FORMULATION AND IMPLEMENTATION OF THE CURRENT FILAMENT METHOD
FOR THE ANALYSIS OF CURRENT DIFFUSION AND HEATING IN
CONDUCTORS IN RAILGUNS AND HOMOPOLAR GENERATORS

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Formulation and Implementation of the Current Filament Method for the Analysis of Current Diffusion and Heating in Conductors in Railguns and Homopolar Generators

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Abstract: Electrical current conductors can be mathematically discretized into current filaments. Within each of the filaments a uniformly distributed current density can be assumed for ohmic loss considerations, and a spatial impulse of current can be assumed for magnetic induction considerations. The continuous diffusion problem can then be solved using a circuit approach. If the discretization is fine enough, a good approximate formulation for the continuous diffusion problem can be obtained. For current diffusion and heating problems, discretization of the current carrying conductors leads to a giant set of simultaneous linear ordinary differential equations. The problem can be solved numerically using a linear equation solver and a robust ordinary differential equation solver.

The method has been applied to solve current diffusion and heating problems in railguns and homopolar generators (HPGs). Results show that the method is a potential candidate for solving some transient electromagnetic (EM) problems and it can be programmed on the computer rather easily. This paper presents the formulation of the current filament model and shows how the method was applied to the diffusion problems in railguns and HPGs in the Center for Electromechanics at The University of Texas at Austin (CEM-UT). Some computational results are presented.

Introduction

In the field of electromechanics where electrical pulses are involved, the skin-effect and proximity-effect eddy currents often play a major role in affecting machine performance and behavior. The transient behavior of electric current and magnetic field is described by Maxwell's Equations. For low frequency problems, Maxwell's Equations can be reduced to diffusion equations. Depending on the problem itself, the diffusion equations describing the diffusion physics of electric current and magnetic field can be solved using finite element methods with judiciously specified boundary conditions. However, for some electromechanic problems, especially where a lot of the components in the system is modeled with lumped parameters, it may be easier to approach the problem from the circuit point of view.

For linear EM problems in which the direction of current can be assumed, solid conductors can be mathematically discretized into a large number of current filaments. In each of the current filaments, a current density distribution described by a spatial impulse of unknown magnitude can be assumed. The integral of the impulse over the cross-section of the filament is the current through the filament. Once the discretization is done, the field problem can be solved with the circuit approach. The mutual inductance between each pair of filament and the self-inductance of each filament can be calculated from the geometry. The ohmic resistance of each filament is determined by the cross-sectional area of the filament and the temperature-dependent resistivity of the material. The

circuit problem can be solved using numerical ordinary differential equation solvers such as Runge-Kutta or Gear methods. A Gear-type method is preferred because arbitrary discretization can cause some circuit branches to have distinctly different time scale than others, leading to a stiff differential system, which would be difficult to solve using single-step methods such as the Runge-Kutta method.

In this paper, the formulation of the method is first briefly introduced. Next, the results of a test case is presented and discussed. Then the application of the method to a self-excited HPG and a railgun design is discussed.

General Formulation

Suppose we have an EM system in which all components except a solid conductor are defined by lumped parameters. We want to solve for the current density distribution in the solid conductor under a certain excitation. If the direction of the current density in the solid conductor can be assumed, we can replace the solid conductor with a fine bundle of current filaments that go in the direction of current density. Physically, the electrical characteristics of a bundle of uninsulated current filaments should not differ from the solid conductor. If the direction of current in the solid conductor is in deed the same as that of the current filaments, we can imagine having infinitesimally thin insulators between walls of the current filaments, and the electrical characteristics of the current filaments would still be the same as that of the solid conductor.

After we replace the solid conductors with current filaments, we assume that each filament has a current density of a spatial impulse. If the cross-section of the current filaments is small, this approximation should be good for computing the inductive effects between filaments. To formulate the induction between each pair of current filaments, we can use some conveniently available handbook formulas such as those published in Grover's Inductance Calculations. The choice of which mutual inductance formulae to use depends on the problem's geometry itself. To compute the resistance of each branch of current filament, we take the cross-section of the whole filament, and compute the resistance as in usual DC resistance calculations. The self-inductance of each filament is also calculated using handbook formulas, assuming uniform current distribution across the cross-section of the filament.

After the discretization procedure described above, the problem becomes a circuit problem in which the solid conductor is now represented by a bunch of current filaments. We define the current in each circuit branch to be a state variable. Writing the circuit equations, we should obtain a set of first order differential equations in which the currents are to be solved. A typical set of equation would look like the matrix differential equation below:

$$\frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix} = \begin{bmatrix} L_{11} & M_{12} & M_{13} & \dots & M_{1n} \\ M_{21} & L_{22} & M_{23} & \dots & M_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & \dots & \dots & L_{nn} \end{bmatrix}^{-1} \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

mutual and self-inductance voltage terms

$$V = M_f(d, \ell) \frac{dx}{dt}$$

Its formula is given by, (Grover, p.31, Eq. (1))

$$M_f(d, \ell) = \frac{\mu}{2\pi} \ell \left[\ln \left(\frac{\ell}{d} + \sqrt{1 + \left(\frac{\ell}{d} \right)^2} \right) - \sqrt{1 + \left(\frac{d}{\ell} \right)^2} + \frac{d}{\ell} \right]$$

Figure 1. A single filament showing the direction of current and the induced voltage

FORMULATION OF FILAMENT METHOD TO COMPUTE CURRENT DIFFUSION PATTERNS OF A RAIL GUN

The application of the filament method to the two-dimensional current diffusion analysis of a railgun is explained below.

Consider a typical simple (as opposed to augmented) railgun. Because of symmetry, both the upper and lower rails can be partitioned in the same manner into a number of filaments of finite cross-sectional areas such that each filament in the upper rail has a corresponding one in the lower rail. Such pairs of filaments then form a collection of circuit loops, each of which can be modeled as a resistance, a self-inductance and a number of mutual inductances with every other loop, in series. While the resistance is easy to compute if we assume uniform current density, the inductance calculation requires some elaboration.

The formulas for calculating these loop inductances can be derived from two basic formulas given by Grover [1]. The first basic formula is for the self-inductance of a single filament of finite length and of circular cross section of finite radius. With reference to figure 1, the self-inductance $S_f(\rho, \ell)$ is defined to be such that

$$V = S_f(\rho, \ell) \frac{dx}{dt}$$

Its formula is given by, (Grover, p.35, Eq. (7))

$$S_f(\rho, \ell) = \frac{\mu}{2\pi} \ell \left[\ln \left(\frac{2\ell}{\rho} \right) - \frac{3}{4} \right]$$

The second basic formula is for the mutual inductance between two parallel filaments of finite length, but of zero cross-sectional area. With reference to figure 2, the mutual inductance $M_f(d, \ell)$ is defined to be such that

Figure 2. A pair of parallel filaments showing the direction of current and the induced voltage

To apply these two basic formulas to calculate the inductances in our loop model of a railgun, we need two assumptions. First we can regard the cross section of a filament to be a circle of the same area as the actual one. Second, when we calculate the mutual inductance between two distinct filaments, regardless whether they belong to the same or different loop(s), we can regard the filaments as having zero cross-sectional area. With these assumptions, and by the principle of superposition, we can repeatedly apply these two basic formulas to calculate the loop inductances.

With these circuit parameters determined, we have a circuit model of the railgun (excluding the armature) as shown in figure 3. By circuit analysis, we can write a system of differential equations to solve for the current in each loop. The more refined we make the model, the bigger the set of differential equations will be. A computer is required to efficiently solve the differential equations numerically.

Test Cases

To verify that the method and the computer implementation do produce useful solutions for the electric current and magnetic diffusion problems, two test cases for which the solution is well-known are investigated.

Figure 3. Equivalent circuit diagram of the loop model of railgun

A Fortran program based on the above formulation is written to solve for current diffusion problem in a rectangular bore railgun. Here, we consider two test cases in which the diffusion pattern is approximately known.

Consider the geometry of a hypothetical copper railgun in which the separation between the upper and lower rails is large compared with the linear dimension of each rail as shown in figure 4. Because of the large separation between the rails, the current diffusion pattern in each rail should be approximately the same as that in a single independent rail. We know that when a single independent rail is subjected to a sinusoidal current source of sufficiently high frequency, skin effect in the current distribution is apparent and the theoretical skin depth can be computed. We use this feature to check the result of the test cases.

Figure 4. Hypothetical railgun configuration used for test simulation

In our two test cases, we use two sinusoidal excitations of amplitude 1 A at different frequencies such that the associated skin depths differ by a factor of two. From our computer results, we obtain contour plots of current distributions across the rail. We observe that the current distribution pattern is symmetric with respect to each diagonal, confirming that each rail acts virtually as an isolated conductor. To see the skin effect,

we obtain, as shown in figures 5 and 6, current density plots as a function of time, at filaments #101 through #106, whose locations are shown in figure 4. We observe that at each filament, the current density profile is sinusoidal with the same frequency as that of the source. Furthermore, there are successive lags in phases and drops in magnitudes as we go from the surface to the center (from filament #101 to #106) of the rail. The diffusion of current is qualitatively depicted by these results.

To calculate the skin depths from the computer results of the two test cases, we plot the phasor magnitude of current density of filament #101 through #106. The plots, figures 7 and 8, show the magnitude of the current density as a function of depth into the conductor because filaments #101 through #106 are progressively deep into the conductor. The expression of current density as a function of depth into the conductor is obtained from the plots using least square curve fitting. The

Figure 5. Current filament test case current density plot #S01703

Figure 6. Current filament test case current density plot #S01704

computed skin depths can be obtained from the expressions of the fitted curves.

In case #S01703 (fig. 7), where the frequency of the source current is 108.9 kHz and the associated theoretical skin depth 0.02 cm, we obtain from the plot a computed skin depth of 0.0213 cm. In case #S01704 (fig. 8), where the corresponding values are 435.6 kHz and 0.01 cm, the computed skin depth is 0.0115 cm. The close agreement between the theoretical and computed skin depths suggests that our filament method and the computer implementation do give correct diffusion pattern.

Figure 7. Current density vs. depth into the conductor for 109 kHz case

Figure 8. Current density vs. depth into the conductor for 436 kHz case

In case #S01703 (fig. 7), where the frequency of the source current is 108.9 kHz and the associated theoretical skin depth 0.02 cm, we obtain from the plot a computed skin depth of 0.0213 cm. In case #S01704 (fig. 8), where the corresponding values are 435.6 kHz and 0.01 cm, the computed skin depth is 0.0115 cm. The close agreement between the theoretical and computed skin depths suggests that our filament method and the computer implementation do give correct diffusion pattern.

Railgun Current Diffusion and Heating at the Gun Breech

Knowledge of how the electric current is distributed over time in a railgun shot is of paramount importance. The distribution of current dictates the magnetic field distribution

which interacts with the current in the armature to propel the armature. It is also a source of ohmic heating in the railgun, one of whose limitations is melting of the rail portion through which very high current density prevails.

Now consider a rectangular railgun of realistic size, subjected to a gun current profile obtained from a railgun simulation. We apply the filament method to calculate the current in each filament. From this, together with other known properties of the material of the rail, we can compute the heat energy ohmically generated, and its associated temperature rise in each filament. The railgun cross-sectional geometry is shown on figure 9. This is typical of tactical railguns. The material is assumed to be high conductivity copper. The current profile is from a simulation simulating a tactical railgun powered by the 60 MJ Balcones HPGs at CEM-UT. The six HPG/inductor power supply modules are discharged at different times to produce a less sharp pulse as shown in figure 10. The computer program includes the change of resistivity and specific heat capacity of copper as temperature rises during the launch. The temperature distribution due to ohmic heating of the rail is computed and plotted in figure 11. The temperature contour plot in figure 11 also shows the distribution on the left half of the upper rail of a rectangular railgun. By symmetry, we know that the distribution on the right half is just the mirror image of that on the left half. We observe that temperature decreases monotonically from the corners towards the inside. Also, most of the temperature drop takes place at the corner and the temperature gradient is steeper at the lower corner than at the upper one. All these results are consistent with what is expected.

While this temperature distribution pattern is in general correct, the actual temperatures calculated at the corner are lower than expected. Probably the partition of the corner into filaments is not fine enough to handle the steep temperature gradient. One way to solve this problem is to use variable-size filaments so that more filaments are used to model the corner. But this may affect the accuracy of inductance calculation between filaments. Whether the new model of variable-size filaments is better than the present one needs further investigation.

Proximity Effect Eddy Current in a Self-Excited Air-Core Homopolar Generator

One of the limitations of traditional iron-core HPGs is the upper limit on the excitation field due to magnetic saturation. In a

Figure 9. Sketch of the railgun geometry used in our real case simulation

Figure 10. Railgun current profile for current filament analysis

Figure 11. Temperature contour plot across the left half of the top rail

typical iron-core HPG, the maximum excitation field is around 2 T. CEM-UT conceived an improved energy density (IED) HPG, as shown in figure 12, that stores magnetic energy in air, with magnetic fields as high as 35 T. The kinetic energy is stored in composite flywheels rotating at very high angular velocity. The high field is created by two opposing air-core field coils as shown in figure 13. The HPG is self-excited and the coils not only excite the field but also store magnetic energy and play the role of an inductor. The surface of the shaft is used for carrying the armature current to the output port. Resistive spacers are put in, as shown in figure 14, to prevent excessive circulating current induced by changing current in the field coils. The output current flows along the shaft mostly in-between high resistance space bars.

One of the designs CEM-UT had has an estimated output current profile as shown in figure 15. This shows only the armature current during charging of the opposing coils, and no load was connected to the HPG. The current peaks at about 600 ms after excitation and the peak current is around 3.2 MA. Depending on what the exact geometry is, the circulation current around the shaft induced by the field generated by the field coil may be significant. The current filament method was employed in a system simulation of the HPG to analyze the magnitude of circulation current around the shaft, which represents an energy loss and lowering of efficiency.

Figure 12. CEM-UT's improved-energy-density homopolar generator

In the analysis, the direction of circulation current is assumed to be around the shaft. The shaft is then modeled as a collection of small coaxial shorted rings that carry the circulation current. Each ring is modeled as a resistance, in series with a self-inductance, and a number of mutual inductances with each of the other rings. The formula for mutual inductance between two coaxial rings is shown in figure 16.

The computer program was developed such that one can easily specify how refined the discretization has to be. Initially, a Runge-Kutta method integrator was used to solve the system differential equations. Later, a Gear-type method was employed because it was found that some material properties and discretization geometry can create circuit branches with distinct time scales, making the differential equations stiff and difficult to solve using ordinary Runge-Kutta integrator. The Gear-type method integrator overcame the stiffness problem.

Parametric studies were performed using the computer program on various HPG designs. With current profiles similar to that on figure 14, the circulation current is generally less than 10 % of the output (transport) current. This means the ohmic loss due to circulation current is about 1% of that due to the transport

Figure 13. Schematic of the improved-energy-density homopolar generator

Figure 14. Squirrel-cage armature construction for improved-energy-density homopolar generator

Figure 15. Estimated piecewise linear profile of field current per-turn

current. Thus, the circulation current can be neglected as far as ohmic loss is concerned. The other concern was that if the circulation current is highly concentrated at some location, heating may be a problem. The computer results showed that it was not problematic.

Conclusion

The current filament method is a conceptually simple and yet useful method to analyze two-dimensional linear magnetic diffusion problems in which the direction of the current density can be assumed. The method is easy to program and can be easily incorporated into lump parameter system models on the

computer. However, some numerical stability problems do emerge occasionally for certain problems. Research is being done to overcome such difficulties. The method can benefit from the availability of supercomputers with a lot of main memory and a robust numerical integrator.

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Figure 16. Mutual inductance between two coaxial circular current filaments

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