**CAPACITOR EFFECTS ON INDUCTION MOTORS FED BY
QUASI-RECTANGULAR CURRENT SOURCES**

Prepared by

J. S. Hsu

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Center for Electromechanics
The University of Texas at Austin
Balcones Research Center
Bldg. 133, EME 1.100
Austin, TX 78758-4497
(512) 471-4496

CAPACITOR EFFECTS ON INDUCTION MOTORS FED BY QUASI RECTANGULAR CURRENT SOURCES

John S. Hsu (Htsui)
Senior Member

Center for Electromechanics
The University of Texas at Austin
Austin, Texas 78712

Abstract: The analytical and experimental studies of the capacitor effects on the steep rising and falling voltage spikes and the current harmonics in induction motors fed by quasi rectangular current sources are conducted through the case of a motor fed with a six pulse current source. The phase current of the motor fed by the six-pulse current source goes through various regions of harmonic current (and voltage) variations. The analytical and experimental results show that as the value of the capacitor reactance is reduced gradually toward the motor magnetizing reactance, the motor encounters the higher frequency current amplifications first, and subsequently, the seventh-harmonic amplification and then the fifth-harmonic amplifications. Analytical result also shows that it is possible to encounter a severe fundamental-frequency current amplification when the capacitor reactance is less than the motor magnetizing reactance.

INTRODUCTION

The effects of capacitors [1,2] connected to the terminals of induction motors fed by a conventional power system supply have been fully investigated. It is known that under that situation, self-excitation can cause over-voltage when the motor is switched off from the three phase, sinusoidal-voltage power supply. The National Electrical Code [2] states

"Capacitors are used for lagging power-factor correction. When these capacitors are connected on the load side of the motor controller, the KVAR capacity of the capacitors shall not exceed that value which is just necessary to bring the power factor up to unity."

However, when both the motor and the capacitors are connected to the conventional power system supply, there is no effect caused by the capacitors on the induction motor. The capacitors act like an independent three-phase load.

With the adjustable speed drives [3-7], the quasi rectangular current source is an important branch among various adjustable frequency power sources. This study shows that unlike the induction motors fed by a conventional power system supply, there are changes in both the motor currents and voltages under a quasi rectangular current source.

It is not certain whether the steep rising and falling voltage spikes associated with the quasi rectangular current actually lower the insulation life expectancy of the motor. However, it is known that capacitors can modify the steepness and the magnitude of the voltage spikes, the currents, and the performance of the motor. The traditional rules of using capacitors for induction motors need to be investigated under the new situation. The objectives of this study are: 1) deriving the analytical method, 2) conducting sample computations and tests, and 3) obtaining conclusions from the results to see the effects caused by the capacitors on induction motors when both the motor and the capacitors are connected to a rectangular current source.

An analytical method based on the 1-2-0 coordinate system for a six-pulse case is developed to calculate the effects on current and voltage of the motor when capacitors are connected to the motor terminals under a quasi rectangular current source supply. A series of computations of a sample motor is conducted to identify the significant harmonic current (and voltage) variations. Experimental results agree with the analytical results.

ASSUMPTIONS

The analysis is based on the following assumptions:

- The motor runs at a constant speed.
- The quasi rectangular current source can be considered as a rectangular current wave.
- Saturated motor parameters are used; otherwise the motor magnetic circuit is considered to be linear.
- Core, friction, windage, and stray-load losses are not considered in the analysis.
- The neutral of the motor is not grounded; consequently, the current and flux linkage of coordinate zero can be neglected. It is worth mentioning that the current and flux of coordinate zero, even if they exist, do not produce torque.

ANALYSIS

The circuit of the analysis is shown in Fig. 1. The rectangular current source is the power supply of the circuit. There are different coordinate systems that can be used for the analysis [8-14]. This paper uses the 1-2-0 coordinate system [10,14] and the per unit values for the analysis. Since the value of coordinate 2 is always the conjugate complex number of the value of coordinate 1, attention can be drawn mainly to the values referring to coordinate 1. Resultant equations established in the available literature, such as the transformation from a, b, c values to 1, 2, 0 values, or vice versa [10,14], and the definition of the operational impedances [14] are mentioned and used directly without repeating the derivations.

Fig. 1. Induction motor connected with capacitors

Fig. 2. Three phase rectangular source currents:
a) i_a, i_b , and i_c
b) real and imaginary parts of i_1

Input Data and Per-Unit (P.U.) Values

The reactances of the induction motor are normally measured under the supply frequency with sinusoidal voltage. For adjustable frequency the inductance can be considered as a constant at different frequencies, but the reactance (a product of $2\pi f$ and the inductance) changes.

The unit (or base) values are defined in Appendix A. The symbols of the input data are chosen as follows:

- p = number of poles
- f = line frequency (Hz)
- X_{ls} = stator leakage reactance (per unit value)
- X_{lr} = rotor leakage reactance (per unit value)
- R_s = stator resistance (per unit value)
- R_r = rotor resistance (per unit value)
- X_m = magnetizing reactance (per unit value)
- X_C = reactance of capacitors connected directly to motor terminals (per unit value)
- I_d = dc current (per unit value)
- N = shaft speed (r/min)

Conversion to 1-2-0 Coordinates Values

The rectangular source currents, i_a, i_b , and i_c , of the analytical model as shown in Fig. 2a can be transformed to the 1-2-0 coordinate values through (1).

$$\begin{aligned} i_1 &= \frac{1}{3} (i_a + a i_b + a^2 i_c) & i_a &= i_1 + i_2 + i_0 \\ i_2 &= \frac{1}{3} (i_a + a^2 i_b + a i_c) & \text{or } i_b &= a^2 i_1 + a i_2 + i_0 \\ i_0 &= \frac{1}{3} (i_a + i_b + i_c) & i_c &= a i_1 + a^2 i_2 + i_0 \end{aligned} \quad (1)$$

$$\text{where } a = e^{j\frac{2\pi}{3}} = -0.5 + j0.866$$

The real and imaginary parts of the coordinate-1 current i_1 of the source currents are shown in Fig. 2b.

Source Currents

The value of the coordinate-1 source current i_1 changes at every $\pi/3$ radians. Using Heaviside unit functions, i_1 can be expressed as

$$\begin{aligned} i_1 &= \left(0 - j \frac{I_d}{\sqrt{3}}\right) u(t-0) + \left(\frac{I_d}{2} + j \frac{I_d}{2\sqrt{3}}\right) u\left(t - \frac{\pi}{3}\right) \\ &+ \left(0 + j \frac{I_d}{\sqrt{3}}\right) u\left(t - \frac{2\pi}{3}\right) + \left(-\frac{I_d}{2} + j \frac{I_d}{2\sqrt{3}}\right) u\left(t - \pi\right) \end{aligned}$$

+ repeats in like manner in opposite direction over every π , up to time t .

where the Heaviside unit function is defined through an example:

$$u\left(t - \frac{\pi}{3}\right) = \begin{cases} 1 & t > \frac{\pi}{3} \\ 0 & t < \frac{\pi}{3} \end{cases}$$

The P-multiplied Laplace transform [15] is used in this study. Using L to represent this transform, the current i_1 can be written according to the definition of the P-multiplied Laplace transform.

$$\begin{aligned} L i_1 &= P \int_0^\infty i_1 e^{-Pt} dt = \frac{(0.5 e^{(\pi/6)P} - 0.5 e^{-(\pi/2)P})}{2 \cosh(\frac{\pi}{2}P)} I_d \\ &+ j \frac{\left(-\frac{1}{\sqrt{3}} e^{(\pi/2)P} + \frac{1}{2\sqrt{3}} e^{(\pi/6)P}\right) (1 - e^{-(2\pi/3)P})}{2 \cosh(\frac{\pi}{2}P)} I_d \quad (3) \end{aligned}$$

where P is a parameter used for the Laplace transform. When applying the Laplace transform to solve a differential equation, P also stands for d/dt of a function while the initial value of the function is 0.

The source current i_1 , as shown in Fig. 2b, contains the motor current i_{1m} and the capacitor current i_{1c} of the coordinate 1 values.

$$L i_1 = L i_{1m} + L i_{1c} = \left[\frac{1}{R_s + P X(P-j\omega)} + \frac{1}{X_C} \right] L v_1 \quad (4)$$

where the operational impedance $X(P-j\omega)$ of the induction motor from [14] at a constant speed, ω , is

$$X(P-j\omega) = \frac{X_{ss} \left[\tau (P-j\omega) + \frac{1}{T_2} \right]}{P-j\omega + \frac{1}{T_2}} \quad (5)$$

and

$$\begin{aligned}\omega &= \frac{\pi N p}{60 \omega_s} & \tau &= 1 - \frac{X_m^2}{X_{ss} X_{rr}} \\ X_{ss} &= X_{ls} + X_m & \omega_s &= 2 \pi f \\ T_2 &= \frac{X_{rr}}{R_r} & X_{rr} &= X_{lr} + X_m\end{aligned}$$

Motor Terminal Voltages

Equation (4) indicates that the combined operational impedance seen by the current i_1 is the parallel values of the motor and the capacitor. The motor terminal voltage, v_1 , of coordinate 1 from (4) can be evaluated

$$L v_1 = \frac{L i_1}{\left[\frac{1}{R_s + P X(P - j\omega)} + \frac{1}{X_C} \right]} \quad (6)$$

Substituting (3) and (5) into (6) gives

$$L v_1 = \frac{N_1(P) I_d + N_2(P) I_d}{2 D_1(P) \cosh\left(\frac{\pi}{2} P\right)} \quad (7)$$

where

$$N_1(P) = \left\{ R_s X_C \left(P - j\omega + \frac{1}{T_2} \right) + P X_{ss} X_C \left[\tau (P - j\omega) + \frac{1}{T_2} \right] \right\} \cdot (0.5 e^{(\pi/6)P} - 0.5 e^{-(\pi/2)P})$$

$$N_2(P) = j X_C \left\{ R_s \left(P - j\omega + \frac{1}{T_2} \right) + P X_{ss} \left[\tau (P - j\omega) + \frac{1}{T_2} \right] \right\} \cdot \left(-\frac{1}{\sqrt{3}} e^{(\pi/2)P} + \frac{1}{2\sqrt{3}} e^{(\pi/6)P} \right) (1 - e^{-(2\pi/3)P})$$

$$D_1(P) = P^3 X_{ss} \tau + P^2 \left[R_s - X_{ss} \tau j \omega + \frac{X_{ss}}{T_2} \right] + P \left[X_C - j \omega R_s + \frac{R_s}{T_2} \right] - j \omega X_C + \frac{X_C}{T_2}$$

The inverse Laplace transform of $L v_1$ of (7) can be solved using the residue theorem [15, 16] or the Heaviside expansion formula [15]. Any appropriate method, such as the Newton-Raphson method, can be used to solve the values of the roots of the denominator of (7). There are three roots, P_Q , where $Q = 1, 2$, and 3, of the denominator term

$$D_1(P) = 0$$

and a series of roots $P_{(k)}$ solved from the other denominator term

$$\cosh\left(\frac{\pi P}{2}\right) = 0$$

where

$$P_{(k)} = (2k + 1)j$$

and $k = 0, \pm 1, \pm 2 \dots$

The inverse Laplace transform of $L v_1$ obtained through Heaviside Expansion Formula is

$$\begin{aligned}v_1 &= \sum_{Q=1}^{Q=3} \frac{[N_1(P_Q) + N_2(P_Q)] I_d e^{P_Q t}}{P_Q \frac{d[2 D_1(P) \cosh(\frac{\pi}{2} P)]}{dP} \Big|_{P_Q}} \\ &+ \sum_{k=-\infty}^{k=+\infty} \frac{[N_1(P_{(k)}) + N_2(P_{(k)})] I_d e^{P_{(k)} t}}{P_{(k)} \frac{d[2 D_1(P) \cosh(\frac{\pi}{2} P)]}{dP} \Big|_{P_{(k)}}}\end{aligned}$$

Since

$$\frac{d[2 D_1(P) \cosh(\frac{\pi}{2} P)]}{dP} = 2 D_1'(P) \cosh(\frac{\pi}{2} P) + \pi D_1(P) \sinh(\frac{\pi}{2} P)$$

v_1 can be written as

$$\begin{aligned}v_1 &= \sum_{Q=1}^{Q=3} \frac{[N_1(P_Q) + N_2(P_Q)] I_d e^{P_Q t}}{P_Q [2 D_1'(P_Q) \cosh(\frac{\pi}{2} P_Q) + \pi D_1(P_Q) \sinh(\frac{\pi}{2} P_Q)]} \\ &+ \sum_{k=-\infty}^{k=+\infty} \frac{[N_1(P_{(k)}) + N_2(P_{(k)})] I_d e^{P_{(k)} t}}{P_{(k)} [2 D_1'(P_{(k)}) \cosh(\frac{\pi}{2} P_{(k)}) + \pi D_1(P_{(k)}) \sinh(\frac{\pi}{2} P_{(k)})]}\end{aligned}$$

where

$$D_1'(P) = 3 P^2 X_{ss} \tau + 2 P \left[R_s - X_{ss} \tau j \omega + \frac{X_{ss}}{T_2} \right] + \left[X_C - j \omega R_s + \frac{R_s}{T_2} \right]$$

Substituting

$$\sinh\left(\frac{\pi}{2} P_{(k)}\right) = j \sin\left[\frac{\pi}{2} (2k + 1)\right] = j (-1)^k$$

gives

$$\begin{aligned}v_1 &= \sum_{Q=1}^{Q=3} \frac{[N_1(P_Q) + N_2(P_Q)] I_d e^{P_Q t}}{P_Q [2 D_1'(P_Q) \cosh(\frac{\pi}{2} P_Q) + \pi D_1(P_Q) \sinh(\frac{\pi}{2} P_Q)]} \\ &+ \sum_{k=-\infty}^{k=+\infty} \frac{[N_1(P_{(k)}) + N_2(P_{(k)})] I_d e^{P_{(k)} t}}{P_{(k)} [2 D_1'(P_{(k)}) \cosh(\frac{\pi}{2} P_{(k)}) + j \pi (-1)^k D_1(P_{(k)})]}\end{aligned}$$

Considering

$$\cosh\left(\frac{\pi}{2} P_{(k)}\right) = \cosh\left[\frac{\pi}{2} (2k + 1)j\right] = \cos\left[\frac{\pi}{2} (2k + 1)\right] = 0$$

and

$$D_1(P_Q) = 0$$

the voltage v_1 is

$$\begin{aligned}v_1 &= \sum_{Q=1}^{Q=3} \frac{[N_1(P_Q) + N_2(P_Q)] I_d e^{P_Q t}}{P_Q [2 D_1'(P_Q) \cosh(\frac{\pi}{2} P_Q)]} \\ &+ \sum_{k=-\infty}^{k=+\infty} \frac{[N_1(P_{(k)}) + N_2(P_{(k)})] I_d e^{P_{(k)} t}}{(2k + 1) \pi (-1)^k D_1(P_{(k)})}\end{aligned} \quad (8)$$

v_2 is the conjugated value of v_1 .

Motor Currents

The motor current, i_{1m} , of coordinate-1 can be calculated through one of the following two approaches. The first approach is to calculate the current through the operational impedances

$$L i_{1m} = \frac{L i_1}{\left[\frac{1}{R_s + P X(P - j\omega)} + \frac{1}{X_C} \right] [R_s + P X(P - j\omega)]}$$

the second approach is from (4)

$$L i_{1m} = \frac{L v_1}{R_s + P X(P - j\omega)}$$

Substituting (7) and (5) into the above equation gives

$$L i_{1m} = \frac{(P - j\omega + \frac{1}{T_2}) [N_1(P) I_d + N_2(P) I_d]}{2 D_2(P) \cosh(\frac{\pi}{2} P)} \quad (9)$$

The denominator term, $D_2(P)$, is represented by

$$D_2(P) = B_1 P^5 + B_2 P^4 + B_3 P^3 + B_4 P^2 + B_5 P + B_6$$

where

$$B_1 = X_{ss}^2 \tau^2$$

$$B_2 = 2 \tau X_{ss} R_s + 2 \tau \frac{X_{ss}^2}{T_2} - 2 j \omega X_{ss}^2 \tau^2$$

$$B_3 = \tau X_{ss} X_C + 2 \tau X_{ss} \frac{R_s}{T_2} - 4 j \omega \tau X_{ss} R_s - \omega^2 X_{ss}^2 \tau^2 - 2 j \omega \tau \frac{X_{ss}^2}{T_2} + \frac{X_{ss}^2}{T_2^2} + R_s^2 + 2 R_s \frac{X_{ss}}{T_2}$$

$$B_4 = -2 j \omega \tau X_{ss} X_C + \frac{\tau X_{ss} X_C}{T_2} - 2 \tau X_{ss} \omega^2 R_s - 2 j \omega \tau X_{ss} \frac{R_s}{T_2} + \frac{X_{ss} R_s}{T_2} + 2 \frac{X_{ss} R_s}{T_2^2} + R_s X_C - 2 j \omega R_s^2 + 2 \frac{R_s^2}{T_2} - 2 j \omega R_s \frac{X_{ss}}{T_2}$$

$$B_5 = -\omega^2 \tau X_{ss} X_C - j \omega \tau X_{ss} \frac{X_C}{T_2} - \frac{X_{ss}}{T_2} j \omega X_C + \frac{X_{ss} X_C}{T_2^2} + 2 R_s \frac{X_C}{T_2} - 2 j \omega R_s X_C - \omega^2 R_s^2 - 2 j \omega \frac{R_s^2}{T_2} + \frac{R_s^2}{T_2^2}$$

$$B_6 = -\omega^2 R_s X_C - 2 j \omega R_s \frac{X_C}{T_2} + \frac{R_s X_C}{T_2^2}$$

The simple poles of (9) are at the points of the roots of the denominator. There are five roots, P_s , where $S = 1, 2, 3, 4$, and 5 , of the denominator term

$$D_2(P) = 0$$

and a series of roots $P(k)$ solved from the denominator term

$$\cosh\left(\frac{\pi P}{2}\right) = 0$$

where

$$P(k) = (2k+1)j$$

and $k = 0, \pm 1, \pm 2 \dots$

The inverse Laplace transform of $L i_{1m}$ obtained through Heaviside Expansion Formula is

$$i_{1m} = \sum_{s=1}^{s=5} \frac{\left(P_s - j\omega + \frac{1}{T_2}\right) [N_1(P_s) + N_2(P_s)] I_d e^{P_s t}}{P_s \left[\frac{d}{dP} \left[2 D_2(P) \cosh\left(\frac{\pi P}{2}\right) \right] \right]_{P_s}} + \sum_{k=-\infty}^{k=+\infty} \frac{\left(P(k) - j\omega + \frac{1}{T_2}\right) [N_1(P(k)) + N_2(P(k))] I_d e^{P(k)t}}{P(k) \left[\frac{d}{dP} \left[2 D_2(P) \cosh\left(\frac{\pi P}{2}\right) \right] \right]_{P(k)}}$$

Substituting

$$\frac{d}{dP} \left[2 D_2(P) \cosh\left(\frac{\pi P}{2}\right) \right] = 2 D_2'(P) \cosh\left(\frac{\pi P}{2}\right) + \pi D_2(P) \sinh\left(\frac{\pi P}{2}\right)$$

into above equation gives

$$i_{1m} = \sum_{s=1}^{s=5} \frac{\left(P_s - j\omega + \frac{1}{T_2}\right) [N_1(P_s) + N_2(P_s)] I_d e^{P_s t}}{P_s \left[2 D_2'(P_s) \cosh\left(\frac{\pi P_s}{2}\right) + \pi D_2(P_s) \sinh\left(\frac{\pi P_s}{2}\right) \right]} + \sum_{k=-\infty}^{k=+\infty} \frac{\left(P(k) - j\omega + \frac{1}{T_2}\right) [N_1(P(k)) + N_2(P(k))] I_d e^{P(k)t}}{P(k) \left[2 D_2'(P(k)) \cosh\left(\frac{\pi P(k)}{2}\right) + \pi D_2(P(k)) \sinh\left(\frac{\pi P(k)}{2}\right) \right]}$$

where

$$D_2'(P) = 5 B_1 P^4 + 4 B_2 P^3 + 3 B_3 P^2 + 2 B_4 P + B_5$$

The motor current, i_{1m} , can be obtained in the like manner as that used for the voltage

$$i_{1m} = \sum_{s=1}^{s=5} \frac{\left(P_s - j\omega + \frac{1}{T_2}\right) [N_1(P_s) + N_2(P_s)] I_d e^{P_s t}}{P_s \left[2 D_2'(P_s) \cosh\left(\frac{\pi P_s}{2}\right) \right]} + \sum_{k=-\infty}^{k=+\infty} \frac{\left(P(k) - j\omega + \frac{1}{T_2}\right) [N_1(P(k)) + N_2(P(k))] I_d e^{P(k)t}}{(2k+1) \pi (-1)^k D_2(P(k))} \quad (10)$$

From (1) the phase-a current i_a is

$$i_a = 2 \text{Real}(i_{1m}) \quad (11)$$

with $i_0 = 0$.

SAMPLE CALCULATION

The parameters of a five-horsepower, 230-V, three-phase, four-pole, Y-connected squirrel-cage motor are derived from the 60-Hz values and given as follows:

f (Hz)	60	30
unit torque (Nm)	19.79	39.58
unit current (A)	9.36	9.36
unit voltage (V)	132.79	132.79
unit impedance (Ω)	14.18	14.18
X_{ls} (p.u.)	0.0628	0.0314
X_{lr} (p.u.)	0.0628	0.0314
X_m (p.u.)	1.7630	0.8814
R_s (p.u.)	0.0395	0.0395
R_r (p.u.)	0.0240	0.0240
I_d (p.u.)		1.0
N (rpm)		886.

A series of computations using (10) and (11) is conducted for various per-unit values of capacitor reactance, X_c , at 30 Hz. When there is no paralleled capacitor connected directly to the motor, the X_c value is infinity; a large value of $X_c = 1,000$ is used to represent this situation. The steady-state current and voltage values are obtained by starting the computation at 200 cycles to allow sufficient attenuations for the initial transients.

Fig. 3a shows the phase-a steady current of the induction motor with $X_c = 1,000$ and 10, respectively. When there is no capacitor ($X_c = 1,000$) connected directly to the motor, the motor current is in a rectangular shape. As compared with the subsequent figures it is seen that when the X_c value is higher (i.e. lesser capacitance), the motor contains higher frequencies of harmonic currents. Fig. 3b shows the voltage of $X_c = 1,000$ with steep rising and falling spikes. The voltage of $X_c = 10$ does not show such sharp spikes.

Fig. 3a. Motor currents with $X_c = 1,000$ and $X_c = 10$ at 30 Hz

Fig. 3b. Motor voltages with $X_c = 1,000$ and $X_c = 10$ at 30 Hz

Fig. 4a. Seventh harmonic current contents with $X_c = 4.0$ and $X_c = 3.0$ under $X_m = 0.88$ at 30 Hz

Fig. 4b. Motor voltages with $X_c = 1,000$, $X_c = 4.0$, and $X_c = 3.0$ under $X_m = 0.88$ at 30 Hz

Fig. 4a shows that the seventh-harmonic current amplification occurs at $X_c = 4.0$. When more capacitance is connected to the motor, this amplification increases significantly as shown by the waveform of $X_c = 3.0$. The corresponding voltage waveforms are shown in Fig. 4b.

If the capacitance connected to the motor keeps increasing, the seventh-harmonic amplification subsides and the fifth-harmonic current amplification occurs. Fig. 5a shows the fifth-harmonic current amplifications when capacitance is progressively increased from $X_c = 2.0$ to $X_c = 1.1$. Fig. 5b shows the corresponding voltages.

Fig. 6 shows that when the X_c value is close to the magnetizing reactance, ($X_c = 0.90$, and $X_m = 0.88$) the current and voltage go through lengthy oscillations after switching on.

Fig. 7 shows the case with $X_c = 0.70$ and $X_m = 0.88$. It also indicates that when the capacitance connected directly to the motor keeps increasing until X_c is less than the magnetizing reactance X_m , a severe fundamental frequency current and voltage amplifications may occur.

EXPERIMENTS

An M68HC11EVB microcontroller is programmed for the generation of the adjustable frequency gating signals. A six-pulse current-source inverter is also built for the test. Fig. 8 shows the tested motor current i_{ma} , and the inverter output current i_a without capacitors. This corresponds to the case of $X_c = 1,000$ in the calculation. Since there is no capacitor connected to the motor, the quasi rectangular currents, i_{ma} and i_a , are the same. The voltage spikes are shown clearly in the tested line voltage v_{line} . The waveforms confirm the analytical results shown in Figs 3a and 3b.

Fig. 5a. Fifth harmonic current content with $X_c = 1.1$ and $X_c = 2.0$ under $X_m = 0.88$ at 30 Hz

Fig. 5b. Motor voltages with $X_c = 1,000$, $X_c = 1.1$ and $X_c = 2.0$ under $X_m = 0.88$ at 30 Hz

Fig. 9 shows the currents i_{ma} and i_a , and the line voltage v_{line} at $X_c = 4.2$. The waveforms of the currents are close to those shown in Fig. 4. The steepness of the voltage is softened significantly as compared with that shown in Fig. 8 for the case without capacitors.

Fig. 10 shows the tested results at $X_c = 1.9$. The fifth current harmonic agrees with the calculated waveform shown in Fig. 5.

CONCLUSION

This study shows that unlike the induction motors fed by the conventional power system supply, there are important effects caused by the capacitors on the induction motor when both the motor and the capacitors are connected to the quasi rectangular current source supply.

Fig. 6. Oscillations of current and voltage after switching on with $X_c = 0.90$, $X_m = 0.88$ at 30 Hz

Fig. 7. Severe fundamental frequency current and voltage amplifications after switching on with $X_c = 0.70$, $X_m = 0.88$ at 30 Hz

An analytical method based on the 1-2-0 coordinate system for a six-pulse case is developed to calculate the effects on currents and voltages of the motor under this situation.

When the supply is a quasi rectangular current, the capacitors are no more acting like an independent load. The capacitors change the motor currents and soften the steepness of the motor voltage.

A series of computations of a sample motor is conducted to identify the significant harmonic current and voltage variations. As the value of the capacitor reactance reduces gradually toward the motor magnetizing reactance, the phase current of the six-pulse current source fed motor goes through the regions of higher

Fig. 8. Tested currents and line voltage at $X_c = 1,000$

Fig. 9. Tested currents and line voltage at $X_c = 4.2$

Fig. 10. Tested currents and line voltage at $X_c = 1.9$

frequencies harmonic current amplifications first. It then encounters the seventh-harmonic current amplification, and subsequently, the fifth-harmonic amplification. Experimental results agree with the analytical results

The analysis shows that when the capacitor reactance is close to the motor magnetizing reactance, a lengthy oscillatory current occurs. It may encounter a severe fundamental-frequency current amplification when the capacitor reactance is less than the motor magnetizing reactance. Under this situation, no experiment is conducted in this study.

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APPENDIX A

The unit (or base) value of time is $(1/2\pi f)$; therefore, the per-unit value of time, t , is given in radians, where f is the nominal frequency. The corresponding per-unit synchronous angular frequency, ω_s , with reference to the per-unit value of a second is 1.

$$\omega_s = \frac{2\pi f}{\frac{1[\text{sec}]}{\frac{1[\text{sec}]}{2\pi f}}} = 1$$

When per-unit equations are used, the value of reactance, x , where $x = (\omega_s \cdot L)$, with $\omega_s = 1$, equals the value of per-unit inductance, L . The rate of change of a variable is with respect to the per-unit value of time calculated from the nominal frequency. Thus, in the per-unit equations, the self and mutual inductances can be represented by the per-unit reactances. Same principle applies to the reciprocal of per unit capacitance.

As far as the equations in this paper are concerned there is no restriction for selecting the unit values based on either the input or the output of the electrical machine. However, since the calculation is mainly for motors, the shaft output is chosen as the unit power for convenience. The unit values under a particular frequency can be defined as follows:

$$\text{Unit power [W]} = 746 \cdot \text{hp} = \text{motor output power}$$

$$\text{Unit voltage [V]} = \text{phase voltage}$$

$$\text{Unit current [A]} = \frac{\text{unit power}}{[\text{Number of phases} \cdot \text{unit voltage}]}$$

$$\text{Unit impedance } [\Omega] = \frac{\text{unit voltage}}{\text{unit current}}$$

$$\text{Unit torque [Nm]} = \text{unit power} \cdot \frac{\left(\frac{p}{2}\right)}{2\pi f}$$

John S. Hsu (or Htsui) (M '64, SM '90) was born in China. He received a BS degree from Tsing-Hua University, Beijing, China, and a PhD degree from Bristol University, England, in 1959 and 1969, respectively. He joined the Electrical and Electronics Engineering Department of Bradford University, England, serving there for nearly two years.

After his arrival in the United States in 1971, he worked in research and development areas for Emerson Electric Company and later for Westinghouse Electric Corporation. He served as head of the Rotating Machines and Power Electronics Program, Center for Energy Studies, The University of Texas at Austin for over four years. Presently, he is the manager of the Industrial Drive Program at the Center for Electromechanics at The University of Texas at Austin.

Dr. Hsu is a chartered engineer in the United Kingdom and a registered professional engineer in Texas, Missouri, and New York.