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ROP Modeling Chronology, Sensitivity Analyses, and Field Data Comparisons

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ROP Modeling Chronology, Sensitivity Analyses, and Field Data Comparisons

by

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Abstract

ROP Modeling Chronology, Sensitivity Analyses, and Field Data Comparisons

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Rate of penetration (ROP), the rate at which a drill bit breaks the rock underneath to deepen the borehole, modeling and measuring is widely used in industry to monitor drilling performance, optimize drilling parameters, detect abnormal pressures, and to improve drilling efficiency.

The objective of this project is to run various simulations and models with field data in order to investigate the relationship amongst the parameters that influence rate of penetration and the limitations and advantages of each model. This paper analyzes six models: Bingham's, Bourgoyne & Young's, Winters, Warren, and Onyia's, Hareland's drag bit, Hareland's roller bit, and Motahhari's. An analysis of the models with respect to changes in lithology and with respect to changes in formation is included as an initial

check for the models. As expected, the analysis done by formation yielded better results, with an improvement of roughly 5% for each model.

When the data sets for wells drilled with drag bits were run for the drag bit models and two other extensive models for comparison, an interesting result occurred. The least amount of errors was always achieved by a non-drag bit model, but Motahhari's model, a drag bit model, always gave the closest physical interpretation. Using a non-bit specific model, however, may lead to a better initial planning, as the non-drag bit models averaged outputted values closer in magnitude to the real data.

This paper provides good practices on how to choose which model to use. As a general assessment, Motahhari's model should be used for drag bits, and Winters, Warren, and Onyia's for roller bits. Using other models is dependent on availability of data, well complexity, and desire to expand on design or confirm calculations.

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1. OVERVIEW

Rate of penetration (ROP), the rate at which a drill bit breaks the rock underneath to deepen the borehole, modeling and measuring is widely used in industry to monitor drilling performance, optimize drilling parameters, detect abnormal pressures, and to improve drilling efficiency.

Common optimization of ROP is based on changing weight on bit, bit diameter and rotary speed. However, a more elaborate analysis can be performed by taking into account hydraulics, drillability, cuttings loading, bit wear, equivalent circulating density, anisotropy, wellbore trajectory, formation type and morphology (Gray 2014).

The objective of this project is to run various simulations and models with field data in order to investigate the relationship amongst the several parameters that influence ROP and the advantages of the models, and to develop more comprehensive quantifications for ROP. These developments would be directly applicable to current drilling practices, with particular significance to real-time, automated drilling operations.

This paper will analyze these models and learn their applicability to various field data as well as determine their shortcomings and advantages, when or where one should be used and determine if any improvements can be made to the method with which the models are implemented in order to accurately depict the drilling design and execution. For all models, there are plots of the interfaces between lithologies: sandstone, limestone, and shale, tables of the calculated model coefficients, and an error analysis.

2. ROP MODELS CHRONOLOGY

ROP models chronology is quite extensive as rate of penetration is one of the key elements of drilling. As to avoid excessive sources, this paper compiled several models that are deemed to fairly represent the ROP models population. An early industry model, a more comprehensive ROP model and models for PDC, roller cone, and drag bits were chosen in order to correctly analyze any field data.

2.1 Bingham, 1965

One of the earliest papers on rate of penetration modeling, Bingham's 1965 paper suggested a model that predicted ROP by using it as simply a function of rotary speed, weight on bit, and bit diameter. The literature on ROP has grown extensively since Bingham's (1965) paper and so have methods of quantification and the overall understanding of what affects ROP. Despite all this, his model is still a very good rough starting point for ROP quantification. His model is:

$$ROP = K \times RPM^{\,e} \times \left(\frac{W}{D}\right)^{d} \qquad (2.1)$$

where D is bit diameter; RPM is rotary speed, W is weight on bit, d is exponent in general drilling equation, e is exponent related to rotary speed, K is constant related to formation.

2.2 Bourgoyne & Young, 1974

Bourgoyne & Young's model revolutionized ROP optimization by proposing a comprehensive model that accounts for the influences to ROP. In the early 1970s, due to developments in onsite well monitoring systems, more accurate methods for quantifying ROP and pore pressure were possible as previously the models had to rely on laboratory data. Thus, developing and applying such a comprehensive model to industry was possible.

Also, prior to Bourgoyne & Young's (1974) paper, a model was being used to determine optimal bit weight and rotary, another for jet bit hydraulics, and another for detection of abnormal pressure. Bourgoyne and Young's model simplifies the need for multiple models. The equations are as follows:

$$ROP = f_1 \times f_2 \times f_3 \times f_4 \times f_5 \times f_6 \times f_7 \times f_8 \qquad (2.2)$$

$$f_1 = e^{2.303 \times a_1} \qquad (2.3)$$

$$f_2 = e^{2.303 \times a_2 \times (10000 - D)} \qquad (2.4)$$

$$f_3 = e^{2.303 \times a_3 \times D^{0.69} \times (g_p - 9)} \qquad (2.5)$$

$$f_4 = e^{2.303 \times a_4 \times D \times (g_p - P_c)} \quad (2.6)$$

$$f_5 = \left[\frac{\left(\frac{W}{d_b}\right) - \left(\frac{w}{d_b}\right)_t}{4 - \left(\frac{w}{d_b}\right)_t} \right]^{a_5} \quad (2.7)$$

$$f_6 = \left(\frac{N}{60}\right)^{a_6} \quad (2.8)$$

$$f_7 = e^{-a_7 \times h} \quad (2.9)$$

$$f_8 = \left(\frac{F_j}{1000}\right)^{a_8} \quad (2.10)$$

where f_1 is the effect of rock drillability, $f_2$is the depth effect, $f_3$is pore pressure effect on ROP, f_4 is the differential pressure effect, f_5 is the effect of changing the weight on ROP, f_6 is the effect of rotary speed, f_7 is the effect of bit wear on ROP, f_8 is the effect of bit hydraulics, w is weight on bit, a_1 models the effect of formations strength, $a_2$and a_3 model the effect of compaction, a_4 models the effect of pressure differential across the hole bottom on ROP, a_5 models the effect of bit weight and bit diameter, a_6 models the effect of rotary speed, a_7 models the effect of tooth wear, a_8 models the effect of bit hydraulics, D is depth in feet, g_p is the pore pressure gradient of the formation in lb/gal, P_c is the equivalent mud density in lb/gal, N is rotary speed in revolutions per minute, W is weight on bit in lbf, $d_b$is the bit diameter in inches,

$\left(\frac{w}{d_b}\right)_t$ is the threshold it weight at which bit begins to drill in 1000 lbf/in, h is the fractional bit tooth wear, F_j is the jet impact force in lbf.

Jet impact force is defined by the following equation:

$$F_j = \frac{Q*V_n*P_c}{1930} \tag{2.11}$$

where Q is flowrate in gpm, V_n is nozzle velocity in ft/sec.

And V_n can be calculated from the equation:

$$V_n = \frac{0.321*Q}{A_n} \tag{2.12}$$

where A_n is the total nozzle area in in^2.

As the model's constants suggest, Bourgoyne & Young's paper expands on the Bingham model by including compaction, pressure differential, bit wear, and hydraulics to the effects on ROP.

2.3 Winters, Warren, and Onyia, 1987

In 1987, Winters, Warren, and Onyia published a paper in which they presented a model relating roller bit penetration rates to the bit design, the operating conditions, and the rock mechanics. They identified rock ductility as a major influence on bit performance, and the cone offset as the most important design feature for drilling ductile rock. The ROP effects encompassed in the model are bit indentation, offset, teeth, and hydraulics (Winters, Warren, & Onyia, 1987).

This is the first model to be presented in this paper that specifically addresses a type of bit; in this particular case Winters, Warren, and Onyia's model focuses on roller cone bits. The model equation is as follows:

$$\frac{1}{ROP} = \frac{\sigma * D^2}{(N * W)} * \left(\frac{a * \sigma * D * \epsilon}{W} + \frac{\Phi}{W}\right) + \frac{b}{N * D} + \frac{c * \rho * \mu * \epsilon}{I_m} \quad (2.13)$$

where σ is rock compressive strength, D is bit diameter, ϵ is rock ductility, N is rotary speed, W is weight on bit, Φ is the cone offset coefficient, a,b,c are model coefficients, ρ is the equivalent mud density which is defined as the apparent mud density which results from adding annular friction to the actual fluid density in the well, μ is mud viscosity, I_m is the modified jet impact force.

The modified jet impact force is defined by the following equation:

$$I_m = \lfloor 1 - A_v^{-0.122} \rfloor * F_j \quad (2.14)$$

where A_v is the ratio of jet velocity to return velocity, F_j is the jet impact force.

And A_v can be calculated, assuming three jets, from the equation:

$$A_v = \frac{v_n}{v_f} = \frac{0.15D^2}{3d_n^2} \tag{2.15}$$

where d_n is nozzle diameter, v_n is nozzle velocity, v_f is return fluid velocity.

Although Winters, Warren, and Onyia's paper appears more simplified than its predecessor the Bourgoyne & Young model, it is actually more advanced because it provides real explanations on how to apply the model to field data. In their paper, they discuss how by generating a continuous rock strength log from interpreting field data, and comparing the rock strength log to the triaxial compressive strength of the rock at a confining pressure equal to the differential bottomhole pressure, one can predict and interpret roller bit performance in offset wells (Winters, Warren, & Onyia, 1987).

Furthermore, the authors explain how by testing a roller cone bit in stepwise increments in weight on bit in a laboratory and measuring ROP, the model coefficients can be calculated.

2.4 G. Hareland's Drag Bit, 1994

Hareland's (1994) model proposed a new way to predict ROP for drag bits. The model expands on previous ones by introducing equivalent bit radius, dynamic cutter action, lithology coefficient, and cutter wear. The model apart from helping with optimization of drilling parameters, also aids in solids control.

Due to the model not accounting for certain theoretical properties that affect ROP, such as bit cleaning, imperfections in bit and cutter geometry, and microscopic variations in rock strength, the paper includes a correlation factor. Here is Hareland's ROP equation for drag bits and the correlation factor:

$$ROP = \frac{14.14 \times N \times RPM}{D} \times \left[\left(\frac{d_s}{2}\right)^2 \times cos^{-1}\left(1 - \frac{4 \times W_{mech}}{(N \times d_s{}^2 \times \pi \times \sigma_c)}\right) - \left(\frac{2 \times W_{mech}}{(N \times \pi \times \sigma_c)} - \frac{4 \times W_{mech}{}^2}{(N \times d_s \times \pi \times \sigma_c)^2}\right)^{0.5} \times \left(\frac{d_s}{2} - \frac{2 \times W_{mech}}{(N \times d_s \times \pi \times \sigma_c)}\right)\right] \tag{2.16}$$

$$COR = \frac{a}{(RPM^b \times W^c)} \tag{2.17}$$

where D is bit diameter in inches, N is number of cutters, RPM is rotary speed in revolutions per minute, d_s is diamond cutter diameter in inches, W_{mech} is weight on bit per diamond cutter in lbs, σ_c is uniaxial compressive strength in pounds per square inch, W is weight on bit, a, b, c are cutter geometry correction factors.

2.5 G. Hareland's Roller Bit, 2010

G. Hareland's (2010) model proposed a different approach to predict ROP for roller cone bits. The paper analyzed the existing drilling models, including Bourgoyne and Young's, and expanded on them by including bit-rock interaction. The added complexity derives itself by relating the roller cone bit and rock interaction to rock failure by a wedge. The model is as follows:

$$ROP = K \times \left(\frac{80 \times n \times m \times RPM^{a}}{(D^{2} \times tan^{2}\psi)}\right) \times \left(\frac{W}{(100 \times n \times CCS)}\right)^{b} \times W_{f} \quad (2.18)$$

where K is the comprehensive coefficient, m is number of insert penetrations per revolution, n is number of inserts in contact with rock at the bottom, RPM is rotary speed, D is bit diameter, ψ is chip formation angle, W is weight on bit, CCS is confined compressive strength, W_f is bit wear, a and b are model coefficients.

2.6 Motahhari's PDC Bit, 2010

Motahhari's (2010) model proposed a new method to accurately predict ROP for polycrystalline diamond compact (PDC) bits and positive displacement motors (PDMs). This model is incredibly useful for directional and horizontal drilling operations with PDMs, as previous models do not as accurately enhance preplanning, reduction of drilling time with ROP optimization According to Motahhari (2010), “PDM performance/selection in the drilling planning phase will help perform a safe and cost-effective operation by preventing motor stalls and maintaining highest average ROP for the section”. The model is as follows:

$$ROP = G \times \left(\frac{W^{\alpha} \times RPM^{y}}{(D \times CCS)}\right) \times W_f \tag{2.19}$$

where G is a coefficient determined by bit geometry, cutter size and design (namely back rake and side rake angles) and cutter-rock coefficient of friction, RPM is rotary speed, D is bit diameter, W is weight on bit, CCS is confined compressive strength, W_f is bit wear, α and y are model coefficients.

3. KHANGIRAN FIELD ANALYSIS USING DRAG BIT MODELS, ALSO INCLUDING TWO OTHER EXTENSIVE MODELS

In this chapter, a case study on the Khangiran field was performed and this section will discuss its results and findings. By comparing the penetration rates given in the Khangiran field data with predicted values calculated from Hareland’s drag bit, Motahhari’s, Winters, Warren, and Onyia’s, and Bourgoyne & Young’s models, each model’s sensitivity and accuracy are calculated and the model differences established. The purpose of the chapter is to study the reasoning behind discrepancies amongst the various models. For reference on the equations for each model, see chapter two.

3.1 Background Information

In this section, background information for the Khangiran field is included as well as information from the technical paper I have extracted the field data from, Bahari et al. (2007).

The Khangiran field is a gas field in the northeast of Iran. It contains three separate gas reservoirs, and the field has been developed since 1968. The data contained in this case study come from the Bahari et al. (2007) paper. In the paper, the authors provide Khangiran formation data points for eight different wells drilled in the Khangiran field and apply Bourgoyne & Young’s model to the data set in order to review the model’s accuracy.

Table 1: Khangiran Formation Data Rows, Obtained from Wells Daily Drilling Progress Reports from Bahari et al. (2007)

Well No.	R, ft/hr	D, ft	W, 1000 lbf	d_b, in.	N, rpm	ρ_c, lbm/gal	h, %	g_p, lbm/gal	F_j, lbf
Well 50	50.6	354	17.5	26	130	8.82	0.25	7.48	960
Well 50	41.5	1411	15	17.5	130	9.96	0.25	8.62	1776
Well 47	24.3	359	15	26	130	8.95	0.25	7.62	1611
Well 47	14.9	1519	10	17.5	110	10.2	0.38	8.82	2123
Well 46	7.3	1772	7.5	17.5	110	10.3	0.25	8.95	1185
Well 42	9.5	1969	10	17.5	110	10.8	0.5	9.49	1324
Well 39	5.7	1900	9	17.5	100	10.5	0.5	9.15	1186
Well 29	25.9	1575	15	17.5	90	10.4	0.38	9.09	2196

Table 1 above shows the field data for the eight wells analyzed in Bahari et al. (2007) and provides information about the penetration rates, R, the depth, D, the weight on bit, W, the bit diameter, d_b, the rotations per minute, N, the equivalent mud density, ρ_c, the bit wear, h, the pore pressure gradient, g_p, and the jet impact force, F_j. From the table, one can notice that the wells are shallow and the penetration rates are not extremely high. Also, due to the authors referring to well 50 and well 47 twice, in the following analyses, the deeper section of well 50 will be referred to as well 49, and the shallower section of well 47 will be referred to as well 48.

The authors calculate coefficients for the Bourgoyne & Young's model by applying, to the data, four different methods: a, b, c, d. Method a, is a multiple regression method, one that had been suggested by Bourgoyne & Young to those that intended to use their model. Method b is a linear square data fitting with non-negativity constraints, where, starting with a set of possible solutions, the method converges to the main solutions, which are not negative. Method c is a non-linear least square data fitting with Gauss-Newton method. The Gauss-Newton algorithm is applied to the data in order to compute the eight coefficients. Method d is a non-linear least square data fitting with trust-region method. It is an optimization algorithm, which minimizes the sum of square errors. In each iteration, the approximate solution of a large linear system is estimated using the method of preconditioned conjugate gradients. This method makes it possible to determine lower and upper bounds for results and limit them to be in the reasonable ranges (Bahari et al. 2007). The authors ran the Bourgoyne & Young model for the various methods and determined values for each model's parameters as seen in table 2.

Table 2: Computed Coefficients Quantities with Four Mathematical Methods from Bahari et al. (2007)

Formation	Mathematical Methods	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Khangiran	method a	2921	-0.3	-0.1	0.22	-2.4	2.07	-19	-3.6
	method b	3.33	0	0	0	2.23	0.85	1.59	0
	method c	249	0	0	0.02	2	0.25	-1.4	-0.8
	method d	1.7	0.00002	0.0000008	0.000011	0.800	1	1	0.15

3.2 Khangiran Field Analysis

In this section, Hareland's drag bit, Motahhari's, Winters, Warren, and Onyia's, and Bourgoyne & Young's models analyses are presented and functional relationships of the parameters common to all models are plotted vs ROP. The functional lines in each plot are created from the equations below.

For Bourgoyne & Young's model:

$$ROP_{Bit} = constant * \left[\frac{\left(\frac{constant}{d_b}\right) - \left(\frac{constant}{d_b}\right)_t}{4 - \left(\frac{constant}{d_b}\right)_t} \right]^{a_5} \quad (3.1)$$

$$ROP_{WOB} = constant * \left[\frac{\left(\frac{W}{constant}\right) - \left(\frac{W}{constant}\right)_t}{4 - \left(\frac{W}{constant}\right)_t} \right]^{a_5} \quad (3.2)$$

$$ROP_{RPM} = constant * \left(\frac{N}{60}\right)^{a_6} \quad (3.3)$$

For Winters, Warren, and Onyia's model:

$$\frac{1}{ROP_{Bit}} = constant * D^2 * (constant * D + constant) + \frac{constant}{D} + constant \quad (3.4)$$

$$\frac{1}{ROP_{WOB}} = \frac{\text{constant}}{W} * \left(\frac{constant}{W} + \frac{\text{constant}}{\text{W}}\right) + constant \quad (3.5)$$

$$\frac{1}{ROP_{RPM}} = \frac{\text{constant}}{N} + \frac{constant}{N} \quad (3.6)$$

For Hareland's drag bit model:

$$ROP_{Bit} = \frac{constant}{D} \quad (3.7)$$

$$ROP_{WOB} = constant \times \left[constant \times cos^{-1}\left(1 - \frac{4 \times W_{mech}}{(constant)}\right) - \left(\frac{2 \times W_{mech}}{(constant)} - \frac{4 \times {W_{mech}}^2}{constant}\right)^{0.5} \times \left(constant - \frac{2 \times W_{mech}}{constant}\right) \right] \quad (3.8)$$

$$COR = \frac{constant}{W^c} \quad (3.9)$$

$$ROP_{RPM} = constant * N \times \left[constant \times cos^{-1}\left(1 - \frac{constant}{(N)}\right) - \left(\frac{constant}{(N)} - \frac{constant}{(N)^2}\right)^{0.5} \times \left(constant - \frac{constant}{(N)}\right) \right] \quad (3.10)$$

$$COR = \frac{constant}{(RPM^b)} \quad (3.11)$$

Where diamond cutter diameter was assumed to be independent of bit diameter.

For Motahhari's PDC bit model:

$$ROP_{Bit} = \left(\frac{constant}{(D)}\right) \quad (3.12)$$

$$ROP_{WOB} = W^{\alpha} * constant \quad (3.13)$$

$$ROP_{RPM} = RPM^{y} * constant \tag{3.14}$$

For Bourgoyne & Young's model, the various coefficients in the model equations were constrained to the limits recommended by Bourgoyne et al. (1973) and also provided in Bahari et al. (2007). Table 3 below shows these upper and lower boundaries for the eight coefficients in the Bourgoyne & Young's model. These bounds are the practical limits for the coefficients in order to achieve physically significant results when applying Bourgoyne & Young's model. It is important to note that none of the methods applied by Bahari et al. (2007) restrict the parameters within these recommended limits. What this entails is that the Bourgoyne & Young analysis presented in this paper will be different from that in Bahari et al. (2007).

Table 3: Recommended Limits to Achieve Meaningful Results in Bourgoyne And Young's Model from Bahari et al. (2007)

Coefficients	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Lower bond	0.5	0.000001	0.000001	0.000001	0.5	0.4	0.3	0.3
Upper bond	1.9	0.0005	0.0009	0.0001	2	1	1.5	0.6

3.2.1 Khangiran Field Analysis With All Wells

Figures 1 through 3 present the parameters common to all models and compare their functional relationships to ROP. In all three models, the models converge at a point where the calculated data was forced to match field data. The point is the well 50 values shown in Table 1: for bit diameter, it is at 26 inches, for bit rotation, it is 130 RPM, and for weight on bit, it is 17500 lbs.

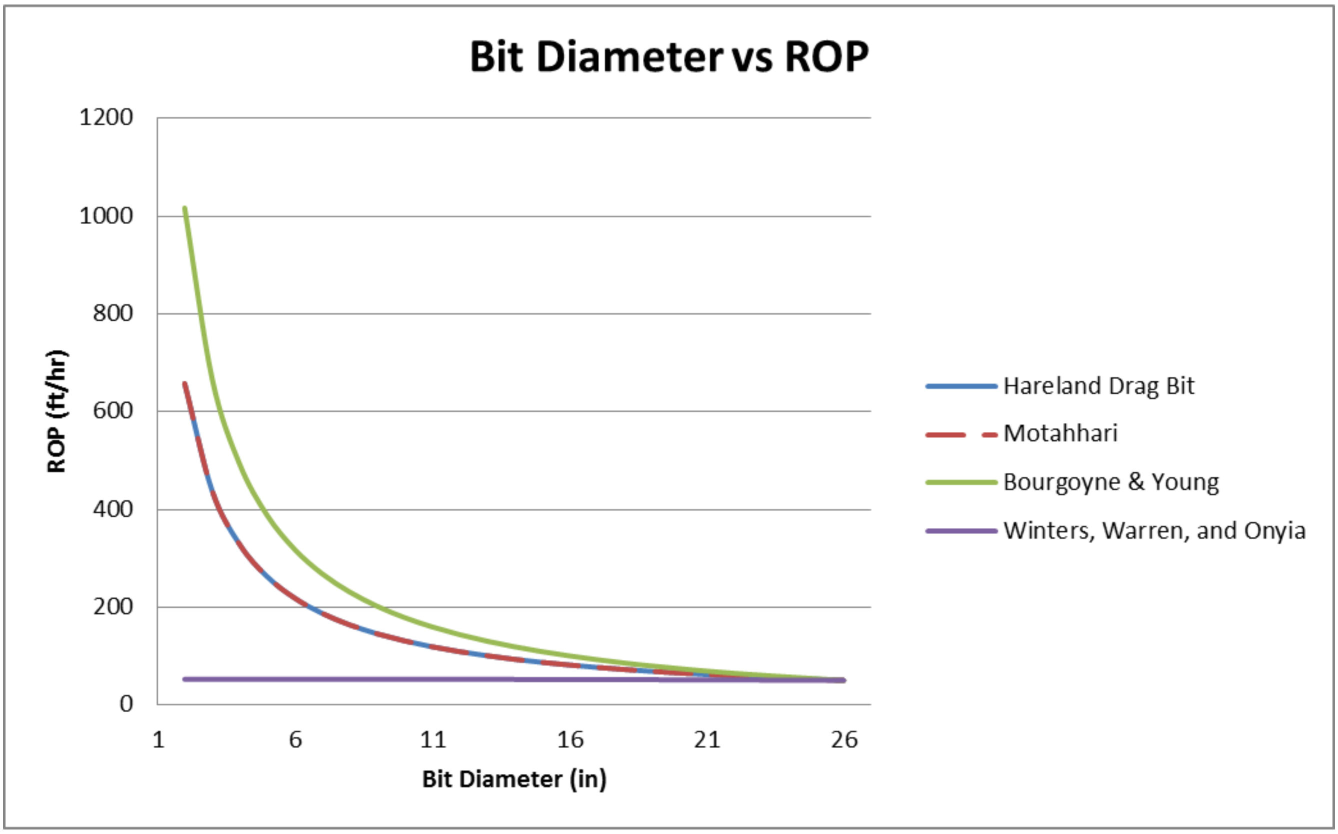

Figure 1: Bit Diameter (in) vs ROP for drag bit models, Winters, Warren, and Onyia and Bourgoyne & Young

In figure 1, Hareland's drag bit model and Motahhari's PDC bit model overlap one another, so Motahhari's line style is a long dash instead of a solid line. This is an important notion, as it signifies that both drag bit models agree on how bit diameter affects ROP when applied to a data set. Winters, Warren, and Onyia's model on the other hand shows a nearly unchanging ROP for a large range of bit diameters. As such, for this data set, it can be said that in Winters, Warren, and Onyia's model, ROP is independent of bit diameter. Before values are applied to equation 3.4, one would not believe otherwise.

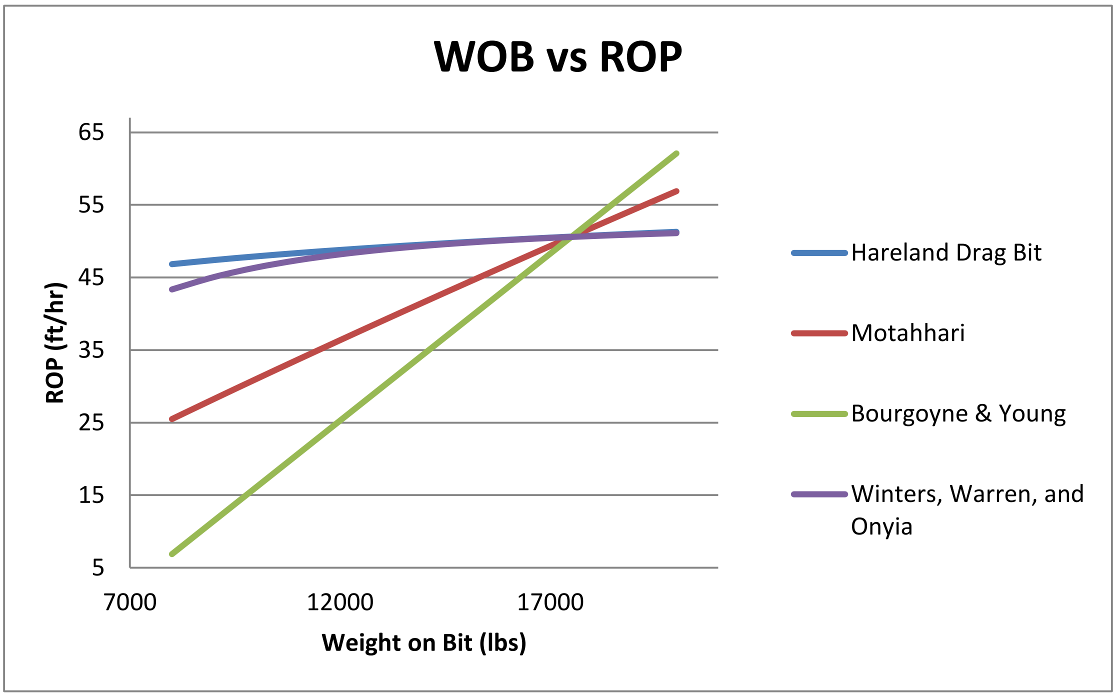

Figure 2: Weight on Bit (lbs) vs ROP for drag bit models, Winters, Warren, and Onyia and Bourgoyne & Young

In figure 2, Motahhari's and Bourgoyne & Young's models show linear relationships between weight on bit and ROP. However, one does not see in the field a one to one relationship between the two. In fact, most likely, one would see a relationship of diminishing returns, as shown by Hareland's and Winters, Warren, and Onyia's models in the plot. For a single, forced data point, these discrepancies would not cause disparities between the models, however, as you apply the model to a largely varying set, for example a large vertical section with the same formation, as the weight on bit values deviate more and more from the median, the larger the associated errors for the Bourgoyne & Young and Motahhari's model.

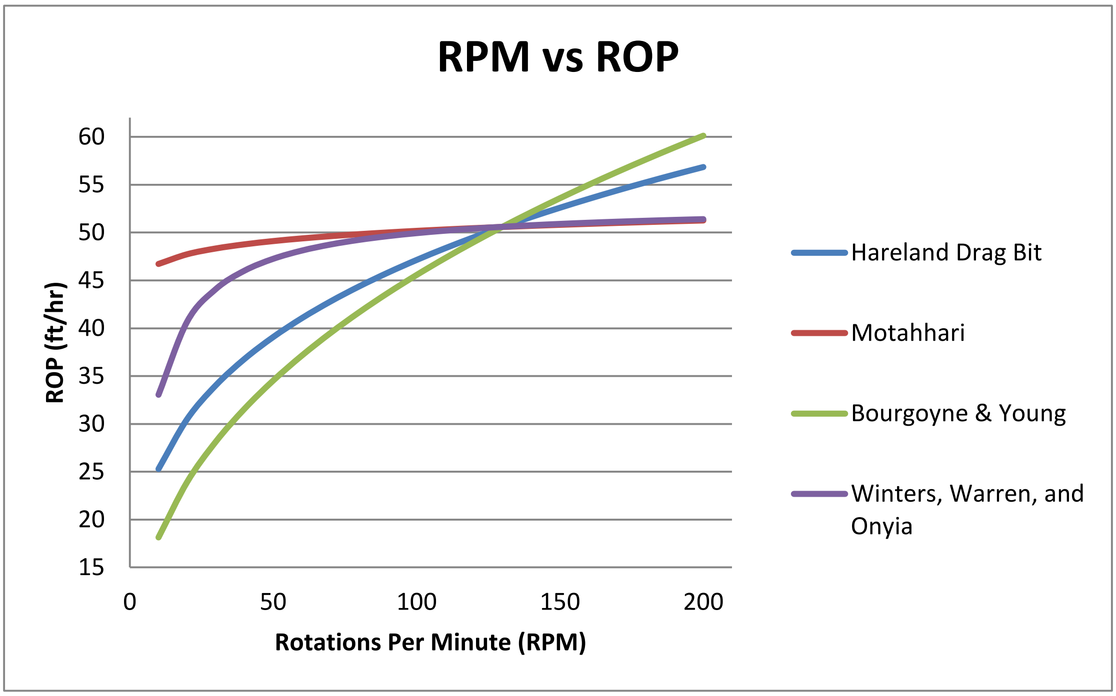

Figure 3: Bit Rotation (RPM) vs ROP for drag bit models, Winters, Warren, and Onyia and Bourgoyne & Young

In figure 3, all models have very close relationships to one another for how bit rotation affects rate of penetration. Also, RPM is usually maintained at a certain value through a section, as it is a drilling parameter one can easily control. Compiling those statements with the fact that even for field data in extremely long formations RPM is nearly unchanging, one can conclude that RPM does not cause the most significant changes in calculated ROP values for the models presented. Unless a slow pump rate operation commences or RPM is greatly increased, the relationship between RPM and ROP through all four models should be almost exactly the same.

Table 4: Average Error Percentages in the Models for the Khangiran Field Data

Well Number	Hareland	Motahhari	Bourgoyne & Young	Winters, Warren and Onyia
Well 50	91.61%	0.00%	54.62%	84.43%
Well 49	84.69%	84.91%	20.88%	68.80%
Well 48	82.40%	82.66%	0.00%	47.05%
Well 47	51.53%	51.54%	2.49%	0.00%
Well 46	0.00%	41.91%	11.75%	12.86%
Well 42	23.97%	0.00%	2.91%	6.82%
Well 39	35.49%	49.62%	17.17%	42.64%
Well 29	68.73%	71.55%	5.74%	41.08%
Average:	54.80%	47.77%	14.45%	37.96%
Median:	60.13%	50.58%	8.74%	41.86%

Table 4 displays the associated errors for each model for the various wells in the Khangiran field and shows the average and median errors for the models in the Khangiran field. As seen from the table, the model that more closely calculates ROP is Bourgoyne & Young's model. A possible reason for that is none of the parameters in the model had to be assumed; Bahari et al. (2007) provided all model inputs. This gives the model an edge when compared to the bit specific models, as no bit geometry was given by the paper and had to be assumed. Another possibility as to why Bourgoyne & Young's model has smaller associated errors is the fact that the data is mostly being affected by weight on bit. By looking at table 1, one sees that for wells drilled with average weight on bit of 15000 lbs or greater, the associated rate of penetration is much larger than that of those drilled with average weight on bit of 10000 lbs or smaller. And the difference between the values is so large that the only model that comes close to having a similar relationship between weight on bit and ROP is Bourgoyne and Young's model, as such, all other models will have much larger associated errors, even if the models themselves were better equipped for real time operations.

The next best model is Winters, Warren,and Onyia's. The reason for that is probably because for this data set, the model suggests that bit diameter has barely any effect on ROP. For that reason, the data set, which has largely varying penetration rates for the two wells of different bit size from the rest, will cause models suggesting lower bit diameters to have exponentially larger drilling rates to have larger errors. This highlights a difficulty in a model's predictability of the bit diameter effect on ROP, as bit size is mostly the same across many formations and for different wells, and thus, when a different bit size occurs, the rate of penetration the model predicts is usually not close to what it should be. This is especially true for this data set, as the only non 17.5" bit diameter is the 26" in bit which is generally used for extremely shallow, unconsolidated sections, thus having large rate of penetrations despite being a much larger bit diameter.

Overall conclusion to be drawn from the models is that if the non-common parameters in the models are assumed to have small effects on the final value of ROP, then the discrepancies in the four models are mostly present due to any differences shown in figures 1 through 3. Although agreeing in bit diameter effects and having almost the same relationships to bit rotation, the models heavily disagree on weight on bit effects. Motahhari's model is better equipped to handle large changes to ROP as effects of weight on bit, but conversely, if field ROP is mostly unchanging, but weight on bit varies often, then the model will generate large errors.

3.2.2 KHANGIRAN FIELD ANALYSIS WITHOUT SHALLOWEST SECTIONS

In this section, the shallowest data points in Wells 50 and 47 were ignored in order to check if an outlier type effect could make significant changes to the model predictions. Figures 4 through 6 present the parameters common to all models and compare their functional relationships to ROP. In all three models, the models converge at a point where the calculated data was forced to match field data. For bit diameter, it is at 17.5 inches, for bit rotation, it is 130 RPM, and for weight on bit, it is 15000 lbs.

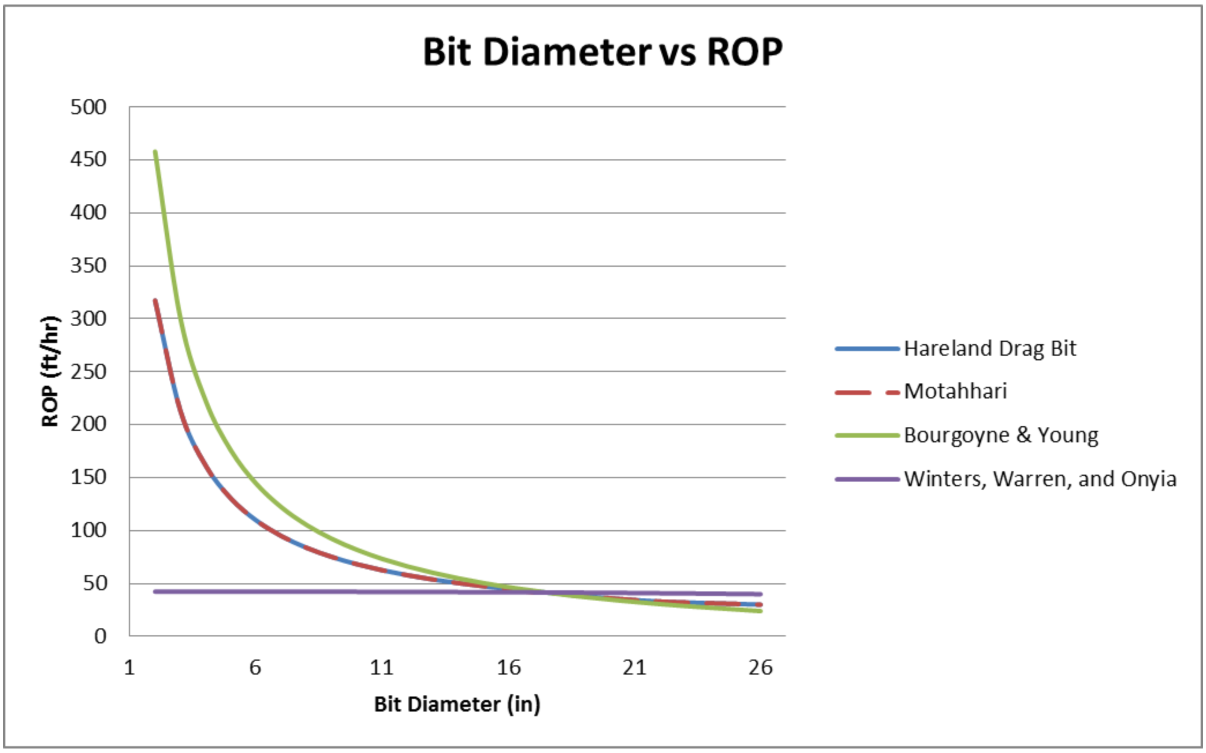

Figure 4: Bit Diameter (in) vs ROP for drag bit models, Winters, Warren, and Onyia and Bourgoyne & Young without shallowest sections

In figure 4, again Hareland's drag bit model and Motahhari's PDC bit model overlap one another, so Motahhari's line style is a long dash instead of a solid line. This is an important notion, as it signifies that both drag bit models agree on how bit diameter affects ROP when applied to a data set. Winters, Warren, and Onyia's model continues to display a nearly unchanging ROP for a large range of bit diameters. As such, for this data set, it can be said that in Winters, Warren, and Onyia's model, ROP is independent of bit diameter. Before values are applied to equation 3.4, one would not believe otherwise. There is no difference in model relationships between figures 1 and 4.

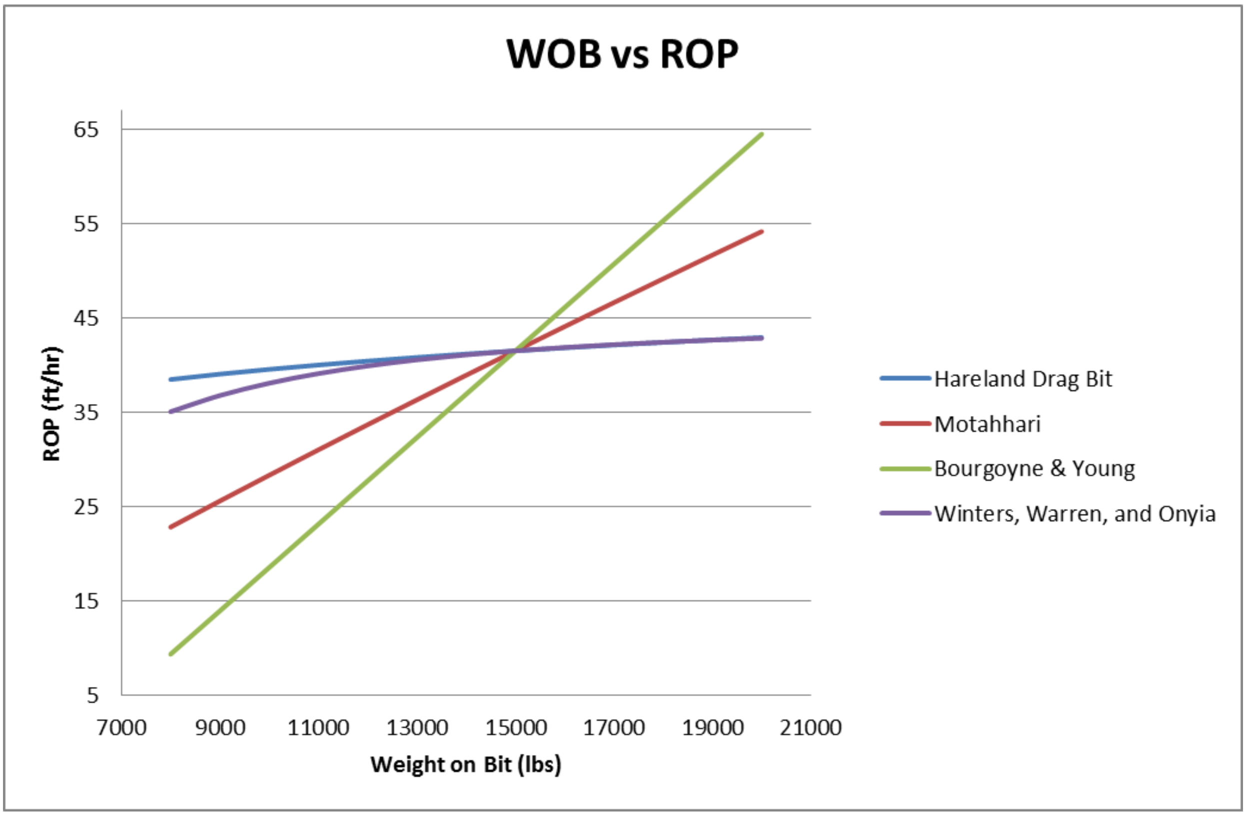

Figure 5: Weight on Bit (lbs) vs ROP for drag bit models, Winters, Warren, and Onyia and Bourgoyne & Young without shallowest sections

In figure 5, Motahhari's and Bourgoyne & Young's models continue to exhibit linear relationships between weight on bit and ROP. As stated earlier, this will cause the models to have large associated error inside a formation where there is a large change in weight on bit, as weight on bit would not cause as much of a change in ROP as predicted by the plot above. A better approach is that of a relationship of diminishing returns, as displayed by Hareland's and Winters, Warren, and Onyia's models in the plot. Again, ignoring the shallow points did not cause any significant change to the model relationships between figures 2 and 5.

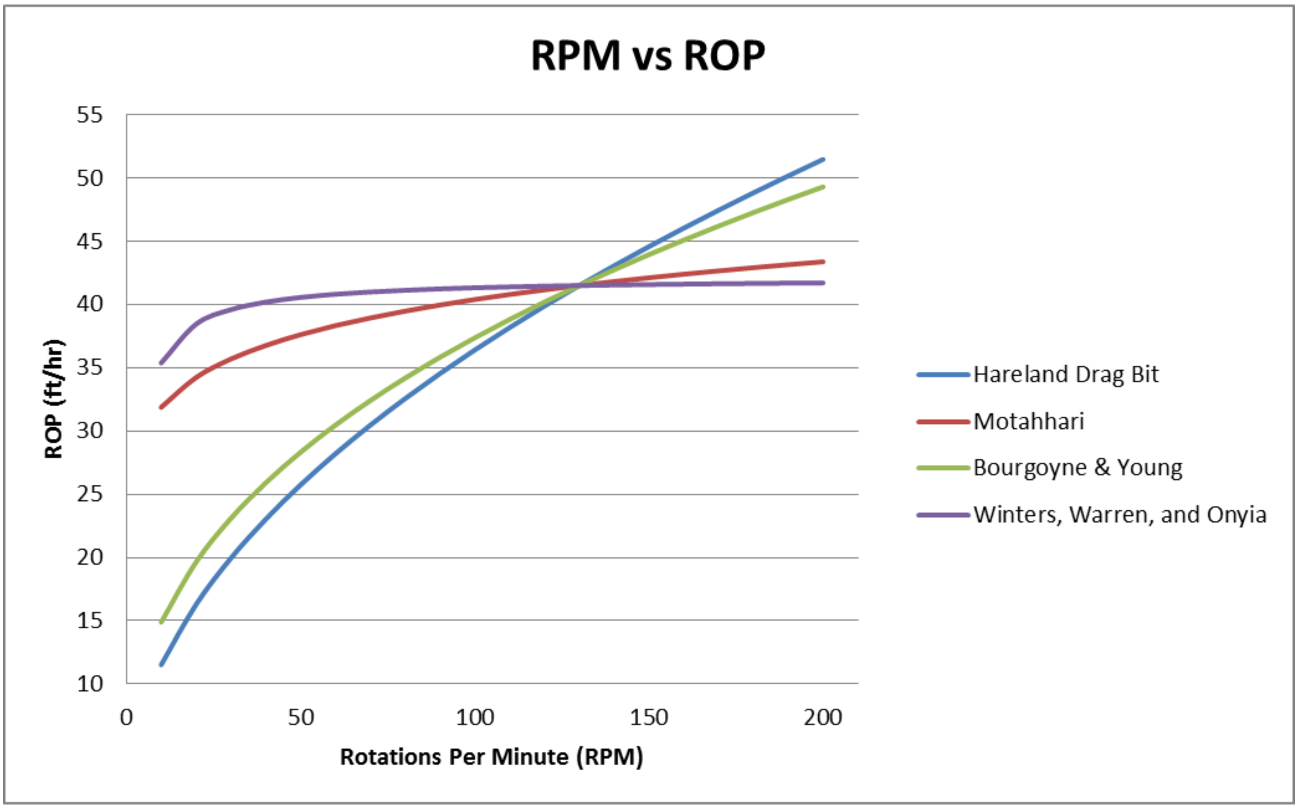

Figure 6: Bit Rotation (RPM) vs ROP for drag bit models, Winters, Warren, and Onyia and Bourgoyne & Young without shallowest sections

In figure 6, all models have very close relationships to one another for how bit rotation affects rate of penetration. Also, RPM does not change much within a formation. As such, one can conclude that RPM does not cause the most significant changes in calculated ROP values for the models presented. Unless a slow pump rate operation commences or RPM is greatly increased, the relationship between RPM and ROP through all four models in a given formation should be almost exactly the same.

Table 5: Average Error Percentages in the Models for the Khangiran Field Data

Well Number	Hareland	Motahhari	Bourgoyne & Young	Warrens, Winter and Onyia
Well 49	84.91%	84.69%	20.88%	21.27%
Well 47	51.54%	51.53%	2.49%	12.86%
Well 46	41.91%	0.00%	11.75%	0.00%
Well 42	0.00%	23.97%	2.91%	34.15%
Well 39	49.62%	35.49%	17.17%	66.14%
Well 29	71.55%	68.73%	5.74%	9.70%
Average:	49.92%	44.07%	10.16%	24.02%
Median:	50.58%	43.51%	8.75%	17.07%

Table 5 displays the associated errors for each model for the various wells in the Khangiran field and shows the average and median errors for the models in the Khangiran field. As seen from the table, the model that more closely calculates ROP is Bourgoyne & Young's model. A possible reason for that is none of the parameters in the model had to be assumed; Bahari et al. (2007) provided all model inputs. This gives the model an edge when compared to the bit specific models, as no bit geometry was given by the paper and had to be assumed. Another possibility as to why Bourgoyne & Young's model has smaller associated errors is the fact that the data is mostly being affected by weight on bit. By looking at table 1, one sees that for wells drilled with average weight on bit of 15000 lbs or greater, the associated rate of penetration is much larger than that of those drilled with average weight on bit of 10000 lbs or smaller. And the difference between the values is so large that the only model that comes close to having a similar relationship between weight on bit and

ROP is Bourgoyne and Young's model, as such, all other models will have much larger associated errors, even if the models themselves were better equipped for real time operations. One key point to note though is that Bourgoyne & Young's model error increases from the analysis with all the wells by a tiny bit, unlike the other models all of which decreased their errors significantly.

The next best model is Winters, Warren,and Onyia's. Unlike in the situation with all the wells, the model's bit diameter to ROP relationship plays no role, as all points have the same bit diameter and thus the model decreases its error the most, as previously it was incorrectly predicting how bit size affects ROP. The main reason the model behaves second best has to do with the fact that both RPM and WOB effects on ROP only have very large incremental effects on the lowest points of the curves in figure 5 and 6. As such, the model, being able to ignore bit size changes, creates an increasing ROP to increase in ROP relationship that closely resembles that of the data set. However, one would expect if the weight on bit varied a bit more, or the data set was larger, the model would intersperse very large errors and very low errors. Motahhari's model is better equipped to handle large changes to ROP as effects of weight on bit, but conversely, if field ROP is mostly unchanging, but weight on bit varies often, then the model will generate large errors.

Hareland's drag bit model has the largest errors. Being quite similar to Motahhari's model, it Hareland's drag bit model will also have large associated errors. The reason as to why it has larger error is due to the data set having largely changing ROP due to changes in weight on bit, and the model relationship flattening out with large weight on bit values while Motahhari's model continues to increase its associated value of ROP.

By limiting the data set to only points with a single bit diameter size, the models' relationships to the three most important parameters affecting ROP is relatively unchanged despite outlier type effects. As mentioned in the last section, the models disagree mostly on how weight on bit affects ROP. Thus, it is not a surprise that even after bit size changes were ignored, the descending list of best models is still the same. Overall conclusion to be drawn from the models is that if the non-common parameters in the models are assumed to have small effects on the final value of ROP and bit size being neglected, then the discrepancies in the four models are mostly present due to the differences shown in figures 2 and 3. What this entails is that the two drag bit models, yielding similar parameter relationships to ROP, should yield almost exact results. And the models do nearly agree for wells 49, 47 and 29, wells with the largest weigh on bits. However, a large difference in associated errors between the two drag bit models persists and is noticeable for the other cases. Thus, the parameters which had to be assumed are also accounting for significant portions of the overall errors between the two drag bit models and thus will be further discussed in the following chapters.

4. SINGLE, VERTICAL WELL ANALYSIS USING ALL THE MODELS

The following analyses were done on a section of a single vertical well in North Dakota. For the first analysis, the models were implemented into Excel and their model coefficients and correlating parameters mathematically determined using Microsoft Excel's built-in add-in Solver's Generalized Reduced Gradient Nonlinear algorithm. A multiple regression was run to find the least squared error of the difference between the models' calculated ROP and the field data ROP. All models had their regressions separated by lithology.

In the second analysis, in order to further understand each model's accuracy and applicability, the models were rerun on Microsoft Excel but had their regressions separated by formation. Also, instead of using a least squared regression for the entire well approach, this time the error was minimized by formation. This last change greatly reduces the error associated with each model.

4.1 Analysis Separated By Lithology

The analysis separated by lithology was divided into a section for each model and a section for an outlook on all the models and how well they were able to compute ROP from the vertical well data given. Each section for the models includes plots showing the interfaces between limestone, sandstone, limestone and shale, and shale and sandstone. The outlook section provides two tables with the model coefficients, and a table with model evaluations.

Assumptions for each model are discussed in the model sections in chapter 4.1, but are also applicable to chapter 4.2.

4.1.1 BINGHAM'S MODEL

Figures 7, 8, and 9 below show the interfaces between limestone and sandstone, limestone and shale and shale and sandstone, respectively.

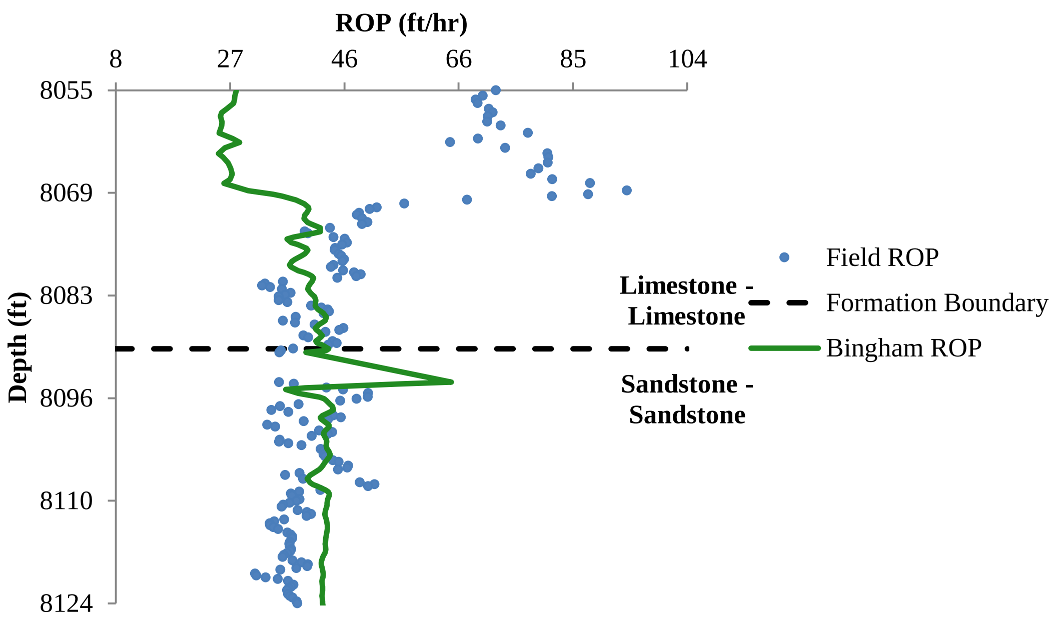

Figure 7: Bingham ROP vs Field ROP Limestone – Sandstone Interface

In Figure 7, one can note that the Bingham model shows not much change in its prediction of ROP between a Limestone and Sandstone interface. The model also underestimates rate of penetration in the top section of the limestone. This occurs because most of the ROP data for the limestone section, which includes a large segment not shown in the plot, varies between 20 ft/hr and 50 ft/hr. As such, the model results are largely skewed by the other lower ROP sections.

Figure 8 shows that Bingham's model accounts for the effect of changing lithology as the bit crosses from limestone to shale. The model mostly underestimates the rate of penetration in the limestone. In the shale, even though the data has a large variance in ROP, the model predicts almost a straight line through it. The error in the shale is mostly due to the statistical method using in determining the coefficients for the models. By minimizing the error, the best coefficients for a formation with highly varying ROP yield an almost unchanging result that goes straight through the data. The coefficients have then made the model not have any physical relevance for the section being analyzed, but it has the smallest error.

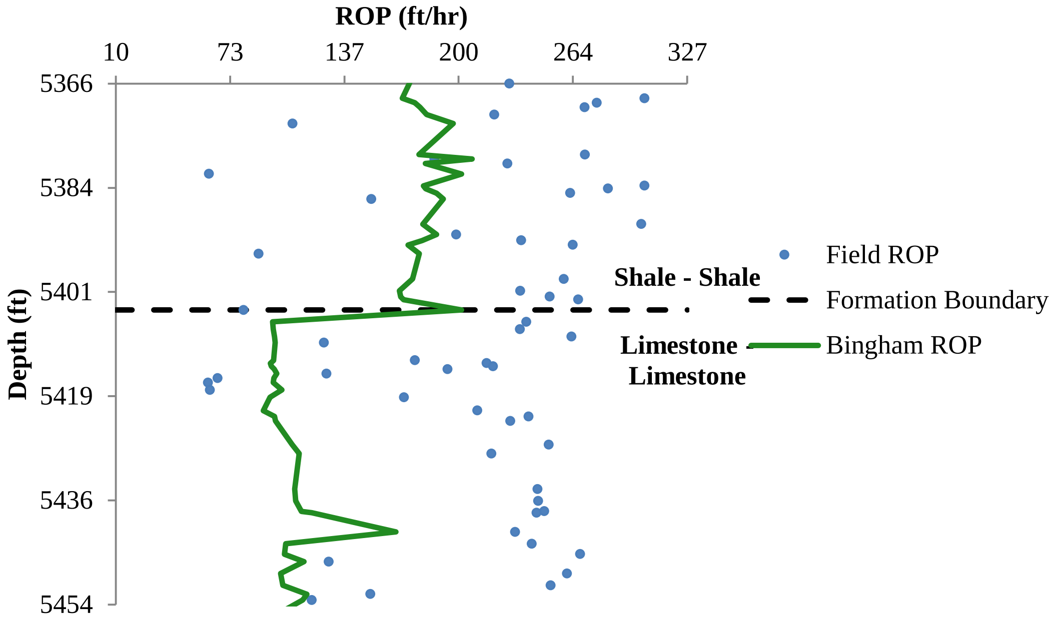

Figure 8: Bingham ROP vs Field ROP Limestone - Shale Interface

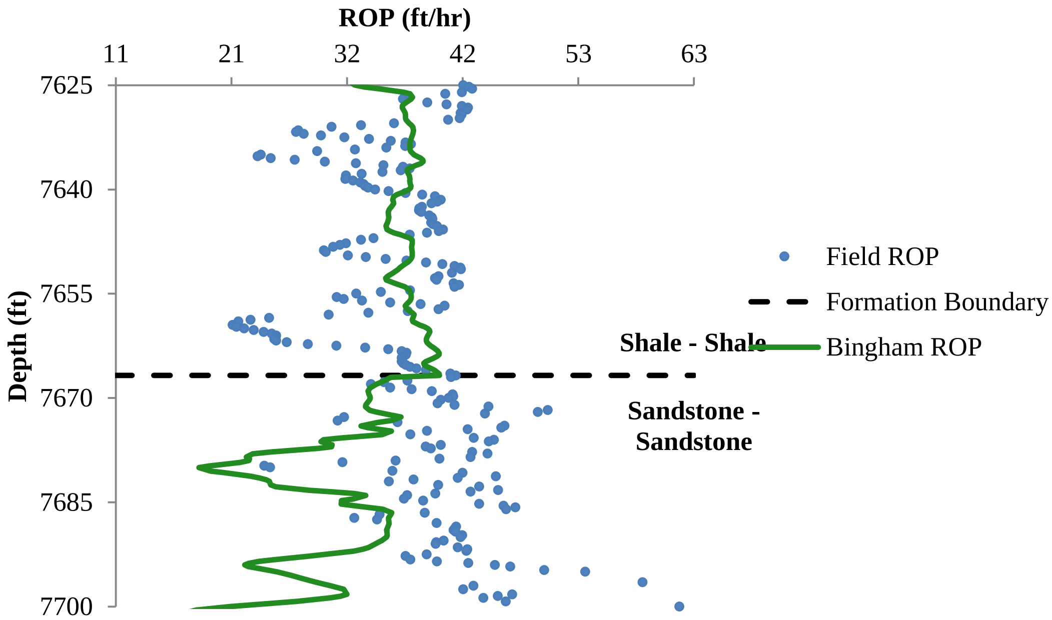

Figure 9: Bingham ROP vs Field ROP Shale - Sandstone Interface

In figure 9, Bingham's model shows a sharp change in rate of penetration as it crosses to a different lithology. And once again the model proves it has limitations, due to its mostly unvarying nature, but it is at least sufficient in determining a shale-sandstone interface. At the sandstone section shown in figure 9, weight on bit is the parameter that varies the most, being reduced from 25 klbs at the boundary between the two lithologies to about 9 klbs just after it Weight on bit then continues to be around 9 klbs, with a few spikes shown in the figure, until it increases again to 25 klbs at 7750 ft. This large reduction in WOB makes for the large underprediction of ROP seen in the plot.

Overall, the model is too rigid for complex wells and will not accurately depict changes in rate of penetration. A more useful variation of the model is Motahhari's model that includes confined compressive strength, a parameter that varies often with the data, and bit wear, another parameter that can be used to model a decrease in ROP while drilling through a formation. With at least those two parameters, the model is more variable to changes within a formation. Otherwise, despite attempts to tweak the coefficients, the results will remain mostly unchanging. And the mostly straight line across the data , that although will greatly reduce error, is not meaningful as a predictive tool. An example of that is for the sandstone section in figure 20 where ROP goes as high as 120 ft/hr just one hundred feet below where the figure stops, however, the model is still predicting an ROP around 70 ft/hr. Thus, the model holds no physical relevance when drilling through a large vertical section, as ROP will not stay as a constant for those intervals.

4.1.2 Bourgoyne & Young's Model

In the Bourgoyne and Young's model, several assumptions for the quantitative section were made. The threshold bit weight at which bit begins to drill was assumed to be 0.25, the bit wear was assumed to be .1875.

Another important distinction between the Bourgoyne & Young model and the other ones calculated in this section, is that this model's coefficients were calculated only once using information from the entire well.

In Figure 10, one can note that the model shows a change in its prediction of ROP between a limestone and sandstone interface even if the change is small. The model also underestimates the rate of penetration at the top sections of the limestone formation, due to the calculated model coefficients being more heavily weighed by the deepest well data for that formation.

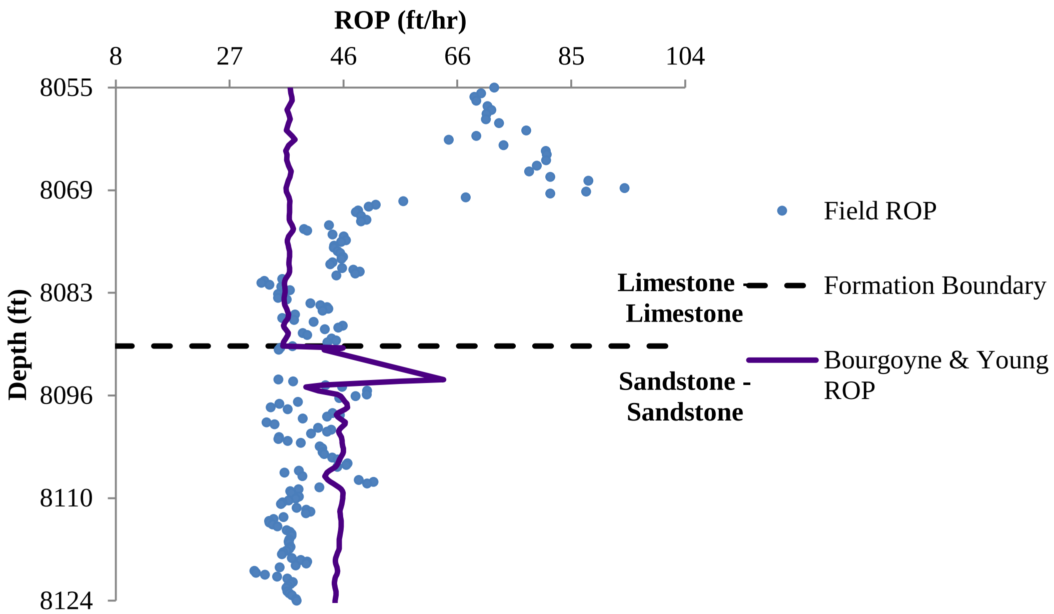

Figure 10: Bourgoyne ROP vs Field ROP Limestone - Sandstone Interface

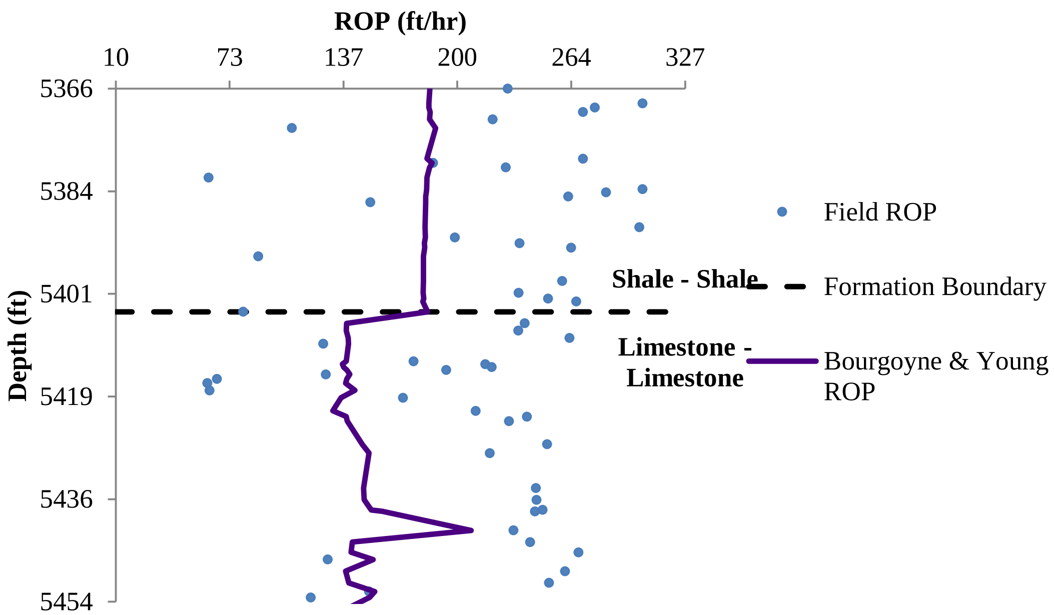

Figure 11: Bourgoyne ROP vs Field ROP Limestone – Shale Interface

Figure 11 shows that the model accounts for the effect of changing lithology as the bit crosses from limestone to shale. However, the model appears to be acting opposite of what the field data would suggest when the data is increasing or decreasing rate of penetration and is mostly a straight line through the middle.

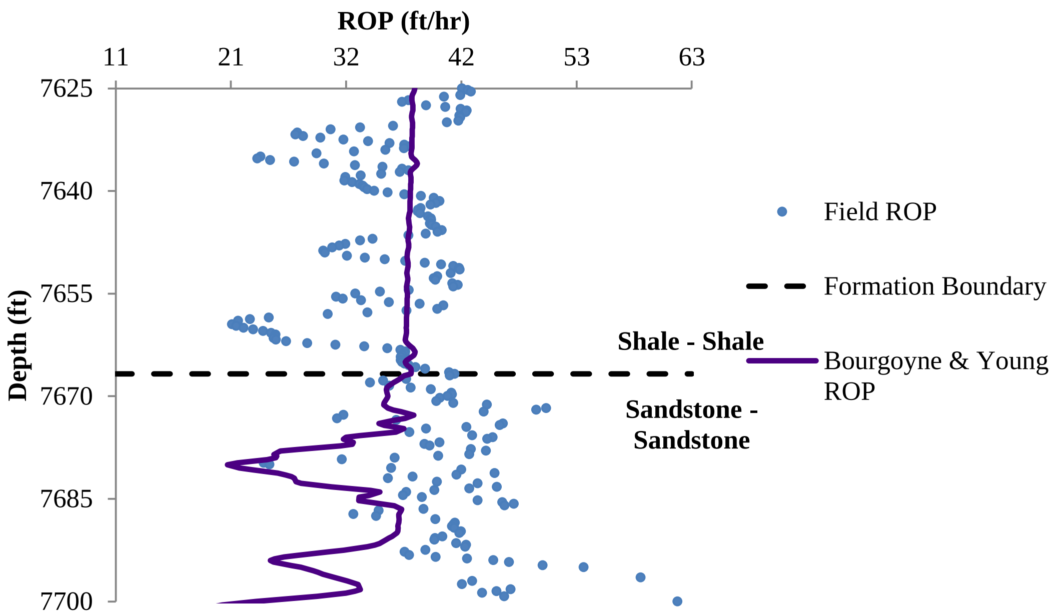

Figure 12: Bourgoyne ROP vs Field ROP Shale -Sandstone Interface

In figure 12, the model shows a smooth transition in rate of penetration as it crosses to a different lithology. Once again, however, the model appears to just be predicting a straight line through the shale section. For the sandstone, Bourgoyne & Young's model is more affected by the variance in the data, but it is underestimating the values a bit, and in the bottom acts opposite to what the data would suggest. Again, the reason the model is underpredicting ROP is due to the sharp change in WOB as the bit crosses from the shale to the sandstone section, where WOB changes from 25 klbs to 9 klbs, and back to 25 klbs at 7800 ft. Due to the model's limitations, and a single coefficient being used for the entire section, the calculated ROP is much smaller than what the data suggests.

Running a single regression through the entirety of the well appears to show that the model is correctly predicting changes, but to the overall pattern to the data, the model is incorrectly predicting the direction of the change, especially since the magnitude of the change is nearly perfect. One factor that may be causing this model, and some of the other ones, to behave opposite to what the data suggests is that in some parts of the data, an increase in ROP is being met by a decrease in the WOB. This relationship is not what is expected from theory and thus the models are not able to correctly calculate ROP. However, the reason why a decrease in WOB is increasing ROP is because of a drilling break at that section. The downhole sensor is measuring a smaller weight being applied at the bit even though surface inputs have not changed.

4.1.3 Winters, Warren, and Onyia Model

For this model, mud density was assumed to be 13 ppg throughout the whole well, viscosity was assumed to be 48 centipoise, and the ductilities were assumed to be 0.8, 0.3, and 0.2 for shale, limestone, and sandstone, respectively. The rock compressive strengths were assumed to be 8000 psi for soft shale, 14000 for hard limestone, and

5000 for sandstone. This last assumption sets the Winters, Warren, and Onyia model apart from the other ones, where CCS was calculated and applied instead.

In Figure 13, one can note that the model does not show a change in its prediction of ROP between a limestone and sandstone interface. This might be due to the values of ROP for the limestone section above the boundary, and the sandstone section below the boundary being approximates of one another. However, the model is relatively unchanging through both formations in the figure, so it is more likely that the model is not a great predictor of ROP for both sections.

In both figures 14 and 15, the model shows a sharp change in ROP prediction from the effect of changing lithology despite an average ROP through each section indicating that the ROP values should be close to one another. The reason for this is due to a single value assumed for rock ductility in each lithology. The differences in the values when crossing lithology will account for the sharp changes in the model calculations, despite a smooth transition in the data. The largest ROP values are then given to the lithology with the largest ductility, shale. In the next section, the results show that by applying formation specific coefficients, instead of lithology specific, sandstone can have larger ROP than some shales or limestones, despite the same assumptions for rock ductility being used. This highlights that, despite the large effect ductility is having on the ROP predictions in this analysis, the coefficients are still the most important parameters in affecting the end result for ROP.

Another key element to be viewed from the plots is that despite large variances in the data, the model continues to behave almost as a straight line through the data. This limitation is assumed to be due to the model being developed for a roller cone bit, however being applied to data from a well drilled by a PDC bit.

An analysis of the model for a well drilled with a roller cone bit would provide real evidence of Winters, Warren, and Onyia's applicability.

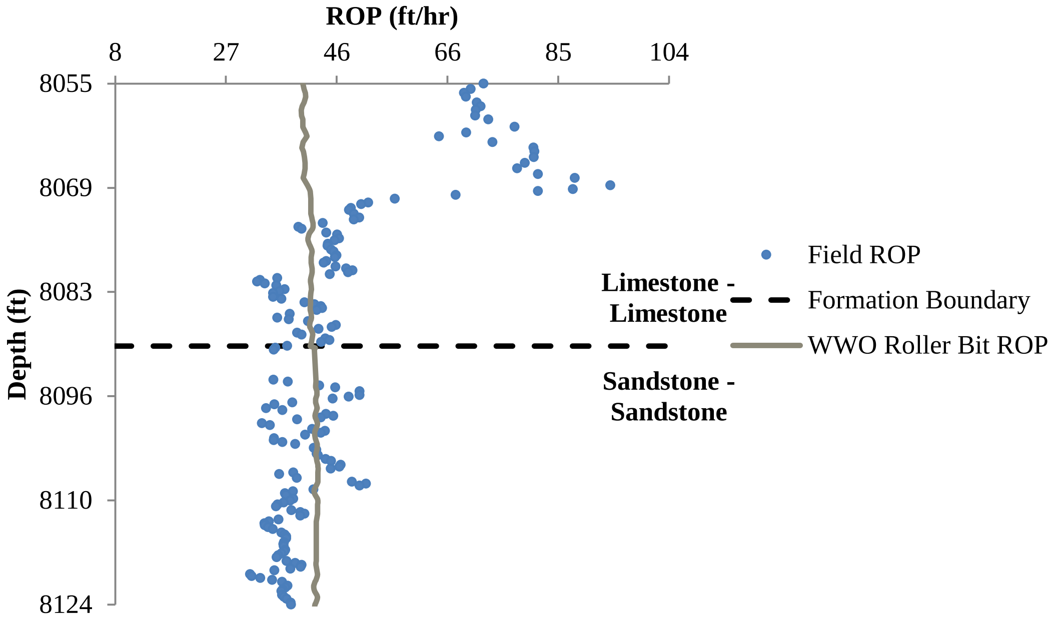

Figure 13: Winters, Warren, and Onyia ROP vs Field ROP Limestone – Sandstone Interface

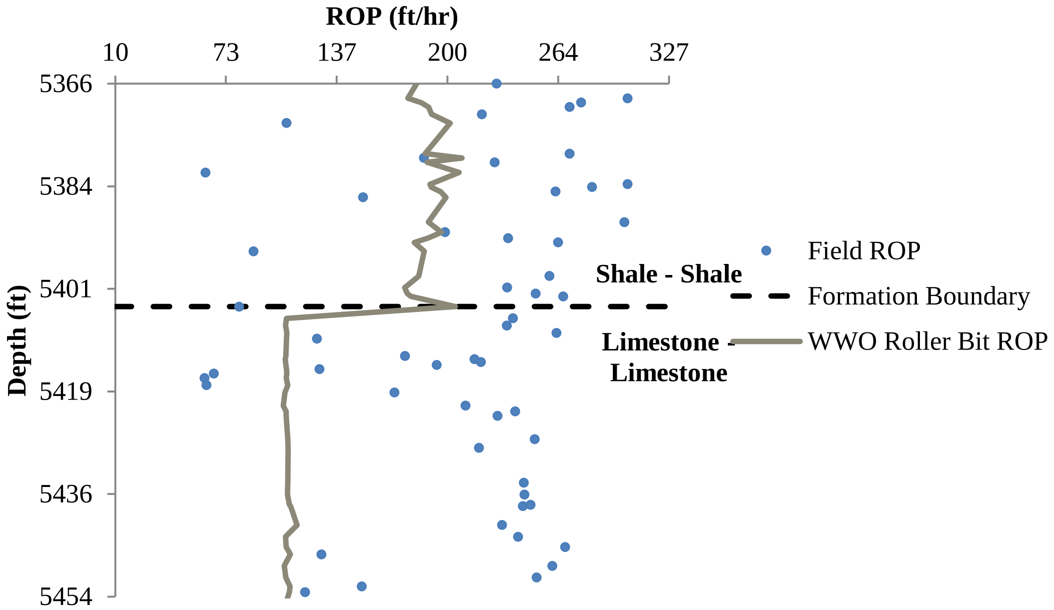

Figure 14: Winters, Warren, and Onyia ROP vs Field ROP Shale – Limestone Interface

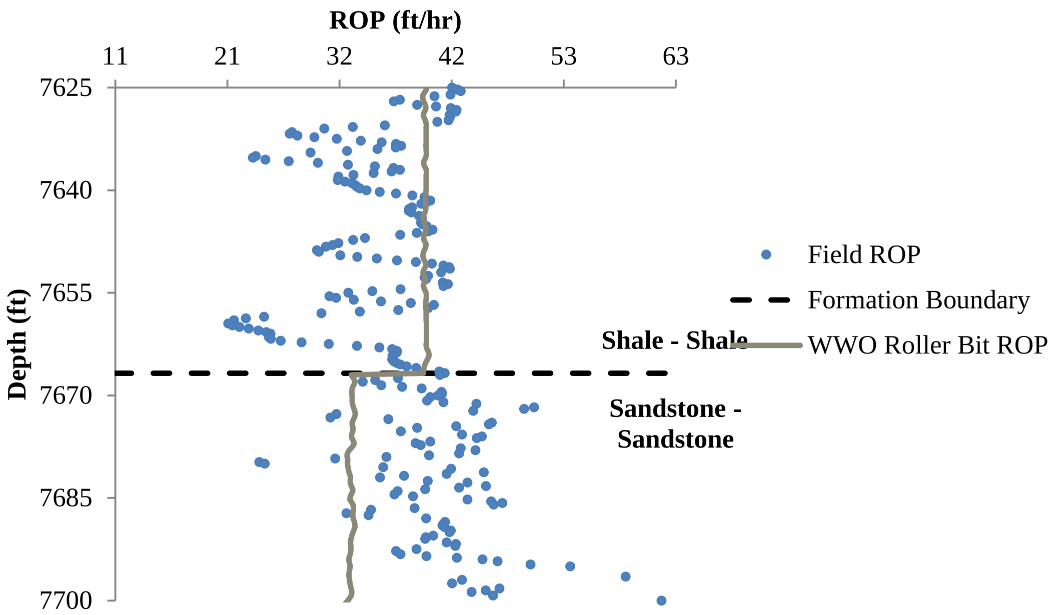

Figure 15: Winters, Warren, and Onyia ROP vs Field ROP Shale – Sandstone Interface

4.1.4 G. HARELAND'S DRAG BIT MODEL

For Hareland's drag bit model, the number of cutters was assumed to be 96 based on the bit being a standard PDC bit. And CCS, the compressive rock strength, was calculated from the equation derived by Ye et al. (2013):

$$CCS = -\frac{(\frac{\tan^2\phi+1}{\tan^2\phi})^2 UCS^2 + 8(\frac{\tan^2\phi-1}{\tan^2\phi}) UCS\, DP}{8(\frac{\tan^2\phi-1}{\tan^2\phi}) UCS + DP} \tag{4.1}$$

where the UCS, confining pressure DP, and angle of internal friction ϕ can be estimated with logging data or measured by lab test, so that nonlinear CCS values can be obtained.

As seen in figure 16, the Hareland's drag bit ROP model estimates ROP quite well for the Sandstone section and the bottom of the Limestone section. However, for the upper part of the Limestone, the model behaves opposite to what is indicated by the field data. Much like the problems encountered with the other models, the opposite behavior that occurs in the limestone section is due to the WOB on the upper section of figure 16 being about half of that in the section between 8070 and 8090 ft whereas ROP roughly doubles.

Figure 17 shows that the model behaves less effectively for a shallower section of the well, grossly underestimating ROP. The model also behaves opposite to what is indicated by the field data, even doing so at the transition between the shale limestone boundary. This pattern of an inverse behavior to the data set is further illustrated in figure 18, a middle section of the well. In this middle section, the model is better predicting the shale section, intersecting the data at times, but the pattern still remains.

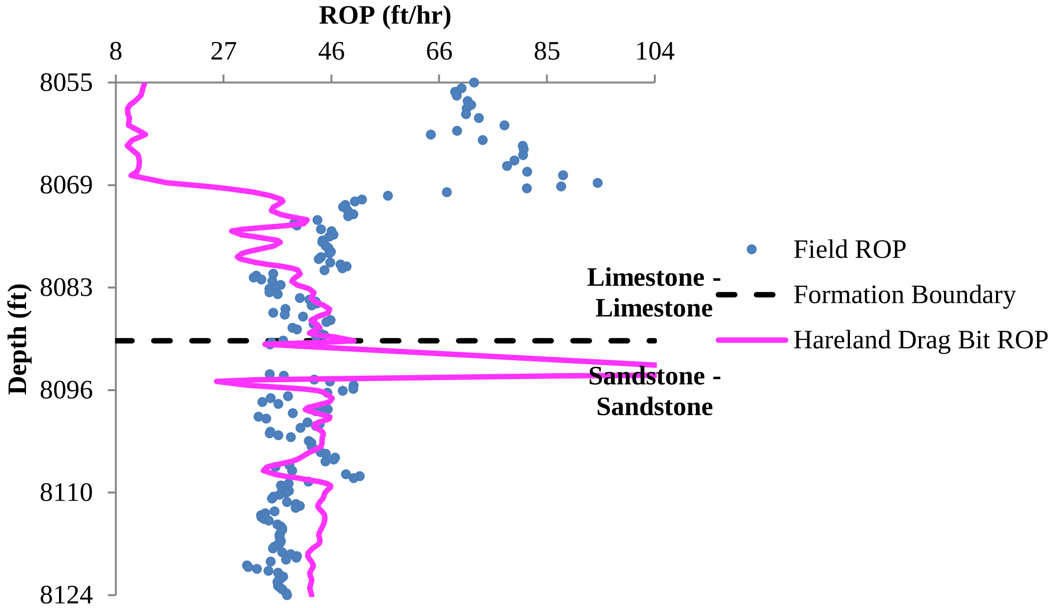

Figure 16: Hareland Drag Bit ROP vs Field ROP Limestone – Sandstone Interface

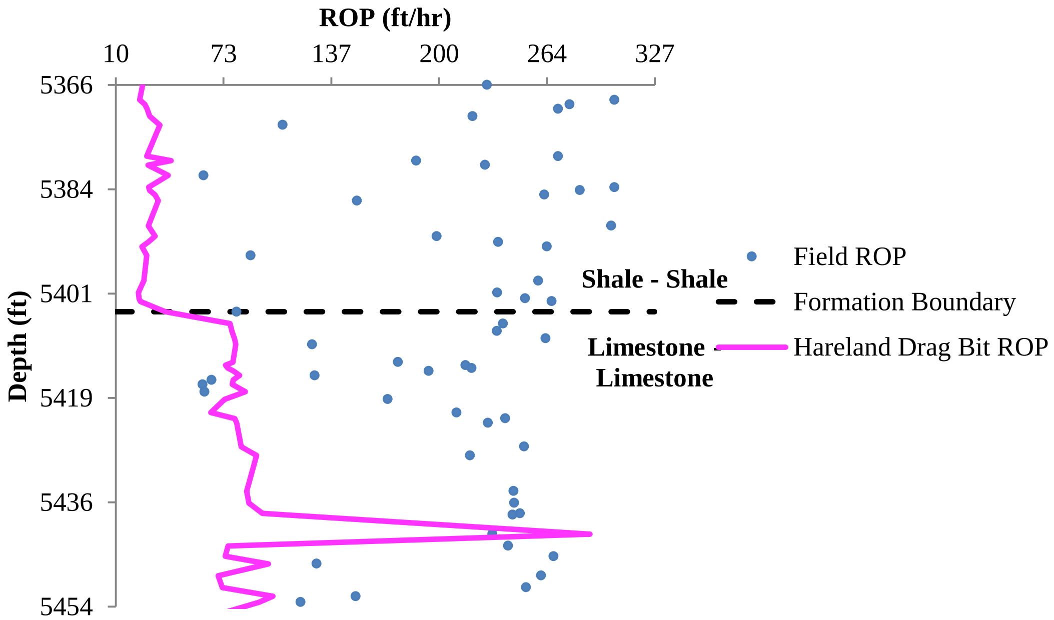

Figure 17: Hareland Drag Bit ROP vs Field ROP Limestone – Shale Interface

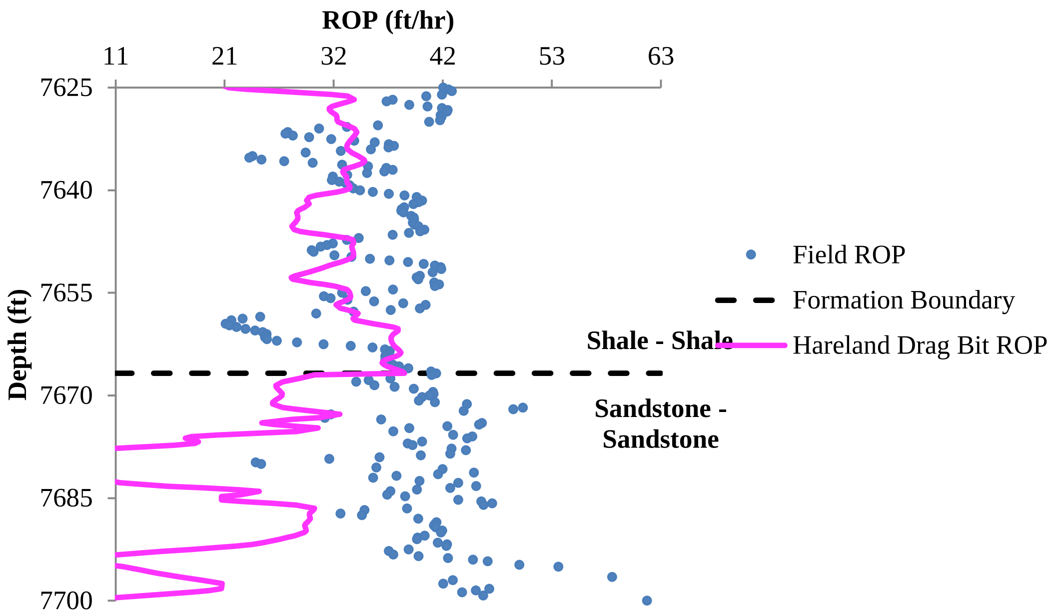

Figure 18: Hareland Drag Bit ROP vs Field ROP Shale - Sandstone Interface

4.1.5 G. Hareland's Roller Bit Model

For this model, the number of insert penetrations was assumed to be 12, the number of insert contacts with rock was assumed to be 6, the chip formation angle was assumed to be ideal at 30 degrees, the compressive rock strength was calculated from the UCS and the friction angle, and bit wear was once again assumed to be .1875.

The worst model in predicting changes for all sections of the well is Hareland's roller bit model, as illustrated by figures 19, 20, and 21. The model is mostly a straight line through all the data sets, and even though it has a low error percentage, it still does not appear to account for the variance in the data.

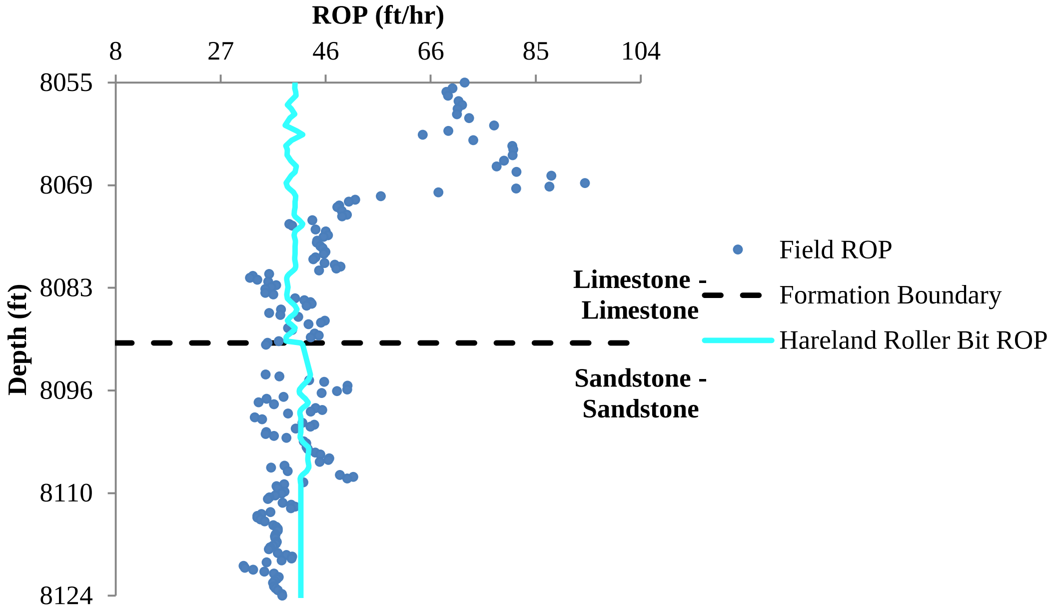

Figure 19: Hareland's Roller Bit vs Field ROP Limestone - Sandstone Interface

Although difficult to perceive in figure 19, Hareland's roller bit model does not behave opposite to what the field data indicates which is a significant improvement when comparing with the previous models discussed.

Despite the limitations, the model averages a good prediction of ROP for the majority of the well, and thus could be used as a starting point for improving drilling efficiency for other wells in the region. The model could also be used for predicting increases or decreases in ROP, even if the magnitude of the change is incorrect.

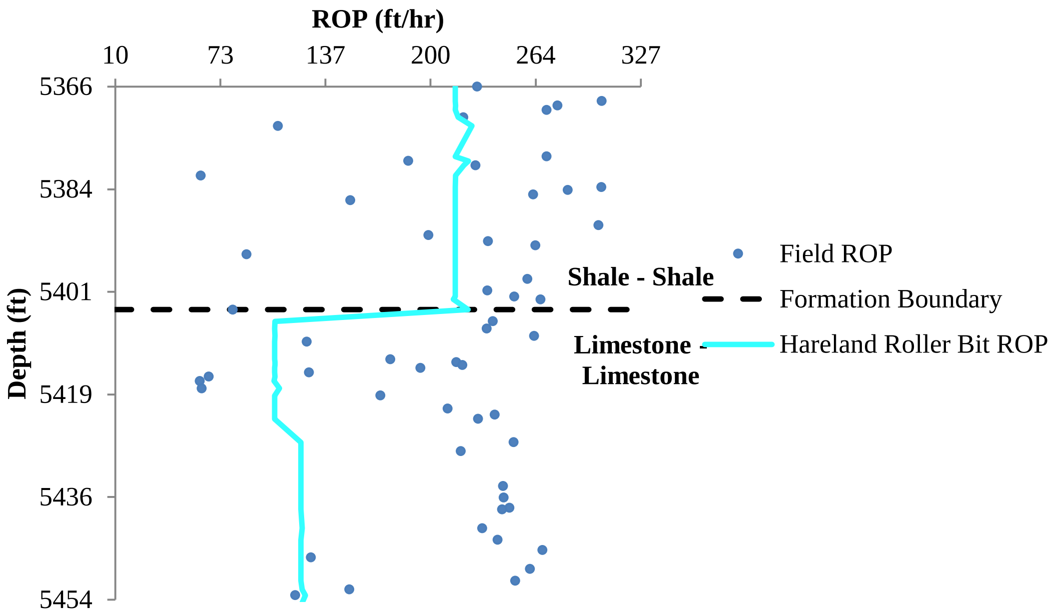

Figure 20: Hareland's Roller Bit ROP vs Field ROP LS-Sh Interface

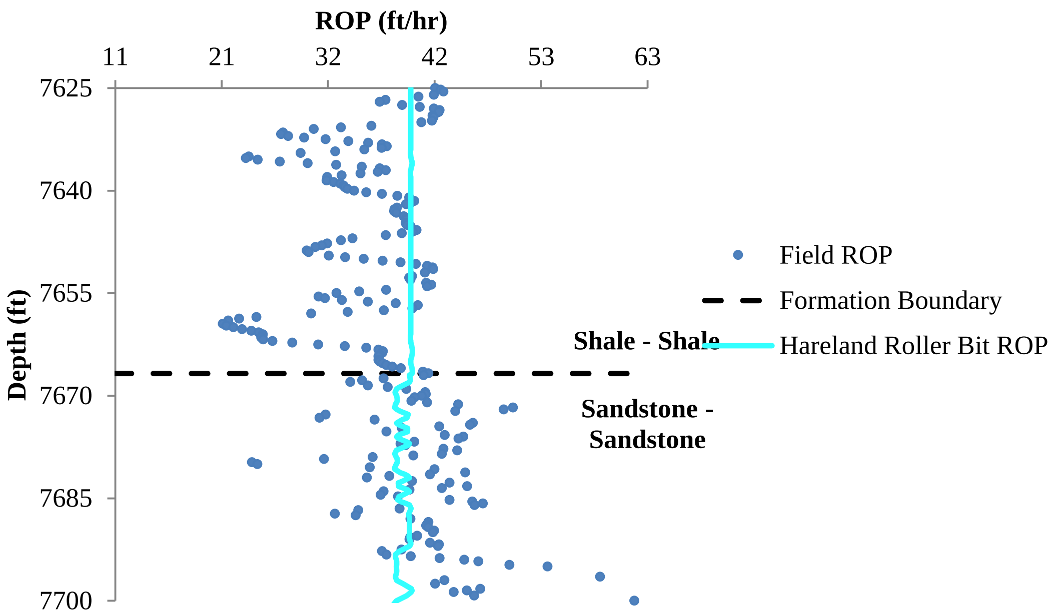

Figure 21: Hareland's Roller Bit ROP vs Field ROP Sh-SS Interface

4.1.6 MOTAHHARI'S PDC BIT MODEL

For Motahhari's method the same assumptions for compressive rock strength and bit wear used in both Hareland's models were applied.

Motahhari's model is a PDC model and thus it is expected to have better results than the previous models, however in the upper part of the deepest limestone section, the model underestimates ROP, as shown in figure 22. Also, in shallower sections, represented by figure 23, the model behaves alike the other ones, and is mostly attempting to go straight down the large variance in the data. Due to all models following this behavior, it is safe to presume that the data set does not provide a deep enough assessment of where there were changes in formations. Most likely, the range of depths represented in figure 23 is comprised of a large number of interbedded lithologies.

In the middle sections and deeper sections of the well, illustrated by figures 22, and 24, the model closely matches the field data ROP thus highlighting the importance of picking a model that accurately describes the drilling environment and equipment.

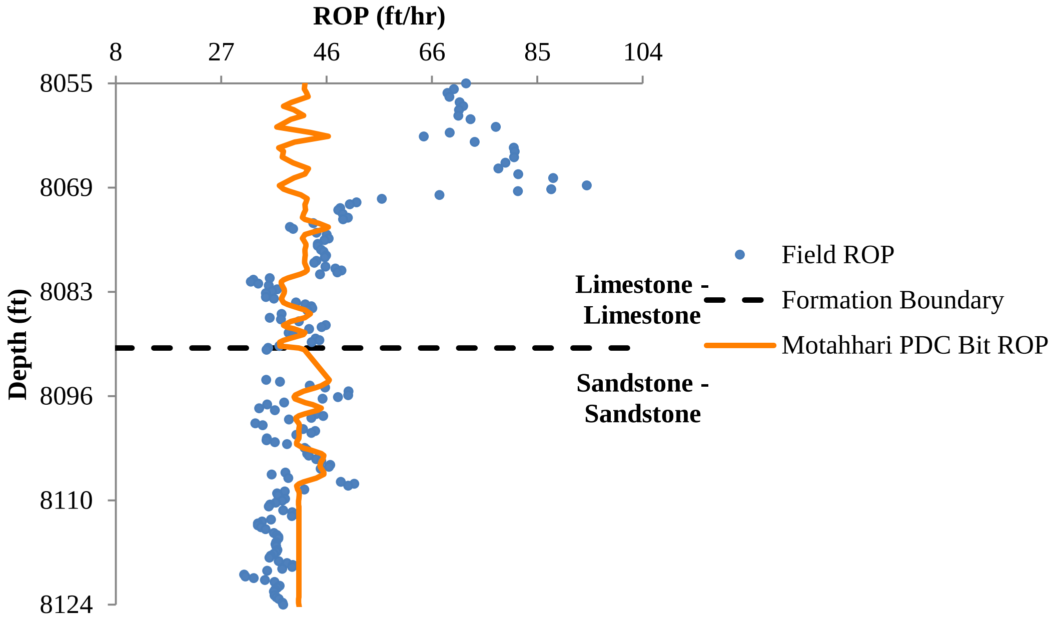

Figure 22: Motahhari ROP vs Field ROP Limestone - Sandstone Interface

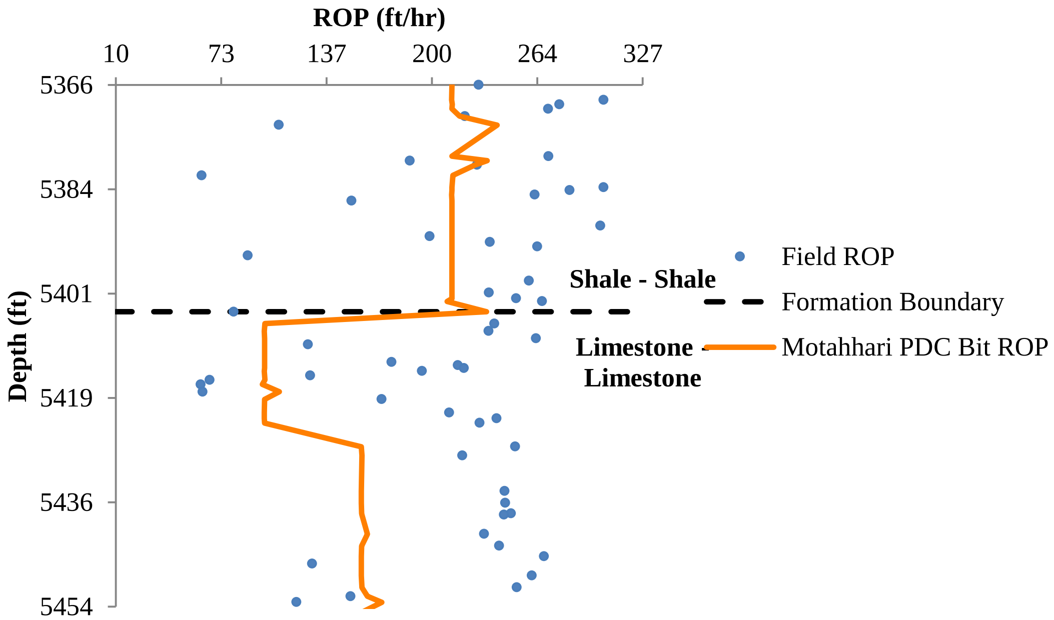

Figure 23: Motahhari ROP vs Field ROP Limestone - Shale Interface

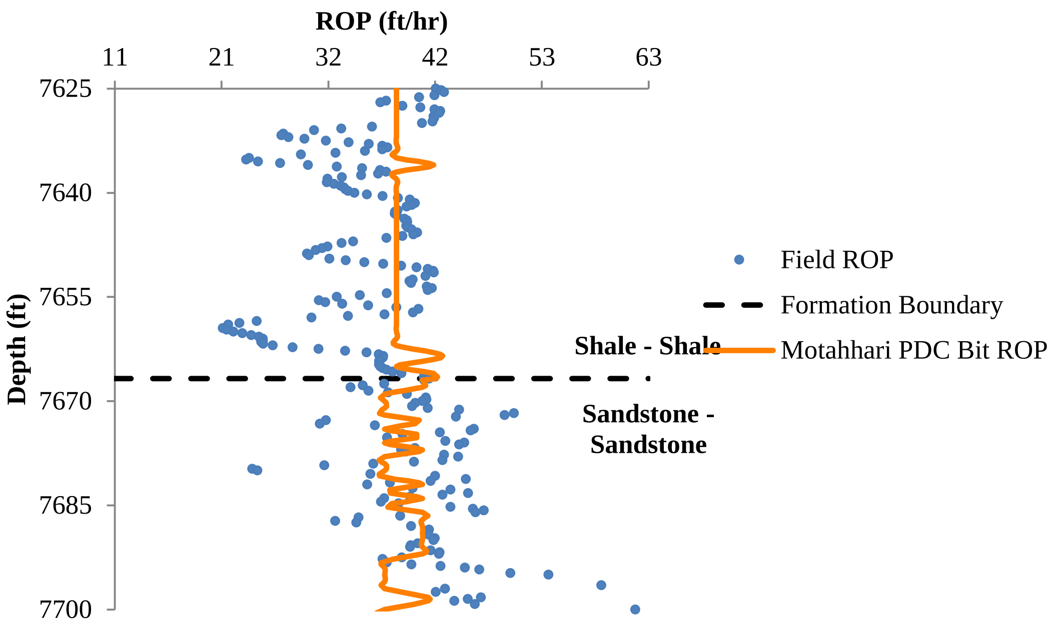

Figure 24: Motahhari ROP vs Field ROP Shale - Sandstone Interface

4.1.7 Outlook on the Models

A reasonable part of the error associated with the models derives itself from the fact a least squared error of the entire well was carried out with each individual lithology being allowed to vary separately. Instead each individual lithology should have its own least squared error, so that the parameters have more freedom in trying to match the data, as well as small sections of the well should have separate model constant estimations.

Table 6: Marathon Separated By Lithology Evaluation

Average percent error of each ROP Model by formation:

Formation Name	Bingham	Bourgoyne & Young	Hareland Drag Bit	Hareland Roller Bit	Motahhari PDC Bit	WWO Roller Bit	Best Model in Formation
Limestone - Limestone	37.56%	32.81%	44.09%	34.60%	39.14%	34.77%	Borgoune & Young
Shale - Shale	30.47%	27.89%	36.62%	26.99%	30.62%	26.93%	WWO Roller Bit
Sandstone - Sandstone	18.28%	20.37%	23.43%	15.36%	15.35%	16.35%	Motahhari PDC Bit
Entire Dataset	29.44%	27.74%	38.86%	25.71%	27.57%	26.69%	Hareland Roller Bit

Best Overall Model: Hareland Roller Bit

Average Percent Error: 25.71%

Although ROP is a function of lithology, lithology is independent of ROP, and the same lithology may have quite different rate of penetrations at a different depth. As the models are rerun using minimum error estimations and stopping the analysis at formation intersections, better results are expected and the models should follow the behavior seen in field data. For these reasons, an analysis separated by formation was performed and is presented in section 4.2.

Table 7: Marathon Data Separated By Lithology All Model Coefficients Part 1 of 3

Coefficients of each ROP Model by formation:

ROP Models	Bingham		B&Y					
Model Coefficients	a	b	a1	a2	a3	a4	a5	a6
Sandstone Average	0.51285	0.5	2.62601	0.0005	1.21E-05	1.00E-06	0.4	0.68457
Limestone Average	1.12042	0.5	2.76159	0.0005	1.39E-05	1.00E-06	0.4	0.68966
Shale Average	2.516	0.5	3.75377	0.0005	1.21E-05	1.00E-06	0.40022	0.69219
Overall Average	1.38309	0.5	3.05E+00	5.00E-04	1.27E-05	1.00E-06	0.40007	0.68881

Table 8: Marathon Data Separated By Lithology All Model Coefficients Part 2 of 3

Coefficients of each ROP Model by formation:

ROP Models			Har Drag			Har Roller			Motahhari
Model Coefficients	a7	a8	a	b	c	K	a	b	G
Sandstone Average	1.04913	0.4977	197.2689	0.00365	0.00682	1.00274	1.29095	0.0001	5.16531
Limestone Average	1.04263	0.4993	625.44748	0.01361	0.0239	2.61263	1.23944	0.0001	7.72577
Shale Average	1.03176	0.5	630.49917	0.00362	0.00496	35.04438	0.88854	0.0001	7.6071
Overall Average	1.04117	0.499	484.40518	0.00696	0.01189	12.88658	1.13964	0.0001	6.83272

Table 9: Marathon Data Separated By Lithology All Model Coefficients Part 3 of 3

Coefficients of each ROP Model by formation:

ROP Models			WWO			
Model Coefficients	α	γ	a	φ	b	c
Sandstone Average	0.0001	3.8399	2.04E-04	1.00E-04	1	0.00323
Limestone Average	0.0001	3.88536	1.00E-04	1.00E-04	0.06241	0.00224
Shale Average	0.0001	3.97491	1.00E-04	1.00E-04	0.03025	0.00183
Overall Average	0.0001	3.90005	1.35E-04	1.00E-04	0.36422	0.00243

As noted in table 6, the overall best model for this well, was Hareland’s Roller bit model. However, due to the model’s limitation of being mostly a straight average line through the data set, the model should only be used as a starting point for future well predictions. A more precise analysis should instead use a different model for each lithology.

Tables 7 through 9 illustrate the calculated model coefficients for each model.

4.2 Analysis Separated By Formation

The analysis separated by formation was divided into a section for each model and a section for an overall look on all the models and how well they were able to compute ROP from the vertical well data given. Each section for the models includes plots showing the interfaces between limestone, sandstone, limestone and shale, and shale and sandstone.

4.2.1 BINGHAM'S MODEL

Figures 25, 26, and 27 below show the interfaces between limestone and sandstone, limestone and shale and shale and sandstone, respectively.

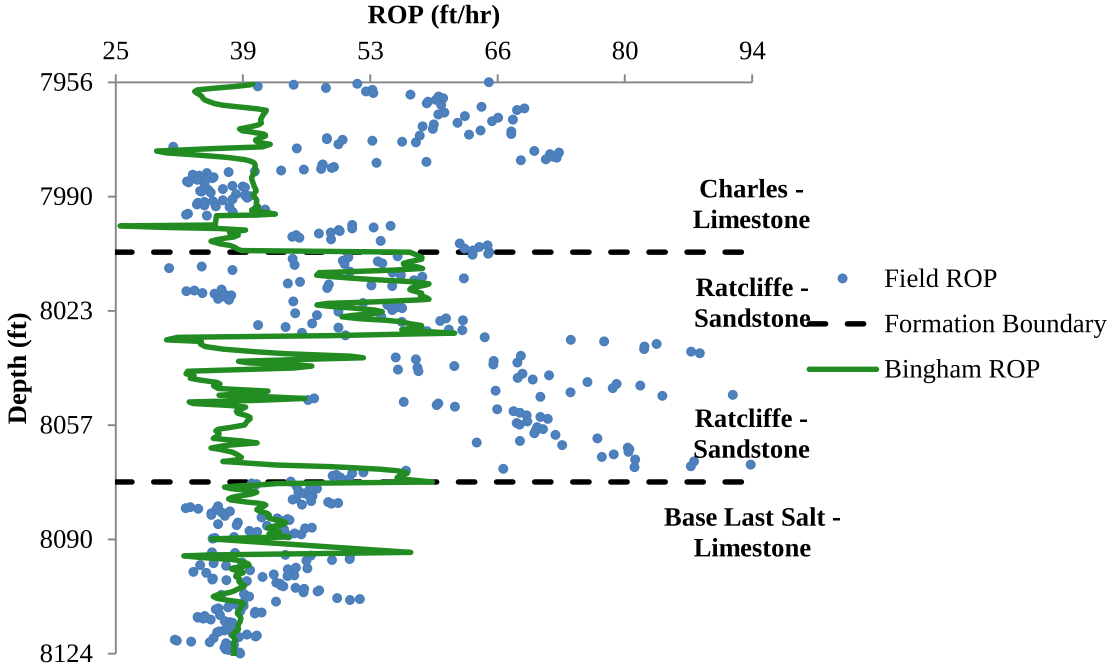

Figure 25: Bingham ROP vs Field ROP Limestone – Sandstone Interface

In Figure 25, Bingham model shows a significant change in its prediction of ROP between a limestone and sandstone interface. The model underestimates rate of penetration towards the end of the sandstone formation, and at the very top, but overall the model is calculating ROP relatively well. Unlike the limestone and sandstone interface for the analysis separated by lithology, Bingham model shows a sharp change in its prediction of ROP between boundaries. Also, now Bingham's model is no longer grossly underestimating rate of penetration across the formations. Despite underestimating ROP at the deepest sections of the sandstone formation and at the top of the plot, an analysis separated by formation shows a great improvement from that separated by lithology.

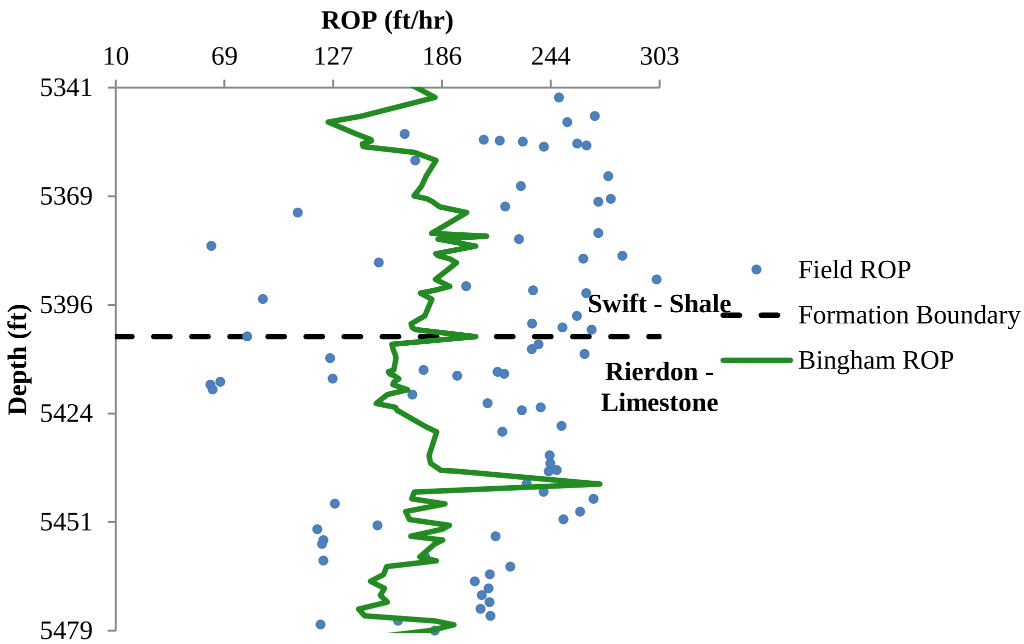

Figure 26: Bingham ROP vs Field ROP Limestone - Shale Interface

Figure 26 shows that Bingham's model accounts for the effect of changing lithology as the bit crosses from limestone to shale, although it is a minor change. Due to the model's simplicity, the plot almost appears to be an average curve of the various data points.

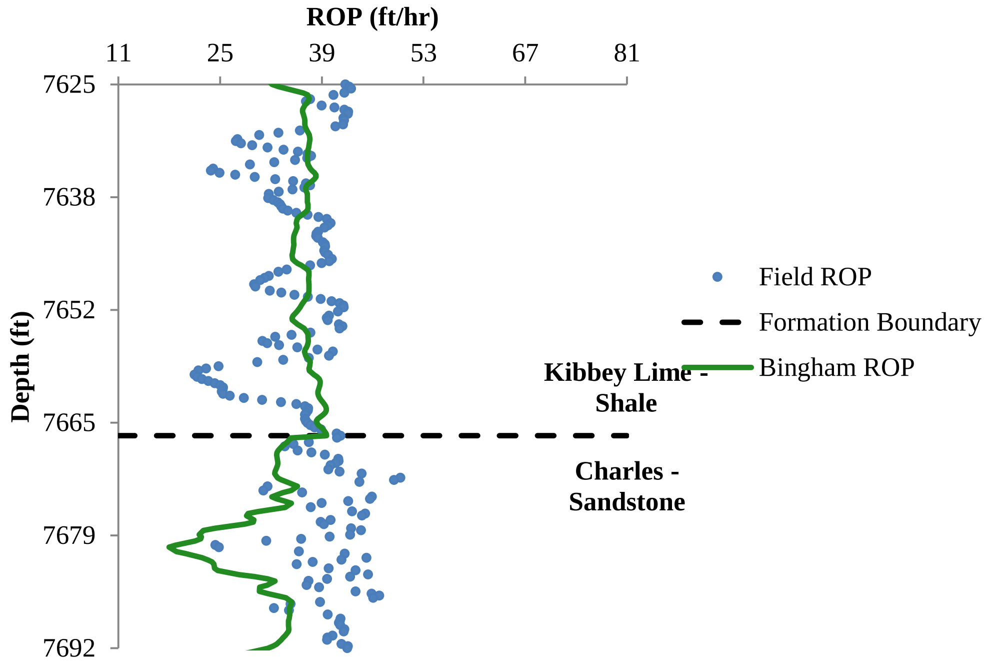

Figure 27: Bingham ROP vs Field ROP Shale - Sandstone Interface

In figure 27, Bingham's model shows a sharp change in rate of penetration as it crosses to a different lithology. Once again the model proves it has limitations, due to its mostly unvarying nature through the shale formation, but it is at least sufficient in determining a shale-sandstone interface. Furthermore, Bingham's model is under predicting ROP for the sandstone formation.

Overall, even when the model is analyzed by formation, it is still too rigid for complex wells and will not accurately depict changes in rate of penetration. Again, despite the model generating a small error, seen in section 4.2.7, because of its straight line across the data, it is not physically meaningful.

4.2.2 Bourgoyne & Young's Model

For the analysis separated by formation, there is no difference in how the calculations were conducted between the analysis for the Bourgoyne & Young's model and the other models.

In Figure 28, one notes that the model shows a change in its prediction of ROP between a limestone and sandstone interface even if at the interface itself it is a smooth transition as is the case for the Ratcliffe, Base Last Salt interface.

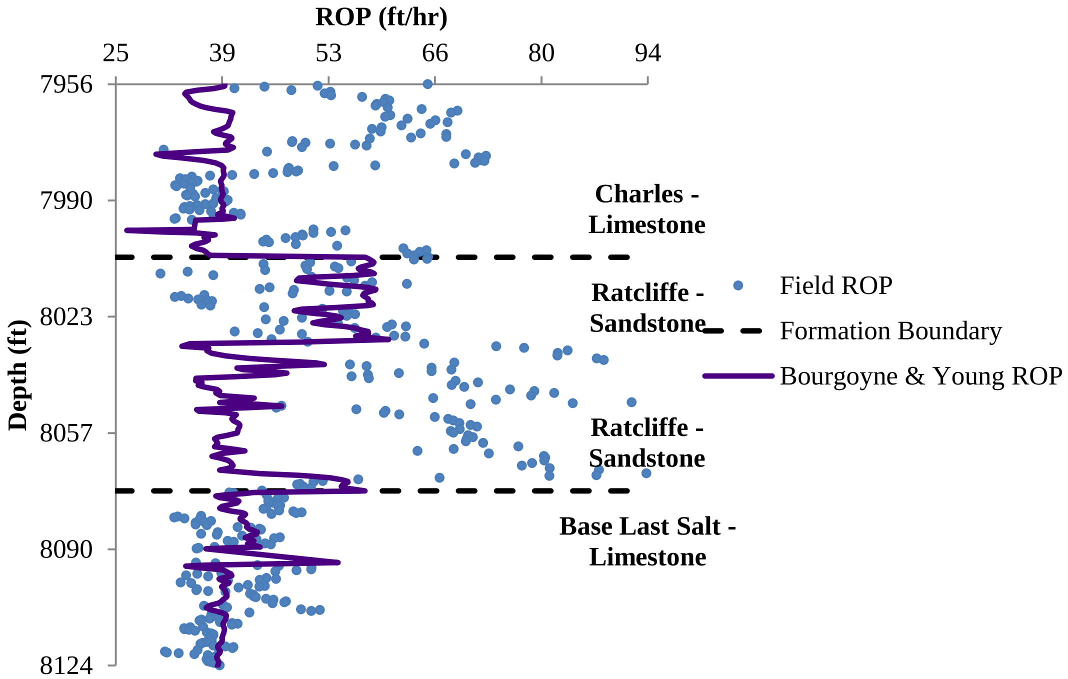

Figure 28: Bourgoyne ROP vs Field ROP Limestone – Sandstone Interface

Figure 29 shows that the model accounts for the effect of changing lithology as the bit crosses from shale to limestone. However, the model responses appear to not be as pronounced as the data suggests, and for parts in the shale and limestone sections, the model acts opposite to what the field data would suggest. Despite running an analysis by formation, the models still encounter a theoretical error when analyzing the data set; at some segments of the data, WOB decreases, but ROP increases. Again, the reason for WOB to decrease is due to the value the downhole sensor is capturing, which is effective weight on bit, rather than the actual weight on bit applied at the surface. As such, whenever one uses these models, one should keep in mind to input the surface values instead of the downhole conditions.

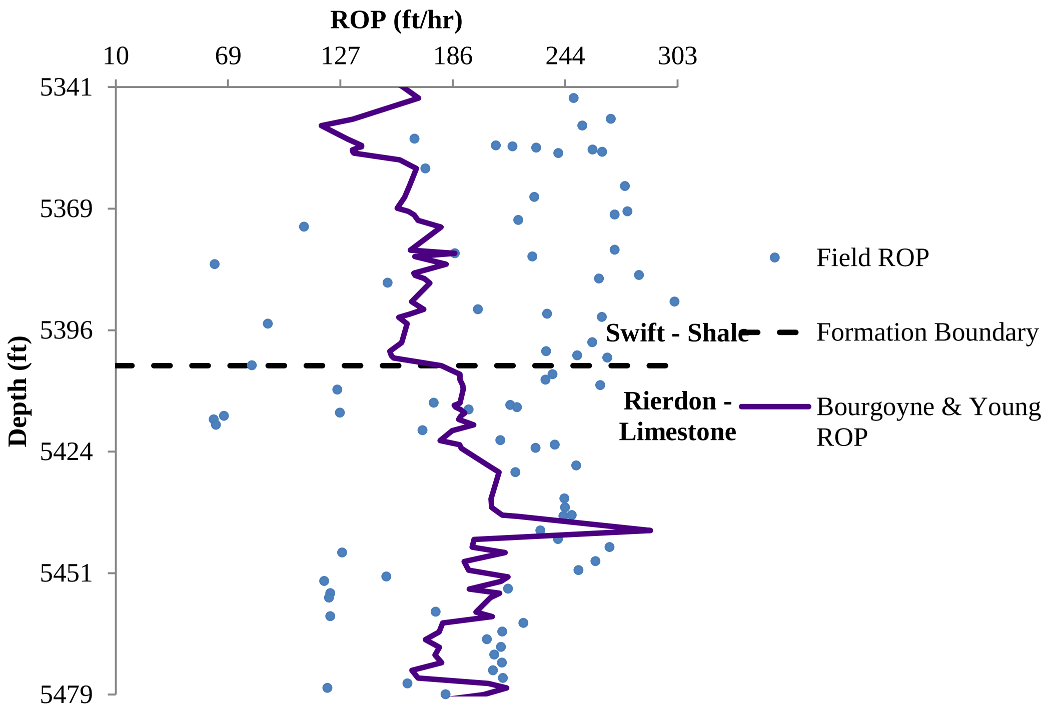

Figure 29: Bourgoyne ROP vs Field ROP Limestone - Shale Interface

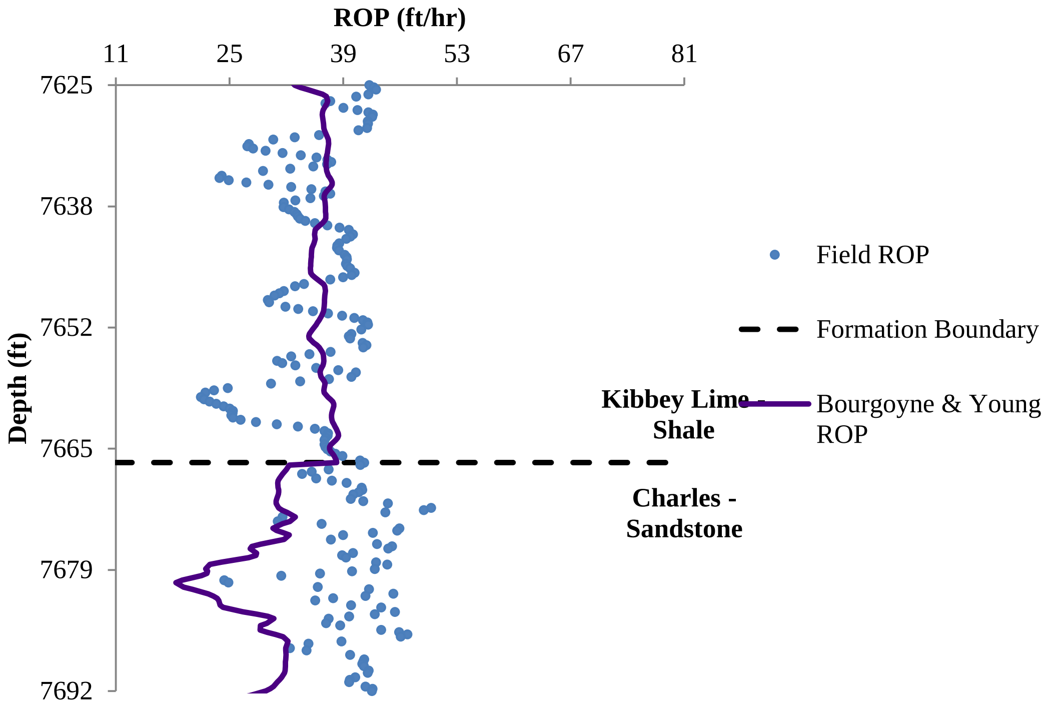

Figure 30: Bourgoyne ROP vs Field ROP Shale - Sandstone Interface

In figure 30, the model shows a sharp change in rate of penetration as it crosses to a different lithology. Once again, the model responses are not as pronounced as the field data would suggest, and throughout most of the shale section, and parts of the sandstone section, the model moves opposite to the field data which is the same issue as seen in figure 29.

Despite running a minimum formation error instead of a single regression through the entirely of the well, Bourgoyne & Young's model appears limited. The more extensive analysis by formation fixed some of the issues with the model's under predictions, but, on segments of the data, the model still behaves opposite to what the data suggests. However, the model is a decent tool for confirming and predicting when a formation change occurs.

4.2.3 WINTERS, WARREN, AND ONYIA MODEL

The same assumptions applied to the analysis separated by lithology are applied in this section.

In figure 31, one notes that the model shows a sharp change in its prediction of ROP between a limestone and sandstone interface. The results on these plots are important because they highlight the fact that despite using assumed ductility values, a lithology with lower ductility, may still have larger ROP from other effects.

As was observed in section 4.1.3, and continues to be seen in figures 31, 32 and 33, despite large variances in the data, the model behaves almost as a straight line through the data. This limitation continues to be assumed to be due to the model being developed for a roller cone bit, however being applied to data from a well drilled by a PDC bit.

Again, an analysis of the model for a well drilled with a roller cone bit would provide real evidence of Winters, Warren, and Onyia's usefulness, but since the analyses presented in this paper are for drag bits, the model's predictability appears underwhelming.

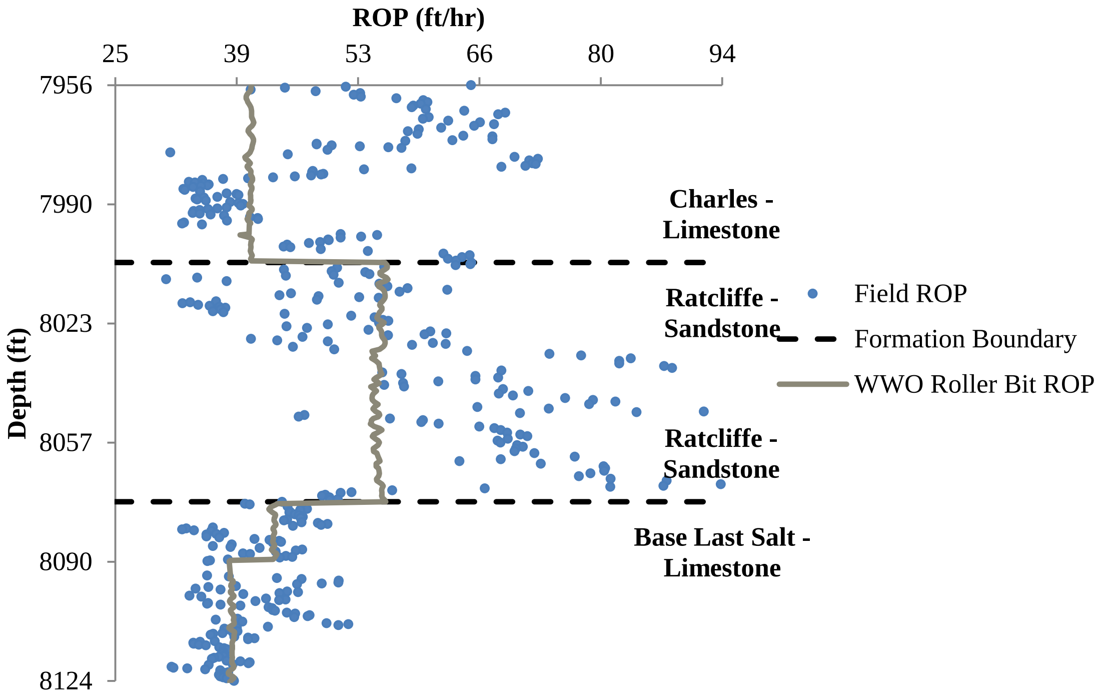

Figure 31: Winters, Warren, and Onyia ROP vs Field ROP Limestone – Sandstone Interface

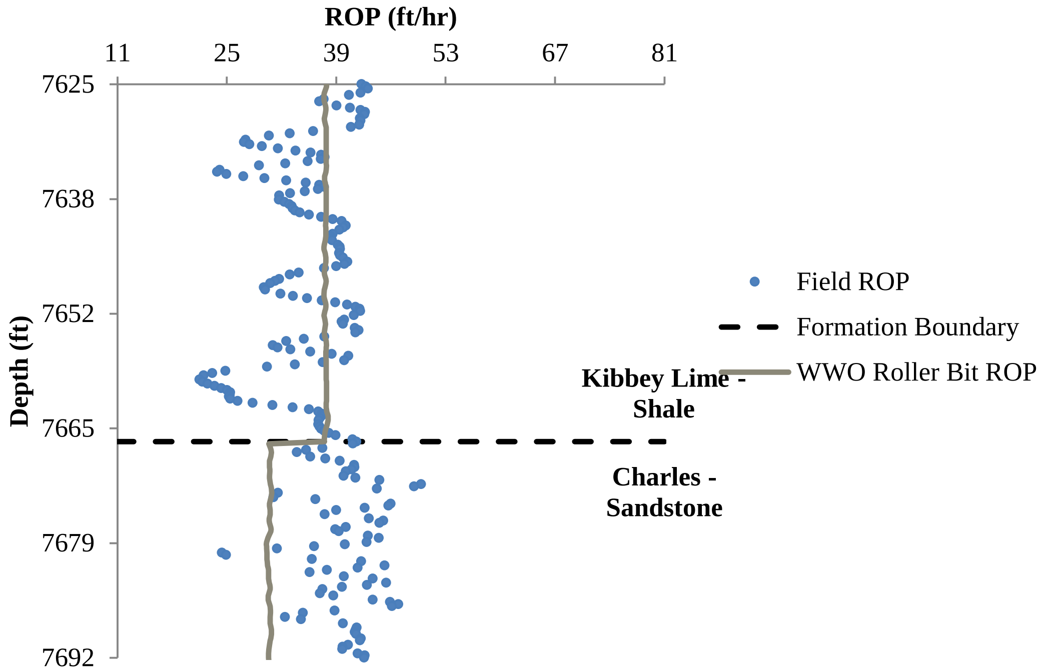

Figure 32: Winters, Warren, and Onyia ROP vs Field ROP Shale – Sandstone Interface

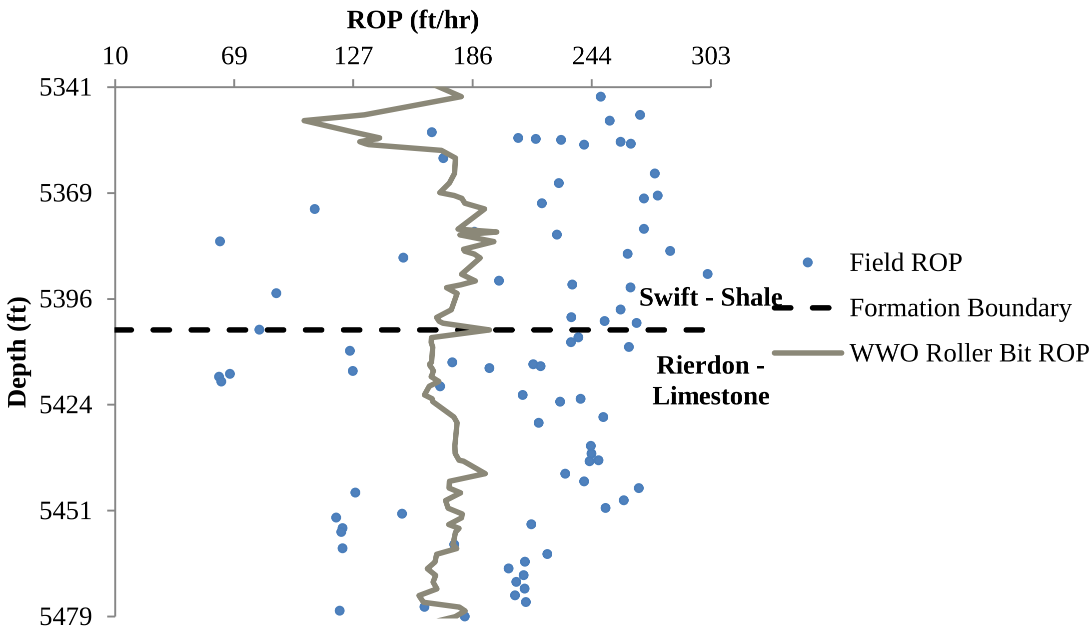

Figure 33: Winters, Warren, and Onyia ROP vs Field ROP Shale – Limestone Interface

4.2.4 G. Hareland's Drag Bit Model

The same assumptions applied to the analysis separated by lithology are applied in this section.

As seen in figure 34, Hareland's drag bit model grossly underestimates the ROP for the shale section of the data and underestimates a bit for the limestone section. This behavior matches that of the analysis separated by lithology, showing that despite separating the model to different formations, the inaccuracy persists. However, unlike the models preceding Hareland's, the error here is not due to an inverse relationship between WOB and ROP, but rather from very low WOB data points for the section being shown. Average WOB for the section is 14 klbs, but the WOB between 5310 and 5400 ft varies from 1.2 klbs to about 2.6 klbs, thus creating the large discrepancy between the model and the actual results. The error here though, may also be caused by the discrepancy between the applied weight on bit, and the instantaneously measured downhole weight on bit. This same issue is seen in figure 36 around 7680 ft, where WOB decreases from 28 klbs at 7565 ft to 7.6 klbs at 7680 ft. The calculated ROP value does not get below 0, but the results are too many standard deviations away from the data to be of any relevance.

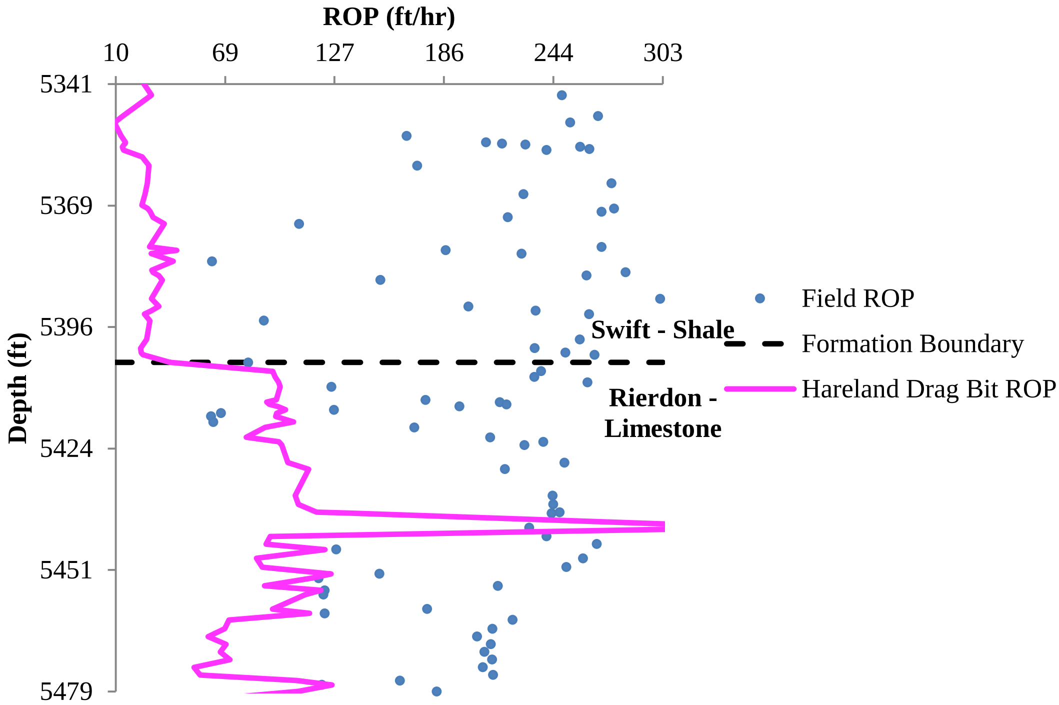

Figure 34: Hareland Drag Bit ROP vs Field ROP Limestone - Shale Interface

Figure 35 shows that the model behaves more effectively for a deeper section of the well, better resembling the changes seen in the field data, as well as being closer in magnitude and correctly predicting the change across the limestone-shale interface. However, figure 35 and 36 demonstrate how the model fails for the sandstone sections depicted. This problem does not occur through all sandstone sections, but whenever there is a large variation in ROP for a given sandstone section, the model fails due to its large sensitivity to change.

Figures 34, 35 and 36, highlight that the model continues to behave opposite to what is indicated by the field data despite utilizing a new quantification method and a new analysis.

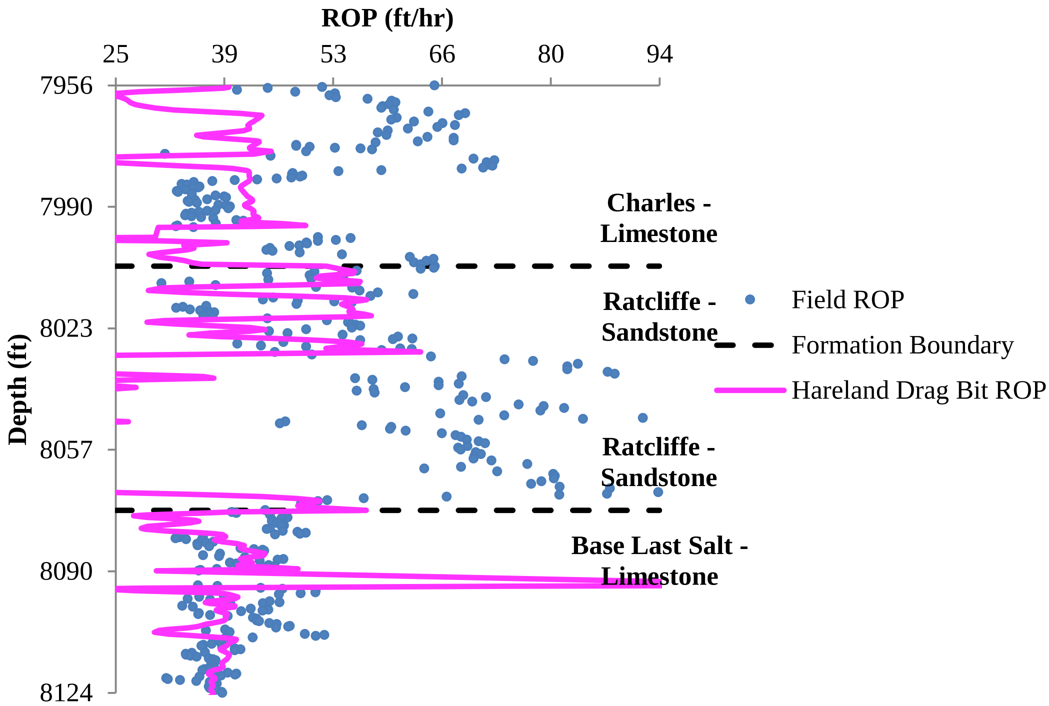

Figure 35: Hareland Drag Bit ROP vs Field ROP Limestone - Sandstone Interface

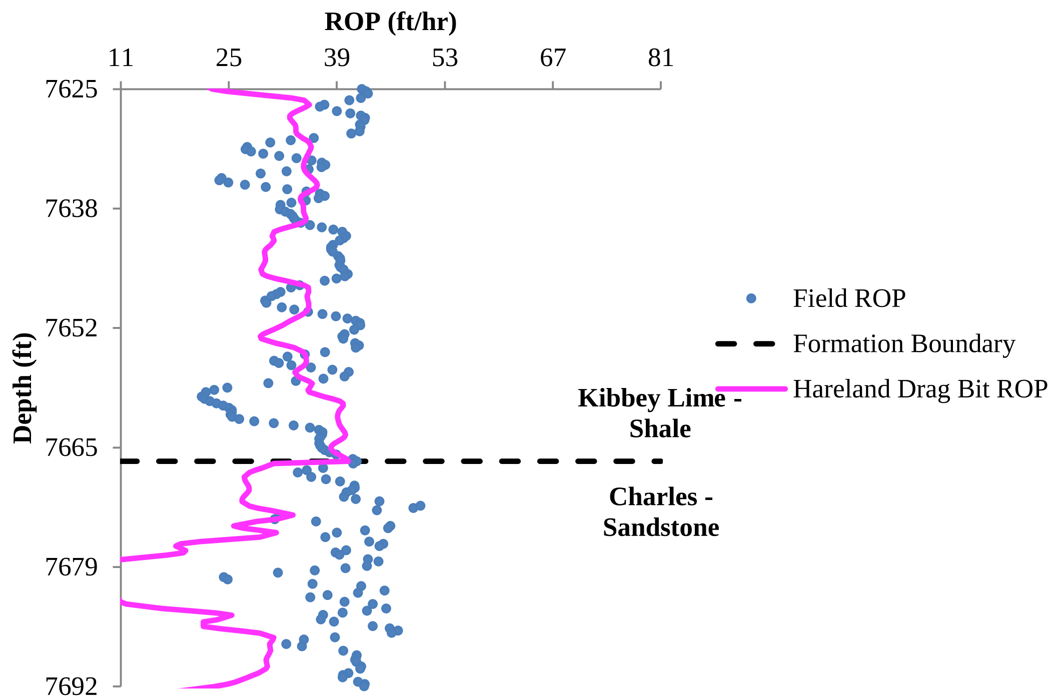

Figure 36: Hareland Drag Bit ROP vs Field ROP Shale - Sandstone Interface

4.2.5 G. Hareland's Roller Bit Model

The same assumptions applied to the analysis separated by lithology are applied in this section.

As illustrated by figure 37, Hareland's roller bit model does not correctly predict changes across the shale limestone interface, despite showing a discontinuity at the boundary. More significantly, the model behaves much like Bingham's throughout the figure and its values therefore have no meaningful interpretations or physical relevance.

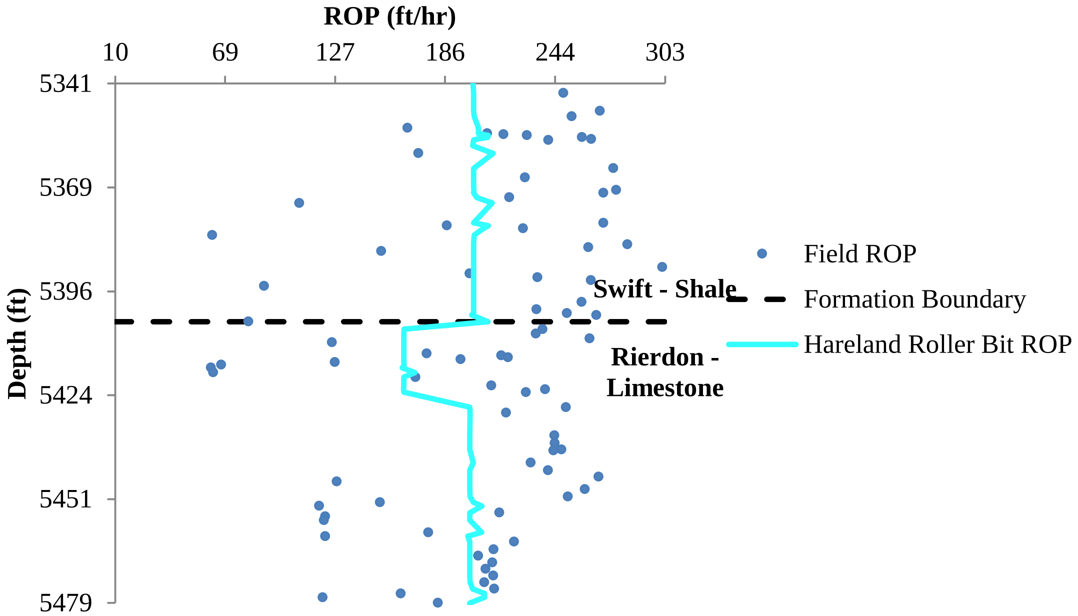

Figure 37: Hareland's Roller Bit ROP vs Field ROP Limestone - Shale Interface

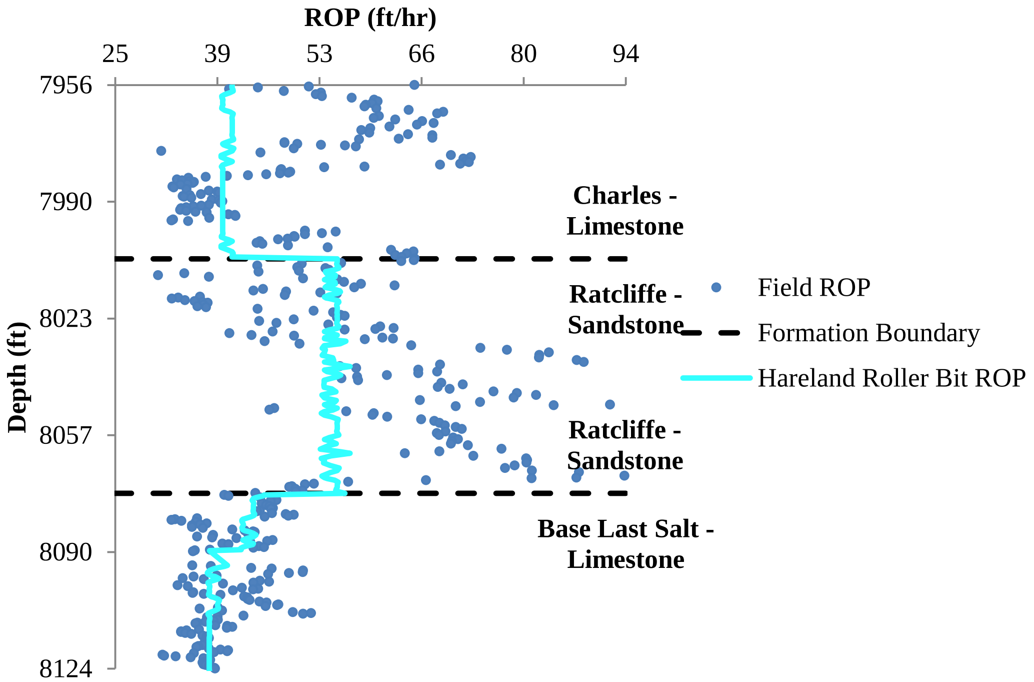

Figure 38: Hareland's Roller Bit ROP vs Field ROP Limestone - Sandstone Interface

Similarly to figure 37, Hareland's roller bit model appears to behave almost as a linear line throughout the data set, as shown in figures 38 and 39. However, in the deeper sections of the well, the model intersects more often the field data, thus the ballpark calculation of ROP for the formation can be of value.

Overall, the model could be used to calculate an average rate of penetration for a section and that initial value be used as a starting point for drilling. The model limitations and failures, although quite significant for this particular well, should not be taken too much into account as the well is being drilled by a PDC bit and not a roller bit.

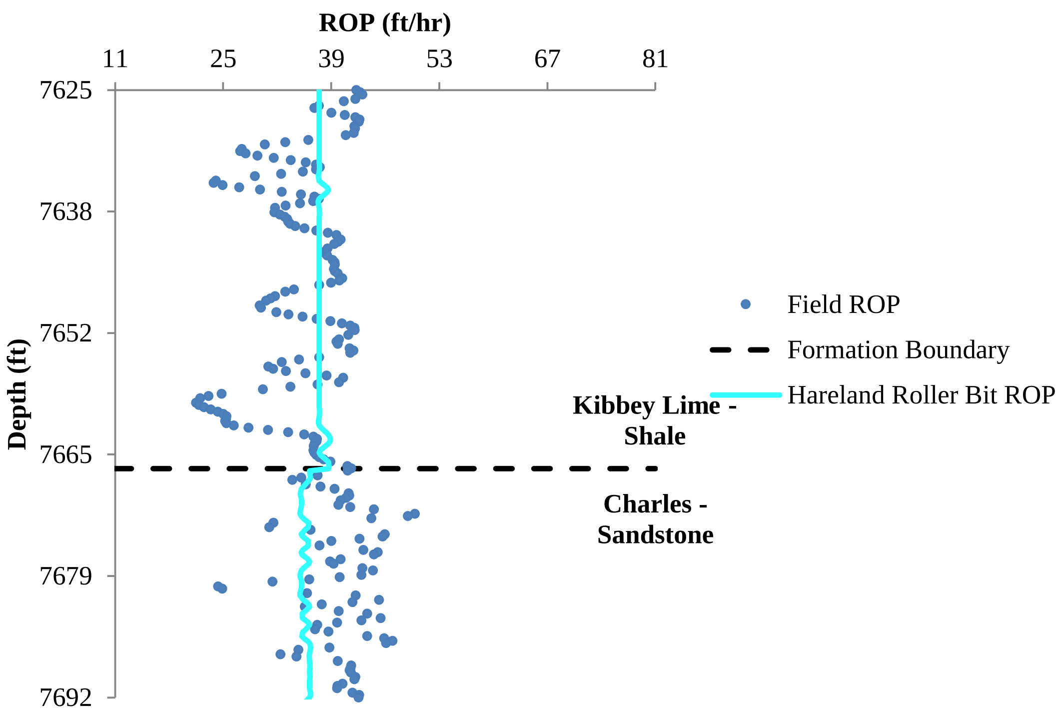

Figure 39: Hareland's Roller Bit ROP vs Field ROP Shale - Sandstone Interface

4.2.6 Motahhari's PDC Bit Model

For Motahhari's model the same assumptions for compressive rock strength and bit wear used in both Hareland's models were applied.

By running Motahhari's model in the analysis separated by formation, a huge improvement was achieved; the model no longer greatly overestimates rate of penetration as seen in figure 40, but the sensitivity of the model to the large range of values for a given section was reduced. In other words, the model predicts rate of penetration better, but will not compute variations within a section very well, seen in figures 41 and 42.

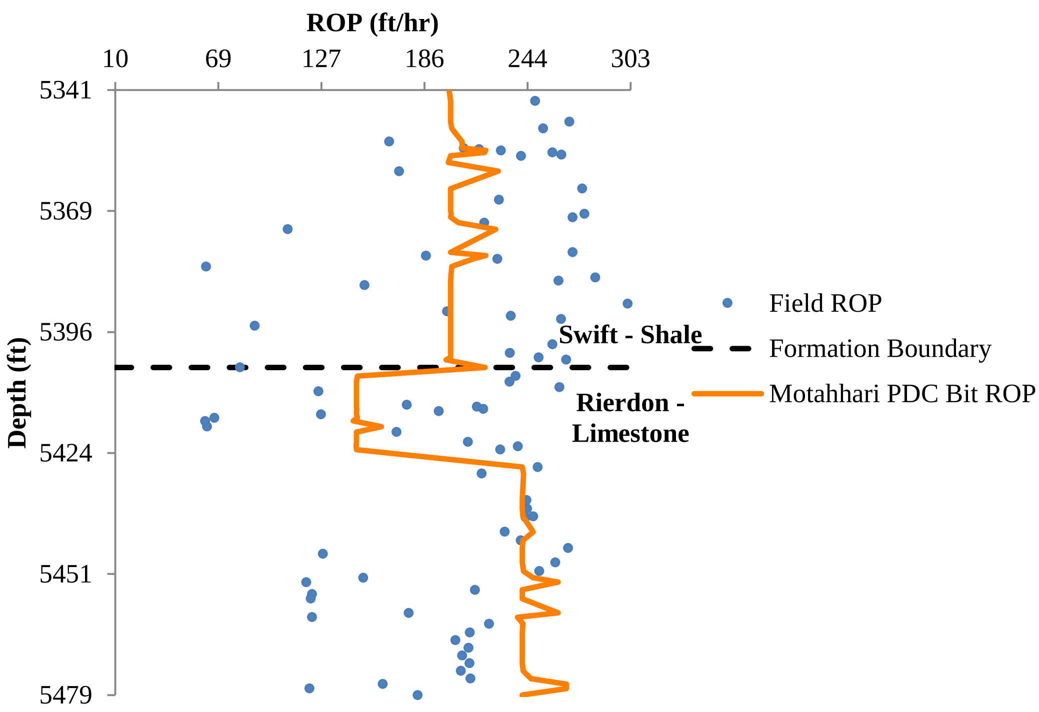

Figure 40: Motahhari ROP vs Field ROP Limestone - Shale Interface

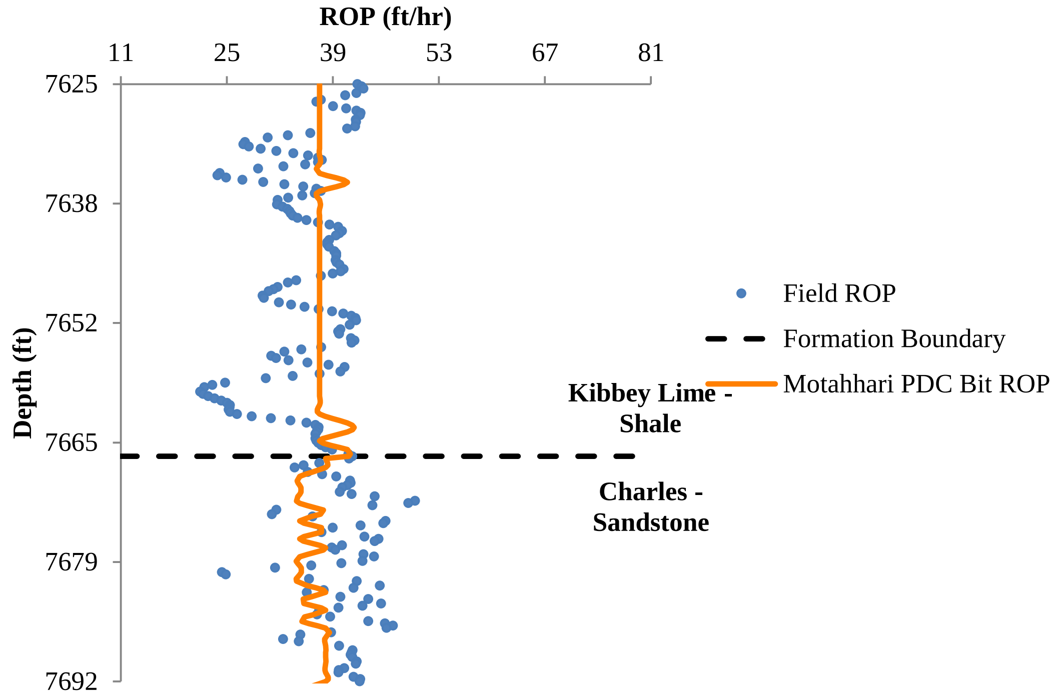

Figure 41: Motahhari ROP vs Field ROP Sh-SS Interface

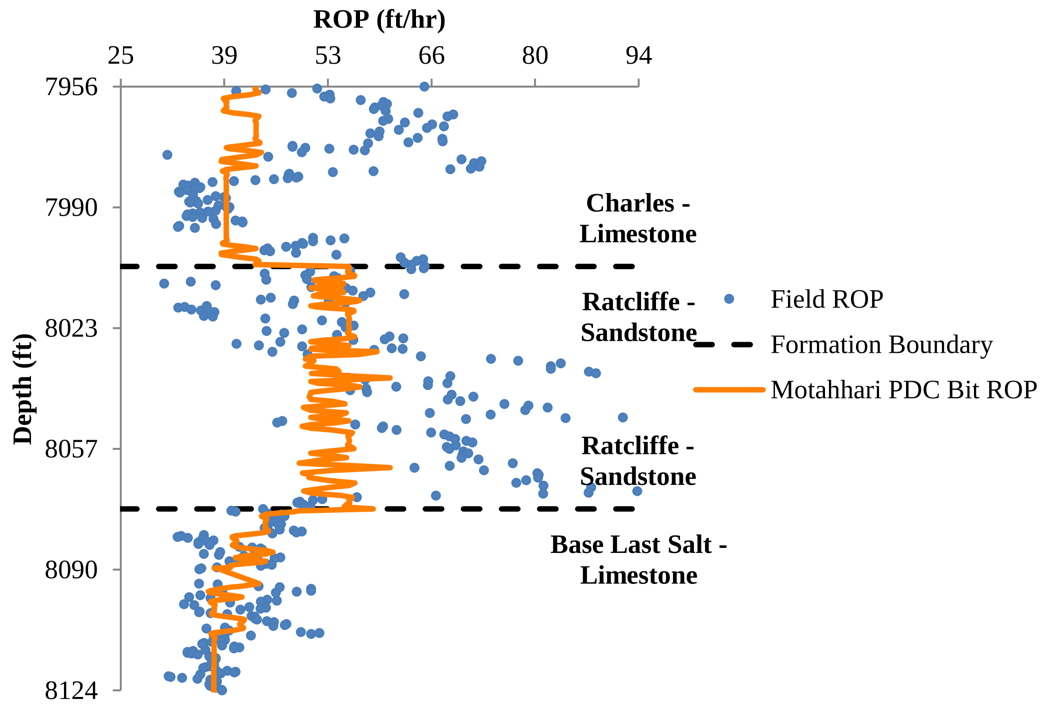

Figure 42: Motahhari ROP vs Field ROP Limestone - Sandstone Interface

4.2.7 OUTLOOK FOR ALL MODELS

As expected from running a minimum formation error analysis, the associated errors with each model have improved, and by roughly 5%. The percent errors for each analysis can be seen in tables 10 and 6. The changes, however, are not as large as one would expect at first. For a well with millions of data points, as opposed to a couple of thousand as is the Marathon well, the need for an analysis separated by formation will be imperative and thus the rest of the analyses in this paper were conducted in such a manner.

Table 10: Marathon Data Separated By Formation Evaluation

Average percent error of each ROP Model by formation:

Formation Name	Bingham	Bourgoyne & Young	Hareland Drag Bit	Hareland Roller Bit	Motahhari PDC Bit	WWO Roller Bit	Best Model in Formation
Greenhorn - Limestone	40.64%	38.20%	58.77%	37.75%	38.70%	37.65%	WWO Roller Bit
Newcastle - Sandstone	20.11%	18.64%	71.68%	21.69%	29.83%	14.93%	WWO Roller Bit
Dakota - Sandstone	24.08%	23.17%	75.19%	19.99%	20.71%	20.29%	Hareland Roller Bit
Swift - Shale	44.88%	44.78%	87.55%	35.87%	36.09%	45.19%	Hareland Roller Bit
Rierdon - Limestone	28.25%	26.97%	46.16%	24.48%	27.66%	25.94%	Hareland Roller Bit
Piper - Limestone	25.89%	25.57%	30.90%	25.53%	32.12%	23.08%	WWO Roller Bit
Spearfish - Sandstone	21.82%	22.75%	42.21%	17.72%	22.67%	19.70%	Hareland Roller Bit
Pine Salt - Sandstone	49.27%	47.51%	55.03%	48.54%	60.35%	48.83%	Borgoune & Young
Broom Creek - Sandstone	42.30%	41.91%	50.14%	37.74%	38.47%	40.38%	Hareland Roller Bit
Tyler - Sandstone	36.36%	33.63%	42.36%	32.74%	32.83%	32.72%	WWO Roller Bit
Kibbey Lime - Limestone	21.54%	19.85%	27.75%	16.72%	35.05%	15.44%	WWO Roller Bit
Kibbey Lime - Shale	15.88%	15.07%	21.55%	13.97%	14.33%	13.92%	WWO Roller Bit
Charles - Sandstone	37.26%	37.94%	45.68%	31.23%	31.19%	36.19%	Motahhari PDC Bit
Charles - Limestone	27.34%	27.19%	30.79%	25.25%	24.73%	25.52%	Motahhari PDC Bit
Ratcliffe - Sandstone	33.84%	32.16%	49.79%	21.59%	22.59%	21.50%	WWO Roller Bit
Base Last Salt - Limestone	11.83%	11.47%	16.47%	9.15%	8.41%	9.79%	Motahhari PDC Bit
Base Last Salt - Sandstone	13.49%	13.33%	17.85%	11.81%	11.98%	11.69%	WWO Roller Bit
Base Last Salt - Limestone	28.66%	25.51%	57.27%	16.05%	17.22%	20.29%	Hareland Roller Bit
Mission Canyon - Limestone	13.95%	16.34%	18.91%	11.95%	11.87%	12.90%	Motahhari PDC Bit
Lodgepole - Limestone	8.15%	7.60%	10.07%	7.40%	7.71%	6.84%	WWO Roller Bit
Entire Dataset	26.14%	25.86%	34.89%	23.37%	25.62%	24.08%	Hareland Roller Bit

Best Overall Model: Hareland Roller Bit

Average Percent Error: 23.37%

As was the case in the analysis separated by lithology, Hareland's roller bit model is the overall best model for the data set. However, again, due the model's limitation, it should not be used as a predictive tool and should only be used as an initial assessment of drilling efficiency. A better model to use is Motahhari's, which even though does not have as small errors as both roller bit models, behaves more naturally to variations in the data. Furthermore, the well was drilled with a PDC bit, and thus by applying the correct model choice for the bit being used, one can effectively improve drilling efficiency through analyzing how drilling parameter changes affect the calculated ROP and the field ROP.

Tables 11 through 13 illustrate the calculated model coefficients for each model.

Table 11: Marathon Data Separated By Formation All Model Coefficients Part 1 out of 3

Coefficients of each ROP Model by formation:

ROP Models	Bingham		B&Y					
Model Coefficients	a	b	a1	a2	a3	a4	a5	a6
Formations								
Greenhorn - Limestone	2.70474	0.5	5	0.0005	1.00E-06	1.00E-06	0.5136	0.7
Newcastle - Sandstone	3.56728	0.5	2.96737	0.0005	1.00E-06	1.00E-06	0.50466	0.70711
Dakota - Sandstone	4.46995	0.5	3.23276	0.0005	1.00E-06	1.00E-06	0.4591	0.74888
Swift - Shale	4.52667	0.5	3.34891	0.0005	1.00E-06	1.00E-06	0.4	0.81704
Rierdon - Limestone	2.66603	0.5	3.1608	0.0005	1.00E-06	1.00E-06	0.4	0.81704
Piper - Limestone	1.41577	0.5	2.71499	5.00E-04	1.00E-06	1.00E-06	0.4	0.8048
Spearfish - Sandstone	1.91098	0.5	3.12938	5.00E-04	1.00E-06	1.00E-06	0.4	0.80902
Pine Salt - Sandstone	0.45075	0.5	1.94263	0.0005	3.06E-05	1.00E-06	0.4	0.80314
Broom Creek - Sandstone	0.59402	0.5	2.33437	0.0005	3.06E-05	1.00E-06	0.4	0.81108
Tyler - Sandstone	0.6098	0.5	2.65123	0.0005	4.25E-05	1.00E-06	0.4	0.80986
Kibbey Lime - Limestone	0.40044	0.5	2.37E+00	5.00E-04	4.25E-05	1.00E-06	0.4	0.72599
Kibbey Lime - Shale	0.5474	0.5	2.63E+00	5.00E-04	4.36E-05	1.00E-06	0.4	0.72589
Charles - Sandstone	0.48111	0.5	2.48E+00	5.00E-04	4.36E-05	1.00E-06	0.4	0.72589
Charles - Limestone	0.49969	0.5	2.63E+00	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589
Ratcliffe - Sandstone	0.72658	0.5	3.05E+00	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589
Base Last Salt - Limestone	0.51899	0.5	2.76E+00	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589
Base Last Salt - Sandstone	0.47025	5.00E-01	2.68E+00	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589
Base Last Salt - Limestone	1.18345	0.5	3.61738	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589
Mission Canyon - Limestone	0.57019	0.5	3.10558	4.90E-04	1.00E-06	1.00E-06	0.4	0.71703
Lodgepole - Limestone	0.51556	0.5	3.23048	0.000486	1.00E-06	1.00E-06	0.4	0.71266
Sandstone Average	1.47564	0.5	2.71919	0.0005	2.64E-05	1.00E-06	0.4182	0.76297
Limestone Average	1.16387	0.5	3.17691	0.0005	1.98E-05	1.00E-06	0.41262	0.73946
Shale Average	2.53703	0.5	2.99049	0.0005	2.23E-05	1.00E-06	0.4	0.77146
Overall Average	1.44148	0.5	2.95229	0.0005	2.30E-05	1.00E-06	0.41387	0.75324

Table 12: Marathon Data Separated By Formation All Model Coefficients Part 1 out of 3

Coefficients of each ROP Model by formation:

		Har Drag			Har Roller			Motahhari
a7	a8	a	b	c	K	a	b	G
1	0.5	1000	0.0001	0.0001	0.95709	1.58366	0.00018	9.16352
1.02535	0.5	1000	0.0001	0.0001	1.62315	1.4912	0.00079	10
1.05563	0.5	1000	0.0001	0.0001	1.18367	1.71555	0.00035	5.26681
1.05563	0.5	1000	0.0001	0.0001	1.09641	1.70252	0.0001	5.75105
1.05563	0.5	1000	0.0001	0.0001	1.09848	1.6465	0.0001	5.18838
1.11546	0.5	7.37E+02	2.13E-03	0.00302	1.12953	1.48E+00	0.0001	10
1.08776	0.5	1.00E+03	1.00E-04	0.0001	1.32258	1.50E+00	0.0001	5.32849
1.10902	0.50095	247.54301	0.01438	0.01379	3.9253	0.84972	0.00282	10
1.07012	0.49944	261.00031	0.00234	0.00406	0.46786	1.51584	0.0001	10
1.07012	0.49944	261.00063	0.00049	0.0034	1.13731	1.27707	0.0001	5.16624
1.36324	0.49944	187.16674	0.0001	0.00011	18.2687	0.45046	0.0002	10
1.36324	0.49944	261.00039	0.00248	0.0043	1.11912	1.24147	0.00011	5.58312
1.36324	0.4934	215.5346	0.00607	0.01528	0.99983	1.25444	0.0001	5.15454
1.36324	0.4934	203.77839	0.00694	0.01325	0.91934	1.30954	0.00018	3.71657
1.36324	0.4934	261.00105	0.00058	0.00342	1.31182	1.29382	0.0001	5.22548
1.36324	0.4934	192.0441	0.00406	0.00801	0.84822	1.34928	0.0001	5.06022
1.36324	0.4934	170.05717	0.00259	0.00677	1.13014	1.32184	0.02706	4.58586
1.36324	0.4934	598.0222	0.00289	0.00324	1.21881	1.36969	0.00048	6.5583
1.35671	0.49259	242.20798	0.01205	0.0174	0.9462	1.36451	0.01043	5.23078
1.3567	0.49206	182.18328	0.00014	0.00013	1.10385	1.32325	0.01758	5.16266
1.16752	0.49778	490.68186	0.00297	0.00522	1.45574	1.35771	0.0035	6.74749
1.25972	0.49603	482.52107	0.00317	0.00504	2.94336	1.3199	0.00326	6.6756
1.20944	0.49972	630.5002	0.00129	0.0022	1.10776	1.47199	0.00011	5.66709
1.2132	0.49719	500.99134	0.00289	0.00484	2.09037	1.35212	0.00305	6.6071

Table 13: Marathon Data Separated By Formation All Model Coefficients Part 1 out of 3

Coefficients of each ROP Model by formation:

		WWO			
α	γ	a	φ	b	c
0.0001	3.6921	1.00E-04	1.00E-04	0.0001	0.00061
0.60835	3.3323	1.00E-04	1.00E-04	0.0876	0.00053
0.0001	4.30728	1.00E-04	1.00E-04	0.0001	0.00063
0.0001	4.25326	1.00E-04	1.00E-04	0.0001	0.00067
0.0001	4.19711	1.00E-04	1.00E-04	1	0.00039
9.91E-01	3.03104	1.00E-04	1.00E-04	0.0001	0.00158
1.00E-04	4.04146	6.18E-04	1.28E-04	0.41355	0.00099
1.64263	2.09774	1.00E-04	8.45E-03	0.0892	0.00238
0.0001	3.68532	1.00E-04	2.66E-04	1	0.00308
0.0001	3.8649	1.00E-04	1.00E-04	0.0001	0.00278
1.5502	2.11738	1.00E-04	1.00E-04	0.0001	0.00334
0.0001	3.80464	1.00E-04	1.00E-04	0.0001	0.00324
0.0001	3.80656	1.00E-04	1.00E-04	1	0.00366
0.0001	3.9277	1.00E-04	1.00E-04	1	0.0033
0.0001	3.90407	1.00E-04	1.00E-04	0.0001	0.00274
0.0001	3.85237	1.00E-04	1.00E-04	1	0.00307
0.03122	3.82899	2.88E-04	1.40E-04	0.1015	0.00391
0.0001	3.90959	1.00E-04	1.00E-04	1	0.00163
0.00266	3.86447	1.00E-04	1.00E-04	1	0.00287
0.0001	3.8478	1.00E-04	1.00E-04	1	0.00301
0.25364	3.65207	1.78E-04	1.05E-03	0.29913	0.0023
0.28276	3.6044	1.00E-04	1.00E-04	0.6667	0.0022
0.0001	4.02895	1.00E-04	1.00E-04	0.0001	0.00195
0.24139	3.6683	1.35E-04	5.29E-04	0.43463	0.00222

5. SENSITIVITY ANALYSIS ON DRAG BIT MODELS, ALSO INCLUDING TWO OTHER EXTENSIVE MODELS FOR COMPARISON

In this section, a sensitivity analysis' results and discussion for drag bits is presented. In order to further the discussion on drag bits and drag bits modeling, two other complex models were included, the Winters, Warren, and Onyia model which is for roller cone bits and the Bourgoyne & Young model which is non-bit specific. The two drag bit models are Hareland's and Motahhari's drag bit models. The reason that non drag bit models were used is to assess if drag bit models will create better results than roller cone bit models when used for a data set drilled with a drag bit, as is expected in theory, or if a roller cone bit model or a non-bit specific model could as easily be used instead. If a roller cone bit model were to give better results, there would be a strong case to show that the models are incorrectly depicting ROP. If a non-bit specific gives better results than the other models, then a general model with a wider range of applicability could always be used instead, thus simplifying the number of models one uses in the well design and the operational process.

For the sensitivity analysis, the trends for WOB, RPM, bit diameter and various parameters versus ROP were assumed to follow the relationships presented in Maurer's 1962 paper on perfect hole cleaning. That is the case because he is one of the few papers providing laboratory experiment data and reasoning as to why it should be that way.

As such, figures 43 and 44 are excerpted from Maurer's paper and shown below as expected basis for the relationships between ROP and RPM and ROP and WOB, two of the most critical relationships for accurately determining ROP. It is important to note however, that although the trends Maurer's paper suggests are quite near what one expects when drilling a well, these trends have their limitations, as an incremental change to an exceedingly large rotary speed or weight on bit will cause a decrease in ROP. Despite these limitations, Maurer's paper is a great indicator of how RPM and WOB affect ROP for most drilling ventures.

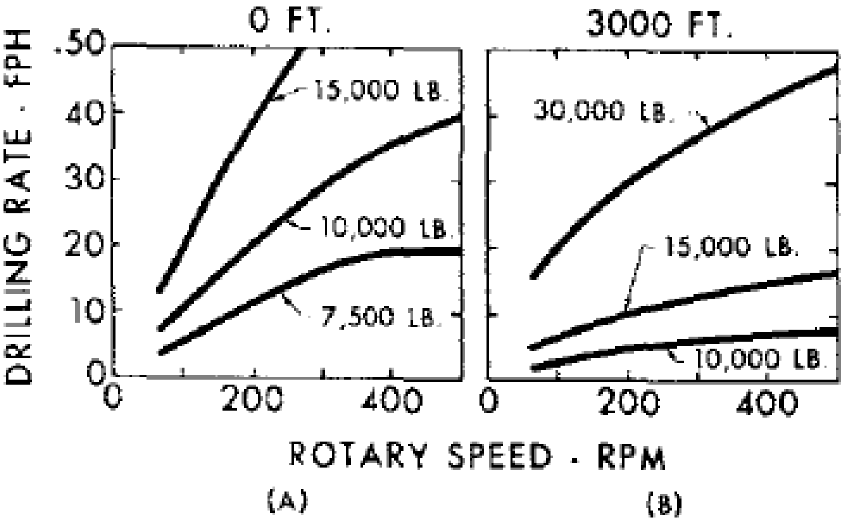

Figure 43: RPM vs ROP relationship from Maurer (1962)

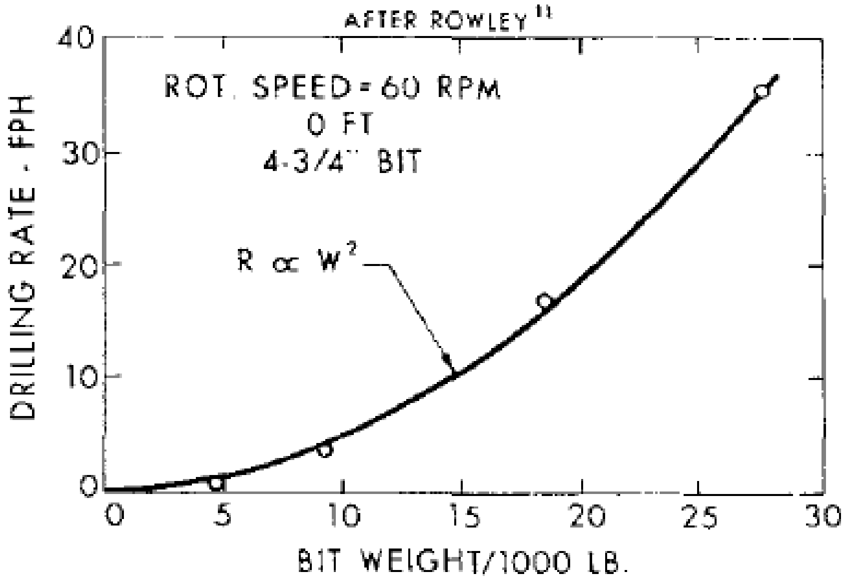

Figure 44: WOB vs ROP relationship from Maurer (1962)

This section will provide plots for all the model parameters and coefficients and discuss the trends expected by the models and the ones expected in a real field as to determine if the methods are correctly predicting the physical effects varying the parameters should have in the predictions of ROP.

As notes, all figures in this section have ROP units in feet per hour and all figures show the model authors first followed by the particular parameter being plotted against ROP. In the special case of Warren, Winters, and Onyia's plots, Warren, Winters, and Onyia have been abbreviated to WW&O. For all the plots the data points are reasonable ranges of values for the various parameters in each of the models. However, for the WOB vs ROP, RPM vs ROP, and bit diameter vs ROP plots, the data was constrained to the first formation in a data set from Marathon, a limestone, in order to study how different model coefficient values affect model comparisons.

5.1 Parameters Common to All Models

In this section, the parameters common to all four models are illustrated. These parameters are, as expected, weight-on-bit, rotations per minute, and bit diameter. Although one expects these parameters to be well defined and understood, seeing that rate of penetration is largely influenced by changes in them, it is interesting to see that not all models agree on how the relationships between these parameters and ROP should be. The probable reason for these differences is that all models were designed based off empirical relationships from the field test data collected. Older models, like Winters, Warren, and Onyia's and Hareland's may have created a relationship based off the entire data curve achieved from the field tests. Motahhari's model though, aware that after a certain WOB, ROP would decrease, may have chosen to only calculate for values within normal drilling situations. As such, the models disagree on how quickly ROP will stop increasing due to an increase in WOB.

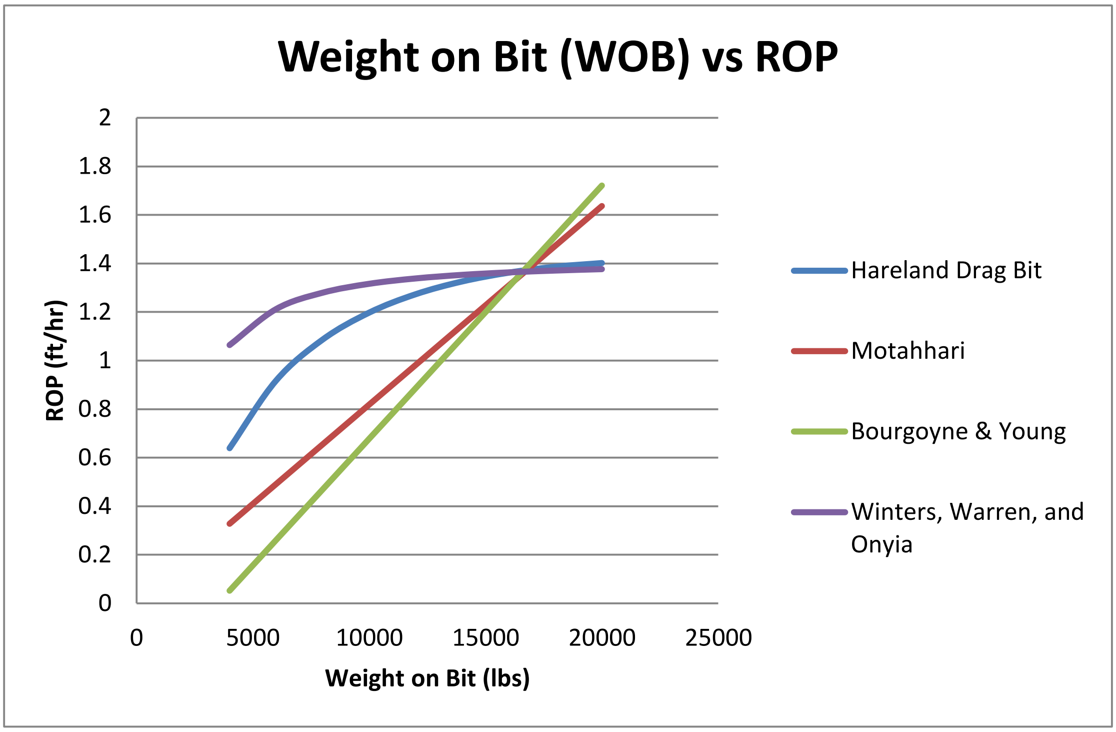

Figure 45: WOB (lbs) vs ROP relationship for drag bit models, Winters, Warren, and Onyia and Bourgoyne & Young

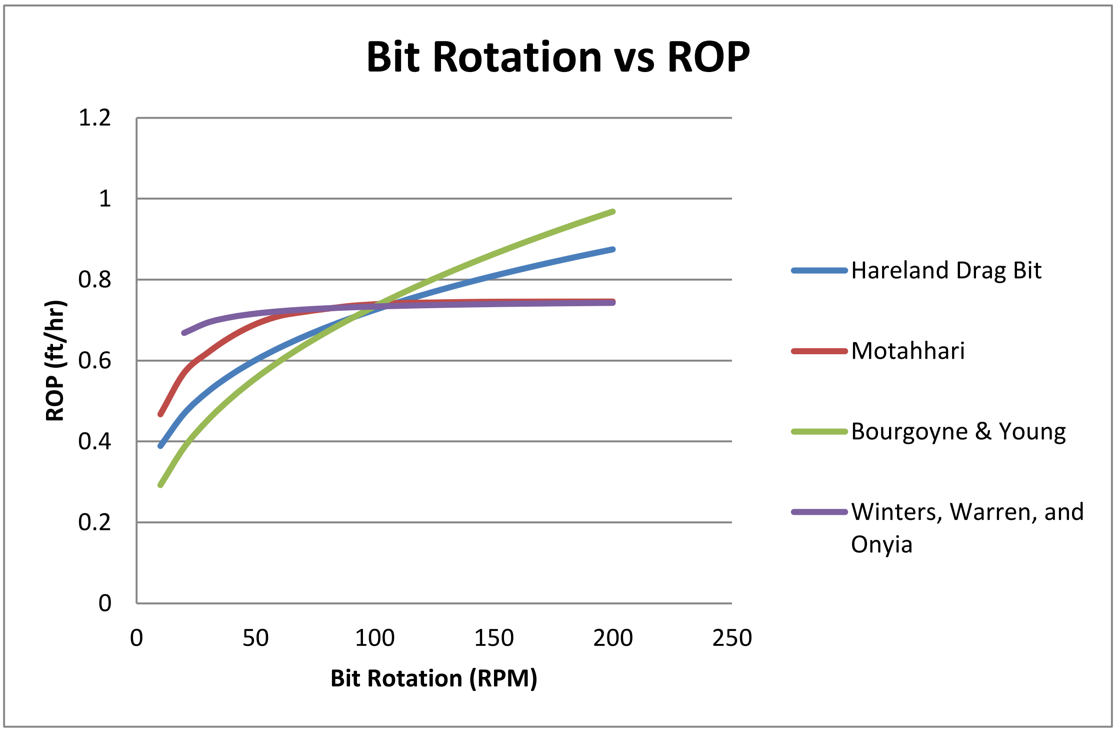

Figure 46: RPM vs ROP relationship for drag bit models Winters, Warren, and Onyia and Bourgoyne & Young

Figure 45 exemplifies one of these discrepancies amongst the models. In the figure, it is easy to notice that the linear trend followed by the Bourgoyne & Young model cannot be accurately depicting how weight on bit affects ROP. However, Bourgoyne & Young's equations were developed more freely, and thus, by tinkering with the coefficient affecting weight on bit, one can still achieve meaningful results. However, one has to limit the range of values the coefficients can be, or too much weight will be placed in a single parameter in the equation. As for Winters, Warren, and Onyia's and Hareland's models which do not appear to follow the same function shown in Maurer's paper, the equations in the plots do suggest the power relationships to ROP as suggested in Maurer's paper. And although difficult to perceive, Motahhari's model matches with Maurer's theoretical relationship.

It is important to note that the reason for large discrepancies in ROP values is due to an entire formation section being analyzed in the creation of figure 46. As mentioned earlier, this was primarily done to further confirm the analysis presented in chapter 3. This graphing mechanism also allows for assessing the sensibility of the two drag bit models when inside the same formation.

Figures 46 and 47 show, respectively, the relationships between RPM and ROP, and between bit diameter and ROP. In both figures, all of the models agree with Maurer's theoretical relationship between the parameters and ROP. For that reason, it is concluded that the discrepancies between the models can only be caused by the unique parameters in them and the model coefficients. Theoretically though, the model coefficients in both drag bit models should be fundamentally the same. In Hareland's model, the a coefficient, a cutter geometry factor, is a multiplication factor for the entire ROP equation, much like the G coefficient in Motahhari's model, a coefficient determined by bit geometry, cutter size and design, and cutter-rock coefficient of friction. Likewise, the b and c coefficients in Hareland's model affect RPM and weight on bit whereas the α and y coefficients in Motahhari's model affect weight on bit and RPM, respectively. Thus, for a bit and formation where the values of the model coefficients are already known, the two models should exactly agree on how small changes affect both models. But given that they do not, the discrepancies can only be caused by the unique parameters. The reason for the existence of unique parameters is that the two papers took different approaches in determining ROP. Hareland's model calculates an effective ROP for each cutter as it rotates about the bit and then sums up those rates in order to find the total ROP, using uniaxial compressive strength as a force against drilling while Motahhari's model calculates ROP based off an entire bit approach, using confined compressive strength as a parameter against drilling. Thus, the main difference between the two models is how rock strength affects drilling. A recommendation is that, although Hareland's model is a better visualization of what each cutter does as it rotates, the entire movement of the bit is controlled by the combined forces around the bit, and a section may hold or slow a cutter down, thus slowing down or stopping all other cutters. For that reason, confined compressive strength is the parameter to use.

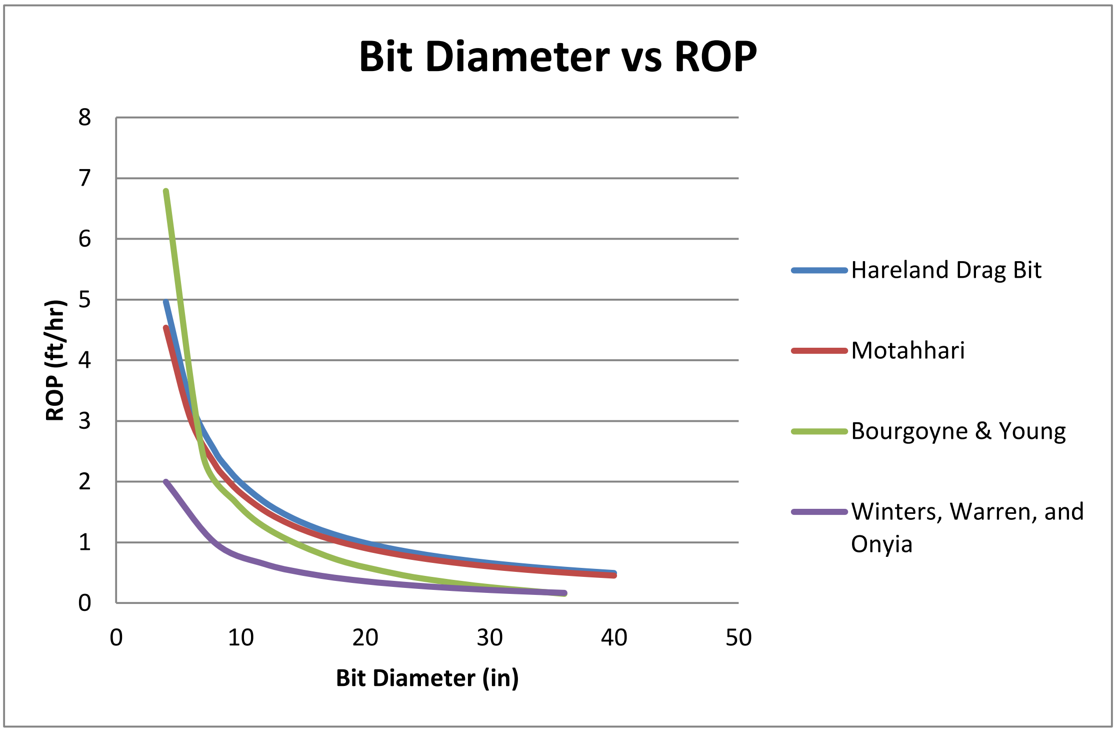

Figure 47: Bit Diameter (in) vs ROP for drag bit models, Winters, Warren, and Onyia and Bourgoyne & Young

5.2 Parameters Common to at Least Two Models

In this section, parameters common to at least two models are illustrated and discussed. These parameters are those that have large influences in ROP, but the relationships to ROP are not well defined mathematically. The parameters are bit tooth wear, jet impact force, compressive strength, and mud density.

Bit tooth wear has been included in both Bourgoyne & Young's and Motahhari's models. Bit wear affects ROP by decreasing the rate of penetration as the wear to the bit teeth increases. However, it is yet to be completely understood how the wear mathematically affects ROP. The amount of bit wear is established after the bit has been pulled from the well. Thus, correctly incorporating bit wear to a real-time ROP prediction model seems currently impossible, as one would have to have a way of assessing tooth wear in real time and a correct measurement of how wear affects ROP. To make matters worse, there are a myriad of ways in which a bit can be dulled, such as broken cutter, or teeth, balled up, cored, chipped teeth / cutters, delaminated cutter, erosion, heat checking, junk damage, lost teeth/cutters, plugged nozzle, ring out, spalled cutter, wash out, or worn teeth. Figure 48 below shows how the two models that include it in their design has decided on relating bit tooth wear to ROP. Both models established bit tooth wear as a multiplier to the total ROP and reducing it proportionally to the wear. Although the models appear to be inverses of each other, the Bourgoyne & Young plot is of an increase of bit wear to ROP while Motahhari's is of an increase of bit integrity to ROP.

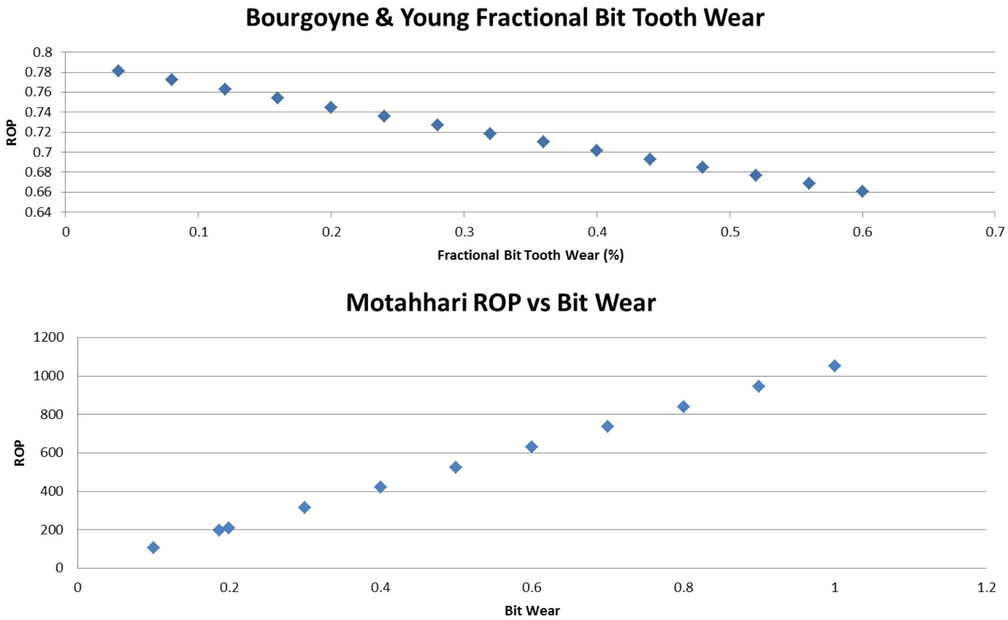

Figure 48: Bit Wear vs ROP relationship for Motahhari and Bourgoyne & Young

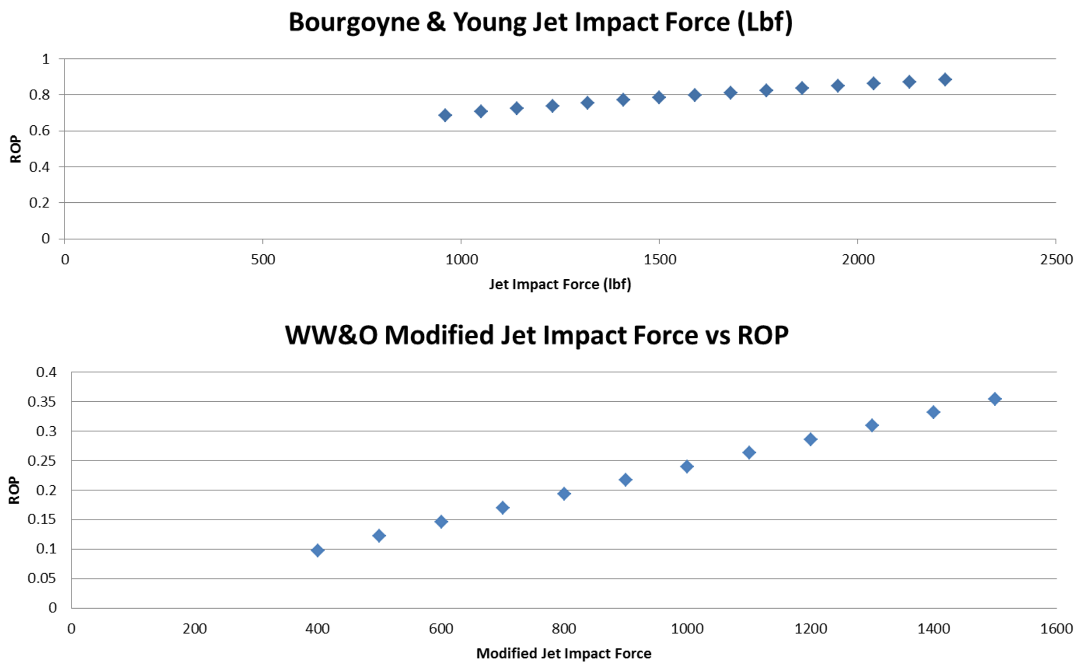

Figure 49: Bourgoyne & Young and Winters, Warren, and Onyia Jet Impact Force (lbf) vs ROP

Although maximizing jet impact force is one the main components of drilling optimization and bit selection, it is interesting to note that both drag bit models chose not to include the term into their model. Both Winters, Warren, and Onyia's and Bourgoyne & Young's model did however. As dictated by drilling optimization theory, an increase or maximization of jet impact force, should yield an increase to rate of penetration. Figure 49 above demonstrates that both models agree on this relationship, however Winters, Warren, and Onyia's model predicts an almost linear increase to ROP with an increase to jet impact force while Bourgoyne & Young's model predicts a more well-defined power function.

Another parameter not directly included in both drag bit models but discussed by the other two models is mud density. Mud density is only an indirect effect to ROP, based on the differential pressure caused by the difference between equivalent mud density and the pore pressure, however, even differential pressure was not included in either drag bit model. In order for the models to be improved for underbalanced drilling operations and managed pressure drilling, a differential pressure effect should be included. Figure 50 shows the relationship between mud density and ROP for the Winters, Warren, and Onyia's and Bourgoyne & Young's models.

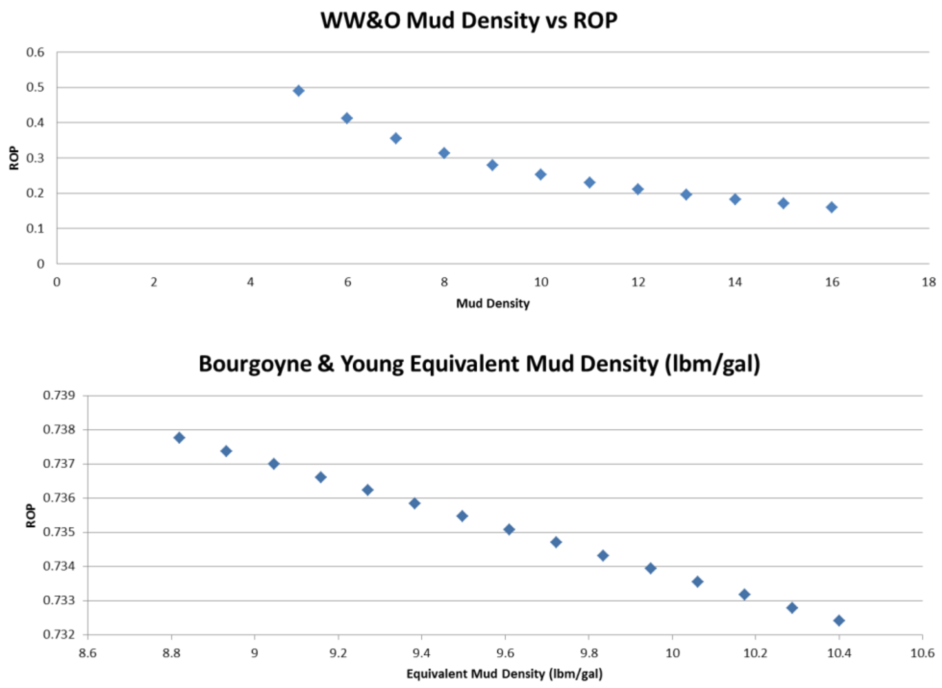

Figure 50: Bourgoyne & Young and Winters, Warren, and Onyia Mud Density (lbm/gal) vs ROP

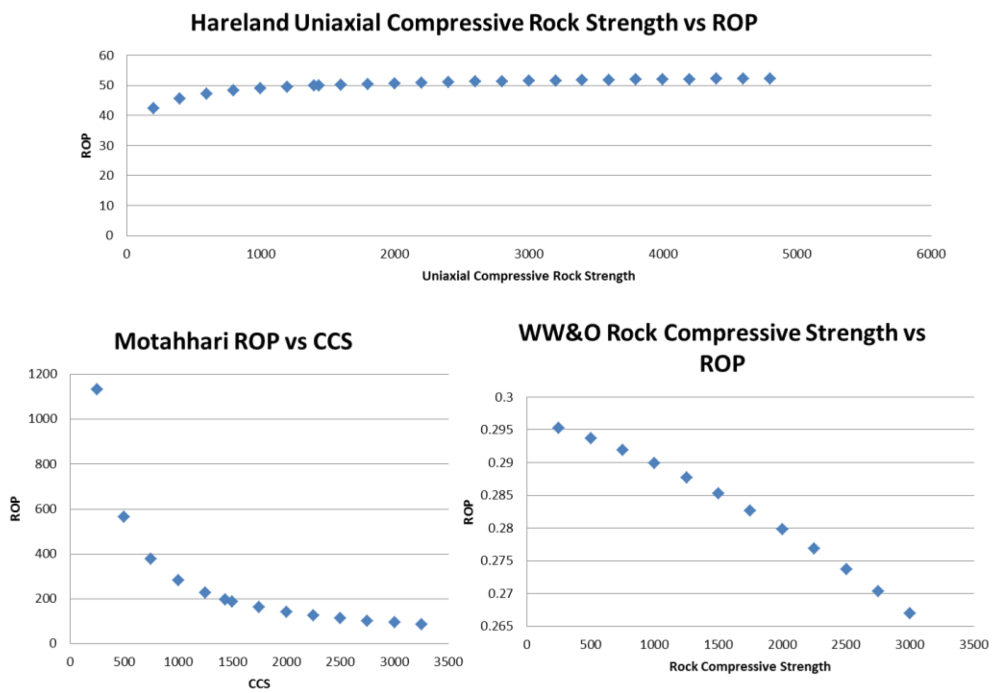

Figure 51: Compressive Strength (psi) vs ROP for Winters, Warren, and Onyia, Bourgoyne & Young, and Hareland

Compressive strength has been included in both drag bit models as well as Winters, Warren, and Onyia's. In all three models, different quantifications of compressive strength were used; Hareland's model implemented uniaxial compressive strength, Motahhari's confined compressive strength, and Winters, Warren, and Onyia's triaxial compressive strength. Curiously, by using an uniaxial compressive strength, an increase in the rock's compressive strength yields a slightly higher ROP, but as observed in field data, and expected by theory, a larger compressive strength, be it confined or unconfined, will make it harder to drill, decreasing ROP. Figure 51 highlights the differences in the effects of compressive strength to ROP based on their choice of which compressive strength to incorporate to the model.

5.3 Parameters Unique to a Model

In this section, the parameters unique to each model are presented. These parameters are for the most part, those that affect ROP indirectly, however Hareland's model includes bit geometry parameters that directly affect ROP.

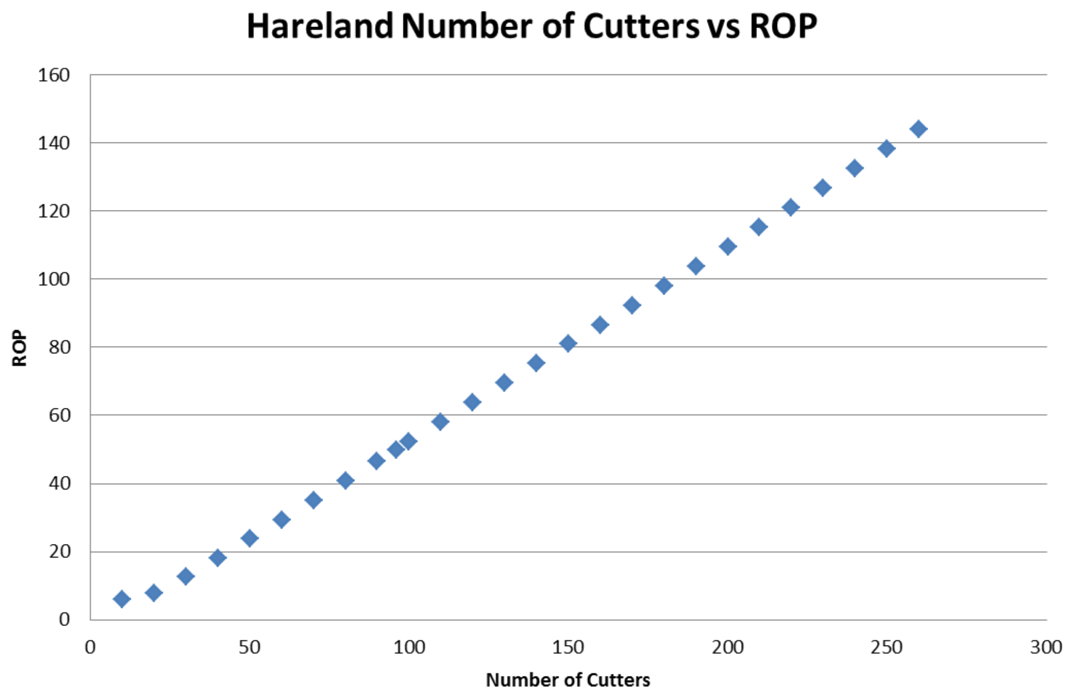

Figure 52: Hareland Number of Cutters vs ROP

Figures 52, 53, and 54 illustrate these bit specific effects to ROP. As expected, an increase to the number of cutters increases ROP, as the surface area contact for the cutters would increase proportionally. Likewise, increasing the diameter of a cutter would have the same effect, except the increase would be larger, due to it following an area function. The last term included is weight on bit per diamond cutter. Weight on bit per diamond cuter follows a sinusoidal relationship to ROP, as seen by the figure.

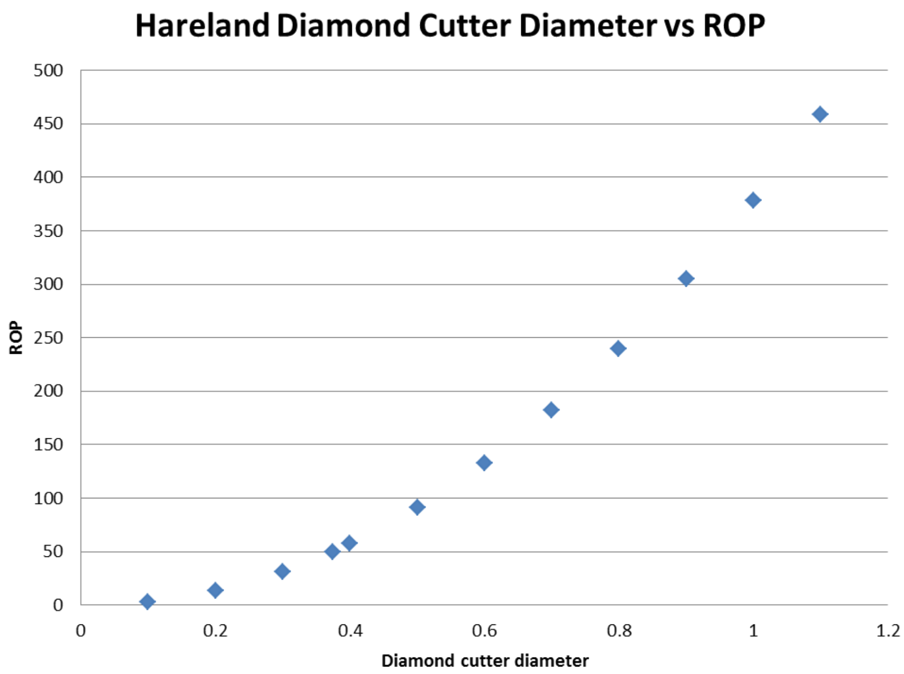

Figure 53: Hareland Diamond Cutter Diameter (in) vs ROP

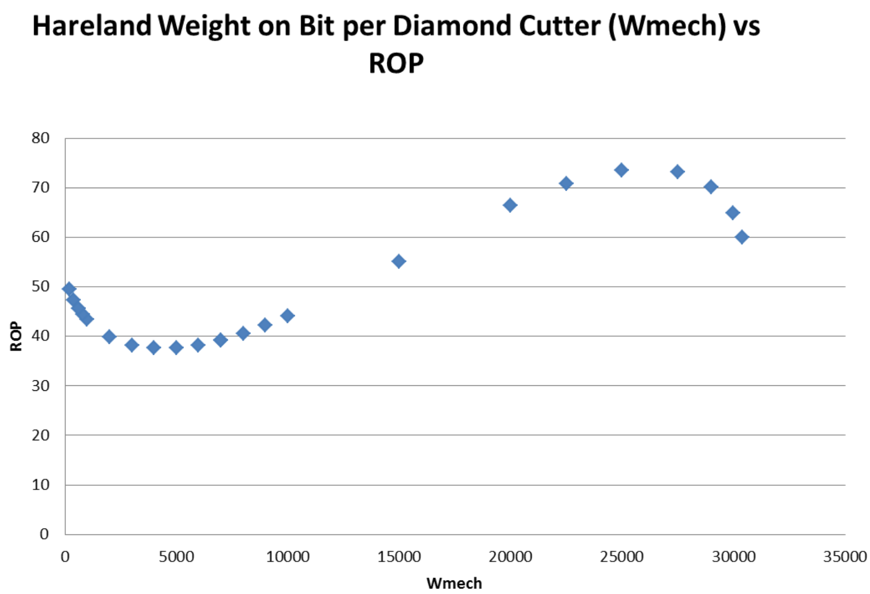

Figure 54: Hareland Weight On Bit Per Diamond Cuter (Wmech) in lbs vs ROP

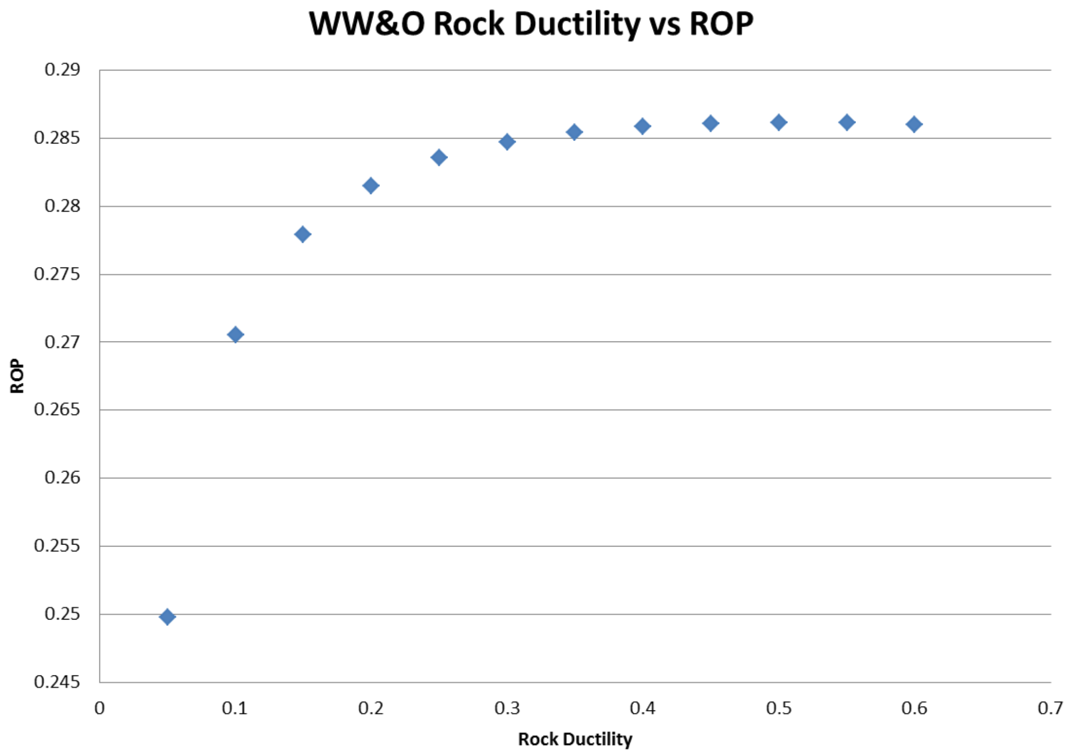

Figure 55: Winters, Warren, and Onyia Rock Ductility vs ROP

Rock ductility is the only parameter analyzed in this paper in which a plot of values of the parameter versus rate of penetration does not yield a very strong correlation. As mentioned in Winters, Warren, and Onyia's paper, an increase in rock ductility should account for a decrease in ROP. However, from figure 55 above, one notices that an increase in rock ductility can cause an increase in ROP. Thus, the simple relationship stated in their paper is not what is described by their model. This is because on the model, Winters, Warren, and Onyia included rock ductility in two terms of the ROP equation. In the bit indentation term, an increase in rock ductility would be associated with a decrease in ROP and in the offset term, an increase in rock ductility would cause an increase in ROP. As such, rock ductility may have different results in affecting ROP based off of how the bit intersects the rock.

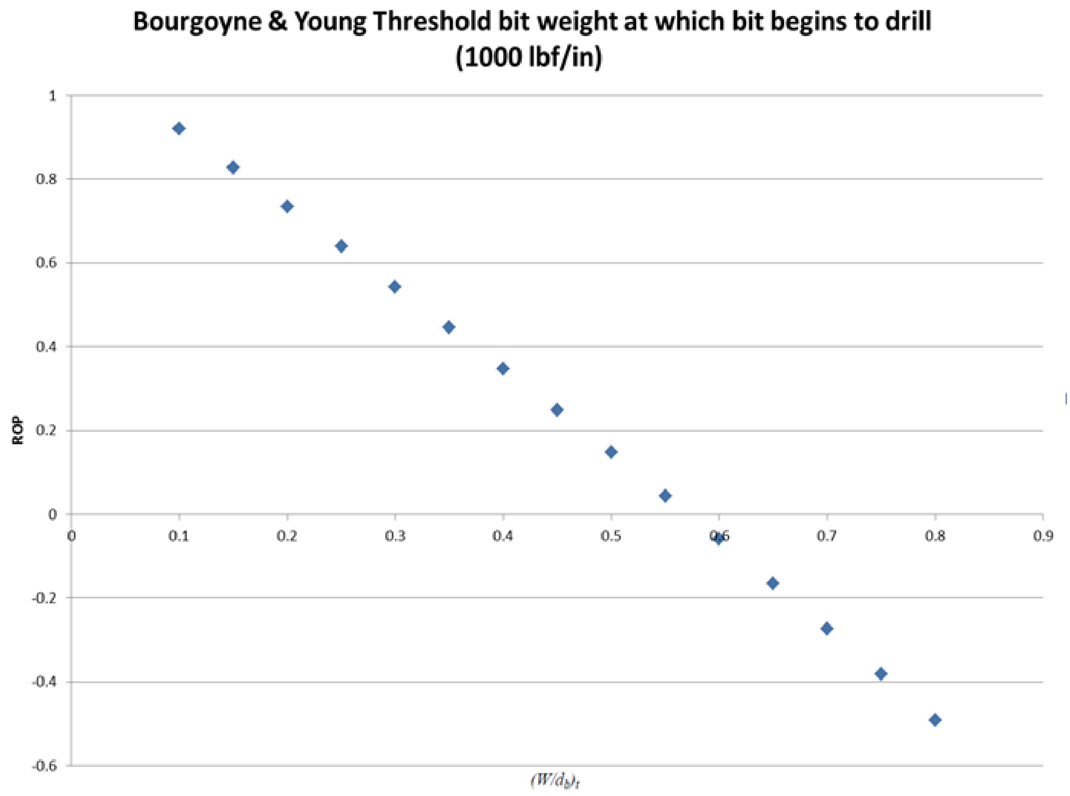

Figure 56: Bourgoyne & Young Threshold bit weight at which bit begins to drill vs ROP

Figure 56 shows another interesting plot; figure 56 is the plot of threshold bit weight at which bit begins to drill versus ROP. The threshold bit weight is a yield point at which surpassing this percentage of weight on bit, the bit will begin drilling. The important point in the plot occurs where a large enough percentage makes it impossible for drilling to begin. The value of the intercept changes based on various other parameters, and thus, will change from well to well, as shown in section 6.1.

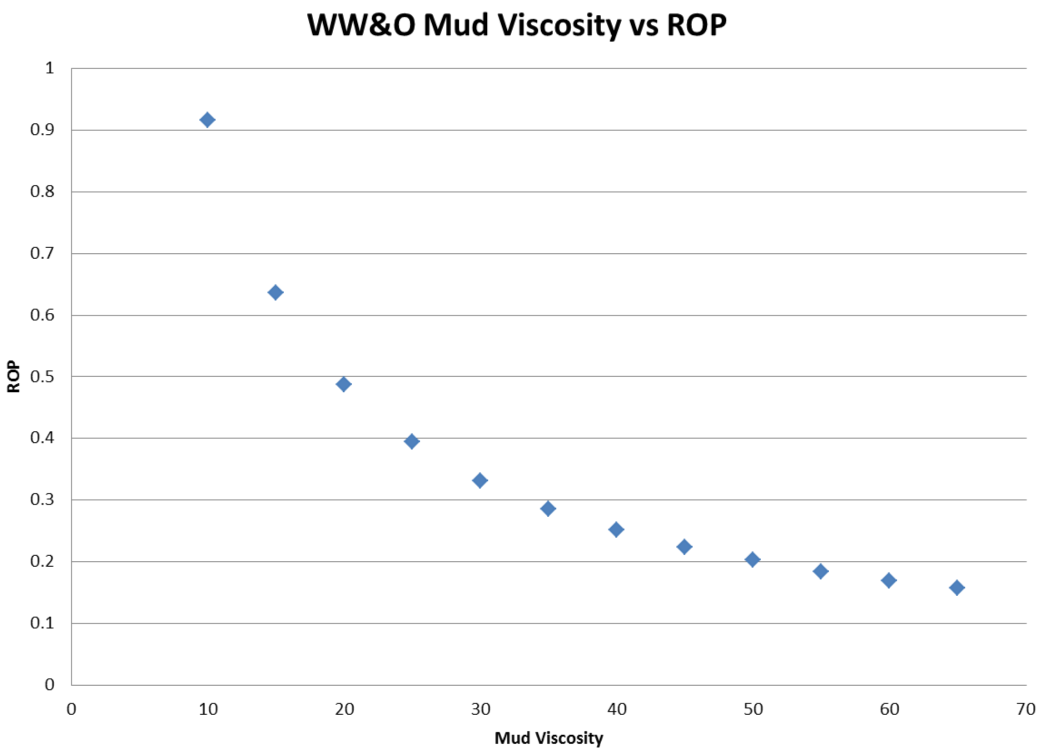

Figure 57: Winters, Warren, and Onyia Mud Viscosity (cP) vs ROP

Mud viscosity, pore pressure gradient, and depth, the parameters discussed in figures 57, 58, and 59, respectively, are all straightforward and their plots correlate their parameters to changes in ROP well. Although, pore pressure gradient and ROP and depth and ROP do not have linear relationships, so a more precise determination is necessary. Depth cannot be correctly quantified, as depth is indirectly related to ROP and there will be too many changes from well to well, but plotting pore pressure gradients and mud gradients for various wells could help develop an analysis for a mathematical computation of their relationship to ROP.

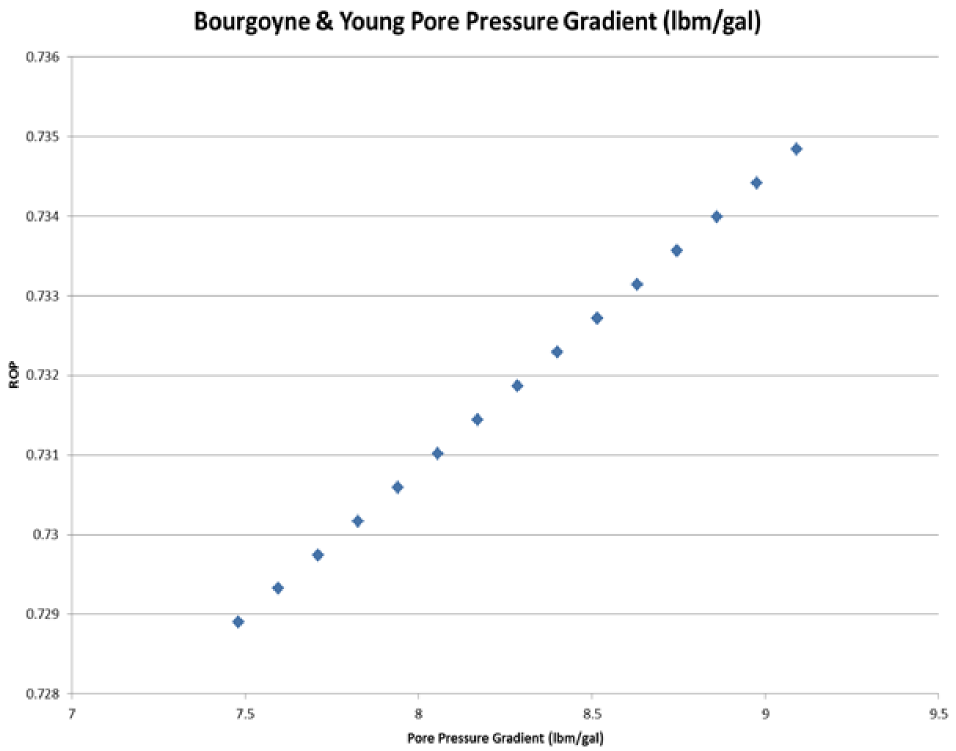

Figure 58: Bourgoyne & Young Pore Pressure Gradient (lbm/gal) vs ROP

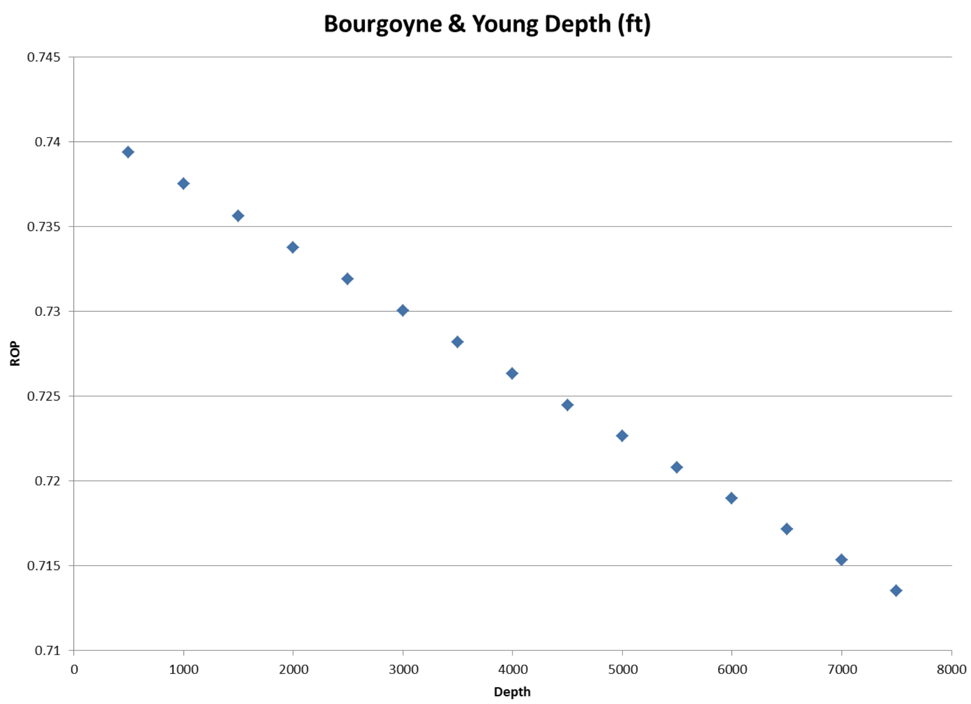

Figure 59: Bourgoyne & Young Depth (ft) vs ROP

5.4 Model Coefficients

In this section, each model's coefficients are discussed and their sensitivity analyses presented.

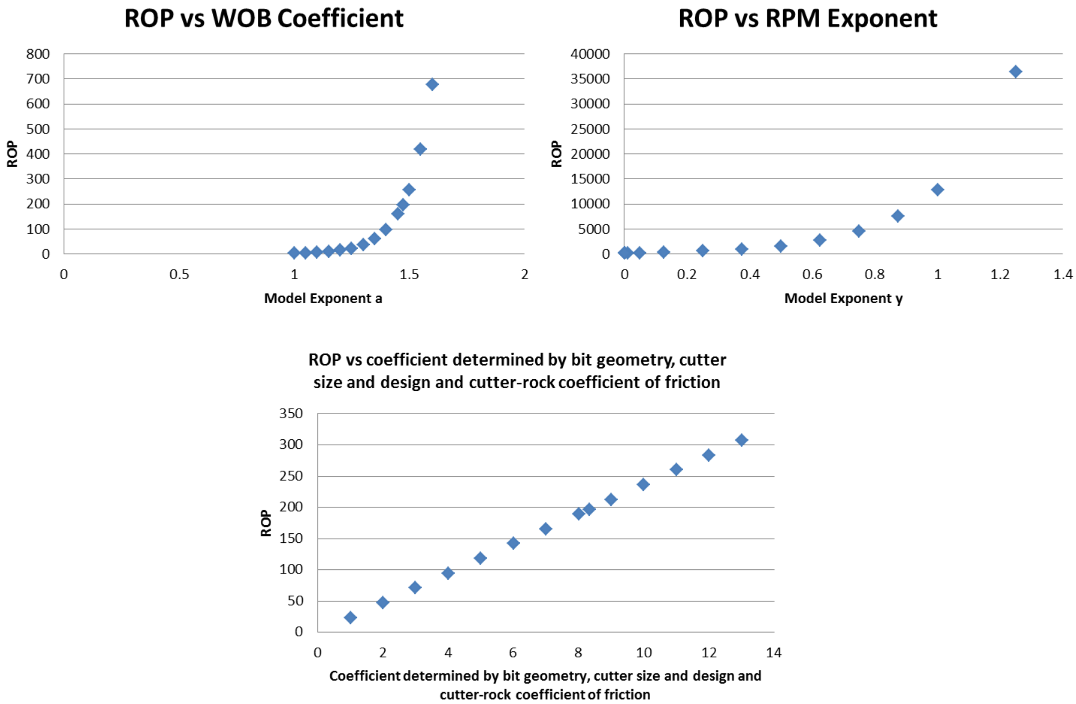

Figure 60: Motahhari Coefficients vs ROP

Motahhari's model coefficients are a weight on bit coefficient, a RPM exponent, and a coefficient determined by bit geometry, cutter size and design and cutter-rock coefficient of friction. The latter coefficient multiplies the overall calculation of ROP by a correction factor for the specific bit. Theoretically, this value should be constant amongst formations if the same bit was used. The weight on bit coefficient determines how sensitive the model is to variations in weight on bit. A value of one would mean that weight on bit has a linear relationship to weight on bit. A value of two would mean quadratic and so forth. A normal range of values for the coefficient is anywhere between 0.2 and 2, however, from figure 60 one can perceive that a small change in the coefficient could cause ROP to double or more if the coefficient was larger than 1.5. The RPM exponent adjusts the relationship between RPM and ROP. Much like the weigh on bit coefficient, a value of one for the RPM coefficient would mean that RPM has a linear relationship to ROP. A value of two would mean quadratic and so forth. A normal range of values for the coefficient could be anywhere between 0.1 and 4, however, from figure 60 one can perceive that a small change in the RPM coefficient could cause ROP to increase many times over itself if the coefficient was increased a small amount. For the reasons above, one should exert caution when tinkering with the coefficients in the Motahhari model or a single parameter in the equation could hold all the weight in future predictions.

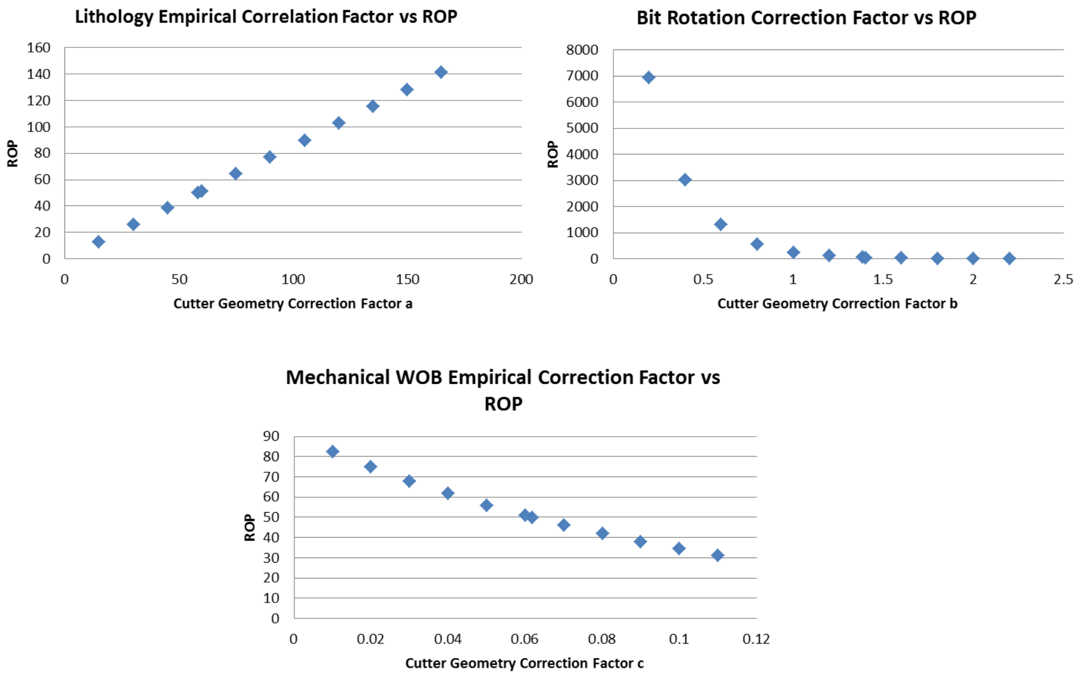

Figure 61: Hareland Coefficients vs ROP

Hareland's drag bit model coefficients are an overall lithology empirical correlation factor, a bit rotation correlation factor, and a mechanical weight on bit empirical correction factor. All three coefficients should not vary much if a study is conducted for a particular zone in a particular, field using the same kind of drag bit. For that reason, the analysis using Hareland's drag bit model should demonstrate the model's accuracy and predictability; the analysis is presented in section 5.5. Similarly to Motahhari's model, too large or low values of one of the coefficients in Hareland's drag bit model, the bit rotation correction factor, can have huge effects on ROP prediction and determination and can end up putting too much weight, or very little weight, on the RPM parameter, as seen in figure 61.

Winters, Warren, and Onyia's model coefficients are: an indentation coefficient, a teeth coefficient, a cone offset coefficient, and a hydraulics coefficient, illustrated by figure 62. All four coefficients are relative to a certain bit, for that reason, similarly to Hareland's drag bit model, an analysis on the Eagle Ford field should help demonstrate the model's accuracy and predictability. The analysis is presented in section 5.5. The hydraulics coefficient should be kept under practical limits for it not to overshadow other parameters in the model.

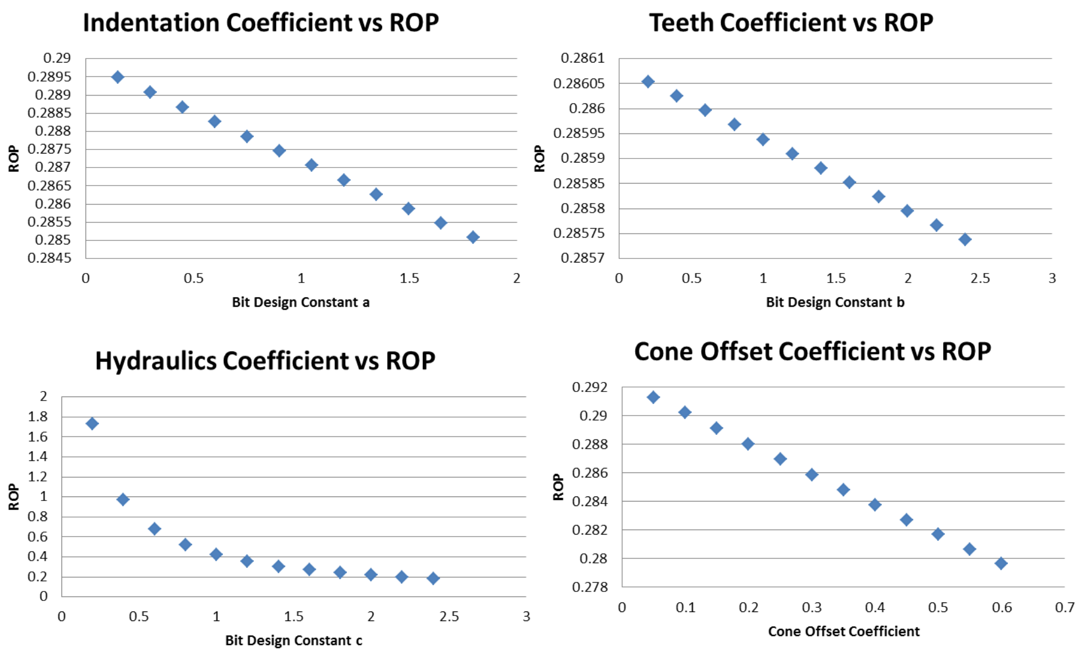

Figure 62: Winters, Warren, and Onyia Coefficients vs ROP

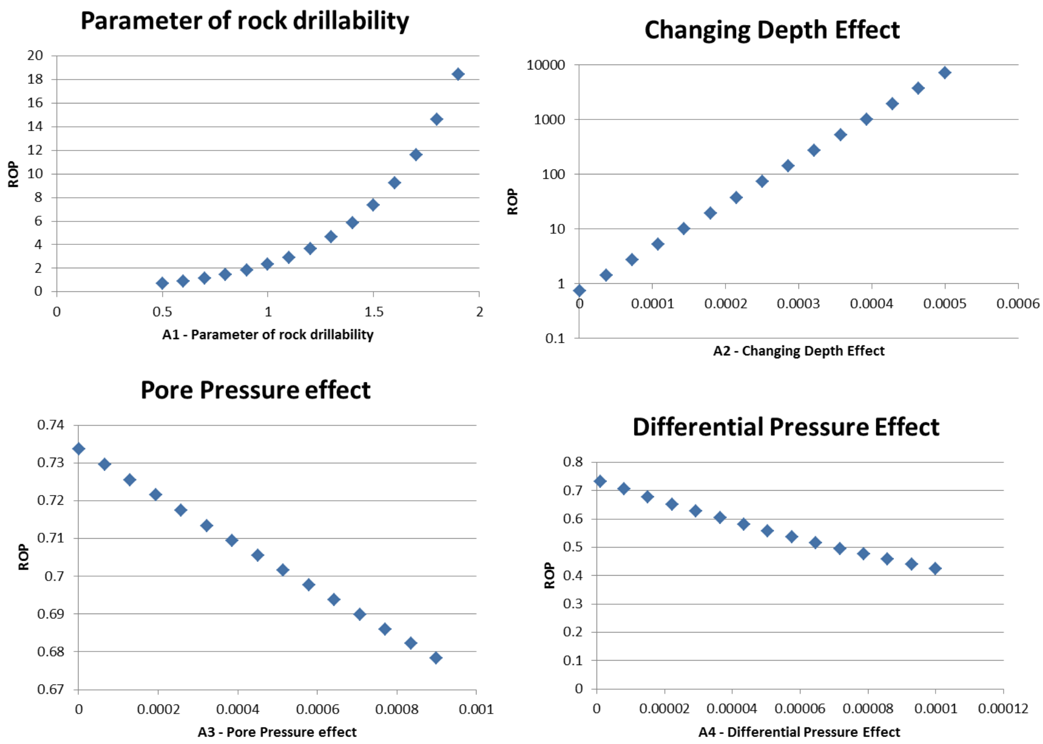

Figure 63: Bourgoyne & Young Coefficients a1 through a4 vs ROP

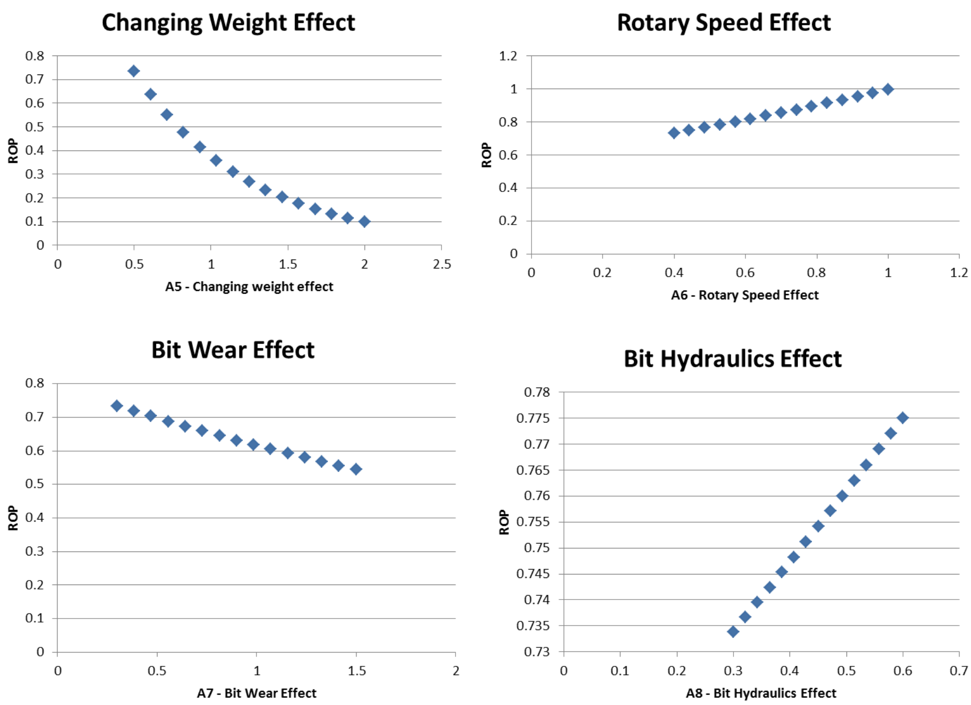

Figure 64: Bourgoyne & Young Coefficients a5 through a8 vs ROP

Bourgoyne & Young's model coefficients are: a rock drillability coefficient, a changing depth effect, a pore pressure effect, a differential pressure effect, a changing weight effect, a rotary speed effect, a bit wear effect, and a bit hydraulics effect. Bourgoyne & Young's model is the most computationally heavy of all the models discussed, as all eight coefficients have to be calculated, however, it is overly simplifying relationships between parameters and ROP. For that reason, the coefficients and their ranges are the primary means of the model accounting for the physical meanings and relationships of its parameters and ROP. A positive to the model limitations, however, is that Bourgoyne & Young provided alongside their paper practical limits for each coefficient, thus allowing parameters to maintain physical significance. These practical limits are provided in chapter 3 and their relationships to ROP are shown in figures 63 and 64.

5.5 Drag Bit Models Comparison And Analysis

The following analyses were done on a section of a vertical well in North Dakota and on a set of horizontal wells in the Eagle Ford shale. For all the analyses, the models were implemented into Excel and their model coefficients and correlating parameters mathematically determined using ROP Plotter. A minimum error regression was run using Hareland's drag bit, Motahhari's PDC bit, Winters, Warren, and Onyia's roller bit and Bourgoyne & Young's models in order to assess the applicability of running specific bit models, and to investigate why a drag bit model may deviate from another in its results. All models had their regressions separated by formation. By comparing the actual penetration rates given in the field data with predicted values calculated from the models, each models' sensitivity and accuracy were calculated.

5.5.1 Marathon Well

This section analyzes the vertical well drilled using a PDC bit in North Dakota. All ROP field values can be seen in figure 65. Also in the figure are the full view calculated values of the drag bit models, and the two other extensive models. From the figure, it is seen that from about 4257 to 5257 feet, Hareland's drag bit model is heavily underestimating ROP and at 4756 ft, both drag bit models appear to be underestimating ROP, as opposed to the other models. However, at 4755 and 4756ft, the recorded rates of penetration are 175 and 118 ft/hr which correspond to much lower values, thus the models are trying to account for these sharp changes, whereas the non-drag bit models are highly unfazed by them. The main reason for the non-drag bit models being unchanging is that the coefficients in those models do not affect the primary parameters, weight on bit, bit diameter, bit rotation, as much as the coefficients in the drag bit models do. As such, whenever a sudden change occurs downhole, the non-drag bit models only change ROP a fraction of what the actual change was.

Another important section to mention is the one between 6500 ft and 6750 ft where all the models underpredict the data. The reason for that is because that small part belongs to a large sandstone formation starting at 6500 ft and ending at 7000 ft and, due to that large thickness, the top part of that sandstone has different properties than the deepest sections. But since the models are interpreting that as a homogeneous formation, in order to minimize the error, all models placed heavier weights on establishing coefficients for the deeper sections which would in part lead to better results for the formation.

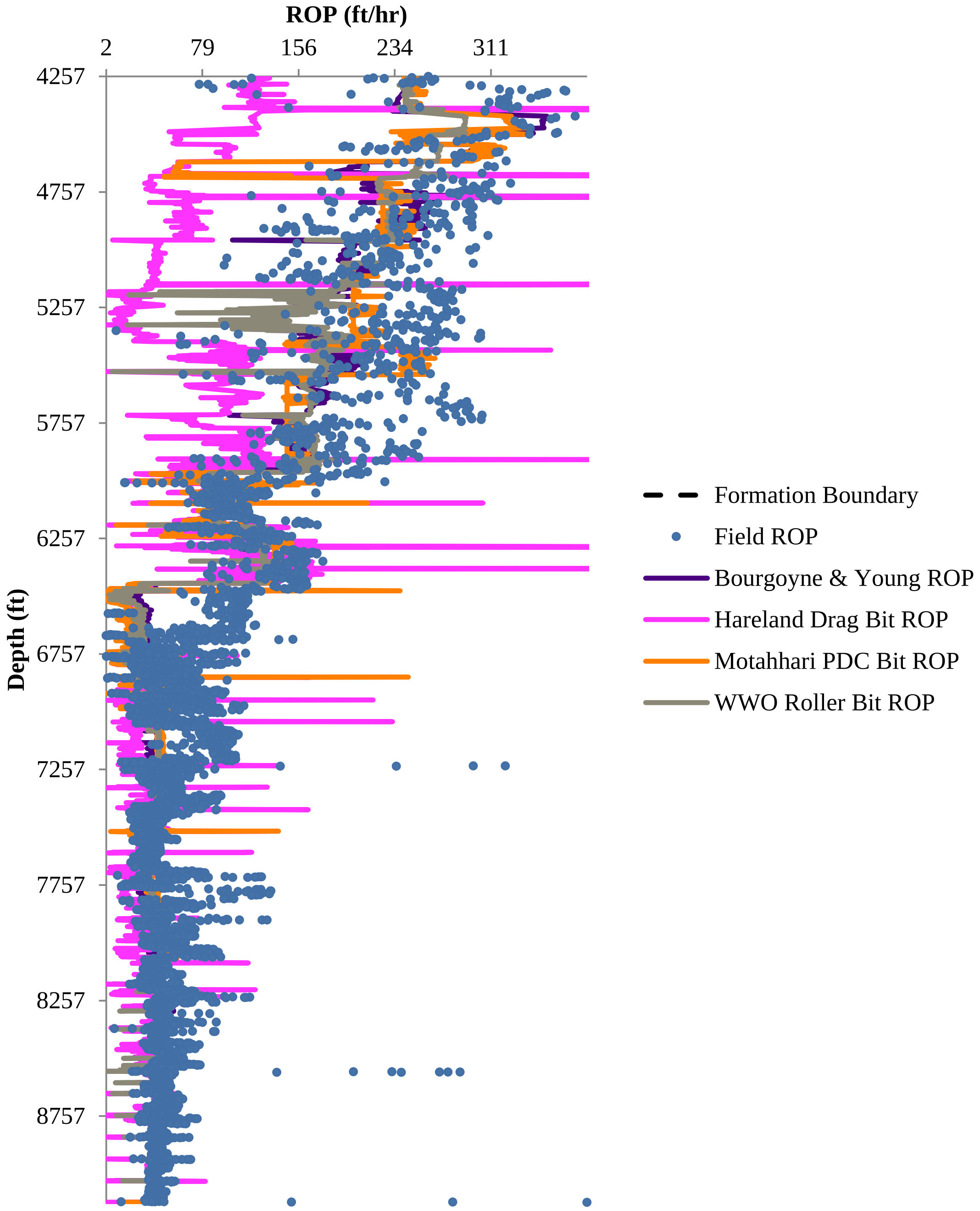

Figure 65: Marathon Full Well View

Figures 66, 67, and 68 below show the interfaces between limestone and sandstone, sandstone and shale and shale and limestone, and sandstone and limestone and limestone and shale, respectively.

In figure 66, all models show significant changes in their prediction of ROP between the limestone and sandstone interfaces. The models underestimate rate of penetration towards the end of the sandstone formation, and at the very top, but overall the models are behaving well to the data. Hareland’s drag bit model grossly underestimates ROP for the deeper section of the sandstone formation. The main reason for that is that the model calibrates itself for the most common weight on bit values. The models coefficients are used in the final calculation in order to multiply the model evaluation by a certain factor. As such, the model becomes highly sensitive to any deviation from the median. Also for that reason, whenever a large fluctuation in weight on bit occurs, Hareland’s model results have large fluctuations, except these fluctuations are more accentuated. Hareland’s model grossly underestimates ROP whenever weight on bit is too low. The problem is even worse for formations where a drilling break occurs, as the measured weight on bit decreases, but ROP increases. Motahhari’s model, on the other hand, has its tinkering coefficients affecting the primary parameters, weight on bit, RPM, bit geometry, directly. The model is thus better equipped to handle large changes in weight on bit for a given formation, since the model will adjust the effect of weight on bit to the final result accordingly. As a result, Motahhari’s model is the best model for the figure. Due to the Winters, Warren, and Onyia model’s simplicity, its plot almost appears to be an average curve of the various data points.

In figure 67, all models appear to have a smooth transition between the sandstone and shale formations. A reason for this might be that the data for the location of the formations specifies a change in formation sooner than is actually encountered when the well was drilled. In the figure, again Hareland's drag bit model underpredicts ROP and Motahhari's appears to more closely match the data.

In figure 68, the models show sharp changes in rate of penetration as they cross to a different lithology. Motahhari's proves it also has limitations, due to the large jumps occurring around 7525 ft, but the model behaves well everywhere else. In that section, rpm increases from 40 rpm to 50 rpm but ROP decreases, and rpm decreases from 50 rpm to 35 rpm, but ROP increases. A cause for the changes might be that the well deviated from vertical at this section and the driller attempted to stop that from occurring. After the bit returned to the intended course, normal drilling parameters were resumed. All models overpredict ROP at the end of the sandstone formation.

Overall, despite Motahhari's model failing at around 7525 ft, it is the best model of the four, which is what one would hope for given that it is a drag bit model. However, the model does not yield the smallest errors, as seen in section 5.6.1.

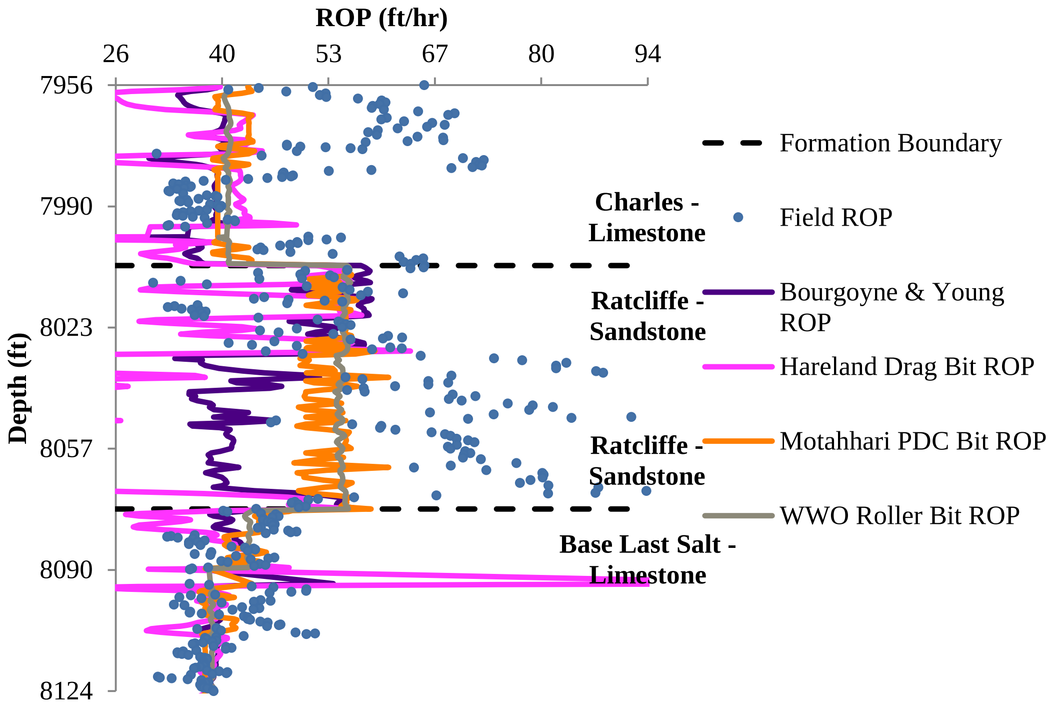

Figure 66: Marathon Sandstone-Limestone Interface

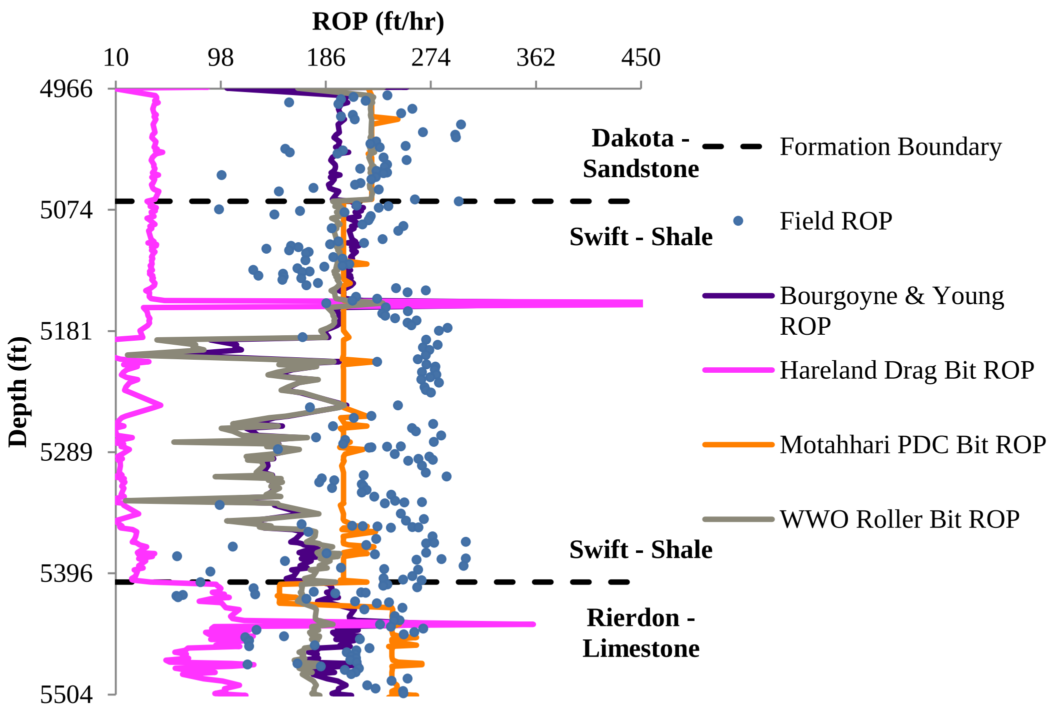

Figure 67: Marathon Sandstone-Shale Interface

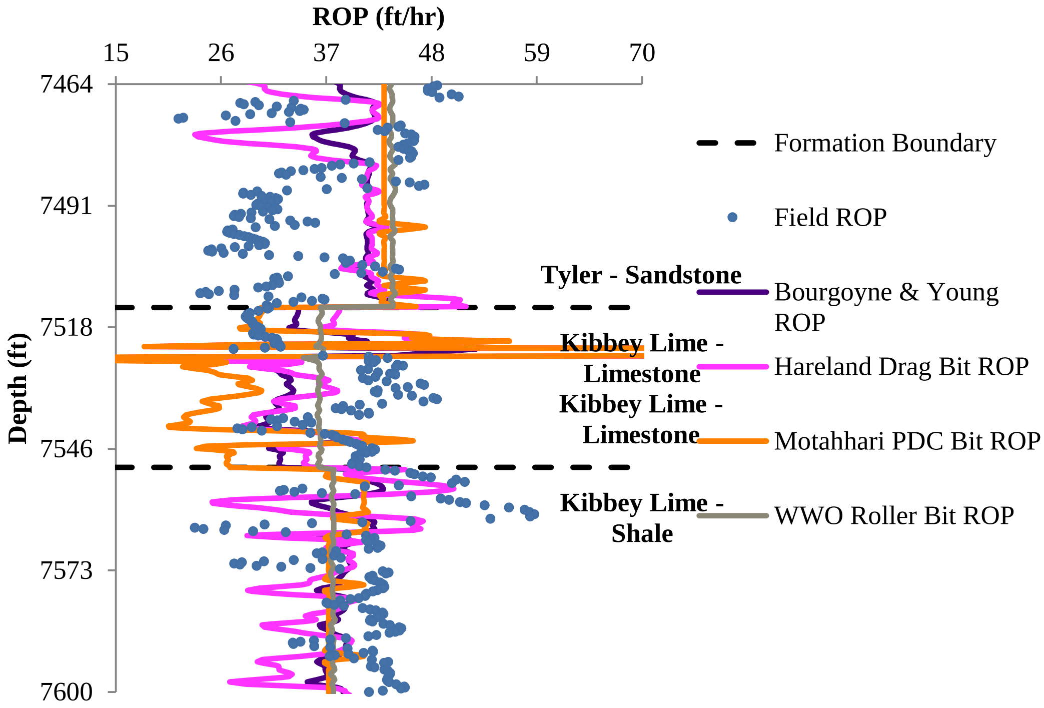

Figure 68: Marathon Limestone-Shale Interface

5.5.2 Baker A4 Well

This section analyzes the horizontal well Baker A4 in the Eagle Ford shale in South Texas. All ROP field values can be seen in figure 69. Also in the figure are the full view calculated values of the drag bit models, and the two other extensive models. From an initial standpoint, all models appear to be computing ROP well.

Another important section to mention is the one between roughly 6000 ft and 7000 ft where appear to underpredict the data. In between 6000 and 7000 ft, there are many sections where ROP changes from around 45 ft/hr to 300 ft/hr. The large spikes in

ROP are only associated by a 100% increase in WOB. The models will not account for a six times over increase in ROP without causing huge errors at later segments of that formation, which is a large limestone formation ending at 9150 ft . Thus, in order to minimize the error, all models placed heavier weights on coefficients to correct for the deeper sections and not for the large ROP jumps.

Figures 70, 71, and 72 below show the interfaces between shale and sandstone, sandstone and limestone and limestone and shale, and sandstone and limestone and limestone and shale, respectively.

In figure 70, all models show significant changes in their prediction of ROP between the shale and sandstone interfaces. All models underestimate ROP around 11575 ft, with Hareland's drag bit model being the most pronounced of them, but overall the models are behaving well to changes in the data. Motahhari's model appears to be the best model for the figure.

In figure 71, all models show a sharp transition between the sandstone and limestone and the limestone and shale formations. In the figure, again Motahhari's model appears to more closely match the data, but it grossly overpredicts ROP at 9480 ft. Also, all models underestimate ROP for the limestone section due to a large variance in the data. The large variance is being caused by sections where easier to drill limestone is being superimposed by a harder to drill limestone which is then being superimposed by an easier to drill limestone and so forth. A possible reason for the easier to drill sections is a karstic landscape.

In figure 72, the models show sharp changes in rate of penetration as they cross to a different lithology. All models predict ROP well throughout the figure.

Overall, despite Motahhari's model failing at around 9480 ft, it is the best model of the four, which is what one would hope for given that it is a drag bit model. However, the model does not yield the smallest errors, as seen in section 5.6.2.

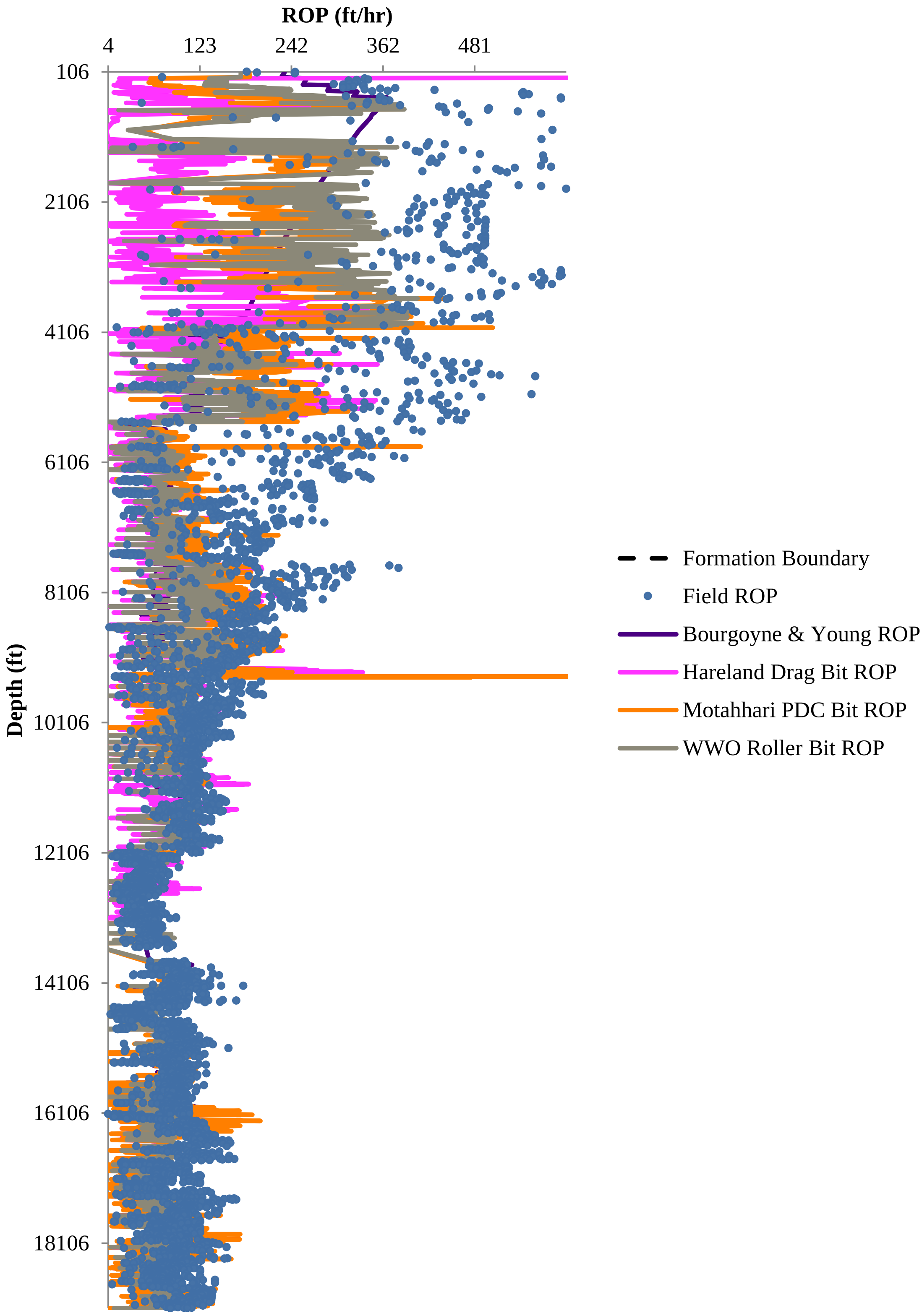

Figure 69: Baker A4 Full Well View

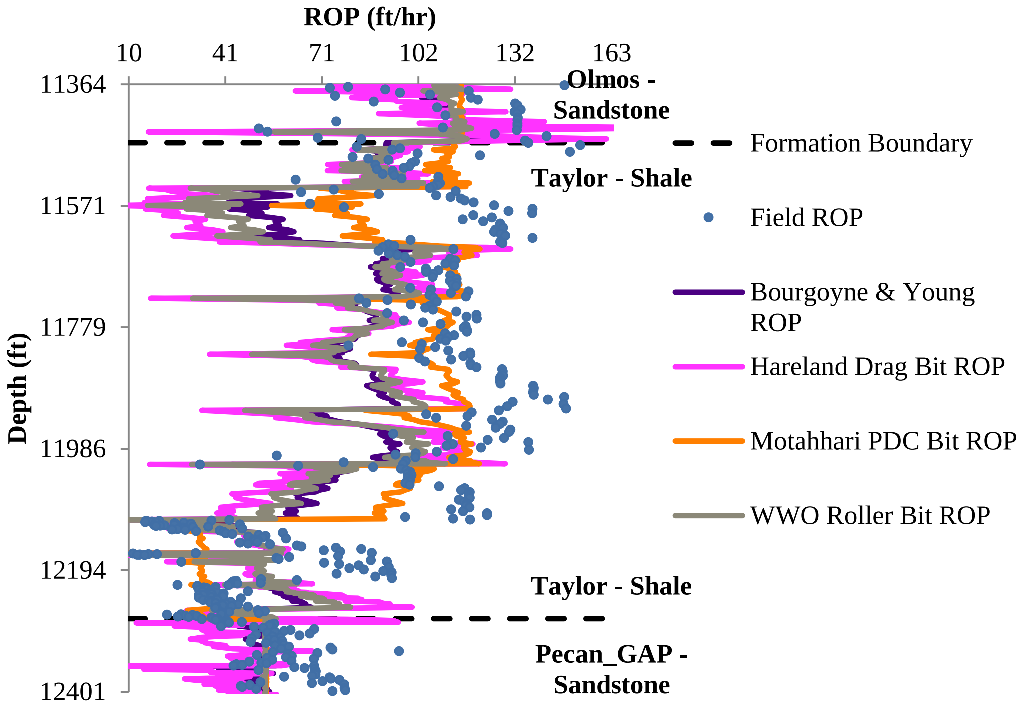

Figure 70: Baker A4 Sandstone-Shale Interface

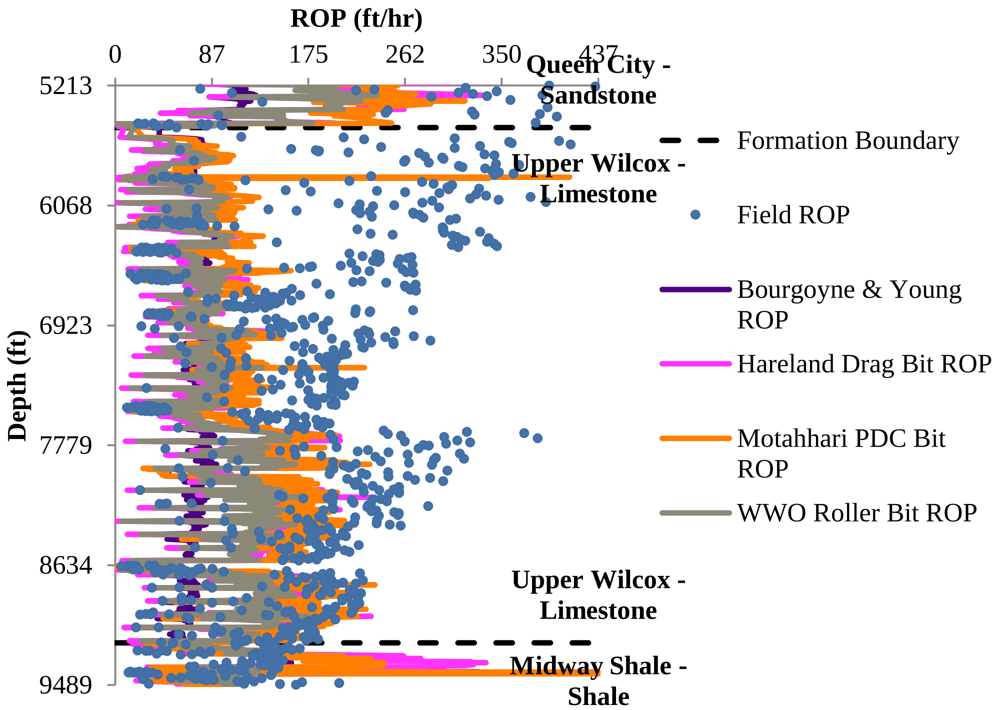

Figure 71: Baker A4 Sandstone-Limestone Interface

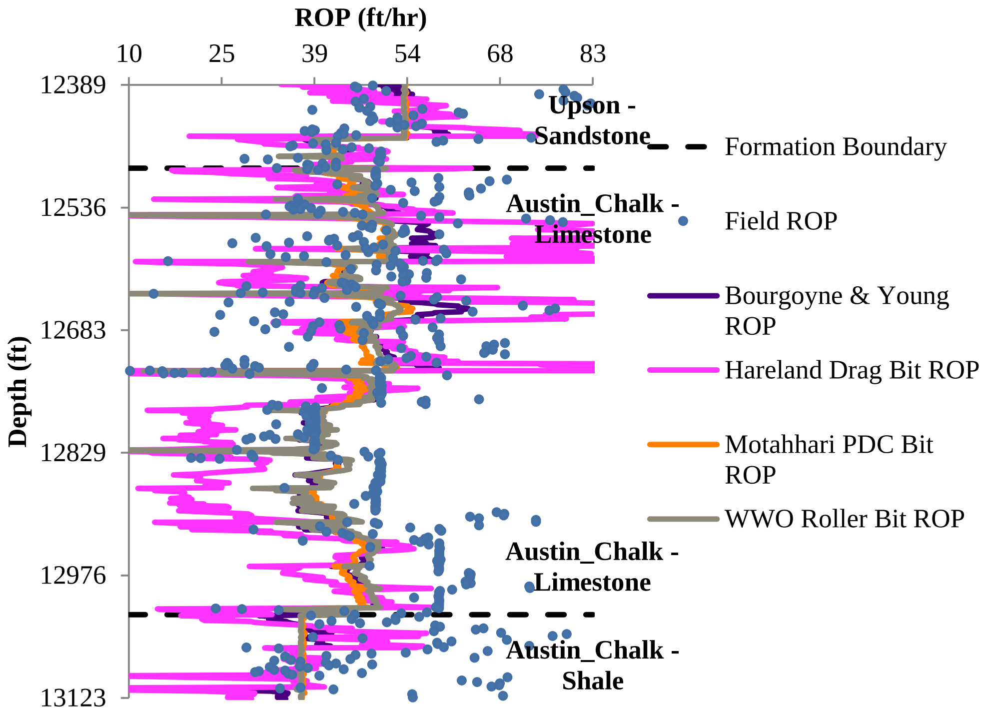

Figure 72: Limestone-Shale Interface

5.5.3 Baker A5 Well

This section analyzes the horizontal well Baker A5 in the Eagle Ford shale in South Texas. All ROP field values can be seen in figure 73. Data from 9740 to 11750 ft is missing. In the figure are the full view calculated values of the drag bit models, and the two other extensive models. In figure 73, all models underpredict ROP for the shallowest section, from 7740 to 9740 ft, and the deepest section, from 15740 ft until the end of the plot.

Figures 74, 75, and 76 below show the interfaces between shale and sandstone, sandstone and limestone, and sandstone and limestone and limestone and shale, respectively.

In figure 74, all models show significant changes in their prediction of ROP between the shale and sandstone interfaces, however the models are underpredicting in the segment shown for the sandstone interface. All models calculate ROP well for the second shale formation with no data being shown in the figure for the upper shale formation, Midway.

In figure 75, all models show an extremely sharp transition between the sandstone and limestone formations, with again the models underpredicting ROP for the deepest formation, this time a limestone one. All models behave well for the sandstone formation. For the limestone formation, neither Hareland's nor Winters, Warren, and Onyia's models could be used; Hareland's model heavily undercalculates ROP while Winters, Warren, and Onyia's model is mostly an unchanging line that also underpredicts ROP. A full view and continuation of the limestone formation can be seen in figure 76. The figure shows that the models have a small transition as they cross from the limestone to the shale formation. All models predict ROP well for the deepest sections, after roughly 12900 ft.

Due to Motahhari's model being highly affected by large variances in the data, and Hareland's model heavily underpredicting ROP at times, one of the drag bit models is not the best choice for the Baker A5 well, despite it being drilled with a drag bit.

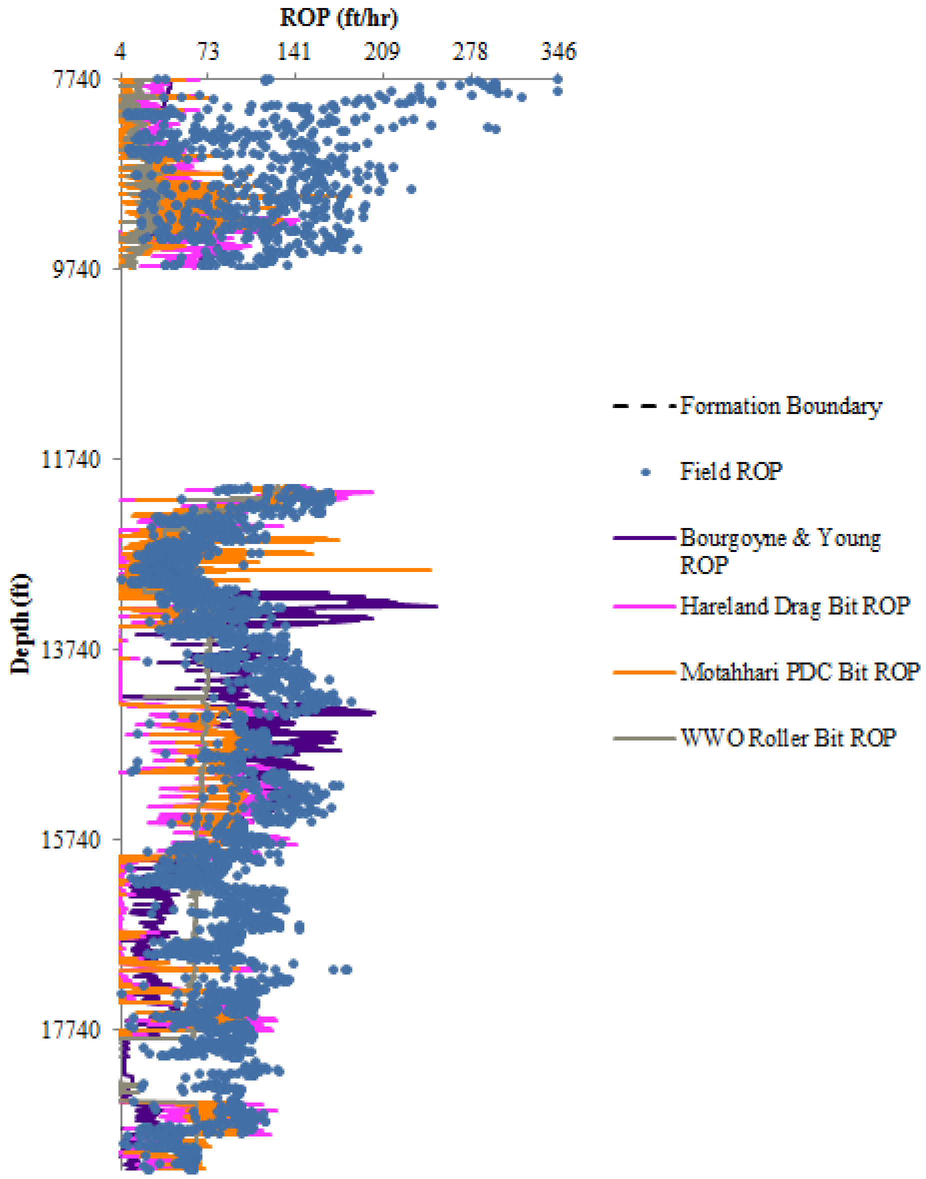

Figure 73: Baker A5 Full Well View

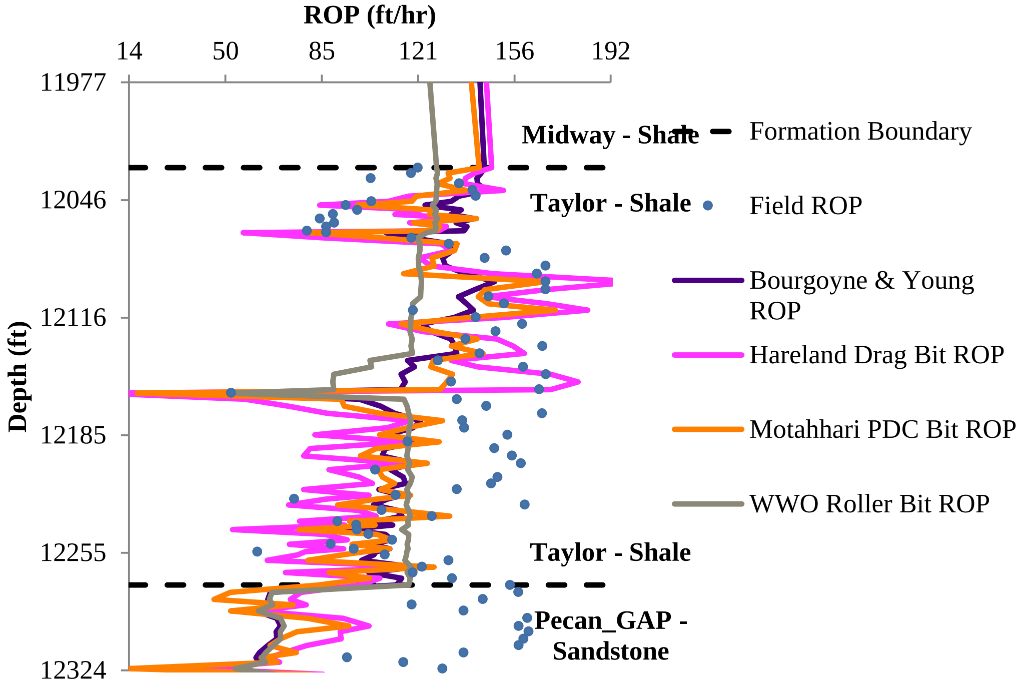

Figure 74: Baker A5 Shale-Sandstone Interface

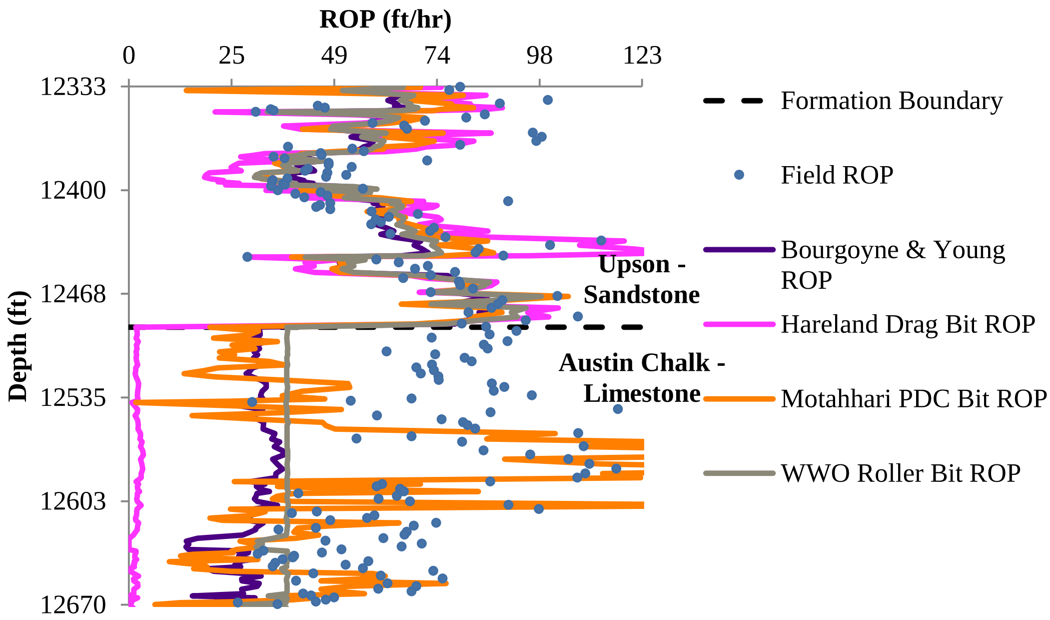

Figure 75: Baker A5 Sandstone-Limestone Interface

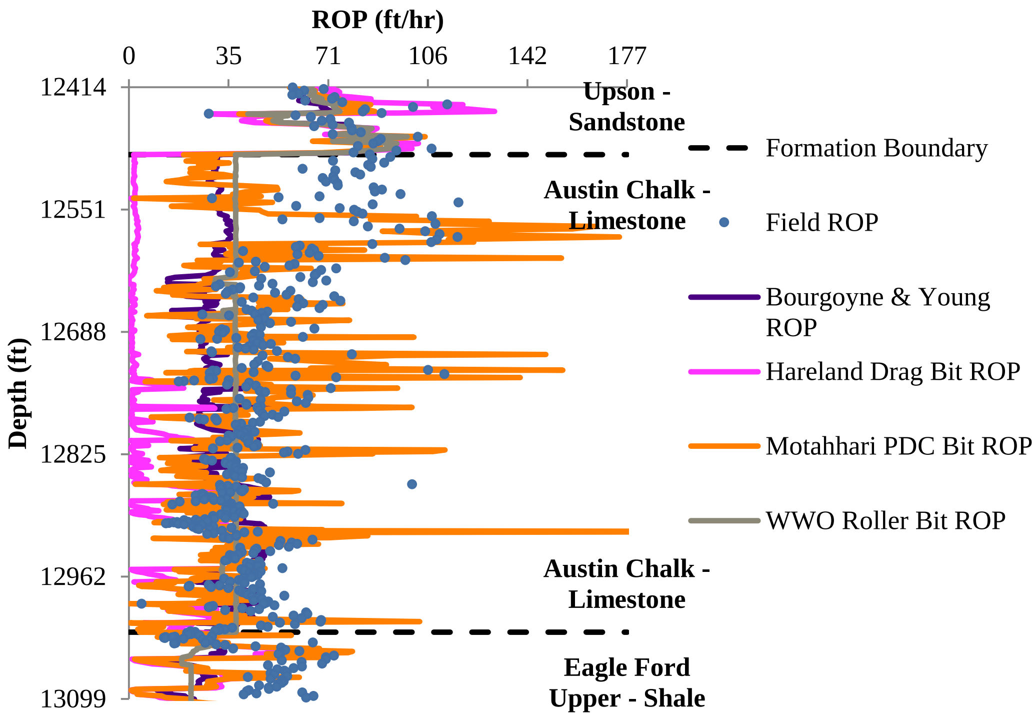

Figure 76: Baker A5 Limestone-Shale Interface

5.5.4 Swallis A6 Well

This section analyzes the horizontal well Swallis A6 in the Eagle Ford shale in South Texas. All ROP field values can be seen in figure 77. Also in the figure are the full view calculated values of the drag bit models, and the two other extensive models. An important section to mention is the one between roughly 5500 ft and 7500 ft where all the models underpredict the data. The reason for that is because that segment belongs to a large formation ending at 9150 ft and, in order to minimize the error, all models placed heavier weights on calculating coefficients that would yield better results for the deeper sections of that formation. Also, twice in the figure, Motahhari's model grossly overestimates ROP, at 8700 and 9200 ft.

Figures 78, 79, and 80 below show the interfaces between sandstone and shale, sandstone and limestone and limestone and shale, and sandstone and limestone and limestone and shale, respectively.

In figure 78, all models show changes in their prediction of ROP between the sandstone and shale interfaces. All models underestimate ROP around 11550 ft, with Hareland's drag bit model being the most pronounced of them, but overall the models are behaving well to changes in the data. Motahhari's model appears to be the best model for the figure.

In figure 79, all models show a sharp transition between the sandstone and limestone and the limestone and shale formations. Also, all models underestimate ROP for the limestone section. Bourgoyne & Young's model increasingly worsens with depth while other models improve.

In figure 80, the models show sharp changes in rate of penetration as they cross to a different lithology. All models predict ROP well throughout the figure however

Hareland’s model overpredicts and underpredicts ROP for the shallower and deeper section, respectively.

Overall, despite Motahhari’s model failing twice, it is the best model of the four, which is what one would hope for given that it is a drag bit model. However, once again the model does not yield the smallest errors, as seen in section 5.6.4.

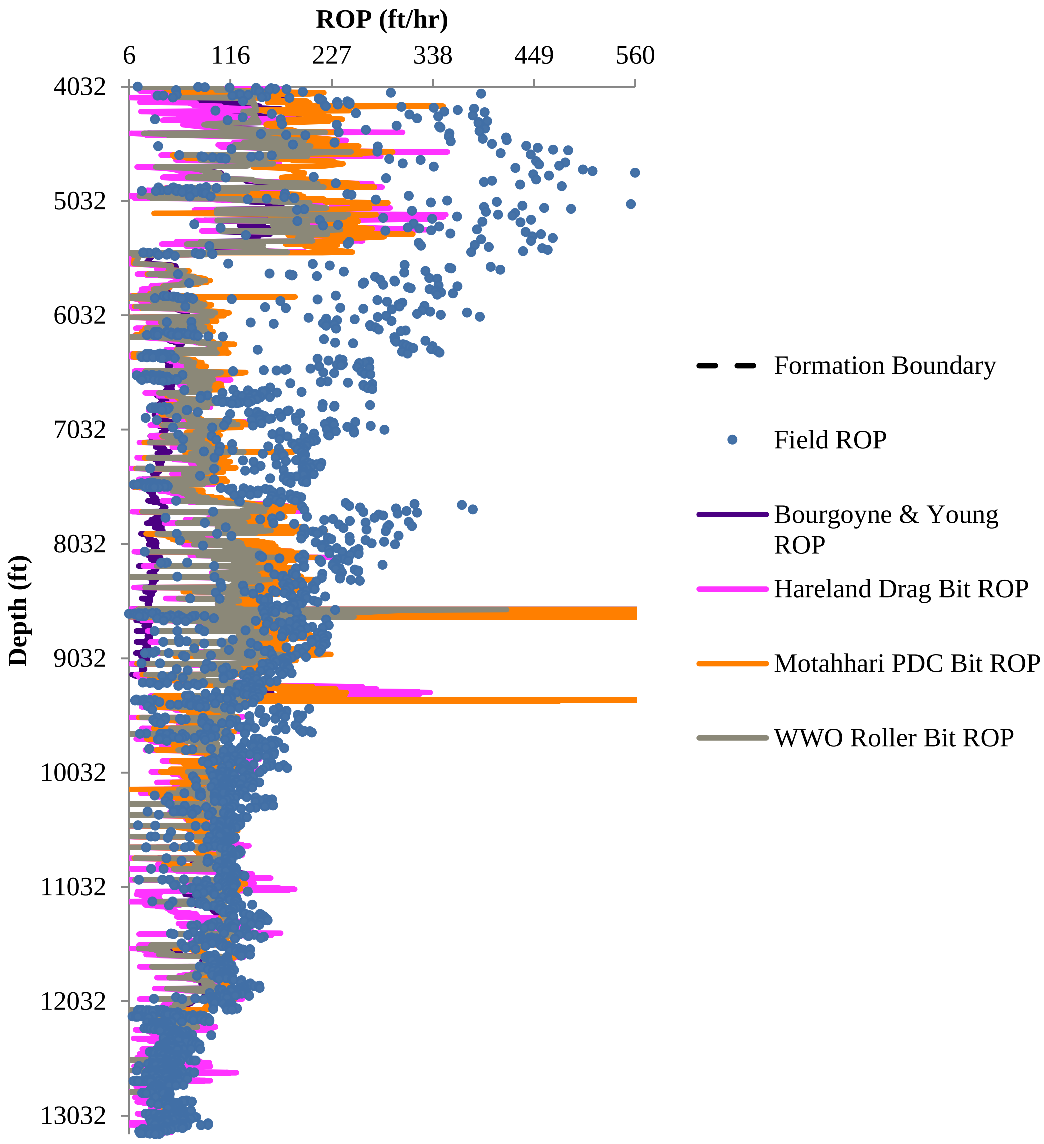

Figure 77: Swallis A6 Full Well View

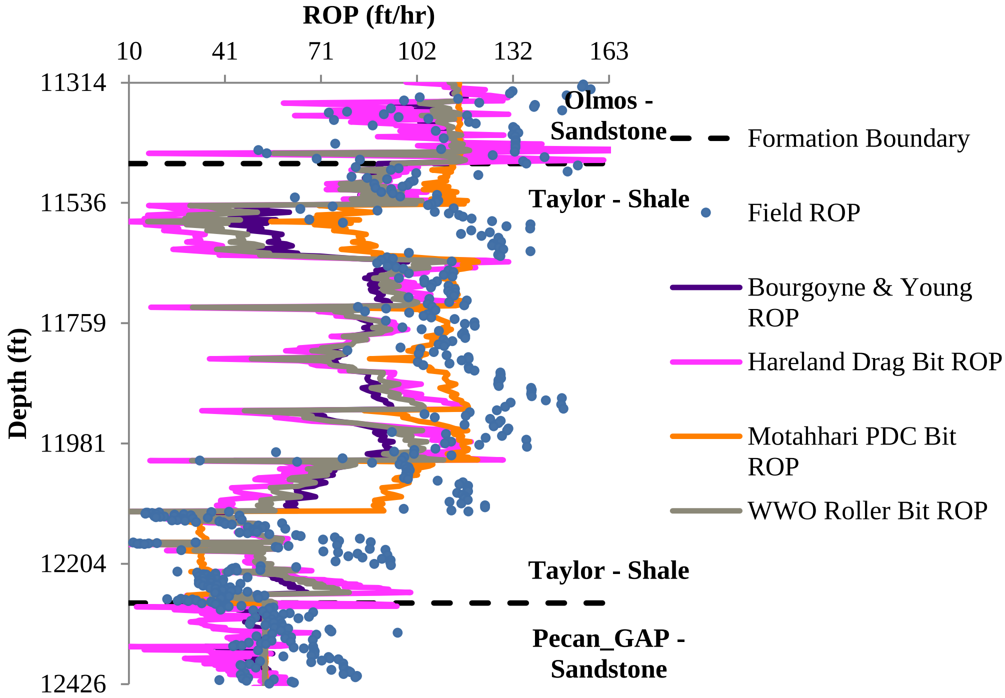

Figure 78: Swallis A6 Sandstone-Shale Interface

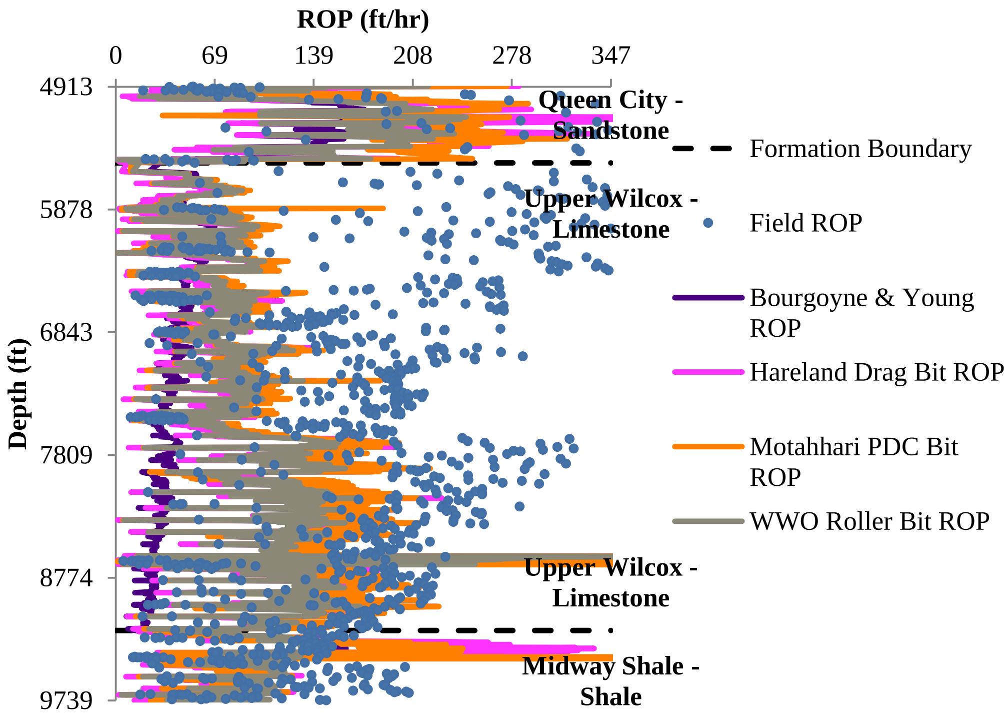

Figure 79: Swallis A6 Limestone-Shale Interface

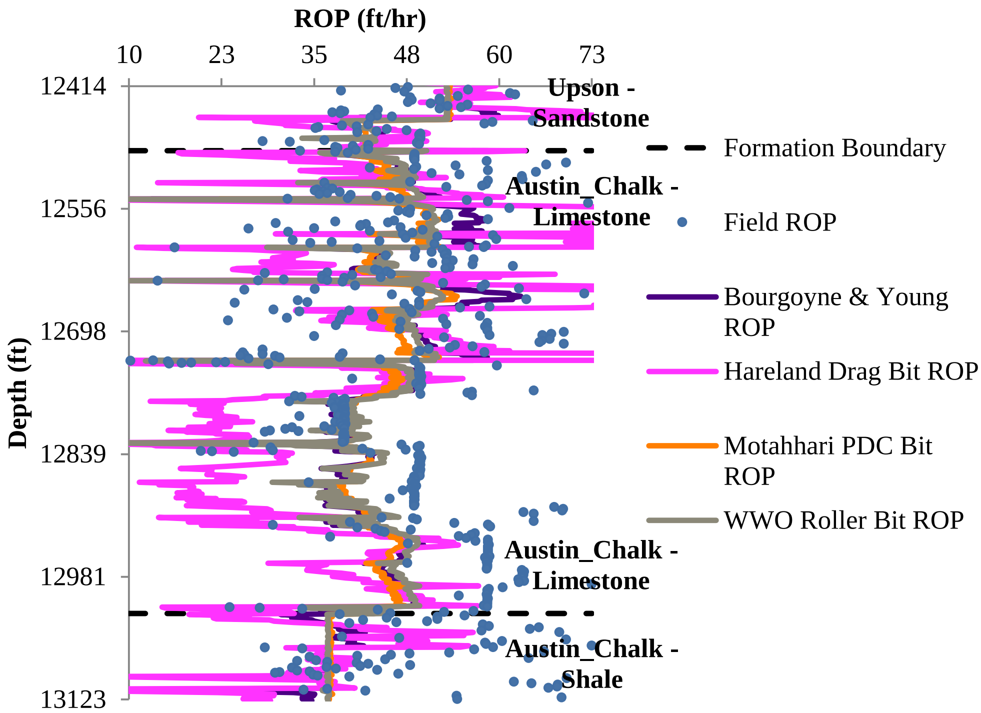

Figure 80: Swallis A6 Sandstone-Limestone Interface

5. 6 Model Results

This section provides the model results for all the wells. The analysis was run using ROPPlotter using a minimum error approach. The wells provided were Baker A4, Baker A5, Swallis A6, and Marathon. Each well has the coefficient results and a table comparing the various models.

5.6.1 Marathon Well

As mentioned in section 5.5.1, the model with lowest associated errors is a roller bit model, as seen in table 14. However, Motahhari's percent error is close to that of Winters, Warren, and Onyia's model. Also interesting is that Hareland's drag bit model does not have the lowest error for any of the formations, despite using CCS data in attempts to reduce the error and despite it being a model specific to the bit used for drilling the well.

Despite Winters, Warren, and Onyia's model having the lowest error, it is by no means a predictive tool and should not be used in order to predict changes downhole or be used as a planning tool for the next well. Motahhari's model should be used instead, as by applying the correct model choice for the bit being used, one can effectively improve drilling efficiency through analyzing how drilling parameter changes affect the calculated ROP and the field ROP.

Tables 15 and 16 illustrate the calculated model coefficients for each model. As noted in the figure, some of the models, especially Winters, Warren, and Onyia's, reach the coefficient limits often, which, despite creating a simulation with lesser error, creates a physically meaningless result.

Table 14: Marathon Evaluation

Average percent error of each ROP Model by formation:

Formation Name	Bingham	Bourgoyne & Young	Hareland Drag Bit	Hareland Roller Bit	Motahhari PDC Bit	WWO Roller Bit	Best Model in Formation
Greenhorn - Limestone	N/A	38.20%	58.77%	N/A	38.70%	37.65%	WWO Roller Bit
Newcastle - Sandstone	N/A	18.64%	71.68%	N/A	29.83%	14.93%	WWO Roller Bit
Dakota - Sandstone	N/A	23.17%	75.19%	N/A	20.71%	20.29%	WWO Roller Bit
Swift - Shale	N/A	44.78%	87.55%	N/A	36.09%	45.19%	Motahhari PDC Bit
Rierdon - Limestone	N/A	26.97%	46.16%	N/A	27.66%	25.94%	WWO Roller Bit
Piper - Limestone	N/A	25.57%	30.90%	N/A	32.12%	23.08%	WWO Roller Bit
Spearfish - Sandstone	N/A	22.75%	42.21%	N/A	22.67%	19.70%	WWO Roller Bit
Pine Salt - Sandstone	N/A	47.51%	55.03%	N/A	60.35%	48.83%	Borgoune & Young
Broom Creek - Sandstone	N/A	41.91%	50.14%	N/A	38.47%	40.38%	Motahhari PDC Bit
Tyler - Sandstone	N/A	33.63%	42.36%	N/A	32.83%	32.72%	WWO Roller Bit
Kibbey Lime - Limestone	N/A	19.85%	27.75%	N/A	35.05%	15.44%	WWO Roller Bit
Kibbey Lime - Shale	N/A	15.07%	21.55%	N/A	14.33%	13.92%	WWO Roller Bit
Charles - Sandstone	N/A	37.94%	45.68%	N/A	31.19%	36.19%	Motahhari PDC Bit
Charles - Limestone	N/A	27.19%	30.79%	N/A	24.73%	25.52%	Motahhari PDC Bit
Ratcliffe - Sandstone	N/A	32.16%	49.79%	N/A	22.59%	21.50%	WWO Roller Bit
Base Last Salt - Limestone	N/A	11.47%	16.47%	N/A	8.41%	9.79%	Motahhari PDC Bit
Base Last Salt - Sandstone	N/A	13.33%	17.85%	N/A	11.98%	11.69%	WWO Roller Bit
Base Last Salt - Limestone	N/A	25.51%	57.27%	N/A	17.22%	20.29%	Motahhari PDC Bit
Mission Canyon - Limestone	N/A	16.34%	18.91%	N/A	11.87%	12.90%	Motahhari PDC Bit
Lodgepole - Limestone	N/A	7.60%	10.07%	N/A	7.71%	6.84%	WWO Roller Bit
Entire Dataset	N/A	25.86%	34.89%	N/A	25.62%	24.08%	WWO Roller Bit

Best Overall Model: WWO Roller Bit

Average Percent Error: 24.08%

Table 15: Marathon Coefficient for Bourgoyne & Young

ROP Models	B&Y							
Model Coefficients	a1	a2	a3	a4	a5	a6	a7	a8
Formations								
Greenhorn - Limestone	5	0.0005	1.00E-06	1.00E-06	0.5136	0.7	1	0.5
Newcastle - Sandstone	2.96737	0.0005	1.00E-06	1.00E-06	0.50466	0.70711	1.02535	0.5
Dakota - Sandstone	3.23276	0.0005	1.00E-06	1.00E-06	0.4591	0.74888	1.05563	0.5
Swift - Shale	3.34891	0.0005	1.00E-06	1.00E-06	0.4	0.81704	1.05563	0.5
Rierdon - Limestone	3.1608	0.0005	1.00E-06	1.00E-06	0.4	0.81704	1.05563	0.5
Piper - Limestone	2.71499	5.00E-04	1.00E-06	1.00E-06	0.4	0.8048	1.11546	0.5
Spearfish - Sandstone	3.12938	5.00E-04	1.00E-06	1.00E-06	0.4	0.80902	1.08776	0.5
Pine Salt - Sandstone	1.94263	0.0005	3.06E-05	1.00E-06	0.4	0.80314	1.10902	0.50095
Broom Creek - Sandstone	2.33437	0.0005	3.06E-05	1.00E-06	0.4	0.81108	1.07012	0.49944
Tyler - Sandstone	2.65123	0.0005	4.25E-05	1.00E-06	0.4	0.80986	1.07012	0.49944
Kibbey Lime - Limestone	2.37E+00	5.00E-04	4.25E-05	1.00E-06	0.4	0.72599	1.36324	0.49944
Kibbey Lime - Shale	2.63E+00	5.00E-04	4.36E-05	1.00E-06	0.4	0.72589	1.36324	0.49944
Charles - Sandstone	2.48E+00	5.00E-04	4.36E-05	1.00E-06	0.4	0.72589	1.36324	0.4934
Charles - Limestone	2.63E+00	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589	1.36324	0.4934
Ratcliffe - Sandstone	3.05E+00	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589	1.36324	0.4934
Base Last Salt - Limestone	2.76E+00	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589	1.36324	0.4934
Base Last Salt - Sandstone	2.68E+00	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589	1.36324	0.4934
Base Last Salt - Limestone	3.61738	5.00E-04	4.35E-05	1.00E-06	0.4	0.72589	1.36324	0.4934
Mission Canyon - Limestone	3.10558	4.90E-04	1.00E-06	1.00E-06	0.4	0.71703	1.35671	0.49259
Lodgepole - Limestone	3.23048	0.000486	1.00E-06	1.00E-06	0.4	0.71266	1.3567	0.49206
Sandstone Average	2.71919	0.0005	2.64E-05	1.00E-06	0.4182	0.76297	1.16752	0.49778
Limestone Average	3.17691	0.0005	1.98E-05	1.00E-06	0.41262	0.73946	1.25972	0.49603
Shale Average	2.99049	0.0005	2.23E-05	1.00E-06	0.4	0.77146	1.20944	0.49972
Overall Average	2.95229	0.0005	2.30E-05	1.00E-06	0.41387	0.75324	1.2132	0.49719

Table 16: Marathon Coefficient for Hareland's Drag Bit, Motahhari's, and Winters, Warren, and Onyia's Models

Har Drag			Motahhari			WWO			
a	b	c	G	α	γ	a	φ	b	c
1000	0.0001	0.0001	9.16352	0.0001	3.6921	1.00E-04	1.00E-04	0.0001	0.00061
1000	0.0001	0.0001	10	0.60835	3.3323	1.00E-04	1.00E-04	0.0876	0.00053
1000	0.0001	0.0001	5.26681	0.0001	4.30728	1.00E-04	1.00E-04	0.0001	0.00063
1000	0.0001	0.0001	5.75105	0.0001	4.25326	1.00E-04	1.00E-04	0.0001	0.00067
1000	0.0001	0.0001	5.18838	0.0001	4.19711	1.00E-04	1.00E-04	1	0.00039
7.37E+02	2.13E-03	0.00302	10	9.91E-01	3.03104	1.00E-04	1.00E-04	0.0001	0.00158
1.00E+03	1.00E-04	0.0001	5.32849	1.00E-04	4.04146	6.18E-04	1.28E-04	0.41355	0.00099
247.54301	0.01438	0.01379	10	1.64263	2.09774	1.00E-04	8.45E-03	0.0892	0.00238
261.00031	0.00234	0.00406	10	0.0001	3.68532	1.00E-04	2.66E-04	1	0.00308
261.00063	0.00049	0.0034	5.16624	0.0001	3.8649	1.00E-04	1.00E-04	0.0001	0.00278
187.16674	0.0001	0.00011	10	1.5502	2.11738	1.00E-04	1.00E-04	0.0001	0.00334
261.00039	0.00248	0.0043	5.58312	0.0001	3.80464	1.00E-04	1.00E-04	0.0001	0.00324
215.5346	0.00607	0.01528	5.15454	0.0001	3.80656	1.00E-04	1.00E-04	1	0.00366
203.77839	0.00694	0.01325	3.71657	0.0001	3.9277	1.00E-04	1.00E-04	1	0.0033
261.00105	0.00058	0.00342	5.22548	0.0001	3.90407	1.00E-04	1.00E-04	0.0001	0.00274
192.0441	0.00406	0.00801	5.06022	0.0001	3.85237	1.00E-04	1.00E-04	1	0.00307
170.05717	0.00259	0.00677	4.58586	0.03122	3.82899	2.88E-04	1.40E-04	0.1015	0.00391
598.0222	0.00289	0.00324	6.5583	0.0001	3.90959	1.00E-04	1.00E-04	1	0.00163
242.20798	0.01205	0.0174	5.23078	0.00266	3.86447	1.00E-04	1.00E-04	1	0.00287
182.18328	0.00014	0.00013	5.16266	0.0001	3.8478	1.00E-04	1.00E-04	1	0.00301
490.68186	0.00297	0.00522	6.74749	0.25364	3.65207	1.78E-04	1.05E-03	0.29913	0.0023
482.52107	0.00317	0.00504	6.6756	0.28276	3.6044	1.00E-04	1.00E-04	0.6667	0.0022
630.5002	0.00129	0.0022	5.66709	0.0001	4.02895	1.00E-04	1.00E-04	0.0001	0.00195
500.99134	0.00289	0.00484	6.6071	0.24139	3.6683	1.35E-04	5.29E-04	0.43463	0.00222

5.6.2 Baker A4 Well

As mentioned in section 5.5.2, the models with lowest associated errors are roller bit models, as seen in table 17. However, Motahhari's percent error is close to that of the two roller bit models. Again, Hareland's drag bit model does not have the lowest error for any of the formations, despite using CCS data in attempts to reduce the error and despite it being a model specific to the bit used for drilling the well.

Despite the roller bit models having the lowest error, as it was discussed in the previous sections, Motahhari's model still yields the most physically meaningful results and should be used instead. Tables 18 and 19 illustrate the calculated model coefficients for each model. As noted in the figure, the roller bit models tend to achieve better results due to them reaching the practical limits imposed, thus putting too much weight on a single parameter rather than varying much alongside the data.

Table 17: Baker A4 Evaluation

Average percent error of each ROP Model by formation:

Formation Name	Bingham	Bourgoyne & Young	Hareland Drag Bit	Hareland Roller Bit	Motahhari PDC Bit	WWO Roller Bit	Best Model in Formation
Sandstone - Sandstone	N/A	60.58%	88.74%	N/A	63.67%	57.08%	WWO Roller Bit
Sparta - Sandstone	N/A	54.66%	66.64%	N/A	52.63%	49.74%	WWO Roller Bit
Queen City - Sandstone	N/A	58.21%	63.90%	N/A	51.03%	58.18%	Motahhari PDC Bit
Upper Wilcox - Limestone	N/A	58.69%	56.90%	N/A	48.42%	56.14%	Motahhari PDC Bit
Midway Shale - Shale	N/A	43.76%	52.72%	N/A	57.29%	40.75%	WWO Roller Bit
Navarro Shale - Shale	N/A	26.85%	52.37%	N/A	23.57%	21.12%	WWO Roller Bit
Olmos - Sandstone	N/A	24.88%	33.92%	N/A	24.06%	23.00%	WWO Roller Bit
Taylor - Shale	N/A	36.47%	40.94%	N/A	22.27%	37.33%	Motahhari PDC Bit
Pecan_GAP - Sandstone	N/A	21.76%	37.51%	N/A	17.13%	17.14%	Motahhari PDC Bit
Upson - Sandstone	N/A	14.63%	17.66%	N/A	14.22%	14.81%	Motahhari PDC Bit
Austin_Chalk - Limestone	N/A	23.59%	44.84%	N/A	22.82%	21.59%	WWO Roller Bit
Austin_Chalk - Shale	N/A	27.87%	31.42%	N/A	26.59%	26.92%	Motahhari PDC Bit
Eagle Ford Lower - Shale	N/A	31.59%	99.92%	N/A	35.77%	33.73%	Borgoune & Young
Eagle Ford - Shale	N/A	36.35%	38.04%	N/A	42.25%	32.57%	WWO Roller Bit
Entire Dataset	N/A	39.66%	59.76%	N/A	40.46%	37.96%	WWO Roller Bit

Best Overall Model: WWO Roller Bit

Average Percent Error: 37.96%

Table 18: Baker A4 Coefficient for Bourgoyne & Young

ROP Models	B&Y							
Model Coefficients	a1	a2	a3	a4	a5	a6	a7	a8
Formations								
Sandstone - Sandstone	5	0.000208	1.00E-06	1.00E-06	0.4	0.2	0.1	0.30048
Sparta - Sandstone	3.32229	0.000208	1.00E-06	1.00E-06	0.4	0.24007	0.16302	0.30048
Queen City - Sandstone	2.90199	0.000208	1.00E-06	1.00E-06	0.4	0.52	0.77136	0.30048
Upper Wilcox - Limestone	2.51169	0.000195	1.00E-06	1.00E-06	0.4	1	1.5	0.3
Midway Shale - Shale	3.20059	0.000001	1.00E-06	1.00E-06	0.4	1	1.10887	0.3
Navarro Shale - Shale	3.38137	1.00E-06	1.00E-06	1.00E-06	0.4	0.9925	1.07549	0.3
Olmos - Sandstone	3.40785	4.00E-06	1.00E-06	1.00E-06	0.4	0.99711	1.07549	0.3
Taylor - Shale	3.08195	0.000051	1.00E-06	1.00E-06	0.586	1	0.41629	0.3
Pecan_GAP - Sandstone	3.25373	0.000051	1.00E-06	1.00E-06	0.4	0.8484	0.41629	0.3
Upson - Sandstone	2.86437	0.000051	1.00E-06	1.00E-06	0.4	0.8484	0.41629	0.3
Austin_Chalk - Limestone	3.07E+00	5.00E-05	2.26E-05	1.00E-06	0.4	0.8479	0.41629	0.3
Austin_Chalk - Shale	2.78E+00	5.10E-05	1.00E-06	1.00E-06	0.4	0.85441	0.4163	0.3
Eagle Ford Lower - Shale	2.74E+00	5.00E-05	3.41E-05	1.00E-06	0.68896	0.88624	0.39764	0.3
Eagle Ford - Shale	2.99E+00	4.80E-05	7.78E-05	1.00E-06	1.15911	0.8278	0.1	0.3
Sandstone Average	3.46E+00	1.20E-04	1.00E-06	1.00E-06	0.4	0.609	0.49041	0.30024
Limestone Average	2.79E+00	1.20E-04	1.18E-05	1.00E-06	0.4	0.92395	0.95815	0.3
Shale Average	3.02837	3.00E-05	1.93E-05	1.00E-06	0.60568	0.92682	0.58577	0.3
Overall Average	3.17845	8.00E-05	1.04E-05	1.00E-06	0.48815	0.7902	0.5981	0.3001

Table 19: Baker A4 Coefficient for Hareland’s Drag Bit, Motahhari’s, and Winters, Warren, and Onyia’s Models

Har Drag			Motahhari			WWO			
a	b	c	G	α	γ	a	φ	b	c
729.04647	0.00068	0.00143	10	0.53616	2.94078	1.00E-04	6.69E-04	0.10042	0.00124
515.06285	0.01253	0.0201	10	0.27725	2.69481	1.06E-04	3.59E-04	0.09994	0.00179
443.83709	0.00012	0.00054	5.16814	0.32526	2.75279	1.00E-04	1.00E-02	1	0.0001
127.36567	0.00136	0.0058	1.59217	0.72018	2.57782	1.00E-04	1.66E-02	0.0001	0.0001
88.72715	0.00014	0.00016	10	1.0118	1.98372	8.61E-03	1.00E-04	0.09834	0.00236
7.33E+01	1.00E-04	0.00018	10	7.18E-02	2.63707	2.54E-03	1.00E-04	0.09999	0.00233
1.04E+02	5.40E-04	0.00056	5.06712	1.00E-04	2.8208	4.78E-02	1.00E-04	0.1129	0.0022
75.54237	0.0001	0.0001	4.85784	0.38973	2.56657	1.00E-04	1.03E-01	1	0.0001
105.02348	0.00735	0.0001	5.08348	0.0001	2.92354	1.00E-04	1.00E-04	0.09996	0.0053
50.19207	0.00013	0.00013	5.07602	0.00086	2.87524	1.42E-01	1.00E-04	1	0.00572
68.66514	0.00067	0.0001	5.07935	0.26294	2.71052	8.26E-03	1.00E-04	0.10593	0.00518
48.94415	0.00652	0.00239	4.93566	0.0145	2.84664	1.00E-04	1.00E-04	0.08782	0.00764
5.07371	0.83661	0.0001	10	0.74675	1.93802	3.09E-02	1.00E-04	0.0001	0.00262
37.00018	0.00107	0.00254	10	2.23491	1.03519	5.65E-02	1.00E-04	0.0001	0.00185
324.60799	0.00356	0.00381	6.73246	0.18995	2.83466	3.17E-02	1.89E-03	0.4022	0.00273
98.01541	0.00102	0.00295	3.33576	0.49156	2.64417	4.18E-03	8.33E-03	0.05302	0.00264
54.77051	0.14076	0.00091	8.29892	0.74491	2.16787	1.65E-02	1.72E-02	0.21439	0.00282
176.59298	0.06199	0.00244	6.91855	0.47088	2.52168	2.13E-02	9.37E-03	0.27183	0.00275

5.6.3 Baker A5 Well

Again, as shown in table 20, the models with lowest associated errors are the roller bit models. In this example, however, all models have large percent errors, with Motahhari’s and Hareland’s drag bit models having similar error percentages. Although, Hareland’s drag bit model still does not have the lowest error for any formation, at the two shallowest formations, the model has lower errors than Motahhari’s model, thus it could be implemented at that region.

As it was noted before, Motahhari’s model should be used instead of the roller bit models, but this time, it is also recommended that Hareland’s model be used for a few of the shallower formations where its associated error is lesser than that of Motahhari’s model. Tables 21 and 22 illustrate the calculated model coefficients for each model.

Table 20: Baker A5 Evaluation

Average percent error of each ROP Model by formation:

Formation Name	Bingham	Bourgoyne & Young	Hareland Drag Bit	Hareland Roller Bit	Motahhari PDC Bit	WWO Roller Bit	Best Model in Formation
Upper Wilcox - Sandstone	N/A	68.54%	69.33%	N/A	84.69%	73.43%	Borgoune & Young
Midway - Shale	N/A	44.02%	54.62%	N/A	81.45%	80.11%	Borgoune & Young
Taylor - Shale	N/A	20.66%	24.40%	N/A	20.37%	22.27%	Motahhari PDC Bit
Pecan_GAP - Sandstone	N/A	28.08%	33.92%	N/A	28.49%	28.13%	Borgoune & Young
Upson - Sandstone	N/A	8.39%	8.86%	N/A	8.55%	7.86%	WWO Roller Bit
Austin Chalk - Limestone	N/A	39.61%	74.28%	N/A	41.93%	30.91%	WWO Roller Bit
Eagle Ford Upper - Shale	N/A	52.17%	48.51%	N/A	44.34%	56.12%	Motahhari PDC Bit
Eagle Ford Lower - Shale	N/A	52.40%	66.64%	N/A	65.01%	41.86%	WWO Roller Bit
Entire Dataset	N/A	51.41%	65.04%	N/A	63.74%	46.35%	WWO Roller Bit

Best Overall Model: WWO Roller Bit

Average Percent Error: 46.35%

Table 21: Baker A5 Coefficient for Bourgoyne & Young

ROP Models	B&Y							
Model Coefficients	a1	a2	a3	a4	a5	a6	a7	a8
Formations								
Upper Wilcox - Sandstone	5	0.0005	1.00E-06	1.00E-06	0.4	0.21975	1	0.5
Midway - Shale	4.50317	0.000406	1.00E-06	1.00E-06	0.4	0.20856	0.99999	0.5
Taylor - Shale	4.45902	0.000406	1.00E-06	1.00E-06	0.4	0.2066	0.98745	0.49916
Pecan_GAP - Sandstone	3.89618	0.000406	1.00E-06	1.00E-06	0.44576	0.22932	0.98377	0.49916
Upson - Sandstone	4.25617	0.000406	1.00E-06	1.00E-06	0.44576	0.22932	0.98377	0.49916
Austin Chalk - Limestone	3.26698	4.09E-04	1.00E-06	1.00E-06	0.48582	0.2	0.1	0.50095
Eagle Ford Upper - Shale	3.0343	4.05E-04	9.69E-05	1.00E-06	0.57563	0.22274	0.1	0.50095
Eagle Ford Lower - Shale	4.73876	0.000375	7.30E-04	1.00E-06	0.92828	0.20982	0.1	0.55277
Sandstone Average	4.38412	0.00044	1.00E-06	1.00E-06	0.43051	0.22613	0.98918	0.49944
Limestone Average	3.27E+00	4.10E-04	1.00E-06	1.00E-06	0.48582	0.2	0.1	0.50095
Shale Average	4.18E+00	4.00E-04	2.07E-04	1.00E-06	0.57598	0.21193	0.54686	0.51322
Overall Average	4.14E+00	4.10E-04	1.04E-04	1.00E-06	0.51016	0.21576	0.65687	0.50652

Table 22: Coefficient for Hareland's Drag Bit, Motahhari's, and Winters, Warren, and Onyia's Models

Har Drag			Motahhari			WWO			
a	b	c	G	α	γ	a	φ	b	c
5.90623	5.8399	0.0001	10	4.0583	0.0001	1.00E-04	1.00E-04	0.0001	0.0001
5.89758	6.40529	0.04785	10	4.1844	0.0001	1.00E-04	1.00E-04	0.0001	0.0001
132.99982	0.01015	0.00044	5.85401	0.83168	2.17603	5.03E-04	1.00E-04	0.09961	0.00257
94.13393	0.00018	0.00733	5.09808	0.75319	2.16354	1.14E-01	4.37E-03	0.1129	0.00342
133.00013	0.0001	0.01807	5.10523	0.66861	2.2552	1.00E-04	3.53E-02	0.09994	0.0001
5.00E+00	4.19E-02	0.03706	10	4.35E+00	0.17187	1.00E-04	1.00E-04	0.0001	0.00809
5.00E+00	2.02E-02	0.0109	10	2.18E+00	0.88779	1.00E-04	1.00E-04	0.0001	0.01051
4.99886	0.05089	0.03112	10	0.72459	1.32084	1.00E-04	5.88E-03	0.0001	0.00365
77.6801	1.94672	0.0085	6.73443	1.8267	1.47295	3.82E-02	1.33E-02	0.07098	0.00121
4.99884	0.04191	0.03706	10	4.34545	0.17187	1.00E-04	1.00E-04	0.0001	0.00809
37.22397	1.62164	0.02258	8.9635	1.97993	1.09619	2.01E-04	1.55E-03	0.02498	0.00421
48.36688	1.54608	0.01911	8.25716	2.21816	1.12194	1.44E-02	5.76E-03	0.03912	0.00357

5.6.4 Swallis A6 Well

The model with lowest associated errors is again a roller bit model, as seen in table 23, but Motahhari's model has an error less than 1% away from Winters, Warren, and Onyia's. For the Swallis A6 well, Hareland's drag bit model does not have a single formation where it has the lowest error, nor does it behave better anywhere than the other drag bit model, Motahhari's model. Thus, Motahhari's model should be the only model used for this particular data set.

Despite Winters, Warren, and Onyia's model having the lowest error, it is by no means a predictive tool and should not be used in order to predict changes downhole or be used as a planning tool for the next well. Motahhari's model should be used instead, as by applying the correct model choice for the bit being used, one can effectively improve drilling efficiency through analyzing how drilling parameter changes affect the calculated ROP and the field ROP. Tables 24 and 25 illustrate the calculated model coefficients for each model.

Table 23: Swallis A6 Evaluation

Average percent error of each ROP Model by formation:

Formation Name	Bingham	Bourgoyne & Young	Hareland Drag Bit	Hareland Roller Bit	Motahhari PDC Bit	WWO Roller Bit	Best Model in Formation
Sandstone - Sandstone	N/A	54.81%	43.50%	N/A	18.40%	55.03%	Motahhari PDC Bit
Sparta - Sandstone	N/A	59.93%	66.64%	N/A	52.63%	49.74%	WWO Roller Bit
Queen City - Sandstone	N/A	58.63%	63.90%	N/A	51.03%	58.18%	Motahhari PDC Bit
Upper Wilcox - Limestone	N/A	68.87%	57.60%	N/A	55.55%	56.63%	Motahhari PDC Bit
Midway Shale - Shale	N/A	41.37%	52.72%	N/A	57.29%	40.75%	WWO Roller Bit
Navarro Shale - Shale	N/A	26.42%	52.37%	N/A	23.57%	21.12%	WWO Roller Bit
Olmos - Sandstone	N/A	24.89%	33.92%	N/A	24.06%	23.00%	WWO Roller Bit
Taylor - Shale	N/A	36.90%	40.94%	N/A	22.27%	37.33%	Motahhari PDC Bit
Pecan_GAP - Sandstone	N/A	21.78%	37.51%	N/A	17.13%	17.14%	Motahhari PDC Bit
Upson - Sandstone	N/A	14.71%	17.66%	N/A	14.22%	14.81%	Motahhari PDC Bit
Austin_Chalk - Limestone	N/A	23.52%	44.84%	N/A	22.82%	21.59%	WWO Roller Bit
Austin_Chalk - Shale	N/A	27.82%	31.42%	N/A	26.59%	26.92%	Motahhari PDC Bit
Entire Dataset	N/A	45.37%	50.32%	N/A	41.42%	40.56%	WWO Roller Bit

Best Overall Model: WWO Roller Bit

Average Percent Error: 40.56%

Table 24: Swallis A6 Coefficient for Bourgoyne & Young

ROP Models	B&Y							
Model Coefficients	a1	a2	a3	a4	a5	a6	a7	a8
Formations								
Sandstone - Sandstone	5	0.000389	2.58E-05	1.00E-05	1	0.69998	1	0.5
Sparta - Sandstone	2.1887	0.000389	2.58E-05	1.00E-05	0.79918	0.64829	1.4773	0.5
Queen City - Sandstone	2.05105	0.000389	1.00E-06	9.31E-06	0.68056	0.76839	1.5	0.4414
Upper Wilcox - Limestone	1.61604	0.000398	3.52E-04	9.31E-06	0.52319	0.52967	1.222	0.38654
Midway Shale - Shale	3.43071	0.000001	1.00E-06	1.00E-06	0.45055	0.50096	1.08483	0.38615
Navarro Shale - Shale	3.55114	1.00E-06	1.00E-06	1.00E-06	0.4	0.50807	1.07378	0.38538
Olmos - Sandstone	3.5756	4.00E-06	1.00E-06	1.00E-06	0.4	0.51272	1.07377	0.38538
Taylor - Shale	2.92549	0.000035	1.00E-06	1.00E-06	0.56167	1	0.98573	0.38405
Pecan_GAP - Sandstone	3.09345	0.000035	1.00E-06	1.00E-06	0.4	0.86757	0.98573	0.38405
Upson - Sandstone	2.70332	0.000035	1.00E-06	1.00E-06	0.4	0.86757	0.98573	0.38405
Austin_Chalk - Limestone	2.90E+00	3.40E-05	7.05E-06	1.00E-06	0.4	0.86243	0.98573	0.38405
Austin_Chalk - Shale	2.61E+00	3.80E-05	1.00E-06	1.00E-06	0.4	0.89648	0.98573	0.38405
Sandstone Average	3.10E+00	2.10E-04	9.27E-06	5.38E-06	0.61329	0.72742	1.17042	0.43248
Limestone Average	2.26E+00	2.20E-04	1.80E-04	5.16E-06	0.4616	0.69605	1.10387	0.3853
Shale Average	3.13E+00	2.00E-05	1.00E-06	1.00E-06	0.45306	0.72638	1.03252	0.38491
Overall Average	2.97E+00	1.50E-04	3.49E-06	3.88E-06	0.5346	0.72184	1.11336	0.40876

Table 25: Swallis A6 Coefficient for Hareland's Drag Bit, Motahhari's, and Winters, Warren, and Onyia's Models

Har Drag			Motahhari			WWO			
a	b	c	G	α	γ	a	φ	b	c
132.99644	0.01539	0.00037	5.31393	3.07014	1.21398	1.00E-04	1.00E-04	0.10467	0.01336
515.06285	0.01253	0.0201	10	0.27725	2.69481	1.06E-04	3.59E-04	0.09994	0.00179
443.83709	0.00012	0.00054	5.16814	0.32526	2.75279	1.00E-04	1.00E-02	1	0.0001
123.73277	0.00523	0.00213	10	1.10564	1.99793	1.00E-04	1.68E-02	0.0001	0.0001
88.72715	0.00014	0.00016	10	1.0118	1.98372	8.61E-03	1.00E-04	0.09834	0.00236
7.33E+01	1.00E-04	0.00018	10	7.18E-02	2.63707	2.54E-03	1.00E-04	0.09999	0.00233
1.04E+02	5.40E-04	0.00056	5.06712	1.00E-04	2.8208	4.78E-02	1.00E-04	0.1129	0.0022
75.54237	0.0001	0.0001	4.85784	0.38973	2.56657	1.00E-04	1.03E-01	1	0.0001
105.02348	0.00735	0.0001	5.08348	0.0001	2.92354	1.00E-04	1.00E-04	0.09996	0.0053
50.19207	0.00013	0.00013	5.07602	0.00086	2.87524	1.42E-01	1.00E-04	1	0.00572
68.66514	0.00067	0.0001	5.07935	0.26294	2.71052	8.26E-03	1.00E-04	0.10593	0.00518
48.94415	0.00652	0.00239	4.93566	0.0145	2.84664	1.00E-04	1.00E-04	0.08782	0.00764
225.26632	0.00601	0.00363	5.95145	0.61228	2.54686	3.17E-02	1.79E-03	0.40291	0.00474
96.19895	0.00295	0.00112	7.53967	0.68429	2.35423	4.18E-03	8.45E-03	0.05302	0.00264
71.63729	0.00171	0.00071	7.44838	0.37196	2.5085	2.84E-03	2.58E-02	0.32154	0.00311
152.54541	0.00407	0.00224	6.71513	0.54418	2.50197	1.75E-02	1.09E-02	0.31747	0.00385

6. CONCLUDING REMARKS

Although rate of penetration has been under scrutiny for decades, some ROP models do not agree with each other on bit rotation's or weight on bit's relationship to ROP. This issue highlights the complex nature that is to fully comprehend just the intrinsic properties affecting ROP. The probable reason for these differences is that all models were designed based off empirical relationships from the field test data collected. Older models, like Winters, Warren, and Onyia's and Hareland's may have created a relationship between a parameter and ROP based off the entire data curve achieved from the field tests. Motahhari's model though, aware that after a certain WOB, ROP would decrease, may have chosen to only calculate for values within normal drilling situations. As such, the models disagree on how quickly ROP will stop increasing due to an increase in WOB.

By analyzing the models with respect to changes in lithology versus changes in formation, the overall conclusion is that the analysis done by formation improves the results of the models sufficiently so that it should always be used. For the case provided in this paper, the improvement was roughly 5% for each model.

When the data sets for wells drilled with drag bits were run for the drag bit models and two other extensive models for comparison, an interesting result occurred. The least amount of errors was always achieved by a non-drag bit model, despite all the wells in the analysis being drilled by a drag bit. However, Motahhari's model, a drag bit model, always gave the closest physical interpretation and was the best model between the two drag bit models in all cases analyzed. Even though the model generated a greater error, it more closely resembled variations in the data. In fact, both drag bit models were most responsive to changes in the data and are better equipped to account for downhole changes while drilling. This confirms the initial assessment that by applying bit specific models to data with a certain bit, better results are to be expected. Furthermore, interpretations and assessments can be made in order to improve drilling efficiency while drilling or for future operations. But using a non-bit specific model may lead to a better initial planning, as the other models outputted averaged values closer in magnitude to the real data for both the Khangiran field and the drag bit data.

Bingham's model is too rigid for complex wells and will not accurately depict changes in rate of penetration. Despite attempts to tweak the coefficients, the results will remain mostly unchanging. The model is not a meaningful predictive tool. More useful variation of the model is Motahhari's model for drag bits and Winters, Warren, and Onyia's for roller bits. These models include other primary factors affecting ROP.

Bourgoyne & Young's model incorrectly predicts the direction of the increase or decrease of ROP when a drilling break occurs. Thus, this model is sensitive as to what data should be inputted, in this case, requiring the weight on bit parameters inputted at the surface. However, the model is a decent tool for confirming and predicting when a formation change occurs.

Winters, Warren, and Onyia's model is highly dependent on having the correct rock ductility values for each formations. If not, the model can predict sharp changes between formations, despite only a smooth transition occuring. However, even with assumed ductility values, a lithology with lower ductility may still have larger ROP from other effects. Primarily, the coefficients are the parameters affecting the end result for ROP the most. Much like Bingham's model, despite large variances in the data, the model behaves almost as a straight line through the data.

Despite separating Hareland's drag bit model analysis to different forations, the model behaves less effectively for very shallow sections of the well, grossly underestimating ROP, and often behaves opposite to what is indicated by the field data as well. The model also generates grossly low ROP values whenever the weight on bit is very small. For those scenarios, the results are too many standard deviations away to be of any relevance.

Hareland's roller bit model is the worst model in predicting changes for all sections of the well. Similarly to Winters, Warren, and Onyia's and Bingham's model, Hareland's is mostly a straight line through all the data sets. However, the model yields very low error percentages, and so whoever wants to apply the various models to a data set, or use them to model a future well, has to double check to make sure that this model is accounting for variance.

By running Motahhari's model in the analysis separated by formation, the model no longer greatly overestimates rate of penetration, but the sensitivity of the model to the large range of values for a given section was reduced. In other words, the model predicts rate of penetration better, but will not compute variations within a section very well. Whoever applies this model to a given data set, must go further and use other models to determine where changes within a formation occur.

As a general assessment, good practices on the decisions for selecting the best ROP model are as follows:

Never utilize Bingham's model. Motahhari's model is the expanded version to run for drag bits, and Winters, Warren, and Onyia's the one for roller bits. If excellent data is expected, Bourgoyne & Young's model should be used regardless of bit choice to further the analysis and confirm sharp changes within a formation and formation changes. Hareland's roller bit model has very low percentage errors regardless of bit selection, and thus can be used as an initial first indicator for designs.

For wells drilled with roller bits, if good rock ductility data is provided, Winters, Warren, and Onyia's model is the main tool while drilling and for future well design. If not, use Hareland's roller bit model. However, Hareland's model can sometimes be quite unchanging, and will need to be verified against another model, Bourgoyne & Young's or even a drag bit model in order to assess if the data itself is not changing much or if Hareland's model is not being responsive enough. In a last case, if there is a lack of rock ductility data, ignoring rock ductility term in Winters, Warren, and Onyia's model can be done to confirm percentage effects of one of the following specifics: bit diameter, weight on bit, bit rotation, or confined compressive strength. However, the magnitude of the change will be incorrect.

For a well drilled with a drag bit, Motahhari's is the main tool while drilling and for future well designs. The model has to be run by formation. Defining formation tops, bottoms, and changes is of utmost importance so that Motahhari's model does not overpredict ROP, however, much like Hareland's roller bit model, running other models will be necessary in order to verify the nature of the data and to help when planning ahead. Hareland's drag bit model can be used as a secondary tool, but keep in mind the model does not behave well for low weight on bit and in shallow formations.

Since none of the models include equations for establishing bit wear in real time, and only include bit wear as a correction factor when the bit is finally pulled, a better of interacting bit wear into the models is by including instead a coefficient on the bit parameter to estimate wear, as this would allow for iterations to be done inside each formation, rather than at each bit change, in order to more closely determine bit wear.

As companies drill deeper wells and utilize of more complicated drilling techniques, more and more the overbalance and underbalance become an important aspect of drilling. And so, rate of penetration modeling must have in its top priority correctly assessing how differential pressure will affect the penetration rates. As such, differential pressure should be included in all future models in order to correctly describe the physical properties at play during drilling.

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