A FUNDAMENTAL LOOK AT ENERGY STORAGE FOCUSING PRIMARILY ON FLYWHEELS AND SUPERCONDUCTING ENERGY STORAGE

By:

K.R. Davey
R.E. Hebner

Electric Energy Storage Applications and Technologies EESAT 2003 Conference Abstracts, San Francisco, California, U.S.A., October 27-29, 2003, pp. 17-18.

PN 280

Center for Electromechanics
The University of Texas at Austin
PRC, Mail Code R7000
Austin, TX 78712
(512) 471-4496

A Fundamental Look at Energy Storage Focusing Primarily on Flywheels and Superconducting Energy Storage

Kent Davey* and Robert Hebner
10100 Burnet Rd, EME 133, J.J. Pickle Research Center, Austin, TX 78758
phone (512) 232-1603, Fax (512) 471-0781, email k.davey@mail.utexas.edu

Abstract - This paper compares energy storage efficiency of Superconducting Energy Storage devices (SMES) with high speed flywheels employing magnetic bearings. Both solid cylinder and shell cylinder flywheels are examined from fundamental physics. Solid cylinder flywheels have a fixed energy density by weight and volume dependent only on the constitutive properties of the flywheel. For a target energy storage, the flywheel's radius, length, and rotation speed are determined given the governing limitation on hoop stress and the requirement that operation will occur below the first bending mode. No design parameters are open for engineering judgment except the margin of safety. Thus the volume necessary to reach a target energy storage is well defined. The shell cylinder has only the thickness of the shell as an open design variable.

The constraint for a SMES system is that the magnetic field density remain below the quench value for the superconductor. This constraint involves the current density, the magnetic field density, and the temperature. A theoretical upper limit can be reached by considering a volume with a B field just under the quench value. In this theoretical upper limit, given the materials available today, the flywheel stores the same energy in a volume 7.4 times smaller than the SMES system even when assuming a 20 T field for the SMES system. Both systems allow for energy to be added and removed rapidly by comparison to battery and capacitive storage, but the flywheel is by far the more efficient choice when examined on a per volume basis.

Flywheel

The design analysis follows two guidelines. First, the frequency should be below the first bending mode. Although power plant turbine generators have been designed to operate above the first 1-3 bending modes, a turbine generator is a constant speed device. The power plant operator attempts to get through the bending modes quickly, albeit with difficulty, and then stays above it. A practical flywheel design should not operate in this fashion. Perhaps advances in magnetic bearings will alleviate this constraint in the future. Second, the primary stress failure is due to hoop stress. Using these two governing restraints, the complete design of a cylindrical flywheel is fixed; under the same conditions all but the thickness of a shell flywheel is fixed.

The flywheels built at the Center for Electromechanics (CEM) are made of high strength carbon composite material having a mass density of about $1.6 \cdot 10^3$ kg/m^3 (0.058 lbs/in^3) and a modulus of elasticity of greater than $8.27 \cdot 10^3$ MN/m^2 ($1.2 \cdot 10^6$ psi), and when loaded with fiberglass, $13.1 \cdot 10^3$ MN/m^2 ($1.9 \cdot 10^6$ psi). Blevins [1] lists the natural bending mode frequency

$$f_i = \frac{\lambda_i^2}{2\pi L^2}\sqrt{\frac{E I_m}{m}} \,. \qquad (1)$$

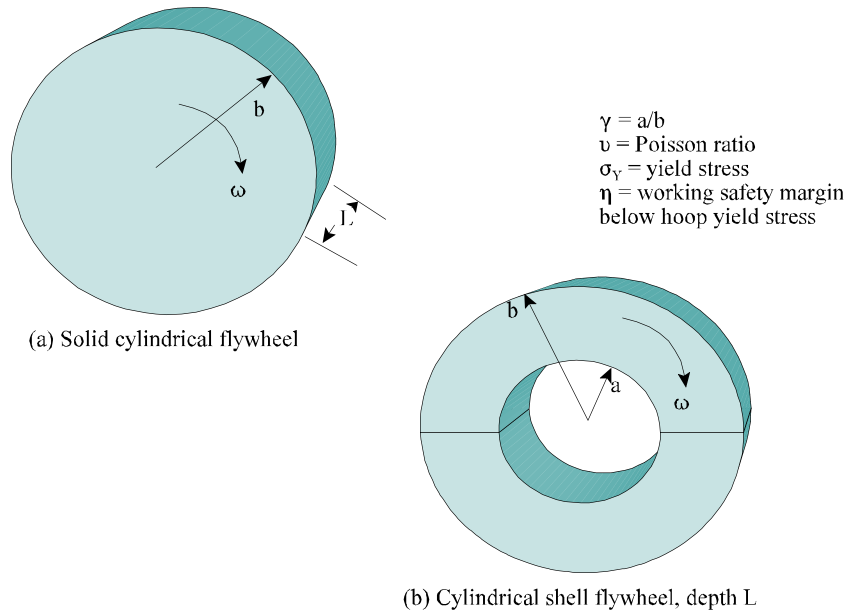

Figure 1 Basic flywheel shapes.

For the first bending mode, λ_1 = 4.73, m is the mass per unit length, L is the length of the beam, m the mass per unit length, E the modulus of elasticity (force/area), and I_m is the moment about the axis. For ring flywheels as shown in Figure 1, with outer radius b and inner radius a,

$$I_m = \frac{\pi}{4}\left(b^4 - a^4\right). \qquad (2)$$

For a ring of thickness δ, i.e., b = a+δ, the moment is approximately

$$I_m \approx \pi \delta a^3 = \pi a^4 \frac{\delta}{a}. \qquad (3)$$

Authur Burr[2] lists the hoop stress in terms of the mass density ρ and the Poisson ratio υ for a ring with radian rotation frequency ω to be

$$\sigma_t = \frac{\rho \omega^2}{4}\left\{(3+\nu) b^2 + (1-\nu) a^2\right\}. \qquad (4)$$

For the ring of thickness δ,

$$\sigma_t \approx \frac{m \omega^2}{4\pi}\left\{2\frac{a}{\delta} + (3+\nu)\right\}. \qquad (5)$$

Here we have used the fact that ρ 2πa δ = mass m.

The weight density for the fibers is $1.58 \cdot 10^3$ kg/m^3 (0.057 lbs/in^3) and the Poisson ratio is

0.45. The ultimate stress σ_Y *is about* $3.1025 \cdot 10^9$ N/m^2 *(450 kpsi*). The energy W stored by the flywheel is

$$W = \frac{1}{2} I \omega^2 = \frac{1}{2} m L \frac{\left(a^2 + b^2\right)}{2} \omega^2 \approx \frac{1}{4} m L \omega^2 a^2 \left(1 + \frac{2\delta}{a} \right) . \qquad (6)$$

Here I is the mass moment of inertia which is mLr^2 for a ring of radius r. For a cylindrical ring this energy depends only on the tip speed v as

$$W = \frac{1}{2} m L v^2 . \qquad (7)$$

The upper limit on the design of a flywheel is reached by extending the radius so that the maximum hoop stress mv^2/r is just under the yield strength of the carbon fiber, and the length is set short enough so that the rotation frequency is less than that in (1).

Cylindrical Flywheel (a=0)

Equations (1), (5), and (6) define the problem. Consider first designing a flywheel that will store the maximum energy, one where a=0. Set the design frequency to a safety fraction β (e.g. 0.9) of the first bending mode in (1), and the hoop stress to a safety fraction η (e.g. 0.9) of the ultimate stress σ_Y for a carbon fiber in (5). In this limit,

$$\omega = \frac{\beta \lambda_1^2 b^2}{2 L^2} \sqrt{\frac{E \pi}{m}} . \qquad (8)$$

$$\eta \sigma_Y = \rho \omega^2 b^2 \frac{(3+\nu)}{8} = \frac{m \omega^2 (3+\nu)}{8 \pi} . \qquad (9)$$

Inserting (8) and (9) into (6) yields the result

$$W = \frac{8 \sqrt{2} \pi L^3 E}{\beta \lambda_1^2} \left(\frac{\eta \sigma_Y}{(3+\nu) E} \right)^{\frac{3}{2}} \qquad (10)$$

The last term in parenthesis is nondenominational, and depends only on the constitutive properties of the material. For a desired energy storage, (10) dictates the length in terms of the material properties. *Note that neither the mass nor the mass density is part of this result.* The commensurate rotation speed and allowed radius which follow from (8) and (9) do depend on the mass.

The energy per unit volume is obtained by dividing (10) by $\pi b^2 L$. It is *dependent only on constitutive properties*,

$$\frac{W}{Volume} = 2\,\frac{\eta\,\sigma_Y}{(3+\nu)}\,. \tag{11}$$

The energy per unit weight is also dependent only on constitutive properties,

$$\frac{W}{Weight} = 2\,\frac{\eta\,\sigma_Y}{\rho\,(3+\nu)}\,. \tag{12}$$

With safety factor η=0.9, carbon fibers have an upper limit on energy density by volume of $1.73 \cdot 10^6$ kW-Hr /m^3 ($6.26 \cdot 10^6$ in-lbs/in^3) and an energy density by weight of $6.55 \cdot 10^6$ kW-Hr /kg ($1.08 \cdot 10^7$ in-lbs/lb). Using these upper bound numbers, a 180 MJ system theoretically only requires 0.0417 m^3 ($2.55 \cdot 10^3$ in^3) of volume. The shell flywheel under design at CEM is rated for 450 MJ, and has a volume of 0.3834 m^3; were it scaled to 180 MJ, it would have a volume of 0.15 m^3. The rotor without the motor generator weighs $2.32 \cdot 10^3$ kg (5100 lbs), delivering an energy density of 0.0539 kW-Hr/kg (0.0245 kW-Hr/lb).

Among the conclusions determined by this result is that for a target energy storage, there is only one optimal design; *none of the design parameters L, b, or rotation frequency are a subject for engineering judgment*. The only judgment comes in determining a priori how close to the stress limit and first bending mode one chooses to operate.

$$\sigma_r = \frac{3+\nu}{8}\,\rho\,\omega^2\,b^2\left(1-\gamma^2\right). \tag{13}$$

Once the storage energy W is specified, the volume $V=\pi\, b^2\,(1-\gamma^2)L$ is determined in terms only of the ratio of the outer radius to the inner radius, γ, as

$$V = \frac{W}{\eta\sigma_Y}\,\frac{\left(3+\nu+\gamma^2-\gamma^2\,\nu\right)}{(1+\gamma^2)}\,. \tag{14}$$

The derivative of this with respect to γ is

$$\frac{dV}{d\gamma} = -\frac{4\,W\,\gamma\,(1+\nu)}{\eta\,\sigma_Y\,(1+\gamma^2)^2}\,. \tag{15}$$

Shown in Figure 2 is the ratio of the terms in parenthesis in (14), showing as expected that the best flywheel will be one in which the inner radius almost equals the outer radius.

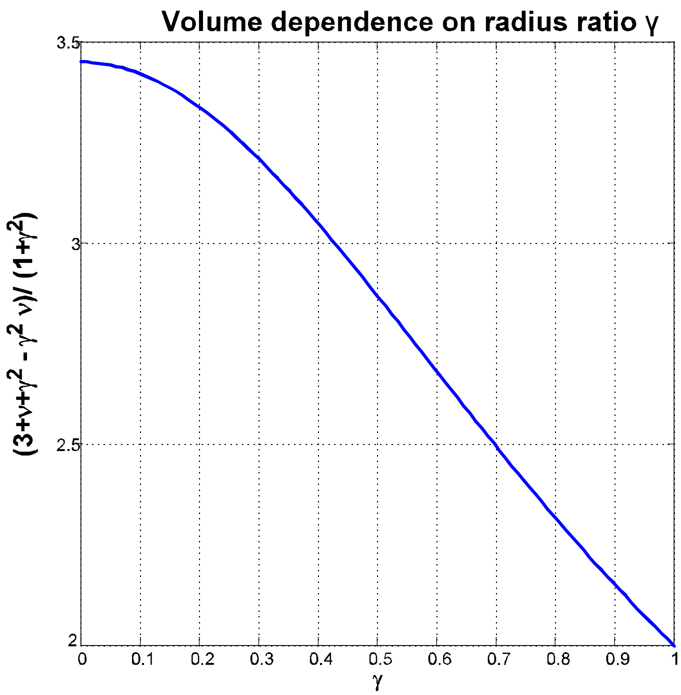

Figure 2 Volume reduction as a function of γ $\alpha = \dfrac{\beta}{\gamma}$.

The energy per unit volume depends now on the thickness of the shell and the constitutive properties,

$$\frac{W}{Volume} = \frac{\eta\ \sigma_Y (1+\gamma^2)}{3+\nu+\gamma^2-\nu\ \gamma^2}\ . \tag{16}$$

For carbon fibers with an ultimate hoop stress of $3.1 \cdot 10^3$ MN/m^2 (450 kpsi), density $\rho = 1.58 \cdot 10^3$ kg/m^3 (0.057 lbs/in^3), and Poisson ratio $\upsilon = 0.45$, and safety margins $\eta = 0.85$ and $\eta = 0.5$, the energy density with radius ratio variation is shown in Figure 3. Note that this represents an upper limit, and depends entirely on constitutive materials. Neither the weight of the magnetic bearing or the motor - generator are considered in this calculation.

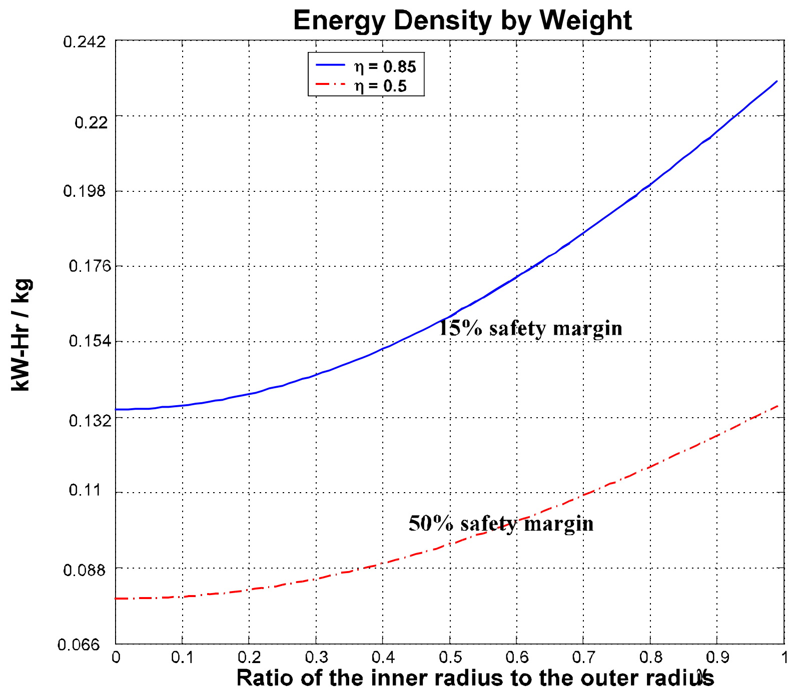

Figure 3 Energy density as a function of radius ratio γ.

Note that all additional parameters are computed directly from the target energy W and the choice of γ,

$$L = 2^{-\frac{2}{3}} \frac{\left(E\,\lambda^4\,\beta^2\,W^2\,(3+\nu+\gamma^2-\gamma^2*\nu)\,\eta^3\,\sigma_Y^3\,\pi^4\,(1+\gamma^2)^5\,(1-\gamma^2)^{10}\right)^{1/6}}{\eta\,\sigma_Y\,\pi\,(1-\gamma^2-\gamma^4+\gamma^6)}\,. \tag{17}$$

$$\omega = 2^{\frac{2}{3}} \frac{(1+\gamma^2)^{\frac{5}{12}}\,(1-\gamma)^{\frac{1}{3}}\,\left(\sigma_Y\,\eta\right)^{\frac{3}{4}}\,\pi^{\frac{1}{3}}\,(\lambda\,\pi)^{\frac{1}{3}}\,E^{\frac{1}{12}}\,\beta^{\frac{1}{6}}}{(3+\nu+\gamma^2-\nu*\gamma^2)\,W^{\frac{1}{3}}\,\sqrt{\rho}}\,. \tag{18}$$

$$b = 2^{\frac{1}{3}} \frac{(1-\gamma^2)^{\frac{2}{3}}\,(1+\gamma^2)^{\frac{7}{12}}\,W^{\frac{1}{3}}\,(3+\nu+\gamma-\nu\,\gamma)^{\frac{1}{4}}}{\beta^{\frac{1}{6}}\,(\pi\,\lambda)^{\frac{1}{3}}\,(\eta\,\sigma_Y)^{\frac{1}{4}}\,E^{\frac{1}{12}}\,(1-\gamma^4)}\,. \tag{19}$$

Superconducting Magnetic Energy Storage (SMES)

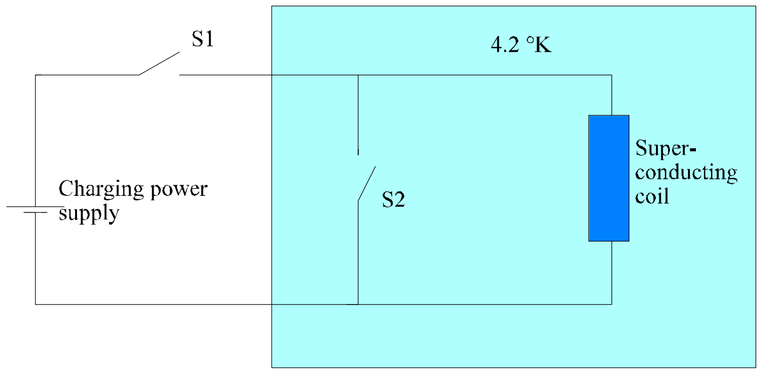

Figure 4 SMES basic design structure.

SMES systems store energy in magnetic fields by means of superconducting wires. Energy is coupled to and from the power grid by means of switches as shown in Figure 4. Although high temperature superconductor ceramics exist, most super conducting wires are made of NbTi, or Nb_3Sn and require temperatures < 20°K for the resistance to drop to zero. To maintain the superconducting state, certain conditions on temperature, the magnetic field density in which the conductor resides, and the current density of the conductor must be maintained. These three conditions form the Achilles heel of the SMES system, a restraint that severely limits the energy density of SMES by comparison to flywheels. Gerald Schoenwetter [3] defines these conditions for NbTi in the plot shown in Figure 5. Anything outside the envelope of this curve causes the superconductor to quench, a condition which is usually commensurate with catastrophic conditions since the energy dissipation in the conductor rises so rapidly. If the temperature for the superconductor is maintained at 4.2 °K, an approximation to the condition on current density and B is

$$|J| = (-6.4\ |B| + 54.0)\ A/mm^2\ . \qquad (20)$$

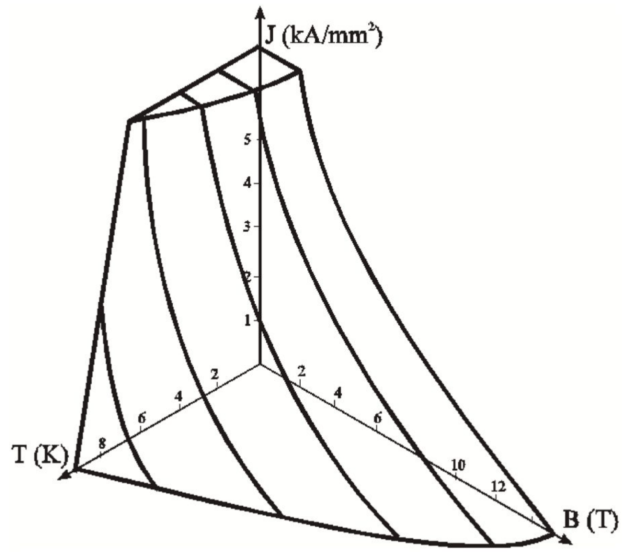

Figure 5 Restraints on temperature, magnetic field density, and current density.

For a typical current density of 22.5 MA/m^2, this restraint translates to insuring that the B field remain below 4.9 T,

$$| B | \leq 4.9219 \ T \tag{21}$$

This result allows an immediate upper limit to be placed on SMES energy storage efficiency. The energy storage density of a magnetic field is ½ $\mu_0 H^2$ = ½ B^2/μ_0. The magnetic field will always be largest adjacent to the superconductor, i.e., the current source. Imagine a solenoid comprised of superconducting wires. As an upper limit on the volume required, imagine the B field to be maintained constant throughout the space within and comprising the solenoid at this upper limit. The volume required to achieve 180 MJ would be

$$V = \frac{180 \cdot 10^6}{\frac{1}{2} 4.92^2 / \mu_0} = 18.69 \ m^3 \ . \tag{22}$$

This is a volume 122 times larger than that for the flywheel storing the same energy. In reality the volume of a practical SMES will be much larger. This author [4] along with two to three other teams have worked through the international TEAM problem 22 [3] which targets an energy storage of 180 MJ within a volume of $4.2 \cdot 10^3$ m^3. The computation of these geometries subject to specified shielding requirements is involved, and the volume greatly increases when shielding is required.

Nb_3Sn offers considerable promise in this area. Recent experiments indicate that the material can support up to 20 T before quenching. A toroidal geometry is the most efficient for volume. Were this achievable, the volume differential would shrink from 122 to 7.4 times the size of comparable flywheel storage.

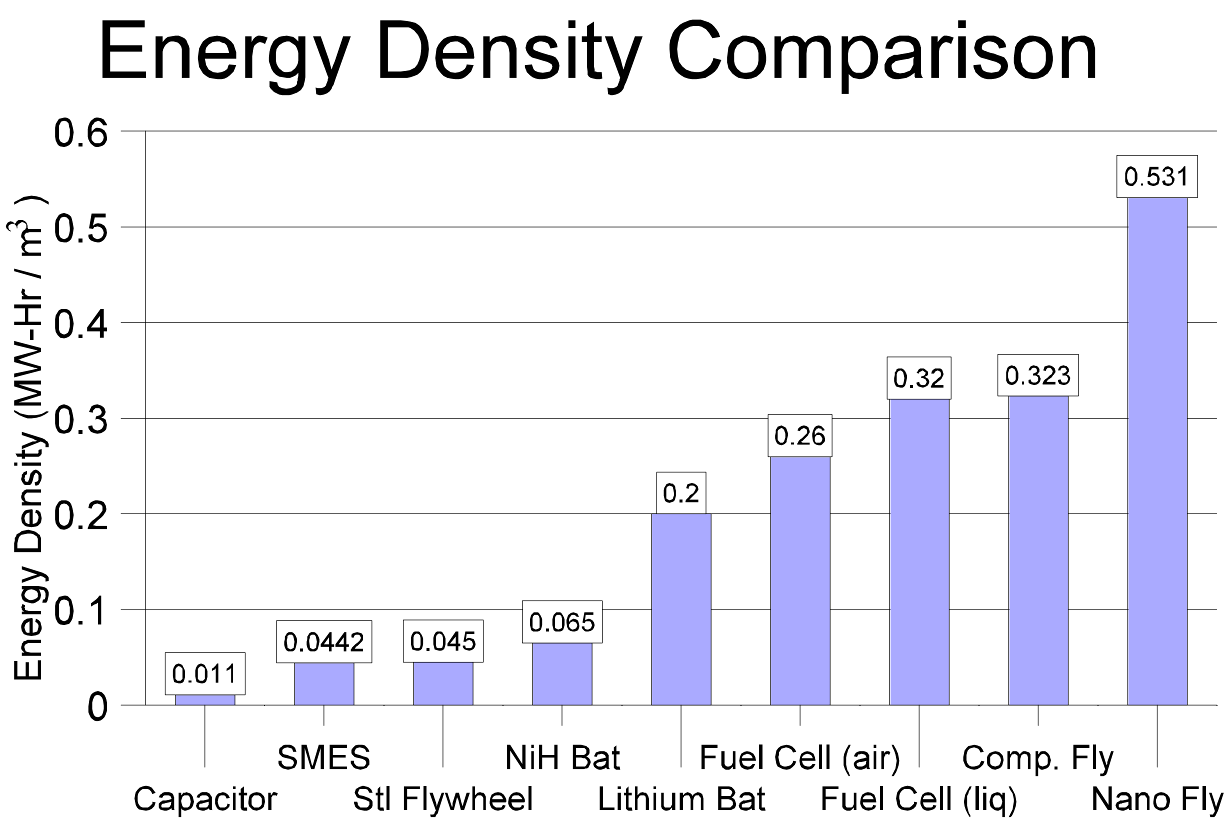

Figure 6 Technology Comparisons by Energy Density.

Technology Comparison

These results allow a comparison of various technologies by energy density. Include in this comparison, steel flywheels, carbon nanotubes, batteries, SMES with Nb_3Sn, and capacitors. McInnis lists 0.29 for the Poisson ratio for structural steel A7, with an ultimate tensile stress of 65 $\cdot 10^3$ psi [5]. Yu [6] reports the ultimate tensile strength for carbon nanotubes as being between 11 and

63 gigapascals. Since little is known about their integration into a practical flywheel, consider setting their ultimate strength at half the lowest value, i.e., 5.5 gigapascals. NEDO, a Japanese based energy storage company focusing on dispersed battery energy storage technology since 1992, reports an energy density of 200 W-Hr/l for lithium secondary batteries, 65 W-Hr/l for NiH batteries, and 40 W-Hr/l for lead acid batteries [7]. The bulk of fuel cell research lists energy / weight. Scamans built and tested an aluminum based fuel cell. Aluminum hydroxide was allowed to precipitate to increase the energy density. Using compressed oxygen, they were able to reach an energy density of 260 W-Hr/l and 320 W-Hr/l using liquid oxygen [8]. In a special issue for IEEE Spectrum, Tom Gilchrist [9] lists 1 kW-Hr/l as the energy density based just on the stack height, affirming the number of 320 W-Hr/l for the hoe system as reasonable. Lastly the Air Force is presently heading a research program for high capacitive energy density storage in excess of 2 J/cc [10]. Consider setting a value of twice that sought, i.e., 4 J/cc. Figure 6 graphically displays the energy densities for all these options using the parameters above.

Conclusions

A well designed flywheel has nearly all the physical and working parameters defined as soon as the energy is specified, based on the hoop stress safety margin and bending mode frequency margin. Key indices such as energy storage per unit volume have very simple relationships both for solid cylinders (11) and for shell cylinders (16). These relations which involve constitutive material properties are well defined. The energy per unit volume storage of SMES systems is quite small by comparison due largely to the rather small permeability of free space. Current sets the H field, and since energy is $\frac{1}{2} \mu H^2$, attempting to store the energy in steel will not work. Although the permeability is high, the H field drops proportionately, not to mention the fact that the permeability will drop precipitously as the B field climbs above 2 T. High temperature superconductors will make the job of storing energy easier since it employs liquid nitrogen, but it will not change volume storage inefficiency.

References

1. Robert Blevens, Formulas for Natural Frequency and Mode Shape, Krieger Publishing, Malabar, Florida, 1995, p. 108.

2. Authur Burr, Mechanical Analysis and Design, Elsevier, New York, p. 318.

3. Gerald Schoenwetter, "International TEAM Benchmark Problem - SMES Optimization Benchmark", http://www-igte.tu-graz.ac.at/team/teamdesc.htm.

4. Kent R. Davey, "Examination of Various Techniques for the Acceleration of Multivariable Optimization Problems", *IEEE Transactions on Magnetics*, accepted for publication - to appear in 2003.

5. Bayliss McInnis and George Webb, Mechanics - Statics and Mechanics of Solids, Printice Hall, Englewood Cliffs, New Jersey, 1971, p. 236.

6. Min-Feng Yu, Oleg Laurie, Mark Dyer, Katerina Moioni, Thomas Kelly, and Rodney Ruoff, "Strength and breaking mechanism of multiwalled carbon nanotubes under tensile load", *Science*, vol. 287, January 28, 2000, pp. 637-640.

7. Internet Web address http://www.nedo.go.jp/itd/fellow/english/project-e/4-1e.html.

8.Geoffrey Scamans, David Creber, John Stannard, and James Tregenza, "Aluminum Fuel Cell Power Sources for Long Range Unmanned Underwater Vehicles", Autonomous Underwater Vehicle Technology, INSPEC Accession Number: 5085587, July 1994, pp. 179-186.

9. Tom Gilchrist, "Fuel Cells to the Fore", IEEE Spectrum, vol. 35, no. 11, Nov. 1998, pp. 35-40.

10. Internet Web address
http://www.fbodaily.com/cbd/archive/2001/05(May)/23-May-2001/asol010.htm.