Optimization Studies in IGNITEX

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ABSTRACT

In the existing budgetary environment, low-cost, high-payoff fusion experiments are increasingly attractive. IGNITEX, a homopolar generator powered single-turn tokamak, has the potential to achieve ignition at relatively low cost. A 60 MJ homopolar generator facility (with an easy upgrade path to 90 MJ) is in place at CEM-UT. Using this facility, CEM-UT researchers have produced a 20 T on-axis magnetic field in a 1/16 scale IGNITEX prototype. This paper presents the results of optimization studies conducted to determine the lowest-cost homopolar generator facility which can power an IGNITEX prototype of any chosen scale operating at 20 T on-axis.

INTRODUCTION

Researchers at the Center For Electromechanics at The University of Texas at Austin (CEM-UT) have designed and constructed a 1/16-scale prototype of the IGNITEX toroidal field (TF) winding. IGNITEX, a compact, single-turn tokamak powered by homopolar generators (HPGs) represents one of the most promising, economical routes to fusion ignition demonstration currently under study.

CEM-UT has successfully operated the 1/16-scale TF winding at 20.3 Tesla on-axis (a record value for a toroidal geometry). An existing 60 MJ homopolar generator installation supplied power for this achievement. However, the 15 GJ HPG facility required to power the full-scale TF winding will represent one of the major cost components of the IGNITEX system. An ongoing optimization effort at CEM-UT seeks to optimize the design of this HPG facility in order to minimize its cost. This paper documents the interim results of this effort.

Work done to date applies to the disk HPG geometry. The first section describes the general operation of disk homopolar generators and discusses the assumptions made in order to facilitate the analysis. The second section presents the technical details of the optimization procedure. This procedure applies not only to IGNITEX but to any inductive load. The third section evaluates the results of the optimization procedure using numerical values pertinent to the IGNITEX system. The fourth and final section discusses these results and indicates avenues of future research toward which they lead.

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THE DISK HOMOPOLAR GENERATOR

Figure 1 illustrates the general operation of a disk homopolar generator. The magnetic field lines produced by the field winding penetrate the conductive rotor parallel to the axis of rotation. As the conductor moves through the magnetic field, a voltage is generated between the inner and outer surfaces of the rotor. This voltage (typically tens of volts) can produce a very large current in a low-impedance load. When the rotor carries current it experiences a torque that tends to bring its rotation to a halt.

Figure 1. Faraday disc

The current carrying capability of a disk HPG is limited primarily by the area of the interface between the brushes and the rotor. A maximum sustainable current density can generally be associated with any particular brush material; throughout this paper the symbol J_b will denote this quantity.

When the rotor spins, it experiences a stress state produced by centripetal acceleration effects. The maximum stress occurring anywhere in the rotor varies with the square of its rotational velocity, but also varies with the radius of the rotor. A maximum *tip speed* can usually be associated with a particular type of rotating machine; the allowable tip speed is generally independent of the radius of the rotor. Throughout this paper the symbol v_t will denote the maximum allowable tip speed of the homopolar generator rotor.

In order to minimize the field winding current necessary to produce the magnetic field that penetrates the rotor, the HPG rotor is commonly formed from a suitable high-permeability magnetic material. Such materials have saturation limits, however, and the intensity of the magnetic field that can be produced in such materials is limited (normally to around 2 T). Throughout this paper the symbol B will denote this quantity.

The quantities J_b , v_t , and B represent fixed limitations that depend on the materials from which the components of the HPG are constructed. In addition, however, there are geometric quantities that can be chosen within broad ranges. The outer and inner radii of the rotor will be denoted by the symbols r_o and r_i respectively and the length of the rotor will be denoted by the symbol l . Finally, the density of the rotor material will be denoted by the symbol ρ .

PERFORMANCE EXPRESSIONS AND OPTIMUM GEOMETRY

In this section expressions for the performance of a disk HPG in terms of its geometry and material characteristics and limitations are derived. These expressions are utilized to determine the optimum disk HPG geometry for any given inductive load. Expressions for HPG current, voltage, stored energy, and peak output power are derived.

First, let the symbol a denote the radius ratio r_i/r_o . It proves convenient to eliminate the inner radius from the performance expressions in favor of the radius ratio (it will be shown that the radius ratio should have a fixed value in all optimum designs, independent of load).

An expression for the peak current of the HPG will now be derived. This quantity is limited by the available brush area at the inner current collection surface of the rotor. The peak current is merely the product of the area available for current collection and the limiting current density of the brush material:

$$I = 2\pi r_i l J_b$$

or, in terms of the radius ratio and outer radius,

$$I = 2\pi r_o l a J_b$$

Next an expression for the maximum HPG voltage will be derived. The voltage generated across a differential element of rotor radial build is given by the product of the magnetic field intensity, the tangential rotor velocity at that radius, and the differential element length:

$$dV = B v_t \left(\frac{r}{r_o} \right) dr$$

Integration of this expression from the inner radius to the outer radius yields

$$V = \left(\frac{B v_t}{2 r_o} \right) (r_o^2 - r_i^2)$$

or, in terms of the radius ratio,

$$V = \left(\frac{B v_t r_o}{2} \right) (1 - a^2)$$

Next an expression for HPG peak stored energy will be derived. The peak kinetic energy of a cylindrical shell of thickness dr and radius r is given by

$$dE = \pi r l \rho v_t^2 \left(\frac{r}{r_o} \right)^2 dr$$

or

$$dE = \pi l \rho \left(\frac{v_t}{r_o} \right)^2 r^3 dr$$

Integration from the inner radius to the outer radius and substitution of the radius ratio yields

$$E = \left(\frac{\pi l \rho v_t^2 r_o^2}{4} \right) (1 - a^4)$$

Finally, the product of the peak voltage and current expressions gives an expression for peak HPG power:

$$P = \pi \left(B J_b v_t l r_o^2 \right) (a - a^3)$$

For each of these four expressions (current, voltage, energy, and power), the radius ratio can be chosen to maximize the value of the expression. Peak current is maximum for $a=1$, while voltage and energy are maximum for $a=0$. Neither extreme represents a workable machine. For maximum power, however, a well-defined and workable value results:

$$a = \frac{1}{\sqrt{3}}$$

In fact, this value is quite close to the value of 1/2 used in many HPG systems built in the past. Substitution of this value into the performance expressions yields

$$I = \frac{2\pi}{\sqrt{3}} J_b r_o l$$

$$V = \frac{1}{3} B v_t r_o$$

$$E = \frac{2\pi}{9} d v_t^2 r_o^2 l$$

$$P = \frac{2\pi}{3\sqrt{3}} B v_t J_b r_o^2 l$$

The IGNITEX TF winding must be operated at a specific current (150 MA) and at that current level stores or dissipates a specific quantity of energy (12 GJ). By setting the expressions for peak current and energy equal to these values and inserting values for the various material characteristics, it is possible to determine values for the radius and length of the rotor which will cause the HPG to reach the current and energy limits simultaneously. Neither current capability nor energy storage capability will be wasted.

To solve these equations, form the ratio of required energy to required current:

$$\frac{E_r}{I_r} = \frac{\sqrt{3} d v_t^2 r_o}{9 J_b}$$

This expression allows determination of the required rotor radius:

$$r_o = \frac{9 J_b E_r}{\sqrt{3} d v_t^2 I_r}$$

Substitution of this value into the expression for peak current allows determination of the required rotor length:

$$l = \frac{d v_t^2 I_r^2}{6\pi J_b^2 E_r}$$

APPLICATION TO IGNITEX

Typical values for the material parameters in the equations above are

$$J_b = 10^7 \text{ A/m}^2$$

$$\rho = 7.8 \times 10^3 \text{ kg/m}^3$$

$$v_t = 1.8 \times 10^2 \text{ m/s}$$

Substitution of these values and the current and energy requirements of IGNITEX (150 MA, 12 GJ) into the expressions for the rotor dimensions yield a radius of 13.3 m and a length of 31 cm. These dimensions are clearly unacceptable; the radius is much too large.

If the HPG radius is reduced to a reasonable value, the energy drops as the square of the reduction factor. In order to recover the energy storage capability, the length of the rotor must rise as the square of the reduction factor. Note that the current capability rises with increasing rotor length in the same manner as the energy storage capability, but decreases linearly with decreasing radius rather than as the square. Decreasing radius and increasing length to compensate energy storage capability, then increases the current capability beyond that required by the mission. The final section of this paper discusses compromise solutions and the direction future research should take to ease these difficulties.

OPTIONS FOR WORKABLE SOLUTIONS

The ideal approach to producing an optimal HPG system that has practical dimensions is to operate the system at higher tip speeds. The rotor radius increases by the square of the tip speed increase factor, while the rotor length decreases by the same amount. Unfortunately, this does not represent a realistic near-term solution. Brush wear in HPGs shows a dramatic upturn at approximately 200 m/s tip speed; higher tip speeds drastically reduce the life of the system.

A pragmatic approach to optimal system design involves incorporation of a flywheel energy storage reservoir into the system. By designing the rotor for optimum current-carrying ability and the flywheel for optimum energy storage ability, an efficient, elegant system emerges. The remainder of this section modifies the analysis presented above to incorporate flywheel energy storage into the HPG system.

Since the rotor and flywheel are mounted to the same shaft, their tip speeds are necessarily proportional. The rotor tip speed is limited to approximately 180 m/s by the brush wear considerations discussed earlier. But the flywheel tip speed is limited by material stress considerations only, and utilization of advanced composite materials allows considerably higher tip speeds to be achieved. Let the ratio of the flywheel tip speed to the rotor tip speed (and also the flywheel outer radius to the rotor outer radius) be denoted by the symbol s . The flywheel radius itself need not be introduced as an independent parameter, but the flywheel length does. Subscripts will be used to differentiate between the rotor length (l_r) and the flywheel length (l_f), and between the rotor density (ρ_r) and the flywheel density (ρ_f).

The performance expressions for the current, voltage, and power are unchanged by the addition of a flywheel to the system. The power maximization procedure performed earlier stands unaltered as well. Only the final expression for maximum stored energy is altered. The performance equations for the HPG system with flywheel energy storage are

$$I = \frac{2\pi}{\sqrt{3}} J_b r_o l_r$$

$$V = \frac{1}{3} B v_t r_o$$

$$E = \frac{2\pi}{9} \rho_r v_t^2 r_o^2 l_r + \frac{\pi \rho_f v_t^2 r_o^2 l_f s^4}{4}$$

$$P = \frac{2\pi}{3\sqrt{3}} B v_t J_b r_o^2 l_r$$

Once again, the expressions for current and energy will be set equal to the values required for the IGNITEX mission. A new free variable has been introduced, however: l_f . So now the two equations contain three unknowns. A third equation can be obtained from rotordynamic considerations by setting the flywheel length to radius ratio equal to a predetermined value f :

$$\frac{l_f}{s r_o} = f$$

Three equations in the variables r_o , l_r , and l_f now exist:

$$I = \frac{2\pi}{\sqrt{3}} J_b r_o l_r$$

$$E = \frac{2\pi}{9} \rho_r v_t^2 r_o^2 l_r + \frac{\pi \rho_f v_t^2 r_o^2 l_f s^4}{4}$$

$$\frac{l_f}{s r_o} = f$$

Using the first and third of these equations, the two length variables can be eliminated from the energy equation, leaving the following cubic equation for the rotor outer radius:

$$\left(\frac{\pi \rho_f v_t^2 f s^5}{4} \right) r_o^3 + \left(\frac{\sqrt{3}}{9} \frac{I \rho_r v_t^2}{J_b} \right) r_o - E = 0$$

Once this equation is solved for the rotor radius, the length variables can be obtained easily from the other two equations, and the flywheel radius is related to the rotor radius by the factor s . The major geometric variables of the entire system are determined by the optimization criteria.

For the flywheel-free case, the number of HPGs among which the load is shared has no effect on the rotor radius. With flywheel energy storage, however, load sharing has an important effect. For application to IGNITEX, the following values of the material properties, limits, and constant ratios have been selected:

$$N = 12$$

$$E = 1 \text{ GJ}$$

$$I = 12.5 \text{ MA}$$

$$J_b = 10^7 \text{ A/m}^2$$

$$\rho_r = 7.8 \times 10^3 \text{ kg/m}^3$$

$$v_t = 1.8 \times 10^2 \text{ m/s}$$

$$f = 2.0$$

$$s = 3$$

$$\rho_f = 1,993 \text{ kg/m}^3$$

Solution of the equations derived above subject to these values yields the following geometry:

$$\text{Rotor radius: } 0.341 \text{ m}$$

$$\text{Rotor length: } 1.01 \text{ m}$$

$$\text{Flywheel radius: } 1.02 \text{ m}$$

$$\text{Flywheel length: } 2.05 \text{ m}$$

These values are quite reasonable, whereas the flywheel-free optimum system had a very unrealistic geometry. This shows that flywheel energy storage is essentially required for IGNITEX. Furthermore, this optimization procedure has quite general applicability, and can be used anytime a given amount of energy must be stored in a specified inductive load.