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A study of undulator induced transparency of magnetized plasma in the linear regime

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Abstract. The phenomenon of Undulator Induced Transparency (UIT) in a magnetized plasma is a particular case of the Electromagnetically Induced Transparency (EIT) effect where the absorption of the EM wave at the electron cyclotron frequency in the plasma is eliminated by adding a pump EM field. The UIT effect takes place when static helical magnetic undulator is used instead of an EM pump wave, thus enabling the coupling of the transverse and longitudinal EM waves in the plasma. The electromagnetic waves with slow group velocity are generated resulting in an extreme compression of the wave energy in the plasma. The polarization of the hybrid compressed waves are primarily longitudinal and their phase velocity is controllable by the undulator period, making UIT attractive for electron and ion acceleration. Numerical simulations (fluid and PIC) reveal several different scenarios of propagation of the EM waves in the UIT regime depending on the strength of the mode coupling between the two polarizations (right-hand and left-hand).

INTRODUCTION

An electromagnetically induced transparency (EIT)[1], the well-known quantum phenomenon in contemporary nonlinear optics, has a classical counterpart in plasma physics, an effect of the magnetized plasma transparency at the cyclotron frequency [2, 3]. This phenomenon is caused by an appearance of a transparency band within the resonance absorption zone in the presence of either a strong electromagnetic wave or a magnetostatic undulator, allowing propagation of electromagnetic waves in the normally opaque medium. A unique feature of this propagation regime is a drastic slowing-down of the wave group velocity which indicates an extreme electromagnetic energy compression. Undulator Induced Transparency (UIT) [4] is the easiest to implement experimentally because it does not rely on high-power pump radiation. A magnetostatic undulator couples the longitudinal plasma wave and the transverse right hand circularly polarized (RHCP) wave resulting in a hybrid wave which has both longitudinal and transverse components of the electric field. The hybrid wave is not absorbed at the cyclotron frequency, which is the signature of the UIT of plasma. The group velocity of the hybrid wave can be very slow for the weak undulator case when undulator magnetic field B_u is much less than axial magnetic field B_0 making the hybrid wave the equivalent of the slow light in quantum optics. If the spatial length of the injected electromagnetic wave is $L_0 = c\tau_0$ (where τ_0 is the temporal pulse duration), the spatial length of the hybrid wave is $L_1 = v_g\tau_0 \ll L_0$, resulting in the energy density increase by a factor $L_0/L_1 \gg 1$.

One of the promising applications of UIT is acceleration of electrons and ions which

is enabled tuning the phase velocity of the longitudinal field. Tunable phase velocity in UIT accelerators is an advantage in comparison with other plasma-based accelerators which are suitable for the electron acceleration only.

GOVERNING SYSTEM OF LINEARIZED FLUID EQUATIONS

We consider a one-dimensional (1-D) fluid model describing electromagnetic wave propagation in a magnetized plasma. The total magnetic field consists of an axial $B_0(z)\mathbf{e}_z$ and a transverse (undulating) $B_u(\mathbf{e}_x \sin k_u z + \mathbf{e}_y \cos k_u z)$ field, where the sign of the undulator parameter (wavenumber) k_u determines the helicity of the undulator, $k_u > (<)0$ for the left (right) helicity. Since the axial magnetic field introduces an asymmetry between the RHCP and LHCP waves, it is convenient to expand the transverse component of the electric field as a superposition of the right- and left-hand circularly polarized components:

$$e\mathbf{E}_\perp/(mc\omega) = \text{Re}[(a_+\mathbf{e}_+ + a_-\mathbf{e}_-) \exp(-i\omega t)] \quad (1)$$

where $\mathbf{e}_\pm = \mathbf{e}_x \pm i\mathbf{e}_y$. We neglect the motion of the ions, and within the framework of the linearized cold electron fluid model we can obtain the following coupled system of equations describing the propagation of a hybrid EM wave in a magnetized plasma inside the undulator:

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} n_+^2(z) \right] a_+ = \frac{\omega^2}{c^2} g(z) e^{2ik_u z} a_- \quad (2)$$

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} n_-^2(z) \right] a_- = \frac{\omega^2}{c^2} g(z) e^{-2ik_u z} a_+, \quad (3)$$

where the refractive indices $n_\pm$ for the RHCP and LHCP polarizations, respectively, are

$$n_\pm^2 = 1 - \frac{\omega_p^2}{\omega} \cdot \frac{2\Omega_R^2 \omega - (\omega^2 - \omega_p^2)(\omega \pm \Omega_0)}{4\Omega_R^2 \omega^2 - (\omega^2 - \omega_p^2)(\omega^2 - \Omega_0^2)}, \quad (4)$$

and $\Omega_R = eB_u/2mc$ is the Rabi frequency proportional to the undulator strength. Without an undulator, the expressions for the indices of refraction revert to the usual formulas [5] for the RHCP and LHCP waves propagating along the magnetic field in the plasma: $n_\pm^2 = 1 - \omega_p^2/(\omega(\omega \mp \Omega_0))$. The spatially-varying coupling coefficient g , responsible for the cross-polarization coupling,

$$g = 2\Omega_R^2 \omega_p^2 [4\Omega_R^2 \omega^2 - (\omega^2 - \omega_p^2)(\omega^2 - \Omega_0^2)]^{-1} \quad (5)$$

vanishes without an undulator (Ω_R) as it should because RHCP and LHCP are the eigenmodes of the uniformly magnetized plasma. The $a_\pm$ amplitudes only characterize the transverse components of the electric field of the hybrid wave. Its longitudinal electric field, $E_z = (mc\omega/e) \times \text{Re}[a_z \exp(-i\omega t)]$, which must significantly exceed its transverse

field for large energy compression to occur, is proportional to the dimensionless parameter a_z given by

$$a_z = \Omega_R^{-1} \left[g(\omega + \Omega_0) a_+ e^{-ik_u z} - g(\omega - \Omega_0) a_- e^{ik_u z} \right]. \quad (6)$$

The "perfect" transparency can be determined by the condition when $\omega^2 = k_z^2 c^2$; namely the plasma is transparent to the wave that would have been absorbed without the undulator. From the expression for the n_+ , RHCP refraction index, the perfect transparency condition for the RHCP wave is [3, 4]:

$$\omega^2 = \Omega_0^2 = \omega_p^2 + \Omega_R^2 = \bar{\omega}_p^2 \quad (7)$$

In other words, the perfect transparency condition is the matching condition of all relevant frequencies: probe frequency ω , cyclotron frequency Ω_0 and renormalized by the undulator electron plasma frequency $\bar{\omega}_p$. Because of the practically achievable axial magnetic field strength B_0 (about 10 Tesla), UIT is a microwave/plasma phenomenon with an upper frequency bound of about 300 GHz. There exist very reliable microwave sources: industrial gyrotrons. We consider a 110 GHz gyrotron with peak power about 1 MW and output window aperture of about 50 mm [6] (gyrotrons have proven to be very reliable in industrial microwave applications). This characteristic gives the normalized electric field amplitude $eE_0/mc\omega = a_0 \approx 1 \times 10^{-3}$. The required axial magnetic field will be $B_0 \approx 4$ Tesla in this case, which is realizable in the modern experimental setups. The helical undulator with the magnetic field strength $B_u = 0.2B_0$ will be $B_u \approx 0.8$ Tesla. Plasma density is $N_0 \approx 1.5 \times 10^{14} \text{ cm}^{-3}$ which corresponds to the plasma frequency $\omega_p \approx 0.995\Omega_0$ fulfilling the perfect transparency condition. The possible experimental setup for the UIT experiment is shown in Fig. 1. It consists of a solenoid generating an axial magnetic field with extra coils on the ends of the system in order to have a "magnetic mirror" configuration (see explanation below). The helical undulator is located inside the solenoid and represents the system of permanent magnets twisted along the axis of the system.

FIGURE 1. Schematic view of the possible experimental setup. Plasma is immersed in the helical undulator structure which is located inside the solenoid. The microwave radiation of the gyrotron incident from the output window into the plasma.

FLUID SIMULATIONS OF THE COUPLED-WAVE EQUATIONS

The system of equations (2)-(3) have been numerically integrated for the case of a RHCP electromagnetic wave incident from the vacuum onto the plasma layer. We used the following electron density profile to represent the plasma slab with the smooth boundaries: $N(z)/N_0 = 0.5 \tanh((\omega z/c - 25)/5) - 0.5 \tanh((\omega z/c - 95)/5)$. We impose the radiation boundary conditions at the boundaries of the computational domain. An EM wave with the amplitude a_+^{inc} incident from the left on the vacuum-plasma boundary. The choice of a_+^{inc} is arbitrary in the context of the linear UIT theory described by Eqs. (2,3). Boundary conditions at the right boundary of the computational domain ($\omega z/c = 120$) ensures that there is no incident radiation from the right.

With the cyclotron resonance condition $\Omega_0 = \omega$ we choose an undulator strength of $B_u = 0.2B_0$ or $\Omega_R/\omega = 0.1$ which, together with the perfect transparency condition (7) gives the plasma frequency: $\omega_p^2 \approx 0.995\omega$.

It is important to properly choose the axial magnetic field profile to enable penetration of the EM wave into the plasma. Analyzing the expression for the index of refraction (4) for RHCP and LHCP modes one can avoid the resonances of $n_{\pm}$ if $\Omega_0 \geq \omega$. By other words it have to be "magnetic mirror" like profile $B(z)/B_0 = 2 - 0.5 \tanh((\omega z/c - 25)/5) + 0.5 \tanh((\omega z/c - 95)/5)$, which satisfies the above criterion. Depending on the undulator wavenumber k_u , three different regimes of wave propagation and mode coupling we identified.

For small undulator numbers ($0 < n_u < 0.5$) the propagation regime is similar to small angle propagation with respect to the constant magnetic field. Incident RHCP electromagnetic wave penetrates into plasma with no reflection and inside the plasma layer partially transforms into LHP polarized wave. The refraction index for RHCP wave inside plasma is not equal to that in vacuum, hence there is no clear transparency effect. The plasma wave, in this case, is running in the same direction as the incident pulse.

For moderate values of the undulator number ($n_u \approx 1$), the situation is drastically different. This is illustrated by Fig. 2(a,b,c), which shows that the plasma is opaque to the RHCP wave, as it does not penetrate into the plateau region. The opaqueness is caused by the strong counter-propagating resonance: the forward-travelling RHCP mode-converts into the backward-travelling LHCP. The plasma wave has a small amplitude because of the small RHCP component of the electric field in the plasma plateau region. Increasing the undulator number we come to the regime where RHCP predominates in the plasma layer and the index of refraction is almost unity, indicating the UIT regime. Transverse and longitudinal field profiles corresponding to $n_u = 1.6$ are shown in Fig. 2(d,e,f).

From Figs. 2(d-e) we observe that the RHCP wave dominates in the plasma plateau region, resulting in a strong harmonic plasma wave shown in Fig. 2(f). Note that the peak plasma wave amplitude is ten times higher than that of the incident RHCP wave: $Q = E_z^{(max)}/E_{inc}^{(max)} \approx 10$. This enhancement demonstrates the energy density compression by a factor $G = Q^2/2 \approx 50$, in good agreement with the single-helicity calculation [4] of the group velocity $v_g/c = 2\Omega_R^2/\Omega_0^2 = 50$. Note that the phase velocity of the plasma wave is opposite to the phase and group velocities of the RHCP wave in the plasma. One can thus envision novel types of plasma-based backward-wave oscillators (BWO), in which electron beams propagating in the minus z direction resonantly interact with

FIGURE 2. The transverse RHCP (a,d), LHCP (b,e) and longitudinal (c,f) components of the electric field in the "opaque" regime ($n_u = 0.9$) (a, b, c) and in the UIT regime ($n_u = 1.6$) (d, e, f). Undulator strength is $B_u = 0.2B_0$, $\omega = \Omega_0$, $\bar{\omega}_p = \Omega_0$. Arrows indicate the direction of the phase velocity.

slow electromagnetic waves propagating in positive z direction.

1D PIC SIMULATIONS OF ENERGY COMPRESSION

In order to confirm the results of the linearized single-frequency fluid calculation of UIT in a cold magnetized plasma, and to study the time-dependent properties of UIT, we used the one-dimensional version of the fully electromagnetic PIC code VLPL [7]. The following simulation parameters have been used: grid size $\delta z = 0.025\lambda$ (where $\lambda = 2\pi c/\omega$ is the vacuum wavelength of the incident radiation), 128 particles per cell. The incident circularly polarized probe pulse has the Gaussian envelope $\exp(-(\omega z/c - \omega t)^2/T^2)$ with the duration $T = 100\lambda/c$ and its amplitude $a_+^{inc} = 10^{-3}$. The undulator wave number $n_u = 1.6$ guarantees the "perfect" UIT regime for the above parameters, i.e. almost non-reflecting regime of propagation.

The electron plasma density profile is chosen to have a smooth vacuum/plasma interface and a constant-density plateau region $15 < z/\lambda < 35$: $N(z)/N_0 = 0.5 \tanh((z/\lambda - 10)/1.67) - 0.5 \tanh((z/\lambda - 40)/1.67)$. The axial magnetic field has the "magnetic mirror" shape: gradually decreasing from a high amplitude ($eB_z/mc = 2\omega$) in the low plasma density regions to the "perfect" UIT value of $eB_0/mc \equiv \Omega_0 = \omega$ in the plasma plateau region. The exact magnetic field profile is $B_z/B_0 = 2 - 0.5 \tanh((z/\lambda - 10)/1.67) + 0.5 \tanh((z/\lambda - 40)/1.67)$. The results of 1D PIC simulations for left and right helicity of the undulator are presented in Fig. 3 and Fig. 4 correspondingly. A time sequence of snapshots t_i (defined in the caption to Fig. (3)) of the transverse and longitudinal electric fields are shown in the left and right panels of Fig. (3), respectively. The long incident pulse is indeed shown to compress in the plasma [see Fig. (3c)]; a fact that could only indirectly inferred from

FIGURE 3. Sequential snapshots at $t = t_i$ of the time evolution of the transverse (left column, (a) - (d)) and longitudinal (right column, (e) - (h)) electric field normalized to the electric field in the incident wave, $a_{+}^{inc} = 10^{-3}$, $n_u = 1.6$, undulator has a left helicity. (a) and (e): $t_1 = 216\lambda/c$; (b) and (f): $t_2 = 600\lambda/c$; (c) and (g): $t_3 = 1104\lambda/c$; (d) and (h): $t_4 = 1488\lambda/c$.

the fixed frequency simulations. At $t = t_2$ the entire pulse is inside the plasma plateau. It propagates with a reduced group velocity through the plasma between $t = t_2$ and $t = t_3$, advancing by $\Delta z = 10\lambda$. Hence, the group velocity $v_{gr} = \Delta z / (t_3 - t_2) \approx 0.02c$ is in excellent agreement with the theoretical estimate of $v_{gr} = cB_u^2 / 2B_0^2$.

While the pulse is inside the plasma, the amplitude of the longitudinal electric field is roughly ten times larger than the amplitude of the incident RHCP wave, as shown in Fig. (3h), i.e. $a_z \approx 10^{-2}$. The large value of the longitudinal electric field makes UIT plasma an attractive candidate for electron and ion acceleration.

Snapshots in Fig. 4 are taken at the same time moments as the snapshots in 3 but with a reversed undulator helicity. The undulator parameter is $n_u = 1.0$. The interesting aspect of the right helicity simulations is that the phase velocity of the plasma wave can be made slow enough to be suitable for heavy ion acceleration and is also controllable by tapering the undulator period as a function of z .

CONCLUSIONS

In conclusion, we have demonstrated, using PIC simulations, that extreme energy compression of electromagnetic radiation is possible in a magnetized plasma made transparent by an addition of a weak helical undulator. For this compression to occur, a right-hand circularly polarized electromagnetic wave has to be coupled into the plasma at the

FIGURE 4. Sequential snapshots at $t = t_i$ of the time evolution of the transverse (left column, (a) - (d)) and longitudinal (right column, (e) - (h)) electric field normalized to the electric field in the incident wave, $a_+^{inc} = 10^{-3}$, $n_u = 1.0$, undulator has a right helicity. (a) and (e): $t_1 = 216\lambda/c$; (b) and (f): $t_2 = 600\lambda/c$; (c) and (g): $t_3 = 1104\lambda/c$; (d) and (h): $t_4 = 1488\lambda/c$.

electron cyclotron frequency. Although without an undulator this wave would be absorbed by the plasma via electron cyclotron heating, the plasma becomes transparent due to the phenomenon of undulator-induced transparency (UIT). Because the electromagnetic wave in the UIT plasma is primarily longitudinal, it can be useful for various accelerator applications. An acceleration gradient of order 10 MeV/m can be obtained, and the controllability of the phase velocity allows acceleration of ions or other heavy particles.

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