PLASMA ARMATURE RAILGUN LAUNCHER SIMULATIONS AT THE
UNIVERSITY OF TEXAS AT AUSTIN

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Abstract

The Center for Electromechanics at The University of Texas at Austin (CEM-UT) has developed, while performing research associated with several Defense Advanced Research Projects Agency (DARPA) contracts, a velocity dependent friction model that accurately predicts the losses associated with a plasma armature railgun.

Test results from CEM-UT's 1 m long, 1.27 cm square bore, plasma-armature railgun have been used to determine the validity of the model. Deviation between calculated and measured performance is typically less than 5% at railgun currents below 500 kA, however, at currents greater than 500 kA, the deviation increases.

Experimental evidence suggests that the railgun's lack of stiffness and subsequent venting of driving pressure rather than the electromechanical model is primarily responsible for the divergence between predicted and measured results. To test this theory a railgun was built using external preloading rings (Ringfeder®) to increase its stiffness. On the first test of the Ringfeder® railgun, 625 kA was discharged into the gun and the projectile was accelerated to 5.9 km/s. Test data indicates that the projectile accelerated through the entire length of the railgun and that a minimum amount of plasma leakage occurred during the test.

An analysis of the 625-kA test was done to compare the results of CEM-UT's frictional loss model to ablation and viscous drag loss models.

Introduction

A variety of loss models have been developed in the community to aid in predicting performance of electromagnetic (EM) railgun type launchers. At the Center for Electromechanics at The University of Texas at Austin (CEM-UT), computer programs were developed that predict shot performance and test theoretical models against experimental data after a railgun has been fired. Data has been accumulating which suggests that there are two loss mechanisms primarily responsible for lowering railgun performance: friction between the projectile and the bore of the railgun, and plasma leakage due to the railgun's lack of stiffness.

This paper will present a derivation of the EM railgun launcher drive force (Lorentz Force) and then review three popular loss models currently used at CEM-UT in modeling EM launchers: friction, ablation, and armature drag, followed by the analysis of one railgun experiment in which the projectile obtained a measured velocity of about 6 km/s. The experiment will be analyzed using the velocity dependent frictional loss, ablative loss, and armature drag both separately and combined for comparison's sake.

Electromechanical Model

Lorentz Force

The conservative equation of motion in a railgun launcher may be derived by a Lagrangian formulation:[1]

$$\mathcal{L} = T - V \quad (1)$$

where

T = kinetic energy

V = electromagnetic potential energy

obeys Lagrange's equation of motion

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}} \right) - \frac{\partial \mathcal{L}}{\partial x} = 0 \quad (2)$$

For a parallel geometry railgun, as shown in figure 1, the energy stored inductively in the rails is:

$$V = \frac{1}{2} L(x) I^2$$

where

V = electromagnetic potential energy

$L(x)$ = inductance as a function of length

I = railgun current

while the kinetic energy stored in the projectile is:

$$T = \frac{1}{2} m \dot{x}^2$$

where

T = kinetic energy

m = projectile mass

x = projectile velocity.

Figure 1. Railgun basic concept

Substituting T and V into equation 1 and further substitution into equation 2 yields

$$m\ddot{x} - \frac{1}{2} I^2 \frac{dL}{dx} = 0$$

which may be rearranged to the more familiar form:

$$F = \frac{1}{2} L' I^2 \quad (3)$$

where

F = force on the projectile
L' = inductance per unit length. [2]

Thermodynamic Force

When a metallic foil is used to initiate a plasma armature in a railgun, it imparts a thermodynamic force on the projectile caused by the rapidly expanding hot gases of the exploding foil. Rather than develop a complicated model, this effect is accounted for by adding an initial velocity of 400 m/s in preshot simulations. This value was chosen from the average values of the y-intercept of post-shot plots of the integral of current squared vs. observed velocity. In post-shot simulations, the observed intercept for each particular shot is used instead of the average value.

Friction Model

To compute frictional losses, the pressure as a function of position in the projectile is determined. A force normal to the railgun bore is calculated from the pressure distribution and then multiplied by a velocity dependent friction coefficient to yield the frictional loss. The pressure in the projectile is assumed to be linearly distributed, starting with the Lorentz force divided by the base area at the back of the projectile and being zero at the front surface. The pressure as a function of axial position is multiplied by the projectile side area as a function of axial position to account for various projectile geometries and then integrated to derive the projectile force normal to the bore as a function of position as shown in figure 2. The effect of Poisson's ratio is included later since it is a constant and unaffected by the integration.

The projectile is loaded on three axes so Poisson's ratio may not be used directly to calculate the stress on the bore due to the projectile drive pressure. Shigley [3] defines the triaxial stress state as when none of the three principal stresses are zero as shown in figure 3. The principal strains are:

$$\begin{aligned} \epsilon_1 &= \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E} \\ \epsilon_2 &= \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} - \frac{\mu\sigma_3}{E} \\ \epsilon_3 &= \frac{\sigma_3}{E} - \frac{\mu\sigma_1}{E} - \frac{\mu\sigma_2}{E} \end{aligned}$$

where

ϵ = the respective principal strain
E = the modulus of elasticity
 σ = the respective principal stress
 μ = Poisson's ratio.

Figure 2. Force distribution in projectile

Rearranging, the principal stresses are:

$$\begin{aligned} \sigma_1 &= \frac{E\epsilon_1(1-\mu) + \mu E(\epsilon_2 + \epsilon_3)}{(1-\mu-2\mu^2)} \\ \sigma_2 &= \frac{E\epsilon_2(1-\mu) + \mu E(\epsilon_1 + \epsilon_3)}{(1-\mu-2\mu^2)} \\ \sigma_3 &= \frac{E\epsilon_3(1-\mu) + \mu E(\epsilon_1 + \epsilon_2)}{(1-\mu-2\mu^2)} \end{aligned}$$

If the projectile is confined along axes 2 and 3, as is the case in a square-bore railgun, then

$$\epsilon_2 = \epsilon_3 = 0$$

and the principal stresses are:

$$\begin{aligned} \sigma_1 &= \frac{E\epsilon_1(1-\mu)}{(1-\mu-2\mu^2)} \\ \sigma_2 &= \sigma_3 = \frac{\mu\epsilon_1 E}{(1-\mu-2\mu^2)} \end{aligned}$$

Figure 3. Triaxial stress in Lexan® projectile

Rearranging and substituting yields σ_2 and σ_3 in terms of σ_1 :

$$\sigma_2 = \sigma_3 = \frac{\mu \sigma_1}{1 - \mu} \quad (4)$$

Due to the triaxial loading of the projectile, the von Mises stress is computed in order to determine if the projectile has exceeded its yield strength. Shigley defines the von Mises stress for a triaxial state as:

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right]^{1/2}$$

where

σ' = the von Mises stress.[3]

Substituting:

$$\sigma_2 = \sigma_3 = \frac{\mu \sigma_1}{(1 - \mu)}$$

yields

$$\sigma' = \frac{\sigma_1 (1 - 2\mu)}{(1 - \mu)} \quad (5)$$

To determine if the projectile has yielded, the von Mises stress, instead of the pressure on the base of the projectile, is compared to the yield strength of the projectile material.

During the simulation, the yielded position of the projectile is calculated and used to find the force normal to the projectile sides for the yielded and unyielded regions. The total normal force per side is determined by individually substituting the unyielded and yielded normal force for σ_1 in equation 4 (since equation 4 merely scales, it may be applied to either pressure, or as in this case, force) and using Poisson's ratio for μ in the unyielded case and $\frac{1}{2}$ for μ in the yielded case, then summing them together. Using the same approach, the frictional loss model can account for various obturator styles

and projectile configurations. A velocity dependent friction coefficient is then applied to the total normal side force to determine the loss in projectile drive.

Projectile-to-bore friction forces have been modeled by light gas gun researchers using a velocity dependent friction coefficient based on the work of Bowden and Tabor.[4] At velocities greater than 400 m/s, the friction coefficient scales according to:

$$h = AV^{-B}$$

where

h = velocity dependent friction coefficient

V = velocity (ft/s)

A = constant scale factor

B = power factor.

Bowden and Tabor's work was done by sliding nylon on steel where they found the coefficient B to be -0.4 . In work published by Powell and associates, it was assumed that B was the same as in Bowden and Tabor's work but A was scaled by the ratio of the static coefficient of friction of Lexan® on steel and nylon on steel yielding A to be 4.0. Powell states that "This approach produces a realistic trend and reasonable values but is known to be approximate." [5] Since a variety of materials are used in railguns, the equation has been rewritten to eliminate the scaling done by Powell by dividing A by the static coefficient of friction for Lexan® on steel and introducing the coefficient of friction of the projectile material on the bore such that:

$$h = 4 \left(\frac{h_1}{h_2} \right) V^{-0.4} \quad (6)$$

where

h = velocity dependent friction coefficient

V = velocity (ft/s)

h_1 = static coefficient of the projectile on the bore

h_2 = static coefficient of the Lexan® on steel.

In CEM-UT railgun simulations, the force normal to the bore is multiplied by the velocity dependent friction coefficient to determine the loss in driving force. A check is performed to insure that the frictional loss does not exceed the drive force and, in the event that it does, a message is printed to the screen indicating to the operator that it has. Table 1 lists the static coefficients of friction for materials used in CEM-UT railguns as determined in CEM-UT labs.

Table 1. Static coefficients of friction of polycarbonate on various bore materials

BORE MATERIAL	COEFFICIENT OF FRICTION ON POLYCARBONATE
Steel	0.23
Copper 110	0.34
Quartz Glass	0.22
Plasma Sprayed Molybdenum	0.24
40% Glass Filled Polycarbonate	0.68

Ablation Model

Analysis of plasma armatures in EM launchers has revealed behavior that suggests the ablation of the railgun bore constituents and subsequent entrainment into the plasma armature.[6] A simple worst-case model for predicting the material mass removed from the bore and added to the armature is:

$$\frac{dm_a}{dt} = \alpha IV_a \quad (7)$$

where

m_a = arc mass
 I = armature current
 V_a = arc voltage
 α = ablation constant.

The ablation constant α is material dependent and can be approximated by calculating the energy per gram required to ionize it. Since the dominant factor in determining the ablation constant is the average atomic weight of the material, a close approximate can be found using the following relation:

$$\alpha = \frac{3n}{4} \text{ g/MJ} \quad (8)$$

where

n = average atomic weight of bore materials.

Table 2 lists ablation constants for typical bore materials determined using the formula above.

Table 2. Average ablation constants for various bore materials

BORE MATERIAL	ABLATION CONSTANT
Steel	41 g/MJ
Copper 110	47 g/MJ
Quartz Glass	21 g/MJ
Plasma Sprayed Molybdenum	71 g/MJ
40% Glass Filled Polycarbonate	8 g/MJ

Ablation affects the performance of a plasma armature railgun in at least two ways: the first is in the increase in the overall launch package mass which may be determined using equation (7), the second being the force associated with changing the launch package mass while moving at velocity, v , which can be determined by [6]:

$$F_A = V \frac{dm_a}{dt} = \alpha IV_a v \quad (9)$$

In CEM-UT railgun simulations, the ablation force is subtracted from the Lorentz force prior to calculating the normal force used in frictional loss calculation.

Armature Drag

The viscous drag of a plasma armature on railgun bore walls further decreases the acceleration force on the projectile. If the plasma armature is assumed to

be uniform along its length, the viscous drag force in a square-bore railgun can be determined from:[6]

$$F_v = \frac{B\rho v^2 C l_a}{2} = \frac{Bv^2 m_a (w+h)}{wh} \quad (10)$$

where

ρ = plasma density
 B = drag coefficient
 C = bore circumference
 l_a = armature length
 m_a = armature mass
 v = armature velocity
 w = width of bore
 h = height of bore.

According to Tidman, Goldstein, and Winsor [7], the drag coefficient can be determined using the following equation:

$$B = [2\log\left(\frac{a}{\delta a_w}\right) + 1.74]^{-2}$$

where

$\frac{a}{\delta a_w}$ = the ratio of the bore size divided by the bore roughness.

For most smooth gun bores, $a/\delta a_w \approx 4000$, so by substitution $B = 0.0125$.

As with the ablation loss, the viscous drag force is subtracted from the Lorentz force prior to calculation of the projectile's velocity dependent friction loss.

Experimental Results

Test results from CEM-UT's 1 m long, 1.27 cm square bore, plasma armature railgun have been compared to data generated using the velocity dependent frictional loss model. Deviation between calculated and measured performance is typically less than 5% at railgun currents below 500 kA; at currents greater than 500 kA, the deviation increases.[8] During post-shot inspections of the railguns, black soot was observed to have penetrated the bore seals in the region of the gun where the current exceeded 500 kA. Since the divergence between the calculated and measured data increased in the sooty areas, it has been suggested that the railgun's lack of stiffness and subsequent venting of driving pressure rather than the electromechanical model is primarily responsible for the divergence between predicted and measured results.[8] To test this theory, a railgun was built using external preloading rings (Ringfeder®) to increase its stiffness. On the first test of the Ringfeder® railgun, 625 kA was discharged into the gun and the projectile was accelerated to 5.9 km/s. Test data indicates that the projectile accelerated through the entire length of the railgun and that a minimum amount of plasma leakage had occurred during the test. Current and magnetic probe traces for this test are shown in figures 4a and b.

An analysis of the 625-kA test was done to compare the results of CEM-UT's frictional loss model to ablation and viscous drag loss models. Measured, pre-shot simulation, and post-shot computation data for the projectiles position and velocity are plotted against time in figures 5 through 8. Measured data was obtained using the methods discussed by Cook.[1] Cook determined the projectile's in-bore position and velocity from the magnetic probe signals. Since

Figure 4a. Instrumentation signals for 625 kA railgun test

Figure 4b. Measured projectile position and velocity with error bars

magnetic probe signal interpretation is very subjective, the error in the calculations of in-bore position and velocity is high. Cook determined the projectile exit time from the sharp rise in muzzle volts and from muzzle flash diodes. Typically, these signals correlate very well and the exit time is accurately determined within a few μs . Exit velocity is determined with 1% using a dual exposure flash x-ray system. Because of the high uncertainty in the in-bore measurements each model's merit will be determined primarily from how asymptotic the traces are at projectile exit. Error bars for the measured data are plotted in figure 4b.

Figure 5. Pre-shot prediction of projectile position

The comparison on measured data and calculated projectile position data from each of the three models separately and combined is shown in figures 5 and 6. The post-shot code calculates the railgun's performance from the measured gun current. In figure 5, the pre-shot simulation was used to determine calculated projectile position and in figure 6, the post-shot computation code was used to determine the calculated projectile's position. Good correlation between measured and calculated position is observed for the friction model alone. It appears that the two traces start to converge in the last half of the gun. The greatest error occurs early in the test and is probably due to a slight inaccuracy in thermodynamic force model (remember the thermodynamic model is used to account for an initial velocity and does not account for displacement). Also, because the projectile is moving very slowly at this time, the accuracy of the velocity dependent frictional model may suffer. The measured position deviates from the calculated position data obtained using the ablation model at the beginning and end of the experiment and do not appear to be converging. It appears the loss is being overpredicted. This makes sense because in all probability, the entire ablated mass is not carried along with the armature. Deviation between measured and calculated position data using the viscous drag of the armature model is small at early times in the test and gets larger near projectile exit. At all times, though, the calculated loss is less than the loss measured. Data calculated using all three models combined is also compared to measured data. It seems that the pre-shot simulation data correlates better than the data computed using post-shot calculations. An explanation for this observation is that post-shot data is generated from the measured railgun current and is more conclusive in determining whether the two data sets correlate, while in the pre-shot simulation the combined losses are so great that the railgun current predicted differs significantly from that measured. Bearing this in mind, the losses are overpredicted when all three models are combined.

Figure 6. Post-shot computed projectile position

The comparison of measured data and calculated projectile velocity data from each of three models separately and combined is shown in figures 7 and 8. In figure 7, the pre-shot simulation was used to determine calculated projectile velocity and in figure 8, the post-shot computation code was used to determine the calculated projectile velocity. In both figures 7 and 8, good correlation between measured and calculated velocity is seen throughout the shot when the friction model is applied only. Some deviation is seen in the first 0.1 m of the gun but is most likely attributable to too few magnetic probes to give required resolution. The comparison of measured data to data calculated using the ablation model alone shows the measured velocity being significantly higher than those calculated in simulations. Measured data showed reasonable correlation to calculated data using the viscous armature drag model; however, during the last half of the gun, the divergence increases. When all three models were combined, the pre-shot simulation data tends to correlate better than the post-shot calculation, due to reasons previously mentioned. It is felt that the losses are greatly overpredicted when the three models are combined.

Analysis

All three loss models (velocity dependent frictional drag, ablation, and armature drag) when evaluated individually, produce realistic results; yet when the models are evaluated together, the total loss is overpredicted. It is probable that most of the error is in the values chosen for the various coefficients used in the models.

At CEM-UT, the velocity dependent friction model is used in simulations because it is an accepted model in light gas guns. Ablation and armature viscous drag have also been incorporated for completeness in the models. It is believed that plasma leakage is the limiting factor in railgun performance and that the magnitude of the effects of the last two models is unclear in railguns when no plasma leakage is observed.

Figure 7. Pre-shot prediction of projectile velocity

Figure 8. Post-shot computed projectile velocity

Conclusions

Railgun performance simulations at CEM-UT were modified to include the effects of friction, ablation, and viscous drag. On experiments at currents below 500 kA using existing railgun design, the friction model alone was acceptable in predicting performance. In an experiment incorporating a railgun structure modified for higher stiffness and a measured peak railgun current of 625 kA, the effects of each of the loss models were compared to the measured results and the greatest success at predicting the final projec-

tile velocity and exit time occurred using the velocity dependent friction model. Experimental evidence is accumulating which suggests that the primary unaccounted for loss mechanism is plasma venting. Future plans include the construction of ceramic re-enforced railgun structures that can be operated at currents in the 800 kA regime with bore deflection of less than 0.13 mm (0.005 in.) to further test these theories.

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