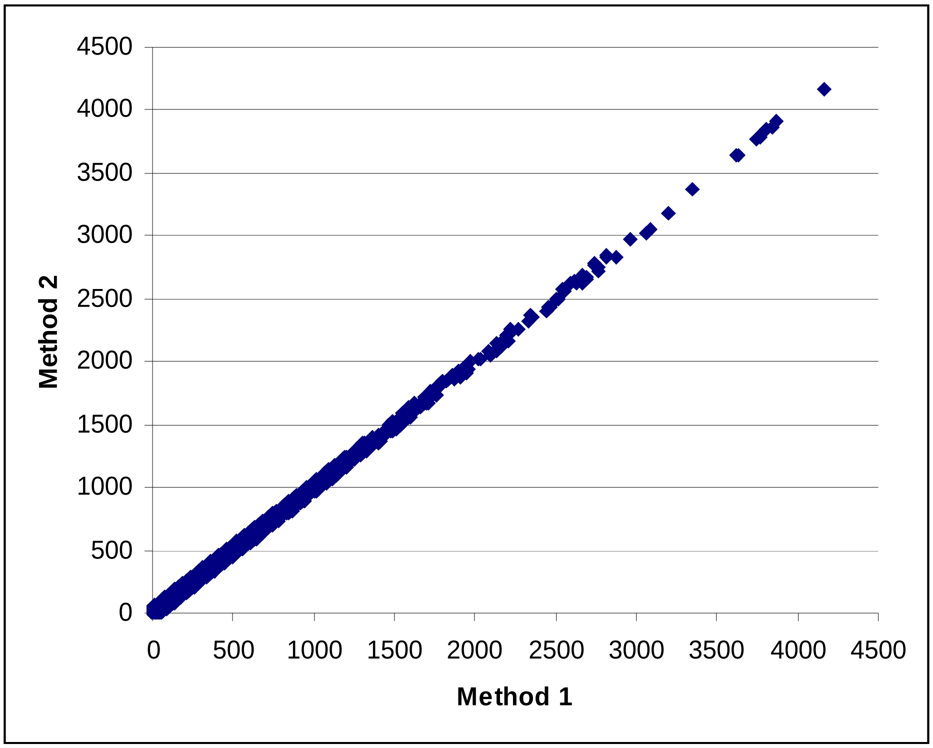

Figure 4.2.5 Scatterplot of feedback methods' final converged hourly link flows

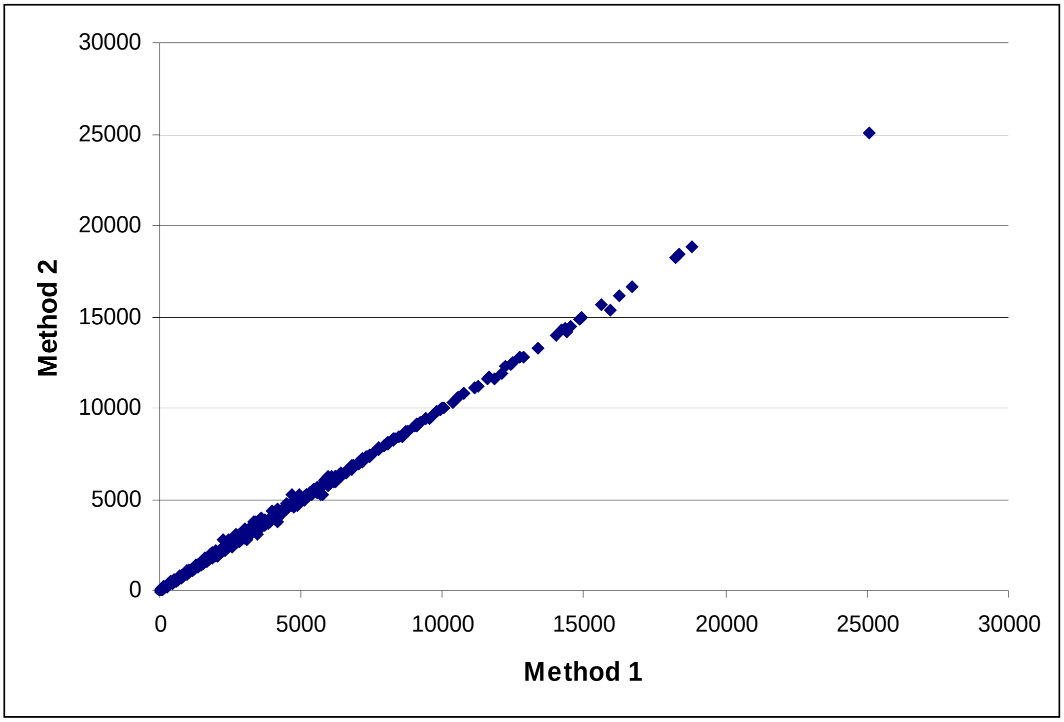

Figure 4.2.6 Scatterplot of feedback methods' final converged trade flows

Cross Comparison

Figure 4.2.3 illustrates one instance of the superiority of Method 2 (in terms of iterations until convergence). However, Method 2 has 5 interior feedback runs, so each iteration of Method 2 consists of 5 UE assignments. For this reason, each iteration in Method 2 may be regarded as 5 iterations in Method 1.

We adopt two measures for the comparison of link predictions (see Boyce, Lupa, and Zhang, 1994). The first one is root mean square error (RMSE) and the second on is a Chi-square value, as follows:

$$RMSE = \sqrt{\sum_k \frac{(T_k - \hat{T}_k)^2}{N}} \tag{4.2.8}$$

$$c^2 = \sum_k \left[\frac{(T_k - \hat{T}_k)^2}{\hat{T}_k} \right] \tag{4.2.9}$$

where $\hat{T}_k$ is the element of the "true" solution (i.e., the fully converged solutions as described above). Both the RMSE and Chi-square terms are very effective in comparing intermediate solutions with the true solutions. In general, RMSE can be considered as a standard error in model results versus true solutions, while Chi-square is a sum of square (and normalized) errors.

The data elements are the pairs of link flows with positive numbers. Zero flows are eliminated from the solution (because the denominator in equation [4.2.9] cannot be zero). Both measures are desired to be close to zero.

Table 4.2.4 presents the results of the two feedback methods for link flows for early iterations. Most feedback applications implement limited iterations (see Miller, 1997). So these early iterations may resemble practice more accurately.

Table 4.2.4 Results of two methods for link flows

Method 1			Method 2		
# of Iterations	RMSE	c^2	# of Iterations	RMSE	c^2
5	276.42	1429304	1	254.39	1817604
10	267.42	966735	2	248.07	729300
15	234.69	890861	3	133.81	173960
20	154.02	508048	4	118.79	90007
25	111.45	205613	5	63.64	89086

It is clear that Method 2 yields better RMSE and Chi-square values except for the earliest iteration noted here. The results also illustrate that Method 2's relatively rapid and efficient convergence, since each iteration of Method 2 lies much closer to the true solution than five iterations of Method 1 (the RMSE and Chi-square values of Method 2 decreases more quickly than those of Method 1).

In summary, the double-loop method is superior to the single-loop method both in computation time and accuracy at most early iterations.

4.3 SUMMARY

In this section, we examined two methods to approach the overall equilibria in ITLUMs. First, we constructed a combined model to synthesize all the choices in ITLUMs: location choices, travel frequency, destination, mode, and route choices. We derived the optimization conditions to show the equivalence to the individual submodels. In addition, the uniqueness of equilibrium solution was discussed. Evans' algorithm was proposed for solutions. Second, we assembled a linked model using two feedback strategies: the first used a single loop to link the UE assignment model to the RUBMRIO model; the second implemented an additional, internal loop, to link the UE model to the mode split model. Our numerical example for the DFW region and network suggests that both feedback methods converge to the unique solutions, but the second method exhibits more efficient, rapid convergence.

However, a comparison between the combined model and the linked model is needed for future studies. The expectation is that the linked model is a practical solution algorithm for the sophisticated, combined model, given the theoretical analysis and/or numerical examples. A similar research processes can be found in expectation maximization (EM) literature (see, e.g., Hartley, 1958; Dempster *et al.*, 1977; McLachlan and Krishnan, 1997).

Chapter 5 Conclusions

5.1 SUMMARY OF FINDINGS

Much interest and effort has been given to the integrated modeling of transportation and land use interactions. This research focuses on equilibrium modeling of such interactions since these equilibria have not been well understood or modeled. The primary objective of this work was to understand, formulate, and assess the equilibria of integrated transportation and land use models (ITLUMs) based on spatial input-output (SIO) theory. Two major tasks undertaken towards this objective were outlined in Section 1.3. In this section, these tasks are reviewed, and related conclusions and findings are discussed.

The first task was to analyze SIO model equilibria. The SIO model is the basis for a number of operational land use-transportation models (e.g., Echenique and colleagues' MEPLAN [Hunt, 1993], de la Barra's TRANUS [1995]). Our analysis relied on the unique solutions for sales prices and trade volumes. A fixed-point formulation was proposed for the uncongestible random-utility-based multiregional input-output (RUBMRIO) model, which originally consisted of a set of model equations. The fixed-point formulation allows one to develop conditions for solution existence and uniqueness. Under weak conditions regarding sales prices, the set of solution prices was shown to exist. And the price solutions are unique under restricted conditions regarding the dispersion parameters. The solution uniqueness is also discussed under more general situations with larger dispersion parameters' values. The logit probability approximation calculation ensures the price fixed-point sequence *almost surely* converges. Once prices and spatial purchase probabilities are known, commodity flows also were found to be unique. The fixed-point formulation established here verifies that the common/original RUBMRIO iterative algorithm always converges if the above conditions are satisfied. In addition, a modified algorithm was demonstrated to be more efficient. In summary, the theoretical analysis of the RUBMRIO model assured solution uniqueness, which underpins the equilibrium solutions to SIO trade volumes.

The second task was to formulate and solve for the *overall* equilibria in ITLUMs. We examined two methods to approach the equilibria of a full, congestible ITLUM based on the RUBMRIO model. First, we constructed a combined model which synthesizes all the ITLUM components including location choices, travel frequencies choices, mode choices, and route choices. The optimization conditions were derived, and they actually presented these choices' models. In addition, the uniqueness of equilibrium solution was discussed. Evans algorithm (see Patriksson, 1994) was proposed to solve this combined model. Second, we assembled a "linked" model using two feedback strategies: the first uses a single loop to link the UE assignment model to the RUBMRIO model; the second implements an additional internal loop to link the UE model to the mode split model. Our numerical example for the Dallas-Fort Worth network suggested both feedback methods converge to the unique solution; but the second one with double loops more efficiently, rapidly converged. In summary, the two methods developed in this task are of theoretical interest and practical application. The combined model allows one to simultaneously solve the set of sub-ITLUM equations, each representing one component of the system. The linked feedback provides a practical way to integrate land use models and network models, and additional feedback loops can reduce solution time. Both methods are rather flexible in describing travel choices and can be extendable to incorporate other behavioral specifications.

5.2 APPLICATIONS AND FUTURE RESEARCH

The findings developed here are applicable for transportation planning and policymaking processes. First, the property of solution uniqueness for the RUBMRIO model makes its application more promising, and removes any concerns about the choice of a start point. The RUBMRIO model is capable of predicting trade interaction across a nation (Kim, 1989), a state (Jin, Kockelman, and Zhao, 2002; Hunt, 1993), or a city (Abraham and Hunt, 1999). Such interactions are essential for human communities. Second, the "fully" integrated and congestible model approach suggests a positive direction for ITLUMs. Once model parameters have been estimated, the model can be solved by Evans' algorithm for determinations of land use and travel choices. Finally, the linked model based on the RUBMRIO model demonstrates a convenient method for MPOs to improve their existing travel demand modeling practices. Specifically, double-loop feedback strategies speed convergence. Overall, these methods create rich and flexible tools for constructing statewide or urban travel forecasting models, which rely on data sets typically available to metropolitan planning organizations (MPOs) and on several existing submodels. These tools are invaluable for economic analysis and transportation planning and can be used to evaluate questions often posed by legislators, the public, and other stakeholders. They also provide a guide for future data collection efforts.

Several limitations in this study are worthy of further research. First, for the RUBMRIO model, permitting substitution across inputs to production will make the problem much more realistic – but also much more difficult, because input choice will become functions of all prices explicitly. Moreover, calibration of such production processes, as functions of the variables tracked in these models, is highly unlikely (due to anonymity, cost, and other issues). However, progress is being made, and model improvements are expected. Another restrictive assumption in this RUBMRIO model is that the sum of technical coefficients for one commodity output, i.e., $\sum_m a_j^{mn} < 1 \forall j, n$. If some or all of these row sums equal one, $\sum_m a_j^{mn} = 1$, then the fixed-point functions $f(\cdot), g(\cdot)$ are non-expensive mappings instead of contractive mappings. Thus, the solution existence and uniqueness conditions will be quite different, and, possibly, only numerical examples can be shown for the uniqueness properties of the RUBMRIO model.

Second, when linking the RUBMRIO model with transportation sub-models on a congestible network, one should study solution uniqueness of the overall system. Although section 4.2.1 described the application of fixed-point problem properties to obtain conditions of solution uniqueness, a formal, theoretical analysis is still needed. The existence of such a proof is fully expected thanks to proofs that exist for congestible travel demand models (Cantarella, 1997) and the RUBMRIO uniqueness proof provided here. Both of these rely on fixed-point problem methods and this approach may continue to be very useful in proving solution existence and uniqueness, and in evaluating solution algorithms, in future extensions to the RUBMRIO model.

Third, for the combined model, although the parameters estimated by individual submodels are acceptable in practical applications, simultaneous parameters estimation procedures are needed for efficient, unbiased calibration. A possible approach is the entropy-maximizing framework of Wilson (1970b), which relies on a highly general information-minimizing solution to spatial interaction parameter estimation (see Fotheringham and O'Kelly, 1989). It has been used as the primary parameter estimation procedure for several combined models (e.g., Shen, 1995; Chu, 1999). In addition, maximum likelihood estimation procedures may be applicable (see, e.g., Oppenheim, 1995).

Finally, it would be very helpful to compare the combined model with the linked model proposed in this study to investigate if their solutions are the same, and examine their efficiency, accuracy in representing observed data, and flexibility in forecasting future land use and transportation changes. The expectation is that the linked model is a practical algorithm, if feedback loops constructed appropriately, to the theoretically correct, combined model.

There exist clear needs for reliable software to implement the combined equilibrium approaches in ITLUMs (Miller, 1997).

Without question, an increased understanding of the ITLUMs equilibria will facilitate improvement in travel demand forecasts (see Boyce, Lupa, and Zhang, 1994). This dissertation represents one step towards wider applications and understanding of the equilibrium approach.

Appendix

A.1 Example Algorithm of The RUBMRIO Model, as Programmed in Fortran

The code is a modified version of the original program developed by Yeonjoo Min, former M.S. candidate, University of Texas, Department of Civil Engineering.

```
c
      program RUBMRIO
c
     implicit double precision (a-h,l,o-z)
      common /data1/ ni,nj,nm,nn,iprint
c     common /mat/ b,d,a,rhamda,y,de
c
  character*15 filein,fileout
      character*50 title
      character*15 tfilein, tfileout
      allocatable :: b(:,:),d(:,:),a(:,:,:)
      allocatable :: lambda(:),y(:,:),ainv(:,:)
      allocatable :: x(:,:),f(:,:),u(:,:,:),umax(:,:)
      allocatable :: fu(:,:,:)
      allocatable :: c(:,:),q(:,:,:),p(:,:),qnew(:,:,:)
c
c  ...I/O Opening
  write(*,10)
 10 format(1x,'Type the input filename  : ',$)
  read(*,*) filein
c
  write(*,20)
 20 format(1x,'Type the output filename : ',$)
  read(*,*) fileout
c
  open(11,file=filein,status='unknown')
      open(12,file=fileout,status='unknown')
c
c   ----------------------------------------------------------------
c  ...Read data
```

```
c     ----------------------------------------------------------------
      read(11,5) title
           write(12,6) title
    5 format(a50)
    6 format(/,a50,/)
      read(11,*) tol,maxiter,iprint
      read(11,*) ni,nj,nm,nn
           read(11,7) tfilein
           write(12,7) tfilein
           read(11,7) tfileout
           write(12,7) tfileout
    7      format(a15)
c
      write(12,30) tol,maxiter,iprint
   30 format(1x,'Tolerance (max. cell change between iterations %):
     1',e8.2,3x,'Max iterations: ',i10,1x, 'Print process=',i2,5x,/,$)
      write(12,40) ni,nj,nm,nn
   40 format(1x,'(zones) i=',i4,1x,'j=',i4,1x,' (sectors) m=',i4,
     1 1x,'n=',i4,1x,/,$)

           open(13,file=tfilein,status='unknown')
           open(14,file=tfileout,status='unknown')

c          print(*.*)
c
c  ...allocate the storage space
      allocate (b(ni,nn),d(ni,nj),a(nj,nm,nn),
     1         lambda(nn),y(nj,nm),ainv(nm,nn))
           allocate (x(nj,nn),f(nj,nm),u(ni,nj,nn),
           1         umax(nj,nn),fu(ni,nj,nn),
     1         c(nj,nm),q(ni,nj,nm),p(nj,nm),
           1         qnew(ni,nj,nm))

c
      ierrflag=0
           itercount=0
           qnew=0.0d0
      call input1(b,d,a,ainv,lambda,x,y)
      call step1(b,d,lambda,u,umax,fu)
      call step2(x,a,ainv,fu,c,q,f,y)
c
           do while (ierrflag .eq. 0)
```

```
c
            itercount=itercount+1
            if (itercount .gt. maxiter) then
              write(*,*) 'Max. number of iteration is exceeded!!'
              stop
            end if
              call step3 (q,p,b,u,a)
              call step1(b,d,lambda,u,umax,fu)
              call step2(x,a,ainv,fu,c,q,f,y)
            call error(q,qnew,d,ierrflag,tol,itercount)
      end do
      close(11)
      close(12)
      stop
 end

subroutine input1 (b,d,a,ainv,lambda,x,y)
      implicit double precision (a-h,l,o-z)
      common /data1/ ni,nj,nm,nn,nj1,iprint
      dimension b(ni,nn),d(ni,nj),a(ni,nm,nn),d1(3),y1(nj),
1          a1(nm,nn),ainv(nm,nn),lambda(nn),x(nj,nm),y(nj,nm)
c  ... Initiate b & x
      do i=1,ni
            do n=1,nn
              b(i,n)=0.0
            end do
      end do

      do m=1,nm
            do j=1,nj
              x(j,m)=0.0
            end do
          end do
c   ... Read from input file
c   ... Rhamda parameters
      read(11,*) (lambda(n),n=1,nn)

c   ... Tech. coeffi.
c       do j=1,nj
            do m=1,nm
              read(11,*) (a1(m,n),n=1,nn)
            end do
```

```
c         end do
          do j=1,nj
            do m=1,nm
              do n=1,nn
                a(j,m,n)=a1(m,n)
              end do
            end do
          end do

c   ... Final Demand
          read(11,*) (y1(j),j=1,nj)
      y=0.0
      m=13
          do j=1,nj
            y(j,m)=y1(j)
          end do

c   ... Highway distance/travel time-shortest path
      do i=1,ni
        do j=1,nj
              read(13,*) (d1(k),k=1,3)
                   d(i,j)=d1(3)
c             write(12,*) i,j,d(i,j)
        end do
          end do
end

subroutine step1 (b,d,lambda,u,umax,fu)
       implicit double precision (a-h,l,o-z)
       common /data1/ ni,nj,nm,nn,iprint
      dimension b(nj,nn),d(ni,nj),lambda(nn),
1     u(ni,nj,nn),umax(nj,nn),fu(ni,nj,nn)
c
c  ...Step 1 : Calculate u(i,j,n)
      do n=1,nn
            do j=1,nj
              do i=1,ni
                u(i,j,n)=1.d0*(b(i,n))+lambda(n)*dlog(dexp(2.75
1                  -0.00393*1.d0*d(i,j))+dexp(-0.00066*0.d0*d(i,j)))
              end do
            end do
          end do
```

```
c
      umax=-10000000.0d0
      do n=1,nn
        do j=1,nj
           do i=1,ni
                if    (umax(j,n) .le. u(i,j,n)) then
                    umax(j,n)=u(i,j,n)
                end if
              end do
           end do
      end do
c
      do n=1,nn
            do j=1,nj
              do i=1,ni
          if ((umax(j,n)-u(i,j,n)).ge. 20.0d0 ) then
                  fu(i,j,n)=-20.0d0
                else
                                  fu(i,j,n)=u(i,j,n)- umax(j,n)-0.01
                end if
              end do
            end do
          end do

      if (iprint .eq. 1) then
            n=13
        write(12,210) n
 210    format(/,1x,'fu(i,j) for ',i3)
        do i=1,ni
          write(12,130) (fu(i,j,n),j=1,1)
            end do
        write(12,220) n
 220    format(/,1x,'d(i,j) for ',i3)

        do i=1,ni
          write(12,130) (fu(i,j,n),j=1,1)
            end do
c         end do

 130    format(1x,50(e10.3,1x))
          end if
c
```

```
      return
end
```

```
subroutine step2 (x,a,ainv,fu,c,q,f,y)
            implicit double precision (a-h,l,o-z)
            common /data1/ ni,nj,nm,nn,iprint
            dimension x(nj,nn),a(nj,nm,nn),fu(ni,nj,nn),
1           c(nj,nm),q(ni,nj,nm),ainv(nm,nn),
1           f(nj,nm),y(nj,nm),xfix(nj,nn)
c     ...Calculate f(j.m)
            f=0.d0
            do j=1,nj
               do m=1,nm
                    f(j,m)=y(j,m)
               end do
            end do

      if (iprint .eq. 1) then
                write(12,230)
  230           format(/,1x,'f(j,m)')
                        do j=1,nj
                                    write(12,130) (f(j,m),m=1,nm)
                        end do
      end if

            x=0.d0
            do m=1,nm
               do j=1,nj
                     do i=1,ni
                        x(j,m)=f(j,m)+q(i,j,m)
                    end do
             end do
            end do

      xfix=0.0
            do j=1,nj
               do m=1,nm
                  do n=1,nn
                     xfix(j,m)=xfix(j,m)+ainv(m,n)*y(j,n)
                  end do
               end do
            end do
```

```
      if (iprint .eq. 1) then
c
      write(12,240)
  240 format(/,1x,'x(j,n)')
      do j=1,nj
            write(12,130) (x(j,n),n=1,nn)
          end do
c
      end if

c     ...Calculate C(j,m)
c     initialize c
      c=0.d0
          do m=1,nm
            do j=1,nj
              do n=1,nn
                c(j,m)=c(j,m)+x(j,m)*a(j,m,n)
              end do
            end do
          end do
c
      if (iprint .eq. 1) then
c
      write(12,250)
  250 format(/,1x,'C(j,m)')
      do j=1,nj
            write(12,130) (c(j,m),m=1,nm)
          end do
c
      end if

c     ...Calculate q(i,j,m)

      do m=1,nm
        do j=1,nj
              denomi=0.0d0
              do i=1,ni
                          denomi=denomi+dexp(fu(i,j,m))
              end do
          if (denomi .eq. 0) then
                print *, 'denom in STEP 2 is zero!!'
```

```
                print *, 'm=',m,'j=',j
             end if
                   do i=1,ni
                     q(i,j,m)=c(j,m)*dexp(fu(i,j,m))
                           q(i,j,m)=q(i,j,m)/denomi
                   end do
           end do
         end do
c
     if (iprint .eq. 1) then
c
     do m=1,nm
           write(12,260) m
  260    format(/,1x,'Q(i,j) for m=',i3)
       do i=1,ni
             write(12,130) (q(i,j,m),j=1,nj)
           end do
         end do
c
     end if
c
     sumc=0.0d0
         sumq=0.0d0
     do m=1,nm
           do j=1,nj
             sumc=sumc+c(j,m)
             do i=1,ni
               sumq=sumq+q(i,j,m)
         end do
           end do
         end do
c
     write(12,300) sumq,sumc
  300 format(/,1x,'Total Q : 'e15.5,3x,'Total C : ',e15.5)
  130    format(1x,50(e10.3,1x))
  120    format(1x,20(e10.3,1x))
c
     return
end
```

```
subroutine step3 (q,p,b,u,a)
         implicit double precision (a-h,o-z)
```

```
      common /data1/ ni,nj,nm,nn,iprint
      dimension q(ni,nj,nm),b(ni,nm),a(ni,nm,nn),p(nj,nm),u(ni,nj,nm)
c  ...Step 3-1 : Evaluate P(j,m)
      do j=1,nj
        do m=1,nm
          denomi=0.0d0
          dnumer=0.0d0
          do i=1,ni
            denomi=denomi+q(i,j,m)
            dnumer=dnumer+q(i,j,m)*(dabs(-1.0d0*u(i,j,m)))
          end do
          if (denomi .eq. 0) then
            p(j,m)=0.00000001
      else
            p(j,m)=dnumer/denomi
          end if

    end do
      end do
c
  if (iprint .eq. 1) then
c
  write(12,240)
 240 format(/,1x,'P(j,m)')
  do j=1,nj
        write(12,120) (p(j,m),m=1,nm)
  end do
c
  end if

c  ...Step 3-2 : Re-evaluate b(i,n)
  b=0.0d0
  do n=1,nn
        do i=1,ni
          sum=0.0d0
          do m=1,nm
            b(i,n)=b(i,n)+a(i,m,n)*p(i,m)
          end do
    end do
      end do
c
  if (iprint .eq. 1) then
```

```
c
      write(12,250)
  250 format(/,1x,'B(i,n)')
      do n=1,nn
            write(12,130) (b(i,n),i=1,ni)
         end do
c
      end if

c
  120      format(1x,5(e10.3,1x))
  130   format(1x,10(e10.3,1x))
c
      return
end
```

```
subroutine error (q,qnew,d,ierrflag,tol,itercount)
         implicit double precision (a-h,o-z)
         common /data1/ ni,nj,nm,nn,iprint
         dimension q(ni,nj,nm),qnew(ni,nj,nm),y(nj,nm),d(ni,nj),
1        sumql(nj),sumqhh(nj),sumqemp(nj)
c
         ierrflag=1
         enorm=0.0d0
         exnorm=0.0d0
c
         do m=1,nm
           do i=1,ni
             do j=1,nj
               e=q(i,j,m)-qnew(i,j,m)
                     enorm=enorm+e**2
               exnorm=exnorm+qnew(i,j,m)**2
               if (q(i,j,m) .eq. 0.) then
                           e=abs(e)
                     else
                 if (abs(q(i,j,m)) .lt. 0.001) then
                       e=0.001d0
                                else
                             e=abs(e)/abs(q(i,j,m))*100.d0
                 end if
           end if
               if (e .gt. tol) ierrflag=0      ! not converged!!
```

```
            end do
          end do
        end do
c
        if (exnorm .ne. 0.0) then

           enorm=dsqrt(enorm/exnorm)
        end if
c
    write(12,10) itercount,enorm
  10 format(1x,'Iteration no. : ',i5,3x,'Error norm : ',e15.5,5x,$)
    if (ierrflag .eq. 0) then
          write(12,20)
  20    format('     Not converged!')
      do m=1,nm
                        do j=1,nj
                                do i=1,ni
                                  qnew(i,j,m)=q(i,j,m)
              end do
           end do
        end do
        else
          write(12,30)
  30    format('Converged!')
        end if
c
    if (ierrflag .eq. 1) then
          write(12,40)
  40    format(/,1x,'============Converged Solutions!==========',/)

    sumqhh=0.0d0
        sumql=0.0d0
        sumqemp=0.0d0
    do m=1,nm
          do j=1,nj
            do i=1,ni
              sumql(j)=sumql(j)+q(i,j,m)
              if (m .eq. 13) then
                           sumqhh(i)=sumqhh(i)+q(i,j,m)
                 sumqemp(j)=sumqemp(j)+q(i,j,m)
          end if
        end do
```

```
            end do
          end do


      do i=1,ni
                  do j=1,nj
                        write(14,105) i, j,q(i,j,13)
                  end do
          end do
  105 format(i3,',',i3,',',e20.12)

      end if

  130    format(1x,20(e10.3,1x))

      return
end
```

A.2 EXAMPLE OF THE LINKED MODEL, AS PROGRAMMED IN TRANSCAD'S GISDK CODE

This GISDK code is a macro script to construct the necessary feedback steps and automatically run TRANCAD modules including mode split, UE assignment models, and shortest path scans for travel cost.

```
Macro "Feedback 1"
   RunMacro("TCB Init")

// STEP 1: PA2OD
   Opts = {{"Input",    {{"PA Matrix Currency",    {"C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\CGRAV1.MTX",
                                        "HBW",
                                        "ROW_IDS",
                                        "COL_IDS"}},
                    {"Lookup Set",          {"C:\\PROGRAM
FILES\\TRANSCAD\\TAB\\HOURLY.BIN",
                                        "HOURLY"}}}},
          {"Field",    {{"Matrix Cores",         {1}},
                    {"Adjust Fields",        {}},
                    {"Peak Hour Field",       {}},
                    {"Hourly AB Field",       {"HOURLY.DEP_HBW"}},
                    {"Hourly BA Field",       {"HOURLY.RET_HBW"}}}},
          {"Global",   {{"Method Type",           "PA to OD"},
                    {"Start Hour",           8},
                    {"End Hour",             8},
                    {"Average Occupancies",   {1.15}},
                    {"Adjust Occupancies",    {"No"}},
                    {"Peak Hour Factor",      {0}}}},
          {"Flag",     {{"Separate Matrices",     "Yes"},
                    {"Convert to Vehicles",   {"Yes"}},
                    {"Include PHF",          {"No"}},
                    {"Adjust Peak Hour",      {"No"}}}},
          {"Output",   {{"Output Matrix",         {{"Label",
                                        "PA to OD"},
                                        {"File Name",
                                        "C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\PA2OD.MTX"}}}}}}
```

```
    if !RunMacro("TCB Run Procedure", 1, "PA2OD", Opts) then goto quit


// STEP 2: MNL Evaluation
   Opts = {{"Input",    {{"View Set",           {"C:\\Documents and
Settings\\yzhao\\My Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\TAPZ919.DBD|TAZ919",
                                        "TAZ919"}},
                    {"Destination Set",     {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\TAPZ919.DBD|TAZ919",
                                        "TAZ919"}},
                    {"Model Table",         {"C:\\DOCUMENTS AND
SETTINGS\\YZHAO\\MY DOCUMENTS\\PROJECTS\\ITLUM\\LINKED
ITLUM\\DATA\\MNL-DFW.BIN"}},
                    {"Matrix Currencies",   {{"C:\\Documents and
Settings\\yzhao\\My Documents\\Projects\\ITLUM\\Linked ITLUM\\Data\\Transit
SPMAT.mtx",
                                        "Shortest Path - [Time *]",
                                        "New",
                                        "New"},
                                        {"C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\PA2OD.MTX",
                                        "HBW (8-9)",
                                        "Rows",
                                        "Cols"},
                                        {"C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\SPMAT.MTX",
                                        "Shortest Path - [Time *]",
                                        "New",
                                        "New"}}}}},
            {"Field",    {{"ID Field",            "TAZ919.ID"}}},
            {"Global",   {{"Number of Modes",     2},
                    {"Model Name",          "Model1"}}},
            {"Flag",     {{"Aggregate",           1},
                    {"Delete Case",         1}}},
            {"Output",   {{"Output Matrix",       {{"Label",
                                        "Output Matrix"},
                                        {"File Name",
                                        "C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\MNL_EVAL.MTX"}}}}}}
```

```
    if !RunMacro("TCB Run Procedure", 5, "MNL Evaluation", Opts) then goto
quit


// STEP 3: Fill Matrices
   Opts = {{"Input",    {{"Matrix Currency",   {"C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\PA2OD.MTX",
                                     "HBW (8-9)",
                                     "Rows",
                                     "Cols"}},
                     {"Core Currencies",   {{"C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\PA2OD.MTX",
                                     "HBW (8-9)",
                                     "Rows",
                                     "Cols"},
                                     {"C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\MNL_EVAL.MTX",
                                     "Auto",
                                     "RIndex",
                                     "CIndex"}}}}},
          {"Global",   {{"Method",           9},
                     {"Cell Range",       2},
                     {"Matrix K",         {1,
                                     1}},
                     {"Force Missing",     "Yes"}}}}

    if !RunMacro("TCB Run Operation", 6, "Fill Matrices", Opts) then goto quit


// STEP 4: Add Matrix Index
   Opts = {{"Input",    {{"Current Matrix",   "C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\PA2OD.MTX"},
                     {"Index Type",      "Both"},
                     {"View Set",        {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
                                    "Endpoints",
                                    "Selection",
                                    "Select * where TAPZ <> null"}},
                     {"Old ID Field",     {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
```

```
                                   "TAPZ"}},
                  {"New ID Field",    {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
                                   "ID"}}}},
         {"Output",   {{"New Index",       "Node"}}}}

   if !RunMacro("TCB Run Operation", 7, "Add Matrix Index", Opts) then goto
quit


// STEP 5: Assignment
  Opts = {{"Input",   {{"Database",            "C:\\Documents and
Settings\\yzhao\\My Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD"},
                  {"Network",            "C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked ITLUM\\Dallas\\Dram-Empal\\DFW
highway.net"},
                  {"OD Matrix Currency",   {"C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\PA2OD.MTX",
                                      "HBW (8-9)",
                                      "Node",
                                      "Node"}}}},
         {"Field",   {{"FF Time",             "[Time *]"},
                  {"Capacity",           "[Capacity *]"},
                  {"Alpha",              "None"},
                  {"Beta",               "None"},
                  {"Preload",            "None"}}},
         {"Global",   {{"Iterations",         40}}},
         {"Output",   {{"Flow Table",         "C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\ASN_LINKFLOW.BIN"}}}}

   if !RunMacro("TCB Run Procedure", 8, "Assignment", Opts) then goto quit

// STEP 6: Fill Dataview
  Opts = {{"Input",   {{"Dataview Set",   {{"C:\\Documents and
Settings\\yzhao\\My Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.dbd|YR95",
                                  "C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\ASN_LINKFLOW.BIN",
                                  "ID",
                                  "ID1"},
```

```
                              "YR95+ASN_LINKFLOW"}}}},
        {"Global",  {{"Fields",       {"BA_Time"}},
                {"Method",       "Formula"},
                {"Parameter",     "(if Ba_Time >60 then 60 else Ba_Time)
       "}}}}

   if !RunMacro("TCB Run Operation", 1, "Fill Dataview", Opts) then goto quit


// STEP 7: Fill Dataview
  Opts = {{"Input",   {{"Dataview Set",  {{"C:\\Documents and
Settings\\yzhao\\My Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.dbd|YR95",
                              "C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\ASN_LINKFLOW.BIN",
                              "ID",
                              "ID1"},
                              "YR95+ASN_LINKFLOW"}}}},
        {"Global",  {{"Fields",       {"AB_Time"}},
                {"Method",       "Formula"},
                {"Parameter",     "(if AB_Time >60 then 60 else AB_Time)
       "}}}}

   if !RunMacro("TCB Run Operation", 2, "Fill Dataview", Opts) then goto quit


// STEP 8: Update Network Field
  Opts = {{"Input",   {{"Database",       "C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked ITLUM\\Dallas\\DFW\\Yr95.DBD"},
                {"Network",        "C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked ITLUM\\Dallas\\Dram-Empal\\DFW
highway.net"},
                {"Link Set",       {{"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked ITLUM\\Dallas\\DFW\\Yr95.dbd|YR95",
                               "C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\ASN_LINKFLOW.BIN",
                               "ID",
                               "ID1"},
                               "YR95+ASN_LINKFLOW"}}}},
        {"Global",  {{"Fields Indices",  "[Time *]"},
                {"Options",        {{"Link Fields",
                               {{"[YR95+ASN_LINKFLOW].AB_Time",
```

```
                                        "[YR95+ASN_LINKFLOW].BA_Time"}}},
                                        {"Constants",
                                        {1}}}}}}}

    if !RunMacro("TCB Run Operation", 3, "Update Network Field", Opts) then
goto quit


// STEP 9: TCSPMAT
   Opts = {{"Input",    {{"Network",          "C:\\Documents and
Settings\\yzhao\\My Documents\\Projects\\ITLUM\\Linked ITLUM\\Dallas\\Dram-
Empal\\DFW highway.net"},
                      {"Origin Set",       {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
                                        "Endpoints",
                                        "Selection",
                                        "Select * where TAPZ <> null"}},
                      {"Destination Set",   {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
                                        "Endpoints",
                                        "Selection"}},
                      {"Via Set",          {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
                                        "Endpoints"}}}},
           {"Field",    {{"Minimize",         "[Time *]"},
                      {"Nodes",            "Endpoints.ID"}}},
           {"Output",   {{"Output Matrix",     {{"Label",
                                        "Shortest Path"},
                                        {"File Name",
                                        "C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\SPMAT1.MTX"}}}}}}

    if !RunMacro("TCB Run Procedure", 4, "TCSPMAT", Opts) then goto quit


// STEP 10: Add Matrix Index
   Opts = {{"Input",    {{"Current Matrix",   "C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\SPMAT1.MTX"},
                      {"Index Type",       "Both"},
```

```
                    {"View Set",        {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
                                    "Endpoints",
                                    "Selection",
                                    "Select * where TAPZ <> null"}},
                    {"Old ID Field",    {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
                                    "ID"}},
                    {"New ID Field",    {"C:\\Documents and Settings\\yzhao\\My
Documents\\Projects\\ITLUM\\Linked
ITLUM\\Dallas\\DFW\\Yr95.DBD|Endpoints",
                                    "TAPZ"}}}},
          {"Output",   {{"New Index",       "New"}}}}

    if !RunMacro("TCB Run Operation", 5, "Add Matrix Index", Opts) then goto
quit

// STEP 11: Intrazonal
   Opts = {{"Input",    {{"Matrix Currency",   {"C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\SPMAT1.MTX",
                                    "Shortest Path - [Time *]",
                                    "New",
                                    "New"}}}},
          {"Global",   {{"Factor",          0.03}}}}

    if !RunMacro("TCB Run Procedure", 1, "Intrazonal", Opts) then goto quit

// STEP 12: TLD
   Opts = {{"Input",    {{"Base Currency",       {"C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\SPMAT1.MTX",
                                      "Shortest Path - [Time *]",
                                      "RCIndex",
                                      "RCIndex"}},
                    {"Impedence Currency",   {"C:\\Documents and
Settings\\yzhao\\My Documents\\Temp\\TransCAD\\SPMAT1.MTX",
                                      "Shortest Path - [Time *]",
                                      "RCIndex",
                                      "RCIndex"}}}},
          {"Global",   {{"Start Option",        1},
                    {"Start Value",        0},
```

```
                    {"End Option",          1},
                    {"End Value",          180},
                    {"Method",             1},
                    {"Number of Bins",      18},
                    {"Size",               1},
                    {"Statistics Option",    1},
                    {"Min Value",           1},
                    {"Max Value",           1}}},
          {"Output",   {{"Output Matrix",       {{"Label",
                                        "Output Matrix"},
                                        {"File Name",
                                        "C:\\Documents and Settings\\yzhao\\My
Documents\\Temp\\TransCAD\\TLD1.MTX"}}}}}}

    if !RunMacro("TCB Run Procedure", 1, "TLD", Opts) then goto quit
   done:
   Return( RunMacro("TCB Closing", 1, "TRUE" ) )
   quit:
   Return( RunMacro("TCB Closing", 0, "TRUE" ) )
endMacro
```

References

Abraham, J.E. 1998. “A Review of the MEPLAN Modeling Framework from a Perspective of Urban Economics.” Working Report at University of Calgary.

Abraham, J.E. and J.D. Hunt. 1999. “Semi-Automated Calibration of the MEPLAN Model of Sacramento”, 6th Computers in Urban Planning and Urban Management Conference, Venice, Italy.

Allen, P.M., G. Engelen, and M. Sanglier. 1986. “Towards a General Dynamic Model of the Evolution of Urban Systems.” In *Advances in Urban Systems Modelling*. Edited by B. Hutchinson, and M. Batty. Amsterdam: North-Holland, pp. 199-200.

Alonso, W. 1964. *Location and Land Use*, Cambridge, MA: Harvard University Press.

Anas, A. 1984. “Discrete Choice Theory and the General Equilibrium of Employment, Housing, and Travel Networks in a Lowry-type Model of the Urban Economy”. *Environment and Planning*, 16A, pp.1489-1502.

Bard, J.F. and J.E. Falk. 1982. “An Explicit Solution to the Multi-level Programming Problem.” *Computer and Operations Research*, 9 (1), pp. 77-100.

Barrett, C. 1994. “TRANSIMS: A Bottoms-up Approach to Travel Forecasting.” Presented at the 73rd Annual Meeting of the Transportation Research Board, Washington, D.C., January.

Baton-Aschman Associates, Inc. 1981. *Development and Calibration of Travel-Demand Models for the New Orleans Area.* Prepared for the Regional Planning Commission, Jefferson, Orleans, St. Bernard, and St. Bernard Parishes, Louisiana.

Bazaraa, M.S. and C. M. Shetty. 1979. *Nonlinear Programming: Theory and Algorithms*. New York: John Wiley and Sons.

Beckmann, M.J., C.B. McGure, and C.B. Winsten. 1956. *Studies in the Economics of Transportation*. New Haven, CT: Yale University Press.

Bell, M.G.H., Y. Iida. 1997. *Transportation Network Analysis*. New York: John Wiley and Sons.

Ben-Akiva, M. and S.R. Lerman. 1985. *Discrete Choice Analysis. Theory and Application to Travel Demand*. Cambridge, MA: MIT Press.

Berechman, J. and P. Gordon. 1986. “Linked Models of Land Use-Transport Interactions.” In *Advances in Urban Systems Modelling*. Edited by Hutchinson, B. and M. Batty. Amsterdam: North-Holland.

Bertsimas, D. and J.N. Tsitsiklis, 1997. *Introduction to Linear Optimization*. Belmont, MA: Athena Scientific.

Bertuglia, C.S. and G. Leonardi. 1980. “Heuristic algorithms for the normative location of retail activity systems.” *Regional Science Association*, 44, pp.149-159.

Bertuglia, C.S., G. Leonardi, S. Occelli, G.A. Rabino, R. Tedai, and A.G. Wilson. 1987. *Urban Systems: Contemporary Approaches to Modelling*. New York: Chapman and Hall.

Bhat, C.R. 1996. "A Generalized Multiple Durations Proportional Hazard Model with an Application to Activity Behavior During the Work-to-Home Commute", *Transportation Research*, 30B, pp. 465-480.

Bhat, C.R., 2001. "Modeling the Commute Activity-Travel Pattern of Workers: Formulation and Empirical Analysis", *Transportation Science*, 35 (1), pp. 61-79.

Border, K.C., 1985. *Fixed Point Theorems with Applications to Economics and Game Theory*. New York: Cambridge University Press.

Bovy, P.H. and G.R.M. Jansen. 1981. “Network Modeling Effects in Equilibrium Assignment: An Empirical Investigation.” *Proceedings of International Symposium on Transportation Equilibrium and Supply Models*, Transportation Research Center, University of Montreal, Montreal.

Boyce, D.E. 1986. “Integration of Supply and Demand Models in Transportation and Location: Problem Formulation and Research Questions”. *Environment and Planning*,18A, pp.485-89.

Boyce, D.E. and M.S. Daskin. 1997. Urban Transportation, Chapter 7 in *Design and Operation of Civil and Environmental Engineering Systems*, edited by

C. ReVelle and A. McGarity, New York: John Wiley and Sons, pp. 277-341.

Boyce, D.E. and T.J. Kim. 1987. “Role of Congestion on Transportation Networks in Urban Land Uses and Travel Choice of Households and Firms.” *Transportation,* 14, pp. 53-62.

Boyce, D.E., K.S. Chon, Y.J. Lee, K.T. Lin, and L.J. LeBlanc. 1983. “Implementation and Computational Issues for Combined Models of Location, Destination, Mode and Route Choice.” *Environmental Planning*, 15A, pp. 1219-1229.

Boyce, D.E, and L. Lundqvist. 1987. “Network Equilibrium Models of Urban Location And Travel Choices. Alternative Formulations for the Stockholm Region.” *Regional Science Association,* 61, pp. 93-104.

Boyce, D.E., M.R. Lupa, and Y-F Zhang. 1994. “Introducing Feedback into The Four-Step Travel Forecasting Procedure Vs. The Equilibrium Solution of A Combined Model.” *Transportation Research Record,* 1443, pp.65-74.

Boyce, D.E. and F. Southworth. 1979. “Quasi-Dynamic Urban Location Models with Endogenously Determined Travel Costs.” *Environment and Planning* 11A, pp. 575-584.

Bureau of Transportation Statistics (BTS), 1997. 1993 Commodity Flow Survey Data CD-ROM, Washington, D.C..

Bureau of Transportation Statistics (BTS), 2001. 1997 Commodity Flow Survey Data CD-ROM, Washington, D.C..

Caindec, E.K. and P. Prastacos. 1995. “A Description of POLIS: The Projective Optimization Land Use Information System.” Working Paper 95-1. Association of Bay Area Governments, Oakland, CA.

Caliper Corporation. 2001. *TransCAD Software Manual*. Newton, MA.

Cantarella, G.E., 1997. “A General Fixed-Point Approach to Multimode Multi-User Equilibrium Assignment with Elastic Demand”, *Transportation Science*, 31 (2), pp. 107-128.

Cantarella, G.E., and E. Cascetta. 1995. “Dynamic Processes and Equilibrium in Transportation Networks: Towards a Unifying Theory”, *Transportation Science*, 29, pp. 305-329.

Cascetta, E. 2001. *Transportation Systems Engineering: Theory and Methods.* Boston, MA: Kluwer Academic.

Chavey, M. 1982. "Nonconvexity of The Dynamic Traffic Assignment Problem." *Transportation Research,* 26B (2), pp. 127-133.

Chavey, M. and E. Subrahmanian. 1987. "An Alternative Model for Dynamic Traffic Assignment." Carnegie-Mellon University, Dept. of Urban and Traffic Affairs, Pittsburgh, PA.

Chu, Y.-L. 1999. "Network Equilibrium Model of Employment Location and Travel Choices." *Transportation Research Record 1667*. pp. 60-66.

Compaq Computer Corp., 2000. Visual Fortran Professional Edition 6.5.

Curran, T.R., T. Faoro, W. Fitz-Simons, B. Freas, B. Nelson, L. Beard, D. Schultz, D. Mobley and W.F.Jr. Hunt. 1992. *National Air Quality and Emissions Trends*. U.S. Environmental Protection Agency. Research Triangle Park, North Carolina.

Dafermos, S.C. 1980. "Traffic Equilibrium and Variational Inequalities", *Transportation Science,* 14 (1), pp. 43-54.

Dafermos, S.C. 1982. "Realization Algorithms for the General Asymmetric Traffic Equilibrium Problem." *Transportation Science,* 16 (2), pp. 231-240.

Dafermos, S.C., and F.T. Sparrow, 1969. "The Traffic Assignment Problem for a General Network." *Journal of Research of the National Bureau of Standards,* 73B, pp. 91-118.

Daganzo, C. and Y. Sheffi, 1977. "On Stochastic Models of Traffic Assignment." *Transportation Science,* 11 (3), pp. 253-274.

Davis, S.C. 1994. *Transportation Energy Data Book. Edition 14.* ORNL Report 6743. Oak Ridge National Laboratory, Oak Ridge, Tennessee.

de la Barra, T. 1994. *Integrated Land Use and Transport Modeling.* New York: Cambridge University Press.

Dempster, A., N. Laird, and D. Rubin. 1977. "Maximum Likelihood from Incomplete Data via the EM Algorithm." *Journal of the Royal Statistical Society*, 39B (1), pp.1–38.

Devarajan, S. 1981. "A Note on Network Equilibrium and Noncooperative Games." *Transportation Research,* 15B, pp. 421-426.

Domencich, M. and D. McFadden, 1975. *Urban Travel Demand: A Behavioral Analysis*. New York: American Elsevier.

Echenique, M.H., A. D. Flowerdew, J.D. Hunt, T.R. Mayo, I.J. Skidmore, and Simmonds, D.C. 1990. "The MEPLAN Models of Bilbao, Leeds and Dortmund." *Transportation Reviews,* 10, pp.309-322.

Echenique, M.H., D. Crowther, and W. Lindsay 1969. "A spatial model for urban stock and activity." *Regional Studies,* 3, pp. 281-312.

Evans, S. P. 1976. "Derivation and Analysis of Some Models for Combining Trip Distribution and Assignment." *Transportation Research,* 10, pp. 37-57.

Ferland, J.A., M. Florian, and C. Achim, "On Incremental Methods for Traffic Assignment." *Transportation Research,* 9, pp. 237-239.

Fernandez, J.E. and T.L. Friesz. 1983. "Equilibrium Predictions in Transportation Markets: the State of the Art." *Transportation Research*, 17B (2), pp. 155-172.

Fisk, D. 1979. "More Paradoxes in the Equilibrium Assignment problem.", *Transportation Research. Part B.* 13 (4) pp. 305-309.

Florian, M. 1977. "A Traffic Equilibrium Model of Travel by Car and Public Transit Modes." *Transportation Science* 11 (2), pp. 166-179.

Florian, M., and S. Nguyen. 1974. "A Method for Computing Network Equilibrium with Elastic Demands." *Transportation Science,* 8, pp. 321–332.

Florian, M. and S. Nguyen. 1977. "A Combined Trip Distribution Modal Split and Trip Assignment Model." *Transportation Research.* 12 (4), pp. 241-246.

Forrester, J.W. 1969. *Urban Dynamics*. MIT Press, Cambridge, Massachusetts.

Frank, C. 1978. A Study of Alternative Approaches to Combined Trip Distribution-Assignment Modeling, Ph.D. Thesis, Department of Regional Science, University of Pennsylvania, Philadelphia, Pennsylvania.

Frank, M. and P. Wolfe. 1956. "An Algorithm for Quadratic Programming." *Naval Research Logistics Quarterly,* 3 (1-2), pp. 95-110.

Friesz, T.L. 1985. “Transportation Network Equilibrium, Design and Aggregation: Key Developments and Research Opportunities.” *Transportation Research*, 19A (5-6), pp.413-427.

Fotheringham, A.S. and M.E. O’Kelly. 1989. *Spatial Interaction Models: Formulations and Applications*. London: Kluwer Academic.

Goldner, W. 1983. “Agency Policy Requirements and System Design.” In *Systems Analysis in Urban Policy-Making and Planning*. Edited by M. Batty and B. Hutchinson. New York: Plenum Press, pp. 271-290.

Golub, G.H. and C.F. Van Loan, 1983, *Matrix Computations*, Baltimore: The Johns Hopkins University Press.

Ham, H., T.J. Kim and D.E. Boyce, 2000. “Implementation of a Combined Trip Generation, Mode Choice, Flow Distribution and Route Choice Model for Interregional Commodity Flows” Proceedings of the 47th Annual Meetings of the North American Regional Science Association, November 9-11, Chicago, IL.

Hartley, H. 1958. “Maximum Likelihood Estimation from Incomplete Data.” *Biometrics*, 14, pp.174–194.

Hartwick, J.M. and P.G. Hartwick. 1974., "Activity Analysis Approach to Urban Model Building", *Papers of Regional Science Association*, pp. 9-31.

Herbert, J.C. and B.H. Stevens. 1960. “A Model for the Distribution of Residential Activity in Urban Areas.” *Journal of Regional Science*, 2, pp. 21-36.

Hillier, F.S. and G. Lieberman. 1995. *Introduction to Operations Research*, Sixth Edition. New York: McGraw-Hill.

Horowitz, J.L. 1984. “The Stability of Stochastic Equilibrium in a Two-Link Transportation Network” *Transportation Research,* 18B (1), pp. 13-28.

Hunt J.D. 1993. “A Description of The MEPLAN Framework for Land Use and Transport Interaction Modeling.” Paper at the 73rd Annual Transportation Research Board Meetings, January, Washington, D.C.

Hunt, J.D. and D.C. Simmonds. 1993. “Theory and Application of An Integrated Land-Use and Transport Modeling Framework.” *Environment and Planning* 20, pp. 221-244.

Isard, W., I.J. Azis, M.P. Drennan, R.E. Miller, S. Saltzman, and E. Thorbecks. 1960. *Methods of Regional Analysis*. Cambridge, MA: MIT Press.

Jin, L., K. Kockelman, and Y. Zhao. 2002. "Tracking Land Use, Transport, and Industrial Production using Random-Utility Based Multizonal Input-Output Models: Applications for Texas Trade." Working Paper submitted to the 82nd Annual Meeting of the Transportation Research Board, January 2002 in Washington, D.C.

Johnson, R.A. 2002, "MPO/COG Modeling Capabilities: Overview and Recommendations", Proceedings of National Association of Regional Councils Conference [online], available from World Wide Web: http://www.narc.org/environment/presentations/modeling.html, accessed on Nov. 11, 2002.

Kanafani, A. 1983. *Transportation Demand Analysis*. New York: McGraw-Hill.

Khamsi, M. A., and W. A. Kirk, 2001. *An Introduction to Metric Spaces and Fixed Point Theory*. New York: John Willy and Sons.

Kim, T.J. 1979. "Alternative Transportation Modes in a Land Use Model: A General Equilibrium Approach." *Journal of Urban Economics*, 6, pp. 197-215.

Kim, T.J. 1986. "Modeling the Density Variation of Urban Land Uses with Transportation Network Congestion." *Journal of Urban Economics*, 10, pp. 1035-1047.

Kim, T.J., 1989. *Integrated Urban System modeling: Theory and Practice.* Norwell, MA: Martinus Nijhoff.

Kim, T.J., D.E. Boyce, and G.J.D. Hewings. 1983. "Combined Input-Output and Commodity Flow Models for Interregional Development Planning: Insights from a Korean Application," *Journal of Geographical Analysis*, 15(4), pp.330-342.

Kim, T. J., H. Ham and D.E. Boyce. 2002. "Economic Impacts of Transportation Network Changes: Implementation of a Combined Transportation Network and Input-Output Model." *Papers in Regional Science*, 81, pp. 223-246.

LeBlanc, L.J. and K. Farhangian. 1981. "Efficient Algorithms for Solving Elastic Demand Traffic Assignment Problems and Mode Split-Assignment Problems", *Transportation Science*, 15, 306-317.

LeBlanc, L. J., E. K. Morlok, and W. Pierskalla. 1975. “An Efficient Approach To Solving The Road Network Equilibrium Traffic Assignment Problem.” *Transportation Research*, 9 (5), pp. 309-318.

Leontief, W.W. 1941. *The Structure of American Economy, 1919-1929*. Cambridge, MA: Harvard University 8.

Leontief, W.W. 1967. *Input-Output Economics*. Oxford University Press, New York.

Leontief, W.W., and A. Strout, 1963. “Multiregional Input-Output Analysis.” In *Structural Interdependence and Economic Development*, edited by T. Barna, pp.119-150. New York: Macmillan.

Lerman, S.R. and C.R. Kern. 1983. “Hedonic Theory, Bid Rents, And Willingness To Pay: Some Comments To Model.” *Journal of Urban Economics* 13, 358-363.

Los, M. 1979. “Combined Residential-Location and Transportation Models.” *Environment and Planning* 11A, pp.1241-1265.

Lowry, I.S. 1964. *A Model of Metropolis*. RM-4035-RC. The RAND Corporation, Santa Monica, California.

Macgill, S.M., and A.G. Wilson. 1979. “Equivalences and Similarities between Some Alternative Urban and Regional Models.” *Sistemi Urbani*, 1, pp. 9-40.

Mackett, R.L.1990. *MASTER Model*. (*Micro-Analytical Simulation of Transport, Employment and Residence*). Report SR 237. Transport and Road Research Laboratory. Crowthorne, England.

Mahmassani, H.S. and G.-L. Chang, 1987. “On Boundedly Rational User Equilibrium in Transportation Systems.” *Transportation Science*, 21 (2), pp. 89-99.

Mannering, F. and C. Winston. 1985. A Dynamic Empirical Analysis Of Household Vehicle Ownership And Utilization. *Rand Journal of Economics*, 162), pp. 215-236.

Martin, W.A., N.A. McGuckin, and Barton-Aschman Associates, Inc. 1998. Travel Estimation Techniques for Urban Planning. *NCHRP Report 365*, Transportation Research Board, National Research Council, Washington, D.C.

Martínez, F.J. 1992. “The Bid-Choice Land Use Model: an Integrated Economic Framework.” *Environment and Planning*, 24A, pp. 871-885.

Martínez, F.J. and P. Donoso. 1995. “MUSSA Model: The Theoretical Framework.” *Modeling Transport Systems. Proceedings 7th World Conference on Transportation Research (WCTR)*, 2, Edited by D. Hensher, J. King and T. Oum, New York: Pergamon, pp. 333-343.

Martínez. F.J. 1996.. “MUSSA: A Land Use Model for Santiago City.” *Transportation Research Record 1552*, pp. 126-134.

McFadden, D., 1974. "Conditional Logit Analysis of Qualitative Choice Behavior," *Frontiers in Econometrics*, Edited by P. Zarembka. New York: Academic Press, pp. 105-142.

McFadden, D.L. 1978. “Modeling The Choice of Residential Location.” In *Spatial Interaction Theory and Planning Models*. Edited by A. Karlqvist, L. Lundqvist, F. Snickars and J. Weibull. Amsterdam: North-Holland, pp. 75-96.

McLachlan, G. and T. Krishnan. 1997. *The EM Algorithm and Extensions*. New York: John Wiley and Sons.

Meyer, M. and E. Miller. 1984. *Urban Transportation Planning*, second edition, New York: McGraw-Hill.

Miller, H. 1997. “Towards Consistent Travel Demand Estimation in Transportation Planning: A Guide to the Theory and Practice of Equilibrium Travel Demand Modeling”, Report prepared for Bureau of Transportation Statistics, [online], available from World Wide Web: http://tmip.fhwa.dot.gov/clearinghouse/docs/miller/, [accessed on June 23, 2002].

Mills, E. S. 1972. “Markets and Efficient Resource Allocation in Urban Areas.” *Swedish Journal of Urban Economics*, 74, pp.100-113.

Min, Y. J., K. M. Kockelman, Y. Zhao, and L. Jin. 2001. “Models of Location, Trade, & Travel: An Application of Random-Utility-Based Interzonal Input-Output Models”. *Proceedings* of the 48th Annual North American Meeting of the Regional Science Association International.

Minnesota IMPLAN Group, Inc. 1997. IMPLAN Professional: Social Accounting & Impact Analysis Software.

Mitchell, R.B., and C. Rapkin. 1954. *Urban Traffic: a Function of Land Use*, New York: Columbia University Press.

Moore, J.E. 1986. "Linearized Optimally Configured Urban System Models: A Dynamic Mills' Heritage Model with Replaceable Capital." Ph.D. dissertation, Dept. of Civil Engineering, Stanford University, CA.

Muchland, J.D. 1969. "Road Traffic Distribution In Equilibrium." Conference Paper, *Mathematical Methods in the Economic Sciences*, Mathematiches forschungsinstitut, Oberwolfack, Germany.

Murchland, J.D. 1970. "Braess's Paradox of Traffic Flow." *Transportation Research* 5, pp. 391-394.

Nagurney, A. 1999. *Network Economics: A Variational Inequality Approach*, Revised second edition, Boston, MA: Kluwer Academic Publishers.

Nagurney, A. and D. Zhang. 1998. "Network Equilibria and Disequilibria." In *Equilibrium and Advanced Transportation Modeling*, edited by P.Marcotte and S. Nguyen. Boston, MA: Kluwer Academic Publishers, pp. 201-243.

NCTCOG. 1999. Dallas-Fort Worth Regional Travel Model (DFWRTM): Description Of The Multimodal Forecasting Process. Report. Transportation Department, North Central Texas Council of Governments, Texas.

Newell, G.F. 1996. "Non-Convex Traffic Assignment On A Rectangular Grid Network." *Transportation Science,* 30 (1), pp. 32-42.

Nguyen, S. 1974. "An Algorithm for The Traffic Assignment Problem." *Transportation Science,* 8 (3), pp. 203-216.

Nwosu, A.D. 1983. "Pre-emptive Hierarchical Programming Problem: A Decentralized Decision Model." Ph.D. dissertation, Rensselaer Polytechnic Institute, Troy, NY.

Oppenheim, N. 1995. *Urban Travel Demand Modeling: from Equilibrium Choices to General Equilibrium.* New York: John Wiley.

Ortege, J.M., and W.C. Rheinboldt, 1970. *Iterative Solution of Nonlinear Equations in Several Variables*, New York: Academic Press.

Oryani, K. and B. Harris. 1996. “Enhancement of DVRPC's Travel Simulation Models Task 12: Review of Land Use Models and Recommended Model for DVRPC”. Report Prepared for Delaware Valley Regional Planning Commission, URS Consultants, Inc., Philadelphia.

Patriksson, M. 1994. *The Traffic Assignment Problem, Models and Methods*, Utrecht:VSP.

Prastacos, P. 1986. “An Integrated Land Use-Transportation Model for the San Francisco Region: 1. Design and Mathematical Structure.” *Environment and Planning,* 18A, pp. 307-322.

Putman, S.H. 1974. “Preliminary Results from an Integrated Transportation and Land Use Models Package.” *Transportation*, 3, pp. 193-224.

Putman, S. H. 1983. *Integrated Urban Models.* London: Poin Limited.

Putman, S. H. 1991. *Integrated Urban Models II.* London: Poin Limited.

Rosenthal, R.W. 1973. “The Network Equilibrium Problem in Integers.” *Networks*, 3, pp. 51-63.

Safwat, K.N., and T.L. Magnanti. 1988. “ A Combined Trip Generation, Trip Distribution, Modal Split, and Trip Assignment Model.” *Transportation Science*, 18, pp. 14-30.

Safwat, K.N., and C.M. Walton. 1988. “Computational Experience with an Application of a Simultaneous Transportation Equilibrium Model to Urban Travel in Austin, Texas.” *Transportation Research*, 22B, pp. 457-467.

Samuelson, P.A. 1952. “Spatial Price Equilibrium and Linear Programming,” *American Economic Review*, 42, pp.283-303.

Sheffi, Y. 1985. *Urban Transportation Network*, Englewood Cliffs, NJ: Prentice-Hall.

Sheffi, Y. and C. Daganzo. 1978. “Another ‘Paradox’ of Traffic Flow.” *Transportation Research,* 12 (1), pp. 43-46.

Sheffi, Y. and W.B. Powell. 1981 “A Comparison of Stochastic and Deterministic Traffic Assignment over Congested Network.” *Transportation Research,* 15 (1), pp. 53-64.

Sheffi, Y. and W.B. Powell. 1982 "An Algorithm for the Equilibrium Assignment Problem with Random Link Times." ***Network,*** 12 (2), pp. 191-207.

Shen, P. 1995. Optimization Network Equilibrium Models of Combined Travel and Residential Location Choices, Ph.D. Dissertation, University of Pennsylvania.

Smith, L.L. 2000. "Status Report on TRANSIMS Development", Presented at the 79th Annual Meeting of the Transportation Research Board, Washington, DC, January.

Smith, M.J. 1979. "The Existence, Uniqueness of Stability of Traffic Equilibria." ***Transportation Research***, 13B, pp. 293-304.

Smith, M.J. 1981. "The Existence of an Equilibrium Solution to the Traffic Assignment Problem When There Are Junction Interactions." ***Transportation Research,*** 15B (6), pp. 443-451.

Sosslau, A.B., A.B. Hassam, M.M. Carter, and G.V. Wickstrom. 1978. Quick Response Urban Travel Estimation Techniques and Transferable Parameters: User Guide. NCHRP Report 187, Transportation Research Board, National Research Council, Washington, D.C.

Southworth, F. 1995. A Technical Review of Urban Land Use-Transportation models as Tools for Evaluating Vehicle Travel Reduction Strategies. Report of Oak Ridge National Laboratory, Oak Ridge, TN.

TMIP, 2001. "Travel Model Improvement Program E-Mail List", [online], available at World Wide Web <http://groups.yahoo.com/group/tmip>, accessed in March 2002.

Waddell, P., ***UrbanSim Reference Guid***e, 2000, University of Washington, Urban Simulation Project.

Wardrop J.G. 1952. "Some Theoretical Aspects of Road Research." ***Proceedings of the Institute of Civil Engineers.*** (2), pp. 325-378.

Watterson, W.T. 1993. "Linked Simulation of Land Use and Transportation Systems: Developments and Experience in the Puget Sound Region." ***Transportation Research***, 27A, pp.193-206.

Wehr, J. and J. Xin, 1996. "White Noise Perturbation of the Viscous Shock Fronts of the Burgers Equation", *Common Mathematics and Physics*, 181, pp. 183-203.

Wilkie, D.F., and R.G. Stafanek. 1971. "Precise Determination of Equilibrium in Travel Forecasting Problems Using Numerical Optimization Techniques." *HRB Highway Research Record 369*, pp. 239-252.

Williams, H.C. and M.L. Senior. 1978. "Accessibility, Spatial Interaction and the Spatial Benefit Analysis of Land Use Transportation Plans." *Spatial Interaction Theory and Planning Models,* Amsterdam: North Holland.

Wilson, A.G. 1967. "A Statistical Theory of Spatial Trip Distribution Models." *Transportation Research,* 1, pp.253-269.

Wilson, A.G. 1969. "Entropy Maximization Models in the Theory of Trip Distribution, Mode Split and Route Split." *Journal of Transportation Economics and Policy,* 3, pp. 108-126.

Wilson, A.G. 1970a. "Interregional Commodity Flows: Entropy Maximizing Procedures", *Geographical Analysis*, 2, pp. 255-282.

Wilson, A.G. 1970b. *Entropy in Urban and Regional Modelling*. London: Pion Limited.

Wilson, A.G., J.D. Coelho, S.M. Macgill, and H.C.W.L. Williams. 1981. *Optimization in Locational and Transport Analysis.* Chichester, England: John Wiley and Sons.

Wilson, A.G. 1981. *Catastrophe Theory and Bifurcation: Applications to Urban and Regional Systems.* , Beckenham, Kent: Croom-Helm.

Wilson, A.G. 1987. "Transport, Location and Spatial Systems: Planning with Spatial Interaction Models." Chapter 7 in *Urban Systems: Contemporary Approaches to Modelling*. Edited by C.S. Bertuglia, G. Leonardi, S. Occelli, G.A. Rabino, R. Tedai, and A.G. Wilson. New York: Routledge, Chapman and Hall, pp. 337-426.

Wilson, A.G., J.D. Coelho, S.M. Macgill, and H.C.W.L. Williams. 1981. *Optimization in Locational and Transport Analysis.* Chichester, England: John Wiley and Sons.

Wrigley, N. 1985. *Categorical Data Analysis for Geographers and Environmental Scientists*, London: Longman.

Zhao, Y. and K.M. Kockelman. 2002. "The Propagation of Uncertainty Through Travel Demand Models: An Exploratory Analysis." *Annals of Regional Science*, 36 (3), pp. 145-163.

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