PHYSICAL SCALE MODELING TO VERIFY ENERGY STORAGE INDUCTOR PARAMETERS

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Summary

In the pulse charging and discharging of energy storage inductors, the influence of I^2R heating, magnetic diffusion skin depth, and eddy current losses must be evaluated. These effects are not easily treated analytically. To address these problems, the system can be constructed according to governing scaling laws. The performance of the model can then be evaluated and scaled up to predict actual system performance. The scaling laws as applied to a 10-MJ homopolar generator charging a 2-MJ cryogenic aluminum Brooks coil are presented in this paper. System parameters measured on the scale model are compared to parameters subsequently measured on the full-size equipment.

Introduction

In July of 1981 a joint project was undertaken by CEM-UT and General Dynamics Corporation to drive an electromagnetic gun with a ten-megajoule (10-MJ) homopolar generator (HPG) charging an energy storage inductor. The HPG was to be the existing 10-MJ generator at The University of Texas at Austin. This generator at the time of the contract had an operating speed of 460 rad/s (4,400 rpm). At an excitation of 0.57 Wb, the open circuit voltage of the generator was 41.5 V. The terminal characteristics of the generator had been measured and verified through a system energy balance. The resistance of the generator was 18.8 $\mu\Omega$ and the inductance was 0.48 μH .

The inductor selected was a Brooks coil design. It is reported in the literature that this configuration gives an optimum inductance for a given amount of material.¹ The coil was wound from aluminum busbar stock. By cooling the inductor to liquid nitrogen temperature, further gains in energy storage could be realized. Table 1 defines the Brooks coil geometry and also presents the design formulas. After selecting a reasonable efficiency of energy transfer from the HPG to the inductor, the desired inductance was found to be 13 μH . In the preliminary design phase the bus resistance was neglected and a coil with a resistance of 4 $\mu\Omega$ would allow operation of the system at maximum generator design current. This design iteration allowed a first pass at determining the number of turns in the coil and the number of coil starts required from skin effect calculations. A drawback of the Brooks coil design is that it must be placed in a shield structure to prevent large magnetic flux levels in the laboratory during pulse charging. With the preliminary design of the coil and shield completed, a parallel-plate busbar was designed to connect the generator to the coil. The calculated resistance and inductance of this buswork are presented in Fig. 1 along with the HPG parameters and the preliminary coil parameters. A linear analysis of this circuit predicted an underdamped current waveform that peaked at 596 kA in 0.45 s. This level of performance was acceptable for the electromagnetic launcher design under consideration.

There were several nonlinearities that might substantially degrade system performance but were not easily treated analytically. These included the I^2R heating in the conductor, the magnetic diffusion skin depth during charging and discharging the coil, and the eddy current losses in the shield structure. Since these cumulative effects are not easily treated analytically,

Fig. 1. Circuit model for full-size system

Table 1. Brooks Coil Formulas²

1. $L = 0.017 \text{ aN}^2 (\mu\text{H})$
  2. $c = 2a/3 (\text{cm})$
  3. $R = \text{resistance} = \frac{\rho l}{A} = \frac{2\pi N^2 a}{2} = 2.69N^2 a/c^2 (\mu\Omega)$
  4. $J = \text{current density} = IN/c^2 (\text{A}/\text{cm}^2)$
  5. $M = \text{weight} = w 2\pi ac^2 = 17 ac^2 (\text{g})$
- $L = \text{coil inductance} (13 \mu\text{H})$
 $I = \text{peak current} (\text{kA})$
 $\rho = \text{resistivity of conductor material} (3/7 \mu\Omega \cdot \text{cm})$
 $w = \text{density of conductor material} (2.7 \text{ g}/\text{cm}^3)$
 $a = \text{dimension shown in figure} (\text{cm})$
 $c = \text{dimension shown in figure} (\text{cm})$
 $N = \text{number of turns in the coil}$

tically, a physical scale model of the system was built and tested.

Model Theory

To make a physical scale model of electromagnetic (or other) phenomena, a model system must be established in which the EM quantities (such as flux density and electric field intensity) of consequence are related by constants to corresponding quantities in the system for which the model is being made.²

The crucial relationships for the proposed model are given by Maxwell's equations

$$\begin{aligned} \nabla \times \vec{H} &= \vec{J}, & \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} &= 0, & \nabla \cdot \vec{J} &= 0 \\ \vec{J} &= \sigma \vec{E}, & \vec{B} &= \vec{F}(\vec{H}) \end{aligned}$$

$$v = \frac{d}{dt} \int_S \vec{B} \cdot \vec{n} da, \quad i = \int_S \vec{J} \cdot \vec{n} da, \quad p = vi$$

where

$\vec{H}$ = magnetic field intensity	$\vec{n}$ = unit normal vector
$\vec{J}$ = current density	da = area differential
$\vec{E}$ = electric field intensity	p = power
$\vec{B}$ = magnetic flux density	v = voltage
σ = conductivity	i = current

$$\nabla = \bar{T}_x \frac{\partial}{\partial x} + \bar{T}_y \frac{\partial}{\partial y} + \bar{T}_z \frac{\partial}{\partial z}$$

$\bar{T}_x, \bar{T}_y, \bar{T}_z$ are unit vectors in the axis directions
 x, y, z are displacements; t is time

If the model is to represent accurately the phenomena that occur in the system for all conditions of interest, a variable in the model must have a constant correspondence with the same variable in the system. Subscript m refers to the model; subscript s refers to the system being modeled. We express this relation as

$$\begin{aligned} x_m &= k_x x_s, & y_m &= k_y y_s, & z_m &= k_z z_s \\ t_m &= k_t t_s, & H_m &= k_H H_s, & J_m &= k_J J_s \\ E_m &= k_E E_s, & B_m &= k_B B_s, & \sigma_m &= k_\sigma \sigma_s \\ v_m &= k_v v_s, & i_m &= k_i i_s, & p_m &= k_p p_s. \end{aligned}$$

Substitution of these quantities into the model equations and collection of the constants on one side of each equation show that in order for the model to be a true replica of the system in space and time the following relations among the constants must be satisfied.

For example, $\nabla_m \times \bar{H}_m = J_m$ because

$$\frac{\partial}{\partial x_m} = \frac{\partial}{k_x \partial x_s} = \frac{1}{k_x} \frac{\partial}{\partial x_s}; \quad \frac{1}{k_x} \nabla_s \times k_H \bar{H}_s = k_J \bar{J}_s;$$

$$\frac{k_H}{k_x k_J} [\nabla_s \times \bar{H}_s] = \bar{J}_s.$$

The point form of Ampere's circuital law for the system being modeled requires that

$$\frac{k_H}{k_x k_J} = 1. \quad (2)$$

Similarly,

$$\frac{k_J}{k_\sigma k_E} = 1 \quad (3)$$

$$\frac{\bar{F}_m(k_H \bar{H}_s)}{k_B} = \bar{F}_s(\bar{H}_s). \quad (4)$$

The last equation ensures proper scaling of magnetic saturation. To be certain that the magnetic properties of air have the same relative effect in model and system,

$$\frac{k_H}{k_B} = 1. \quad (5)$$

With this constraint, the relation between $\bar{F}_m$ and $\bar{F}_s$ is set as a scaling by equal factors along both coordinates. For example, a magnetization curve will be scaled by equal factors on flux and mmf.

Using this last relation ($k_B = k_H$) we eliminate k_H and obtain the three equations

$$\frac{k_B}{k_x k_J} = 1; \quad \frac{k_E k_t}{k_B k_x} = 1; \quad \frac{k_J}{k_\sigma k_E} = 1. \quad (6a,b,c)$$

There are six constants. Three may be set arbitrarily and the rest will be determined by these equations.

Which three constants are to be set arbitrarily is a question to be answered in terms of the objective of the modeling process.

Further derivations of modeling constants relating voltage, current, resistance, inductance, and capacitance are presented in Appendix A.

Model System

As mentioned earlier, the HPG terminal characteristics are well known. We are interested in the electrical characteristics of the coil situated in the shield structure and its associated interconnecting buswork. In this case the model is a hybrid in which the HPG is modeled by an electrolytic capacitor bank having a low equivalent series resistance. The rest of the circuit will be a scale model of the energy storage inductor, shield structure, and interconnecting buswork.

When the individual components of the model system are lumped, the circuit becomes simply an RLC series circuit (Fig. 2).

Fig. 2. Model system circuit

$$R_m i_m + L_m \frac{di_m}{dt_m} + \frac{1}{C_m} \int i_m dt_m - V_{om} = 0. \quad (7)$$

where R_m is model resistance, L_m is model inductance, C_m is model capacitance, i_m is model current, and V_{om} is model open-circuit voltage.

By introducing the scaling factors this may be rewritten as

$$k_R R_s k_i i_s + k_L L_s \frac{d k_i i_s}{d k_t t_s} + \frac{1}{k_C C_s} \int k_i i_s d k_t t_s - k_v V_{os} = 0 \quad (8)$$

Dividing Eq. (8) by $k_R k_i$ and substituting the identities for k_T, k_L, k_C, k_i , and k_v (derived in Appendix A) into Eq. (8) gives

$$R_s i_s + \frac{k_\sigma^2 k_\sigma}{k_t} L_s \frac{di_s}{dt_s} + \frac{k_\sigma^2 k_\sigma}{k_t} \frac{1}{C_s} \int i_s dt_s - \frac{k_\sigma^2 k_\sigma}{k_t} V_{os} = 0. \quad (9)$$

If $k_\sigma^2 k_\sigma / k_t$ is equal to 1, then the model is a true replica of the system. By manipulation of Eqs. (6), this identity is seen to hold under the already established scaling laws. Therefore, Kirchhoff's law has put no further constraints on the model system.

During construction and experimentation the coil material is heated, cooled, and work hardened. All of these processes affect the conductivity of the material. To insure that these effects are incorporated in the model, the model coil is constructed from the same aluminum alloy used to build the system coil. This insures that K_σ is unity.

The next parameter to examine is K_B , which determines the effect of the shield on the charging circuit. As mentioned in the Model Theory section, a magnetization curve must be scaled by equal factors on flux and mmf. The full-size system shield is made from 2.54-cm (1-in.) thick hot rolled plate, while the model system shield is made from cold rolled sheet. Steady-state flux plots of the full-size system show that the system shield is driven into saturation at one-fourth of the peak charging current. By making the magnetic field intensity the same in both model and full-size system, $K_H = 1$, the model shield will also saturate early in the charging sequence. Even if the magnetization curves for the two shield materials are not the same, changes in the field intensity in the saturated materials will produce equal changes in the flux density in the model shield and the system shield. This

changing flux density produces potentials in the shield material that drive eddy currents. The losses associated with these eddy currents are reflected into the coil charging circuit. By measuring the impedance of the model circuit during pulse charging, the performance of the full-size system can be predicted. This establishes the second scaling factor $K_B = 1$. Substituting this into Eqs. (6a) gives

$$k_\ell k_J = 1 \quad k_J = \frac{1}{k_\ell} \quad (10)$$

Also,

$$k_i^2 = k_J k_\ell \quad (\text{Appendix A}) \quad (11)$$

Substituting Eq. (10) into Eq. (11),

$$k_i = k_\ell \quad (12)$$

Furthermore, from Eq. (6c) with $k_\sigma = 1$,

$$k_E = k_J \quad (13)$$

Substituting Eq. (10) into Eq. (13),

$$k_E = \frac{1}{k_\ell} \quad (14)$$

By examining Eq. (6b) and using $k_B = 1$ and $k_E = 1/k_\ell$, it is found that

$$k_t = k_\ell^2 \quad (15)$$

The scaling factor for the voltage is

$$k_v = \frac{k_B k_\ell^2}{k_t} \quad (\text{Appendix A}) \quad (16)$$

Substituting Eq. (15) and $k_B = 1$ into Eq. (16) yields

$$k_v = 1 \quad (17)$$

From Appendix A, the scaling for the capacitance is

$$k_c = \frac{k_t^2}{k_\ell} \quad (18)$$

Substituting Eq. (15) into Eq. (18) gives

$$k_c = k_\ell^3 \quad (19)$$

Relating the various scaling factors to the dimensional scale for the system helps the modeler see various tradeoffs as the physical realization of the model progresses. For instance, either a heavy fast-acting high-current mechanical switch or an extremely fast, moderate-current electronic switch can be used to discharge the capacitor bank into the inductor.

The mechanical switch can close in ~3 ms and switch hundreds of kiloamperes. The switching time is designed to be a small percentage of the current rise time, say five percent. Therefore, the rise time will be greater than $0.003/0.05 = 0.6$ s; k_t will be approximately $t_m/t_s = 0.06$ s/ 0.410 s = 0.146 and $k_\ell = 0.382$; $k_c = k_\ell^3 = 0.056$, and C_m will be 436 F.

If the electronic switch, a high-current capacity SCR, is used, the switching time will be 50×10^{-6} s. Therefore the rise time of the model circuit can be as low as 50×10^{-6} s/ 0.05 s = 1×10^{-3} s. Then, k_t becomes 1×10^{-3} s/ 0.41 s = 2.44×10^{-3} and $k_\ell = 0.049$. $k_c = k_\ell^3 = 1.20 \times 10^{-4}$ and $C_m = 0.939$ F.

The choice is obvious. The electronic switch was chosen so that a reasonably sized capacitor bank results. An International Rectifier 7641 SCR with a half-cycle surge rating of 35 kA has been used at CEM-UT to switch 20 kA in a pulse width of 1 to 2 ms. Higher currents at this pulse width have damaged this

particular device. Attempts to parallel these devices have not proven successful in past experiments. The model current is therefore restricted to 20 kA.

With this decision k_i will be $20 \text{ kA}/596 \text{ kA} = 0.034$. But, Eq. (12) requires $k_i = k_\ell$, which has already been fixed at 0.049 by this semiconductor device's turn-on time. This will ultimately lead to an error in the modeling.

In addition it was pointed out earlier that the large coil would be fabricated from 1.27×20.3 -cm ($1/2 \times 8$ -in.) busbar stock. The thickness of the model conductor would then be 0.049×0.5 in. $\times$ (2.54 cm/in.) = 0.062 cm (0.0245 in.). Aluminum alloy 6101 is not available in thin sheets. A piece of the 1.27×20.3 -cm ($1/2 \times 8$ -in.) stock was cut to produce material for the model coil. The minimum conductor thickness that could be obtained was 0.079 cm (0.0312 in.). Therefore k_ℓ was actually

$$k_\ell = \frac{0.0312 \text{ in.}}{0.5 \text{ in.}} = 0.0625 \quad (20)$$

A second error was introduced at this point. With $k_\ell = 0.0625$, $k_c = 2.44 \times 10^{-4}$. The required capacitance value for the model is then 1.91 F. The capacitor required a low equivalent series resistance (ESR) in order to permit large pulsed currents to be delivered by the bank. This type of capacitor is a long-lead-time item, and only thirty 0.037 F, 50 -V low-ESR cans were available at the time. The decision was made to build a 30×0.037 F = 1.11 -F bank and evaluate the scaling error when the model testing began. The characteristic dimension c of the full-size Brooks coil (Table 1) is 20.3 cm (8 in.). This dimension is 20.3 cm $\times$ $0.062 = 1.27$ cm (0.5 in.) for the model. The 0.079 -cm (0.0312 -in.) stock was sheared into 1.27 -cm (0.5 -in.) widths. The coil was wound on a G-10 form machined in the shape of a spool. The hub of the spool was machined with three equally spaced axial slots. The coil starts and finishes were laid in these slots for bracing against discharge forces. A strip of 0.0127 -mm (0.005 -in.) Mylar was used to insulate the individual starts from one another. The complete coil was then banded to provide additional mechanical strength.

The capacitor bank was built by constructing a heavy copper bus to parallel thirty 0.037 -F, 50 -V, 240×10^{-6} -ESR electrolytic cans. The capacitance of the bank is therefore 1.11 F.

The parallel-plate busbar conceived for connecting the HPG to the full-size coil was constructed to 0.0625 scale. The SCR switch was clamped to the positive output of the capacitor bank. The model busbar then connected the coil terminals to the switch and the negative terminal of the capacitor bank.

A scale-model shield was constructed from 0.065×2.54 -cm = 0.159 -cm (0.0625 -in.) sheet steel. A 10.46 -cm (4 -in.) PVC end cap was used as a cryogenic tank for cooling the model coil in liquid nitrogen.

With the model coil cooled and situated in the shield, the capacitor bank was charged to 34.7 V and discharged (Fig. 3). The current rose to 20.8 kA in 1.65 ms. The following circuit parameters result after fitting the data to a linear circuit model (Fig. 4).

Conclusions

The realized scaling parameters are:

$$k_\ell = 0.0625 \quad (21)$$

$$k_\sigma = 1 \quad (22)$$

$$k_i = \frac{20,800}{596,000} = 0.0349 \quad (23)$$

Fig. 3. The model experiment

Fig. 4. Measured model circuit parameters

$$k_J = \frac{k_i^2}{k_\ell} = 8.93 \quad (24)$$

$$k_B = k_\ell k_J = 0.558 \quad (25)$$

$$k_t = \frac{0.00165}{0.410} = 0.004 \quad (26)$$

$$k_v = \frac{34.7}{41.5} = 0.836 \quad (27)$$

$$k_E = \frac{k_v}{k_\ell} = \frac{0.836}{0.0625} = 13.3 \quad (28)$$

$$\text{From Eq. (6)} \quad \frac{k_B}{k_\ell k_J} = \frac{0.558}{0.0625 \times 8.93} = 0.99, \quad \frac{k_E k_t}{k_B k_\ell} = 1.52,$$

$$\frac{k_J}{k_\ell k_E} = \frac{8.93}{1 \times 13.3} = 0.67$$

The inaccuracies were that the shield was not driven quite as hard into saturation and the conductor did not heat as much. The scaling for the magnetic saturation was good because the time, conductivity, and dimensional scaling related well. k_t should equal k_ℓ^2 with $k_\sigma = 1$ (Eq. (15)).

In actuality $k_t = 0.004$

$$\text{and} \quad k_\ell^2 = (0.0625)^2 = 0.0039.$$

With the model parameters measured, the scaling factors can be used to transfer these quantities into the full size system. The busbar and coil parameters were scaled in this manner. The generator parameters were measured with the generator under load. The resulting circuit is shown in Fig. 5. In this circuit the current peaks at 470 kA in 0.35 s. The model has predicted that the impedance of the coil will be larger during the pulse charging than the steady-state predicted value. Because of the limited diffusion depth of current into the conductor, the charging current and total energy stored will be reduced. Testing on the

Fig. 5. Full-size system with scaled model parameters

full-size system confirmed this prediction, and the measured parameters are presented in Table 2.

Table 2. Model and Full-Size System Comparison

	Model	Model Scaled to Full-Size System	Full-Size System
<u>Cryogenic Brooks Coil in Shield:</u>			
Time to peak, ms	1.50	408	340
Resistance, $\mu\Omega$	159	9.9	10.4
Inductance, μH	0.88	14	10
<u>Room Temperature Brooks Coil in Shield:</u>			
Time to peak, ms	1.41	361	300
Resistance, $\mu\Omega$	669	41.8	45
Inductance, μH	0.9	14.4	10.82

Just as important, it can be seen that the model predicts an inductance very close to the Brooks coil formula of Table 1. The coil in the full-size system was measured to have a smaller value. The reason for this is that the model coil was wound and banded by hand, whereas the full-size coil was banded with a machine that held a constant tension of 2,200 N (100 lbf). The full-size system coil could be seen to tighten as the banding was applied. Equation (1) of Table 1 predicts a larger value of inductance for the loosely wound coil, and this is what was measured. The importance of holding the scaling factors in the model system is well demonstrated with this measurement.

APPENDIX A

$$\begin{aligned} i_m &= k_i i_a & v_m &= k_v v_s & R_m &= k_R R_s \\ k_i &= k_J k_\ell^2 & k_v &= \frac{k_E k_\ell}{k_t} & k_R &= \frac{1}{k_\ell k_p} \\ L_m &= k_L L_s & C_m &= k_C C_s & & \\ k_L &= k_\ell & k_C &= \frac{k_t^2}{k_\ell} & & \end{aligned}$$

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