SIMULATION AND PREDICTED PERFORMANCE OF THE BALCONES GEDI EXPERIMENT

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Abstract

The guided electromagnetic defense interceptor (GEDI) railgun launcher system is powered by six Balcones homopolar generators (BHPG), each of which stores 10 MJ in kinetic energy. Each BHPG is connected to a coaxial inductor which stores about half of the BHPG's energy when charged to 1.2 MA. Six two-stage opening switches open in sequence or simultaneously after current peak, directing the current to the railgun armature. As much as 7.2 MA can be delivered to the railgun. A computer simulation code is developed to perform design parameter studies, sensitivity studies, and to predict GEDI system behaviors. The code is general enough for analyzing any railgun driven by multi-HPG inductors coupled with two-stage opening switches. A Gear type, multi-step algorithm is used to overcome stiffness of the differential system caused by the way the switches are modeled. GEDI system performances are predicted with the code based on most updated component parameters.

INTRODUCTION

The GEDI railgun experiment is a railgun system powered by six, 10-MJ BHPG's, each of which is connected in series with an energy storage inductor capable of storing about 45% of kinetic energy originally stored in the BHPG. The BHPG charges up the inductor which is then discharged to a railgun by opening a two-stage switch. To design, study, and predict the performance of the GEDI experiment, a system simulation code has been developed.

This paper first introduces and discusses the mathematical models on which the code is based. Then, a discussion explains the operation of the simulation code. After that, a description of the present conception and design of the GEDI system's physical parameters follows. Finally, those physical parameters are fed into the simulation to generate performance predictions.

MATHEMATICAL MODELS OF SYSTEM COMPONENTS

The electrical circuit model for the GEDI system consists of six power supply modules whose outputs are connected in parallel to the railgun breech. A circuit representation of a typical power module is shown on Fig. 1.

Fig. 1. Circuit model for GEDI power supply Module #1

Fig. 1 shows the HPG as a voltage source, V_{HPG1} . This voltage depends on the flux and the angular velocity of HPG#1. The equations governing the transformation of mechanical energy to electromagnetic energy in the HPG are not modeled in the circuit. They will be explained below.

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HPG Model

Each HPG is characterised by

- i) ϕ_i = the flux linked by HPG #i brushes,
- ii) J_i = the polar moment of inertia of the HPG #i's rotating parts coupled to the electromagnetic circuit,
- iii) T_{bi} = the brush drag torque applied on HPG #i's rotor retarding rotating motion,
- iv) ω_{oi} = initial angular velocity of the HPG #i's rotating parts, and
- v) the number of brushes and the brush voltage drop characteristics as a function of current per brush.

If we index each BHPG module with corresponding subscripts, each BHPG is governed by Equations (1) and (2), where

ω_i = the instantaneous angular velocity of BHPG # i

$$V_{HPGi} = \frac{\omega_i \phi_i}{2\pi} \quad (1)$$

$$\frac{d\omega_i}{dt} = -\frac{1}{J_i} \left[\frac{\phi_i i_{HPGi}}{2\pi} + \left| \frac{\omega_i}{\omega_i} \right| T_{bi} \right] \quad (2)$$

Based on previous experience with the Center for Electromechanics iron core, all iron rotating, compact homopolar generator (CHPG), the flux linked by the HPG's brushes varies generally as a function of the output current, while the excitation current stays constant. This phenomenon is a characteristic of individual HPG's, and it needs to be modeled for any particular HPG under consideration. For the BHPG, ϕ_i is modeled as a function of HPG #i's output current, i_{HPGi} , as follows:

$$\frac{\phi_i}{\phi_o} = 1 - [2.6 \times 10^{-14} (i_{HPGi} - 200,000)^2 + 2.3 \times 10^{-7} (i_{HPGi} - 200,000)] \quad (3)$$

where ϕ_o is the open circuit flux of the HPG.

The voltage drop in HPG brushes also varies as a function of output current. Based on previous experience with the type of brushes used in the BHPG, the variable brush resistance, R_{brush} , is modeled as

$$|R_{brushi}| = \frac{1.105 \left[1 - e^{-\frac{|i_{HPGi}|}{1,360}} \right]}{|i_{HPGi}|} \quad (4)$$

Inductor Model

The inductors are assumed to be circular, coaxial, and multi-turn. They are made from non-magnetic materials. In our applications, especially during discharge, the frequencies of the transient are probably in the range where eddy current in the inductor conductors is significant. For that reason, a transient electrical model has been formulated for this type of coaxial inductor. The formulation is implemented in the code.

The charging current profile is treated as a quarter of a sinusoid in determining the transient resistance. Apart from the initial value, a time

varying equivalent resistance model is obtained based on Equations (5a) to (5e):

$$P_c(t) = \frac{K_m^2}{\sigma d} [\sin^2 \omega t + \sum_{n=1}^{\infty} 2f_n^2(t) + \sum_{n \text{ odd}} 8N(N+1)f_n^2(t)] \quad (5a)$$

where

$$K_m = \frac{I_{\text{peak}}}{2\pi R} \quad (5b)$$

$$\bar{R} = \frac{R_{\text{in}} + R_{\text{out}}}{2} \quad (5c)$$

$$f_n(t) = \frac{\omega \tau_n [\cos \omega t + \omega \tau_n \sin \omega t] - \omega \tau_n e^{-\frac{t}{\tau_n}}}{\omega^2 \tau_n^2 + 1} \quad (5d)$$

$$\tau = \frac{\mu_o d^2}{n^2 \pi^2} \quad (5e)$$

In the above equations, R_{in} and R_{out} are the inner and outer radii of each cylindrical conductor in the inductor, d is conductor thickness, I_{peak} is the inductor peak current, and n is the integral multiple of the current enclosed by the conductor under consideration. Equation (5a) gives the instantaneous ohmic power loss in each conductor in the inductor due to the calculated total current distribution (which is a sum of the transport and circulation current) in the conductor. At each integration time step, this instantaneous power is divided by the square of the current calculated at that time step to compute a transient resistance for the next time step. At the next time step, that transient resistance is substituted in the lumped resistance parameter for the inductor, and the circuit problem is solved for that time step based on that. Naturally, based on this scheme, we do not have a transient resistance calculated for the initial time step because there is no last step from which the transient resistance can be computed. To overcome this problem, the dc resistance is taken as the initial resistance of the inductor! While the overall current of the HPG is not very sensitive to the inductor resistance for the first time step, we believe this approximation should not cause significant error in the overall results.

During discharge, the current profile is assumed to be a decaying exponential. Clearly, the discharge current profile is probably not exactly a decaying exponential, but this assumption should give us a good estimation of the transient resistance of the inductor. At the code level, we implement a scheme similar to the one for charging to calculate transient resistance as follows.

Generally, for any simple exponential, $A_0 e^{-\alpha t}$, given the value of the function at two points, and the value of the independent variable at those points, one should be able to recover the assumed exponential function. We take the current peak point as one point, and the current at the current time step as the other to recover an assumed exponential function. With the exponential function assumed, dividing the ohmic power loss in Equation (6a) by the square of the current at the time march front, an equivalent inductor resistance is calculated for each subsequent time step for the lumped circuit model.

$$P_c(t) = \frac{I_{\text{peak}}}{\pi(R_{\text{in}} + R_{\text{out}})\sigma d} (e^{-2\alpha_d \Delta t} + \sum_{n=1}^{\infty} 2g_n(\Delta t) + \sum_{n \text{ odd}} 8N(N+1)g_n(\Delta t)) \quad (6a)$$

$$g_n(\Delta t) = [e^{-\frac{2\Delta t}{\tau_n}} - 2e^{-(\alpha_d + \frac{1}{\tau_n})\Delta t} + e^{-2\alpha_d \Delta t} (\frac{\alpha_d \tau_n}{\alpha_d \tau_n + 1})^2] \quad (6b)$$

where α is the time constant of the assumed exponential.

Notice that in both formulations for charging and discharging, the infinite sums are implemented in the code as finite sums, accounting for about 30 terms of the series.

Clearly, this scheme has an averaging effect such that, for instance, the calculated impedance that a sudden drop in current encounters is probably less than what actually occurs. To get a more accurate prediction of the transient resistance, the program must take into account the total history of the current since the zero state. That means if we want to attack the problem more rationally, we must have the code store the history of the current profile as the essential information to compute impedances for the state under consideration. In our approach, we store only the peak current and the peak time, assume a simple exponential decay from the peak current to the point being considered, and we neither store nor consider intermediate values of the current for transient resistance calculations. This approach, though not perfect, is convenient and should be much better than using an equivalent constant resistance calculated *a priori* based on a presumed current profile, as far as modeling the transient behavior of the inductor is concerned.

Since the transient behavior of the inductor has significant influence on the amount of energy that we can recover from the inductor and deliver to the armature, we will improve our model as we have more experimental results to indicate how valid the present model is. The logical next order of improvement is as follows. First, we implement the current distribution and power loss due to a step (or a ramp) of current rise in the program. Then, as the integration marches on, the program stores the pointwise representation of the current profile. Treating each current rise from one time step to the next as a scaled step (or ramp) of current, we can superimpose the responses of all the steps due to the linear properties of the material to find the power loss at the time-marching front.

Opening Switch Model

The opening switch has two stages. The first stage opens by sliding contact from a conductor to an insulator. The second stage opens by exploding aluminum conductor plates. Prior to opening, the mechanical stage is modeled as a constant resistance in series with a constant inductance. The explosive stage is modeled as a time-varying resistance, which depends on the temperature of the conductor plates, in series with a constant inductance. Obviously, the inductance of the opening switch necessarily varies with time because of changes in geometry, not to mention the change of current distributions in the conductors. We think our models should be good first order approximations as far as the macro-system behavior is concerned.

In the course of opening, the resistance of each stage of the switch increases from its dc value to an unbounded value, at which point the stages are open circuits. For the code, instead of allowing the resistance to become infinite, we bound the resistance at a relatively high value.

The time-varying resistance parameters for the opening switch are obtained from tests on actual switches. We expect the electrical characteristics of the switches to vary as the level of current through them. For that reason, we will improve and update our switch models from more experimental data to be obtained in future railgun shots.

Ignitron Model

Each ignitron is modeled as a "high-or-zero" resistance in series with a constant inductance. When an ignitron is turned off, it is open, and in the code, a very high resistance is assigned to it. When an ignitron is turned on, the ignitron's resistance is replaced with a 50 V constant arc drop. The inductance of the ignitrons is based on manufacturers' specifications.

The ignitrons are to be triggered individually such that they can be turned on sequentially to create a flatter and longer railgun current pulse. Each ignitron is set to be triggered when the voltage across the corresponding opening switch reaches a certain value after opening.

Railgun Model

The railgun circuit is modeled as a resistance, R_{gun} , in series with an inductance, L_{gun} , and an armature arc drop. The value of the arc drop depends on the armature material and its thermodynamic state. Since the voltage at the gun breech is probably significantly higher than the armature drop, we choose 50 V as the arc drop for convenience. Rigorously, the values of R_{gun} and L_{gun} are functions of position of the armature, and the current distribution across the rail's cross-section. At present, no skin depth model for the rail is implemented in the code. The user has to bear that in mind and use equivalent R_o , R' , L_o , and L' to take skin depth into account. Ignoring the skin effect, for a "constant cross-sectioned" gun, R' and L' , the per-unit length resistance and inductance are constant. R_{gun} and L_{gun} are approximated according to Equation (7).

$$R_{gun} = R_o + R' x; L_{gun} = L_o + L' x \quad (7)$$

Projectile Model

We model the projectile as a solid which experiences a drag force that is a fraction of the acceleration force applied on the back of the projectile. The magnitude of the fraction is a user input, and thus can be specified by the user easily. For plasma shots, we use zero drag. For solid projectiles, we use a 20% drag so that the net acceleration force is

$$F = (0.80)(0.5)L' i^2_{gun} \quad (8)$$

System Governing Equations

Using the currents at circuit branches as the state variables, a matrix differential equation in the following form can be obtained:

$$L \frac{d}{dt} I = V \quad (9)$$

where L is a spare matrix containing all the inductance terms, I is a vector containing all currents in all branches, and V is a vector containing linear combinations of voltages across all resistances.

In addition to (9), each HPG's mechanical to electrical energy transformation, namely Equations (1) and (2), also constitute the governing equations. Also, the integrator integrates the power of all circuit resistances for three purposes. First, we want to examine the mechanical and electrical energy losses in the system. Second, by summing up all the energies at every time step, we get an energy

balance. That balance should indicate the relative magnitude of signed errors accrued due to numerical computations. Third, if we made an error in implementing the differential equations to the code, the energy balance term will probably not be close to zero, and it may grow to values that cannot be handled by the computer, making detection of such an implementation error inevitable. Totally, about 50 first order differential equations must be solved by the integrator.

PROBLEM STIFFNESS

Initially, a seventh order Runge-Kutta integration subroutine was used to solve the set of time-varying ordinary differential equations. Stiffness (or numerical instability) becomes a big problem as the integration reaches the range where the resistances of the switches become very big (subsequent to their opening). The computation time taken to go through the stiff region is unacceptably long even for a single HPG case. It is possible to take the opening switches out of the circuit when the current through them becomes low relative to other currents, and the problem should become less stiff. However, that is undesirable because of the complications on the code. To overcome the stiff problem, a Gear-type method is used. The multi-step Gear-type method used is more difficult to control but does solve the stiff problem. The Jacobian of the governing equation, required for the integrator, is quite tedious to formulate in closed form. For that reason, the Jacobian is calculated by the code using finite differences.

VERIFICATION OF THE CODE

Since the code is to be general enough for different switches, railguns, and HPG's, it has been written in a highly modular structure so that modifications and checking can be done with less chance of causing program errors. Each module is carefully and thoroughly tested against the mathematical models to insure that the code does solve the model problem with acceptably small numerical errors. We intend to verify the code against experimental results so that we can improve our mathematical models in case experimental results show evident discrepancies. So far, railgun shots made with the CHPG yield results consistent with the code's predictions, within the error bounds of the measuring devices. While the GEDI system is still being built, the code will further be verified against more carefully instrumented railgun experiments powered by the CHPG.

GEDI SYSTEM PARAMETERS

The code was used to perform design and performance studies for the GEDI experiment. The BHPG has the following design parameters:

$J_{BHPG} = 49.2 \text{ kg-m}^2$	Open circuit $\phi_{BHPG} = 1.085 \text{ Wb}$
$T_b = 1370 \text{ N-m}$	Open circuit voltage = 110 V
Rated $\omega_o = 637 \text{ rad/s}$	(180 m/s brush slip speed)
$R_{BHPG} = 8.88 \mu\Omega$	$L_{BHPG} = 175 \text{ nH}$

The code was used to investigate the design parameter space of the inductor. The objective is to find an inductor geometry to match the BHPG to satisfy the electrical requirements of the system, as well as other requirements. After considering various configurations, materials and a very wide range of sizes with the aid of a CDC mainframe computer, taking into account the cost factors, environmental restrictions, and the system's usual modes of operation, the inductors are designed as follows:

Material: ETP copper	
Configuration: Circular coaxial two-turn	
Length = 15 ft, Outer diameter = 55 in.	
Conductor thicknesses: 1/2 in. to 3 in.	
Total dc resistance: 9.1 $\mu\Omega$	Inductance: 6 μH

The opening switches have the following parameters at room temperature:

$$\begin{aligned} R_{\text{mech sw}} &= 4.1 \mu\Omega & L_{\text{mech sw}} &= 80 \text{ nH} \\ R_{\text{expl sw}} &= 28 \mu\Omega & L_{\text{expl sw}} &= 70 \text{ nH} \end{aligned}$$

The explosive stages open about 4 ms after the mechanical stages.

Bus resistances: $4 \mu\Omega$ (this accounts for the bus from HPG to inductor and from inductor to switches)

The ignitron's inductance is assumed to be 90 nH based on the specifications of a possible choice of available ignitrons.

GEDI SYSTEM PERFORMANCE PREDICTIONS

Based on the system parameters described above, the code is used to predict the performance of the GEDI railgun experiment. Assuming that the inductors are operating at room temperature, over the course inductor charging, the code predicts the following:

Current risetime = 162 ms , Peak current in each BHPG = 1.2 MA
Angular velocity at current peak, $\omega(t_{\text{peak}}) = 266 \text{ rad/s}$

Figure 2 shows the computed charging current profile.

Fig. 2. HPG current during charging

At t_{peak} , the energy balance is accounted as follows:

Loss in brush friction	1.1 %
Loss in BHPG resistances	13.2 %
Loss in bus resistances	5.3 %
Loss in inductor resistance	13.5 %
Loss in switch resistances	5.1 %
Kinetic energy remaining in BHPG rotor	15.3 %
Inductive energy in the inductor	44.7 %
Inductive energy in other inductances	1.8 %

All the above are percentages of the kinetic energy initially stored in the BHPG, which is totally 10 MJ per HPG.

Using the code, we examine two discharge cases: one with a 2-g projectile, the other one with a 2-kg projectile. In both cases, we will open the switches sequentially in time. That will give a relatively flatter gun current profile at the expense of a longer gun to realize the same launch velocity.

Case 1:

Gun characteristics:

$$R_o = 20 \mu\Omega \quad R' = 600 \mu\Omega/\text{m} \quad L_o = 0.1 \mu\text{H} \quad L' = 0.36 \mu\text{H}/\text{m}$$

Projectile mass = 2 gram ; Friction factor = 20%

Object launch velocity = 50 km/s

Limiting the peak gun current to 2 MA , find the gun length and number of BHPG modules needed with all BHPG at full speed.

Opening the switches in about $200 \mu\text{s}$ intervals, the code finds that three BHPG's are needed. As shown on Fig. 3a, the 50 km/s object velocity is reached in about 10 m of the gun, about $460 \mu\text{s}$ after the first ignitron is triggered. Fig. 3b shows the voltage rise at the gun breech as ignitrons #1 to #3 are triggered consecutively. The maximum voltage is about 72 kV . Fig. 3c shows the predicted gun current profile. The peak current is about 1.8 MA .

Fig. 3a. Projectile velocity vs. gun length for Case 1

Fig. 3b. Gun breech voltage vs. time for Case 1

Fig. 3c. Gun current vs. time for case 1

Case 2

Gun characteristics:

$$R_0 = 50 \mu\Omega \quad R' = 45 \mu\Omega/m \quad L_0 = .1 \mu H \quad L' = .4 \mu H/m$$

Projectile mass = 2 kg; Friction factor = 20 %

Object launch velocity = 2.7 km/s

Use all six BHPG modules at full speed.

Opening the switches in about 1 ms intervals, computed results are plotted on Fig. 4a to 4c. Fig. 4a shows that 2.7 km/s is reached in about 6.8 m, about 174 ms after the first ignitron is triggered. Fig. 4b shows the voltage rise at the gun breech as ignitrons #1 to #6 are triggered consecutively. The maximum voltage is about 36 kV. Fig. 4c shows the saw-toothed gun current profile due to staging of the switches. The peak current is about 3.4 MA.

Summary and Future Improvements

With some simplifications, mathematical models have been developed for general multi-HPG charged inductor railgun launchers with two-stage opening switches. For the GEDI railgun experiment which is powered by the BHPG, the models plus some additional energy equations yield a set of about 50 ordinary differential equations. A code for solving the equations is developed and used to perform predictions, parameter studies, and sensitivity studies. Computer results show that with three BHPG, a 2-g projectile can be accelerated to 50 km/s in a 10 m, 360-nH/m railgun. With all six BHPG at full speed, a 2-kg projectile can be accelerated to 2.7 km/s in a 7 m, 400-nH/m gun.

Future refinements of the code will include:

1) At the present state, the switch model is a constant inductance in series with a time-varying resistance which is obtained from experimental results. We will verify against experimental results to find out if this model is good for the space of system parameters that the GEDI experiments are in. If this model is basically sound, we will just have to modify the time-varying resistance to accommodate different loads and current levels. If we find significant differences between

predictions and experimental results, we need to establish a more sophisticated circuit model for the switches.

2) The amount of energy retrievable from the storage inductors is quite sensitive to the discharge current profile for inductors which have relatively high internal inductance. For that reason, a good and practical inductor discharge model is important. In the present implementation, we know that our numerical solution should converge to the exact solution if, a) the time steps tend to zero, and b) the discharge current is a simple exponential decay. We will pursue some study to find out how erroneous the results can be for discharge to railguns which does not necessarily follow a decaying exponential path. Eventually, we will want to modify the code to take into account the history of the current so that any arbitrary current profile would yield reliable results.

3) The code uses a lumped constant friction factor to account for the mechanical losses in accelerating the projectile. If experimental results show inadequacies in this model, further studies will be called for to formulate a rational model.

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References

- [1] H. H. Woodson and S. B. Pratap, "Effects of Diffusion on Resistive Losses in Conductors of a Coaxial Inductor," CEM-UT Technical Note Number 002, January 1983.
- [2] H. H. Woodson and J. R. Melcher, *Electromechanical Dynamics*, Part II, John Wiley and Sons, New York: 1968.
- [3] R. L. Stoll, *The Analysis of Eddy Currents*, Clarendon Press, Oxford: 1974.
- [4] R. A. Willoughby, *Stiff Differential Systems*, Plenum Press, London: 1974.
- [5] M. D. Driga, et. al., "Homopolar Generator Charged Inductors," Pulsed Power Conference, Washington, D. C.: 1985.

Fig. 4a. Projectile velocity vs. gun length for Case 2

Fig. 4b. Gun breech voltage vs. time for Case 2

Fig. 4c. Gun current vs. time for Case 2