**TRANSIENT EDDY CURRENT DISTRIBUTION IN THE
SHIELD OF THE PASSIVELY COMPENSATED, COMPENSATED
PULSED ALTERNATOR - IRON CORE MACHINES**

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Abstract*

A compensated pulsed alternator (compulsator) is a single-phase generator capable of delivering high power, energy pulses to a low impedance load. Various types of compulsators have evolved since its inception at the Center for Electromechanics, The University of Texas at Austin in 1978. One variant of this machine, which is the topic of this paper, is called the passively compensated compulsator. In this machine, a continuous conductive shield spins relative to the armature. The action of the shield reduces the inductance of the armature winding considerably, thus enabling it to deliver high power pulses to low impedance loads. In this respect, the passive compulsator is very similar to a single phase short-circuit generator. With all its variants, the compulsator has a variety of applications in areas such as fusion, electromagnetic launching, particle beam accelerators, or simply as a short-circuit generator for testing power system equipment under fault conditions.

The transient eddy current distribution in the shield of the passive compulsator is discussed. Analytical solutions of the two-dimensional (2-D) convective diffusion equation is obtained. This solution is then used to interpret various experimental

* This work was supported by U.S. Army AMCCOM under contract number DAAAK10-83-C-0126.

observations obtained from test data on this machine. The solution is also used for preparing design data for passive compulsators. This paper pertains to iron-core machines. A follow-on paper considers superconducting or "air" core machines which do not use ferromagnetic materials as the main magnetic flux path.

Introduction

Figure 1 shows the basic geometry of the passive compulsator. This particular geometry has a stationary armature and a rotating field coil and shield. The analysis would be equally valid for a stationary shield and a rotating armature.

There are two sources for eddy current generation in the shield: (a) switching of the current in the field coil and (b) current flow in the armature winding. Both these are analyzed with similar methods. Figures 2a and b show the schematic of the problem definition.

In most machines the thickness of the shield is small compared to its average radius. This allows a true cylindrical geometry to be represented by the Cartesian coordinate system, which greatly simplifies the expressions. Figure 3 shows the coordinate system and the relative location of the shield and armature winding. The shield considered is nonmagnetic with a magnetic permeability of free-space (μ_0) and an electrical conductivity (σ). The shield has a thickness (h) and a relative velocity (v) with respect to the armature winding. Coordinates fixed with respect to the armature winding are x and y , while x_r and y_r are fixed coordinates with respect to the shield.

The following assumptions have been made while seeking a solution to the problem :

(a) The axial length of the machine is considered to be large compared to the radius. Thus, the effects of the end turns of the winding are neglected. The solution is then invariant with respect to the axial coordinate z .

(b) Change in the impedance of the machine, due to the diffusion process, is considered to be a small fraction of the total circuit impedance for the duration of interest. The current in the winding is then determined by an impedance which is constant in time.

(c) Material properties are constant in space and time; thus, the problem being solved is linear. This allows the application of superposition. The effects of saturation of iron are also neglected.

(d) It is assumed that the source of excitation can be represented by a superposition of space harmonics. This is not much of a limitation especially for rotating machinery.

(e) The shield is assumed to have high permeability material on both sides (top and bottom, fig. 3). The other boundary condition that is possible is highly permeable on one side (i.e. the side of the excitation coil) and highly conductive on the other side. Typically, the rotor of the compulsator (fig. 1) is made from solid iron. Therefore, one might consider the second boundary condition as more appropriate. However, experimental observations indicate that the perturbation in the field coil current during a high current discharge corresponds more closely to the fully permeable boundary condition. A solution can be obtained for a highly conductive boundary condition. However, this is of less interest since it excludes the possibility of any coupling between the armature and field coil during a transient armature discharge. In reality, the behavior of the boundary at the rotor surface lies in between these two extremes.

In obtaining the solution, a response to a step in excitation is first solved. This is then convoluted with specific time functions corresponding to the nature of the excitation current. The reason behind this process is to present a general solution, i.e. response to a step followed by a procedure describing the convolutions, thus enabling the reader to convolute with any specific time function of interest.

Formal Solution

The partial differential equation to be solved is

$$\frac{1}{\mu_0\sigma}\nabla^2\vec{B} = \left(\frac{\partial}{\partial t} + v \cdot \nabla\right)\vec{B} \quad (1)$$

i.e. the convective diffusion equation for the shield. This equation is written with respect to fixed coordinates on the stator relative to which the shield moves at 'v' m/s.

The component equations can be written as follows:

$$\frac{1}{\mu_0\sigma}\left\{\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2}\right\} = \frac{\partial B_x}{\partial t} + v \frac{\partial B_x}{\partial x} \quad (2)$$

$$\frac{1}{\mu_0\sigma}\left\{\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2}\right\} = \frac{\partial B_y}{\partial t} + v \frac{\partial B_y}{\partial x} \quad (3)$$

Since the excitation is periodic in x, we can assume that the solution takes the form

$$B_x(x, y, t) = \text{Re}[e^{j\lambda x} p(y, t)]$$

$$B_y(x, y, t) = \text{Re}[e^{j\lambda x} q(y, t)]$$

$$\Lambda = \frac{2\pi}{W_p}$$

where

W_p = the pitch of a pole pair.

Then substitute into equations (2) and (3) to obtain component equations as follows:

$$\frac{\partial^2 B_x(y, t)}{\partial y^2} = \left\{ \mu_0 \sigma \frac{\partial B_x(y, t)}{\partial t} + (\Lambda^2 + j\Lambda v \mu_0 \sigma) B_x(y, t) \right\} \quad (4)$$

$$\frac{\partial^2 B_y(y, t)}{\partial y^2} = \left\{ \mu_0 \sigma \frac{\partial B_y(y, t)}{\partial t} + (\Lambda^2 + j\Lambda v \mu_0 \sigma) B_y(y, t) \right\} \quad (5)$$

Consider equation (5) for the y component first and separate the solution into a steady state and transient part.

$$B_y(x, y, t) = \text{Re} \left[e^{j\Lambda x} \{ q_s(y) + q_t(y, t) \} \right]$$

Here the subscript 's' denotes the steady-state solution and the subscript 't' denotes the transient solution. For brevity 'Re[]' will not be used in the following text. Recognize that only the real part will be used from the final solution.

The steady-state solution for the two components under the boundary conditions

$$B_{xs} = 0 \text{ at } y = h$$

and

$$B_{xs} = -\mu_0 K_z e^{j\Lambda x} \text{ at } y = 0 \text{ (a step of magnitude } K_z \text{ at } t = 0)$$

is given as

$$B_{xs} = -\frac{\mu_0 K_z e^{j\lambda x}}{[1 - e^{2\lambda h}]} \{e^{\lambda y} - e^{2\lambda h} e^{-\lambda y}\}$$

$$B_{ys} = j \frac{\mu_0 K_z e^{j\lambda x}}{\left(\frac{\lambda}{\Lambda}\right)[1 - e^{2\lambda h}]} \{e^{\lambda y} + e^{2\lambda h} e^{-\lambda y}\}$$

Here we have used a constant λ defined as

$$\lambda = \Lambda \sqrt{1 + jS}$$

$$S = \frac{v\mu_0\sigma}{\Lambda}$$

The solution for the transient part is obtained by the separation of variables and using the boundary conditions:

$$B_{xt} = 0 \text{ at } y = 0$$

$$B_{yt} = 0 \text{ at } y = h$$

We get the transient solution for the two components as :

$$B_{xt} = e^{j\lambda x} e^{-t/\tau_n} \left\{ b_n \sin\left(\frac{n\pi}{h} y\right) \right\} \quad (6)$$

$$B_{yt} = e^{j\lambda x} \left\{ a_0 e^{-t/\tau_0} + a_n e^{-t/\tau_n} \cos\left(\frac{n\pi}{h} y\right) \right\} \quad (7)$$

the expression for τ_n is given by :

$$\tau_n = \frac{\mu_0 \sigma}{\lambda^2 + \left(\frac{n\pi}{h}\right)^2}$$

The constants in equations (6) and (7) are evaluated using the initial condition $B_x, B_y = 0$ at $t = 0$ for all x and y .

The complete solution for the case with a constant relative velocity of v is then given as:

$$B_x = -\frac{\mu_0 K_z e^{j\Lambda x}}{[1 - e^{2\lambda h}]} \left\{ (e^{\lambda y} - e^{2\lambda h} e^{-\lambda y}) - \sum_{n=1}^{\infty} b_n e^{-t/\tau_n} \sin\left(\frac{n\pi}{h} y\right) \right\} \quad (8)$$

$$B_y = \frac{j\mu_0 K_z e^{j\Lambda x}}{(\lambda / \Lambda)[1 - e^{2\lambda h}]} \left\{ (e^{\lambda y} + e^{2\lambda h} e^{-\lambda y}) - \left[a_0 e^{-t/\tau_0} + \sum_{n=1}^{\infty} a_n e^{-t/\tau_n} \cos\left(\frac{n\pi}{h} y\right) \right] \right\} \quad (9)$$

$$\Lambda = \frac{2\pi}{W_p}$$

$$\lambda = \Lambda \sqrt{1 + jS} \quad (10)$$

$$S = \frac{v\mu_0\sigma}{\Lambda}$$

$$\tau_n = \frac{\mu_0\sigma}{\lambda^2 + \left(\frac{n\pi}{h}\right)^2}$$

$$a_0 = -\left(\frac{1 - e^{2\lambda h}}{\lambda h}\right)$$

$$a_n = -\frac{\left(\frac{2\lambda}{h}\right)(1 - e^{2\lambda h})}{\lambda^2 + \left(\frac{n\pi}{h}\right)^2}$$

$$b_n = \frac{\left(\frac{2n\pi}{h^2}\right)(1 - e^{2\lambda h})}{\lambda^2 + \left(\frac{n\pi}{h}\right)^2}$$

The eddy current density distribution is obtained by using the relation $J = (1/\mu_0)\nabla \times B$.

$$J_z = -\frac{K_z e^{j\lambda x}}{[1 - e^{2\lambda h}]} \left\{ \left(\frac{\Lambda^2 - \lambda^2}{\lambda} \right) (e^{\lambda y} + e^{2\lambda h} e^{-\lambda y}) - \left(\frac{\Lambda^2}{\lambda} \right) \left(a_0 e^{-t/\tau_0} + \sum_{n=1}^{\infty} a_n e^{-t/\tau_n} \cos\left(\frac{n\pi}{h} y\right) \right) \right. \\ \left. + \sum_{n=1}^{\infty} b_n \left(\frac{n\pi}{h} \right) e^{-t/\tau_n} \cos\left(\frac{n\pi}{h} y\right) \right\} \quad (11)$$

Case 1: Stationary Shield ($v = 0$)

Equation 12 gives the current density distribution and figure 4 is a plot of the current density distribution for various depths in the shield as a function of

$$J_z(x, y, t) = -K_z e^{j\lambda x} \left\{ \frac{e^{-t/\tau_0}}{h} + \frac{2}{h} \sum_{n=1}^{\infty} e^{-t/\tau_n} \cos\left(\frac{n\pi}{h} y\right) \right\} \quad (12)$$

normalized time, when the shield is stationary. Time is normalized with respect to the largest time constant (τ_1) under the summation of equation (8).

$$\tau_1 = \frac{\mu_0 \sigma}{\Lambda^2 + \left(\frac{\pi}{h}\right)^2}$$

As an example, a 12.7 mm thick shield (40% IACS) on a six-pole machine is considered. The average radius of the shield is 40 cm. Figure 4 shows that initially the current density is highest at $y = 0$, which is closest to the excitation. Until $0.5\tau_1$ there is no significant current at a depth beyond 9.53 mm. However, after $t = \tau_1$ the current is present through the entire thickness, but as yet higher nearer the excitation. As time proceeds, the current density through the entire thickness approaches the same value. It is not surprising that the uniform current density approaches a normalized value of unity as this is an indication that $J_z = K_z/h$ or simply that the current in the shield is equal and opposite to the excitation. Figure 4 considers time only until $4\tau_1$ in order to illustrate the manner in which the current density approaches a uniform value. The time it takes to reach this uniform distribution is roughly $3\tau_1$. Beyond $3\tau_1$, the current density through the entire thickness decays to zero with a time constant

$$\tau_0 = \frac{\mu_0 \sigma}{\Lambda^2}$$

It is interesting to note how the time constant τ_0 is related to the circuit parameters of the shield. For this purpose, we rewrite τ_0 as obtained from the field equations as

$$\tau_0 = 4\mu_0 \sigma \frac{r^2}{N_p^2} \quad (13)$$

where

N_p = number of poles in the machine.

Now consider the shield as a monolithic winding with one turn per pole and let us calculate the inductance and resistance of this winding.

Inductance:

$$L = \frac{\mu_0 N_p}{2h} \left[\frac{W_p}{12} \right] \text{ H / m}$$

$$W_p = 2 \left(\frac{2\pi r}{N_p} \right)$$

$$L = \frac{\mu_0 \pi r}{6h}$$

Resistance:

$$R = \frac{N_p^2}{\sigma(2\pi r h)} \text{ } \Omega / \text{ m}$$

Time Constant:

$$\tau_{0c} = \frac{L}{R} = \left(4\mu_0 \sigma \frac{r^2}{N_p^2} \right) \frac{\pi^2}{12}$$

τ_{0c} agrees in form to τ_0 except for a factor $\pi^2/12$ which has a value of 0.822. Even though the two values are not exactly equal, the circuit approach sheds light on the nature of the time constant τ_0 .

Now consider the first term under the summation of equation (12) for $n = 1$. It indicates a variation in current density distribution given by $\cos(\pi y/h)$. This variation is better illustrated in figure 5. From this figure, it is apparent that the eddy currents corresponding to $n = 1$ flow in a direction to aid the constant current ($n = 0$) on the top half ($0 - h/2$) and in the opposite direction in the bottom half ($h/2 - h$) of the shield.

Calculating the circuit inductance and resistance of this eddy current path yields a time constant which agrees well with τ_1 obtained from the field equations. Similar analogies between circuit parameters and field equations can be drawn for all τ_n .

Case 2: Moving Shield ($v \neq 0$)

Comparison of equations (11) and (12) indicates the presence of a steady state eddy current term for the case where v is other than zero. This is to be expected since a continuous motion of the shield relative to the excitation will result in a steady-state eddy current.

Figure 6 illustrates the eddy current distribution at $x = 0$ and $y = 0$ for a time period of approximately $4\tau_1$ (same as that in fig. 4). For a very small time ($t < \tau_1$), the behavior of the distribution with and without relative motion is about the same. The difference is apparent as the eddy currents approach a steady-state value. The two main differences are: (a) a steady state is attained earlier, and (b) under steady state, the eddy current density decreases rapidly at locations away from the excitation.

The first of the two differences can be better understood by considering the term containing the time constant τ_0 as this was the largest time constant in equation (11) for case 1. It must be noted that all time constants are complex, thus implying a decay as well as a phase shift in time.

The term containing the time constant τ_0 is written as:

$$\left(\frac{\Lambda^2}{\lambda^2} \right) \left(-\frac{K_z e^{j\lambda x}}{h} e^{-t/\tau_0} \right)$$

i.e. a factor of $(\Lambda/\lambda)^2$ is present in this term as compared to equation (14). This factor can be further rewritten as

$$\left(\frac{\Lambda}{\lambda}\right)^2 = \frac{1}{1+jS} = \frac{1}{1+j\frac{v\mu_0\sigma}{\Lambda}}$$

For this specific example:

$$S = \frac{v\mu_0\sigma}{\Lambda} \gg 1$$

$$\left|\left(\frac{\Lambda}{\lambda}\right)^2\right| \approx \frac{1}{|S|}$$

and

$$\left|\left(\frac{\Lambda}{\lambda}\right)^2\right| \ll 1$$

The higher the conductivity and relative velocity, the more accurate this approximation. Therefore, this term plays a very inconsequential role in determining the distribution of eddy currents. Thus, one is left with the terms containing τ_n ($n = 1, 2, \dots$). The largest time constant in these terms is τ_1 . We see in figure 6 that the steady state is attained in a time of the order of τ_1 seconds.

It is to be noted that the factor (Λ/λ) is also present in the equation for the component of the field which is normal to the conductor in motion. This implies that the normal flux density is also lower compared to the case where $v = 0$. Therefore, the flux bridging the magnetic gap is also lower and whatever flux does bridge this gap establishes itself with the time constant τ_0 . In the case where $h \gg \delta$, this flux approaches zero and also the term containing τ_0 .

The second of the differences can be better understood by considering the steady-state equation for $S \gg 1$:

$$J_{zs} = \frac{\sqrt{2}}{\delta} K_z e^{-y/\delta} \cos\left(\Lambda x - \frac{y}{\delta} + \frac{\pi}{4}\right) \quad (14)$$

$$\delta = \sqrt{\frac{2}{\omega_e \mu_0 \sigma}}$$

$$\omega_e = v\Lambda$$

Equation (14) illustrates that the steady state eddy current density decays exponentially with depth in a manner which is typical of steady state ac.

Another important fact can be derived from the steady state current density distribution. We define a line current density as

$$K_{zs} = \int_0^h J_{zs} dy$$

This is found to be

$$K_{zs} = K_z e^{j\Lambda x} \left\{ -\frac{jv\mu_0\sigma}{\Lambda + jv\mu_0\sigma} \right\}$$

if $v\mu_0\sigma \gg \Lambda$, then

$$K_{zs} = -K_z e^{j\Lambda x}$$

i.e. for the case when either conductivity or the velocity is high, image currents (which are equal in magnitude and opposite in direction to the excitation) would flow in the shield.

Convolution of a Step Response -- Sinusoidal Excitation

The discussion in the previous sections was limited to the response to a step. In what follows, use is made of the solutions discussed in case 2 to arrive at the response to a sinusoidal excitation current flowing in the armature winding. Typically, the current in the armature winding can be represented by equation (15), when the generator is applied for a pulsed duty.

$$i_a(t) = \frac{V_m}{|Z|} \left\{ e^{-t/\tau_c} \sin(\varphi - \phi) - \sin(\omega_e t + \varphi - \phi) \right\} \quad (15)$$

Here,

$$|Z| = \sqrt{R_c^2 + (\omega_e L_c)^2}$$

$$\tau_c = \frac{L_c}{R_c}$$

$$\phi = \tan^{-1} \left(\frac{\omega_e L_c}{R_c} \right)$$

where

V_m = voltage of the generator

L_c = total circuit inductance

R_c = total circuit resistance

ω_e = electrical frequency of the machine

φ = initiation phase angle with respect to zero voltage

The corresponding line current density for a lap wound armature can be represented as:

$$K_z = \frac{i(t)}{w_{\text{cond}}} = \frac{V_m}{N_p |Z| w_{\text{cond}}} \left\{ e^{-t/\tau_c} \sin(\varphi - \phi) - \sin(\omega_e t + \varphi - \phi) \right\} \quad (16)$$

where

w_{cond} = width of the armature conductor,

N_p = number of poles in the machine

Then, the response to the sinusoidal excitation is obtained as:

$$l(t) = \int_0^t L(t-\tau) \frac{dK_z(\tau)}{d\tau} d\tau$$

Here, the function 'L' denotes either the step response of the component fields or the current density. The complete solution for the eddy current distribution is given by equation (17).

$$J_z = -\frac{K_m e^{j\lambda x}}{[1 - e^{2\lambda h}]} \left\{ \left(\frac{\Lambda^2 - \lambda^2}{\lambda} \right) (e^{\lambda y} + e^{2\lambda h} e^{-\lambda y}) T_s - \left(\frac{\Lambda^2}{\lambda} \right) \left(a_0 T_0 + \sum_{n=1}^{\infty} a_n T_n \cos\left(\frac{n\pi}{h} y\right) \right) \right. \\ \left. + \sum_{n=1}^{\infty} b_n \left(\frac{n\pi}{h} \right) T_n \cos\left(\frac{n\pi}{h} y\right) \right\} \quad (17)$$

Here the time functions T_s and T_n are given by:

$$T_s = \{e^{-t/\tau_c} \sin(\psi - \phi) - \sin(\omega_e t + \psi - \phi)\}$$

and

$$T_n = \left\{ \frac{-\omega}{1/\tau_n^2 + \omega_e^2} \left[\omega (\sin(\omega_e t + \phi - \phi) - e^{-t/\tau_n} \sin(\phi - \phi)) \right] + \frac{\sin(\phi - \phi)}{1 - \tau_c/\tau_n} [e^{-t/\tau_c} - e^{-t/\tau_n}] \right\}$$

In order to understand the distribution of the magnetic field and current density in a meaningful manner, it is better to resort to 2-D contour mapping. Figure 7 shows a typical current time profile for the compulsator under pulsed duty. Figures 8 and 9

show the flux lines (lines of constant magnetic vector potential) and lines of constant current density, respectively, at various instants during the current pulse.

Consider now the true nature of the excitation in the armature winding. The excitation line current density distribution at a given time can be represented in space harmonic form as

$$K_a = \sum_{n(\text{odd})} C_n K_z(t) \cos(n\Lambda x)$$

where

$$\begin{aligned} C_n &= \text{a constant which is a function of } n \\ K_z(t) &= \text{as described in equation (16)} \end{aligned}$$

We can refer all the coordinates to an axis fixed on the rotor using the relation $x = x_r + v \cdot t$.

$$K_a = \sum_{n(\text{odd})} C_n K_z(t) \cos(n\Lambda x_r + n\omega_e t)$$

Consider now the first harmonic term;

$$\begin{aligned} &C_1' \left\{ e^{-t/\tau_c} \sin(\varphi - \phi) - \sin(\omega_e t + \varphi - \phi) \right\} \cos(\Lambda x_r + \omega_e t) \\ &= C_1' \left\{ \begin{aligned} &e^{-t/\tau_c} \sin(\varphi - \phi) \cos(\Lambda x_r + \omega_e t) \\ &-\frac{1}{2} \sin(2\omega_e t + \varphi - \phi + \Lambda x_r) - \frac{1}{2} \sin(\varphi - \phi - \Lambda x_r) \end{aligned} \right\} \end{aligned} \quad (18)$$

The dc offset in the discharge pulse therefore appears to the rotor as a wave of fundamental frequency, which is to be expected. A response to this wave can be obtained by convoluting equations (8) to (11) with an exponential rise function and subtracting it from a response to a step at $t = 0$. The sinusoidal term decomposes into two waves, one moving with a fundamental frequency in a direction opposite to the rotor and the other in a direction of the rotor with the same frequency. The former therefore appears to the rotor as a wave of twice the fundamental frequency and the

latter as a step with no relative motion. The response to this step would be as indicated in equation (12). It is possible therefore to express the response in equation (17) as a sum of the responses to the individual excitation terms in equation (18). The response to these individual excitations have already been determined under cases 1 and 2 above.

Higher harmonic terms result in waves which have relative motion with respect to the shield. These waves, along with the two waves obtained from the first harmonic, can be prevented from fully penetrating the shield by the proper selection of the thickness. However, the wave moving in the direction of the rotor with the same speed will establish itself across the air gap with a time constant τ_0 [eqn. (13)].

Therefore, the field coil which is shielded would experience a current transient associated with the penetration of the half amplitude fundamental frequency wave. The magnitude of this transient would be determined by the conductivity of the shield material.

Consider equation (17) now for large times $t \gg \tau_0$, in order to determine the steady state eddy current distribution. Equation (17) reduces to

$$J_z = \frac{-jK_m e^{j(\Lambda x_r + 2\omega t)}}{2(1 - e^{2\lambda_2 h})} \left\{ \frac{\Lambda^2 - \lambda_2^2}{\lambda_2^2} \left(e^{\lambda_2 y} + e^{2\lambda_2 h} e^{-\lambda_2 y} \right) \right\} \quad (19)$$

Here λ_2 is a new constant defined by

$$\lambda_2 = \Lambda \sqrt{1 + j \frac{2v\mu_0\sigma}{\Lambda}}$$

Note that equation (19) has only the double frequency component of the eddy current. This is so because for large times, the dc offset of the armature current has decayed to zero. Also, the wave moving along with the rotor has established itself across the air

gap and so does not induce eddy currents in the shield. In fact, comparison of equation (19) with equation (11) indicates that the former is the steady-state term in the latter, with all the constants adjusted to reflect a velocity $2v$ instead of v .

Correlation of Experimental Observation with Theory

Under a project funded by the U.S. Army ARDEC and DARPA, a passively compensated compulsator was successfully designed and built. It has been undergoing extensive testing since 1985 at CEM-UT. A cross section of this machine is shown in figure 10. Parameters of this machine are shown in table 1. The typical current pulse from this machine is shown in figure 7.

Table 1: Machine parameters

PARAMETERS	UNITS	VALUE
Peak open circuit voltage	kV	2
Peak current (pulsed duty)	kA	950
Pulse width	ms	2
Rotor speed	rpm	4,800
Electrical frequency	Hz	240
Number of poles	-	6
Overall length	m	1.6
Overall diameter	m	1.2

Shielding in this machine is accomplished by a 12.7 mm thick aluminium cylinder shrunk on the periphery of the rotor. The conductivity of this aluminium is

40% IACS. In order to provide the extra support to the field coil end turns, Inconel™ end caps are used. These end caps cover 60 mm of axial length on both ends of the rotor, leaving 88% of the length for the aluminium shield. The conductivity of the Inconel™ end caps is lower by a factor of almost 30 compared to the aluminium shield. In this machine, the rotor is made from solid 4340 steel, whereas the stator is laminated.

Correlation of experimental observations with theory can be accomplished more easily through voltages and currents than the magnetic fields, as the former are easier to measure. The transient field current during a pulse and the open-circuit voltage after a pulse are therefore considered in the following sections.

Transient Field Current

Figure 11 shows the observed transient component of the field current during a pulse. Also shown in this figure is the armature current. This particular pulse was obtained at 2,300 rpm when the dc excitation current was about 620 A, which is 41% of the rated excitation current. The transient portion of the field current is seen to reach a peak of about 1,200 A.

The transient flux linking the field coil due to penetration through the aluminium shield is small (20% of total) compared to that caused by the penetration through the Inconel™ end plates, in spite of the fact that the aluminium shield occupies 88% of the axial length.

Figure 12 shows the simulated field current using the model developed in the previous section. Since this model assumes a permeable boundary at the rotor iron surface, the transient field-coil inductance for the simulation has also been calculated under the same assumption for the purpose of consistency. This results in a much higher transient field coil inductance. In reality, since the rotor iron is solid, the field

coil inductance is much lower and correspondingly the flux linking the field coil due to the discharge currents is also lower. After allowing for the solid rotor iron effects, the field current is much closer in shape to the observed field current transient. The magnitudes are not in good agreement because of the approximations involved in determining the discharge flux linking the field coil with a conductive rotor iron boundary.

It is important to consider the transient current in the field coil for two reasons. First, it affects the duty and therefore the design of the brushes and slip rings on the rotor. Second, it affects the power supply and leads used to drive the field coil. Nominally, temperature rise is not a great concern since the transient pulse is present only for a short duration.

Open Circuit Voltage After a Pulse

From figure 11, it is observed that the armature current does not go through a natural zero, but is interrupted prematurely. After the armature current stops flowing, the transient current in the field coil decays almost exponentially to zero. Another effect which occurs is the field wave which is rotating with the rotor has partially established itself across the air gap. Once the armature current goes through zero, this flux cannot disappear instantaneously, as it is supported by eddy currents in the shield.

The presence of this discharge related flux after an armature current discharge results in the distortion of the radial excitation flux density distribution near the armature winding. This is shown in figure 13 along with the prepulse radial field distribution. The distortion is quite small after a single pulse and diminishes with time as the eddy currents in the shield, supporting the field, decay. If the discharge is carried out over a number of pulses, more time is available for the half amplitude fundamental

frequency wave to establish itself across the air gap. Radial field distribution would therefore be more distorted. A low field current, continuous discharge over 40 pulses was carried out on the prototype machine. The machine speed was 3,600 rpm. Results of this test are shown in figure 14. Figure 14a shows the continuous train of 40 current pulses, while figure 14b shows the open circuit voltage before, during, and after a continuous discharge; discharge duration was about 110 ms. One can see the open circuit voltage recovers in an almost exponential form after the armature current has stopped flowing. During this time, the shape of the open circuit voltage trace is also distorted as shown in figure 14c. Figure 14d shows the open circuit voltage trace a long time after the eddy currents in the shield have decayed.

This effect has a potential use from the standpoint of helping the interruption process. If the effect can be significantly enhanced without greatly diminishing the effectiveness of the shield over the duration of the pulse, then a lower open circuit voltage is obtained immediately following the interruption of the armature current; thereby allowing the interrupting device to regain its voltage standoff capability.

Shield Design Considerations

So far, we have used the solution to the 2-D diffusion equation in order to understand the physical phenomenon and also to interpret observed data. From an engineering standpoint, the next question in the design of these machines is how is one to determine an optimum shield thickness? There is no general answer that can cover the design of all passively compensated compulsators. The application, to a large extent, will govern that optimum. As in all electromechanical devices, the mechanical stresses must also be considered in determining the thickness of the shield. This model, however, helps to provide the electromagnetic parameters in the optimization process.

In order to demonstrate the use of this model, some of the design considerations which could be used for the example machine (specified in table 1) are presented.

In figures 15 and 16, several shield thicknesses, ranging from 3.18 mm (0.125 in.) to 25.4 mm (1 in.), have been considered. Two parameters have been plotted -- the peak value of the transient field current and the effective shield resistance. As one would expect, the peak field current and the equivalent resistance go up as the shield becomes thinner. The effective shield resistance has been calculated using the formulation

$$R_{eq} = \frac{\iiint \left[\int_0^t \frac{J_z^2}{\sigma} dt \right] dV}{\int_0^t i^2 dt}$$

One would prefer to minimize the shield thickness as this lowers the field excitation requirement because of a shorter air gap. Lower field excitation in turn implies lower dissipation and a smaller power supply for excitation purposes. From figures 15 and 16, it is clear that there is no significant penalty to be paid in going from a 25.4 mm to a 12.7-mm shield. In fact, from a purely electromagnetic design standpoint, a shield thickness between 6.35 and 12.7 mm may have been acceptable. Mechanical constraints, however, may have limited the thickness to 12.7 mm, considering that the shield for the example machine is shrunk onto the rotor surface.

Figure 17 shows the temperature distribution in the shield after a discharge pulse (fig. 7). This is the full rated discharge duty of the machine. The thickness range is 12.7 mm. This data could be used to determine temperature related stresses. It would also be used, along with a heat diffusion model, to determine the temperature of the shield under a multiple discharge duty cycle. The temperature could be a concern when the repetition rate is high.

Conclusion

Presented above is a model describing the transient eddy current distribution in the shield of a passively compensated compulsator. The response to a step excitation with and without relative motion of the shield have been computed. The step response has also been used to calculate the response to a sinusoidal excitation which is typically the problem of interest. Also, the step response could be used to calculate the response to an excitation with a different time profile.

This model has been used to understand and interpret experimental observations. The agreement between the model and the experimental observations is not very close because of the limitations in representing the boundary conditions at the shield rotor-iron interface. However, the model could be used to calculate the relative merits of different shield designs and materials. In a design where the shield-rotor interface is indeed either permeable or highly conductive, the model would serve very well. Appendix A gives a solution for the step response with relative motion of the shield when the shield-rotor interface is conductive.

The transient response to a sinusoidal excitation with an electrical frequency ω_e was shown to be the sum of the following three responses:

- (a) Response to a step and exponential decay corresponding to the dc offset in the discharge current.
- (b) Response to a step with shield moving at twice the mechanical frequency corresponding to an excitation wave moving in a direction opposite to the rotor with the same frequency.

- (c) Response to a step with no shield motion corresponding to an excitation wave moving in the direction of the rotor with the same frequency.

Further Related Work

Recently, there has been great interest in compulsators for mobile and space-based applications. Thus, there is a continuing effort to develop compulsators with very high power and energy densities. These densities have been enhanced primarily by the use of composites in self-excited compensated pulsed alternators. These machines are air-cored because there is an absence of iron in the magnetic circuit.

A repetitive, pulsed duty, air core, passively compensated compulsator is being developed at CEM-UT at present. A model for the diffusion process in the shield of an air-core compulsator has been completed. This will be the subject of another paper which will also discuss the differences in the shielding used in air-core and iron-core machines.

APPENDIX A

Case 3: Conductive Boundary Condition at $y = h$, Shield Moving with a Velocity v

In this appendix, a solution to the convective diffusion equation (eqn. A-1) is sought. The difference is that one of the boundaries, specifically, the one on the other side of the shield compared to the excitation, is considered conductive rather than permeable. The purpose is to express the differences in the solutions and the physical implications. The true nature of that boundary is as mentioned, in between these two extremes.

The solution is given in equation (A-1)

$$J_z = \frac{-K_z e^{j\Lambda x}}{(1 + e^{2\lambda h})} \left\{ \left(\frac{\Lambda^2 - \lambda^2}{\lambda} \right) (e^{\lambda y} - e^{2\lambda h} e^{-\lambda y}) - \frac{1}{\lambda} \sum_{n(\text{odd})} a_n \left(\Lambda^2 + \left(\frac{n\pi}{2h} \right)^2 \right) e^{-t/\tau_n} \cos\left(\frac{n\pi}{2h} y \right) \right\} \quad (\text{A-1})$$

$$a_n = \frac{-(2\lambda / h)(e^{2\lambda h} + 1)}{\lambda^2 + \left(\frac{n\pi}{2h} \right)^2}$$

$$\tau_n = \frac{\mu_0 \sigma}{\lambda^2 + \left(\frac{n\pi}{2h} \right)^2}$$

$$\lambda = \Lambda \sqrt{1 + j \frac{v\mu_0 \sigma}{\Lambda}}$$

Figure A-1 shows the eddy current distribution at $x = 0$ for a time period τ_1 (τ_1 as given above). It must be noted that $\tau_{1c} = 4\tau_{1p}$ where the subscript 'c' and 'p' denote conductive and permeable boundaries, respectively. For the parameters considered ($v\mu_0\sigma \gg 1$), there is no significant difference in the distribution over this period for a step response. The difference is most apparent at larger depths until at $y = h$ there are no eddy currents for the case with the conductive boundary.

The other differences, though not discernible for the condition $v\mu_0\sigma \gg 1$, are in the transient distribution modes. In the case with the conductive boundary, there is a complete absence of the terms involving $a_0 e^{-t/\tau_0}$. This term, in the permeable boundary case, implies a constant current density distribution over the depth and is associated with the flux which jumps across the 'air gap'. A highly conductive rotor-shield interface does not allow the existence of flux and correspondingly that distribution

mode. Figures A-2a and b show the distribution modes for $n = 1$ and $n = 2$ for the permeable boundary case. Correspondingly, the first two distribution modes for $n = 1$ and $n = 3$ for the conductive boundary case are shown in figures A-3a and b, respectively.

From the above, one can conclude that for the pulsed type of applications considered, either the permeable boundary or the conductive boundary solution would give comparable result for the calculation of energy losses and temperature rise in the shield. However, when computing the transients in the field coil with solid rotors, one is compelled to use the permeable boundary solution in spite of its inaccuracy.

Acknowledgments

The author would like to thank Dr. H. H. Woodson for his guidance and comments on this work and also the Center for High Performance Computing for the use of the Cray computer.

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2701.1383a

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2701.1383b

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