PARAMETER STUDIES FOR TRAVELING WAVE
COAXIAL LAUNCHERS

A. Y. WU

Presented at the
5th Symposium on Electromagnetic
Launch Technology
Eglin AFB, Florida
April 2-5, 1990

Publication No. PR-112
Center for Electromechanics
The University of Texas at Austin
Balcones Research Center
EME 1.100, Building 133
Austin, TX 78758-4497
(512) 471-4496

PARAMETER STUDIES FOR TRAVELING WAVE COAXIAL LAUNCHERS

A. Y. Wu
Center for Electromechanics
The University of Texas at Austin
10100 Burnet Rd., Bldg. 133
Austin, TX 78758-4497

Abstract: The traveling wave coaxial launcher is a complex machine that requires very extensive parameter studies to optimize. Most of previous attempts to realize hypervelocity using coaxial launchers have failed partly due to inadequate analyses. This paper reports the results of very extensive "air-core" coaxial launcher parameter studies performed using computers. These results and the methodology introduced should help future researchers on this topic.

In the course of studying the feasibility of accelerating a 1-kg projectile to 10 km/s with an 18 m air-core multiphase coaxial launcher powered by a rising frequency generators (RFGs), a complete simulation code based on the current filament method was developed. Results from the simulation code indicate rather chaotic behavior of an arbitrary coaxial launcher design. More fundamental studies were then conducted using various current filament method based computer codes.

An extensive study was performed to analyze the variation in coaxial launcher behavior due to perturbations in launcher geometry such as pole pitch, coil pitch, stator-coil thickness, number of phases in the stator current, and armature thickness, armature injection velocity, armature material properties, and stator current frequencies. A normalized study was performed on the relationship among stator-coil length, stator and armature radii, mutual inductance, and its gradient.

Introduction

Electromagnetic (EM) launchers are considered viable alternatives to explosive driven conventional launchers. Electromagnetic launchers have several forms. Between the two main classes of EM launchers (railguns and coilguns), most attention has been paid to railguns. Railguns are relatively easy to build and test. Also, power supplies for railguns, such as homopolar generators (HPGs) and capacitors, are more available. Coilguns are more complex to design and a practical power supply of interesting power levels has not yet been built.

Because of coilguns' complexity, it is of paramount importance to perform detailed simulations in the design phase before engaging in fabrication. Previous failures of coilguns to perform may be attributable to inaccurate analyses. Therefore, sophisticated analytical tools are critical to coilgun research. This paper introduces state of the art analytical tools developed for a traveling wave passive coaxial accelerator, and the results generated with them are presented for future references.

The Passive Coaxial Accelerator Project

A project targeted at accelerating a 1-kg projectile to 10 km/s with an 18 m air-core multiphase passive coaxial accelerator (PCA) powered by RFGs was undertaken at the Center for Electromechanics at The University of Texas at Austin (CEM-UT). [1]

The operation of the PCA is rather complex, particularly during start-up. Our research strategy was to first develop the most detailed yet cost effective analytical tools. Extensive parameter studies would then be conducted. The parameter studies' results would give insights on the feasibility of the idea. Afterwards, full systems of particular gun geometries with integrated power supplies would be identified and simulated. Detailed designs of individual hardware components would then follow.

Passive Coaxial Accelerator Configuration and Operation

The basic geometry of the PCA investigated is shown in figure 1. The stator is composed of an array of coaxial solenoids located discretely along the z-axis. The armature is a round tubular conductor centered inside the solenoids. The armature has no continuous contact with the stator and its current is totally induced. Change of stator currents over time induces a proximity effect eddy current in the

armature. The EM force between the stator currents and the eddy current propels the armature along the z-axis. The fundamental principle of PCA operation can be explained based on electromechanical interactions between two coaxial circular current filaments in a nonferromagnetic environment as shown in figure 2.

4401.0061.1

Figure 1. Basic coaxial launcher geometry

1001 0267

Figure 2. Two coaxial circular current filaments

In figure 2, the two filaments (of radii r_s and r_a) represent an infinitesimal partition of the stator (carrying current i_s) and the armature coils (carrying current i_a), separated by distance z . The mutual inductance, M , between the two filaments is a function of r_s , r_a , and z , expressed as

$$M = \mu \sqrt{r_s r_a} \left[\left(\frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right] \quad (1)$$

The constant μ is the magnetic constant or the magnetic permeability of air. The terms $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and the second kinds. They can be expressed as

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \quad (2)$$

and

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta \quad (3)$$

The variable k is a function of r_s and r_a

$$k = \sqrt{\frac{4r_s r_a}{z^2 + (r_s + r_a)^2}} \quad (4)$$

The z-axial force between the two filaments, F , is

$$F = i_a i_b \frac{\partial M}{\partial z} \quad (5)$$

where $\partial M/\partial z$, the mutual inductance gradient, is

$$\frac{\partial M}{\partial z} = \frac{\mu k s}{4(1-k^2)\sqrt{r_a r_b}} \left[2(1-k^2)K(k) - 2(1-k^2)E(k) \right] \quad (6)$$

A typical plot of M and $\partial M/\partial z$ as a function of the coil separation, z , is shown in figure 3

1001.0268

Figure 3. Plot of M and $\partial M/\partial z$ vs. z

Principles of Operation of Traveling Wave Passive Coaxial Accelerator

The schematics of a three phase traveling wave PCA is shown in figure 4. The stator solenoids are connected in such a way that adjacent solenoids have a 60° phase difference. For such a spatial time distribution of current, a sinusoidal current waveform travels in space as exemplified in figure 5. In turn, it causes a similarly shaped waveform of radial B field as shown in figure 6

1001.0269

Figure 4. Schematics of a three-phase PCA

1001.0270

Figure 5. PCA stator current distribution at four different times

1001.0271

Figure 6. Radial B field vs. z

The radial and axial B fields at a point located at (r, z) due to one filamentary coil current, I , are:

$$B_r = \frac{\mu_0 I}{2\pi} \frac{z}{r\sqrt{(a+r)^2 + z^2}} \left\{ -K(k) + \frac{a^2 + r^2 + z^2}{(a-r)^2 + z^2} E(k) \right\} \quad (7)$$

$$B_z = \frac{\mu_0 I}{2\pi} \frac{1}{r\sqrt{(a+r)^2 + z^2}} \left\{ K(k) + \frac{a^2 - r^2 - z^2}{(a-r)^2 + z^2} E(k) \right\} \quad (8)$$

$$k = \sqrt{\frac{4ar}{z^2 + (a+r)^2}} \quad (9)$$

Again, $E(k)$ and $K(k)$ are complete elliptic integrals, I is the current in the coil filament, and a is the coil radius.

Both waveforms travel at the synchronous speed, v_s , which is related to, f , the frequency of the power supply and τ , the pole pitch according to the equation

$$v_s = 2\tau f \quad (10)$$

The armature would travel at a speed minus the slip speed. If the armature travels at the synchronous speed (that is, if the slip speed is zero) the armature sees a static magnetic environment and no current will be induced in the armature. When the slip is nonzero, the armature will see a change of radial B field over time and a proximity effect eddy current will be induced. This eddy current is the induced current in the armature. A $\vec{J} \times \vec{B}$ body force density in the armature results from the interaction between the armature current and the radial magnetic field, B_r . This force accelerates the armature in the z direction.

Current Filament Method

The current filament method [2] is based on modeling the current-carrying elements (the electrical conductors) in the system with current filaments of very small cross sections. For the consideration of inductive effects external to each filament, it is assumed that the current flows through the centroid of the cross section as a spatial impulse. In this case, the filament can be considered a current that occupies a one-dimensional (1-D) space and the current density, $\vec{J}$, through the cross section of the filament is an impulse function located at the centroid. For the consideration of self-inductive effect, the actual cross section of the filaments are used. For the consideration of the ohmic resistance of the filament, the current is assumed to be uniformly distributed over the actual cross section of the filament.

Passive Coaxial Accelerator Equivalent Circuits

Two sections of a three-phase PCA is shown in figure 7. The armature is a hollow aluminum cylinder modeled with three current filaments. Each stator current phase is modeled as one solenoid whose magnetic couplings with others are computed by modeling each solenoid as a number of filaments. The solenoid's ohmic resistance is computed based on uniform current density over the solenoid's cross section. More sophisticated formulation would include a temperature-dependent resistivity

Figure 7. Sections of a three-phase PCA

The governing equation for the PCA system can be written as a set of first order simultaneous ordinary differential equations that have to be solved by numerical methods. Because all the mutual inductances and mutual inductance gradients change over time as the armature moves, the set of ordinary differential equations is time varying.

Computer Implementation

A robust ordinary differential equation solver that can automatically switch between stiff and nonstiff methods was used to solve the system equations numerically. A complete simulation code modeling an arbitrary PCA with an integrated power supply was initially developed and executed on a Cray XMP/24. The complexity of PCA behavior made it inefficient, however, to study the fundamentals with the complete code. Two simpler simulation codes based on the same methodology were created for parameter studies

The first one is called a "voltage source simulation code." Instead of modeling a real power supply with machine parameters, the stator coils are assumed to be fed by ideal sinusoidal voltage sources. The second simplified code is called a "traveling wave simulation code." This code assumes an ideal sinusoidal stator currents that produce a well behaving traveling wave. Furthermore, the armature is uncoupled so that the armature magnetic field does not interfere the traveling wave. The code helps studying the relationship between PCA geometry and the traveling waveform.

Passive Coaxial Accelerator Parametric Variations and the Base Case

Optimizing a PCA design requires the consideration of the pole pitch, coil separation, coil thickness, layering of coils, frequency of the excitation, number of phases, armature radius to stator radius ratio, armature initial velocity, armature length, armature conductor thickness, initial conditions for the stator and armature currents, and other parameters. A parameter sensitivity study was conducted using the simulation codes mentioned.

Figure 8 depicts the geometry of one section of the base case for which perturbations of each significant factor are applied. The figure shows one section of the base-case PCA. The armature is a 20 cm long aluminum cylinder with 8.8 cm outer diameter (OD) and 7.3 cm inner diameter (ID). The stator coils are copper windings with 20-cm OD and 9-cm ID.

Pole Pitch

The pole pitch controls the separation between two phase coils, which significantly affects the smoothness of the traveling waveform. The larger the pole pitch, the more undulating the waveform is and the jerkier the acceleration is as the armature slips.

Figure 8. Geometry of the base-case PCA

Two cases of different pole pitches were analyzed and compared using the traveling wave simulation code. The code generates a time series of plots for the current distribution along the z-axis and a time series of plots of the radial B-field at a certain radius specified by the code user. A ten-phase system is considered for this study. Each phase coil of each section is modeled as one current filament. The sinusoidal stator current is 300 kA/phase coil at 5,000 Hz. The armature is magnetically decoupled from the stator. Plots for both current and B-field were made for time zero, 0.02, and 0.04 ms.

Figure 9 shows the stator current filaments' spatial distribution at $t = 0, 0.02$, and 0.04 ms over a length of 2 m, which constitutes two sections of the PCA. This waveform travels at the synchronous speed, $v_s = 2\pi f$, where $\tau = 1$ m, and $f = 5,000$ Hz. Figure 10 shows the radial B-field at the radius of 3.65 cm (which is the mean radius of the armature in the base case) due to the stator current. The radial B-field wave travels at the same synchronous velocity, but it is phase shifted compared to the current wave.

Figure 9. Stator current filaments spatial distribution

Figure 10. Radial B field vs. z

Figures 11 and 12 show the current and B-field waves for a $\tau = 0.5$ -m case. The reduction in pole pitch can be achieved by compressing the whole launcher axially from 2 to 1 m in length. As expected, the synchronous velocity has decreased. Moreover, the B-field waveform is smoother and the peak B-field more than tripled. These imply that the acceleration will be less jerky. The synchronous velocity is halved and the body force in the armature is almost tripled.

There is one more interesting observation worth mentioning. When figure 9 is overlaid on figure 11, there is perfect coincidence. However, if figure 10 is overlaid on figure 12, the general waveform coincide, but there are some visible harmonics in figure 12. The difference is because the spatial scales are different on the two plots. If only 1 m of the base case (the 1 m pole pitch case) is plotted, the fact that the coils are farther apart in the base case is more evident. Some insights on the effects of variation in coil separation are introduced and explained next.

Figure 11. Current distribution for $\tau = 0.5$ m case

Figure 12. Radial B-field vs. z for $\tau = 0.5$ m case

Sensitivity of Coil Separation

The following explains the reason for the harmonics that appear in figure 10, where the coils are farther apart, but not on figure 12. In the absence of ferromagnetic materials, the B-field is the sum of the B-field due to each coil. To explain the harmonics in the long pole pitch case, the B-field due to each coil in space is plotted on the same graph.

A six-phase case with 0.5 m section length is examined. Figure 13 displays its initial current distribution. Figure 14 shows B_r due to all coils plotted for one section. Figure 15 shows six curves representing B_r due to each coil (filament) alone. It is apparent from figure 15 that B_r dies down within about 15 cm from a filament. The summation of all six curves gives the curve in figure 14. In examining figure 15, it is obvious that the jacked waveform in figure 14 is due to the relatively wide coil separation (compared to the inductance length). Figure 16 shows the B_r for a case with half the pole pitch and also shows a smoother waveform, which is due to the closer distance between phase coils.

Figure 13. Initial distribution for the six-phase case

Figure 14. Plot of z vs. total B_r for a six-phase section

Figure 15. B_r due to each filament

Frequency of the Stator Current

Since the synchronous velocity of the traveling wave is directly proportional to the pole pitch and frequency of the excitation current, changing the frequency would have similar effect on the synchronous velocity as varying the pole pitch does. However, changing the frequency does not cause changes in the smoothness of the waveform. This fact is evident comparing the radial B-field plots for 4- and 6-kHz cases in figure 17.

Figure 16. Total B_r for halved pole-pitch case

Figure 17. B_r vs. z at $t = 0.04$ ms

Number of Phases

The bigger the number of phases, the smaller the phase difference between the two adjacent phase coil, and therefore, the smoother the waveform. This is evident in figure 18, which shows the radial B-field traveling waveforms for ten- and five-phase cases. Note that in the five-phase case, two filaments are used to simulate each phase coil. Because of that, there are some discretization errors such that the waveform computed is less smooth than it actually is. If the two filaments are replaced by a solenoid model, the curve should look smoother.

Figure 18. Ten- and five-phase systems

Stator and Armature Diameters

The ratio between the stator and armature diameters is one of the most important parameters for a PCA. This ratio controls the coefficient of magnetic coupling between the armature and the stator coils. A detailed study was done using normalized parameters. Two families of cases were studied.

The first family (depicted in fig. 19a) pertains to the case of two circular current filaments of radius R and r , separated by a distance z . The radius ratio is defined as $r_n = r/R$. The normalized separation, z_n , is defined as $z_n = z/R$. The mutual inductance gradient was computed for z ranging from -1.5 to 1.5, which is equivalent to 1.5 times R in distance on either side of the coil, letting the radius ratio vary from 0.55 to 0.90. The graph of the curves (fig. 20) shows that as the armature gets farther from the stator, the maximum mutual inductance gradient goes down significantly, but the $\partial M/\partial z$ curve is smoother. In figure 21, two curves were plotted to summarize the data in figure 20. The curve that slopes down to the right-hand side is the location (in terms of normalized coil separation, z_n) of the peak mutual inductance gradient. The other curve, the solid line, is the peak mutual inductance gradient. The mutual inductance gradient curve shows that the peak mutual inductance gradient increases rapidly as the gap between the armature and the stator decreases (i.e., as r_n approaches unity).

Figure 19. Two cases for normalized study

Figure 20. $\partial M/\partial z$ vs. z

Figure 21. Maximum $\partial M/\partial z$ and z_n of minimum $\partial M/\partial z$ vs. r_n

The second family (fig. 19b), pertains to the case of a thin solenoid (a circular current sheet) of radius R and length ℓ and a circular current filament of radius r . The distance, z , is between the midpoint of the solenoid and the center of the filament. The normalized parameters (r_n and z_n) are defined as before. The normalized coil length, ℓ_n , is defined as L/R . The mutual inductance gradient was computed for z_n ranging from -1.5 to 1.5, and for a normalized solenoid length ranging from 0 to 2.8. A series of graphs were generated for the radius ratio ranging from 0.55 to 0.90. Figures 22 and 23 summarize the results and they are self-explanatory.

Figure 22. Maximum dM/dz for solenoids with $\ell_n = 0.0$ to 2.8

Figure 23. Plot of z_n with minimum $\partial M/\partial z$ for solenoid with $\ell_n = 0.0$ to 2.8

Conclusions

The PCA is a complicated machine. Detailed simulation should be done before a full-scale PCA is built and tested. A strong foundation of analytical work has been built in CEM-UT's PCA project. It had been shown that the traveling PCA is analytically feasible. However, numerous nontrivial engineering problems remain to be solved. Further understanding of the subject based on more analytical results should lead to more insights, based on which a complete optimization should then be established. An optimization scheme including thermal and structural considerations should be implemented on a supercomputer. After analytically satisfactory results are obtained on a design, tests should be performed on individual components to verify against analytical results. Following successful tests of individual components, a full-scale PCA can be fabricated and tested.

Acknowledgments

Funding for this research has been provided by the Defense Advanced Research Projects Agency and the U. S. Army Armament Research, Development, and Engineering Center

References

- [1] M. D. Driga, et. al., "Electromagnetic Induction Launchers," *IEEE Transactions on Magnetics*, vol 22, no. 6, November 1986, pp 1453-1458.
- [2] A. Y. Wu and K. S. Sun, "Formulation and Implementation of the Current Filament Method for the Analysis of Current Diffusion and Heating in Conductors in Railguns and Homopolar Generators," *IEEE Transactions on Magnetics*, vol 25, no. 1, January 1989, pp 610-615