**COMPARISON OF THE NATURE OF TORQUE PRODUCTION
IN RELUCTANCE AND INDUCTION MOTORS**

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COMPARISON OF THE NATURE OF TORQUE PRODUCTION IN RELUCTANCE AND INDUCTION MOTORS

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Abstract

The nature of torque production is different in reluctance and induction motors. One significant difference occurs in a reluctance motor that has nonsalient stator punching and a salient rotor. When the flux per pole is small in such a motor, the torque can still be high, as long as the rate of energy change with respect to the rotor angular displacement at the rotor pole fronts and pole ends is high. This study provides a theoretical foundation to improve the torque capability of reluctance motors. Effects of saturation and stray-load loss are also studied. Experimental results show agreement with theoretical conclusions.

Introduction

Reluctance motors have several advantages, such as no rotor copper loss and a simple rotor structure. For these reasons, reluctance motors have been an active topic among researchers over the past several decades. From the late fifties to the early seventies, various segmental-type reluctance motors, whose original concept can be traced back to the Kostko arrangement [1], were the center of interest [2-4]. At present, switched reluctance motors, with their high efficiency and torque, attract significant notice [5, 6]. The switching of power fed to individual coils of a switched reluctance motor is controlled by the shaft position encoders. Because of the salient structure of the stator punching and the switching of power, the motor's air-gap torque is generally pulsating. Driven equipment with high enough inertia is well suited for the switched reluctance motor drive. However, preventing possible mechanical oscillation and reducing noise by improvement of torque quality are of general interest.

Unlike the switched reluctance motor, the peaked-mmf (magnetomotive force) reluctance motor [7] has a nonsalient stator bore. The peaked mmf of the stator is produced by the proper arrangement of the stator winding and the current fed to the winding. The peaked-mmf reluctance motor does not require a shaft position encoder. Furthermore, there is no abrupt change of power from one salient pole to the other; the air-gap torque is smooth.

Since the most popular motor is the induction motor, it is logical to compare the nature of torque production and the expected performance between induction and reluctance motors.

This study shows that the torque capability of induction motors goes up when the flux per pole increases. With reluctance motors, on the other hand, increasing the flux per pole is not necessarily the best approach for increasing torque capability, because magnetic saturation is a serious problem for conventional reluctance motors. A better approach is to lighten the saturation, as is done in the single-peaked-mmf reluctance motor [7]. In this motor, the magnitude of a squeezed air-gap flux is strengthened to increase the rate of energy change with respect to the rotor angular displacement at the rotor pole fronts and pole ends. Consequently, the lower flux per pole can ease up the core saturation problem.

Another approach is to increase the number of peaks under a pole of a multiple-peaked-mmf reluctance motor [8,9] to produce higher torque with relatively low tooth flux densities. For a given motor frame, in order to gain higher torque, a reluctance motor should have a higher current loading than its conventional design. The high-torque reluctance motor requires more copper and, consequently, less net iron.

This study shows that stray-load loss is also a serious factor for torque reduction of reluctance motors, particularly for two-pole motors. Stray-load loss in rotating machines has an extensive literature. Dating back to 1912, Olin [10] suggested estimating the stray-load loss by relating it empirically to the no-load loss. Several years ago, Jimoh, Findlay, and Poloujadoff [11,12] outlined the state of the art in understanding and measuring stray-load loss in induction machines with an impressive number of references. It is commonly agreed that the investigation of stray-load loss in rotating machines is far from complete. However, a reasonable experimental method is derived in this paper for the stray-load-loss estimation. Extensive experimental work has been conducted to verify the accuracy of the analysis.

Ideal Torque Capability of Conventional Reluctance Motor

The ideal concept of a conventional, groove-type reluctance motor is to assume that the conductor distributions are represented in simple cosine functions; only fundamental mmf exists in the air gap; no saturation and no mmf drops exist in the iron portions; the tooth harmonics are neglected; the fringing and groove air-gap fluxes are not considered; and the friction, windage, and stray-load losses are not included.

Fig. 1a shows a rotor pole and the stator mmfs for a conventional and a single-peaked-mmf reluctance motor, respectively. The ideal air-gap torque, TRQ_0 , of a single-peaked-mmf reluctance motor derived in ref. [7] is

$$TRQ_0 = -\frac{\pi D^3 L 10^{-7}}{pG} \left(\left(A_1 \sin\left(\frac{\pi}{2} - \frac{\gamma}{2} - \delta\right) - \frac{A_3}{3} \sin\left[3\left(\frac{\pi}{2} - \frac{\gamma}{2} - \delta\right)\right] \right)^2 - \left(A_1 \sin\left(\frac{\pi}{2} + \frac{\gamma}{2} - \delta\right) - \frac{A_3}{3} \sin\left[3\left(\frac{\pi}{2} + \frac{\gamma}{2} - \delta\right)\right] \right)^2 \right) \quad (1)$$

where the subscript 1 refers to fundamental, and the subscript 3 refers to third harmonic for the calculation of A values. The current loading, A, is

$$A = \frac{2mNlK_p K_d}{\pi D} \quad (2)$$

D = stator bore diameter in meters
G = effective air gap in meters
I = rms (root mean square) phase current
 K_p = pitch factor
 K_d = distribution factor
L = core length in meters
m = number of phases
N = $2pnT/(\text{no. of parallels})$ = turns in series/phase
n = slots per phase belt per pole
p = number of pole pairs
T = turns per coil
 δ = angle between stator and rotor pole in electrical degrees
 γ = rotor pole arc in electrical degrees

When the effect of harmonic current of a conventional reluctance motor is neglected, the ideal air-gap torque equation can be simplified to contain the fundamental current loading, A_1 , only:

$$TRQ_0 = -\frac{\pi D^3 L 10^{-7}}{pG} \left\{ [A_1 \sin(\frac{\pi}{2} - \frac{\gamma}{2} - \delta)]^2 - [A_1 \sin(\frac{\pi}{2} + \frac{\gamma}{2} - \delta)]^2 \right\} \quad (3)$$

When γ is $\pi/2$ and δ is increased to $\pi/4$, the value of TRQ_0 reaches its maximum.

Fig. 1 (a) Rotor pole and stator mmfs for reluctance motors
(b) Stator and rotor mmfs of induction motor

$$TRQ_{ol_{max}} = -\frac{\pi D^3 L 10^{-7}}{pG} [A_1]^2 \quad (4)$$

The ideal torque given in eqn. 3 can also be derived from

$$TRQ_0 = -\lim_{\Delta(\frac{\delta}{p}) \rightarrow 0} \frac{\left(\text{Energy loss at rotor pole front} - \text{Energy gain at rotor pole end} \right)}{\Delta(\frac{\delta}{p})} \quad (5)$$

This relationship means that regardless of the width of the rotor pole arc of the reluctance motor, the torque depends mainly on the rate of energy change with respect to the rotor angular displacement at the rotor pole fronts and pole ends.

Ideal Torque Capability of Induction Motor

The ideal assumptions similar to those used for reluctance motors are repeated here for the analysis of induction motors. Unlike a reluctance motor, which does not have rotor current, an induction motor has both stator and rotor currents. Fig. 1b shows the stator mmf and rotor mmf of an induction motor. The rotor current is assumed to be distributed sinusoidally to create a sinusoidal rotor mmf.

The energy [13,14], W, stored in the air gap of an induction motor is

$$W = \frac{pDLG10^7}{8\pi} \int_0^{\frac{\pi}{p}} [B_s \sin p\theta + B_r \sin p(\theta + \frac{\delta}{p})]^2 d\theta \quad (6)$$

where

B_s is the amplitude of the stator flux density
 B_r is the amplitude of the rotor flux density
 θ is the mechanical angle along the rotating direction
 δ is the angular displacement in electrical degrees between stator and rotor mmfs.

The ideal torque of induction motors is derived from the derivative of the energy stored in the air gap of an induction motor with respect to the angular displacement, δ/p , of the rotor.

$$TRQ_0 = -\frac{dW}{d(\frac{\delta}{p})} = \frac{\pi D^3 L 10^{-7}}{pG} [\pi A_s A_r \sin \delta] \quad (7)$$

where A_s and A_r denote the stator and rotor current loadings, respectively. They can be calculated by substituting the motor parameters, and the stator and rotor currents, respectively, into eqn. 2. When the motor is loaded, the stator current is summed up with the rotor current to form the magnetizing current, which is generally smaller than the stator current for a squirrel-cage motor, as shown by the vector diagram in Fig. 2.

As indicated in eqn. 7, the maximum value of ideal torque, $TRQ_{ol_{max}}$, corresponds to the maximum value of $\sin \delta$. For an induction motor, the angular displacement, δ , between stator and rotor currents (or mmfs) is normally greater than 90 electrical

Fig. 2 Current vector diagram of induction motor

degrees, which results in a smaller current than the stator current. The maximum value of $\sin\delta$ is somewhat less than 1. Therefore, for an induction motor, the maximum ideal torque is

$$TRQ_{ol_max} = \frac{\pi D^3 L 10^{-7}}{pG} [\pi A_s A_r K] \quad (8)$$

where K represents the maximum value of $\sin\delta$, which in realistic situations is always less than one.

Since both the stator and rotor currents of an induction motor are distributed around the periphery, eqn. 7 can be viewed as the stator current, (represented by stator current loading A_s) interacting with rotor flux, (represented by rotor current loading A_r). Their interacting relationship is represented by $\sin\delta$. For a given δ angle, the ideal torque capability is increased when A_s and/or A_r are increased. Referring back to the vector diagram of an induction motor shown in Fig. 2, the resultant magnetizing current under a given δ angle is increased while the stator and/or the rotor current are increased. Consequently, this increase in ideal torque corresponds to the increase in flux per pole.

Comparison of Ideal Torque Capabilities Between Induction and Reluctance Motors through Sample Calculations

Comparing eqn. 4 and 8, the maximum ideal torque ratio (induction over conventional reluctance) is $\frac{\pi A_s A_r K}{A_1^2}$. With the same stator current in both induction and reluctance motors (i.e. $A_s = A_1$), the ratio becomes $\frac{\pi A_r K}{A_s}$. As mentioned before, for a conventional squirrel-cage motor, the rotor current is less than the stator current, and the value of this ratio is less than π and normally greater than 1. For the following sample motors, the upper ratio boundary is 1.2 for 6-pole and 1.6 for 2-pole.

The parameters of a 3-phase, 6-pole, 1-horsepower (hp), conventional reluctance motor, and a similar peaked-mmf reluctance motor are given as follows:

p	= 3	L	= 0.1254 meter (m)
D	= 0.1135 m	KG	= Carter coefficient = 1.31
$m1$	= 3	G	= $KG0.000330 = 0.000432$ m
$m3$	= 2	$Kd1$	= 0.966

$Kp1$	= 1.0	$Kd3$	= 1.0
$Kp3$	= 1.0	No. of parallels	= 1

The same stator is used for both the induction and the reluctance motor. For the reluctance motor, a rotor identical to that of the induction motor is milled with 90-electrical-degree pole arcs. In like manner a 2-pole stator with the following parameters is used for the calculations.

p	= 1	L	= 0.125 m
D	= 0.1135 m	KG	= Carter coefficient = 1.22
$m1$	= 3	G	= $KG0.000406 = 0.000495$ m
$m3$	= 2	$Kd1$	= 0.956
$Kp1$	= 1.0	$Kd3$	= 0.911
$Kp3$	= 1.0	No. of parallels	= 1

For the reluctance rotor, the grooves are milled, with 90-electrical-degree pole arcs, on a solid mild-steel rotor. Substituting the parameters and currents into eqn. 1, 3, and 7 gives the ideal torques of reluctance and induction motors. The ideal torques of the 2-pole sample motors are plotted in Fig. 3. The figure shows that the ideal torque of an induction motor is always higher than that of a reluctance motor when the stator winding and current remain unchanged. Under a loaded condition, the rotor currents of the induction motor in the calculation are obtained through vector diagrams according to the no-load and loaded tests under given stator currents. The calculated results of ideal torques for the sample motors are compared with experimental torques in the following sections.

Experimental Results

The shaft output torques and the estimation of stray-load losses are the focus of this section. Fig. 4a shows the experimental setup. The dc machine, coupled to the tested motor through a torque gauge (Lebow/Eaton model 1104-1k), is operated as a generator. The steady-state torque output of the motor is measured by the torque gauge. A full-pitch search coil located on tops of slots of the tested motor, along with a search conductor with return path out-

Fig. 3 Ideal torques of sample induction and conventional reluctance motors versus stator rms phase currents

Fig. 4 (a) Experimental setup for load-torque test
(b) Arrangements for stray-load torque estimation

side the stator frame placed at the bottom of a stator slot, are used for air-gap and core flux density measurements.

Fig. 4b shows the experimental arrangement for the stray-load torque estimation. The energy flows into the motor partially through the stator winding and partially through the shaft as indicated by the arrows. For friction and windage loss measurement, the tested motor is not energized; the torque indicated by the torque gauge gives this loss. For stray-load loss estimation, the torque of the dc motor injected to the tested motor is adjusted to the point that the rotor direct axis of the tested motor is in line with the peak of the stator mmf rotating at synchronous speed. Theoretically, there should be no torque generated by the tested motor; the torque indicated by the torque gauge represents the friction, windage, and stray-load loss at no load. This torque, subtracting friction and windage losses, is plotted against voltage and current, respectively. Since the no-load voltage and current of the tested motor are not both the same as those at the load point, it is roughly assumed that the stray-load loss under the loaded condition is the average of the two points matching the loaded voltage and the loaded current, respectively, from the no-load stray-load-loss curves. This assumption seems justified for small motors because the no-load current of small motors is closer to the full-load current than that of the large motors.

Figs 5 and 6 show the experimental shaft torques of the 6-pole, and the 2-pole motors, respectively. In these two figures, the torque capability of the induction motor under a given stator winding is higher than that of the conventional reluctance motor. The torque capability of single-peaked-mmf reluctance motor is better than that of a conventional reluctance motor, particularly in the 2-pole case shown in Fig. 6. Fig. 7 shows that under a given maximum air-gap flux density, the mmf drop in iron (which is the total mmf subtracted by the air-gap mmf corresponding to the 2-pole sample conventional reluctance motor) is significantly higher than that of the single-peaked-mmf reluctance motors. In other words, the single-peaked-mmf reluctance motor significantly

Fig. 5 Experimental shaft torque for 6-pole sample motors

Fig. 6 Experimental shaft torque for 2-pole sample motors

Fig. 7 mmf drops in iron of 2-pole reluctance motors versus air-gap flux density

Fig. 8 Summations of experimental shaft and stray-load-loss torque versus phase rms currents

Fig. 9 Comparison between ideal and experimental torques of 6-pole motors without considering stray-load-loss torques

lightens the magnetic saturation problem. Fig. 8 shows the comparison of summations of experimental shaft torque and stray-load-loss torque among a 2-pole induction motor, a conventional reluctance motor, and a single-peaked-mmf reluctance motor. The torque differences between the induction motor and the reluctance motors become smaller when the stray-load-loss torques are taken into consideration. Comparison of Fig. 6 and 8 shows that the stray-load-loss torques can be quite large for reluctance motors, especially when a solid rotor, as installed in the 2-pole prototype motor, is used. Fig. 9 shows the comparison among ideal torques and experimental torques for the 6-pole sample motors. The differences in torque values are not as large as those for 2-pole motors because 6-pole motors have lower degrees of saturation and less stray-load losses.

The large differences among ideal and tested curves as shown in Fig. 10, without considering stray-load-loss torque, and Fig. 11, with consideration of stray-load-loss torque, indicate that the effects of magnetic saturation and fringing cannot be ignored [7].

Conclusions

1. For induction motors, the torque capability goes up when the flux per pole increases.
2. For reluctance motors, flux per pole is not a major determining factor for torque production. The rate of energy change with respect to the angular displacement at the rotor pole fronts and pole ends is the major determining factor. Therefore, a high-torque-capability reluctance motor can be built with less flux per pole.
3. In an induction motor, rotor current is available to act on the stator current to form the magnetizing current, while in a reluctance motor, no rotor current is available. Therefore, under a fixed stator current, the magnetizing mmf of a reluctance motor is higher than that of an induction motor. Consequently, the tooth flux densities and the mmf drops in iron are higher. Saturation is a severe drawback for conventional reluctance motors.
4. For the same stator winding and core, the torque produced by an induction motor is generally higher than that produced by a reluctance motor.
5. To produce higher torque from reluctance motors, higher

Fig. 10 Comparison between ideal and experimental torques of 2-pole motors without considering stray-load-loss torque

current loading per pole than that of induction motor and/or a higher number of peaked mmfs per pole are essential.

6. Stray-load-loss torque significantly reduces the shaft output torque of 2-pole reluctance motors—especially when a solid rotor is used. Stray-load-loss torque is relatively insignificant with higher numbers of poles, such as 6-pole reluctance motors.

7. Experimental results show good agreement with theoretical conclusions.

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Fig. 11 Comparison between ideal and experimental torques, including stray-load-loss torques, of 2-pole motors

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