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STRESS ANALYSIS FOR DESIGN OF ELECTROMAGNETIC LAUNCHERS

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The recent development of very high current pulsed power supplies provides an opportunity for order of magnitude improvements in electromagnetic launcher performance. Consideration of design objectives and constraints suggests that a ceramic-steel construction offers significant advantages over conventional composite-steel railguns. Structural analysis of such a system focuses upon material specific stress concentrations produced by dynamic and asymmetric electromagnetic loads.

INTRODUCTION

The development of electromagnetic railguns [1] and light gas guns [2] has been motivated by a variety of research and defense applications for hypervelocity launchers. A recent symposium [3] detailed interest in the development of improved capabilities for the experimental study of hypervelocity impact phenomena, for kinetic energy penetrator [4], equation of state [5], meteorite impact [6], and other research. Although extensive studies of the mechanics of target penetration have been performed [7, 8], a better understanding of the mechanics of high velocity launchers is needed.

Although it appears that light gas guns have reached the practical limit of their capabilities, the relatively recent development of very high current pulsed power supplies [9] has made it possible to attempt order of magnitude improvements in electromagnetic launcher performance. Hence this analysis considers the structural design of railguns for strength and stiffness, under loadings associated with significant increases in available input electrical energy.

RAILGUN STRUCTURE

The operation of a railgun [10] is illustrated in Figure 1. A large electrical current provided by an appropriate power supply is passed through parallel metal rails, the circuit completed by a conducting armature which may be solid or plasma in form. A projectile may be located just forward of the armature. A Lorentz force created by the interaction of the current (I) in the armature and the magnetic flux density (B) between the rails propels the projectile down the bore of the railgun. Since a similar electromagnetic force acts to drive the rails apart, a stiff supporting structure is needed to maintain the desired system geometry.

A schematic of a cross section of a representative round bore electromagnetic launcher is shown in Figure 2. This drawing depicts the essential features of the structure, namely: (1) parallel metal rails of high electrical conductivity, (2) nonconducting sidewall insulators which serve to separate the rails and confine the projectile and any

plasma armature in the bore, (3) an inner nonconducting support structure, here labeled as ceramic, which serves to hold the rails and insulators in place, and (4) an outer supporting structure, here shown as steel, which may provide preloading, secondary containment, or longitudinal stiffness. A launcher of the type shown is under development and will have a bore diameter near 100 mm, an outer diameter of 500 mm or more and a length of ten meters.

DYNAMIC LOADING

The principal structural loading is provided by the electromagnetic body force which acts on the rails, with an approximate magnitude per unit length

$$F' = \mu_0 I^2 / (2 \pi d) \quad (1a)$$

where μ_0 is the magnetic permeability of free space, I is the current in the rail, and d is the bore diameter. Due to the brief transit time of the projectile in the bore (less than ten milliseconds), the diffusion of current away from the rail surface during firing is limited. Hence the primary structural load is modeled as a uniform pressure applied normal to the rail surface, with a magnitude

$$P_m = F' / (d \sin \alpha) \quad (1b)$$

where 2α is the angle subtended by the arc of a single rail. The time dependence of this load is determined by the current pulse produced by the power supply. Since the rail current flows only behind the projectile, the axial extent of the load varies with time in accordance with the armature velocity. Predicted current profiles and projectile velocity histories for a high performance railgun are provided by Wu et al. [11].

The Lorentz force applied to the conducting projectile or plasma armature has a magnitude

$$F = L' I^2 / 2 \quad (1c)$$

where L' is the inductance per unit length of the railgun. In the case of a plasma armature device, transmission of this force to the projectile via a gas pressure gives rise to an additional structural load in the form of a uniform pressure

$$P_a = F / (\pi d^2 / 4) \quad (1d)$$

applied to the bore surface in the immediate vicinity of the projectile.

DESIGN OBJECTIVES, CONSTRAINTS, AND ALTERNATIVES

Although the use of steel to withstand the loads just described is attractive from a strength and stiffness point of view, the introduction of such material in the vicinity of the rails, in the presence of intense transient electromagnetic fields, results in the creation of eddy currents in the steel and the dissipation of input energy intended for use in accelerating the projectile. This problem motivates the use of composites and other low conductivity materials in the fabrication of inner supporting structures for railguns. Although ceramic has been proposed as an appropriate material for the inner supporting structure [12], to date most railguns have employed composite materials such as Kevlar or E-glass in this role. Conducting materials may be employed for the outer supporting structure. However some performance penalty can be associated with the use of steel in the outer supporting structure, and may be quantified as a reduction in the inductance per unit length of the railgun. Recent work [13] has studied this penalty for the round bore geometry of interest here.

The primary structural design objective is to minimize the system compliance, subject to various material, electromagnetic, thermal, and assembly constraints. Cost and weight limitations might also be imposed, however such considerations will not be discussed here. In this problem system compliance may be defined as the vertical rail deflection per unit applied load, since the magnitude of bore deformation under loading is indicative of the extent of performance degradation due to frictional drag and bore ablation as well as the probability of occurrence of severe problems such as projectile tumbling. With structural stiffness a primary concern, ceramics offer a considerable improvement over conventional composite structures, in that ceramics can provide an isotropic Young's modulus matched only in the fiber direction by the orthotropic continuum elastic moduli of typical high quality carbon fiber reinforced epoxy composites.

For plasma armature guns, an additional objective is to maintain the integrity of the rail-sidewall seals, since experiments [14] indicate that gun performance is strongly dependent upon the extent of plasma leakage. This requirement, as well as the need to close assembly clearances present in the nonhomogeneous structure, may be met by preloading. Consideration of high performance launchers suggests two additional preloading considerations. First, the significant time, expense, and uncertainty associated with the design and construction of very large high pressure guns emphasizes that provisions must be made for disassembly and replacement or modification of the structural components based upon operational experience. Second the attractiveness of ceramics for use as stiff nonconducting rail support may require preloading to bias the ceramic stress state towards mean compression, in order to avoid brittle failure in tension under operating loads.

Available techniques for the application of the required preload depend upon the choice of composite or ceramic as an inner support structure. One approach [15] is to fabricate an outer steel support structure in several parts, bolting these parts together around the remainder of the railgun in order to close assembly clearances, since a thermal shrink fit of a steel outer support tube on a composite inner support tube is limited by the thermal operating range of most fiber reinforced epoxy composites. Alternative preloading techniques used for composite structures, such as fiber winding on a rail-insulator assembly [16], may not satisfy an assembly-disassembly requirement. In contrast the excellent thermal properties and compressive strength of ceramics mean that shrink fit preloading of a ceramic cylinder using a steel tube is limited only by the thermal and strength properties of the steel.

These design considerations suggest that a ceramic-steel launcher of the type shown in Figure 2 is an attractive alternative to conventional designs, and can take full advantage of new pulsed power supplies in extending the capabilities of hypervelocity launchers.

QUALITATIVE ANALYSIS

The principal effects of electromagnetic body force loading on a fiber reinforced composite structure are illustrated by the two dimensional model shown in Figure 3, developed using the finite element code ABAQUS [17]. The asymmetric load tends to oval the structure, introducing large tangential stress concentrations in the inner supporting structure, compressive under the rail and tensile under the sidewall. In addition a large radial compressive bearing stress appears beneath the rail. The tensile stress concentration is of primary concern for a ceramic design, while the relatively low yield stress for hoop wound composites in a direction transverse to the fibers suggests a material limitation to the allowable bearing stress in that case. The model also depicts vertical rail motion which can lead to poor electrical contact with solid armatures or axial gas leakage with plasma armatures, sidewall motion which tends to squeeze the projectile, and loss of the rail-sidewall seal. The use of frictional interface elements allows a quantitative study of these effects, as well as sliding and stress discontinuities along the junction of the inner and outer support structures. The model effects observed here have been identified as problems in experimental studies [18].

DESIGN FOR STRENGTH

The feasibility of a ceramic construction is based upon the avoidance of tensile failure beneath the sidewall, by means of a compressive prestress introduced by shrink fit of an outer steel tube onto a stack of ceramic disks. Difficulties associated with the manufacture of very large high strength ceramic parts eliminate the possibility of a monolithic ceramic tube. The tensile stress concentration of interest, associated preload requirements, and dimensions for the structural members may be estimated by analytical methods and the results evaluated with a numerical model.

Each ceramic disk may be analyzed as an isotropic elastic ring, subjected to a normal pressure load over two symmetrically placed arcs of angular extent 2α , located along the disk's inner circumference (Figure 4). The appropriate boundary pressure is

$$P = P_m (d / 2a) \quad (2a)$$

A general solution to the stress boundary value problem for an isotropic ring has been obtained by Buchwald [19] using the method of complex variables. Jaeger and Cook [20] have presented results to the problem of interest here for the case $\alpha = 7.5$ degrees. Direct application of Buchwald's method yields the additional data shown in Figure 5, for the stress concentration factor

$$f = \sigma_{\theta\theta} / P \quad (2b)$$

which determines the tensile tangential stress at the point labeled "S" in Figure 4. The magnitude of the stress concentration depends upon the rail width and the ceramic wall thickness.

This stress concentration determines the required preload pressure and the corresponding steel tube dimensions, which may be estimated based upon the ultimate strength of the ceramic in tension (σ_{max}) and a shrink fit calculation for dissimilar isotropic elastic materials (Figure 6). Burr [21] gives the radial interference (δ) - interface pressure (P_i) relation for the latter problem, in plane stress, as

$$P_i = (2\delta / b) / (F + G) \quad (3a)$$

where

$$F = [(1+\nu_1)(a^2/b^2)+(1-\nu_1)]/[(E_1/2)(1-a^2/b^2)] \quad (3b)$$

$$G = [(1+\nu_2)(b^2/c^2)+(1-\nu_2)]/[(E_2/2)(1-b^2/c^2)] \quad (3c)$$

with ν_1 , E_1 , ν_2 , and E_2 the values of Poisson's ratio and Young's modulus for the inner and outer cylinders respectively. The

compressive tangential stress developed at the inner surface of the ceramic under preload is

$$\sigma_{\theta\theta} = -2 P_1 / (1 - a^2 / b^2). \quad (4)$$

The allowable radial interference, and hence the available preload, is limited by the thermal and strength properties of the steel. In addition some allowance must be made for the shrink fit assembly clearance, particularly in the case of extremely long steel cylinders. A thermal limit to the preload pressure (P_t) is calculated from equations (3) with

$$\delta = (\alpha_t T_m - \delta') b - \delta_0 / 2 \quad (5)$$

where α_t and T_m are the thermal expansion coefficient and maximum allowable temperature for the steel, δ_0 is a minimum thermal growth which allows for manufacturing tolerances, and δ' is an additional assembly clearance per unit inner radius of the steel cylinder. A strength limit to the preload pressure (P_s) is determined by the maximum shear stress developed in the steel cylinder, and its corresponding shear strength (τ_{max}), in the form

$$P_s = (1 - b^2 / c^2) \tau_{max}. \quad (6)$$

The preceding calculations, in combination with the choice of a maximum acceptable inductance penalty associated with the use of a steel outer structure, may be used to identify feasible ceramic-steel configurations and estimate factors of safety. Dynamic effects may be quantified using finite element analysis, and will be discussed in a later section. In the absence of numerical simulation, a conservative allowance for dynamic effects is provided by designing for a factor of two increase in the displacements and stresses calculated from a static analysis [22].

NUMERICAL EXAMPLE

Consider a solid armature railgun design, constructed of aluminum oxide disks, a steel outer tube, polycarbonate sidewalls, and molybdenum rails. Material properties are listed in Appendix 1. Experience has proven polycarbonate to be an effective sidewall insulator, while the strength, stiffness, frictional, and conductivity properties of molybdenum make it an attractive alternative to standard copper rails. Given a bore diameter and the maximum available current, a minimum rail thickness is determined from resistive heating calculations. Figure 7 compares the preloading requirements and design constraints calculated from the preceding discussion of structural loads and stresses, for the numerical example detailed in Appendix 2. The "incipient yielding" curve defines the minimum preload pressure which avoids tensile failure of the ceramic under static loading, based upon a maximum normal stress criterion, as a function of the ceramic wall thickness. The "incipient tension" curve represents a conservative design, defining the preload required to avoid the development of a tensile stress in the ceramic under static loading. "Strength" and "thermal" limits to available preload are also shown, for a particular steel tube thickness. Finally an "electromagnetic limit" is indicated, based upon the acceptability of a fifteen percent reduction in inductance per unit length [13] due to proximity of the outer steel support structure to the rails. In this example the intersection of the electromagnetic and strength limit curves defines a conservative design. The active constraints determine the magnitude of the interference fit and the preheat temperature for the steel.

FINITE ELEMENT ANALYSIS

A finite element model of the structure sized in the preceding example is shown in Figure 8. Static analysis with this model verifies the accuracy of the approximate calculations, as indicated in Table 1, despite their neglect of the presence of a rail-sidewall assembly, ovaling of the steel tube, frictional loads, and other effects. The tensile

stress concentration as well as the compressive bearing stress are accurately described by the analytical model. The interference pressure is shown to have little azimuthal dependence under the asymmetric load and to be perturbed only slightly by the ceramic deformation. In addition the predicted bore deformations, shown in Table 2, compare quite favorably to those expected from composite railguns [16].

The dynamic response of the structure was determined for the current profile shown in Figure 9, based upon the characteristics of a sixty megajoule pulsed power supply now under construction. Abrupt variations in the input current are due to the successive discharge of energy storage inductors into the gun with the use of explosive opening switches. The maximum current value shown in Figure 9 is that used previously to calculate a pressure load for the static analysis. The time variation of the vertical displacement of the rail centerline and the tangential normal stress at a position corresponding to the point "S" in Figure 4 are shown in Figures 10 and 11. Comparison of these results with those listed in Table 1 shows that the maximum displacement and stress increments from the preloaded state are increased by a factor of 1.28, as compared to the static case.

CONCLUSION

The stress analysis described here offers an alternative to purely numerical modeling of round bore railgun geometries [23] for the case of a ceramic-steel design. In principle a similar approach is possible for any orthotropic material, such as a continuum model of a fiber reinforced composite, again based upon complex variable methods [24]. However the complications associated with the introduction of anisotropy are considerable.

The design of rather successful launchers [14] on the basis of two dimensional stress analysis suggests that the three dimensional effects not considered here, such as longitudinal bending, may be of secondary importance. However, three dimensional finite element models (Figure 12) are under study to quantify these effects. Compared to two dimensional models, cross-sectional mesh density may be reduced due to computer memory limitations, and computer time requirements for dynamic simulation are large, even with the availability of supercomputer resources.

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APPENDIX 1

The material properties used in the numerical examples are as follows: for the ceramic, $\rho = 3605 \text{ kg/m}^3$, $E = 276 \text{ GPa}$, $\nu = 0.22$, and $\sigma_{\max} = 221 \text{ MPa}$; for the steel, $\rho = 7833 \text{ kg/m}^3$, $E = 207 \text{ GPa}$, $\nu = 0.29$, $\tau_{\max} = 414 \text{ MPa}$, $\alpha_t = 12.6 \times 10^{-6} / ^\circ\text{C}$, and $T_m = 482^\circ\text{C}$; for the molybdenum, $\rho = 10241 \text{ kg/m}^3$, $E = 324 \text{ GPa}$ and $\nu = 0.325$; and for the polycarbonate, $\rho = 1522 \text{ kg/m}^3$, $E = 11.6 \text{ GPa}$, and $\nu = 0.25$.

APPENDIX 2

The parameters describing the numerical example shown in Figures 7, 10, and 11 and Tables 1 and 2 are as follows: $d = 0.09 \text{ m}$, $\alpha = \pi/4$, rail thickness = 0.0381 m , $a = 0.0831 \text{ m}$, $I = 3.27 \text{ MA}$, $P_m = 373 \text{ MPa}$, ceramic wall thickness = 0.135 m , $b = 0.2181 \text{ m}$, $\delta_o = 2.54 \times 10^{-4} \text{ m}$, $\delta' = 0.001$, steel wall thickness = 0.0762 m , $c = 0.2943 \text{ m}$, design interface pressure = 186 MPa , and design interference fit = 0.0009 m .

Fig. 1 Electromagnetic launcher concept

Fig. 2 Cross section of a round bore ceramic-steel railgun

Fig. 3 Response of a finite element model of a composite railgun structure to static loading

Fig. 4 Isotropic elastic ring model for the inner support structure of a ceramic-steel railgun

Fig. 6 Thermal shrink fit of a steel tube onto an inner ceramic railgun structure

Fig. 5 Stress concentration factor for the tensile tangential normal stress at the point "S" in Fig. 4

Fig. 7 Design constraints for a ceramic-steel railgun

Fig. 8 Response of a two dimensional finite element model of a ceramic-steel railgun to static loading

Fig. 10 Time variation of the rail centerline vertical displacement under dynamic loading

Fig. 9 Design input current pulse for a high performance electromagnetic launcher

Fig. 11 Time variation of the tangential normal stress at a point corresponding to point "S" in Fig. 4

Fig. 12 Response of a three dimensional finite element model of a segment of a round bore electromagnetic launcher to static loading

Table 1

Comparison of Predicted Stress Values

Stress Component [$\sigma_{ij}(x,y) / P_m$]	Analytical Model	FEM Model
<u>Preload:</u>		
$\sigma_{xx}(a,0) / P_m$	0.00	-0.07
$\sigma_{yy}(a,0) / P_m$	-1.17	-1.12
$\sigma_{yy}(0,a) / P_m$	0.00	-0.02
$\sigma_{xx}(b,0) / P_m$	-0.50	-0.50
$\sigma_{yy}(0,b) / P_m$	-0.50	-0.50
<u>Static load:</u>		
$\sigma_{xx}(a,0) / P_m$	0.00	-0.04
$\sigma_{yy}(a,0) / P_m$	0.00	-0.07
$\sigma_{yy}(0,a) / P_m$	-0.54	-0.48
$\sigma_{xx}(b,0) / P_m$	-0.50	-0.52
$\sigma_{yy}(0,b) / P_m$	-0.50	-0.52

Table 2

Predicted Displacements (mm)

Displacement [$u_i(x,y)$]	Preload [Δu_i]	Static load [Δu_i]
$u_x(d/2,0)$	-0.109	-0.077
$u_y(0,d/2)$	-0.131	+0.183