AN INVESTIGATION INTO THE POTENTIAL FOR
MULTIPLE RAIL RAILGUNS

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AN INVESTIGATION INTO THE POTENTIAL FOR MULTIPLE RAIL RAILGUNS

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Abstract - A simple numerical technique for determining current distribution and inductance in parallel conductor geometries is outlined. Interim results of an ongoing study which applies this technique to analyze and compare two and multi-rail railgun geometries are presented. Interim conclusions are made.

Introduction

As railgun technology progresses toward higher energy devices with the capability of surviving many shots, motivation for the use of guns with more than two rails increases. The primary benefit of such a gun is distribution of the outward forces exerted on the rails over more than the single plane found in a two-rail railgun. While this benefit is extremely significant, the capabilities and limitations of a multi-rail railgun must be compared to that of the two-rail railgun to determine its ultimate usefulness. This paper presents some results of an ongoing study to investigate the potential of a multi-rail railgun. (The term "multi-rail railgun" is not intended to include what is usually termed an "augmented" gun. See figure 1 for a typical geometry.)

A study of multi-rail alternatives necessarily involves investigating an assortment of railguns with different numbers of rails, and different rail cross sectional shapes. The only candidate geometries for multi-rail railguns investigated to date incorporate symmetrical bore cross sections with an even number of rails. Adjacent rails carry equal and opposite currents. Additionally, studies to date have only investigated the capabilities of accelerators isolated from power supply and projectile considerations. It has been temporarily assumed that a power supply exists which can drive the accelerators with any current desired. Under these conditions, the multi-rail candidates were compared to comparably sized two rail counterparts. The railguns were compared on their ability to produce projectile force (where $F = 1/2 L'i^2$ in all cases), while current density was restricted to a specified peak value (arbitrarily set to 1MA per inch). This approach attempted to compare accelerator capabilities under conditions in which accelerator damage would be comparable.

Studies have been done assuming uniform current densities along inner rail surfaces to an appropriate skin depth, and assuming infinite conductivity rails with non-uniform surface current on all rail surfaces. Since the latter case more closely approximates the pulsed railgun, and the calculated inductances per unit length (L') more closely approximate those observed during actual two-rail railgun firings, only these results will be presented.

Computational Method

Experimental and modeling work done at the Center for Electromechanics at The University of Texas at Austin (CEM-UT) has shown that the measured inductance per unit length which most correctly predicts projectile force, is that which is measured using a high frequency source. Additionally, of the various methods used for calculating the

inductance per unit length, the one which most correctly predicts projectile force is that which is based on the current distribution in the middle of long perfectly conducting rails. Due to the above considerations, determination of the force producing capability of an accelerator involves determining the current distribution and inductance per unit length of long parallel conductors remote from the projectile. As discussed by Kerrisk [1], if one assumes infinite rail conductivity, calculating the current distribution is equivalent to finding the electrostatic charge distribution on long parallel conductors when the electric potential, V , for each conductor is known. For a two rail railgun, the "known" rail potentials can be of any magnitude as long as the two potentials are equal and of opposite sign. By electrostatic theory, the surface charge density obeys the following integral equation:

$$V = \int \frac{\sigma(s')}{r-r'} da' \quad (1)$$

In this equation, $\sigma(s)$ is the surface charge density expressed as a function of arc length along the surface of the conductor (see figure 1 for an example), and physical constants have been absorbed into V for algebraic simplicity. The primes denote charge coordinates, and the integration is over the surface of the rails (where the electrostatic charge is located).

The above integral equation can be solved for $\sigma(s)$ by the method of moments. The solution procedure used below is equivalent to using Dirac delta functions as testing functions and unit pulse functions as subsectional basis functions in the method of moments. The advantage to this procedure is the direct and simple physical interpretations of the equations and integrals involved. A more detailed discussion of the method of moments is provided by Harrington [2].

SQUARE BORE GEOMETRY

Figure 1: Square bore railgun. Dark lines show the two-rail railgun, defining the rail height (h), rail thickness (t), bore dimension (b), and arc length along the rail perimeter (s). The arc length origin is at inside center of the right rail. Dashed lines show additional rails for the square bore four-rail railgun.

Using a two-rail railgun as an example, each rail is divided in half, and the perimeter of each half-rail is divided into N sections, Δs (figure 1). Each section represents a strip of charge, which extends into the paper one unit length. Now, functions g_n are defined as 1 on Δs_n , and 0 on all other Δs_m . The charge density can then be represented as

$$\sigma(s) \approx \sum_{n=1}^N a_n g_n(s) \quad (2)$$

where the a_n are unknowns to be determined. Substituting this expression into the integral equation (1) yields the following equation for the potential at the midpoint of each Δs_m :

$$V = \sum_{n=1}^N b_{mn} a_n \quad (3)$$

where

$$b_{mn} = \int \frac{1}{r_{m-r}} da_n' \quad (4)$$

In the above expressions, all subscripted positions refer to the charge segment midpoint. Since V is a known constant, equation (3) represents a set of linear equations which can be solved for the a_n coefficients, once the b_{mn} are known. The a_n then represent the magnitude of the charge (current) density over strip Δs_n .

The equation for b_{mn} shows that they have a simple physical interpretation. Each b_{mn} is the potential at the center of strip Δs_m due to a uniform unit charge density over strip Δs_n . These integrations are somewhat involved if done analytically. While they can easily be done numerically, considerable simplification results with very little loss in accuracy, by using expressions for the potential at the midpoint of Δs_m due to a unit length line charge of appropriate magnitude located at the midpoint of Δs_n . These formulas are not difficult to derive and eliminate the need for numerical integrations. The equations, for $m \neq n$, are of the form

$$b_{mn} = \Delta s_n \ln \left(\frac{\frac{z}{2} + \sqrt{d^2 + \left(\frac{z}{2}\right)^2}}{-\frac{z}{2} + \sqrt{d^2 + \left(\frac{z}{2}\right)^2}} \right) \quad (5)$$

where z is the length of the conductors (unit length), and d is the distance between the midpoints of sections m and n .

For $m=n$, b_{mm} represents the potential at the surface of a strip of charge which is Δs wide. This can be integrated analytically to yield

$$b_{mm} = 2 \left\{ \Delta s \sinh^{-1} \left(\frac{z}{\Delta s} \right) + z \sinh^{-1} \left(\frac{\Delta s}{z} \right) \right\} \quad (6)$$

where z is again the length of the conductors (unit length).

Due to symmetry, it is only necessary to solve for the a_n coefficients in the first quadrant. Each b_{mn} in the system of equations, then has four contributions (one from each quadrant) of the form shown in equations (4), (5) or (6).

A FORTRAN program, based on the above approach, has been developed which calculates surface current

density, and then uses the results to find inductance per unit length. Results, which are obtained for the two-rail railgun, agree with other previously published results found by different methods. (See [1] and [3]). Numerous trial runs have shown the method to converge quickly as the number of sections is increased until answers change very little with further increases in the number of sections. For rectangular rail cross sections with corners which have a 1 mm radius rounded edge, 250 to 300 sections provide adequate accuracy for bore cross sections as large as 4 square cm. Under these conditions, a 15% increase in the number of sections produces less than a 0.1% change in final answers for total current or inductance. Trial runs have also shown that convergence is slower for geometries with sharper corners - as can be expected by the much higher localized peak densities at such corners.

Two and Four Rail Results

Calculations have been done for two and four-rail railgun geometries. For the results presented, all corners have a 1 mm radius unless otherwise mentioned. All current plots are normalized so that a density of 1 is assigned to the minimum density region. Total currents for use in force calculations all were made with the point of maximum density being assigned a density of 1MA per inch of rail surface and other regions being scaled in accordance with their relative density. This was done so that comparisons between various railguns can be based on similar operating conditions.

First, the square bore results are presented. Figure 2 shows the current density of the top half of the right rail of a two-rail square bore gun. The position scale (in cm) has its origin at the center of the inside face of the rail, and proceeds around the top perimeter of the rail, ending at the center of the outside face of the rail. This particular plot is for a gun with a 4 cm square bore, in which the rail height is only 3.2 cm - so the rail does not completely span the bore. The current density peaks occur at the corners of the rail, with the highest peak on the inside corner. The peak current density (position ≈ 1.5) is approximately 2.5 times as large as the density on the center of the inside of the rails (position $=0$), and more than 6 times the density on the backside of the rails (position ≈ 4.2). Figures 3 and 4 show similar plots for 4 cm, square bore, four-rail, railguns. The rail cross sectional geometries for the square four-rail gun are identical with the two-rail guns, except for the varied rail height. Figure 3, for a four-rail railgun with a rail height of 1.2 cm, displays ratios of current density peaks to valleys which are not too different from the two-rail case. This is contrasted with figure 4, in which the rail height is 3.2 cm so the corners of two adjacent rails are considerably closer together. Here, the effect of neighboring currents (the proximity effect) is very pronounced, as can be seen by the relative magnitude of the current density peak at the inside corner of the rail (position ≈ 1.5 cm in figure 4).

The inductance gradient (L') of the square bore four-rail railgun varies with rail height, as depicted in figure 5. All calculations were for a 4 cm bore, so increased rail height means that a larger percentage of the bore surface is covered with rail. While it would be possible to have a rail height of up to 4 cm, the plot only extends to approximately 3.25 cm.

Figure 2: Square bore, 2-rail railgun. Rail height = 3.2 cm. Rail thickness = 1 cm. Bore = 4 cm. Corner rad.=1 mm.

Figure 3: Square bore, 4-rail railgun. Rail height = 1.2 cm. Rail thickness = 1 cm. Bore = 4 cm. Corner rad. = 1 mm.

Figure 4: Square bore, 4-rail railgun. Rail height = 3.2 cm. Rail thickness = 1 cm. Bore = 4 cm. Corner rad.=1mm.

Figure 5: Square bore, 4-rail railgun. Bore = 4 cm. Rail Thickness = 1 cm. Corner rad = 1 mm.

The effect of the variation of current density and inductance with rail height is manifested in the railgun total current and projectile force, as represented in figure 6. For these calculations, the peak values of the current densities (such as occurred at the inside corners in figures 4 and 5) were set at 1 MA per inch of rail surface. Each dot on the plots represent the current or force (in mega-units) for a square bore four-rail railgun. As the rail height of a railgun is increased, the current carrying surface of the rails increases, and the total current initially increases. When the rail height reaches approximately 65% of the bore, the proximity effect becomes dominant. Now, the result of fixing the peak current density is to actually reduce the total current as the current carrying surface area increases. Increasing the current, at this point, would require accepting higher peak densities and, therefore, higher rail damage. The effect of the falling inductance as rail height is increased, is to cause the projectile force to reach its maximum value with a rail height of only 1.5 cm, even though the current has not yet peaked. This maximum force on the projectile is only approximately 60% of the force developed by a 4 cm two-rail railgun with 4 cm rails (i.e., the rails completely cover one face of the bore, as is frequently done). The peak current densities for the two-rail railgun were fixed at the same levels as for the four-rail railgun for this comparison.

Figure 6: Square bore, 4-rail railgun. Bore = 4 cm. Rail thickness = 1 cm. Corner rad = 1 mm.

As has been noted in previous sources [1], peak current densities can be considerably reduced by using a greater radius of curvature on the corners of the rails. Although inductance and current density plots are not provided, figure 7 shows the final current and force results for a four-rail railgun with a larger radius of curvature on the rail corners. Here, the rails have a thickness of 1 cm and a radius of curvature for the corners of .45 cm. This means that the top and bottom of the rail is very nearly a continuous semi-circle, with only a 1 mm flat area. The results are very similar to the 1 mm corner radius case, except that the projectile force is increased. The rail height at which the maximum force occurs is relatively unchanged by the corner radius. It should be noted, that the maximum force developed for this case is approximately 70% of the force for a two-rail railgun with 4 cm rails and corner radii of .45 cm. Again the two and four-rail railguns had the same peak current densities for this comparison.

Figure 7: Square bore, 4-rail railgun. Bore = 4 cm. Rail thickness = 1 cm. Corner rad = .45 cm.

The above results lead to the conclusion that an attempt to improve the distribution of the outward rail forces by using four rails exacts a high price. If a four-rail railgun is operated at the same peak current levels as the two rail counterpart, a 30 to 40% reduction in projectile force must be accepted. Operating at higher peak currents, means accepting higher rail damage. These factors lead to consideration of alternate cross sectional geometries, which increase current carrying surface area (and therefore total current) without leading to an aggravated proximity effect. The only such geometry which has been investigated at this time, involves rails with a triangular, or wedge shaped cross section (fig. 8), which creates a 'X' shaped bore.

For the wedge shaped rail calculations which are presented, the three corners of the triangle all have a radius of 1 mm. The 'X' bore railguns considered all have a bore area of 16 sq. cm. This means that as rail height is increased and each rail gets larger, the separations between rails decreases. This is an attempt to provide some basis of comparison with the two-rail, 4 cm, square bore railgun. (Otherwise, one simply learns that bigger guns can provide more force than smaller guns.)

Figure 8: Railgun with wedge or triangular shaped rails, defining rail height (h), bore dimension (b) and arc length along the rail perimeter (s).

Figure 9 depicts a typical current density plot of one rail in the four-triangular-rail railgun. The position origin is at the center of the inside surface of the rail, which puts it at the corner of the triangle nearest to the center of the bore. For this particular plot, the rail height is 7.5 cm, which makes the rail separation equal to 1.03 cm (b in figure 8). This provides a cross sectional area of 16 sq. cm. Interestingly, as the rail height is increased so that the rails get closer together, the ratio of the peak current density (on the inside corner of the triangle) to the current density along the remainder of the inside of the rail, does not change significantly. In other words, the general shape of the current density plot does not change with rail height. Therefore, as the rail height increases, the total current continues to climb (figure 11) -- in contrast to the four-rectangular-rail case previously considered.

Figure 9: X bore, 4-triangular-rail railgun. Rail height = 7.5 cm. Bore area = 16 sq. cm. Corner rad = 1 mm.

In figure 10, the inductance variation with rail height is presented (with the bore cross sectional area still fixed at 16 sq cm). Figure 11 shows the results of current and force calculations for several different rail heights. As the rail height increases, the force developed continues to increase. Eventually, the force exceeds that of the conventional two-rail railgun (with a 4 cm bore, 4 cm rails and corner radii of 1 mm). As a point of reference, the four-wedge-rail railgun with a rail height of approximately 6 cm produces equivalent force as the two-rail counterpart. The rail separation (b in figure 8) for this gun is 1.3 cm.

4 WEDGE RAIL INDUCTANCE GRADIENT

Figure 10: X bore, 4-triangular-rail railgun. Bore area = 16 sq cm. Corner rad = 1 mm.

4 WEDGE RAIL FORCE AND CURRENT

Figure 11: X bore, 4-triangular-rail railgun. Bore area = 16 sq cm. Corner rad = 1 mm.

Conclusions

The results presented indicate that a four-rail, square bore railgun has serious performance shortcomings (a 30 - 40% reduction in projectile force). Four triangular rails have a different drawback. Although figure 11 indicates that an 'X' bore four-rail railgun can produce more force than the two-rail counterpart (or it can produce the same force with lower current densities), the bore cross-sectional shape is not useful for all types of projectiles.

The key to the better force producing capabilities of the 'X' bore railgun, is that the rail current carrying surface area is increased in a manner which does not aggravate the proximity effect of currents in neighboring rails. This permits offsetting lower inductances with larger total currents, while not increasing local current densities. Rounding the rail corners on the bore interior (blunting the apex of the triangle) will provide for more usable area within the bore, while still providing for increased rail surface area. This, and other related rail shapes are the object of the present investigations aimed at finding a good compromise geometry for a four-rail railgun. The optimized four-rail results will then provide a good starting point for considering railguns with more than four rails.

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