PRODUCTION AND INELASTIC SCATTERING OF NEUTRINOS BY NUCLEI AT EXTREME TEMPERATURES¹

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ABSTRACT

The role of neutrino interactions with nuclear states is investigated at high excitation energies during stellar collapse. Cross sections for inelastic scattering from nuclei are determined. These cross sections remain finite as the neutrino energy goes to zero, and for sufficiently high temperatures, the usual low energy window becomes totally closed. The rate of neutrino production from highly excited nuclear states is calculated. The resultant cooling from this process limits the temperature to $\sim 10^{10}$ K in the transparent regions during collapse. During collapse this process is dominant in producing neutrinos other than ν_e . After neutrino trapping, since ν_e production is diminished, it becomes the dominant neutrino-production mechanism.

Subject headings: dense matter — neutrinos — nuclear reactions — stars: collapsed — stars: interiors

I. INTRODUCTION

The significant role played by neutrino processes in the final collapse of massive stars is well known (see review by Barkat 1975), and modifications in such processes due to neutral currents in weak interactions are under intensive study (Bond 1978; Freedman, Schramm, and Tubbs 1977). In this communication we examine the rate of neutrino production by excited nuclei, and inelastic neutrino/nucleus scattering under the extreme conditions of stellar collapse.

The current scenario (see Arnett 1973; Weaver, Zimmerman, and Woosley 1978) for the final evolution of stars in the range $10 \lesssim M/M_\odot \lesssim 70$ is the following: An inert core of iron-peak elements forms in the center. As the core grows to $\sim 1.5 M_\odot$ it becomes unstable due to the combined effect of electron capture and nuclear dissociation. The electrons in the core are highly degenerate, and the beta captures decrease the dominant source of pressure. With increasing densities the neutrinos become trapped (Mazurek 1976; Yueh and Buchler 1977; Arnett 1977; Tubbs 1978). At densities above $\sim 10^{12}$ g cm⁻³ beta equilibrium in reactions of the form $e^-[(A, Z), (A, Z - 1)]\nu_e$ is established, where A denotes the mass number and Z the charge of a nuclear species. The net result is that two seas of relativistic and extremely degenerate leptons (e^- and ν_e) exist throughout the collapse.

The existence of the degenerate electron-type leptons has important consequences for the matter. The matter is kept at cooler temperatures, since the gravitational energy of collapse is tied up in zero-point energy rather than being turned into heat. The net result is that nuclei can exist up to very high densities (Mazurek, Lattimer, and Brown 1979; Bethe *et al.* 1979). Thus the neutrino/nuclei processes at the extreme temperatures expected in stellar collapse need to be determined.

Our considerations of inelastic neutrino/nucleus scattering show that this process produces a new feature for neutrino transport at high temperatures. The usual neutrino cross sections considered (Freedman *et al.*) are proportional to the incident neutrino energy, and hence go to zero at sufficiently low neutrino energies. The inelastic process, on the other hand, goes to a finite value, dependent only on the temperature. For sufficiently high temperatures, this type of scattering can trap the low energy neutrinos that would usually escape. At higher neutrino energies the cross sections from inelastic scattering are negligible in comparison to those of coherent scattering from nuclei. While the mean energy loss per inelastic collision with a nucleus is large ($\sim 25\%$), the collision mean free path is much longer than that of electron scattering, which involves comparable energy losses.

The production of neutrinos through the decay of excited nuclear states was first considered by Bahcall, Treiman, and Zee (1974). They considered the cooling effects on white dwarfs that would arise from emission of neutrino pairs in the decay of low-lying excited states. Such neutrino emission at states of higher excitation for temperatures

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in the range $0.1 \leq T_{10} \leq 0.6$ ($T_{10} = T/10^{10}$ K) was considered by Crawford, Hansen, and Mahanthappa (1976). They applied their results to a precollapse stellar model and concluded that the energy losses were negligible in comparison to electron pair, photon, and plasmon processes. However, the stellar model which they considered was in an early precollapse phase. In these early phases, the star's central regions are only partially electron degenerate while the temperature in these regions is relatively low. At the time of collapse the stellar cores become quite degenerate (see temperatures and densities in Weaver *et al.*). With increasing degeneracy the other neutrino processes diminish dramatically, and only electron bremsstrahlung produces significant neutrino pairs (Dicus *et al.* 1976). Our results indicate that at the point of dynamic collapse, neutrino pair production by the decay of excited nuclear states overwhelmingly dominates that of the bremsstrahlung process. It is true that neutrino production by electron capture may be greater, but electron capture heats rather than cools the matter (see § III). Thus the dominant cooling is due to the neutrino pairs from decay of excited states. In fact, such cooling is so efficient in regions where matter is transparent, the temperatures in these regions will be limited to $T_{10} \lesssim 2$.

Section II presents our calculations of the cross sections for inelastic neutrino/nuclear collisions and of the neutrino pair emission from highly excited nuclear states. Approximate analytical expressions for the relevant quantities are derived, and plots giving the results of detailed numerical calculations are presented. Section III discusses the roles of these processes in stellar collapse.

II. CALCULATIONS

a) Inelastic Neutrino/Nucleus Scattering

First, we calculate the cross section for inelastic neutrino/nucleus scattering (INNS),²

$$\nu[(A, Z), (A, Z)^*]\nu. \quad (1)$$

Reaction (1) proceeds through the weak neutral current. Assuming e - μ - τ universality, the neutrino in relation (1) can be an electron neutrino, or a muon neutrino, or a tau³ neutrino. At the level applicable here, e - μ universality is experimentally verified. If neutrinos beyond ν_i exist, they also will be scattered via reaction (1).

The effective Lagrangian can be expressed in terms of a hadronic neutral current J_Z^μ (see Walecka 1977)

$$\mathcal{L} = \frac{G}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu J_Z^\mu, \quad (2)$$

where

$$J_Z^\mu = \bar{N}_f [\gamma^\mu (C_{V0} + C_{V1} t_3) - \gamma^\mu \gamma_5 (C_{A0} + C_{A1} t_3)] N_i. \quad (3)$$

The operator t_3 is the third component of the isospin, while the subscripts on the form factors refer to the space-time and isospin structure. Subscripts V and A represent the vector and axial-vector parts of the current, while 0 and 1 separate the isoscalar and isovector parts. In the nonrelativistic limit

$$\begin{aligned} J_Z^0 &= N_f^\dagger (C_{V0} + C_{V1} t_3) N_i, \\ J_Z^j &= -N_f^\dagger (C_{A0} + C_{A1} t_3) \sigma^j N_j, \quad (j = 1, 2, 3), \end{aligned} \quad (4)$$

where σ^j are the Pauli spin matrices. The J_Z^0 part is the neutral current analog of Fermi transitions. In the nonrelativistic approximation, the operators in the Fermi part are diagonal. Therefore, in the dipole approximation, if $i \neq f$ (as will be our case), the Fermi transitions do not contribute. The J_Z^j parts in the dipole approximation are the allowed Gamow-Teller transitions

$$g_A^2 |M_{GT}|^2 \equiv \langle (C_{A0} + C_{A1} t_3) \sigma \rangle_{fi}^2, \quad (5)$$

with selection rules $\Delta J = 0, \pm 1, 0 \nrightarrow 0$; $\Delta\pi = \text{no}$; $\Delta T = 0, \pm 1$. The cross section for $\text{INNS}(\sigma_i)$ may be expressed in terms of $|M_{GT}|^2$ as

$$\sigma_i = \sigma_0 (E_\nu - Q_{fi})^2, \quad (6)$$

where

$$\begin{aligned} \sigma_0 &= \frac{G_F^2}{\pi} |M_{GT}|^2 g_A^2 \\ &= \frac{10^{-10}}{m_p^4 \pi} (\hbar c)^2 |M_{GT}|^2 g_A^2 \\ &= 2.5 \times 10^{-44} |M_{GT}|^2 \text{ cm}^2 \text{ MeV}^{-2}, \end{aligned}$$

² The INNS cross section, $\nu_i(A^*, A)\nu_i^*$ dominates the higher-order process $\nu_i(A^*, A)\nu_i\nu_j\bar{\nu}_j$.

³ We assume that the recently discovered lepton $\tau(1780)$ —for a review see Feldman and Perl (1977)—has associated with it a massless neutrino which couples to the neutral intermediate vector boson with the same strength as ν_e or ν_μ .

where $g_A = 1.24$. In equation (6), G_F is the Fermi constant, E_ν is the incident neutrino energy, and Q_{fi} is the Q value of the reaction (the energy difference between the final and initial nuclear levels). In the Weinberg-Salam model (Weinberg 1967; Salam 1968),

$$|M_{GT}|^2 = \frac{1}{4} |\langle \sigma t_3 \rangle|^2, \quad (7)$$

(see Bahcall *et al.*).

We are interested in transitions between nuclear states at high excitation. For such transitions the nuclear level parameters, and hence $|M_{GT}|^2$, are generally not known. However, at the high excitation energies the level multiplicity becomes so large that a statistical treatment becomes appropriate (Bethe 1937). In this case, only an average value $\overline{|M_{GT}|^2}$ is required, along with a means of estimating the probability that a given transition between two nuclear levels satisfies the Gamow-Teller selection rules. The latter probability can readily be estimated on the basis of statistical theories of highly excited nuclear states. For numerical estimates below, we use $\overline{|M_{GT}|^2} = 0.03$, the average value of known matrix elements as tabulated by Crawford *et al.* (their Table 1).

In such a statistical approach, σ_0 in equation (6) is constant, and the total INNS cross section for a given nucleus is obtained by integration of equation (6) over a Boltzmann distribution of initial nuclear levels times the allowed transitions to final levels. The latter is just the strength function for transitions between levels E and E' . Following Mazurek, Cameron, and Truran (1974), we use a strength function of the form $\sigma_0 \Theta(E, E') \omega(E') \Lambda(E')$, where $\Theta(E, E')$ is the fraction of transitions between E and E' that corresponds to the Gamow-Teller selection rules, $\omega(E')$ is the level density (per MeV) at energy E' , and

$$\Lambda(E') = \min [\omega(E'), \omega_{\max}] / \omega(E'), \quad (8)$$

and $\omega_{\max} = 30 \text{ MeV}^{-1}$. The functional form of $\Lambda(E')$ is meant to represent the effects of configuration mixing at high energies (see discussion in Mazurek, Cameron, and Truran 1974). With this approximation for the strength function, the total INNS cross sections become

$$\sigma_I^T(E_\nu) = \sigma_I^{\text{endo}}(E_\nu) + \sigma_I^{\text{exo}}(E_\nu), \quad (9)$$

$$\sigma_I^{\text{endo}}(E_\nu) = \frac{\sigma_0}{\Omega} \int_0^{E_{\max}} dE \omega(E) \exp(-E/kT) \int_E^{E+E_\nu} dE' \Theta(E, E') \omega(E') \Lambda(E') [E_\nu - (E' - E)]^2, \quad (10)$$

$$\sigma_I^{\text{exo}}(E_\nu) = \frac{\sigma_0}{\Omega} \int_0^{E_{\max}} dE \omega(E) \exp(-E/kT) \int_0^E dE' \Theta(E, E') \omega(E') \Lambda(E') [E_\nu - (E' - E)]^2, \quad (11)$$

$$\Omega = \int_0^{E_{\max}} dE \omega(E) \exp(-E/kT), \quad (12)$$

where we have separated endothermic and exothermic neutrino collisions, and introduced a cutoff in the level densities at $E_{\max}$ [$\omega(E) = 0$ for $E > E_{\max}$]. The physical basis for the cutoff is discussed in § III. Note that the integration over E' in equation (10) should also terminate for $E + E_\nu > E_{\max}$. However, for $E_\nu \ll E_{\max}$, neglecting the latter cutoff should cause only a small error.

In numerical evaluations, we use the spin probability distributions and level densities given by Gilbert and Cameron (1965). We are interested in transitions between highly excited states with $E, E' \gtrsim 5 \text{ MeV}$. Evaluation of $\Theta(E, E')$ with the spin probabilities of Gilbert and Cameron using the selection rules following equation (5) shows that it is roughly constant. For a nucleus with $A \sim 50$ and transitions between arbitrary energies in the range $5 \leq E/\text{MeV} \leq 40$, the value $\bar{\Theta} = 0.1$ is accurate to better than 20%. In the numerical evaluations below we assume that $\Theta(E, E') = \bar{\Theta}$.

For high excitation energies an analytic estimate for $\sigma_I^{\text{endo}}(E_\nu)$ is obtainable that is independent of $\omega(E)$. Most of the contribution to the integral over E' comes from high energies where $\Lambda(E') = \omega_{\max}/\omega(E')$. The integral can then be done analytically, yielding $\bar{\Theta} \omega_{\max} E_\nu^3/3$. This can be taken outside the integral over E , which yields the partition function of equation (12). The final result is

$$\sigma_I^{\text{endo}}(E_\nu) = \frac{1}{3} \sigma_0 \bar{\Theta} \omega_{\max} E_\nu^3 = 2.5 \times 10^{-41} \overline{|M_{GT}|^2} \left(\frac{E_\nu}{10 \text{ MeV}} \right)^3 \text{ cm}^2. \quad (13)$$

Our numerical evaluation of $\sigma_I^{\text{endo}}(E_\nu)$ is shown in Figure 1. Also given in the figure is the coherent scattering cross section (Freedman 1974). It is evident that coherent scattering dominates $\sigma_I^{\text{endo}}(E_\nu)$ for all energies of interest. While the level densities depend on A , for energies greater than $\sim 10 \text{ MeV}$ the strength function bound is saturated, and the numerical results are only weakly dependent on A . The results are also only weakly dependent on the maximum excitation energy $E_{\max}$. Above $E_\nu \sim 10 \text{ MeV}$ the results are not sensitive to the ratio E_ν/kT in the range $1 \leq E_\nu/kT \leq 10$. Almost over the entire range, σ_I^{endo} is proportional to $\bar{\Theta} \omega_{\max}$.

FIG. 1.—Neutrino scattering cross sections as a function of incident neutrino energy

An analytic expression for the energy loss per collision can also be obtained by the procedure leading to equation (13). Inserting an additional factor of $(E' - E)$ in the integral over E' of equation (10) defines a new quantity, $S(E_\nu)$. The mean energy loss per collision is just

$$\langle \Delta E \rangle \equiv \frac{S(E_\nu)}{\sigma_I^{\text{endo}}(E_\nu)} \approx \frac{1}{4} E_\nu. \quad (14)$$

An analytic estimate for $\sigma_I^{\text{exo}}(E_\nu)$ is also possible. Again assuming that E is sufficiently large so that most of the contribution comes from the region where the strength function saturates gives

$$\sigma_I^{\text{exo}}(E_\nu) = \frac{\sigma_0 \bar{\Theta} \omega_{\text{max}}}{3\Omega} \int_{E_{\text{min}}}^{E_{\text{max}}} dE \omega(E) \exp(-E/kT) [(E_\nu + E - E_s)^3 - E_\nu^3], \quad (15)$$

where E_s is the energy above which $\omega > \omega_{\text{max}}$, and where E_{min} is the lowest energy at which the Fermi gas approximation of $\omega(E)$ is valid. This E_{min} has to be introduced to eliminate the divergence of the Fermi gas $\omega(E)$ at $E = 0$. In reality, at the high energies of interest, the contributions from regions with $E < E_{\text{min}}$ are totally negligible. This integral can be evaluated by the method given in the Appendix. The final result is

$$\sigma_I^{\text{exo}}(E_\nu) = \frac{\sigma_0 \bar{\Theta} \omega_{\text{max}}}{3} (kT)^3 \left[\left(\frac{E_\nu}{kT} + a_F kT - \frac{E_s}{kT} \right)^3 - \left(\frac{E_\nu}{kT} \right)^3 \right], \quad (16)$$

where a_F is the Fermi gas level density parameter (see Appendix). Note that for $E_\nu/a_F(kT)^2 \rightarrow 0$,

$$\sigma_I^{\text{exo}}(E_\nu) \rightarrow \sigma_{I0}^{\text{exo}} = \frac{1}{3} \sigma_0 \bar{\Theta} \omega_{\text{max}} (kT)^3 \left(a_F kT - \frac{E_s}{kT} \right)^3. \quad (17)$$

E_s may be approximated by solving equation (A1) for the energy at which $\omega = 30 \text{ MeV}^{-1}$. For $A = 40$, $E_s \approx 2.3 \text{ MeV}$. The analytic expressions here and below are underestimates, since the contribution from states with $E < E_s$ are neglected. The significance of this for stellar collapse is discussed in § III.

b) Neutrino Production by Nuclear De-Excitation

We now calculate the neutrino production from the decay of nuclear states at high excitation,

$$(A, Z)^* \rightarrow (A, Z) + \nu_i + \bar{\nu}_i, \quad (18)$$

where $i = e, \mu, \tau, \dots$. The rate of reaction (18) between two nuclear states E_i and E_f is given by Bahcall *et al.*,

$$\lambda_i(E_i, E_f) = \frac{G_F^2}{60\pi^3 \hbar} (E_i - E_f)^5 [g_V^2 |M_F|^2 + g_A^2 |M_{GT}|^2], \quad (19)$$

where $g_v = 1$, $g_A = 1.24$, and $|M_F|^2$ is the matrix element corresponding to Fermi transitions. Again, in the non-relativistic and dipole approximation the Fermi transitions do not contribute (Crawford *et al.*). As in subsection (a) above, we will again be using statistical means to evaluate the sums over many levels. We therefore replace $|M_{GT}|^2$ by the average $\overline{|M_{GT}|^2}$ and write equation (19) as

$$\lambda_i(E_i, E_f) = \lambda_0(E_i - E_f)^5, \quad (20)$$

where $\lambda_0 = 1.3 \times 10^{-4} \overline{|M_{GT}|^2} \text{ MeV}^{-5} \text{ s}^{-1} = 3.9 \times 10^{-6} \text{ MeV}^{-5} \text{ s}^{-1}$.⁴ We are interested in the rates at sufficiently high densities where the contribution of the discrete states included in the calculations of Crawford *et al.* are negligible. We again make the same strength function approximation as was made to obtain equations (10) and (11). The production rate of neutrino pairs ν_i per nucleon can then be written as

$$\lambda_i = \frac{\lambda_0}{\Omega} \int_0^{E_{\max}} dE \omega(E) \exp(-E/kT) \int_0^E dE' \Theta(E, E') \omega(E') \Lambda(E') (E - E')^5. \quad (21)$$

To obtain an analytic approximation at high energies, we note that most of the contribution to the integration over E' will come from regions where the strength function is saturated. Therefore,

$$\lambda_i = \frac{\lambda_0 \overline{\Theta} \omega_{\max}}{6\Omega} \int_{E_{\min}}^{E_{\max}} dE \omega(E) \exp(-E/kT) (E - E_s)^6, \quad (22)$$

where we have introduced $E_{\min}$ and E_s for the same reasons as was done in equation (15). Evaluation by the method given in the Appendix gives

$$\lambda_i = \frac{\lambda_0 \overline{\Theta} \omega_{\max}}{6} [a_F(kT)^2 - E_s]^6. \quad (23)$$

The rate of energy production Q_i in neutrino pairs ν_i is given by an expression like that of equation (20) with an additional factor of $(E - E')$. A similar procedure to that above yields

$$Q_i \approx \frac{\lambda_0 \overline{\Theta} \omega_{\max}}{7} [a_F(kT)^2 - E_s]^7. \quad (24)$$

Hence the mean energy of the neutrino pair is

$$\langle E_{\nu_i} + E_{\bar{\nu}_i} \rangle \approx \frac{6}{7} [a_F(kT)^2 - E_s]. \quad (25)$$

III. DISCUSSION

a) Modifications at High Excitation Energies

At sufficiently high excitation energies, the cross sections and rates will be limited by two effects that are not directly accounted for in the statistical method used in § II. First, at very high excitation energies it is not clear whether a given nucleus exists as an entity distinct from the surrounding bath of external nucleons. Second, at the higher excitations many nucleons will share the energy of the nucleus and behave independently. Thus the rates given by a statistical approach which ignores these problems will be too large. To correct these problems in a very crude way we have introduced the cutoff in partition functions at energy $E_{\max}$. In addition, we modify the rates of § II to account explicitly for the second effect. Let us now discuss each of the two physical limitations on the rates in more detail.

At very high excitation energies single particles within the nucleus can have energies in excess of that required for their separation. The level densities in the Fermi gas model are determined by summing over the total number of single particle states of a spherical well potential (see Bethe 1937). At high nuclear excitation, some fraction of the total level density contains configurations with a residual nucleus and single particles above separation energies. A part of the level multiplicity associated with the single particle can lie in the continuum. As suggested by Fowler, Engelbrecht, and Woosley (1978), such levels should be removed before doing the combinatorics for determining the nuclear level densities. The introduction of a level density cutoff at $E_{\max}$ is meant to represent this subtraction of continuum levels in a very crude way. Physically, $E_{\max}$ is the excitation energy at which the probability for a single particle excitation in the continuum approaches unity for a given nucleus.

The value of $E_{\max}$ is currently under debate. The results of Fowler *et al.* would indicate $E_{\max} \sim 10 \text{ MeV}$. This low value is probably not realistic, since removing all levels with particles above separation energies ($\sim 8 \text{ MeV}$) at excitation energies around 10 MeV alters the Fermi level density by less than 10% (Mazurek,

⁴ The matrix element can be compared to the competing electromagnetic transition; $\lambda_{M1} \approx \alpha E_\gamma^3 \mu_0^2 |M_{GT}|^2$ (μ_0 = nuclear magnetic moment). For a reaction Q value of 1 MeV, $\lambda_i/\lambda_{M1} \approx 10^{-14}$.

Lattimer, and Brown 1979). As noted by Fowler *et al.*, continuum levels that are sharp resonances should be kept. Thus the value of $E_{\max}$ should be considerably higher. A *minimum* value of $E_{\max} \sim 25$ MeV is not unreasonable.

The second limiting factor comes from the fact that, at high excitation energies, many nucleons share the total excitation energy of the nucleus, and the excited nucleons tend to behave independently. But in the formalism of § II we assume that the total excitation energy is available. However, for high excitations, the nuclear configuration involved in the transition corresponds to several independent nucleons being excited and sharing the total energy. Therefore, the transition energy will be limited to the average excitation energy of a single nucleon. This effect thus also limits the maximum excitation energy available for a transition. It will be far more important for the neutrino production than for the neutrino scattering, due to the much higher power of the transition energy involved in the former reaction. We therefore consider the limit implied for neutrino production in detail.

In order to estimate this effect, we assume an independent particle model (IPM) for the nucleus that consists of A noninteracting fermions confined in a potential well of depth V_0 . The nucleus has a Fermi energy ϵ_F (here taken to be 36 MeV). The initial nucleon has an energy ϵ , and the final nucleon has an energy ϵ' . In this model, the single particle rate is

$$\lambda_i = \lambda_0(\epsilon - \epsilon')^5, \quad (26)$$

where λ_0 is given by equation (20). In the independent particle model $|M_{GT}|^2$ is of order unity (cf. eq. [7]). The total rate is found by summing equation (26) over the Fermi distribution of occupied initial states and the available final states.

In the limit that the sum may be replaced by an integral, the total rate is proportional to

$$\left[2 \int dr \int dp f(\epsilon) \right] \left\{ 2 \int dr' \int dp' [1 - f(\epsilon')] \right\} / (2\pi\hbar)^6 \quad (27)$$

where $f(\epsilon)$ is the Fermi-Dirac distribution function

$$f(\epsilon) \equiv [1 + \exp(\epsilon - \epsilon_F)/kT]^{-1}. \quad (28)$$

The factor $\int dr$ may be replaced by the nuclear volume (proportional to A), and the total rate becomes

$$\lambda = \lambda_0 \left[0.155 \left(\frac{A}{40} \right) \right]^2 \int_0^\infty d\epsilon (\epsilon)^{1/2} f(\epsilon) \int_0^\epsilon d\epsilon' (\epsilon')^{1/2} [1 - f(\epsilon')] (\epsilon - \epsilon')^5. \quad (29)$$

Equation (29) was evaluated for $A = 40$, and is shown by the dashed line in Figure 2. The average energy was also calculated in the independent particle model; it is shown as the dashed line in Figure 3.

The figures show that when the excitation energy is sufficiently high, considerable modifications result from the sharing of energy among independent nucleons. At low temperatures, the IPM calculation agrees well with that of the approach in § II. The agreement between the two would probably improve if shell effects were included in the IPM calculation. This inspires confidence that the procedure which we have borrowed from electron capture

FIG. 2.—The number of $(\nu_i \bar{\nu}_i)$ emitted by $A^* \rightarrow A \nu_i \bar{\nu}_i$ as a function of temperature for different maximum excitation energies. The dashed line is the result for the independent particle model discussed in § III.

FIG. 3.—The average energy of the $(\nu_i\bar{\nu}_i)$ emitted by $A^* \rightarrow A\nu_i\bar{\nu}_i$ as a function of temperature for different maximum excitation energies. The dashed line is the result for the independent particle model discussed in § III.

work (Mazurek, Cameron, and Truran 1974) gives reliable results for low temperatures. However, for temperatures greater than $\sim 1.5 \times 10^{10}$ K, significant modification in the rates are indicated.

The needed modifications can be incorporated in our analytic formulae of § II through the following considerations. The number of nucleons that participate in a transition (N_{ex}) is roughly equal to those in the thermal region at the top of the Fermi sea, i.e.,

$$N_{\text{ex}} \sim A \left(\Phi \frac{kT}{\epsilon_F} \right), \quad (30)$$

where Φ is a number of order unity that depends on integrals over the allowed phase space and weight factors for the process being considered. The analytic approximations of § II are obtained by evaluation of the final phase space factor at the average excitation energy of the nucleus, $a_F(kT)^2$ (see Appendix). In the IPM, the energy is shared by N_{ex} nucleons and the phase space factor must be evaluated at $N_{\text{ex}}^{-1}a_F(kT)^2$. As noted above $|M_{\text{GT}}|^2$ is in this case unity. Carrying out these substitutions in the appropriate equations of § II, one obtains the rates and cross sections predicted by the IPM.

Adopting the empirical value for the Fermi gas constant $a_F = (A/8) \text{ MeV}^{-1}$ (see Bohr and Mottelson 1969), we obtain $a_F = 5 \text{ MeV}^{-1}$ and $E_S \sim 2.3 \text{ MeV}$ for $A = 40$. With the replacements described in the preceding paragraph, we obtain the production rate of neutrino pairs and energy per gram of element A (these equations should be compared with eqs. [23] and [24])

$$R_i = \frac{N_A}{A} \lambda_i = 9.8 \times 10^{17} \left(\frac{40}{A} \right) \left[3.9 \left(\frac{\epsilon_F}{36 \text{ MeV}} \right) T_{10} - 2.3 \right]^6 \text{ g}^{-1} \text{ s}^{-1}, \quad (31)$$

and

$$\dot{E}_i = \frac{N_A}{A} Q_i = 1.3 \times 10^{12} \left(\frac{40}{A} \right) \left[3.9 \left(\frac{\epsilon_F}{36 \text{ MeV}} \right) T_{10} - 2.3 \right]^7 \text{ ergs g}^{-1} \text{ s}^{-1}, \quad (32)$$

where Φ was taken to be unity. Equation (31) reproduces the results of the IPM of Figure 2 quite well for $T_{10} > 1$. For the mean energy, we find that a similar procedure with equation (25) but with $\Phi = 0.5$ yields

$$\langle E_{\nu_i} + E_{\bar{\nu}_i} \rangle \approx 0.86 \left[7.8 \left(\frac{\epsilon_F}{36 \text{ MeV}} \right) T_{10} - 2.3 \right] \text{ MeV}. \quad (33)$$

This analytic expression adequately reproduces the IPM results of Figure 3.

For the limiting cross section ($kT \rightarrow 0$) of equation (17), an identical approach to that above with $\Phi = 1$ yields

$$\sigma_{10}^{\text{exo}} = 2.5 \times 10^{-44} \left[3.9 \left(\frac{\epsilon_F}{36 \text{ MeV}} \right) T_{10} - 2.3 \right]^3 \text{ cm}^2. \quad (34)$$

b) Implications for Stellar Collapse

The inelastic scattering cross section is significant in stellar collapse to the extent that it becomes independent of the neutrino energy at sufficiently low values of the latter. This is not true of the usual cross sections involved in stellar collapse, which vanish at low neutrino energies (see Freedman, Schramm, and Tubbs 1977). Using equation (34), $T_{10} \sim 3.5$, and $\rho \sim 10^{12} \text{ g cm}^{-2}$, we find that the mean free path of any neutrino is $< 2 \times 10^6 X_A^{-1}$

cm, where X_A is the heavy element fraction and $A = 40$ was taken. This is below the dimensions of the core ($\sim 10^7$ cm) when $X_A > 0.2$, indicating that even the neutrinos with vanishing small energies will be trapped. Thus the INNS effectively eliminates the low energy window.

Let us now examine the implications of the neutrino energy loss due to $A^* \rightarrow A\nu\bar{\nu}$. Its effect on the precollapse evolution should be negligible, as was noted by Crawford *et al.* However, as the higher densities of dynamic collapse are approached, the matter becomes increasingly more degenerate. With increasing electron degeneracy the dominant electron pair, photon, and plasma neutrino-production processes diminish drastically, so that only neutrino pair bremsstrahlung is significant (Dicus *et al.* 1976). The total neutrino energy production in ν_μ and ν_e pairs by bremsstrahlung is given by these authors in the extremely degenerate limit

$$\dot{E}_{\text{brems}} \approx 10^{13} \left(\frac{Z}{10} \right) \left(\frac{Z}{A} \right) T_{10}^6 \text{ ergs g}^{-1} \text{ s}^{-1}. \quad (35)$$

A comparison with Figures 2 and 3 shows that for $T_{10} \gtrsim 1$, the production of neutrinos by processes other than electron capture is totally dominated by that due to the decay of excited nuclear states.

It is true that neutrinos from electron capture may carry away more energy. However, under conditions of extreme electron degeneracy, the energy carried away by these neutrinos is zero-point energy, which does not result in cooling. In such conditions, electron capture, in fact, heats the matter in two ways. First, holes are created below the Fermi surface by the captures. Electrons must down-scatter in energy to fill these holes, heating the matter. The second reason why electron capture heats the matter is that the high energy of the electrons that are captured favors transitions to daughter states above those that are populated thermally. These states then decay to equilibrium, giving up their energy to heat the matter. Thus, at the high electron degenerate conditions of interest, the dominant cooling process is that of equation (32).

First let us consider the cooling in regions transparent to the neutrinos that are produced. When T_{10} exceeds unity at the start of the collapse, the production given by equation (32)—when multiplied by 3 for the different neutrinos—becomes comparable to the cooling processes (see Weaver *et al.*, the $\dot{S}_{\nu p}$ in their Figs. 1b and 2b). As noted above, with increasing degeneracy and temperature the production of neutrinos by the decay of excited states overwhelmingly dominates all other processes that result in cooling. The pair production is quite sensitive to the temperature. As T_{10} approaches 2, the energy rate given by equation (32)—after multiplication by 3 (or possibly more) for the different neutrino types—approaches 2×10^{17} ergs $\text{g}^{-1} \text{ s}^{-1}$. Regions dominated by $A^* \rightarrow A\nu\bar{\nu}$ lie at densities $\gtrsim 3 \times 10^9 \text{ g cm}^{-3}$ (see Weaver *et al.*, and discussion at the start of this paragraph). On the other hand, the core center becomes opaque to neutrinos for densities $\gtrsim 10^{11} \text{ g cm}^{-3}$ (see Mazurek 1976; Arnett 1977). Hydrodynamic calculations by one of us (T. J. M.) show that the time required for the core center to traverse the density range between $3 \times 10^9 \text{ g cm}^{-3} \leq \rho \leq 10^{11} \text{ g cm}^{-3}$ is of the order of 0.3 s. Thus, if $T_{10} \sim 2$ in the transparent region, the energy loss could be of order 6×10^{16} ergs g^{-1} . This loss roughly equals the energy in the ion gas, and the cooling effect of neutrino pairs from nuclei at higher excitation limits the temperature in the transparent regions to $T_{10} \lesssim 2$.

Let us now consider the implications for the opaque regions where the neutrinos are trapped. In these regions, the electron capture produces neutrinos so rapidly that the electron-type neutrinos become degenerate and come to equilibrium with the degenerate electrons (Mazurek 1976; Arnett 1977). Thus the degeneracy of the electron-type leptons persists throughout the collapse. This degeneracy severely limits the rate of production of electron-type neutrinos by the usual processes (see Mazurek 1978). Therefore the production of ν_μ and ν_τ pairs will be overwhelmingly dominated by the decay of highly excited nuclear states in the opaque regions where nuclei exist.

In summary, the interactions of neutrinos with nuclear states at high excitation may play an appreciable role in stellar collapse. Inelastic scattering from nuclei eliminates the low energy window at sufficiently high temperatures. The production of neutrino pairs by nuclear states at high excitation restricts temperatures to $T_{10} \lesssim 2$ in the transparent regions, and dominates the production of neutrinos in the opaque region.

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APPENDIX

This Appendix summarizes the procedure used in § II to obtain analytic approximations of the Boltzmann averages over the level densities of the Fermi gas model. The level densities have the form (actually the proper terminology is "state" density; see Huizenga and Moretto 1972)

$$\omega(E) = \frac{\sqrt{\pi}}{12} (a^{1/4} E^{5/4})^{-1} \exp [2(a_F E)^{1/2}], \quad (A1)$$

where the level density parameter a_F is a constant. In general, we are interested in Boltzmann averages of an energy dependent quantity $g(E)$. These have the form

$$\langle g(E) \rangle = \frac{C}{\Omega} \int_{E_{\min}}^{E_{\max}} dE \frac{g(E) \exp [2(a_F E)^{1/2} - E/kT]}{(a_F E)^{5/4}} = \frac{\int_{x_{\min}}^{x_{\max}} g(kTx) x^{-5/4} \exp [2(a_F kT)^{1/2} x^{1/2} - x] dx}{\int_{x_{\min}}^{x_{\max}} x^{-5/4} \exp [2(a_F kT)^{1/2} x^{1/2} - x] dx}, \quad (A2)$$

where Ω is the partition function given in equation (12) above, C is a constant, and $x = E/kT$. The exponential is a rapidly varying function and its argument has a maximum at $x_0 = a_F kT$. Assuming that $x_{\min} < x_0 < x_{\max}$, and if $g(kTx)$ is slowly varying, we can evaluate it at x_0 and take it outside the integral. This gives

$$\langle g(E) \rangle \approx g[a_F(kT)^2]. \quad (A3)$$

Note that average excitation energy is just $a_F(kT)^2$, so that $\langle g(E) \rangle$ is given by evaluating g at the average excitation energy.

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