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On the Numerical Design of Synchronous Generators

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Abstract – High power density applications require special treatment in design optimization. Among the critical questions is how the working torque angle will change through an optimization analysis. A new method for computing and using the synchronous reactance is presented which accounts for the torque angle in the midst of complex parameter changes. The challenge is to tally this effect along with load power factor into a variable metric optimization. The contribution offered by this paper is a new technique for both computing this reactance under heavy saturation and integrating it into the machine design. Attention is also given to appropriate stator current excitation densities in transient and steady state excitation.

Keywords – synchronous, optimization, reactance, deterministic, superconducting

Introduction

Chari was among the pioneers to apply finite element analysis to the design of synchronous generators [1]. The emphasis in the early stages was to compute representative performance indexes such as torque and inductance. A decade later the analysis tools had advanced to the point where they could be used profitably in magnetic optimization. Russenchuck[2] demonstrated an optimization technique for a synchronous permanent magnet motor, minimizing the magnet volume and harmonic loss vector with a Lagrange penalty function used to satisfy power requirement; the quasi-Newton methods such as that by Davidson, Fletcher, and Powell is cited as being the best convergent technique at that time[3]. Anderson[4] suggested one of the more robust approaches for complex optimization. His technique consisted of using Monte Carlo based search directions in his parameter space, and included complex factors such as short circuit reactance and thermal constraints by the use of penalty functions. Analytic approximations had to be employed for these complex functions. Wu [5] shows an optimization for a permanent magnet generator using a hybrid numerical analytic – numerical approach on one variable at a time.

Synchronous machine designers are well aware of the importance of synchronous reactance and torque angle in design. Most commercial field codes are now equipped with parametric capability. How can this feature best be used in generator design? More importantly, what is the correct procedure for satisfying torque demand, minimizing weight, incorporating the right torque angle, and constraining slot conductor force. This paper discloses one technique involving flux linkage in two positions to incorporate the synchronous reactance effect into the generator optimization. Stator fields along two key segments are constrained during the optimization to curtail lamination loss. Methods to determine appropriate slot current densities are discussed.

Key Points

Important observations gained through this team's study are as follows:

1. Accounting for the synchronous reactance x_d and the load power factor through the parameter variation is critical to getting a meaningful design.
2. The synchronous reactance must be computed in saturation with both the rotor and the stator excited.
3. A correct parametric optimization will link the variables. For example if one of the variables is the slot depth, and another the back iron behind the slot, the back iron should step concurrently with the slot change, and then the back iron adjusted again by a separate variable. The actual scaling for the back iron becomes an equation involving the tooth depth scale and a dummy variable scale for the incremental back iron changes. Poor results are realized if this is not done, because the effects cannot be isolated and much of the variable space is banal.
4. The design criterion is to insure that the generator makes torque at operational speed, but the torque angle between the rotor and stator is critical to this calculation, and unknown a priori.
5. Stator current excitation is dictated by thermal constraints. Optimization proceeds by assigning a current density to the slot, and holding that consistent with the duty cycle and cooling employed.
 - a. For very short duty cycle machines, the current density is determined adiabatically.
 - b. Allowed current densities for steady state armatures ultimately require a separate thermal analysis to offer any confidence that insulation ratings are not compromised; in practice this assignment leans heavily on historical precedent.
6. A proper optimization should involve maximum field densities both in the teeth, and the critical back iron locations. This is not because the numerical codes are not able to compute the field correctly in saturation; proper use of the Frolich-Kenelley relations [6][7][8] takes care of this difficulty. The two reasons for imposing a maximum allowed B field within the optimization are as follows:
 - a. The long term force on the conductors in the slots climbs considerably with saturation, ultimately causing conductor insulation fatigue failure.
 - b. The loss in the laminations grows as B^2 .

Generator Excitation Current Phase Angle

Consider the 48 slot two pole generator shown in Fig. 1. The machine has four slots per pole per phase with an $11/12^{\text{th}}$ pitch span. The objective is to find the minimum weight geometry that delivers 5 MW at 15,000 RPM. The rotor field comes from a HTS superconductor, either BISCO or YBCO. The HTS superconductor must have vacuum insulation and an eddy current shield, the thickness of which is about 2.2 cm. Mechanical stress considerations limits the tip velocity of the shield to 235 m/s, a condition dictates an outer shield radius of just under 15 cm.

Although vector diagrams representing generator operation are common in elementary motor textbooks, the corresponding current pattern within the generator for a numerical modeling view is often misunderstood. Because the proposed optimization method depends on an understanding of this concept, a brief discussion is warranted. Consider the two pole, 6 phase generator with clockwise rotation shown in Fig. 1. For the phase pattern shown, the rotor is drawn at the maximum torque angle, 90° ahead of the stator. The snapshot of currents on the rotor and stator in Fig. 1 can only be observed under two conditions:

1. The power factor of the load is unity, and the internal reactance x_d of the generator is negligibly small.
2. Capacitance is added to the generator load; negative capacitor vars are added to exactly cancel the positive vars from x_d .

Fig. 1 Two pole, six phase superconducting generator.

A more typical snapshot of the relative rotor and stator currents is shown in Fig. 2. Both the load shown with a positive power factor, and the internal generator reactance contribute to the additional angle by which the rotor leads the ideal 90° position witnessed in Fig. 1. In terms of the synchronous reactance x_d , the electrical angle by which the rotor must be advanced past the maximum torque position is

$$\theta = \cos^{-1}(pf) + \sin^{-1}\left(\frac{Ix_d \cdot pf}{E_f}\right) = \cos^{-1}(pf) + \tan^{-1}\left(\frac{Ix_d \cdot pf}{\sqrt{V^2 + (Ix_d \sqrt{1 - pf^2})^2}}\right) \quad (1)$$

Fig. 2 Picture of rotor and stator currents feeding an inductive load.

The torque angle under load is dependent on nearly all design variables.

Magnetizing Reactance

The reactive voltage drop in a phase winding is due to the self inductance drop, $\omega L I_A$, and the mutual inductance drop due to the other phases, $\omega M_{AB} I_B$, $\omega M_{AC} I_C$, and so on. For balanced excitation, it is desirable to express this voltage drop in terms of only the phase A current, i.e., to incorporate the effects of the other line currents within the phase A circuit. This is usually approached by assuming that phase A current can be broken into a leakage component L_ℓ and a component that accounts for flux actually crossing the gap L_m . An approximation commonly employed is to assume that the mutual coupling to the other phases n for an m phase machine is $\cos(2\pi n/m) \cdot L_m$. Since the current in the other phases is reduced by the same factor $\cos(2\pi n/m)$, the equivalent synchronous inductance L_s of all the windings is

$$L_s = L_\ell + \sum_{n=1}^m \cos^2\left(\frac{2\pi n}{m}\right) L_m = L_\ell + \frac{m}{2} L_m \quad (2)$$

The reactance x_d is simply ωL_s . With only the stator excited, the energy of a three phase system would be

$$W = \frac{1}{2}(L_\ell + L_m)I_{peak}^2 \sum_{n=0}^{m-1} \cos\left(\omega t - n \cdot \frac{2\pi}{3}\right)^2 - \frac{1}{2}L_m \left[\cos(\omega t) \cos\left(\omega t - \frac{2\pi}{3}\right) + \cos(\omega t) \cos\left(\omega t - \frac{4\pi}{3}\right) + \cos\left(\omega t - \frac{2\pi}{3}\right) \cos\left(\omega t - \frac{4\pi}{3}\right) \right] \quad (3)$$

Combining terms and switching to rms current yields the result

$$W = 3 \left(\frac{1}{2} L_s I_{rms}^2 \right) \quad (4)$$

The leading “3” is replaced by m for an m phase machine.

Computing X_d

Regardless of the method, all steel should be close to operational saturation state to compute x_d properly. There are at least three techniques available, involving flux coupling between phases, system magnetic energy, and direct and quadrature flux coupling. The latter two are discussed briefly.

Energy Computation of X_d

Determining the magnetizing reactance accurately using energy requires three calculations. Magnetic energy is obtained by integrating the product of magnetic vector potential with current. Repeat this calculation with the stator current increased by $\gamma\%$ and with the rotor field current increased by $\gamma\%$. The energy for the machine is known at three excitations. Call these $W_0(I_s, I_r)$, $W_1((1+\gamma)I_s, I_r)$, and $W_2(I_s, (1+\gamma)I_r)$, each being equal to $\int \vec{A} \cdot \vec{J} dV$. The synchronous motor is to be thought of as a two terminal system for the purpose of getting the synchronous reactance,

$$W = \frac{1}{2} L_s I_s^2 + \frac{1}{2} L_r I_r^2 + M I_s I_r \quad (5)$$

When this expression is written three times with the rotor and stator current adjusted appropriately, the synchronous reactance for an m phase machine can be computed as

$$L_s = \frac{m}{2} L_m = m \frac{W_1 + W_2 - 2W_0 + \gamma(W_1 - W_0(3 + \gamma))}{I_s^2 \gamma^2 (2 + \gamma)} \quad (6)$$

Note that the rotor and stator are excited with instantaneous current matching the load conditions, but the correct rotor position is unknown. Closing this loop requires the procedure to be repeated at a number of rotor positions for a range of angles θ . Such an approach forces the optimization to compute all quantities of interest at a number of rotor positions, and then adding the constraint that the rotor angle θ as determined from (1) match the position angle in the optimization, a computationally intensive procedure.

Direct and Quadrature Axis Flux Calculation.

The proposed procedure requires only two, not three calculations to get x_d . The method involves the calculation of the flux linking the armature winding at zero degree and 90 degree torque angles as shown in Fig. 3. Inset (a) shows the rotor in the maximum torque position; at this position it links no flux with the highlighted phase A coil. The currents on the stator are excited commensurate with their amplitude at $t=0$. The flux linking the coil is entirely due to the stator excitation, but accurately reflects the rotor's saturation. The product of this flux with the electrical radian frequency, $\omega\Phi$, is identically equal to the product of slot current I and x_d .

Fig. 3 Method for numerically computing synchronous reactance.

When the rotor is advanced 90 electrical degrees as in inset (b), the $\omega\Phi$ product (again with phase A) equals the field voltage less the synchronous reactance drop $E_f - I x_d$. The angle θ by which the rotor must be advanced past the maximum torque position when driving a load with power factor pf is dictated by (1). One key observation is that this angle is independent of active stack length. The operating voltage will be

$$V = \sqrt{E_f^2 - (x_d I \cdot pf)^2} - x_d I \sqrt{1 - pf^2} \quad (7)$$

The per unit synchronous reactance is

$$x_{d-pu} = \frac{x_d I}{V} \quad (8)$$

Rotor and Stator Excitation

Assigning appropriate current densities is the final discussion topic required before performing the optimization. There is no substitute for experience in choosing current density properly. The primary concern in the superconductor is the material B field exposure. Tests show current densities of 200 MA/m² are appropriate for 1 cm thick conductors, but the current should be reduced linearly to 150 MA/m² for 2 cm thick conductors.

Stator Current Density – Transient

Short duty transient machines should be rated by adiabatic considerations. Consider a one turn winding having cross sectional area A, length L, conductivity σ , mass density ρ , carrying current density J. The resistive dissipation in the winding is

$$P = (J \cdot A)^2 \frac{L}{\sigma A} \quad (9)$$

Adiabatic heating demands a commensurate temperature change ΔT in δt seconds of

$$P \delta t = \rho C_p (AL) \Delta T \quad (10)$$

Substituting (9) into (10) yields the current density in this one turn winding independent of area to be

$$J = \sqrt{\frac{\rho \sigma C_p \cdot \Delta T}{\delta t}} \quad (11)$$

Irrespective of the conductor size, the current density is dictated by the adiabatic temperature jump allowed. The conductivity, density, and specific heat for copper are respectively $5.8 \cdot 10^7$ S/m, 8.96 kg/m^3 (0.323 lbs/in^3), and 383 J/kg/K . The conductivity drops to $4.3 \cdot 10^7$ S/m at 100°C . The total loss in the stator slot is complicated by the slot induced eddy currents. The loss has two origins. The first is due to the B field across the slot hitting the edge of the slot conductor. The second is due to circulating current induced by the same flux that passes between conductors to be shorted. This latter contribution can be eliminated by transposing the conductors. Fig. 4 shows geometry suitable for estimating the first component. The x directed H field in the slot is due to the number of conductors with current below it,

$$H_x = \frac{nI}{a} \quad (12)$$

n slots each carrying current I
 Frequency ω , conductivity σ
 Permeability μ_0
 Depth D

Fig. 4 Geometry used for computing excess conductor slot loss.

Using Faraday's law at excitation frequency ω , it follows that the z directed E field within a conductor at position $n+1$, having n conductors below it is

$$E_z = \frac{j\omega\mu_0 n I}{a} y \quad (13)$$

The real power dissipation per unit volume is $P_v = \Re\{\sigma E \cdot E^*\}$. Integrating this over the conductor thickness from $-b$ to b gives the conductor loss

$$P = Da \int_{-b}^b \sigma \left(\frac{\omega\mu_0 n I}{a} \right)^2 y^2 dy = \left(\frac{DI^2}{\sigma ab} \right) \frac{(\omega\mu_0 \sigma n b^2)^2}{12} \quad (14)$$

The first term in parenthesis is the dc loss within the conductor. Labeling this the dc loss P_{dc} , the total loss in the slot would be

$$P_{total @ position n+1} = P_{dc} + P_{ac} = P_{dc} \left[1 + \frac{(\omega\mu_0 \sigma n b^2)^2}{12} \right] \quad (15)$$

Typical loss values for a 32 conductor slot with $b=0.0702$ cm (0.0276") are shown in Fig. 5.

Fig. 5 Typical slot losses for 0.07 cm thick conductors at 443 Hz (0.59-15,000 RPM)

Stator Current Density – Steady State

Selecting a proper current density for steady state operation is anything but easy; considerable thermal analysis must be performed to gain any confidence in a custom application. It depends on multiple issues such as whether the back iron can be cooled, how much air can be pumped down the slot, whether the coils are water cooled, and whether the coolant can be run through the center of the conductor. Programs indexing current density at the University of Texas's Center for Electromechanics are summarized in Table I. The 3rd column is the current density in the copper. The final row is actually a General Electric generator, having an equivalent current density of 4 MA/m² (31 % packing factor).

Table I Benchmarks for current density.

Program	Cooling	J (MA/m ²)
EMALS	Air Cooled pulsed	5
Megagen	Air Cooled	8.8
Geco & Balcones Homopolar	water cooled steady state	25
GE 138 kV	Water cooled stator	13 (4 in slot)

The current densities listed assume a 100 % duty cycle. If the active duty is less than one minute, the current densities can be multiplied by the square root of (100/(% duty on)). For high

frequency conductors, it is important to employ full transposition of the stator conductors to avoid high circulation current. Insulation standoff thicknesses of 1.9 to 3.9 kV/mm (50 - 100 V/mil) are reasonable industry standards. Experience at this center indicates that packing factors much greater than 30 % are unreasonable for 3-4 kV generators. The packing factor which depends on the insulation demand is necessary to pinpoint the appropriate slot current density required in the numerical insulation. The maximum expectation for water cooled Litz conductors with a 60% duty cycle is about 21 MA/m² with a packing fraction no greater than 30%. This translates to about 5 MA/m² in the slot, the number assumed in the analysis to follow.

Optimization

Optimization is performed on the superconducting generator shown in Fig. 6. The objective is to minimize weight subject to the constraint that the torque required for 5 MW is delivered at 15,000 RPM. The outer radius of the rotor is dictated by the mechanical yield strength of the outer shield. A copper shield's tip speed should be approximately 200 m/s. For the moment consider the number of poles fixed at two, and the number of stator teeth for six phases fixed at 48. Within those constraints lies the question "What is the best choice of the rotor steel, the thickness of superconductor, the depth of the stator tooth, and the thickness of the back iron?" Since weight is a premium, the stator steel will be constructed of Hyperco steel. The superconductor current density decreases from 200 MA/m² for a 1 cm thick medium to 155 MA/m² for a 2 cm thick medium.

Fig. 6 Synchronous machine to be optimized.

In addition to the four unknowns x , y , z , and u in Fig. 6, the operating torque angle is unknown. Why is the torque angle critical to the optimization? Optimizations that do not account for it favor deep slots with high synchronous reactance. The length and thus the weight must be selected with the correct torque angle.

Approach

The four parameters in Fig. 6 are allowed to vary within a nested loop of 256 permutations. Sequential quadratic programming is employed to determine the local minimum for weight using a random starting guess in the variable space examined. The process is repeated multiple times using Monte-Carlo methods to insure the avoidance of local minima. The procedure is as follows:

1. Construct a nested loop of all unknown parameters. A numerical analysis is performed for each configuration. The optimization in this study involves several steps each for the field back iron thickness, the stator slot depth, and the stator back iron thickness, and the rotor superconductor thickness. Because of the changes considered and the saturation within the steel, adaptive meshing must be employed for each analysis. This background computation usually requires several days of continuous analysis. The full nested loop computation must be performed twice, once at the maximum torque angle, and once at zero torque angle. The loop structure is as follows:

For $x=x_{\min}$ to $x_{\max}$

Set field steel thickness

For $y=y_{\min}$ to $y_{\max}$

Change superconductor thickness, increasing the thickness inward radially

Repositioning the rotor steel to maintain a 6.35 mm ($\frac{1}{4}$ ") thermal gap

Reduce the superconductor current density appropriately with thickness

For $z=z_{\min}$ to $z_{\max}$

Change the stator slot depth, but keep stator current density fixed

Reposition the back iron boundary, preserving initial thickness

For $u=u_{\min}$ to $u_{\max}$

Reduce the stator back iron thickness

End adjustment on back iron thickness u

End adjustment on slot depth z

End adjustment of superconductor thickness y

End adjustment of field steel thickness x

2. After each analysis compute the following:
 - a. Stator current I based on stator slot dimensions
 - b. Voltage

- i. Compute the voltage linking the stator winding both when the field is positioned for maximum and zero torque (90 degree torque angle).
 - ii. Based on the flux linkage, determine the synchronous reactance x_d and the back emf from the field E_f .
 - iii. Compute both the terminal voltage and the torque angle between the field current and the armature current based on the power factor of the load, E_f , and $I \cdot x_d$ using (7) and (1).
 - c. Torque
 - i. Determine the length of the generator L required to deliver the power required at the lowest rotation speed for which power is required, i.e. set the length to be the power required divided by the product of the rotation speed and the cosine of the torque angle.
 - d. Weight based on slot size, length L , the rotor superconductor thickness and the steel for both the rotor and the stator.
3. Use a quad-variate spline to fit the computed quantities for weight and torque angle to the four unknowns shown in [9].
4. Use a trust region optimization algorithm based on the interior – reflective Newton method to determine the minimum weight [10][11]. The following three points are in order:
 - a. The length has already been incorporated into the optimization at step 2.c.i.
 - b. The optimization is dependent on the power factor of the load through step 2.b.iii.
 - c. The real problem has only been computed at 256 separate points in the search space. The intermediate space is only approximated using the multivariate spline. It is an approximation demanded by practical considerations.
5. Incorporate inequality constraints to limit slot leakage and conductor force. These types of unconstrained optimizations typically promote designs with rather high flux densities in the steel.
 - a. Place an upper limit on the allowed slot conductor force, specifically the force on the copper wires within the slot.
 - i. Both conductor force limits and maximum B field limits in the teeth achieve the same end.
 - ii. A good design should naturally push the field heavily into saturation. The field in the teeth is elevated.
 - b. Limit the lamination losses. The mid pole back iron position serves well in this regard.

Fig. 7 The B field is computed along key segments for each parametric position.

Results

This design analysis has been performed on both a two pole and a four pole design. Key observations are as follows:

1. When the torque angle calculation is brought into the optimization, both the slot depth and the stator back iron thickness decrease.
2. Even though the rotor steel is in saturation, it is useful, and more than pays for its weight in contributing to torque.
3. The thickness of the HTS winding enters the optimization in two ways.
 - a. The optimization drives the solution to shallow stator teeth and back iron both to reduce weight and x_d . In order to make torque with this reduced stator MMF, the optimization drives the HTS thickness up.
 - b. The thickness of the HTS is perhaps the most sensitive parameter to the constraint on the allowed maximum B field in the stator.

Criteria 5 in the optimization approach is accomplished by computing the radial B field along the mid-tooth segment at the maximum torque position, i.e., the direct axis shown in Fig. 7. The

azimuthal B field is also computed in the back iron at the quadrature axis position. If a maximum B field of 2.5 T is imposed along these two segments, the optimized two pole generator shown in Fig. 8 results. The optimization used a 256 parameter investigation (4 nested sets of four variables), and requires approximately three days of computational time on a 3 GHz Pentium IV. The 5 MW load is a six phase bridge converter with 0.95 power factor. Note the relatively small synchronous reactance and the commensurate torque angle of 26.3° .

Fig. 8 Optimized two pole generator.

The approach outlined has the advantage that the optimization algorithm runs quickly once the data has been computed. The effect of the constraint on the maximum B field can be computed rather quickly. Fig. 9 shows how much the superconductor thickness grows, and the active length shrinks as this constraint is relaxed.

Fig. 9 Effect of the maximum B field constraint on generator weight, length, and HTS conductor thickness.

The parametric optimization was repeated for a four pole generator with the same 2.5 T constraint on the tooth and back iron B field. Fig. 10 shows the resulting parameter set. The weight drops from 203 kg (447 lbs) for the two pole generator to 143 kg (315 lbs). The weight includes the rotor, stator, and end turns, but not the housing, shield, or refrigeration equipment. The two pole's additional weight primarily comes from the rotor steel and the end turns. The test is a bit unfair to the two pole design in the following respect. The ac to dc resistance of the stator conductor is proportional to frequency squared. Since the speed is fixed, the four pole stator will witness four times the additional slot loss. When the current density is adjusted up for the two pole generator to account for this effect, the weight difference essentially vanishes.

Fig. 10 Optimized 4 pole design.

Conclusions

Integrating the synchronous reactance into generator optimization is critical to meaningful design. It can be computed in saturation through flux linkage with the stator at 90° and 0° torque angles. For HTS superconductors, current density should be decreased with increasing conductor thickness. The primary consideration is the B field within the HTS material. By contrast, the primary consideration in the stator is heating. Circulating current and ac/dc resistance ratios dominate coil layout in high speed machines. Multiple small conductors are mandatory for minimizing cross slot field loss. A full transposition of slot conductors is required to minimize circulating current of the parallel conductors. Litz wire coils are a natural choice for frequencies above 400 Hz. The technique has been applied to the design of two and four pole synchronous generators.

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References

- [1] M. Chari, "Finite element analysis of electrical machinery and devices", *IEEE Trans. Magn.*, Vol. 16, No. 5, September 1980, pp. 1014-1019.
- [2] S. Russenschuck, "Mathematical optimization techniques for the design of permanent magnet synchronous machines based on numerical field calculation", *IEEE Trans. Magn.*, Vol. 26, No. 2, May 1990, pp. 638-641.
- [3] D.M. Himmelblau, Applied Nonlinear Programming, McGraw Hill, 1972.
- [4] O.W. Andersen, "Optimized design of electric power equipment", *IEEE Computer Applications in Power*, Vol. 4, No. 1, Jan. 1991, pp. 11-15.
- [5] C. Gu, W. Wu, K. Shao, "Magnetic field analysis and optimal design of DC permanent magnet coreless disk machine", *IEEE Trans. MAG-30*, No. 5, pp. 3668-3671, 1994.
- [6] O. Frohlich, "Investigations of dynamoelectric machines and electric power transmission and theoretical conclusions therefrom", *Electrotech Z.*, Vol. 2, 1881, pp. 134-141.
- [7] A.E. Kennelly, "Magnetic reluctance", *Trans. Am. IEE.*, vol. 8, 1891, pp. 485-517.
- [8] Brian F. Phelps, An Inclusive Model of Ferromagnetic Hysteresis, Ph. D. Thesis, Dept. of Physics, Queen's University, Ontario, Canada, 1999. p. 2.1.3.
- [9] C. de Boor, C., A Practical Guide to Splines, Springer-Verlag, 1978.
- [10] Coleman, T.F. and Y. Li, "An Interior, Trust Region Approach for Nonlinear Minimization Subject to Bounds," *SIAM Journal on Optimization*, Vol. 6, pp. 418-445, 1996.
- [11] Coleman, T.F. and Y. Li, "On the Convergence of Reflective Newton Methods for Large-Scale Nonlinear Minimization Subject to Bounds," *Mathematical Programming*, Vol. 67, Number 2, pp. 189-224, 1994.