RAILGUN CURRENT DENSITY DISTRIBUTIONS

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Abstract - A closed form solution for the magnetic field intensity and current density in an electromagnetic railgun is presented for a simple geometrical design. A constant velocity for the armature and constant material properties for the rail and armature have been assumed in all calculations. Plots of the magnetic field, current densities, and current streamline are presented for a sample velocity and conductive material.

NOMENCLATURE

A_n = Fourier coefficients of the solution for region #1 of the model
 B_n = Fourier coefficients of the solution for region #2 of the model
 C_n = Fourier coefficients of the solution for region #2 of the model
 d = width of the armature
 D_n = Fourier coefficients of the solution for region #2 of the model
 F_n = Fourier coefficients of the solution for region #3 of the model
 G_n = Fourier coefficients of the solution for region #4 of the model
 H_z = magnetic field intensity in the z direction (A/m)
 J_x = electric current density in the x direction (A/m²)
 J_y = electric current density in the y direction (A/m²)
 k = constant used in a separation of variables solution to a differential equation
 $K = J_x(-\infty, y)$
 l = half of the armature length
 M_n = poles for the linear differential equation in region #2 of the model
 n = index used for series summations
 N_n = poles for the linear differential equation in region #2 of the model
 P_n = poles for the linear differential equation in region #2 of the model
 v = velocity of the armature
 w = width of the rails
 x = x coordinate
 y = y coordinate
 σ = conductivity
 μ = permeability

INTRODUCTION

There are railgun applications in which it is desirable to have a solid conducting armature complete the current path between the rails. In order to prevent railgun damage due to ohmic heating or internal forces, it is necessary to control the current distribution

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within the conducting medium. The current distributions cannot be predicted without an accurate model of the magnetic field and current density. The purpose of this paper is to describe some of the preliminary research that is being conducted at the Center for Electromechanics (CEM) in order to develop a model which will accurately predict the current distributions. Although the model is very simple, the information provided by the results can provide insight into railguns which have more complex designs. This model can also be employed as an analytical check against a finite element or finite difference model which would be used for curved armature shapes and different material designs.

MODEL

Figure 1 depicts the two dimensional railgun model that has been used for the calculations. The system is considered to be infinitely long in the z direction.

Fig. 1. The two dimensional model for the railgun.

The magnetic field is a constant value along the inside walls and zero along the outside walls of the rails and armature. In the limiting case, this arrangement is not too different from the case of a coaxial railgun with a large radius compared to the rail spacing. The direction of the magnetic field is out of the plane of the paper. Due to the symmetrical design, half of the railgun system will be analyzed. A simpler analysis is possible if the armature is selected as the reference frame. The equivalent problem to the armature moving to the right would be to have the rails moving to the left and holding the armature stationary. Since the rails are moving, all $q(\mathbf{v} \times \mathbf{B})$ forces will be applied to the rails.

The starting point for calculating the magnetic field is the diffusion equation.

$$\nabla^2 \mathbf{H} + \sigma \mu [\nabla \times (\mathbf{v} \times \mathbf{H})] = 0 \quad (1)$$

For this analysis, steady state conditions have been assumed and the displacement current term associated with the diffusion equation has been neglected.

RAIL REGION ANALYSIS

It is convenient to divide the rail into three regions as shown in Fig. 2.

Fig. 2. The rail regions.

Since the magnetic field and velocity vectors are normal, the diffusion equation for the rail reduces to the following;

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} - v\sigma\mu \frac{\partial H_z}{\partial x} = 0 \quad (2)$$

The separation of variables method will be used to solve this differential equation.

The electrical boundary conditions that must be satisfied due to the geometry of the problem for each region are as follows;

Region 1:

$$\begin{aligned} J_y(x, 0) &= J_y(x, d) = 0 \\ J_x(-\infty, y) &= -K \\ H_z(x, 0) &= K \\ H_z(x, w) &= 0 \end{aligned} \quad (3)$$

Region 2:

$$\begin{aligned} J_y(x, d) &= 0 \\ H_z(x, w) &= 0 \end{aligned} \quad (4)$$

Region 3:

$$\begin{aligned} J_y(x, 0) &= J_y(x, d) = 0 \\ J_x(\infty, y) &= 0 \\ H_z(x, 0) &= H_z(x, d) = 0 \end{aligned} \quad (5)$$

At the interfaces, the normal components of the current density and the tangential components of the electric field must be continuous.

Letting $H_z(x, y) = X(x)Y(y)$, equation (2) in cartesian coordinates becomes;

$$\begin{aligned} \frac{d^2 X(x)}{dx^2} - v\sigma\mu \frac{dX(x)}{dx} - k^2 X(x) &= 0 \\ \frac{d^2 Y(y)}{dy^2} + k^2 Y(y) &= 0 \end{aligned} \quad (6)$$

The selection of the sign of k^2 has been chosen according to the geometrical considerations of the problem. In addition, a value of $k^2 = 0$ must be considered for region #1 and region #2. In order to solve the interface conditions between region #2 and the armature, k^2 values of the opposite sign must also be used in region #2. Considering only the geometry of the problem, the magnetic field and current density equations in each region of the rail become;

Region 1:

$$\begin{aligned} H_z(x, y) &= \left[\sum_{n=1}^{\infty} A_n \sin(n\pi y/w) e^{M_n x} \right] - (K/w)(y-w) \\ J_x(x, y) &= \left[\sum_{n=1}^{\infty} A_n (n\pi/w) \cos(n\pi y/w) e^{M_n x} \right] - (K/w) \\ J_y(x, y) &= \left[\sum_{n=1}^{\infty} -A_n M_n \sin(n\pi y/w) e^{M_n x} \right] \end{aligned} \quad (7)$$

Region 2:

$$\begin{aligned} H_z(x, y) &= \left[\sum_{n=1}^{\infty} (B_n e^{M_n x} + C_n e^{N_n x}) \sin(n\pi y/w) \right] \\ &+ \frac{(K/w)(y-w)(e^{v\sigma\mu x} - e^{v\sigma\mu d})}{(e^{v\sigma\mu d} - 1)} \\ &+ \left[\sum_{n=1}^{\infty} D_n (e^{P_n y} - e^{P_n(2w-y)}) e^{(v\sigma\mu x/2)} \sin(n\pi x/d) \right] \end{aligned}$$

ARMATURE REGION ANALYSIS

A diagram of the armature region is shown in Fig. 3.

Fig. 3. The armature region.

Half of the armature will be considered for the analysis since this is a symmetric problem. The diffusion equation in cartesian coordinates for the armature is;

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = 0 \quad (12)$$

The electrical boundary conditions that must be satisfied due to the geometry of the problem are as follows;

$$\begin{aligned} J_y(x, y) &= J_y(x, -y) \\ J_x(0, y) &= J_x(d, y) = 0 \\ J_x(x, 0) &= 0 \\ H_z(0, y) &= K \\ H_z(d, y) &= 0 \end{aligned} \quad (13)$$

Across the boundary region between the rail and the armature, the normal component of the current density and the tangential component of the electric field must be continuous.

Letting $H_z(x, y) = X(x)Y(y)$, equation (12) becomes;

$$\begin{aligned} \frac{d^2 X(x)}{dx^2} + k^2 X(x) &= 0 \\ \frac{d^2 Y(y)}{dy^2} - k^2 Y(y) &= 0 \end{aligned} \quad (14)$$

The selection of the sign of k^2 for each differential equation has been chosen according to the geometrical considerations of the problem. Considering only the geometry of the problem, the magnetic field and current density in the armature become;

$$\begin{aligned} J_x(x, y) &= \left[\sum_{n=1}^{\infty} (B_n e^{M_n x} + C_n e^{N_n x}) \left(\frac{n\pi}{w} \right) \cos \left(\frac{n\pi y}{w} \right) \right] \\ &+ \frac{(K/w) (e^{v\sigma\mu x} - e^{v\sigma\mu d})}{(e^{v\sigma\mu d} - 1)} \\ &+ \left[\sum_{n=1}^{\infty} P_n D_n (e^{P_n y} + e^{P_n(2w-y)}) e^{(v\sigma\mu x/2)} \sin \left(\frac{n\pi x}{d} \right) \right] \\ J_y(x, y) &= \left[\sum_{n=1}^{\infty} (-B_n M_n e^{M_n x} + C_n N_n e^{N_n x}) \sin \left(\frac{n\pi y}{w} \right) \right] \\ &+ \frac{(K/w) (y-w) (v\sigma\mu e^{v\sigma\mu x})}{(e^{v\sigma\mu d} - 1)} \\ &- \left[\sum_{n=1}^{\infty} D_n (v\sigma\mu/2) (e^{P_n y} - e^{P_n(2w-y)}) e^{(v\sigma\mu x/2)} \sin \left(\frac{n\pi x}{d} \right) \right] \\ &- \left[\sum_{n=1}^{\infty} D_n \left(\frac{n\pi}{d} \right) (e^{P_n y} - e^{P_n(2w-y)}) e^{(v\sigma\mu x/2)} \cos \left(\frac{n\pi x}{d} \right) \right] \end{aligned} \quad (8)$$

Region 3:

$$\begin{aligned} H_z(x, y) &= \left[\sum_{n=1}^{\infty} F_n \sin \left(\frac{n\pi y}{w} \right) e^{N_n(x-d)} \right] \\ J_x(x, y) &= \left[\sum_{n=1}^{\infty} F_n \left(\frac{n\pi}{w} \right) \cos \left(\frac{n\pi y}{w} \right) e^{N_n(x-d)} \right] \\ J_y(x, y) &= \left[\sum_{n=1}^{\infty} -F_n M_n \sin \left(\frac{n\pi y}{w} \right) e^{N_n(x-d)} \right] \end{aligned} \quad (9)$$

$$\begin{aligned} \text{where: } M_n &= (v\sigma\mu/2) + \sqrt{(v\sigma\mu/2)^2 + (n\pi/w)^2} \\ N_n &= (v\sigma\mu/2) - \sqrt{(v\sigma\mu/2)^2 + (n\pi/w)^2} \\ P_n &= -\sqrt{(v\sigma\mu/2)^2 + (n\pi/d)^2} \end{aligned} \quad (10)$$

For all current density calculations in this model,

$$J_x = \frac{\partial H_z}{\partial y} \quad J_y = -\frac{\partial H_z}{\partial x} \quad (11)$$

The values of A_n, B_n, C_n, D_n , and F_n are determined by continuity of the electric field and current density at the interfaces between the three rail regions and the region #2/armature boundary.

$$H_z(x, y) = \left[\sum_{n=1}^{\infty} G_n (e^{(n\pi y/d)} + e^{-(n\pi y/d)}) \sin(n\pi x/d) \right] - (K/d)(x-d) \quad (15)$$

$$J_x(x, y) = \left[\sum_{n=1}^{\infty} G_n (n\pi/d) (e^{(n\pi y/d)} - e^{-(n\pi y/d)}) \sin(n\pi x/d) \right]$$

$$J_y(x, y) = \left[\sum_{n=1}^{\infty} -G_n (n\pi/d) (e^{(n\pi y/d)} + e^{-(n\pi y/d)}) \cos(n\pi x/d) \right] + (K/d)$$

The values of G_n are determined by matching the tangential component of the electric field and the normal component of the current density at the interface between the rail and armature.

INTERFACE CONDITIONS BETWEEN REGIONS

In order to satisfy Maxwell's equations, continuity of the tangential electric field and normal current density must exist at all points across interface areas. By matching these conditions at the three interfaces, the values of A_n, B_n, C_n, D_n, F_n , and G_n may be obtained. The following results are obtained by the continuity requirements;

Region 1/2 Interface: Matching J_x

$$A_n = B_n + C_n \quad (16)$$

Region 1/2 Interface: Matching J_y

$$C_n = \frac{(2K/n\pi)(v\sigma\mu)}{(N_n - M_n)(e^{v\sigma\mu d} - 1)} - \frac{(2n\pi/w^2) \left[\sum_{q=1}^{\infty} \frac{D_q(q\pi/d)(1 - e^{v\sigma\mu d})}{(P_q)^2 + (q\pi/w)^2} \right]}{(N_n - M_n)} \quad (17)$$

Region 2/3 Interface: Matching J_x

$$F_n = B_n e^{M_n d} + C_n e^{N_n d} \quad (18)$$

Region 2/3 Interface: Matching J_y

$$B_n = \frac{(2K/n\pi)(v\sigma\mu)e^{v\sigma\mu d}}{(N_n - M_n)(e^{v\sigma\mu d} - 1)e^{M_n d}} - \frac{(2n\pi/w^2) \left[\sum_{q=1}^{\infty} \frac{D_q(q\pi/d)(1 - e^{v\sigma\mu d})(-1)^q}{(P_q)^2 + (q\pi/w)^2} \right]}{(M_n - N_n)e^{M_n d}} \quad (19)$$

The J_x and J_y fields are matched at the rail armature interface. This will result in two

expressions for G_n . The equations for G_n are set equal to each other, and equations (16) - (19) are substituted for A_n, B_n, C_n , and F_n . The result will be one equation in terms of D_n which is suitable for solution using linear equations and matrix techniques. Once D_n has been determined, the other Fourier coefficients are obtained from equations (16) - (19).

SAMPLE PROBLEM

A sample problem has been conducted with the rail and armature width equal to 1.0 cm and the armature length equal to 2.0 cm. The velocity of the projectile has been assumed to be 50 m/sec with both materials having the permeability and conductivity of copper. The steady state current density injected into the rails at $x = -\infty$ is assumed to be 100 A/m². Figures 4-10 depict the J_x , J_y , H_z , and a current streamline plot in the rail and armature. For comparison purposes, a current streamline plot is presented in Fig. 11 for a velocity of 0.1 m/sec.

The rail-armature corner is the area where the highest concentration of current will occur in the system. For this particular case, which is an extremely slow velocity for railguns, the current density has increased approximately thirty times from the steady state value at a far distance away from the armature.

Fig. 4. J_x in the rail. The peak value occurs at the interface corner with the armature. Peak value is -2900 A/m². Origin is shown in Fig. 2.

Fig. 9. Magnetic field intensity in the armature. The origin is shown in Fig. 3.

SINGLE LAYER RAIL SYSTEM RAIL CURRENT STREAMLINE PLOT

% OF TOTAL CURRENT
 — CURRENT = 20%
 — CURRENT = 40%
 — CURRENT = 60%
 — CURRENT = 80%

ARMATURE WIDTH = 1.00 CM
 ARMATURE LENGTH = 2.00 CM
 RAIL WIDTH = 1.00 CM
 VELOCITY = 0.1 M/S
 $\mu = 4\pi \times 10^{-7}$ H/M
 $\sigma = 5.75 \times 10^7$ MHQS/M
 $J_x(-\infty, Y) = 100$ A/M²

Fig. 11. Current streamlines for $v = 0.1$ m/sec

CONCLUSION

This paper has presented one possible approach to solving for the magnetic field and current densities in a simple geometry railgun. Complex geometries require numerical solutions of the magnetic diffusion equation. For a solid conducting armature, the skin effect which occurs at the rail-armature interface corner is significant at slow speeds. An estimate of the diffusion length is provided by the quantity M_n discussed in the paper. High armature speeds will cause damage to the railgun through ohmic heating and/or internal forces. Significant changes to rail and armature design must be considered if a solid conducting medium is to be used between the rails.

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SINGLE LAYER RAIL SYSTEM RAIL CURRENT STREAMLINE PLOT

% OF TOTAL CURRENT
 — CURRENT = 20%
 — CURRENT = 40%
 — CURRENT = 60%
 — CURRENT = 80%

ARMATURE WIDTH = 1.00 CM
 ARMATURE LENGTH = 2.00 CM
 RAIL WIDTH = 1.00 CM
 VELOCITY = 50.0 M/S
 $\mu = 4\pi \times 10^{-7}$ H/M
 $\sigma = 5.75 \times 10^7$ MHQS/M
 $J_x(-\infty, Y) = 100$ A/M²

Fig. 10. Current streamlines for $v = 50$ m/sec

Fig. 5. J_x in the armature. The peak value occurs at the interface corner with the rail. Peak value is -1300 A/m^2 . Origin is shown in Fig. 3.

Fig. 7. J_y in the armature. The peak value occurs at the interface corner with the rail. Peak value is 2800 A/m^2 . Origin is shown in Fig. 3.

Fig. 6. J_y in the rail. The peak value occurs at the interface corner with the armature. Peak value is 2800 A/m^2 . Origin is shown in Fig. 2.

Fig. 8. Magnetic field intensity in the rail. The origin is shown in Fig. 2.