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ABSTRACT

This paper discusses the eddy current distribution in the shield of an air core, passively compensated pulsed alternator (compulsator). The action of the shield reduces the inductance of the armature winding considerably, thus enabling it to deliver high power pulses to low impedance loads. By virtue of its function, the shield is stressed mechanically as well as thermally. It is stressed mechanically because the transient electromechanical torque (as well as the radial loading due to flux compression) act on it. The shield is stressed thermally, because of the transient and steady state eddy currents flowing in it. A knowledge of the eddy current distribution is essential in determining the stresses in the shield and therefore in the proper selection of the shield parameters.

An analytical solution of the two-dimensional diffusion and convective diffusion equations is obtained. The solution is obtained by the extraction of the poles and corresponding residues of the transformed equations. This solution technique is fairly general and can be applied to a wide variety of field problems.

The theory was developed specifically for the shield of an air-core compulsator which is a single-phase alternator. By using superposition, the results can also be applied to determine the transient eddy current distribution in the shield of superconducting synchronous generators. The transient performance of the shield during fault conditions and power angle oscillations is of considerable importance from the standpoint of the superconductor. Wherever appropriate, this discussion is presented.

A previous paper [1] discussed the transient eddy current distribution in the shield of iron-core machines. A comparison of the eddy current distribution in air core, iron core, and hybrid machines is also presented. Hybrid machines are those which use iron only partially in the magnetic circuit (e.g. the iron shield of superconducting machines).

INTRODUCTION

Various types of compulsators have evolved since the inception at the Center for Electromechanics at The University of Texas at Austin (CEM-UT) in 1978. One variant of this machine is the passively compensated compulsator. In this machine, a continuous conductive shield spins relative to the armature. In this respect the passive compulsator is very similar to a single phase, short-circuit generator. The compulsator with all its variants has a variety of applications in areas such as fusion, electromagnetic launching, particle beam accelerators, or simply as a short-circuit generator for testing power system equipment under fault conditions.

Figure 1 shows the basic geometry of the passive, air-core compulsator. This particular geometry has a stationary field coil, shield, and a rotating armature winding. The analysis would be equally valid for a stationary armature and a rotating shield.

There are two sources for eddy current generation in the shield: (a) switching of the current in the field coil, and (b) current flow in the armature winding. Both are analyzed with similar methods. When considering the interaction between the field coil and the shield, the relative motion between the two components is set to zero.

In most machines the thickness of the shield is small compared to its average radius. This allows a true cylindrical geometry to be represented by the Cartesian coordinate system, which greatly simplifies the expressions. Figure 2 shows the coordinate system and the relative location of the shield and the armature winding. The shield considered is nonmagnetic with a magnetic permeability of free-space (μ_0) and an electrical conductivity (σ). The shield has a thickness (h) and a relative velocity (v) with respect to the armature winding. X and Y are the coordinates fixed with respect to the armature winding. X_r and Y are the coordinates fixed with respect to the shield.

The following assumptions have been made while seeking a solution to the problem:

- (A) The axial length of the machine is considered to be large compared to the radius. Thus the effects of the end turns of the winding are neglected. The solution is then invariant with respect to the axial coordinate 'Z'.
- (B) The change in the impedance of the machine, due to the diffusion process, is considered to be a small fraction of the total circuit impedance for the duration of interest. The current in the winding is then determined by an impedance which is constant in time.

- (C) The material properties are constant in space and time, thus the problem being solved is linear.
- (D) It is assumed that the source of excitation can be represented by a superposition of space harmonics. This is not much of a limitation especially for rotating machinery.
- (E) The shield is assumed to have free space on both sides. Another possible boundary condition is highly permeable on one side (i.e. the side of the armature coil) and free space on the other side. The second boundary condition is applicable in those machines which use a ferromagnetic shield in order to contain stray magnetic fields, as is commonly done with superconducting synchronous generators.

In obtaining the solution, the response to a step in excitation is first solved. This is then convoluted with specific time functions corresponding to the nature of the excitation current. The solution with mixed boundary conditions can be solved in a manner similar to the one described in this paper. The characteristic equation corresponding to the mixed boundary conditions has been discussed.

FORMAL SOLUTION

Shield Moving with a Velocity 'v' with Respect to the Excitation

From figure 2 it is seen that the solution for the three regions must be obtained simultaneously. The partial differential equations for the three regions are as follows:

Region 1 (y < 0) - Excitation

$$\nabla^2 \bar{B} = 0, \text{ Laplace's equation}$$

Region 2 (0 < y < h) - Shield

$$\frac{1}{\mu_0 \sigma} \nabla^2 \bar{B} = \left(\frac{\partial}{\partial t} + \bar{v} \cdot \nabla \right) \bar{B}, \text{ convective diffusion equation}$$

Region 3 (y > h) - Free space

$$\nabla^2 \bar{B} = 0, \text{ Laplace's equation}$$

These equations in which $\vec{B}$ and $\vec{v}$ are vector quantities must be solved under the following boundary conditions:

$$\begin{aligned} B_{x1} - B_{x2} &= \mu_0 K_z e^{jkx} S_0(t) & \text{at } y = 0 & \text{ i.e., a step excitation at } t = 0 \\ B_{y1} &= B_{y2} & \text{at } y = 0 \\ B_{x2} &= B_{x3} & \text{at } y = h \\ B_{y2} &= B_{y3} & \text{at } y = h \end{aligned} \quad (1)$$

Here the subscripts x and y refer to the two components of the field and the subscripts 1, 2 and 3 refer to the corresponding regions specified above. Also a constant k has been introduced which is defined by:

$$k = \frac{2\pi}{\tau_p}$$

where

τ_p = pitch of a pole pair

Expansion of the equations for the three regions gives the following:

Regions 1 & 3:

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} = 0$$

$$\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} = 0$$

Region 2:

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} = \mu_0 \sigma \left\{ \frac{\partial B_x}{\partial t} + v \frac{\partial B_x}{\partial x} \right\}$$

$$\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} = \mu_0 \sigma \left\{ \frac{\partial B_y}{\partial t} + v \frac{\partial B_y}{\partial x} \right\}$$

Since the excitation is periodic in the x direction it is assumed that the solution for three regions is also periodic in the same direction. The solution therefore takes the form:

$$B(x, y, t) = e^{jkx} \tilde{B}(y, t)$$

Performing the Laplace transform of the above partial differential equations with respect to the variable 't', the equations can be rewritten as:

Regions 1 & 3:

$$\frac{d^2 \tilde{B}_x(y,s)}{dy^2} - k^2 \tilde{B}_x(y,s) = 0$$

$$\frac{d^2 \tilde{B}_y(y,s)}{dy^2} - k^2 \tilde{B}_y(y,s) = 0$$

Region 2:

$$\frac{d^2 \tilde{B}_x(y,s)}{dy^2} = \{k^2 + \mu_0 \sigma s + jv\mu_0 \sigma k\} \tilde{B}_x(y,s)$$

$$\frac{d^2 \tilde{B}_y(y,s)}{dy^2} = \{k^2 + \mu_0 \sigma s + jv\mu_0 \sigma k\} \tilde{B}_y(y,s)$$

Here we define a new complex variable:

$$p = \sqrt{k^2 + \mu_0 \sigma s + jv\mu_0 \sigma k}$$

Applying the boundary conditions stated above and the initial condition that the field is zero everywhere prior to $t=0$ gives the following solutions for the three regions.

Region 1:

$$B_{x1}(x,y,s) = \frac{\mu_0 K_z e^{jkx} e^{ky} kh}{s} \left\{ \frac{ph \cosh(ph) + kh \sinh(ph)}{[(ph)^2 + (kh)^2] \sinh(ph) + 2(ph)(kh) \cosh(ph)} \right\}$$

$$B_{y1}(x,y,s) = \frac{\mu_0 K_z e^{jkx} e^{ky} kh}{s} \left\{ \frac{ph \cosh(ph) + kh \sinh(ph)}{[(ph)^2 + (kh)^2] \sinh(ph) + 2(ph)(kh) \cosh(ph)} \right\}$$

Region 2:

$$B_{x2}(x,y,s) = \frac{\mu_0 K_z e^{jkx} ph}{s} \left\{ \frac{-ph \sinh(p(h-y)) - kh \cosh(p(h-y))}{[(ph)^2 + (kh)^2] \sinh(ph) + 2(ph)(kh) \cosh(ph)} \right\}$$

$$B_{y2}(x,y,s) = -\frac{\mu_0 K_z e^{jkx} jkh}{s} \left\{ \frac{ph \cosh(p(h-y)) + kh \sinh(p(h-y))}{[(ph)^2 + (kh)^2] \sinh(ph) + 2(ph)(kh) \cosh(ph)} \right\}$$

Region 3:

$$B_{x3}(x,y,s) = -\frac{\mu_0 K_z e^{jkx} (kh) e^{k(h-y)}}{s} \left\{ \frac{(ph)}{[(ph)^2 + (kh)^2] \sinh(ph) + 2(ph)(kh) \cosh(ph)} \right\}$$

$$B_{y3}(x,y,s) = -j \frac{\mu_0 K_z e^{jkx} (kh) e^{k(h-y)}}{s} \left\{ \frac{(ph)}{[(ph)^2 + (kh)^2] \sinh(ph) + 2(ph)(kh) \cosh(ph)} \right\}$$

The current density in region 2 is given by:

$$J_{z2} = \frac{K_z h}{s} (k^2 - p^2) e^{jkx} \left[\frac{ph \cos(p(h-y)) + kh \sin(p(h-y))}{[(ph)^2 + (kh)^2] \sinh(ph) + 2(ph)(kh) \cosh(ph)} \right] \quad (2)$$

Having obtained the expressions for the various electromagnetic quantities in terms of the transform, the next step is to obtain the inversion and express the quantities in the time domain.

The Inversion Integral

The expressions for the field quantities can be converted into the time domain using the inversion integral:

$$f(t) = \frac{1}{2\pi j} \int_{\alpha-j\infty}^{\alpha+j\infty} e^{st} F(s) ds$$

Here the path of integration is the Bromwich path and is such that α is greater than the largest real part of the poles of the integrand. In order to evaluate the integral, it is necessary to determine the poles of the transforms of the field quantities.

All the expressions have poles corresponding to $s=0$. Evaluation of the residues at this pole yields an expression which corresponds to the steady-state response to a step excitation. The other set of poles correspond to the zeros of the expression:

$$[(ph)^2 + (kh)^2] \sinh(ph) + 2(ph)(kh) \cosh(ph) \quad (3)$$

which is the solution to the equation:

$$\tanh(z) = -\frac{2(z)(kh)}{(z)^2 + (kh)^2} \quad (4)$$

$$z = \lambda + j\mu = ph$$

It can be shown that the roots of equation (4) are purely imaginary ($\lambda=0$). Equation (3) then reduces to:

$$\tan(\mu) = \frac{2\mu(kh)}{\mu^2 - (kh)^2} \quad (5)$$

where

μ is real in the above equation

Figure 3 shows the two functions, i.e. $y_1 = \tan(\mu)$ and the function $y_2 = 2\mu(kh) / (\mu^2 - (kh)^2)$. The latter has been plotted for various values of k , corresponding to the different harmonics of the armature or different radii of the armature. The intersection of these curves yields the roots of equation (5). Since the tangent is a periodic function with a period of π , equation (5) has infinitely many roots. For large values of μ , y_2 tends to zero and therefore for large μ equation (5) reduces to $\tan(\mu)=0$ with roots approaching $n\pi$.

Once the poles of the integrand have been determined, the solution can be expressed as the infinite sum of a series.

$$f(t) = \text{res}_{s=0}(F(s)) + \sum_{n=1}^{\infty} e^{s_n t} \text{res}_{s=s_n}(F(s)) \quad (6)$$

Here 'res' implies the residue of the argument at the corresponding pole and s_n are the poles obtained from the solution of equation (5) and are given by:

$$s_n = \frac{(z_n / h)^2 - k^2 - jv\mu_0\sigma k}{\mu_0\sigma}$$

where

z_n = zeros of equation (5)

This inversion integral may be applied to each of the electromagnetic quantities in order to obtain an expression in the time domain. This procedure is fairly straightforward and amenable for evaluation. The expressions however are not being rewritten here in the time domain as they would not serve any purpose.

STEP RESPONSE

CASE 1: Stationary Shield ($v=0$)

In order to illustrate the transient diffusion phenomenon associated with the shield, consider a specific example. For this example let the shield be made of aluminum with an average radius of 20.53 cm and a thickness of 12.7 mm. The conductivity is assumed to be 40% IACS. Also the excitation is assumed to be in the form of a two pole armature winding.

The eddy current distribution in the shield in response to a step excitation is shown in figure 4. The time axis is normalized with respect to the time constant $\tau_2=1/s_2$. From equation (2) it is seen that the steady state term vanishes because when $v = 0$, $k^2 - p^2 = -\mu_0\sigma s$, and therefore the pole at $s = 0$ is removed. Thus only the transient part of the solution needs to be considered. This is to be expected, because with the shield stationary there can be no steady state currents under a step excitation. Solution of equation (5) yields an infinite number of zeros. The first five of these are listed in table 1. Equation (5) also has a root at $\mu=0$; however, the residues corresponding to this root are all zero and therefore of no consequence. Table 1 also lists the time constants corresponding to the roots of equation

(5). These time constants are of importance since they indicate the manner in which the eddy currents in the shield decay (equation (6)) after the excitation is switched on.

To each of these zeros, there corresponds an eddy current distribution mode over the thickness of the shield. The first two of these modes are shown in figures 5 and 6. The total response is, of course, the superposition of all these modes. Consider the first mode, which indicates an almost uniform distribution over the entire thickness. Assuming that this distribution corresponds to a two-pole winding, the inductance and resistance of this winding may be calculated and a time constant obtained.

Inductance:

$$L = \frac{2\mu_0}{\pi} \left[\frac{\sin(\pi/2)}{\pi/2} \right]^2 = 324.2 \text{ nH}$$

Resistance:

$$R = \frac{2.0}{\sigma(\pi 0.205) 0.0127} = 10.54 \text{ } \mu\Omega$$

Time Constant:

$$\tau = L/R = 32.4 \text{ ms}$$

This time constant calculated from circuit considerations agrees well with the time constant calculated using the field equations. Similarly it can be shown that the other time constants in table 1 corresponding to the higher distribution modes can be calculated using circuit parameters.

Transient Shielding Effectiveness

The eddy currents generated in the shield delay the establishment of a magnetic field on the side of the shield away from the excitation. This effect is used to protect material which is sensitive to high rates of change in the magnetic fields. An example is the superconducting material which forms the field coil in many air core generators. Figure 7 illustrates the shielding effectiveness for the example machine. Shield thicknesses of 12.7 and 8.5 mm have been considered. The ordinate in this figure is the transient magnetic field strength normalized with respect to the field strength in the absence of the shield.

For the first 1 ms the field strength is insignificant. This time depends on the second mode (fig. 6) of the eddy current distribution. Then the field slowly rises to its "unshielded" value in a time of about 0.1 s. This second time depends on the first mode (fig. 5) of the eddy current distribution. Considering that the excitation in this case was a step function, the presence of the shield has considerably reduced the rate of change of magnetic field.

CASE 2: Shield Moving with a Velocity - 'v'

When $v \neq 0$, steady state eddy currents are present in the shield. The steady state eddy current distribution in a shell rotating in a heteropolar field is discussed at length in various references [2,3]. Here the discussion will be focused on the transient eddy currents. Figure 8 illustrates the eddy current distribution in the shield, in response to a step excitation, when it is moving with a constant velocity. The time axis is normalized with respect to the second time constant in table 1. The distribution is very similar to case 1 until $t = \tau_2$. Thereafter, it is seen that the distribution is considerably different from case 1, which has an almost uniform current density throughout the cross section, whereas case 2 has a sharply graded distribution. The graded distribution is a typical response obtained with a steady state ac excitation.

Another difference that is not directly apparent by comparison of figures 4 and 8 is that the rotating shield attains steady state earlier than the stationary shield. In the case where the shield is stationary, the current after uniformly distributing itself across the cross section of the shield, decays to zero with a decay time constant equal to the first time constant in table 1. When the shield is moving at a considerably high velocity (as it is in the present example) the first mode of current distribution plays an insignificant role in determining the overall current distribution. This is because the first mode of current distribution is related to the flux that passes through the shield, which is insignificant in the present case. This is more clearly illustrated in figure 9, which shows the shielding effectiveness when the shield is in motion. The ordinate is the ratio of the field strength with and without the shield. For the shield thickness (12.7 mm) and the velocity (510 m/s) considered, the field strength on the shielded side is negligible.

SINUSOIDAL EXCITATION WITH RELATIVE MOTION

The previous section discussed the response to a step excitation in the armature current. That helped to understand the underlying diffusion process and the associated time constants. The response which is of interest in most cases is when sinusoidal currents flow in the armature winding. The solution can be obtained by convolution of the armature current waveform with equation (6), i.e. the step response. During the pulsed duty that the compulsator is mostly used, the armature current waveform can be expressed as:

$$i_a(t) = \frac{V_m}{Z} \left\{ e^{-t/\tau_c} \sin(\varphi - \phi) - \sin(\omega_e t + \varphi - \phi) \right\} \quad (7)$$

where

- V_m = peak generator voltage
- Z = circuit impedance
- ω_e = electrical frequency of the machine
- φ = initiation phase angle with respect to zero voltage
- ϕ = circuit phase angle
- τ_c = circuit time constant

The response to this excitation can be written as:

$$F(t) = \frac{1}{\beta} \int_0^t f(t - \tau) \frac{di_a(\tau)}{d\tau} d\tau \quad (8)$$

In this expression, $f(t)$ is obtained from equation (6) and β is a constant which depends on the machine connection scheme and the width of the armature conductors. For example, with a lap wound machine, β is the product of the number of poles and the conductor width.

The transient current has two components, one is the dc offset and the second is the steady state sinusoidal term. The dc offset appears to the shield as a step. The eddy currents corresponding to the dc offset (for times much less than the circuit time constant) can be obtained as in case 2 above. Using the revolving field theory, a stationary oscillating field can be resolved into two fields revolving in opposite directions with a magnitude which is half the peak value of the stationary oscillating field. With this in mind, the steady state sinusoidal term appears to the shield as two further components. One is a field which is

stationary with respect to shield and therefore the response to this field can be obtained from case 1. The other component is a field which is spinning in a direction opposite to the shield and at the same speed. The response to this can be obtained from case 2 with the velocity term increased to $2v$. Thus for currents having the same frequency as the machine electrical frequency, the solution can be obtained directly by superposition of results from the step response. Hence either equation (8) or the superposition of the various step responses may be used to obtain the solution to a sinusoidal excitation. In what follows, equation (8) has been utilized.

Figure 10 shows the variation of the field strength on the shielded side for sinusoidal excitation. Various shield velocities have been considered for a shield 12.7 mm thick at a radius of 20.5 cm. The value is normalized with respect to the amplitude of the first harmonic of the field strength in the absence of the shield. The component of the field produced by the excitation which is stationary with respect to the shield will penetrate it with a time constant corresponding to the first mode of current distribution. In that respect the curve corresponding to 510 m/s in figure 10 is similar to the one in figure 7. The main difference is that the value approaches 0.5, since the component of the field stationary with respect to the shield has half the amplitude of the total field. The other component of the field which is spinning in the opposite direction to the shield will penetrate the shield to a greater or lesser extent depending on the frequency of the machine. The curves for 100 and 10 m/s in figure 10 clearly show this double frequency component superimposed on the stationary component.

Figure 11 shows a typical current pulse that is obtained from a compulsator during a pulsed discharge. In order to visualize the diffusion process during this pulse, two-dimensional contour maps are used. Figure 12 shows the contours of constant magnetic vector potential (flux lines) at various instants of time. Three regions are shown: 1) excitation side of the shield, 2) the shield, and 3) the shielded side. Figure 13 shows the lines of constant current density in the shield.

Figure 14 shows the diffusion process for a three-phase machine under a symmetrical, three phase, short-circuit condition. The machine has the same dimensions and parameters as the single phase machine of figures 12 and 13. Since the view is fixed with respect to the stator the field is seen to travel from left to right. After a sufficient time (50 ms) the field is seen to penetrate the shield completely.

PRACTICAL APPLICATIONS

Under a project funded by the U.S. Army AMCCOM, Picatinny Arsenal, New Jersey, an air core, passively-compensated compulsator is being developed at CEM-UT. This machine has a rotating armature and a stationary aluminum shield for compensation. It has a distributed field coil which surrounds the shield. Figure 11 shows the full performance current pulse of this machine and table 2 indicates some of the parameters of this machine. The shield is made of 40% IACS aluminum and is 8.6 mm thick. At the rated angular speed, the tip speed of the armature winding is 500 m/s.

During the design of this machine the model developed in the paper was applied for various purposes such as: i) determine the current transient in the field coil during the discharge pulse, ii) determine the temperature distribution in the shield after a discharge, and iii) determine the optimal charging rate of the field coil in a self excitation mode.

Field Current Transient

As discussed in the previous sections, there is a component of the field produced by the armature during a discharge which is stationary with respect to the shield. This component of the field will penetrate the shield and, being time variant, it induces a current in the field coil. The magnitude of this transient current must be known in order to protect the components in the field-coil circuit. The thickness of the shield governs the magnitude of the transient.

Figure 15 shows the magnitude of the field-current transient for various shield thicknesses. Also shown in this figure is the current through the armature windings during a pulse. The transient current is quite sensitive to the shield thickness. Decreasing the shield thickness from 12.7 to 3.18 mm (i.e., by a factor of four) increases the peak transient current in the field coil by almost an order of magnitude.

Temperature Distribution in the Shield

The eddy currents in the shield during a pulse result in heat dissipation. The pulse is of a sufficiently short duration that the heating can be considered adiabatic since there is very little time for the heat to diffuse. After a single pulse, the temperature is normally not of much significance. However most of these applications require a number of pulses at high repetition rates. It is therefore necessary to determine the temperature

distribution in order to ensure that the deflections and stresses are within acceptable levels. This machine is particularly sensitive to deflections since the gap between the armature and the shield is typically quite small to achieve good compensation. Figure 16 shows the temperature distribution in the shield of the machine described above.

Determining the Optimal Charging Rate for the Field Coil

This air-core machine uses a room temperature field coil. Also, it is self-excited with the help of full wave rectifiers. The reason is that the application requires it to be portable, therefore carrying cryogenics and a cooling system would be undesirable. Also, a room temperature field coil would require a large excitation unit if it is separately excited, thus offsetting the benefits of an air-core machine to some extent. In order to increase the field coil charging efficiency, it is necessary to accomplish the charging in as short a time as possible.

When the charge time becomes small and approaches the value of the first time constant in table 1, then the eddy currents in the shield begin to interfere with the charging process. The result is that the eddy currents in the shield delay the establishing of a magnetic field in the armature conductor region. The field current then must be held constant at the peak value in order to allow the eddy currents to decay in the shield before making a discharge. This also lowers the charging efficiency. Thus an optimal charging rate which maximizes efficiency can be found.

COMPARISON OF EDDY CURRENT DISTRIBUTIONS IN AIR CORE AND IRON CORE MACHINES

In order to make a comparison, three cases will be considered: an all-iron machine, an all-air machine, and a hybrid machine where there are iron laminations backing the armature windings but the rotor is not made of iron. The first case has been discussed at length in a prior publication [1]. The second case is discussed in this paper. The third one has not been treated to any depth; however, it is of practical value since it would apply in those cases where iron is used as a shield in air-core machines. The third case can be treated in a manner similar to the all air machine.

The solution of all these cases is split into two parts: one is the steady state and the second is the transient. The steady-state solution is obtained from the residue of the pole at $s=0$. The transient part is obtained by determining the roots of a characteristic equation. It

can be shown that these roots are purely imaginary and the total solution can be obtained by evaluating the residues at the poles corresponding to these roots. The three characteristic equations are as follows.

$$\begin{aligned}
 \tan(\mu) &= \frac{2G\mu}{(\mu^2 - G^2)}, & \text{all - air machine} \\
 \tan(\mu) &= \frac{G}{\mu}, & \text{hybrid machine} \\
 \tan(\mu) &= 0, & \text{all - iron machine} \\
 G &= \left(\frac{N_p}{2}\right)\frac{h}{r}, & \text{a constant}
 \end{aligned} \tag{9}$$

where

- N_p = the number of poles of the machine
- h = the thickness of the shield
- r = the radius of the shield

The significance of these characteristic equations is better understood by considering how they relate to the time constant of each mode of current distribution. The time constant of mode n is given by:

$$\tau_n = \frac{\mu_0 \sigma}{\left(\frac{\mu_n}{h}\right)^2 + k^2}$$

Here the subscript n denotes the mode number of the current distribution. Figure 17 shows the tangent function and the function on the right hand side of equation (9). This curve helps to locate the roots of equation (9). For the all iron machine the first root is at zero, for the hybrid machine it is higher in value, and finally for the all air machine it is the highest. Since μ_n is in the denominator, this implies that the time constant diminishes from the all iron to the all air configuration. This is to be expected since for the same conductor geometry the inductance diminishes.

Figure 17 also illustrates the fact that the three curves merge as μ becomes larger. This implies that for the higher modes of current distribution, the time constants approach the same value. Table 3 gives a comparison of the time constants for the three cases. The difference in the time constant of the second mode is less than 2.5%. The behavior of these higher modes is the same in all cases and the major difference is seen in the first mode

only. This can also be deduced directly by considering the second and higher modes of current distribution. The magnetic fields external to the shield produced by these modes is negligible and are therefore unaffected by the boundary conditions imposed external to the shield.

The implication of this on the eddy current density distribution is as follows: for pulse widths that are short compared to the first mode time constant, there will not be any appreciable difference in the current density distribution in the shield for the three cases. The pulse width for the present application is $1/20^{\text{th}}$ the time constant of the first mode, therefore the above criterion is definitely valid.

CONCLUSIONS

An analytical model for the diffusion process in the shield of a compulsator has been presented. A general method of solving the diffusion and the convective diffusion equations for these types of machines has been presented. This method is applicable for air core, iron core, and hybrid machines. The theory developed is applicable to both single phase and multiphase machines. The diffusion process in both these types of machines has been illustrated.

When considering the step response, time constants of the various modes of current distribution were obtained. A qualitative argument on transient current distribution can be based on these modes and the relative magnitude of their time constants. The main difference in the three types of machines is in the time constant of the first mode. Therefore one would not observe a significant difference in the transient eddy current distribution when the pulse width considered is very short with respect to the first time constant. The differences become prominent as the pulse widths approach the time constant of the first mode.

The application of the model to various aspects of the compulsator has been illustrated. These include calculation of the transient current in the field coil and the temperature distribution during a discharge. Also, a means for optimizing the field coil charging in a self-excitation scheme was indicated.

The model could also be used for other purposes not illustrated here. These include calculating the effective resistance of the shield during a discharge and calculating the mechanical load distribution in the shield under various short circuit situations.

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- Figure 15. Variation in the field current transient with shield thickness
- Figure 16. Temperature distribution over one pole of the shield after a pulse (numbers indicate increase in temperature in °C over one pulse)
- Figure 17. Location of the roots of the characteristic equation for different types of machines

Table 1. Time constants

ZEROS	TIME CONSTANTS (ms)
0.35	37.200
3.18	0.465
6.32	0.118
9.44	0.053
12.60	0.030

Table 2. Air core machine parameters

<i>Parameters</i>	<i>Units</i>	<i>Value</i>
Peak open circuit voltage	kV	2.25
Peak current (pulsed)	kA	390
Pulse width	ms	1.9
Rotor speed	rpm	25,000
Electrical frequency	Hz	416
Number of poles	-	2
Overall length	m	0.76
Overall diameter	m	0.67

Table 3. Comparison of the roots and associated time constants for the three types of machines

Mode Number n	All Air Machine		Hybrid Machine		All Iron Machine	
	μ_n	τ_n	μ_n	τ_n	μ_n	τ_n
1.0000	0.35000	0.037221	0.24620	0.072966	0.0000	1.2284
2.0000	3.1805	0.00046466	3.1612	0.00047035	3.1416	0.00047624
3.0000	6.3028	0.00011835	6.2930	0.00011872	6.2832	0.00011909
4.0000	9.4379	5.2787×10^{-5}	9.4313	5.2860×10^{-5}	9.4248	5.2933×10^{-5}
5.0000	12.576	2.9729×10^{-5}	12.571	2.9752×10^{-5}	12.566	2.9776×10^{-5}
6.0000	15.716	1.9038×10^{-5}	15.712	1.9047×10^{-5}	15.708	1.9057×10^{-5}
7.0000	18.856	1.3225×10^{-5}	18.853	1.3229×10^{-5}	18.850	1.3234×10^{-5}
8.0000	21.997	9.7178×10^{-6}	21.994	9.7203×10^{-6}	21.991	9.7229×10^{-6}
9.0000	25.138	7.4411×10^{-6}	25.135	7.4426×10^{-6}	25.133	7.4441×10^{-6}
10.0000	28.279	5.8799×10^{-6}	28.277	5.8808×10^{-6}	28.274	5.8818×10^{-6}

1001.0526

Figure 1. Typical air core compensated pulsed alternator

1001.0527

Figure 2. Definition of the coordinate system

Figure 3. Determination of the roots of the characteristic equation

1001.0529

Figure 4. Current density distribution with step excitation and a stationary shield

1001.0530

Figure 5. First mode of current distribution

1001.0531

Figure 6. Second mode of current distribution

1001.0532

Figure 7. Transient shielding effectiveness - stationary shield

1001.0533

Figure 8. Current density distribution with step excitation and shield motion

1001.0534

Figure 9. Transient shielding effectiveness - rotating shield

1001.0535

Figure 10. Transient shielding effectiveness with sinusoidal excitation - rotating shield

1001.0536

Figure 11. Typical current pulse from the pulsed alternator

Figure 12. Diffusion of the magnetic field over one pulse, lines of constant magnetic vector potential

Figure 13. Current density distribution over one pulse, lines of constant current density

Figure 14. Diffusion of the magnetic field in the shield of a synchronous generator under three phase short circuit conditions

1001.0537

Figure 15. Variation in the field current transient with shield thickness

Figure 16. Temperature distribution over one pole of the shield after a pulse
(numbers indicate increase in temperature in $^{\circ}\text{C}$ over one pulse)

1001.0538

Figure 17. Location of the roots of the characteristic equation for different types of machines

BIOGRAPHY

Siddharth B. Pratap (M'88) was born in Bombay, India in 1955. He received his Bachelor of Electrical Engineering degree from the University of Bombay in 1979 and his Master of Science in Engineering from The University of Texas at Austin in 1982.

From 1979 to 1981, he worked at the Tata Engineering and Locomotive Company in Pune, India as an Engineer in the Central Plant Engineering Department. Currently, he works at the Center for Electromechanics at The University of Texas at Austin as a Special Research Associate. His area of research is pulsed power systems and electrical machinery. He has been with CEM-UT since 1982 and has worked on various aspects such as design, analysis, and testing of pulsed power systems.

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