Magnetic Field Stimulation – The Brain as a Conductor

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Summary

The brain is an inhomogeneous conductor consisting of white matter, grey matter, and cerebral spinal fluid with conductivities, 0.48 S/m, 0.7 S/m, and 1.79 S/m respectively [1][2]. The skull is essentially a zero current density region since its conductivity about 100 times smaller, 32-80 mS/m [3]. Analysis shows that for the purposes of magnetic stimulation, the brain can be treated as a homogeneous conductor; differences in the computations of the induced electric field for the homogeneous and inhomogeneous models are insignificant.

The currents induced in the brain are induced by a changing magnetic field, but they are too small to influence that field. The induction is a one way coupling, and thus the problem is not a true eddy current problem. Faraday and Ampere's laws are easily applied to predict the induced

current subject to the condition that the normal component of current density go to zero at the scalp interface. Iron core stimulators constructed as tape cores are more efficient than air core stimulators. With both air and iron core stimulators, the field will always be higher on the scalp than within the white and grey matter. The higher induced surface fields are the cause of pain in some patients. This effect can be mitigated by shields and stimulator topologies that spread out the field, but it can never be eliminated. No inversion can ever be realized wherein the induced field is larger at depth than at all places on the scalp.

A properly designed brain stimulation system starts with the target stimulation depth, and it should incorporate the neural strength – duration response characteristics. Higher frequency pulses require stronger electric fields. At the heart of the process is the transfer of charge across the nerve membrane commensurate to raise its intracellular potential about 30-40 mV. Think of this membrane as a capacitance that behaves more like a short circuit at high frequency. A nerve's chronaxie and rheobase values can be used to dictate the electric field required for stimulation as a function of frequency. The system's parameters can then be chosen to minimize stimulator energy and size.

Changes to TMS stimulators are not likely to come from the superconducting community, or the ultracapacitor and supercapacitor community. Because of the large air gaps involved, 3% grain oriented steels and vanadium cores are about as suitable for standard C cores as one might expect. The malleability offered by powdered cores might, however, offer interesting penetration and flexibility options for the TMS market.

Background Information

An important question for researchers in this arena is determining exactly where in the brain TMS induces electrical activity, and whether this shifts as a function of differences in conductivity and organization of grey matter, white matter and CSF [4][5][6][7]. A number of effective homogeneous models of the TMS magnetic field have been proposed [8][9][10]. Liu and Ueno [11] proposed that when current flow from a lower conductivity region to a higher one, the interface acts as a virtual cathode. An analogy is then drawn to infer the similarities between conventional electric stimulation and magnetic.

This author would support that inference and underscore the fact that at the point that positive ions are driven into a nerve cell, its intracellular potential will rise, and if the rise is sufficient, an action potential results. The nerve cell cannot distinguish whether the rise occurred because of a rapidly changing magnetic field or an imposed electric field. The inner skull boundary condition insures that the normal component of current density is essentially zero on that boundary.

The cortex is characterized by neural bends and terminations, both of which activate on the electric field, not its gradient. Because of the small conductivity of the cortex, the induced B field is considerably smaller than the source field. For air core stimulators, the magnetic field is dictated entirely by the source current $\vec{J}_s$. With time harmonic stimulation at frequency ω , the electric field is determined by combining Ampere's law and Faraday's law,

$$\nabla \times \nabla \times \vec{E} = j\omega\mu_0\vec{J}_s \quad (1)$$

The electric field boundary condition $\hat{n} \cdot \vec{E} = 0$ must be imposed to insure no normal component current exists at the skull interface.

Fig. 1 shows a typical stimulation circuit in which low voltage ac is transformed to a higher voltage and then rectified. This higher voltage dc charges a capacitor which is fired via a thyristor switch into the stimulator core.

Fig. 1 Typical stimulation circuit.

This circuit goes through one complete resonance cycle before the diode thyristor shuts down and further current flow is prohibited by the diode. During the firing cycle, the circuit can be treated as a simple RLC resonance circuit. The current is

$$I(t) = \frac{V}{\omega L} e^{-(\alpha t)} \sin(\omega t),$$

where

$$\alpha = \frac{R}{2L} \tag{2}$$

$$\omega = \sqrt{\frac{1}{LC} - \alpha^2}$$

This is the equation for a damped sinusoid. A typical trace with $V=1.5$ kV, $C=15$ μ F, $L=11$ μ H, and $R=0.2$ Ω is shown in Fig. 2.

Fig. 2 Characteristic transcranial magnetic stimulation current.

Of particular interest is the time and value of the current peak,

$$t_p = \frac{\tan^{-1}\left(\frac{\omega}{\alpha}\right)}{\omega} \quad (3)$$

$$I_p = V e^{-\left(\frac{\tan^{-1}\left(\frac{\omega}{\alpha}\right)}{\omega/\alpha}\right)} \sqrt{\frac{C}{L}} \quad (4)$$

Neural Response

Motor and sensory thresholds for time varying magnetic fields are related to the rheobase and chronaxie strength through strength duration curves. For magnetic stimulation Geddes reports the rheobase and chronaxie results summarized in Table I [12].

Table I Neural magnetic stimulation response parameters

	Strength Duration Curve Parameters			
	Rheobase (β)		Chronaxie (γ)	
	Median (V/m)	Std. Dev. (V/m)	Mean (μ s)	Std. Dev. (μ s)
Sensory	6.75	2.06	329	78.4
Motor	16	6.1	203	78.5

Duration was defined as “onset to zero”, or one-half cycle. In terms of the stimulus frequency f , and the table parameters β and γ , the electric field is

$$E = \beta \cdot (1 + 2\gamma f) \quad (5)$$

Fig. 3 shows the required induced electric field as a function of frequency.

Fig. 3 Neural Stimulation threshold as a function of frequency.

Conductivity Considerations

The intent of this section is to give the reader a feel for the shape of the electric field induced in the brain, and to defend the thesis that attention to the exact conductivity distribution within the brain is unwarranted.

Homogeneous Model

Consider first the homogeneous model shown in Fig. 4. The brain is depicted as a homogeneous sphere with radius 7.5 cm. The outer surface corresponds to the skull. 1/4 of the problem is worked due to symmetry. The arrows depict how the E field curls around the flux face of the magnet. When the conductivity is dropped in half to 0.37 S/m, the resulting E field differs by a maximum of 0.17%. This was computed by breaking the 1/4 brain region into 778 sub volumes and computing the E field at the center of those volumes.

Fig. 4 Induced electric field arrow plot modeling the brain with a homogeneous conductivity of 0.75 S/m.

Inhomogeneous Model 1 - Concentric spheres

Consider positioning the grey and white matter a number of concentric spherical bands as shown in Fig. 5. The band pattern was intentionally altered so that the two 1/8 sections would themselves exhibit a contrast.

Grey matter = blue

White matter = yellow

Fig. 5 Concentric sphere distribution of white and grey matter.

Fig. 6 Surface E field plot for the spherical distribution of white and grey matter.

The surface E field obtained from this model is shown in Fig. 6. The mean absolute value of the difference between this model from the homogeneous model is 6.8%.

Inhomogeneous Model 2 - Concentric Wedges with CSF

A final test might suffice to underscore the point being made that paying attention to the distribution of conductivity in the brain is unwarranted. Baumann [2] has shown that the electrical conductivity of cerebral spinal fluid is 1.45 S/m at room temperature (25 EC) and 1.79 S/m at body temperature (37 EC) across the frequency range 10 to 10 kHz. Using the latter value, consider analyzing another wedge shaped model of the brain, this time with CSF distributed in equal volume with white and grey matter as suggested in Fig. 7.

Grey matter = blue

White matter = yellow

Cerebral Spinal Fluid = red

Fig. 7 Combination of white, grey and cerebral spinal fluid in a wedge shaped model of the brain.

In this extreme case, the volume of CSF is assumed to be equal to that of grey and white matter. The difference between the E fields increases to a mean absolute value of 15.2% from the homogeneous model due to the higher conductivity of the cerebral spinal fluid. The mean of the difference is only 2.6%. The fact that such an extreme distribution of the three returns such a small difference supports the claim that efforts to accurately model the brain's composition is unwarranted. If it is necessary to predict the exact site of stimulation, Wagner makes a case for building individualized models with precise locations of the CSF, white matter, and grey matter [13].

Conclusions about Conductivity Effects

As long as the tissue conductivity differences are small, two homogeneous models will deliver the same induced E field regardless of the conductivity distribution. The word small applies when the magnetic field generated by the currents induced is insignificant compared to the stimulation field. The total integrated E field around a loop is fixed by the primary B field. The models analyzed contained well defined borders between white and grey matter. Rather extreme distributions of matter in the brain were assumed to determine that the induced fields have a mean variance from the homogeneous field of about 7 % for white and grey matter, and

15 % when cerebral spinal fluid is added. The boundary condition that the normal current density must be continuous dictates the maximum departure from the homogeneous electric field case of $\frac{1}{2}$ the ratio of the conductivities of the media involved.

Suppressing the Surface Field

The use of strong electric fields in the treatment of many neurological disorders is well established. Both in the treatment of incontinence and clinical depression, the electric field should be sufficiently strong to initiate an action potential. A changing magnetic field induces an electric field within a conducting medium.

(a) Typical capacitor discharge circuit

(b) Air core stimulator

(c) Iron core stimulator

Fig. 8 (a) Typical magnetic stimulation circuit, (b) $\frac{1}{2}$ of an air core stimulator for TMS, and (c) an iron core stimulator showing a winding wrapped around a tape wound laminated core..

The simplest magnetic stimulator is a simple coil of wire, such as that shown in inset b of Fig. 8. Iron core stimulators require a specially constructed tape wound core to suppress eddy currents. These tape wound cores require about 25% of the energy of air core stimulators to realize the same induced electric field.

Magnetic induced electric fields are effective for clinical depression treatment, but their use is sometimes accompanied by pain, especially with transcranial magnetic stimulation. Both the

magnetic field and the induced electric field fall off exponentially with distance from the core in the near field region. Neurons in the soft scalp are subjected to the strongest field. Another area which registers problems is within neurons passing through an opening in bone. This paper focuses on the first problem. It is an oxymoron to think about a no surface field magnetic stimulation. There are however means by which the ratio of target depth electric field to surface field can be reduced considerably.

The electric field results displayed are computed using a boundary element solver [14][15]. For simplicity and speed, the results are displayed and computed in two dimensions, through a midplane cut through the stimulation core.

Fig. 9 Stimulator core next to the brain in quarter plane perspective.

Methods Considered

Surface Shield

A surface conductor such as that shown in Fig. 10 has the ability to suppress the local electric field, and in fact drive it to zero under the conductor if it is in electrical contact with the scalp. The integral form of Faraday's law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot \hat{n} dS \quad (6)$$

makes it clear that this technique will both move and increase the magnitude of the peak field. A better approach is to insulate the conductor from the surface.

Fig. 10 Transcranial magnetic stimulation B field plot without (a) and with (b) a passive copper shield.

Reverse Excited Secondary Coils

Fig. 11 shows how a secondary coil can be inserted within the primary coil. If the secondary coil has a reverse excitation from the primary winding, it will lower the surface field. Because the secondary coil is an air core winding, and the winding spread is small, the field penetration into the brain is reduced.

Fig. 11 Reverse excited secondary coil between the core and the brain to suppress the surface E field.

Stretched Core

Shown in Fig. 12 is a magnetic core stimulator with the angle opened considerably from Fig. 9. Keep the 3% grain oriented steel within the core. If an inversion of the excitation current is allowed spanning an angle β , the electric field is suppressed along the vertical midline.

Fig. 12 Opening the angle of the core yields deeper field penetration.

Stretched Core, No reversed excitation winding

Fig. 13 shows a simpler core without reversed excitation. Consider the exercise of opening the core angle from 90° to 140° . At each opening compute the correct excitation under saturation so that the induced E field 3 cm down remains at 1 V/cm.

Fig. 13 Core opened to 140°, 3790 AT excitation at 5280 Hz.

A Comparison of the Three Methods

Three methods have been suggested for mitigating the surface field, using a passive conducting shield, actively exciting a smaller coil with opposite field polarity, and opening up the excitation core angle. Of the three methods, the third is favored. The reverse field excitation has the drawback that it requires either another power supply or additional load to the existing supply. It suffers the safety issue that if that reverse field excitation fails or suffers a phase lag, the patient will suddenly experience much pain. The passive shield is rated second because there is little probability that it will fail. However it suffers the problem that an additional load is imposed on the stimulator supply as well. The more serious difficulty is the heat dissipation within the shield. Continuous excitation will register a possibly dangerous rise of the shield temperature. The open angle core poses the least additional burden on the stimulator power supply, and it does not suffer the safety problems of the reverse excitation or thermal shield.

One option for suppressing the surface field mentioned only in passing is to superimpose a secondary electric field with electrodes. Surface electric fields are placed normal to the induced surface field. Every other electrode must be excited with independent potential sources, and

chosen to inject a current opposite to the induced current. The skull insures that the injected current not penetrate into the white and grey matter. Implementation problems will probably preclude any development of this system.

Optimization of Magnetic Stimulators

What constitutes an optimized system? Among the items that might be optimized are the following:

- Capacitor Size
- Stimulation Coil Size
- Voltage
- Energy

Energy involves both the capacitor and the voltage. The number of turns N increases the resistance, and lowers the peak current in (4).

Air Core

An air core optimization is simplest. Many finite element and boundary element programs are suitable for analyzing this type of problem. Since the air core represents a linear analysis, a three dimensional boundary element analysis [16] is employed to predict the electric field as a function of depth for various core sizes.

Analytic Optimization

Consider an air core in which energy is to be minimized, and the core shape is known. If the shape in Fig. 9 is known, then the problem can be solved using a numerical solver for the induced field E_0 at desired depth, at current I_0 and radian frequency ω_0 . The actual induced electric field will scale from this value by the number of turns N , the actual peak current I_p and the frequency ω ,

$$E = NE_0 \left(\frac{I_p}{I_0} \right) \cdot \left(\frac{\omega}{\omega_0} \right) \quad (7)$$

The induced electric field is required to satisfy the requirement dictated in (5); this can be interpreted as a requirement on voltage V ,

$$V = NI_0 \left(\frac{\omega_0}{\omega} \right) \left(\frac{\beta(1+\gamma\omega/\pi)}{E_0} \right) e^{-\left(\frac{\tan^{-1}(\frac{\omega}{\alpha})}{\omega/\alpha} \right)} \sqrt{\frac{L}{C}} \quad (8)$$

Let L_0 represent the inductance of the core with 1 turn. The resistance is actually a bit complicated because it must account for that lost in the thyristor and the wire. As will be seen shortly, it must also account for the eddy and hysteresis loss in the core if it is magnetizable. For the moment consider only the loss from the wire, and consider the core to be filled with wire so that additional turns are added at the expense of a smaller cross-section. In this approximation, the inductance and resistance will scale as N^2 ,

$$\begin{aligned} L &= N^2 L_0 \\ R &= N^2 R_0 \end{aligned} \quad (9)$$

The energy can be written in terms of the two remaining unknowns C and N as

$$W = \frac{1}{2} L_0 I_0^2 \left(\frac{\omega_0}{\omega} \right)^2 \left(\frac{\beta(1+\gamma\omega/\pi)}{E_0} \right)^2 e^{-2 \left(\frac{\tan^{-1}(\frac{\omega}{\alpha})}{\omega/\alpha} \right)} \quad (10)$$

Consider the one turn air core stimulator shown in Fig. 9. The inductance is 0.004 μ H for an ID=3.214 cm, OD=10.66 cm, and height = 5.9 cm. The core induces an electric field of 4.273 mV/m with 1A of excitation with characteristic frequency 5.208 kHz. Using these parameters in (10) yields the energy requirement shown in Fig. 14 for a spread of capacitance values and number of turns. The equations clearly suggest the use of a small number of turns and a large capacitor. As will be seen shortly, when more realistic relationships are employed to relate resistance and inductance to the number of turns by incorporating parasitic lead inductance and resistance loss from the thyristor and core, this trend will change.

Numerical Optimization

When the problem is considered as a four parameter optimization in the variables C, V, N, and core size x it can no longer be solved analytically. A numerical approach allows the parameters such as resistance to be treated more realistically, with the inclusion of proper bounds on voltage. Assume the core size to be a scale parameter x, scaling all the dimensions equally from the core origin. If ζ_0 represents the length of the core winding with one turn, then the length ζ of the winding with N turns, scaled by a value x is

$$\zeta = Nx\zeta_0 \quad (11)$$

The combined resistance of both the parasitic core resistance and the diode R_0 with one turn is about 20 m Ω . A reasonable approximation to the resistance to be used in (2) is

$$R = R_0 \left(0.9 + 0.1 \frac{\zeta}{\zeta_0} \right) \quad (12)$$

The leads have a parasitic inductance $L_{\text{parasitic}}$ equal to about 3 μ H. Allow the core to vary through a spread of sizes and compute the inductance as the flux linkage per amperage for each size $L_0(x)$. The inductance with N turns is

$$L = N^2 L_0(x) + L_{\text{parasitic}} \quad (13)$$

Energy for an Air Core Stimulator

Fig. 14 Energy required for stimulation at a depth of 1 cm for a spread of capacitance and turns options.

Compute the induced electric field $E_0(x)$ with a current of I_0 A at radian frequency ω_0 for a spread of stimulation depths. Equation (7) dictates the induced electric field as delivered by the stimulator. If the inductance and induced electric field E_0 are fitted to the core size using a smooth spline, its derivative can be approximated and a variable metric procedure can be used to minimize an optimization index. If a combination of energy and stimulator core size are involved in the design objective, the optimization problem becomes

$$\begin{aligned} \text{Min } \mathfrak{J} &= \frac{1}{2} CV^2 x \\ \text{Subject to} & \\ NE_0 \left(\frac{I_p}{I_0} \right) \cdot \left(\frac{\omega}{\omega_0} \right) &= \beta \left(1 + \frac{\gamma \omega}{\pi} \right) \end{aligned} \quad (14)$$

The peak stimulator current I_p is determined from (4). This index is one of many options open to the designer. One of the applications motivating this research was the use of these stimulators in the field for alertness assistance. In such mobile contexts, minimizing size and energy consumption is warranted.

Results of the Air Core Numerical Optimization

Stimulation depth is a key parameter in the optimization. Fig. 15 shows how the system energy changes with target stimulation depth. Here a core shell with ID=1.836 cm, OD=6.096 cm, and height = 3.38 cm is scaled in all dimensions by a scale factor which varied from 1 to 1.75. The capacitance was allowed to vary from 5 to 75 μF , the voltage from 500 V to 3 kV, and the number of turns from 2 to 18. The problem has many local minima. A Monte Carlo method is employed to randomly vary the starting guess to increase the probability that the global minimum is found.

Fig. 15 Optimized energy as a function of various stimulation depths.

Table II shows the results of the optimization for each of the parameters. Among the key lessons are the following:

1. Smaller cores are desired for the lower stimulation depths.
2. Deeper stimulation target depths drive both the capacitance and the voltage up. The voltage comes up slower since it affects the optimization by its square.
3. When parasitic losses such as the switching and lead resistance are considered, the optimization always favors a higher number of turns. The neural response depicted in Fig. 3 is driving the frequency down with depth, and the inductance up.

Table II Optimization results for an air core stimulator.

Stimulation Depth (cm)	Scale Parameter	Capacitance (μF)	Voltage (kV)	Number of Turns	Frequency (kHz)	Stimulation Current (kA)
1.75	1.1621	5	2.1368	18	26.3319	30.6324
2.25	1.3918	5	2.6897	18	24.9194	36.3856
2.75	1.6752	6.1056	3	18	21.2232	41.8804
3.25	1.75	10.9834	3	18	15.5839	54.3823
3.75	1.75	21.7783	3	18	11.0567	74.5978
4.25	1.75	47.6275	3	18	7.4599	105.7719

Steel Core

Tape wound cores substantially reduce the required system size and energy requirements [17] [18], although their construction is more difficult [19]. The advantage is introduced with the price that the problem is nonlinear. The nonlinear element complicates the optimization in two respects. First, (2) no longer describes the current. The magnitude will be dictated by the degree of saturation. Second, the frequency is no longer a simple index. A core in saturation is characterized by a higher frequency and a lower amplitude. A fourier decomposition must be performed to determine the fundamental frequency amplitude and at least the first harmonic.

Linear

Much is to be learned by examining what should be expected from a steel core. The gap is very large. Treating the core as linear with a relative permeability of 1000 is a reasonable approximation. Fig. 15 shows how the energy drops with steel core in this approximation. Since the inductance is so high, the optimization parameters take a different posture. Important trends with iron cores are the following:

1. Deeper stimulation target depths require larger cores as with air cores.
2. Because of the high inductance, low capacitance is desirable.
3. As with air cores, voltage must increase with target stimulation depth.
4. Deeper target depths are commensurate with lower stimulation frequencies, and a lower frequency.
5. The required stimulation current increases nearly linearly with depth ($4.25/1.75=2.43$; $20.49/8.09=2.53$). By contrast the required air core amp-turn excitation increases by $105.7/30.6=3.45$. The iron core field does not fall off as rapidly with distance.

Table III Optimization parameters for a linear iron core stimulator using $\mu_r=1000$.

Stimulation Depth (cm)	Scale Parameter	Capacitance (μF)	Voltage (kV)	Number of Turns	Frequency (kHz)	Stimulation Current (kA)
1.75	1.0022	5	1.0616	17.9702	13.7545	8.0951
2.25	1.2508	5	1.2975	15.2522	14.4552	8.8111
2.75	1.3283	5	1.6264	14.5454	14.4532	10.5291

3.25	1.3579	5	2.0476	14.334	14.4539	13.0631
3.75	1.3738	5	2.575	14.2388	14.4533	16.3166
4.25	1.3913	5.9533	3	13.7745	13.558	20.4981

Saturable Cores

The analysis becomes nonlinear with real magnetizable cores. This complicates the analysis details, but the approach is unchanged. Discussion of this can be found in [20]. Table IV shows the results of the nonlinear analysis allowing the capacitance to vary from 5 to 35 μF , the number of turns from 1 to 18, the voltage from 400 V to 1.5 kV, and the core scale parameter from 1 to 1.75, using a parasitic inductance of 4.5 μH . Please note that the saturable core requires fewer joules; the linear core was modeled with a permeability of 1000; the saturable M-19 cores have a low field permeability of nearly 6000.

Table IV Results for a nonlinear core analysis

Target Depth (cm)	Scale Parameter	Capacitance (μF)	Voltage (kV)	Number of Turns	Frequency (kHz)	Stimulation Current (kA)	Energy (J)
1.75	1.7167	15.8764	0.4206	13.655	6.2797	1.4495	1.4041
2.25	1.75	35	0.4	13.984	4.0998	2.0442	2.8002
2.75	1.5021	34.9972	0.4002	18	3.54	2.3122	2.8027
3.25	1.75	34.9537	0.5752	18	3.2554	3.3454	5.7818
3.75	1.75	34.9579	0.6896	18	3.2552	4.0112	8.3118
4.25	1.75	34.9994	0.8136	18	3.2533	4.7351	11.583

Cortical Stimulation

The results quoted are consistent with the strength duration information of Table I, and dependent on the optimization criteria targeted which could in general be different from (14). The presence of a myelin sheath will increase both β and γ in Table I. Geddes shows some of the variation of γ in [21]. More reasonable values of β and γ for the cortex are suggested by this team to be 32 V/m and 406 μs , and yield the optimization results in Table V. Doubling these parameters from their former values in Table I increases the energy by more than an order of magnitude. Until data analogous to Table I is available for the cortex, the higher values of 32 V/m and 406 μs for β and γ appear reasonable, since they are consistent with excitation levels in the laboratory.

Table V Results for the nonlinear core analysis after doubling the rheobase and chronaxie values.

Target Depth (cm)	Scale Parameter	Capacitance (μF)	Voltage (kV)	Number of Turns	Frequency (kHz)	Stimulation Current (kA)	Energy (J)
1.75	1.7353	34.9637	1.0384	17.867	3.2801	5.685	18.851
2.25	1.747	27.5194	1.3949	18	3.6591	6.8373	26.772
2.75	1.75	27.176	1.75	18	3.7858	8.3957	41.613
3.25	1.75	62.4074	1.75	18	2.772	13.5831	95.561
3.75	1.7501	112.4891	1.75	18	1.794	27.2075	172.25
4.25	1.7502	207.656	1.75	18	1.342	45.9637	317.97

Conclusions of Optimization Considerations

The frequency, system voltage, capacitance, core stimulator size, and the number of turns should be treated as unknowns in a TMS stimulation design. Based on the neural magnetic stimulation response parameters, and the electric field as computed through a boundary element solver, the ideal parameters for the system can be derived. A trust region technique is used to solve the four parameter optimization problem. The result is target depth dependent, and certainly dependent on the shape of the stimulation coil. Deeper targets are commensurate with lower excitation frequency, and higher amp-turn products. Rheobase and chronaxie values of 32 V/m and 406 μ s appear consistent with laboratory data.

Possible Topological Changes to Consider in the Future

The pedagogical nature of this book warrants contemplation of changes likely to affect this field. Vanadium and 3% grain oriented steel cores are already near the top of what a researcher might desire for C shaped cores. Ac superconductors are not likely to help soon, since their magnetic field limitations in frequency and field are below what room temperature devices are doing. Five kHz operation would pose serious problems to the superconductor's stability. Both super and ultracapacitors are characterized by low voltage. There is however at least one topological change that might prove interesting.

Consider the 1.5" thick iron based C core shown in Fig. 16 using a 20,000 At excitation at 5740 Hz. The induced electric field is predicted in the center plane, the most favorable plane for the C core (Fig. 17). The field falls off rapidly from this center plane. If the excitation plane is moved 0.75" from the midplane the excitation field drops by 8%. The excitation from the C core weakens away from this midplane.

Fig. 16 C core analyzed in three dimensions.

Fig. 17 Induced electric field within the brain region (0.47 S/m).

Consider a bowl of powdered iron with a doughnut shaped cavity as suggested in Fig. 18. The cavity will house a toroid excitation coil. Tape wound cores are typically C shaped with the winding in the center of the “C”. Among the advantages of the core proposed in Fig. 18 is a much improved magnetic circuit, with lower magnetic reluctance than that possible with the C core. Iron powder also has a higher field saturation than 3% grain oriented silicon steel, (2.15 versus 1.85 T) albeit with higher hysteresis loss. The field is able to spread out in 360° to return around the excitation coil as shown in Fig. 19. The 7.5 cm brain is modeled with conductivity 0.48 S/m. Note how the magnetic field concentrates in the center of the core but is quite small on the perimeter. Since the stimulator is bowl shaped, the analysis is performed with an axysymmetric computation.

Fig. 20 shows the induced electric field due to this stimulator. Fig. 21 shows the same picture annotated with radius markers. Since cranial threshold occurs near 1 V/cm, it is clear that for the excitation chosen, stimulation would occur down to 3.5 cm. By contrast, the C core in Fig. 16 stimulates down to only 3 cm under the same excitation (Fig. 17).

Fig. 18 Proposed powder iron stimulator core.

The excitation region generated by the rotational symmetric geometry of Fig. 18 is disk shaped. Suppose an asymmetric excitation is desired. Assume the powder core region is built with partitions. The simplest partition would be a dual chamber geometry as suggested in Fig. 22. Note how the induced electric field focuses on the right chamber when the left chamber is not filled with iron powder. It should be clear that splitting the bowl into 4 chambers and filling one of the four will further focus the volume of neurons being excited.

Fig. 19 Magnetic field for a 4" OD stimulator excited with 20,000 AT at 5740 Hz.

Fig. 20 Electric field due to the powder stimulator.

Fig. 21 Induced electric field with depth annotation.

Fig. 22 Induced electric field with a split chamber core, filled on one side.

In practice the powder would be housed in a bag, perhaps a plastic reinforced and molded around the excitation coil. An inherent advantage would be the ability to shape the surrounding structure to the contour of the patient's head in a customized fashion.

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