POSSIBLE ERRORS IN MEASUREMENT OF AIR-GAP TORQUE PULSATIONS OF INDUCTION MOTORS

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Presented at

The IEEE Power Engineering Society 1991 Summer Meeting
San Diego, CA

July 28-August 1, 1991

Publication PR-142

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Abstract - The torque produced by adjustable speed induction motors fed by inverters inherently contains harmonics. Most mechanical torque sensors have their own natural frequencies and bandwidths. The shaft of a motor can be considered as a spring. Consequently, the measured pulsating torques obtained by using strain gauges coupled to the motor shaft are different from the air-gap torques.

This paper discusses the possible errors associated with the air gap torque measurements. The methods based on the direct measurement of the air-gap fluxes are not error proof. The convenient method using motor-terminal data is further studied for the purpose of understanding the sources of its possible errors.

INTRODUCTION

The adjustable speed induction motors fed by either voltage-source or current-source inverters have undoubtedly gained in popularity. Torques associated with the chopped supplies inherently contain various of harmonics. Torque pulsations affect the noise level and the smoothness, particularly at low speed, of the torque produced by the adjustable-speed drives. Most mechanical torque sensors have their own natural frequencies and bandwidths.[1,2] The shaft of a motor can be considered as a spring with a certain moment of inertia, torsional stiffness, and damping. Consequently, unlike the smooth torque of a motor fed with a balanced sinusoidal supply, the measured pulsating torques of an inverter-fed motor obtained by using strain sensors coupled to the motor shaft or to the stator support are different from the actual air gap pulsating torques.[3-10]

In order to measure the air gap pulsating torque, various methods have been developed to measure the air-gap flux directly.[9-12] These methods include search coils, partial coils of main windings, and Hall-effect generators. After the air-gap fluxes and the stator currents are measured, the air-gap torque is calculated according to the equations that are based on various coordinate systems, such as the torque equations [13-17] corresponding to the a-b-c, d-q-0, 1-2-0, and α-β-0 coordinate systems.

Although it is known that the unbalanced leakage inductances affect the accuracy of the torque equations [10,17], and when superimposition is involved, the derivations of the torque equations in the existing literature assume the magnetic circuit of the motor is linear. It is felt that a derivation to demonstrate where the saturation affects the torque equations used for the computation of the measured torques might be worthwhile for those, who are engaged in the air gap pulsating torque measurements.

Possible Errors Associated with Torque Measurements Using Air-Gap Fluxes

Figure 1a shows a two-phase motor having two symmetrical stator windings, labeled as α α'and β β' respectively. The effective numbers of turns is N for each winding. The cylindrical rotor contains no winding and does not have hysteresis losses. The energy associated with the motor does not change while the rotor's angular position is changed. From a physical standpoint, the motor does not produce torque. With reference to figure 1a, the torque, T, produced at the stator windings is the product of the total current-carrying conductors, $2Ni_\alpha$ and $2Ni_\beta$ interacting with the flux densities B_β and B_α. The air-gap radius is D/2 and the "Length" denotes the core length of the motor.

$$T = 2N\, i_\beta\, B_\alpha \frac{D}{2} \text{Length} - 2\,N\, i_\alpha\, B_\beta \frac{D}{2} \text{Length}$$

Substituting the flux densities, B_β and B_α,

$$B_\alpha = 2\,N\, i_\alpha\, \lambda_\alpha, \qquad \text{and} \qquad B_\beta = 2\,N\, i_\beta\, \lambda_\beta$$

into the torque equation, where λ_α and λ_β are the permeances along the two axes and the signs are determined according to the positive directions shown in figure 1a. The torque can be written as

$$T = 2\,N^2\, i_\beta\, i_\alpha\, \lambda_\alpha \frac{D}{2} \text{Length} - 2\,N^2\, i_\alpha\, i_\beta\, \lambda_\beta \frac{D}{2} \text{Length}$$

$$= 2\,N^2\, i_\beta\, i_\alpha \frac{D}{2} \text{Length}\left(\lambda_\alpha - \lambda_\beta\right)$$

$$= i_\beta\, \psi_\alpha - i_\alpha\, \psi_\beta \qquad (1)$$

The flux linkages ψ_α and ψ_β for the two axes are:

$$\psi_\alpha = 2\,N^2\, i_\alpha\, \lambda_\alpha \frac{D}{2} \text{Length}, \quad \text{and} \quad \psi_\beta = 2\,N^2\, i_\beta\, \lambda_\beta \frac{D}{2} \text{Length} .$$

Indeed, the zero torque is confirmed by the torque equation (1) under the given balanced condition of $\lambda_\alpha = \lambda_\beta$ at any instant, regardless what values of i_α and i_β are.

Figure 1b shows a similar motor, except it is exaggerated for the unbalance of the magnetic paths caused by the unequal magnetic saturations along the two axes at this instant. The permeance λ_α and λ_β for both axes are not identical. λ_α is less than λ_β. The measured air-gap torque computed from (1) is not zero. This is contrary to the fact that since there is no current in the cylindrical rotor, the energy associated with the motor does not change when the rotor angular position is changed, the real air-gap torque is zero. There is an error at this instant between the actual nonexisting air-gap torque and the measured air-gap torque computed from the measured air-gap fluxes through (1). The magnitude of the error depends upon the difference between λ_α and λ_β at the instant being considered. In practice, the fluxes along the two axes are generally not identical. This error occurs.

The existing air gap torque measurements do not differentiate whether the air-gap flux is produced by the stator currents or the rotor currents. In other words, at a given instant if the magnetic paths along the two axes are not identically saturated, the torque equation for the computation of the measured air-gap torque includes a nonexisting torque associated with the fluxes produced by the stator currents interacting with the same stator currents. Thus, even the air-gap fluxes can be measured directly and accurately using either the search-coil or the Hall-effect approach, the air gap flux methods are not the error-proof methods. There is a limitation of assuming the magnetic circuit is balanced at any instant and the portion of torque associated with the fluxes produced by the stator currents interacting with the same stator currents is zero. A numerical evaluation is planned for studying the effects of the unbalanced magnetic paths further.

For most practical situations, the pulsating torque measurements using only the motor terminal data without the requirement of additional search coils or Hall-effect sensors are relatively the most convenient approach to obtain the air gap torque value.[11,16] Based on this intention, a thorough understanding of the torque equations might help to assess the error associated with the air gap torque measurement using only the terminal data.

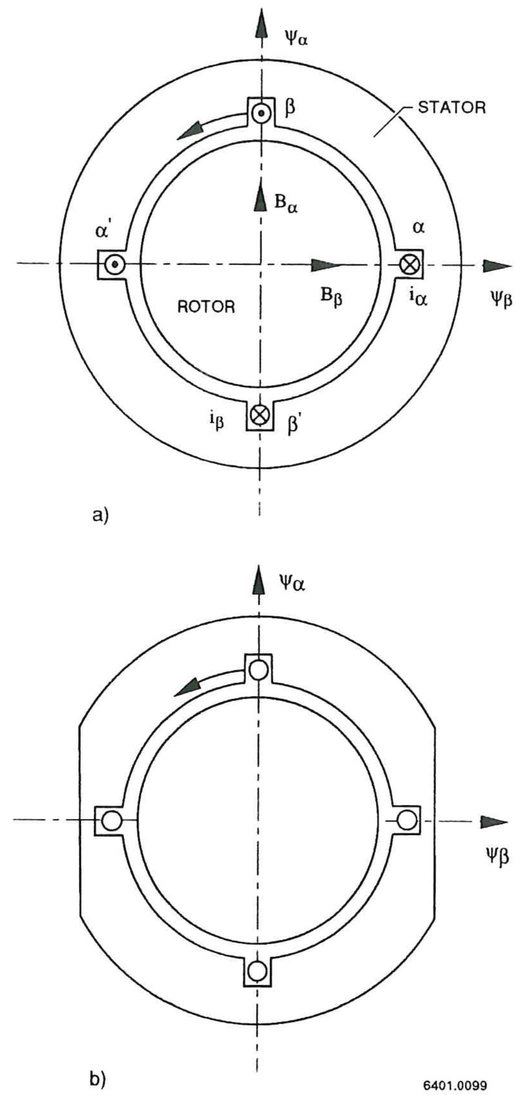

Figure 1.A two-phase motor (a) with balanced magnetic paths (b) with unbalanced magnetic paths

<u>Possible Errors of Air Gap Pulsating Torque Measurement Using Terminal Data</u>

The per-unit power of a three phase induction motor is the summation of the products of the per unit phase voltages and currents.

$$\text{power} = \frac{2}{3}\left(v_a i_a + v_b i_b + v_c i_c\right) \qquad (2)$$

The coefficient 2/3 in (2) gives a unity power while a balanced three-phase motor is operated at a unity power factor and with unity magnitudes of phase voltages, v_a, v_b, v_c, and phase currents, i_a, i_b, i_c. If the equation is written for actual unit in Watts, Volts, and Amperes, the coefficient, 2/3, is omitted.

In order to see how the power is distributed, the following voltage equations, where ψ_a, ψ_b, and ψ_c, are flux linkages of windings a, b, and c respectively,

$$v_a = \frac{d\,\psi_a}{d\,t} + r\,i_a, \qquad v_b = \frac{d\,\psi_b}{d\,t} + r\,i_b, \quad \text{and} \quad v_c = \frac{d\,\psi_c}{d\,t} + r\,i_c$$

are substituted into (2) to give

$$\text{power} = \frac{2}{3}\left[i_a\left(\frac{d\,\psi_a}{d\,t} + r\,i_a\right) + i_b\left(\frac{d\,\psi_b}{d\,t} + r\,i_b\right) + i_c\left(\frac{d\,\psi_c}{d\,t} + r\,i_c\right)\right] \quad (3)$$

When the per-unit values are used in the equations, the unit value of time is $1/(2\pi f)$, where f is the nominal frequency. The corresponding per unit angular frequency, ω, with reference to the per-unit value of a second is one.

$$\omega = \frac{2\,\pi\,f}{\dfrac{1\,[\text{Sec.}]}{\left(\dfrac{1\,[\text{Sec.}]}{2\,\pi\,f}\right)}} = 1$$

The flux linkage of each winding, for instance, ψ_a, of the phase-a winding, is not only produced by the current, i_a, of the winding a. It also includes the projections of the fluxes produced by the phase-b and c windings and the rotor bars. Figure 2 shows the assigned positive directions of the currents and fluxes. Since the rotor of an induction motor is cylindrical, the self inductances of either stator windings or rotor bars are not affected by the rotor position. The mutual inductance among stator windings or among rotor bars are also not affected by the rotor position. However, the mutual inductance between a stator winding and a rotor bar is affected by the rotor position. The rotor position is represented by the electrical angle, γ, that is between the phase-a winding centerline and the half-winding centerline of an arbitrarily chosen bar denoted as the number 1 bar. The electrical angle between any adjacent two bars is ζ.

$$\zeta = \frac{2\,\pi}{S_2}\frac{P}{2}$$

where

$S2$ = number of rotor bars
p = number of poles

When per-unit equations are used, the value of reactance, x, where $x = (\omega \cdot L)$, with $\omega = 1$, equals the value of per-unit inductance, L. The rate of change of a variable is with respect to the per-unit value of time calculated from the nominal frequency. Thus, in the per-unit equations, the self and mutual inductances can be represented by the per-unit reactances, x_{aa}, x_{ab}, etc. The first subscript of a reactance indicates the flux linkage of what winding is being considered. The second subscript represents the source of the flux generated by the current in which winding. Therefore, x_{ab} represents the flux linkage of winding a, generated by the current in winding b. For a balanced three-phase winding, The flux linkages presented in terms of per-unit reactances are

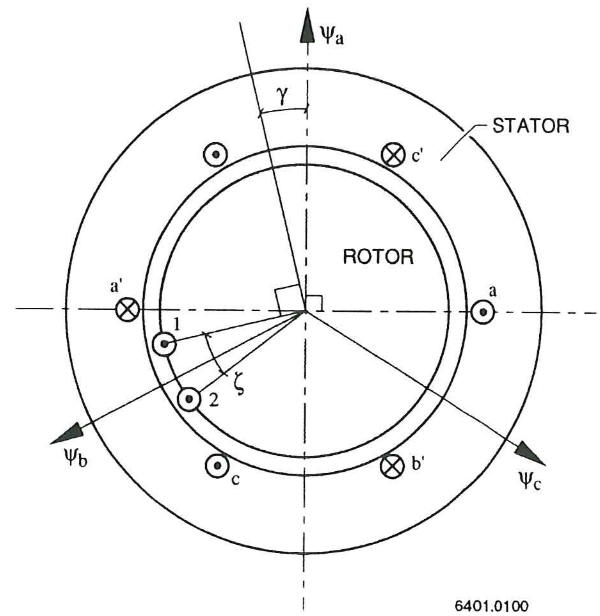

Figure 2. Assigned positive directions of currents and flux linkages of a three phase induction motor

$$\psi_a = -\,x_{aa}\,i_a - x_{ab}\,i_b - x_{ac}\,i_c + x_{a1}\,i_1 + x_{a2}\,i_2 + \ldots + x_{aS_2}\,i_{S_2}$$

$$\psi_b = -\,x_{ba}\,i_a - x_{bb}\,i_b - x_{bc}\,i_c + x_{b1}\,i_1 + x_{b2}\,i_2 + \ldots + x_{bS_2}\,i_{S_2}$$

$$\psi_c = -\,x_{ca}\,i_a - x_{cb}\,i_b - x_{cc}\,i_c + x_{c1}\,i_1 + x_{c2}\,i_2 + \ldots + x_{cS_2}\,i_{S_2} \quad (4)$$

Substituting equation group (4) into (3) gives

$$\text{power} = \frac{2}{3}\,\Big\{$$

$$i_a\left[\frac{d\,(-\,x_{aa}\,i_a - x_{ab}\,i_b - x_{ac}\,i_c + x_{a1}\,i_1 + x_{a2}\,i_2 + \ldots + x_{aS_2}\,i_{S_2})}{d\,t}\right]$$

$$+\,i_b\left[\frac{d\,(-\,x_{ba}\,i_a - x_{bb}\,i_b - x_{bc}\,i_c + x_{b1}\,i_1 + x_{b2}\,i_2 + \ldots + x_{bS_2}\,i_{S_2})}{d\,t}\right]$$

$$+\,i_c\left[\frac{d\,(-\,x_{ca}\,i_a - x_{cb}\,i_b - x_{cc}\,i_c + x_{c1}\,i_1 + x_{c2}\,i_2 + \ldots + x_{cS_2}\,i_{S_2})}{d\,t}\right]$$

$$+\;r\left(i_a^2 + i_b^2 + i_c^2\right)\Big\} \quad (5)$$

The projection of flux linkage to a stator winding from the flux produced by the current of a rotor bar varies according to the rotor position, γ. The flux produced by a winding can be represented by a series of spatial harmonics, for simplification purpose, only the fundamental spatial flux is presented in the following equations.

$$x_{a1} = x_{a10}\cos(\gamma) \qquad x_{b1} = x_{b10}\cos\left(\gamma - \frac{2\,\pi}{3}\right)$$

$$x_{c1} = x_{c10}\cos\left(\gamma + \frac{2\,\pi}{3}\right)$$

$$x_{a2} = x_{a10}\cos(\gamma + \zeta) \qquad x_{b2} = x_{b10}\cos\left(\gamma - \frac{2\,\pi}{3} + \zeta\right)$$

$$x_{c2} = x_{c10}\cos\left(\gamma + \frac{2\pi}{3} + \zeta\right)$$

The general equations of mutual reactances between stator winding and rotor bar are:

$$x_{an} = = x_{a10}\cos(\gamma + n\,\zeta) \qquad x_{bn} = x_{b10}\cos\left(\gamma - \frac{2\pi}{3} + n\,\zeta\right)$$

$$x_{cn} = x_{c10}\cos\left(\gamma + \frac{2\pi}{3} + n\,\zeta\right) \tag{6}$$

where

x_{a10}, x_{b10}, and x_{c10} = amplitudes of the fundamental components of the mutual reactances

n = rotor bar number

It is within the range of 1 to the total number of rotor bars, S_2.

According to equations of (6), equation (5) contains two different kinds of reactances; either not affected or affected by the rotor position, γ. The nonaffected type, such as x_{ab}, can be treated as a constant in the differentiation with respect to time while the rotor position γ changes.

$$\frac{d\, x_{ab}\, i_b}{d\,t} = x_{ab}\frac{d\, i_b}{d\,t}.$$

The affected type, such as x_{a1}, is a function of γ, and can be treated in the differentiation as follows:

$$\frac{d\, x_{a1}\, i_1}{d\,t} = x_{a1}\frac{d\, i_1}{d\,t} + i_1\frac{d\, x_{a1}}{d\,t} = x_{a1}\frac{d\, i_1}{d\,t} + i_1\frac{d\, x_{a1}}{d\,\gamma}\frac{d\,\gamma}{d\,t}$$

Rewriting (5) gives

$$\text{power} = \frac{2}{3}\{$$

$$i_a\left(-x_{aa}\frac{di_a}{dt} - x_{ab}\frac{di_b}{dt} - x_{ac}\frac{di_c}{dt} + x_{a1}\frac{di_1}{dt} + x_{a2}\frac{di_2}{dt} + \ldots + x_{aS_2}\frac{di_{S_2}}{dt}\right)$$

$$+i_b\left(-x_{ba}\frac{di_a}{dt} - x_{bb}\frac{di_b}{dt} - x_{bc}\frac{di_c}{dt} + x_{b1}\frac{di_1}{dt} + x_{b2}\frac{di_2}{dt} + \ldots + x_{bS_2}\frac{di_{S_2}}{dt}\right)$$

$$+i_c\left(-x_{ca}\frac{di_a}{dt} - x_{cb}\frac{di_b}{dt} - x_{cc}\frac{di_c}{dt} + x_{c1}\frac{di_1}{dt} + x_{c2}\frac{di_2}{dt} + \ldots + x_{cS_2}\frac{di_{S_2}}{dt}\right)$$

$$+i_a\left(i_1\frac{dx_{a1}}{d\gamma} + i_2\frac{dx_{a2}}{d\gamma} + \ldots + i_{S_2}\frac{dx_{aS_2}}{d\gamma}\right)\frac{d\gamma}{dt}$$

$$+i_b\left(i_1\frac{dx_{b1}}{d\gamma} + i_2\frac{dx_{b2}}{d\gamma} + \ldots + i_{S_2}\frac{dx_{bS_2}}{d\gamma}\right)\frac{d\gamma}{dt}$$

$$+i_c\left(i_1\frac{dx_{c1}}{d\gamma} + i_2\frac{dx_{c2}}{d\gamma} + \ldots + i_{S_2}\frac{dx_{cS_2}}{d\gamma}\right)\frac{d\gamma}{dt}$$

$$+ r\left(i_a^2 + i_b^2 + i_c^2\right)\} \tag{7}$$

Referring to (4) the first three long lines of (7) can be presented as

$$\left.\frac{i_a\, d\psi_a + i_b\, d\psi_b + i_c\, d\psi_c}{d\,t}\right|_{\text{fixed mutual reactances}}$$

This is similar to the primary winding of a three-phase transformer. The power goes for the storage of energy in the windings and for the consumption of energy in the secondary winding circuits. No torque is generated. The next three lines represent the power that is converted into the product of the air-gap torque and the angular speed, $\left(\frac{d\gamma}{dt}\right)$. This product is the rotation power. The final line gives the stator copper loss.

The terms relating to the torque in (7) are the focus of the subsequent derivation.

Substituting group equations (6) into the air gap torque portion of (7)

$$T_{p.u.} = \frac{2}{3}\Big[\, i_a\left(i_1\frac{dx_{a1}}{d\gamma} + i_2\frac{dx_{a2}}{d\gamma} + \ldots + i_{S_2}\frac{dx_{aS_2}}{d\gamma}\right)$$

$$+i_b\left(i_1\frac{dx_{b1}}{d\gamma} + i_2\frac{dx_{b2}}{d\gamma} + \ldots + i_{S_2}\frac{dx_{bS_2}}{d\gamma}\right)$$

$$+i_c\left(i_1\frac{dx_{c1}}{d\gamma} + i_2\frac{dx_{c2}}{d\gamma} + \ldots + i_{S_2}\frac{dx_{cS_2}}{d\gamma}\right)\Big]$$

gives the per-unit air-gap torque as a function of γ.

$$T_{p.u.} = \frac{2}{3}\{\, i_a\,[-i_1\, x_{a10}\sin(\gamma) - i_2\, x_{a10}\sin(\gamma + \zeta) - \ldots$$

$$- i_{S_2}\, x_{a10}\sin(\gamma + S_2\,\zeta)\,]$$

$$+i_b\,[-i_1\, x_{b10}\sin\left(\gamma - \frac{2\pi}{3}\right) - i_2\, x_{b10}\sin\left(\gamma - \frac{2\pi}{3} + \zeta\right) - \ldots$$

$$- i_{S_2}\, x_{b10}\sin\left(\gamma - \frac{2\pi}{3} + S_2\,\zeta\right)]$$

$$+i_c\,[-i_1\, x_{c10}\sin\left(\gamma + \frac{2\pi}{3}\right) - i_2\, x_{c10}\sin\left(\gamma + \frac{2\pi}{3} + \zeta\right) - \ldots$$

$$- i_{S_2}\, x_{c10}\sin\left(\gamma + \frac{2\pi}{3} + S_2\,\zeta\right)]\,\} \tag{8}$$

The term

$$[-i_1\, x_{a10}\sin(\gamma) - i_2\, x_{a10}\sin(\gamma + \zeta) - \ldots - i_{S_2}\, x_{a10}\sin(\gamma + S_2\,\zeta)\,]$$

in the first and second lines of (8) represents the portion of air-gap flux produced by the rotor currents in the magnetic axis normal to the magnetic axis of phase-a winding. The same understanding can be applied to the remaining terms for phases b and c. This understanding is in line with the law of physics that only the interaction between stator currents and the flux produced by the rotor currents yields torque. The interactions of the stator currents with the fluxes produced by the same stator currents do not produce torque. This is not to say that the stator current cannot affect the saturation of the magnetic paths. In fact, the values of the reactances must be those under the influence of both stator and rotor currents.

The portions of air-gap fluxes produced by the rotor currents alone cannot be measured directly. This is because the air-gap flux contains the portions of fluxes produced by the stator currents. In order to have the torque equation presented in measurable data, the following manipulations to include the fluxes produced by the stator currents are used.

Figure 3 shows that by changing sine terms in the above equation to two cosine terms. For instance,a flux linkage, $-i_1 x_{a10}$,

$$-i_1 x_{a10} \sin(\gamma) = -\frac{1}{\sqrt{3}} i_1 x_{a10} \left[\cos\left(\gamma - \frac{2\pi}{3}\right) - \cos\left(\gamma + \frac{2\pi}{3}\right) \right], \quad (9)$$

projected to the axis normal to the magnetic axis of phase a is equal to $\frac{1}{\sqrt{3}}$ of the scalar difference of the same flux linkage projected to the magnetic axes of phase b and phase c windings.

Since the motor windings, and the magnetic paths are assumed to be symmetrical while the torque equations are derived, $x_{aa} = x_{bb} = x_{cc}$, $x_{ab} = x_{ba} = x_{ac} = x_{ca} = x_{bc} = x_{cb}$. It is also assumed that their values remain unchanged at any rotor position.

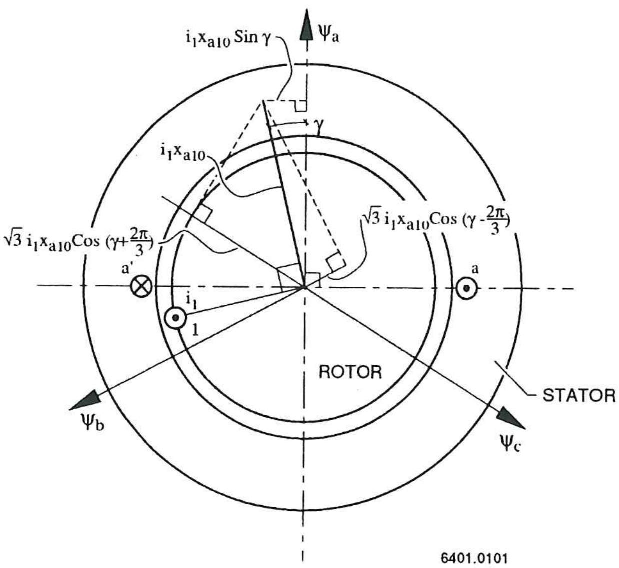

Figure 3. Changing a sine term of one phase to two cosine terms of other two phases

Using the manner described for (9) and adding the following cancellable terms that reflect the nonexisting torque associated with the stator currents and the fluxes produced by the same stator currents

cancellable terms =

$$i_c x_{aa} i_a - i_a x_{cc} i_c + i_a x_{bb} i_b - i_b x_{aa} i_a + i_b x_{cc} i_c - i_c x_{bb} i_b$$
$$+ i_b x_{cb} i_b - i_b x_{ab} i_b + i_c x_{ab} i_b - i_b x_{ac} i_c + i_c x_{ac} i_c - i_c x_{bc} i_c$$
$$+ i_a x_{ba} i_a - i_a x_{ca} i_a + i_a x_{bc} i_c - i_c x_{ba} i_a + i_b x_{ca} i_a - i_a x_{cb} i_b \quad (10)$$

gives the per-unit air-gap torque:

$$T_{p.u.} = \frac{2}{3\sqrt{3}} [\, i_a (\psi_c - \psi_b) + i_b (\psi_a - \psi_c) + i_c (\psi_b - \psi_a)] \quad (11)$$

The flux linkages, ψ_a, ψ_b, or ψ_c, of the stator windings include leakage fluxes. Only when the leakage reactances of three phases are identical at any instant, the effect of these leakage fluxes can be cancelled. Under this condition the term $i_a (\psi_c - \psi_b)$ in (11) represents that the phase-a current is interacting with the air-gap flux normal to the magnetic axis of the phase-a winding. The remaining two terms of (11) bear the same torque producing nature for the phase-b and phase-c currents.

Under the magnetically balanced assumption, the terms in (10) can all be cancelled. For instance, when $x_{aa}=x_{cc}$, the first term of (10) that is the phase a self-linkage flux interacting with phase-c current is cancelled by the second term.

The self-linkage reactance, x_{aa}, contains two components, one is the self-leakage reactance, the other one is the self air-gap reactance, corresponding to the flux that goes through the air gap. In practice, mainly because of the magnetic saturation, together with the possibility of the asymmetrical air gap, the unbalance of end winding magnetic paths, and the material asymmetrical factors, the self inductances, represented by the self reactances, x_{aa}, x_{bb}, x_{cc} in the per-unit equations, may not be identical at every instant. The same situation, except without leakage reactance, may apply to the mutual reactances, x_{ab}, x_{bc}, and x_{ca}. The cancellable terms for the creation of (11) may therefore no longer be cancellable. A torque value is generated for the nonexisting portion of torque. An error is thus generated.

Locations of Search Coils

The above derivation also shows that if search coils are used to measure the flux linkages, ψ_a, ψ_b, and ψ_c the locations of the search coils can either be at the tops of the slots or at the bottoms of the slots. The former ones contain less slot leakage fluxes. As long as the locations of the three phase search coils are symmetrical, and the three-phase leakage and air-gap reactances are balanced, either locations of the search coils give the same results. Otherwise, the locations at the top of the slots (fig. 4) to exclude the slot leakages give more accurate results. However, either locations cannot exclude the error caused by the saturated unbalanced magnetic paths.

Torque Equation in Newton Meters

The air gap torque equation in Newton Meters (NM) is similar to the per unit torque equation (11), except that a factor of 3/2 and the number of pole pairs p/2, where p is the number of poles, must be included. The units of Amperes and Weber-turns are used for the currents and flux linkages.

$$T_{N.M.} = \frac{p\sqrt{3}}{6} [\, i_a (\psi_c - \psi_b) + i_b (\psi_a - \psi_c) + i_c (\psi_b - \psi_a)] \quad (12)$$

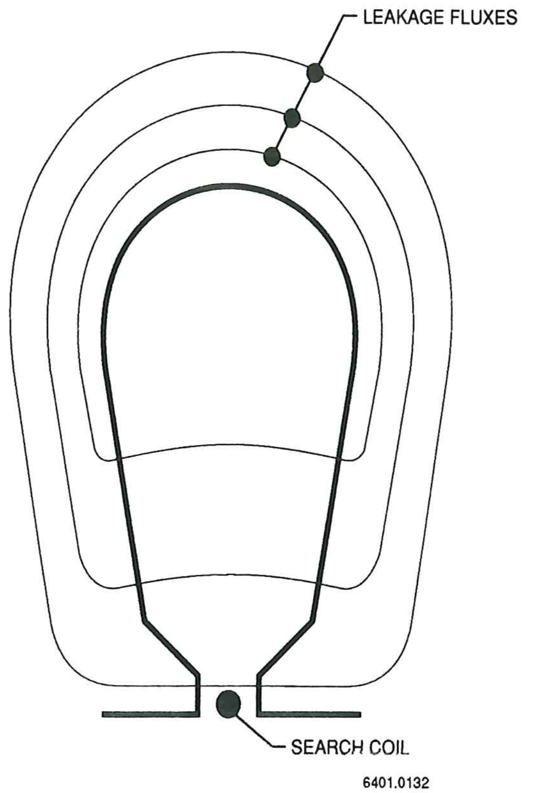

Figure 4. Slot leakage fluxes and search coil location

Using Line Data for Air-Gap Torque Computation

The torque equation written for the per unit phase data, equation (11), can be derived for the line data as indicated in figure 5. Substituting flux linkage equations, where r is the phase resistance,

$$\psi_a = \int (v_a - r\, i_a)\, dt \qquad \psi_b = \int (v_b - r\, i_b)\, dt$$

and

$$\psi_c = \int (v_c - r\, i_c)\, dt \tag{13}$$

into (12) and simplifying give

$$T_{P.U.} = \frac{2}{3\sqrt{3}} \{ (i_A - i_B) \int [v_{CA} - R\,(i_C - i_A)]\, dt - (i_C - i_A) \int [v_{AB} - R\,(i_A - i_B)]\, dt \} \tag{14}$$

Equation (14) is valid for either Wye- or Delta-connected motors. This equation is the same as (34) from reference [11] which is used for the computation of starting torque of salient pole synchronous motors.

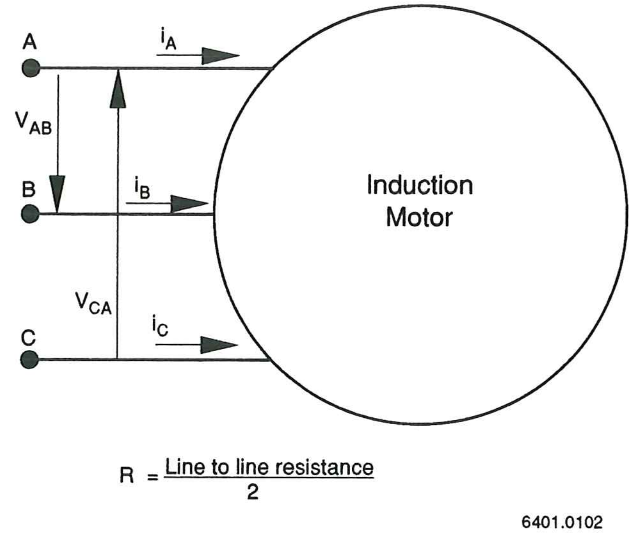

Figure 5. Line data for air gap torque computation

where

i_A, i_B, and i_C = line currents
R = half of the line-to-line resistance value

For the Wye-connected motor: R = phase resistance = r, and for delta-connected motor: R = r/3.

CONCLUSION

Unlike an induction motor fed with balanced sinusoidal supply, the adjustable-speed induction motors fed by inverters inherently contain various harmonics. Most mechanical torque sensors have their own natural frequencies and bandwidths, and the shaft or frame of a motor can be considered as a spring. Consequently, the measured pulsating torques of an inverter-fed motor obtained by using strain sensors coupled to the motor shaft or to the stator support are different from the air-gap torques.

This paper discusses the possible errors in the known methods for measuring the air gap pulsating torques. From the law of physics that only the interaction between stator currents and the fluxes produced by the rotor currents yields torque; however, the portion of air-gap fluxes produced by the rotor currents alone cannot be measured directly. This is because the air-gap flux contains the portions of fluxes produced by the stator currents. The air-gap fluxes produced by the rotor currents alone can be obtained indirectly through the differences of the flux linkages of the stator windings. By doing so, the unbalanced leakage reactances and magnetic paths become the sources of errors. The error occurs when the supposedly cancellable terms (10) is not zero at any instant.

The measurements based on the accurately obtained air gap flux data are not error proof. This is because even the unbalanced leakage effects can be reduced to minimal by measuring the air-gap flux at the tip of the slots. The magnetic paths of different axes may not be balanced at any instant. This generates a computed "measured" torque value that includes an uncancellable error.

The relatively convenient method for measuring the air gap pulsating torque is the terminal-data method. The unbalanced saturated stator mutual and self reactances that includes the leakage reactances at any instant are the sources of possible errors.

A numerical evaluation is planned for studying the effects of the unbalanced magnetic paths further.

ACKNOWLEDGMENT

The author would like to thank the State of Texas for financial support through a grant under Texas Advanced Technology Program, Grants no. 1591 and 003658-181. This work has been partially supported by NSF Grant no. ECS-880884, which is gratefully acknowledged. Thanks are due to the Center for Electromechanics, The University of Texas at Austin, for the support staff and facilities provided for the research work.

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