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Transmission Expansion Planning Considering Substation Arrangements

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Transmission Expansion Planning Considering Substation Arrangements

by

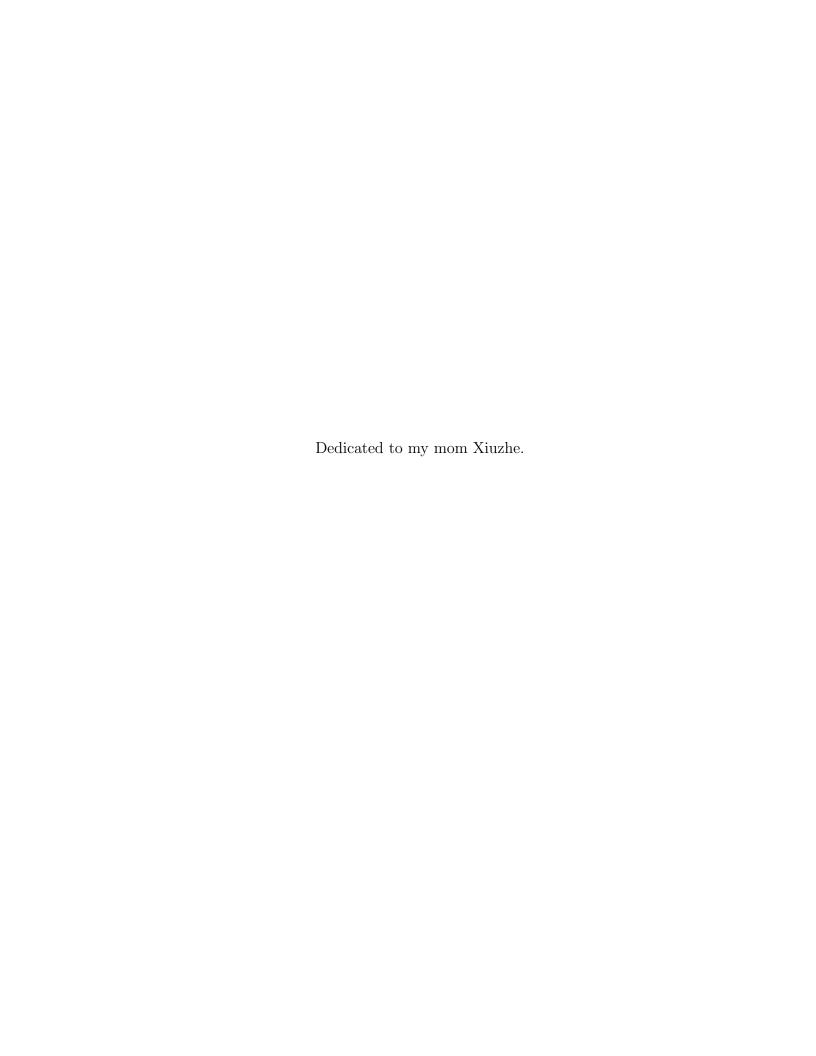
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THESIS

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Transmission Expansion Planning Considering

Substation Arrangements

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The University of Texas at Austin, 2017

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Transmission expansion planning (TEP) is aimed at expanding the ex-

isting transmission system to satisfy potential power demand growth and fu-

ture power plant expansion. Generally speaking, the TEP problem can be

mathematically modeled as a large scale, non-convex, and non-linear opti-

mization problem. Uncertainties causing by development of renewable energy,

electricity market, and load fluctuations are also taken into consideration. The

tradition TEP problem can be solved using stochastic mixed integer linear

programming and contingency analysis. However, the practical application of

TEP problems generates some questions.

This thesis mainly focuses on certain restrictions ignored by traditional

TEP problem formulation, which are important in practice and will change the

optimal solution completely. By adding certain restrictions based on spacing

arrangements on substations, TEP problems can be solved more efficiently and

will be more valuable for industry.

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Chapter 1

Nomenclature

Sets	
N_b	set of all buses, the indices are denoted by k
N_q	set of all generators
N_l	set of all existing and candidate lines, denoted by l
N_{le}	set of all existing lines
N_{ln}	set of all new lines
N_{lk}	set of all lines connected to bus k
N_{gk}	set of all generators connected to bus k
N_{wk}	set of all wind generators connected to bus k
S_{stq}	set of all scenarios for stage stg , denoted by s_{stg}
\ddot{FBUS}	set of from buses
TBUS	set of to buses

Variables	
	binary variables to represent if line l is built for stage
	stg, 1 for built, 0 otherwise
$CD_{k,c}^{s_{stg}}$	MW load curtailment at bus k under stage stg , contin-
и,с	gency c and scenario s
$CW_{k,c}^{s_{stg}}$	MW wind curtailment at bus k under stage stg , contin-
	gency c and scenario s
$Pg_{k,c}^{s_{stg}}$	MW generator output at bus k under stage stg , contin-
	gency c and scenario s
$f_{l,c}^{s_{stg}}$	active power flow in line l under stage stg , contingency
	c and scenario s
$f_{l,c}^{s_{stg}}$	MW power flow on line l under stage stg , contingency c
,	and scenario s
$ heta_{k,c}^{s_{stg}}$	voltage angle at bus k under stage stg , contingency c and
•	scenario s

Parameters

 γ_k penalty cost on load shedding per MW at bus k penalty cost on wind curtailment per MW at bus k

 α_l cost of building lines l per mile

 L_l length of line l

 $Pd_k^{s_{stg}}$ demand at bus k for stage stg

 B_l admittance of line l

 Pw_k^{max} maximum capacity of of wind generators connected to bus k Pw_k^{min} minimum capacity of of wind generators connected to bus k

 Pg^{max} maximum capacity of generator Pg^{min} minimum capacity of generator f_l^{max} maximum capacity of line l minimum capacity of line l

M a large positive constant number

C number of contingency, which is $N_l + 1$

 $Pw_{k,c}^{s_{stg}}$ MW wind generator output at bus k under contingency c and

scenario s_{stg}

 $Limit_k$ Substation spacing limitation at bus k

Chapter 2

Introduction

2.1 Definition

The transmission expansion planning problem can be formulated as an optimization problem aimed at finding out an optimal choice of lines and equipments to expand the existing transmission network[1]. This type of problem is generally subject to several constraints involving power balance, generation limitation, security, and reliability. TEP problem in nature is a non-linear and non-convex problem. [1] Together with the fact that it usually is a large-scale problem, to solve this type of the problem, solution techniques and decomposition methods are critical.

2.2 Solution Techniques

Based on [2], transmission expansion problem can mostly be formulated as linear programming problem [3], dynamic programming problem [4], nonlinear programming problem[5], and mixed integer programming problem [6]-[7]. Decomposition methods including Bender's and hierarchical decomposition are widely applied in TEP problems; see [8]-[9].

In recent years, there is a worldwide trend of integration of renewable energy sources, wind and solar included. Renewable energy sources especially wind and solar are usually highly variable due to their uncertain nature. Together with electricity markets and demand fluctuation, uncertainties in transmission networks cannot be ignored in transmission expansion planning. As a consequence, a stochastic model is introduced to cover the uncertainty in the transmission system; see [10].

In this thesis, the problem is formulated as a stochastic mixed-integer linear programming problem. General model for the problem will be discussed later.

2.3 General Model

A general transmission optimization model can be formulated as

Objective = min Losses/Investment/Reliability Cost + E[Uncertainty Related]
s.t. Power Balance Constraints
Power Flow Limits
Generation Output Limits
Other limits

The description will be elaborated in detail in the following sections.

2.3.1 Objective

The objective in a TEP problem can be varied under different purposes. Some TEP problems are primarily designed to lower transmission losses, and some focus on enhancement of reliability but most commonly, TEP problems are formulated for the purpose of minimizing the capital cost.

In this thesis, the main concern is to obtain an optimal capital cost. Meanwhile a reliability problem will be considered. Moreover, to fully exploit renewable energy source, specifically wind power in this problem, minimization of wind curtailment will be also addressed.

2.3.2 Constraints

The constraints of the TEP problem include power balance, flow limits, output limits and several particular limitations applied under different situations.

Power Balance Constraints In general, power balance means for every node in the system, the power, both active and reactive power injected and sent out should be balanced, which is

$$\forall k \in N_b P_k = P_{Gk} - P_{demand} \tag{2.1}$$

$$Q_k = Q_{Gk} - Q_{demand} \tag{2.2}$$

Since in transmission system, reactive power balance is assumed to be fully satisfied, (2.2) can be ignored.

 P_k is total real power injected into bus i, and wind and demand curtailments are taken into consideration, so (2.1) can be reformulated as

$$-\sum_{i \in N_{lk}} f_i + Pg_k + Pw_k - CW_k + CD_k - Pd_k = 0$$
 (2.3)

Linearized Power Flow By nodal equations,

$$P_k = V_k \sum_{j=1} V_j (G_{kj} \cos \theta_{kj} + B_{kj} \sin \theta_{kj})$$
(2.4)

$$Q_k = V_k \sum_{j=1} V_j (G_{kj} \sin \theta_{kj} + B_{kj} \cos \theta_{kj})$$
(2.5)

can be derived.

However, in transmission system, as reactive power is well supplied, the voltage is maintained almost constant at 1.0 p.u.. Moreover, resistance in transmission network is in generally much smaller than reactance, so R can be ignored. Furthermore, phase angles are generally close to zero in transmission system, which means $\lim_{\theta\to 0} \sin\theta = \theta$. As a consequence, a linearized simplification of (2.4) can be obtained after considering curtailments of wind generation and demand,

$$f_l = B_l(\theta_{fbl} - \theta_{tbl}) \tag{2.6}$$

Generation Output Limitations There is a limitation for generation plants, which can be represented as

$$Pg^{min} \le Pg \le Pg^{max} \tag{2.7}$$

$$Pw^{min} \le Pw \le Pw^{max} \tag{2.8}$$

Other limitations There are other constraints due to the nature of parameters. For example, wind curtailment could never be negative. Phase angles

are limited within certain range. There are shown as follows:

$$0 \le CW \le P_w \tag{2.9}$$

$$0 \le CD \le P_d \tag{2.10}$$

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \tag{2.11}$$

(2.12)

Chapter 3

Formulation and Decomposition

The problem can be formulated as a two stage stochastic optimization problem. In a two-stage stochastic optimization problem, the first stage decision will be influenced by second stage actions. Second stage actions will be under the influence of the uncertainties in the problem and also first-stage decisions.

To solve the problem, several techniques are introduced, including Sample Average Approximation (SAA). Scenarios of the problem are sampled using Monte Carlo.

3.1Original Formulation

Model and Model Explanation

3.1.1.1Model

Here presents the original model, which is consistent with the framework shown in 2.3 in Chapter 1:

$$\min \sum_{l \in N_{ln}} \alpha_l L_l x_l + E[h(x, \epsilon)]$$
(3.1)

s.t.
$$x_l \in \{0, 1\}$$
 $\forall l \in N_l$ (3.2)
 $x_l = 1$ $\forall l \in N_{le}$ (3.3)

$$x_l = 1 \qquad \forall l \in N_{le} \tag{3.3}$$

The line-building decisions are determined by minimizing the total cost: investment cost, wind curtailment cost, and reliability cost. Wind curtailment cost and reliability cost depend on the uncertainties on loads and wind generation output, which forms the second stage problem. Together with limitations on (2.3),(2.4),(2.6)-(2.11), second stage problem can be formulated (shown as following).

where,
$$(3.4)$$

$$h(x,\epsilon) = \min \sum_{c \in C} \sum_{k \in N_b} (q_k C W_{k,c} + \gamma_k C D_{k,c})$$
(3.5)

s.t.
$$M(1 - C_{l,c}x_l) \ge f_{l,c} - B_l(\theta_{fbl,c} - \theta_{tbl,c})$$
 $\forall l \in N_l, c \in C$ (3.6)
 $-M(1 - C_{l,c}x_l) \le f_{l,c} - B_l(\theta_{fbl,c} - \theta_{tbl,c})$ $\forall l \in N_l, c \in C$ (3.7)

$$-\sum_{i \in N_{lk}} f_{i,c} + \sum_{j \in N_{gk}} Pg_{j,c} + \sum_{n \in N_{wk}} (Pw_{n,c}(\epsilon) - CW_{n,c})$$

$$+CD_{i,c} - Pd_{i,c}(\epsilon) \quad \forall k \in N, c \in C$$

$$(3.8)$$

$$+CD_{k,c} = Pd_k(\epsilon) \qquad \forall k \in N_b, c \in C$$

$$(C_{l,c}x_l)f_l^{min} \leq f_{l,c} \leq (C_{l,c}x_l)f_l^{max} \qquad \forall l \in N_l, c \in C$$

$$(3.9)$$

$$Pg^{min} \le Pg_c \le Pg^{max} \qquad \forall c \in C \tag{3.10}$$

$$Pw^{min} \le Pw_c(\epsilon) \le Pw^{max} \quad \forall c \in C$$
 (3.11)

$$0 \le CD_{k,c} \le d_k \qquad \forall k \in N_b, c \in C \tag{3.12}$$

$$0 \le CD_{k,c} \le d_k \qquad \forall k \in N_b, c \in C$$

$$-\frac{\pi}{2} \le \theta_{k,c} \le \frac{\pi}{2} \qquad \forall k \in N_b, c \in C$$

$$(3.12)$$

$$0 \le CW_{k,c} \le Pw(\epsilon) \qquad \forall k \in N_w, c \in C \tag{3.14}$$

3.1.1.2Model Explanation

The explanation of the model is given as following.

For (3.9), When line is not built or it is under contingency analysis, there should be no power flow, which means $f_l = 0$. Otherwise, power flow should be limited by its upper and lower limits.

(3.6) and (3.7) are transformed from (2.6), which is power flow constraints. Similar analysis is applied. When the line is built and it is not under contingency analysis, which means $x_l = 1$ and $C_l = 1$, the combination of (3.6) and (3.7) is the same as (2.6). In contrast, if the line is not built or it is under contingency analysis, $x_l = 0$ or $C_l = 0$ and by (3.9), $f_{l,c} = 0$. Since M is large enough, there is approximately no limitation for the difference in phase angle between the two unconnected buses that would have been joined by line l if it were built and in-service.

Limitations on dispatchable generators and wind generators are given in (3.10) and (3.11) separately. Moreover, (3.12) and (3.14) represent load and wind curtailment. Furthermore, phase angle is limited in (3.13).

3.1.2 Model After Monte Carlo Sampling

Since the approximate distribution for wind speed (Weibull Distribution) and demand is continuous, for the sake of calculation, Monte Carlo sampling method is used to approximate the original problem. By Monte Carlo method, the objective can be reformulated as:

$$\min \sum_{l \in N_{ln}} \alpha_l L_l x_l + \sum_{s \in S} p^s \left[\sum_{c \in C} \sum_{k \in N_b} (q_k C W_{k,c}^s + \gamma_k C D_{k,c}^s) \right]$$
(3.15)

$$-\sum_{i \in N_{lk}} f_{i,c}^s + \sum_{j \in N_{gk}} Pg_{j,c}^s + \sum_{n \in N_{wk}} (Pw_{n,c}^s - CW_{n,c}^s) + CD_{k,c}^s$$
(3.16)

$$=Pd_k^s \quad \forall s \in S, k \in N_b, c \in C$$

$$-M_l(1 - C_{l,c}x_l) \le f_{l,c}^s - B_l(\theta_{fbl,c} - \theta_{tbl,c}) \qquad \forall s \in S, l \in N_l, c \in C$$

$$(3.17)$$

$$M_l(1 - C_{l,c}x_l) \ge f_{l,c}^s - B_l(\theta_{fbl,c} - \theta_{tbl,c}) \qquad \forall s \in S, l \in N_l, c \in C \quad (3.18)$$

$$(C_{l,c}x_l)f_l^{min} \le f_{l,c}^s \le (C_{l,c}x_l)f_l^{max} \qquad \forall s \in S, l \in N_l, c \in C$$

$$(3.19)$$

$$Pg^{min} \le Pg_c^s \le Pg^{max} \quad \forall s \in S, c \in C$$

$$(3.20)$$

$$0 \le CD_{k,c}^s \le d_k^s \qquad \forall s \in S, k \in N_b, c \in C$$

$$(3.21)$$

$$0 \le CD_{k,c}^{s} \le d_{k}^{s} \qquad \forall s \in S, k \in N_{b}, c \in C$$

$$-\frac{\pi}{2} \le \theta_{k,c}^{s} \le \frac{\pi}{2} \qquad \forall s \in S, k \in N_{b}, c \in C$$

$$(3.21)$$

$$0 \le CW_{k,c}^s \le Pw^s \qquad \forall s \in S, k \in N_w, c \in C$$
(3.23)

3.2 Modified Formulation

Traditional TEP formulation only concerns the optimal decision to build transmission lines. Generally, the spacing problem to build a line is ignored. This mostly will not cause problems. However, there are situations where existing substations were built in a narrow neighborhood or somewhere that is too expensive to expand the original substations. Given that, in certain planning problems, the implementation of a spacing constraint is necessary. As a result, a new constraint is introduced in (3.24).

To solve the problem, an additional constraint shown below is added

into (3.16)-(3.23).

$$\sum_{l \in N_{lk}} x_l \le Limit_k \qquad \forall k \in N_b \tag{3.24}$$

3.3 Decomposition

For the formulation from (3.15)-(3.24), the main goal is to find the optimal solution for all contingencies. It will work successfully for small amount of lines to be built (shown in 4-bus test in the following chapter). However, it will be time-consuming and also have strict requirements of memory for large amount of promising lines. To solve the problem, decomposition is applied.

The problem becomes as following:

$$\sum_{c \in C} \min \sum_{l \in N_{ln}} \alpha_l L_l(x_l^c - x_l^{c-1}) + \sum_{s \in S} p^s \left[\sum_{k \in N_b} (q_k C W_{k,c}^s + \gamma_k C D_{k,c}^s) \right]$$
(3.25)

$$-\sum_{i\in N_{lk}}^{s} f_{i,c}^{s} + \sum_{j\in N_{gk}} Pg_{j,c}^{s} + \sum_{n\in N_{wk}} (Pw_{n,c}^{s} - CW_{n,c}^{s}) + CD_{k,c}^{s}$$
(3.26)

$$=Pd_k^s \quad \forall s \in S, k \in N_b, c \in C$$

$$-M_l(1 - C_{l,c}x_l) \le f_{l,c}^s - B_l(\theta_{fbl,c} - \theta_{tbl,c}) \qquad \forall s \in S, l \in N_l, c \in C$$

$$(3.27)$$

$$M_l(1 - C_{l,c}x_l) \ge f_{l,c}^s - B_l(\theta_{fbl,c} - \theta_{tbl,c}) \qquad \forall s \in S, l \in N_l, c \in C$$
(3.28)

$$(C_{l,c}x_l)f_l^{min} \le f_{l,c}^s \le (C_{l,c}x_l)f_l^{max} \qquad \forall s \in S, l \in N_l, c \in C$$

$$(3.29)$$

$$Pg^{min} \le Pg_c^s \le Pg^{max} \quad \forall s \in S, c \in C$$
 (3.30)

$$0 \le CD_{k,c}^s \le d_k^s \qquad \forall s \in S, k \in N_b, c \in C$$

$$(3.31)$$

$$0 \le CD_{k,c}^s \le d_k^s \qquad \forall s \in S, k \in N_b, c \in C$$

$$-\frac{\pi}{2} \le \theta_{k,c}^s \le \frac{\pi}{2} \qquad \forall s \in S, k \in N_b, c \in C$$

$$(3.31)$$

$$0 \le CW_{k,c}^s \le Pw^s \qquad \forall s \in S, k \in N_w, c \in C$$

$$x_l^c \ge x_l^{c-1}, c \in C \qquad \forall l \in N_l$$

$$(3.33)$$

$$x_l^c \ge x_l^{c-1}, c \in C \qquad \forall l \in N_l \tag{3.34}$$

Shown in the formulation, instead of solving all the contingencies at one time, the problem is reformulated so that at each iteration, only one contingency is taken into account, and at next iteration, the decision made in last iteration is preserved.

 $\forall x^* \in argmin\{(3.15)\text{-}(3.24)\}, \ x^* \text{ is a feasible solution of } (3.25)\text{-}(3.34).$ This means minimum of $(3.25)\text{-}(3.34) \geq \text{minimum of } (3.15)\text{-}(3.24).$ Although the value may not be exactly the same, the optimal solution of (3.25)-(3.34) can still provide valuable information of the optimal solution of (3.15)-(3.24). In the meantime, calculating using the formulation of (3.25)-(3.34) will save more time and memory.

To be noticed, the lines constructed in one contingency should be considered for the next contingency, which is (3.34).

Chapter 4

Tests and Conclusion

In this chapter, a few tests and results are displayed.

4.1 4-Bus Test

4.1.1 Problem Description

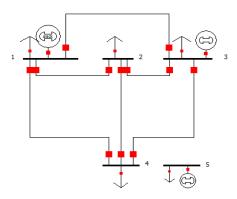


Figure 4.1: 4 Bus Test Original

Originally, the network is a 4-bus network. An industrial load is going to be constructed. To meet demand growth, a wind farm is built and is planned to connect with the existed transmission network. The optimal connecting plan should be found with the consideration of security. Fig. 4.1 shows the original structure.

4.1.2 Data Set

Table 4.1: Bus Data for 4 Bus Test

Bus No.	Max Gen Output(MW)	Line No. Limit
1	400	3
2	0	6
3	500	4
4	0	6
5	300	6

Table 4.2: Exist Line Data for 4 Bus Test

Line No.	From Bus	To Bus	Reactance(p.u.)	Flow Limit(p.u.)
1	1	2	0.02	1
2	1	4	0.03	0.8
3	1	3	0.01	1
4	2	3	0.01	1
5	2	4	0.02	1
6	3	4	0.01	1

The data of scenarios and new lines are shown in the Appendix 1.

4.1.3 Solution Without Consideration of Security

4.1.3.1 Without Spacing Limit

The solution to the problem when spacing limit is not considered is connecting the new bus 5 with original bus 1, which can be shown in the Fig. 4.2.

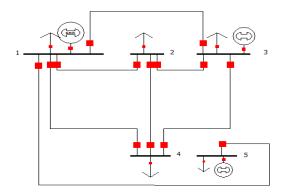


Figure 4.2: 4 Bus Test Without Security Without Spacing

The total cost of that is $$1.10025933 \times 10^{12}$. The solution time without spacing limit is 0.23s.

4.1.3.2 With Spacing Limit

After considering spacing constraint, the total cost becomes $$1.10025934 \times 10^{12}$. The new strategy will be connecting bus 5 with bus 3 shown below in Fig. 4.3. The solution is obtained in 0.20s.

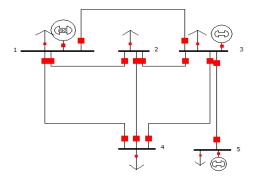


Figure 4.3: 4 Bus Test Without Security With Spacing

4.1.4 Solution With Consideration of Security

4.1.4.1 Without Spacing Limit

The total cost of the problem becomes $\$2.97069253 \times 10^{13}$. The optimal solution requires the construction of four lines where two are from bus 1 to bus 5, and others are from bus 3 to bus 5. The program ends in 10.45s.

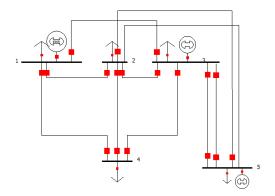


Figure 4.4: 4 Bus Test With Security Without Spacing

4.1.4.2 With Spacing Limit

 $$2.97069256 \times 10^{13}$ is the optimal cost. It takes 7.08s to achieve the optimum. Three lines are being built. Two of them are from bus 2 to bus 5. The other one is from bus 3 to bus 5.

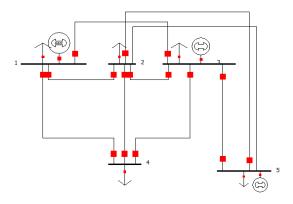


Figure 4.5: 4 Bus Test With Security With Spacing

4.1.5 Solution Summary Without Decomposition

Table 4.3: Final Result Without Decomposition for 4 Bus Test

Security	Spacing	Optimum	Time
No	No	$$1.10025933 \times 10^{12}$	0.23s
No	Yes	$$1.10025934 \times 10^{12}$	0.20s
Yes	No	$$2.97069253 \times 10^{13}$	10.45s
Yes	Yes	$$2.97069256 \times 10^{13}$	7.08s

Interestingly, the cases with a spacing constraint solve faster than the cases without.

4.1.5.1 Solution Summary After Decomposition

Similar analysis is completed using decomposition method. The results are shown on the table below.

Table 4.4: Final Result With Decomposition for 4 Bus Test

Security	Spacing	Optimum	Time
Yes	No	$$2.97069253 \times 10^{13}$	29.62s
Yes	Yes	$$2.9707082 \times 10^{13}$	29.10s

4.1.6 Analysis

From above, it is clear that with spacing limits, the optimal solution can be obtained more quickly than that without spacing limits when decomposition is not applied. When decomposition is applied, operational time is mainly determined by the number of promising lines, but it helps with time saving. The results from method with decomposition are close to that from method without decomposition.

4.2 14 Bus Test

4.2.1 Problem Description

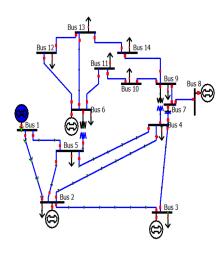


Figure 4.6: 14 Bus Test Original

An offshore wind farm is planned to be connected to the existing 14-bus transmission system. The wind farm is going to meet a upcoming large industrial load nearby and growing loads in the network. The problem is to find out an optimal strategy with contingency analysis. The original test system is presented in Fig. 4.6.

4.2.2 Data Set

Table 4.5: Bus Data for 14 Bus Test

Table 4.5: Bus Data for 14 Bus Test					
Bus No.	Max Gen Output(MW)	Line No. Limit			
1	800	6			
2	700	8			
3	500	5			
4	0	8			
5	0	7			
6	500	9			
7	0	5			
8	500	6			
9	0	6			
10	0	4			
11	0	4			
12	0	4			
13	0	4			
14	0	5			
15	0	4			
16	0	4			

Table 4.6: Exist Line Data for 14 Bus Test					
From Bus	To Bus				
1	2	0.0023668	1		
1	5	0.0089216	1		
2	3	0.0079188	0.8		
2	4	0.0070528	1		
2	5	0.0069552	0.6		
3	4	0.0068412	1		
4	5	0.0016844	0.95		
4	7	0.0083648	1		
4	9	0.0222472	1		
5	6	0.0100808	0.8		
6	11	0.007956	1		
6	12	0.0102324	0.7		
6	13	0.0052108	1		
7	8	0.007046	0.9		
7	9	0.0044004	1		
9	10	0.00338	1		
9	14	0.0108152	1		
10	11	0.0076828	0.7		
12	13	0.0079952	1		
13	14	0.0139208	1		

All of the scenarios and new line list are on Appendix 2.

4.2.3 Result

Table 4.7: Final Result for 14 Bus Test

Security	Spacing	Optimum	Time
No	No	$$1.37 \times 10^{12}$	16.72s
No	Yes	$$1.80 \times 10^{12}$	8.55s
Yes	No	$$1.92 \times 10^{14}$	476.56s
Yes	Yes	$$1.94 \times 10^{14}$	466.41s

4.2.4 Analysis

In 14-bus case, due to the high amount of promising line, without decomposition method, the transmission optimal planning cannot be obtained in certain system setting. Similar observation can be found in 14-bus test. With spacing constraint, it will take less time to get the optimal solution.

4.3 Conclusion

From the analysis in both 4-bus case and 14-bus case, the inclusion of spacing limit can reduce the running time. If the behavior is repeated for other systems, it will be beneficial in large scale transmission expansion planning problem. Besides, with the constraint, the optimizer of the problem can be completely different from that without the constraint. The implementation of the constraint is necessary and practical for certain type of projects. The connection between timing and constraint can be an intriguing topic to study. Moreover, the necessities and feasibility of applying substation planning to transmission expansion planning can be further studied.

Appendices

Appendix A

Data Sets for 4 Bus Test

Table A.1: Scenarios-Probability for 4 Bus Test

Probability Scen 1		Scen 2	Scen 3	Scen 4
	0.2	0.3	0.3	0.2

Table A.2: Scenarios-Wind for 4 Bus Test

Bus No.	Scen 1	Scen 2	Scen 3	Scen 4
1	160	144	80	16
2	0	0	0	0
3	200	160	100	20
4	0	0	0	0
5	200	160	100	20

Table A.3: Scenarios-Demand for 4 Bus Test

Bus No.	Scen 1	Scen 2	Scen 3	Scen 4
1	75.2	136	160	192
2	225.6	408	480	576
3	37.6	68	80	96
4	150.4	272	320	384
5	225.6	408	480	576

	Table A.4: New Line Data for 4 Bus Test						
Line No.	From Bus	To Bus	Reactance	Flow Limit	Length		
1	1	2	0.02	1	40		
2	1	3	0.019	1	38		
3	1	4	0.03	0.8	60		
4	1	5	0.01	1	20		
5	2	3	0.01	1	20		
6	2	4	0.02	1	40		
7	2	5	0.0155	1	31		
8	3	4	0.0295	0.82	59		
9	3	5	0.01	1	20		
10	4	5	0.0315	0.9	63		
11	1	2	0.02	1	40		
12	1	3	0.019	1	38		
13	1	4	0.03	0.8	60		
14	1	5	0.01	1	20		
15	2	3	0.01	1	20		

0.02

0.0155

0.0295

0.01

0.0315

0.82

0.9

Appendix B

Data Sets for 14 Bus Test

Table B.1: Scenarios-Probability for 14 Bus Test

Probability Scen 1		Scen 2	Scen 3	Scen 4
	0.2	0.3	0.3	0.2

Table B.2: Scenarios-Demand for 14 Bus Test

Bus No.	Scen 1	Scen 2	Scen 3	Scen 4
1	0	0	0	0
2	108.5	151.9	217	260.4
3	471	659.4	942	1130.4
4	239	334.6	478	573.6
5	38	53.2	76	91.2
6	56	78.4	112	134.4
7	0	0	0	0
8	0	0	0	0
9	147.5	206.5	295	354
10	45	63	90	108
11	17.5	24.5	35	42
12	30.5	42.7	61	73.2
13	67.5	94.5	135	162
14	74.5	104.3	149	178.8
15	70	98	140	168
16	0	0	0	0

Table B.3: Scenarios-Wind for 14 Bus Test Bus No. | Scen 1 | Scen 2 | Scen 3 | Scen 4

	Table B.4: New Line Data for 14 Bus Test-Part1						
Line No.	From Bus	To Bus	Reactance	Flow Limit	Length		
1	1	2	0.0023668	1	79		
2	1	3	0.0023668	0.9	120		
3	1	4	0.0023668	1	115		
4	1	5	0.0089216	1	40		
5	1	6	0.0023668	1	39		
6	1	7	0.0089216	0.85	126		
7	1	8	0.0089216	1	149		
8	1	9	0.0089216	1	137		
9	1	10	0.0023668	1	98		
10	1	11	0.0089216	1	87		
11	1	12	0.0089216	1	73		
12	1	13	0.0023668	1	89		
13	1	14	0.0089216	1	102		
14	1	15	0.0089216	1	160		
15	1	16	0.0089216	0.87	111		
16	2	3	0.0079188	1	85		
17	2	4	0.0070528	1	92		
18	2	5	0.0069552	1	45		
19	2	6	0.0070528	0.947	84		
20	2	7	0.0079188	1	114		

Table B.5: New Line Data for 14 Bus Test-Part2						
Line No.	From Bus	To Bus	Reactance	Flow Limit	Length	
21	2	8	0.0069552	1	133	
22	2	9	0.0079188	1	135	
23	2	10	0.0069552	1	136	
24	2	11	0.0070528	1	133	
25	2	12	0.0079188	1	100	
26	2	13	0.0070528	0.75	137	
27	2	14	0.0069552	1	140	
28	2	15	0.0079188	1	141	
29	2	16	0.0070528	1	154	
30	3	4	0.0068412	0.695	77	
31	3	5	0.0068412	1	93	
32	3	6	0.0068412	1	102	
33	3	7	0.0068412	1	94	
34	3	8	0.0068412	1	97	
35	3	9	0.0068412	1	103	
36	3	10	0.0068412	1	103	
37	3	11	0.0068412	1	116	
38	3	12	0.0068412	0.75	135	
39	3	13	0.0068412	1	138	
40	3	14	0.0068412	1	128	
41	3	15	0.0068412	1	44	
42	3	16	0.0068412	1	153	
43	4	5	0.0016844	1	73	
44	4	6	0.0016844	0.98	54	
45	4	7	0.0083648	1	29	
46	4	8	0.0083648	1	25	
47	4	9	0.0222472	1	41	
48	4	10	0.0222472	1	53	
49	4	11	0.0222472	1	67	
50	4	12	0.0016844	1	104	

1	1able B.0: New Line Data for 14 Bus Test-Part3						
Line No.	From Bus	To Bus	Reactance	Flow Limit	Leng		
51	4	13	0.0083648	0.74	97		
52	4	14	0.0222472	1	85		
53	4	15	0.0222472	1	46		

Line No.	From Bus	To Bus	Reactance	us Test-Part3 Flow Limit	Length
51	4	13	0.0083648	0.74	97
52	4	14	0.0222472	1	85
53	4	15	0.0222472	1	46
54	4	16	0.0222472	1	134
55	5	6	0.0100808	0.96	29
56	5	7	0.0100808	1	84
57	5	8	0.0100808	1	100
58	5	9	0.0100808	1	92
59	5	10	0.0100808	1	79
60	5	11	0.0200808	0.85	64
61	5	12	0.0100808	1	67
62	5	13	0.0100808	1	94
63	5	14	0.0080808	1	109
64	5	15	0.0100808	1	108
65	5	16	0.0100808	1	109
66	6	7	0.007956	1	63
67	6	8	0.0102324	1	85
68	6	9	0.0052108	1	61
69	6	10	0.0102324	0.75	43
70	6	11	0.007956	1	38
71	6	12	0.0102324	1	47
72	6	13	0.0052108	1	48
73	6	14	0.0052108	1	50
74	6	15	0.0102324	1	104
75	6	16	0.007956	0.89	43
76	7	8	0.007046	1	21
77	7	9	0.0044004	1	22
78	7	10	0.0222472	1	30
79	7	11	0.0222472	1	45
80	7	12	0.0222472	1	131
81	7	13	0.0016844	1	134

Τ	Table B.7: New Line Data for 14 Bus Test-Part4						
Line No.	From Bus	To Bus	Reactance	Flow Limit	Length		
82	7	14	0.0083648	0.9	78		
83	7	15	0.0222472	1	82		
84	7	16	0.0222472	1	129		
85	8	9	0.0222472	1	16		
86	8	10	0.0100808	1	39		
87	8	11	0.0100808	1	62		
88	8	12	0.0100808	1	125		
89	8	13	0.0100808	1	117		
90	8	14	0.0100808	1	73		
91	8	15	0.0200808	1	62		
92	8	16	0.0100808	1	125		
93	9	10	0.0100808	1	21		
94	9	11	0.0102324	0.7	59		
95	9	12	0.007956	1	105		
96	9	13	0.0102324	1	109		
97	9	14	0.0108152	1	57		
98	9	15	0.0076828	1	71		
99	9	16	0.0108152	1	110		
100	10	11	0.0076828	1	27		
101	10	12	0.0052108	1	61		
102	10	13	0.0102324	1	67		
103	10	14	0.007956	1	37		
104	10	15	0.007046	1	84		
105	10	16	0.0044004	1	79		
106	11	12	0.0222472	0.8	59		
107	11	13	0.0222472	1	37		
108	11	14	0.0222472	1	25		
109	11	15	0.0016844	1	104		
110	11	16	0.0083648	1	57		

Table B.8: New Line Data for 14 Bus Test-Part5

Line No.	From Bus	To Bus	Reactance	Flow Limit	Length
111	12	13	0.0079952	1	27
112	12	14	0.0079952	1	53
113	12	15	0.0109952	1	139
114	12	16	0.0079952	0.85	40
115	13	14	0.0139208	1	41
116	13	15	0.0139208	1	139
117	13	16	0.0052108	1	37
118	14	15	0.0102324	0.9	103
119	14	16	0.0044004	1	47
120	15	16	0.007046	1	140

Bibliography

- [1] H. Zhang, V. Vittal, G. Thomas, and J. Quintero. A mixed-integer linear programming approach for multi-stage security constrained transmission expansion planning. *IEEE Transactions on Power Systems*, 27(2):1125– 1133, May 2012.
- [2] G. Latorre, R. Cruz, J. Areiza, and A. Villegas. Classification of publications and models on transmission expansion planning. *IEEE Transactions on Power Systems*, 18(2):938–946, May 2003.
- [3] R.Villasana, L.L. Garver, and S.J. Salon. Transmission network planning using linear programming. *IEEE Transactions on Power Systems*, PAS-104(2):349–356, Feb. 1985.
- [4] Y.P. Dusonchet and A.H. El-Abiad. Transmission planning using discrete dynamic optimization. *IEEE Transactions on Power Apparatus System*, PAS-92:1358–1371, July 1973.
- [5] H.K. Youssef and R. Hackam. New transmission planning model. *IEEE Transactions on Power Systems*, 4(1):9–18, Feb. 1989.
- [6] N. Alguacil, A.L. Motto, and A.J. Conejo. Transmission expansion planning: a mixed-integer lp approach. *IEEE Transactions on Power Systems*, 18(3):1070–1077, Aug. 2003.

- [7] A. Santos, P.M. Franca, and A. Said. An optimization model for longrange transmission expansion planning. *IEEE Transactions on Power* Systems, 4:94–101, Feb. 1989.
- [8] S. Binato, M.Pereira, and S. Grancille. A new benders decomposition approach to solve power transmission net ork design problem. *IEEE Transactions on Power Systems*, 16:235–240, May 2001.
- [9] R. Romeo and A. Monticeli. A hierarchical decomposition approach for transmission network planning. *IEEE Transactions on Power Systems*, 9(1):373–380, Feb. 1994.
- [10] H. Park, R. Baldick, and D.P. Morton. A stochastic transmission planning model with dependent load and wind forcasts. *IEEE Transactions* on Power Systems, 30(6):3003–3011, Nov. 2015.
- [11] S.D.L. Torre, A.J. Conejo, and J. Contreras. Transmission expansion planning in electricity markets. *IEEE Transactions on Power Systems*, 23(1):238–248, Feb. 2008.
- [12] A. Seifu, S. Salon, and G. List. Optimization of transmission line planning including security constraints. *IEEE Transactions on Power Sys*tems, 4:1507–1513, Oct. 1989.
- [13] P. Tsamaspyhrou, A. Renaud, and P.Carpentier. Transmission network planning: An efficient benders decomposition scheme. *Proc.* 13th PSCC in Trondheim, pages 487–494, 1999.

[14] G.C. Oliverira, A.P.C. Costa, and S. Binato. Large scale transmission network planning using optimization and heuristic techniques. *IEEE Transactions on Power Systems*, 10:1828–1834, Nov. 1995.