Lumped Parameter Model to Describe Dynamic Translational Interaction for High Temperature Superconducting Bearings

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Abstract—This paper discusses a lumped parameter modeling methodology to describe the dynamic vertical and translational force interaction between a bulk superconductor and levitated permanent magnet. The model is formulated to be easily incorporated into larger rigid body system models to aid design of superconducting bearings for flywheel energy storage applications. The proposed modeling technique will significantly reduce the computational expense in order to shorten the design cycle process. The validity of the proposed lumped parameter model is demonstrated by comparing results from Finite Element Method analysis and measurements of the force displacement interaction between a permanent magnet and bulk high temperature superconductor.

Index Terms— electromagnetic coupling, high temperature superconductors, magnetic levitation, simulation

I. INTRODUCTION

RESEARCHERS are pursuing high temperature superconducting bearings for applications such as flywheel energy storage where low losses are critical for retaining stored energy [1], [2], [3]. Typically, these bearing systems consist of an array of permanent magnets that are stably-levitated over an array of bulk high temperature superconductors (HTSC) such as yttrium barium copper oxide (YBCO). The exterior magnetic field induces pinning currents within the bulk HTSC, that produce stabilizing levitation forces on the permanent magnets. The bearing must provide sufficient lifting capacity to support the weight of the flywheel rotor.

Understanding the translational dynamics of permanent magnet material levitated over bulk superconductor is critical for the design of HTSC bearings. Along with vertical lifting capability, the lateral stiffness affects dynamic performance and may limit operation. For applications such as flywheel energy storage, operation above critical speed may be required

to meet energy storage requirements [4]. The low lateral stiffness and damping capabilities of HTSC bearings have been shown to impede operations above critical speeds [5]. In such cases, active control bearings, or passive damper systems have been implemented to counter these conditions [6], [7].

During the design phase, determining the lateral stiffness of a HTSC bearing requires modeling three dimensional electromechanical effects, since axisymmetric boundary conditions cannot be assumed. Analysis of these cases requires 3-D Finite Element Method (FEM) models [8], [9] which can become computationally expensive. Control volume techniques may also be applicable, but these also require iterative solution steps like FEM [10], [11]. Currently, there are no low-order methods that can capture the stiffness, dynamic, and hysteretic behavior induced by the translational motion.

Tests performed by Hikihara and Isozumi demonstrated a force-displacement hysteresis curve for translational motion of a permanent magnet over a bulk HTSC [12]. These authors suggested a non-dimensional model to replicate the observed dynamics from the experiment, but these models did not consider the physical characteristics or properties of the bulk HTSC or the permanent magnet. Since then, mirror image and advanced mirror image models have also been proposed by Kordyuk [13] and by Hull and Cansiz [14], [15] to model the dynamics of a levitated permanent magnet over bulk HTSC. These techniques model the permanent magnet as a dipole with a stationary image reflected through the top plane of the bulk HTSC, and a diamagnetic image which mirrors the position of the PM. The image models can be used for reduced order modeling to characterize the force displacement characteristics in field-cooled and zero-field cooled conditions. However, image modeling techniques assume a perfect conductor and do not consider the energy loss mechanisms that contribute to the unique dynamics and hysteretic behavior reflected in measured force-displacement curves.

Previously we have developed a technique for modeling the vertical dynamics and force displacement interaction of a permanent magnet (PM) levitated over a bulk HTSC in an axisymmetric frame, [16]. The procedure modeled the bulk HTSC as a series of nested concentric superconducting rings which interacted with the changing magnetic fields produced

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by the movement of a permanent magnet. The dynamic modeling technique predicted the magneto-mechanical force interaction between the PM and bulk HTSC and could be implemented into larger system models. Sub-dividing the bulk HTSC into a mesh of nested concentric ring elements was also performed by Davey *et al.* to perform quasi-static models which predicted trapped fields during flux-pumping [17].

This paper proposes a method of modeling a single bulk HTSC as a stacked array of individual overlapping HTSC discs that interact with the fields produced by the permanent magnet. The permanent magnet is represented by discrete surface current loops with strength I_i , that produce the equivalent magnetic fields. The model assumes each superconducting disc element carries a uniformly distributed circulating current, I_j , that is induced by movement of the PM. The overlapping discs allow non-circular currents to flow in a loop within the bulk HTSC material, and enable the use of lumped parameter modeling techniques to better estimate hysteretic force behavior. The resulting model order (i.e., number of states in dynamic model) depends on the number of discs used to represent the bulk HTSC, where each disc has a single degree of freedom. Figure 1 shows a bulk HTSC represented as proposed. The goal of this lumped parameter model formulation is to provide designers with a tool for estimating the nonlinear stiffness and hysteretic force displacement relationship for a range of displacements relevant to the design of high temperature superconducting bearings.

Fig. 1. Proposed model of permanent magnet levitated over bulk HTSC. Bulk HTSC is represented as discrete disc elements with circulating current I_j , and the permanent magnet is represented by surface current loops of strength I_i .

The second section of this paper discusses the proposed modeling technique and derivation of the equations. Additional supporting equations for the model are also provided in the Appendix. Section III compares modeling results with the translational lumped parameter model versus FEM analysis results and test measurements. Section IV summarizes the findings.

II. MODEL DEVELOPMENT

A. Modeling Methodology

The bulk HTSC is represented by N number of individual superconducting disc elements, each identified as subscript j , as shown in Fig. 1. The permanent magnet is represented by S discrete current loops, I_i , on the cylindrical surface of the permanent magnet, that produce the equivalent magnetic field strength. Multiple current loops are used to represent the equivalent surface currents and shape the magnetic field produced by the permanent magnet. This modeling configuration was chosen to take advantage of the methodology proposed by Smythe for determining the magnetic potential and flux linkage between current loops in free space [18].

Per Faraday's law, the total varying magnetic flux through each disc, ϕ_j , due to movement of the permanent magnet, induces an electromotive force, ε_j , on each disc,

$$\varepsilon_j = -\frac{d\phi_j}{dt} \quad (1)$$

For the proposed model, the net induced electromotive force on a disc element can be expressed as a summation of spatial change in magnetic flux with respect to the permanent magnet component velocity, v , in the x , y , and z directions,

$$\varepsilon_j = -\frac{d\phi_j}{dt} = -\left[\frac{d\phi_j}{dx}v_x + \frac{d\phi_j}{dy}v_y + \frac{d\phi_j}{dz}v_z\right] \quad (2)$$

The gradients of magnetic flux expressed in (2) are identified as gyrator moduli, representing the ideal electromagnetic conversion of power between the mechanical movement of the permanent magnet and the induced electrical currents in the bulk superconductor. Gyrators are lumped-parameter modeling elements routinely used in dynamic system models to represent lossless electromechanical conversion of energy [16], [19]. By identifying the spatial gradients as gyrator moduli, a power-conserving assumption leads to an expression for the net force on the permanent magnet as the sum of the induced currents through each element times the spatial gradient in the relevant component direction,

$$\mathbf{F} = \left(\sum_{j=1}^N -\frac{d\phi_j}{dx}I_j\right)\hat{i} + \left(\sum_{j=1}^N -\frac{d\phi_j}{dy}I_j\right)\hat{j} + \left(\sum_{j=1}^N -\frac{d\phi_j}{dz}I_j\right)\hat{k} \quad (3)$$

From these relationships, the following dynamic equations are proposed (4) to determine the current in each individual disc, I_j , which makes up the bulk HTSC in Fig. 1. For each disc, the time varying current equals the total induced electromotive force, due to movement of the permanent magnet, minus the nonlinear losses, e_j , times the inverse inductance matrix, $\mathbf{L}$. The inductance matrix is a symmetric matrix with self-inductance terms on the diagonal, and mutual

inductance terms, due to the array of disc elements, on the off-diagonal. The following sections discuss deriving the components for this dynamic equation in more detail.

$$\begin{bmatrix} \frac{dI_1}{dt} \\ \frac{dI_2}{dt} \\ \vdots \\ \frac{dI_N}{dt} \end{bmatrix} = -\mathbf{L}^{-1} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} + \begin{bmatrix} \frac{d\phi_1}{dx} \\ \frac{d\phi_2}{dx} \\ \vdots \\ \frac{d\phi_N}{dx} \end{bmatrix} v_x + \begin{bmatrix} \frac{d\phi_1}{dy} \\ \frac{d\phi_2}{dy} \\ \vdots \\ \frac{d\phi_N}{dy} \end{bmatrix} v_y + \begin{bmatrix} \frac{d\phi_1}{dz} \\ \frac{d\phi_2}{dz} \\ \vdots \\ \frac{d\phi_N}{dz} \end{bmatrix} v_z \quad (4)$$

B. Induced Voltage

The spatial change in flux is a critical step in the formulation of this model. To determine $\frac{d\phi_j}{dx}$, $\frac{d\phi_j}{dy}$, and $\frac{d\phi_j}{dz}$, we first consider two ring elements in space as shown in Fig. 2. The first ring element is located at (x_i, y_i, z_i) , has source current I_i , radius, r_i , and is traveling with component velocities v_{xi} , v_{yi} and v_{zi} in the x , y , and z directions, respectively. This ring element represents one of the surface current elements on the PM. The second element represents one of the passive filament elements that make up the HTSC disc elements, and it is located at (x_j, y_j, z_j) with radius r_j . Both loops are separated by a distance D from the centers,

$$D = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \quad (5)$$

Fig. 2. Image of source current loop i with respect to passive filament loop j .

Smythe describes formulations for determining the magnetic potential vector, $\mathbf{A}_i$, at an axial and radial displacement from a circular current loop [18]. Utilizing Stokes Theorem, the magnetic flux due to current I_i that passes through the passive circular filament loop j , is equal to the closed loop integral of the dot product of the magnetic potential vector, and passive filament loop path, $\mathbf{l}_j$,

$$\phi_{ij} = \oint \mathbf{A}_{ij} \cdot d\mathbf{l}_j \quad (6)$$

Based on the formulations from Smythe, the magnetic potential vector at any location on the passive filament circle,

as a function of θ , can be derived as shown by (7). In (7), μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$), and $K(k)$ and $E(k)$ are elliptical integrals of the first and second kind, with modulus k , (8). This radius r_{ij} is the distance from the center of the circular current source to any point on the passive loop, as a function of θ , (9).

$$\mathbf{A}_{ij} = \frac{\mu_0 I_i}{\pi k} \left(\frac{r_i}{r_{ij}} \right)^{\frac{1}{2}} \left(\left(1 - \frac{1}{2} k^2 \right) K(k) - E(k) \right) \left(-\frac{r_j \sin \theta}{r_{ij}} \hat{\mathbf{i}} + \frac{r_j \cos \theta + D}{r_{ij}} \hat{\mathbf{j}} \right) \quad (7)$$

$$k = \sqrt{\frac{4r_i r_j}{(r_i + r_j)^2 + (z_j - z_i)^2}} \quad (8)$$

$$r_{ij} = \sqrt{r_j^2 + 2Dr_j \cos \theta + D^2} \quad (9)$$

With the differential length along the passive filament ring (10), the total flux through the filament ring can be calculated by the integral (11) with respect to θ .

$$d\mathbf{l}_j = r_j (-\sin \theta \hat{\mathbf{i}} + \cos \theta \hat{\mathbf{j}}) d\theta \quad (10)$$

$$\phi_{ij} = \int_0^{2\pi} \frac{\mu_0 I_i r_j}{\pi k} \left(\frac{r_i}{r_{ij}} \right)^{\frac{1}{2}} \left(\left(1 - \frac{1}{2} k^2 \right) K(k) - E(k) \right) \left(\frac{r_j - D \cos \theta}{r_{ij}} \right) d\theta \quad (11)$$

Utilizing the Leibnitz rule (12), the change of flux with respect to x_i , y_i , and z_i can be determined by (13), (14), and (15) respectively. Formulations for additional component derivatives in these formulae are listed in the Appendix.

$$\frac{d\phi_{ij}}{dx_i} = \int_0^{2\pi} \frac{\partial}{\partial x_i} \left[\frac{\mu_0 I_i r_j}{\pi k} \left(\frac{r_i}{r_{ij}} \right)^{\frac{1}{2}} \left(\left(1 - \frac{1}{2} k^2 \right) K(k) - E(k) \right) \left(\frac{r_j + D \cos \theta}{r_{ij}} \right) \right] d\theta \quad (12)$$

$$\begin{aligned} \frac{d\phi_{ij}}{dx_i} = \int_0^{2\pi} \frac{\mu_0 I_i r_j}{\pi k} \left(\frac{r_i}{r_{ij}} \right)^{\frac{1}{2}} \left[\left(\frac{r_j + D \cos \theta}{r_{ij}} \right) \left[\left(1 - \frac{1}{2} k^2 \right) \frac{\partial K(k)}{\partial x_i} \right. \right. \\ \left. \left. - K(k) k \frac{\partial k}{\partial x_i} - \frac{\partial E(k)}{\partial x_i} \right] \right. \\ \left. - \left(\frac{1}{k} \frac{\partial k}{\partial x_i} + \frac{3}{2r_{ij}} \frac{\partial r_{ij}}{\partial x_i} \right) \left(\left(1 - \frac{1}{2} k^2 \right) K(k) - E(k) \right) \right] \\ \left. + \frac{\cos \theta}{r_{ij}} \frac{dD}{dx_i} \left[\left(1 - \frac{1}{2} k^2 \right) K(k) - E(k) \right] \right] d\theta \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{d\phi_{ij}}{dy_i} = \int_0^{2\pi} \frac{\mu_0 I r_j}{\pi k} \left(\frac{r_i}{r_{ij}} \right)^{\frac{1}{2}} & \left[\left(\frac{r_j + D \cos \theta}{r_{ij}} \right) \left[\left(1 - \frac{1}{2} k^2 \right) \frac{\partial K(k)}{\partial y_i} \right. \right. \\ & - K(k) k \frac{\partial k}{\partial y_i} - \frac{\partial E(k)}{\partial y_i} \Big] \\ & - \left(\frac{1}{k} \frac{\partial k}{\partial y_i} + \frac{3}{2 r_{ij}} \frac{\partial r_{ij}}{\partial y_i} \right) \left(\left(1 - \frac{1}{2} k^2 \right) K(k) \right. \\ & \left. \left. - E(k) \right) \right] \\ & + \frac{\cos \theta}{r_{ij}} \frac{dD}{dy_i} \left[\left(\left(1 - \frac{1}{2} k^2 \right) K(k) \right. \right. \\ & \left. \left. - E(k) \right) \right] d\theta \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d\phi_{ij}}{dz_i} = \int_0^{2\pi} \frac{\mu_0 I_i r_j}{\pi k} \left(\frac{r_i}{r_{ij}} \right)^{\frac{1}{2}} & \left(\left(1 - \frac{1}{2} k^2 \right) \left(\frac{dK(k)}{dz_i} - \frac{K(k)}{k} \frac{dk}{dz_i} \right) \right. \\ & + \left(\frac{E(k)}{k} - kK(k) \right) \frac{dk}{dz_i} \\ & \left. - \frac{dE(k)}{dz_i} \right) \left(\frac{r_j + D \cos \theta}{r_{ij}} \right) d\theta \end{aligned} \quad (15)$$

The integrals in (13) thru (15) can be numerically integrated, but this operation may be computationally intensive to incorporate into a dynamic model. Instead, these integrals should be calculated offline and supplied to the dynamic model as a lookup table based on the x_i , y_i , and z_i location of the PM. If S current loops are used to represent the surface currents on the PM, superposition can be used to sum the effects, i.e.,

$$\frac{d\phi_j}{dx} = \sum_{i=1}^S \frac{d\phi_{ij}}{dx_i} \quad (16)$$

$$\frac{d\phi_j}{dy} = \sum_{i=1}^S \frac{d\phi_{ij}}{dy_i} \quad (17)$$

$$\frac{d\phi_j}{dz} = \sum_{i=1}^S \frac{d\phi_{ij}}{dz_i} \quad (18)$$

The formulation presented so far is for flux passing through a single filament in free space, but the individual disc elements used to model the bulk HTSC have a fixed height, h_i and outer radius, R_i . In order to increase the accuracy of computing the flux through a disc element, the disc element is partitioned into M equally sized sub-elements, with a filament located in the center, Fig. 3. At each filament location in the disc, the spatial flux derivative per (13) thru (15) is calculated (where q_i is x_i , y_i , or z_i respectively), and the average sum is taken per (19). This methodology has been used to calculate coil inductances using Smythe's equations, as previously discussed in the literature [20], [21], [22].

Typically, the calculation the spatial gradients can be a time consuming process. To improve model performance,

these calculations should be performed off-line and saved as look-up tables for the primary simulation.

Fig. 3. Sub-divided disc element into discrete ring current loops to improve estimation of flux gradients and inductance calculations.

$$\frac{d\phi_{ij}}{dq_i} = \frac{\sum_{m=1}^M \frac{d\phi_{im}}{dq_i}}{M} \quad (19)$$

C. Inductance Calculations

The self and mutual inductance calculations of all disc elements must be carried out to determine the matrix $\mathbf{L}$. To accurately estimate the self and mutual inductances between individual disc elements, the discs should be subdivided, or meshed, into nested rings with a filament loop at the center, as shown in Fig. 3. The inductance between two current loops, assuming one turn per loop, is defined simply as;

$$L_{ij} = \frac{\phi_{ij}}{I_i} \quad (20)$$

Equations (5) thru (11) can be reused here to calculate the flux linkage, ϕ_{ij} , and mutual inductance between two filament current loops. The methodologies presented by Kim [20], Babic [21], and Akyel [22] should also be pursued to sum and average the self and mutual inductances of the disc elements. For the self-inductance of an individual filament loop, Smythe recommends the following equation, where w_i is the width of a sub-divided mesh element.

$$L_{ii} = \mu_0 r_i \left(\ln \left(\frac{16 r_i}{w_i} - 2 \right) + 0.25 \right) \quad (21)$$

D. Voltage Loss

From (2), each disc element that makes up the bulk HTSC contains a nonlinear voltage loss, e_j , which is a function of the bulk current flow through each element, plus the current flow in neighboring overlapping elements. Type II superconductors have an upper critical field at which the material loses superconductivity and becomes resistive. The induced current flow is limited to the critical current density, J_c [23]. The use of a power-law has been suggested in the literature to model the nonlinear voltage-current characteristics and the rapid rise of resistivity at the critical current density [9], [24].

In order to apply this methodology to this model description, we must first consider that we have overlapping

currents from overlapping the disc elements, as shown in Fig. 4. Each disc element in the model is assumed to have a uniform current that flows circumferentially. To estimate the voltage loss, each disc element is sub-divided into six 60° arc segments, where the net current flow through each segment is calculated due to the primary disc, I_j , and current flow through neighboring discs, I_n , as shown in (22).

Fig. 4. Schematic view of overlapping disc elements with overlapping current flows.

$$e_j = \frac{2\pi r_j E_c}{6J_c^n} \sum_{n=1}^6 \left[\left(\frac{|I_j - I_n|}{A_j} \right)^n \text{sign}(I_j - I_n) \right] \quad (22)$$

In (22), r_j is the mean disc radius, A_j is the cross sectional area for current flow of each disc, and E_c is the electric field threshold ($E_c = 1 \mu V/cm$). Grilli *et al.* [9], suggested values for the exponential constant n can range between 5 to 30. Higher values of n will increase the computational stiffness of the final set of equations, and can increase solver time depending on the numerical integration routine used. For the analyses presented in this paper, a value of n equal to 10 is assumed.

III. MODEL VERIFICATION

In order to verify model performance, comparisons were made to results from both FEM analysis and experimental test data to verify the force-displacement interaction due to the translational movement of a permanent magnet over a bulk HTSC. Such experiments have been performed previously that describe the translational force vs. position and hysteresis characteristics [12].

For model comparisons, the FEM analysis is based on the T-Omega formulation in three dimensions, which uses the electric vector potential $\vec{T}$ and the magnetic scalar potential Ω in the superconductor and the magnetic scalar potential alone in the region outside the conductor. The highly non-linear problem is solved using the Newton-Raphson iteration process using the tangent stiffness matrix. This matrix is updated at each iteration, thus helping in the faster convergence of the problem. The boundary conditions are enforced using the technique of Lagrangian multipliers thus increasing the

number of degrees of freedom being solved for on the boundary surfaces.

In the current setup, the bulk HTSC is placed in a cryostat on a moveable platform below a permanent magnet. The magnet is held stationary and force measurements are made via a load cell, as shown in Fig. 5. A set of bearings are included in-plane of the load cell to isolate directional force. The test setup can be configured, as shown in Fig. 5, to measure translational force on the magnet with respect to translational position, or vertical lifting force on the magnet with respect to translational position. Fig. 6 shows a picture of the test rig in the lab.

Fig. 5. Test apparatus to measure translational (as shown) and vertical force on a permanent magnet with respect to translational motion of a bulk HTSC.

Fig. 6: Photograph of test apparatus in laboratory with sensor setup for horizontal force measurement.

The tests were performed with a 47 mm OD x 15 mm H YBCO bulk superconductor. An N48 grade, neodymium magnet, which measured 3/4" (19 mm) OD x 1/4" (6.4 mm) ID x 3/4" H (19 mm), was used. The field strength of the magnet at liquid nitrogen temperatures (77 K) was measured at 0.4 T on the surface and 0.28 T at a 2 mm gap from the surface. Since the strength of a neodymium magnet can decrease by about 15% at cryogenic temperatures compared to room temperature [25], the magnet was pre-cooled so that a near constant field was maintained during cooling of the bulk HTSC.

Before collecting test data, model verification was performed against the three dimensional T-Omega FEM analysis of the experimental setup. In the analysis, the magnet is positioned 2mm above the surface of the bulk HTSC. The critical current density, J_c , of the bulk superconductor was assumed to be 9 kA/cm², and the magnet field strength was approximated as 760 kA/m based on field measurements for both analyses. The magnet is originally positioned over the

center of the HTSC and is swept ± 20 mm, over a period of 30 seconds.

The lumped parameter model used 654 overlapping disc elements to represent the bulk HTSC. These elements were divided into 6 layers axially, with the top three layers having a thickness of 1.3mm and the bottom three layers having a thickness of 3.8mm. All elements had a diameter of 7.9 mm. The inductance matrix, $\mathbf{L}$, and spatial gradients $\frac{d\phi_j}{dx}$, $\frac{d\phi_j}{dy}$, and $\frac{d\phi_j}{dz}$ were solved off-line and took 75 minutes and 20 minutes, respectively, to calculate on a 64 bit laptop computer with an Intel(R) Core(TM)2 Duo P8700 processor. Using a fixed step 4th order Runge-Kutta solver with a time step of 0.5 ms, a solution could be attained within 10 to 12 minutes for the system of equations in (4), where v_x represented the translational velocity of the magnet.

In comparison, the 3D FEM code used 26,000 elements and required 46.8 hours to solve at a larger time step of 0.75 s over the 30 s sweep. Each time step required 14 iterations on average to converge to a solution using a frontal solver. The FEM code ran on a 64-bit desktop with an Intel(R) Xenon(R) X5472 processor. This comparison shows a significant increase in computational speed for the lumped parameter model, which may be further increased by removing disc elements which are shielded by the exterior flux. Plots of translational force and versus distance for sweeps of ± 20 mm and ± 10 mm are shown in Figs. 7 and 8, respectively, to compare predictions between the FEM and lumped parameter methods. The sweep period for both distances was 30 seconds.

Compared to the FEM analysis, the lumped parameter model shows good tracking of the translational force-displacement profile near the center of the bulk HTSC. As the magnet is moved closer to, and over the edge of the bulk HTSC, there are greater divergences in the force estimation. Due to the use of individual disc elements, the edge is not very well replicated by the lumped parameter mesh. In addition, the proposed lumped parameter method only considers current flow in directions parallel to the surface plane, so current flow near the edges, which may not be parallel, is not replicated.

Experimental tests were performed with the magnet positioned at a 2 mm height above the surface of the bulk HTSC, and sweeps of ± 20 mm and ± 10 mm were performed over a period of 26 seconds. Comparisons of the lumped parameter model predictions to translational and vertical force measurements are shown in Figs. 9-12. In these figures, Lumped Model 1, refers to model performance with the original estimated critical current density and magnet strength used in the FEM analyses. These original values showed good matching in the vertical force direction, but the translational force predictions were lower than measured results.

For a second pass, Lumped Model 2 was formed using a magnet strength slightly increased from 760 kA/m to 812 kA/m, and the critical current density was increased from 9 kA/cm² to 10 kA/cm². These slight changes in material properties produce much closer agreement in translational

force estimates, although the vertical force is slightly over predicted further away from the center. The critical current densities for YBCO material can range from 10-100 kA/cm² [2].

Fig. 7. Translational force comparison of lumped parameter to FEM analysis for ± 20 mm magnet displacement

Fig. 8. Translational force comparison of lumped parameter to FEM analysis for ± 10 mm magnet displacement

Fig. 9. Translational force of lumped parameter model compare to data for ± 20 mm displacement.

Fig. 10. Vertical force of lumped parameter models compared to data for +/- 20mm displacement

Fig. 11. Translation force of lumped parameter model compared to data for +/- 10mm displacement

Fig. 12. Vertical force of lumped parameter model compared to data for +/- 10mm displacement

As compared to the FEM analysis, the lumped parameter model shows a significant falling off of force near the edges of the bulk HTSC, which is not observed in the test data. The lumped parameter model continues to show good performance

for smaller displacements near the center of the bulk HTSC. For integrated flywheel applications where tight air gaps are required for efficient motor-generator performance, expected displacements should not exceed 1 mm. For these applications the lumped parameter methodology provides sufficiently accurate results at a fraction of the computational time.

IV. CONCLUSION

This paper presents a methodology to estimate the lateral and vertical forces developed on a levitated permanent magnet over bulk superconducting material. This model is intended for use in aiding the design of high temperature superconducting bearings for flywheel energy storage applications. Although the proposed model shows results that differ from FEM and test analyses at large displacements, the proposed technique has been validated to be highly accurate within a displacement range that encompasses the allowed tolerance for bearing design. Compared to 3D FEM analysis, the lumped parameter model significantly reduces computational time from days to minutes, while also incorporating inherent losses that produce the hysteretic force loops induced between the interacting permanent magnets and superconductors. The proposed lumped parameter model may also be applied to larger system models that feature rigid body dynamics, to help predict rotor dynamic behavior for proposed advanced flywheel energy storage systems.

APPENDIX

Evaluation of elliptical modulus with respect to x_i , y_i , and z_i .

$$\frac{\partial k}{\partial x_i} = -\frac{k^3(r_i^2 - r_{ij}^2 + z^2)(D + r_j \cos \theta)(x_j - x_i)}{8r_i r_{ij}^3 D} \quad (23)$$

$$\frac{\partial k}{\partial y_i} = -\frac{k^3(r_i^2 - r_{ij}^2 + z^2)(D + r_j \cos \theta)(y_j - y_i)}{8r_i r_{ij}^3 D} \quad (24)$$

$$\frac{dk}{dz_i} = \frac{k^3(z_j - z_i)}{4r_i r_{ij}} \quad (25)$$

Evaluation of change in center distance with respect to x_i and y_i .

$$\frac{\partial D}{\partial x_i} = \frac{-(x_j - x_i)}{r_{ij}} \quad (26)$$

$$\frac{\partial D}{\partial y_i} = \frac{-(y_j - y_i)}{r_{ij}} \quad (27)$$

The following derivatives of elliptical integrals with respect to modulus are given by Smythe [16].

$$\frac{dK}{dk} = \frac{E}{k(1-k^2)} - \frac{K}{k} \quad (28)$$

$$\frac{dE}{dk} = \frac{E}{k} - \frac{K}{k} \quad (29)$$

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