A CONTROL ALGORITHM FOR RELUCTANCE ACCELERATORS

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**Presented at the
5th Symposium on Electromagnetic
Launch Technology
Eglin AFB , Florida
April 2-5, 1990**

**Publication No. PR-103
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Abstract: A brief review of the theory of linear reluctance machines is presented, followed by the derivation of a control algorithm capable of tailoring the acceleration vs. time profile of the device to user specifications. This control algorithm is based upon finite element techniques, numerical integration, and the solution of nonlinear equations. Finally, practical implementation considerations are addressed, and an intuitive picture of the operation of the control algorithm is presented.

Introduction

The Center for Electromechanics at The University of Texas at Austin (CEM-UT) is currently engaged in the design and construction of a linear reluctance accelerator which will accelerate a large mass to a relatively low velocity. A primary design objective is the incorporation of an active control system which will allow the device to follow a user-specified acceleration vs. time profile. This paper addresses the theory and implementation of this control system. Other aspects of the launcher are covered in other papers (see Bibliography).

The control algorithm described here can be applied to a wide range of electromagnetic devices. For instance, it could be used to control the velocity of a magnetic levitation train or to tailor the torque-speed curve of a rotary induction machine.

Theoretical Background

This section presents a brief review of the theory of linear reluctance machines for readers not familiar with these devices. In a reluctance machine the movable member is simply a yoke formed of magnetic material. The stationary member usually contains magnetic material as well, and is energized by a set of current-carrying windings (fig. 1). The effective reluctance of the flux path depends upon the position of the iron yoke. This in turn makes the inductance, as observed from the stator terminals, a function of yoke position.

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Figure 1. A typical reluctance accelerator stage

The magnetic materials normally used in these devices have nonlinear B-H curves, so the usual expression for stored energy in terms of inductance and current is no longer applicable. Instead, the stored coenergy is given by:

$$W'(x, I) = \int_0^I \lambda(x, i) di$$

where

λ = magnetic flux linkages
 x = yoke offset
 I = winding current

A change in yoke position changes the functional form of the integrand and therefore changes the stored coenergy. In order to satisfy the conservation of energy, it is necessary that a mechanical force appear which is given by:

$$F(x, I) = \frac{\partial W'(x, I)}{\partial x}$$

This force tends to move the yoke to the position which offers the lowest reluctance to the magnetic flux. Reluctance machines operate by producing a set of currents in their various windings so that the magnetic force or torque is always in the desired direction.

Finite Element Analysis

The calculation of the inductance of electrical circuit configurations is normally carried out by evaluating the total flux linkages arising from a unit current and then defining the inductance such that $\lambda = LI$ holds. Systems that contain nonlinear magnetic materials can be analyzed in the same manner, except that the inductance becomes a function of current (or flux linkages), as well as of position. The calculation of the inductance of such a system is very difficult and is normally carried out using finite element techniques.

Such a calculation has been made for the reluctance launcher geometry chosen by the CEM-UT design team. Figure 2 illustrates the geometry of the fixed and movable members of a single stage of the machine, while figure 3 shows the inductance of the winding as a function of yoke position and total current. It proves convenient to have inductance as a function of position and flux linkages as well; this information can be derived easily from figure 3. The CEM-UT launcher will consist of a large number of such stages, each of which will deliver to the yoke a portion of its final kinetic energy.

Relation Between Flux Linkages And Position

The inductance surface depicted in figure 3 serves as the foundation upon which the control algorithm is erected. This surface relates terminal inductance to the

Figure 2. The geometry of the CEM-UT accelerator

Figure 3. Inductance vs. position and current for the CEM-UT accelerator -- position zero is the centered position

position of the system in the state space (λ, x). Next, two quite reasonable assumptions allow the determination of the trajectory of the system through this state space (i.e., the specification of λ as a function of x). These assumptions are:

1. Only one winding will be active at any given instant. Simultaneous activation of two or more windings complicates enormously the calculation of the system inductances, since superposition does not hold for this nonlinear system.
2. Each stage makes a relatively small change in the yoke's kinetic energy level. This assumption allows yoke velocity to be treated as a constant during the operation of any one stage.

Assumption 2 is not valid for the first few stages. However, it becomes valid very early in the launch. The algorithm described here can be extended to allow for

nonconstant velocities. These extensions, however, will not be addressed in this paper.

Now a differential equation giving flux linkages as a function of position relative to a typical stage will be derived. A simple voltage loop equation including the winding and its power supply gives:

$$V = \frac{R\lambda}{L} + \frac{\partial \lambda}{\partial t}$$

The first term on the right gives the voltage drop due to the winding resistance and the second is the drop due to Faraday's Law. Multiplication of this equation by the reciprocal of the (constant) velocity gives:

$$\frac{d\lambda}{dx} + \frac{R}{vL}\lambda = \frac{V}{v}$$

which gives the trajectory through the state space produced by a specified voltage waveform $V(x)$. The winding is open-circuited prior to application of the supply voltage V , thus the required initial condition is $\lambda(x) = 0$ for $x < x_{on}$. Here x_{on} denotes the yoke position when the supply voltage is activated.

The CEM-UT launcher will operate with a single supply voltage level. The supply will be applied to a stage; after an active interval, the stage will be crowbarred to allow the current to circulate until it decays. No attempt to open a crowbar switch will be made, so the trajectory of the stage through stage space is completely specified by two parameters: x_{on} , (defined above) and x_{cb} (which denotes the position at which the crowbar switch is activated).

The differential equation above and the essence of the procedures outlined below are valid for any applied voltage waveform. However, the specific discussion below applies to the fixed voltage, once-on, once-off system described in the preceding paragraph. The job of the control algorithm is the determination of the proper values of x_{on} and x_{cb} for each stage of the launcher.

Control Criteria

The first step in control system design is the choice of an optimality criterion which the system will attempt to satisfy. An important design objective of the CEM-UT reluctance launcher is the incorporation of a control system that will cause the yoke to follow an arbitrary user-specified acceleration vs. time profile. However, it proves convenient to express this profile as an energy vs. position profile. The two profiles are equivalent as long as the yoke position is a monotonically increasing function of time.

An energy vs. position profile gives the desired value of the yoke kinetic energy as it leaves the domain of each winding. Each winding must deliver just enough energy to the yoke to give it the desired exit energy. A set of sensors measures the yoke energy as it approaches the winding. The control system must then choose values for (x_{on}, x_{cb}) to give the proper energy boost.

The specification of a kinetic energy vs. position profile does not uniquely specify the set of (x_{on}, x_{cb}) for all stages. Many values for these parameters will give the

necessary energy boost. A further optimality criteria is required. An obvious candidate is the minimization of resistive energy losses during the launch. If $E(x_{on}, x_{cb})$ denotes the energy boost produced by the specified parameter choices and $H(x_{on}, x_{cb})$ denotes the resistive losses incurred during the operation of the stage under consideration, then the optimal control criterion can be stated as:

$$\text{Minimize } H(x_{on}, x_{cb}) \text{ subject to } E(x_{on}, x_{cb}) = E_0$$

Here, E_0 denotes the energy boost required to bring the yoke up to the desired kinetic energy.

Determination Of The Optimal Parameters

For a moment assume that H and E can be evaluated for any value of their arguments. Then the method of Lagrange multipliers can be used to solve the optimal control problem. Consider the augmented function:

$$U(x_{on}, x_{cb}, \kappa) = H(x_{on}, x_{cb}) + \kappa[E(x_{on}, x_{cb}) - E_0]$$

The optimal selection of the switching positions can be found by solving the three nonlinear equations obtained by setting the partial derivatives of U with respect to each of its parameters to zero. Many FORTRAN subroutines exist to solve just such a problem. The programmer specifies the FORTRAN functions that evaluate the partial derivatives, and the library routine does the rest.

All that remains is to describe the evaluation of the functions H and E for a specified (x_{on}, x_{cb}) pair. The next section addresses this task.

Evaluation Of $H(x_{on}, x_{cb})$ And $E(x_{on}, x_{cb})$

The evaluation of these functions is straightforward. Clearly, the heat loss in a single stage can be expressed as:

$$H = \int_{x_{on}}^{\infty} R I^2 dt$$

which can be written in the following form (since the velocity is treated as constant):

$$H = \frac{R}{v} \int_{x_{on}}^{\infty} \left[\frac{\lambda}{L} \right]^2 dx$$

Remember that λ is a known function of x , given by the differential equation presented earlier. Also, L is a known function of x and λ , given by finite element analysis. The parameter x_{cb} enters the integration by its effect upon the function $\lambda(x)$. Since the winding current decays exponentially to zero, the infinite upper limit is not a problem.

The calculation of $E(x_{on}, x_{cb})$ is a bit more complex. Clearly, its value is given by:

$$E = \int_{x_{on}}^{\infty} F(x, \lambda) dx$$

but the mechanical force $F(x, \lambda)$ must be evaluated. Since λ is a known function of x , the integrand is really a function only of x .

The force can be evaluated as the partial derivative of coenergy with respect to position, as discussed earlier. The partial derivative can be taken numerically and the coenergy can be calculated as:

$$W = \frac{\lambda^2}{L} - \int_0^{\lambda} \frac{\lambda}{L} d\lambda$$

which is expressed in terms of known quantities. This gives a complete algorithm for the evaluation of the functions H and E and therefore the augmented function U . This provides the necessary computational framework for the application of the method of Lagrange multipliers, as discussed earlier.

Practical Considerations And Intuitive Insights

While the technique described above provides a complete solution to the optimal control problem, its computational complexity prevents real-time implementation of the algorithm on a stage-by-stage basis. To circumvent this problem, table look-up techniques will be used.

Once an acceleration vs. time profile is chosen, conversion to an energy vs. position profile is straightforward. This defines the desired kinetic energy level from which the velocity can be calculated. Prior to the launch, a table can be constructed for each stage, giving the control parameters (x_{on}, x_{cb}) for a range of velocities in a velocity window centered on the nominal desired velocity.

The real time control algorithm is then straightforward. The sensor system measures velocity as the yoke approaches a given winding stage. The table for that stage is then consulted to obtain the proper (x_{on}, x_{cb}) pair and the switches are activated at these positions. To change the acceleration profile requires only that a new set of tables be generated.

Techniques described above obviously provide a conceptually simple approach to the control problem. However, the approach is highly mathematical. It is helpful to take an intuitive look at the effect of the control parameters (x_{on}, x_{cb}) on the operation of the system.

Figure 4 illustrates some typical trajectories of the system through its state space. These clearly show the effect of changing the control parameters (x_{on}, x_{cb}) . The dashed curves indicate the level surfaces of force calculated using the techniques of the previous section. It is important to remember that the force is always directed toward $x = 0$, so the force contours to the left of the λ axis form a "mountain", while those to the right of the λ axis form a "valley".

To obtain a large energy transfer, the trajectory should pass near the peak of the forward force "mountain", while avoiding the depths of the reverse force "valley" (trajectory A). A smaller transfer can be obtained in several ways; for instance, by passing near the peak of the "mountain" and descending slightly into the "valley" (trajectory B) or by traveling partially up the "mountain"

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Figure 4. Trajectories through state space and their force effects

and avoiding the "valley" (trajectory C). Trajectory C, however, will produce less heat dissipation than trajectory B. Finally, trajectory D would cause the yoke to slow down, since it passes through large reverse force regions of the state space, but not through large forward force regions. This trajectory might be chosen if the actual energy level exceeded that called for by the chosen profile.

Conclusions

This paper has presented the theory and implementation of a control algorithm applicable to a wide range of machines. While the algorithm itself is rather computationally intensive, involving finite element techniques, numerical integration, and the iterative solution of nonlinear equations, the real-time requirements (i.e., velocity measurements and table look-ups) can easily be met using standard microprocessor technology. Furthermore, the off-line computations are complex, but not extremely so, and should be well within the abilities of any machine for which a finite element software package is available.

Acknowledgment

This work is supported by Department of the Navy/Applied Research Labs, under Contract no. N00039-88-C-0043, Patti Hall, Contract Monitor Representative.

Bibliography

- [1] D.A. Bresie and J.A. Andrews, "Design of a Reluctance Accelerator," to be presented at the 5th EML Conference, Eglin AFB, FL, April 2-5, 1990.
- [2] A.E. Fitzgerald, C. Kingsley, and A. Kusko, Electric Machinery. McGraw Hill Book Company, 1971, 3rd Edition, pp. 80-103.
- [3] R.D. Pillsbury, "Texas Magnetic Analysis Program (Texmap)," Publication No. XM-5, September, 1976.
- [4] F.P. Emad, D.B. Steen, D.H. Waltman, W.R. Ruby, M.J. Superczynski, "DTRC Laboratory EM Accelerator System Design, Modeling and Simulation," IEEE Transactions on Magnetics, vol. 25, no. 1, pp. 170-173, January 1989.
- [5] M. Petrovic and I. Mancho, "Numerical Procedure for Determining parameters of Electrical Machines," IEEE Transaction on Magnetics, vol. 24, no. 1, pp. 508-511, January 1988.