Effect of noise and pump depletion on the plasma beat wave accelerator

W. Horton and T. Tajima

Citation: [AIP Conference Proceedings](#) **130**, 179 (1985); doi: 10.1063/1.35297

View online: <http://dx.doi.org/10.1063/1.35297>

View Table of Contents:

<http://scitation.aip.org/content/aip/proceeding/aipcp/130?ver=pdfcov>

Published by the [AIP Publishing](#)

Articles you may be interested in

[Evolution of the laser beam envelope in the beat wave accelerator](#)

AIP Conf. Proc. **130**, 156 (1985); 10.1063/1.35322

[Multiple scattering and synchrotron radiation in the plasma beat-wave accelerator](#)

AIP Conf. Proc. **130**, 146 (1985); 10.1063/1.35321

[Some nonlinear processes relevant to the beat wave accelerator](#)

AIP Conf. Proc. **130**, 138 (1985); 10.1063/1.35320

[Efficiency factors in the beat wave accelerator](#)

AIP Conf. Proc. **130**, 134 (1985); 10.1063/1.35319

[Study of beat-wave growth and saturation](#)

AIP Conf. Proc. **130**, 172 (1985); 10.1063/1.35296

Effect of Noise and Pump Depletion on the Plasma Beat Wave Accelerator

W. Horton and T. Tajima
University of Texas at Austin
Austin, Texas 78712

ABSTRACT

The limits on the effective growth of the plasma beat wave due to noise in the plasma density and the depletion of the laser driving fields are investigated theoretically.

GROWTH OF THE PLASMA WAVE AND PUMP DEPLETION

We consider the self-consistent growth of the plasma wave ω_p driven up from the beating of two high frequency, ω_1, ω_2 , linearly polarized laser beams in the presence of a noisy background plasma density. The laser fields are $E_j \sin(k_j x - \omega_j t + \phi_j) \hat{e}_y$ and $B_j \sin(k_j x - \omega_j t + \phi_j) \hat{e}_z$ with $B_j = (ck_j/\omega_j) E_j$. The plasma wave is $E_p \cos(k_p x - \omega_p t + \phi_p) \hat{e}_x$ with ideal, resonant $\omega_1 - \omega_2 = \omega_p$, growth rate given by $\mathcal{E} = eE_p/m\omega_p c = \lambda t \omega_p$, where $\lambda \cong a_1 a_2/4$ is the ideal growth rate given in terms of the dimensionless quiver velocities defined in Eq. (1). We now investigate how this ideal growth can be limited due to variations in the laser-plasma wave phase relation $\psi = \phi_1 - \phi_2 - \phi_p$ due to variations in the plasma density and pump depletion.

From the transverse electromagnetic wave dispersion relation $\omega_j = (c^2 k_j^2 + \omega_p^2)^{1/2}$ it follows that the phase velocity of the beat wave $v_p = (\omega_1 - \omega_2)/(k_1 - k_2) \simeq d\omega_1/dk_1 = c(1 - \omega_p^2/\omega_1^2)^{1/2}$ equals the group velocity of the laser pulse for $\omega \gg \omega_p$. The Lorentz transformation to the beat wave frame is given by $\beta_p = \Delta\omega/\Delta kc = \omega_p/k_p c$ and $\gamma_p = (1 - \beta_p^2)^{1/2} = \omega_1/\omega_p$. The energy gain in the simple beat wave accelerator¹ occurs from reflection off the electrostatic potential $\varphi_{-wave} = \gamma_p \varphi_p$ in the wave frame and is given by $\Delta E = 2mc^2 \gamma_p^2 \mathcal{E}_{max}$ where $\mathcal{E}_{max} = \max(eE_p/m\omega_p c)$. We now consider two effects which can determine $\mathcal{E}_{max}$.

The energy density in the transverse laser waves is $W_j = \langle (E_j^2 + B_j^2)/8\pi + \frac{1}{2} m n v_{y_j}^2 \rangle = (E_j^2/16\pi) [1 + (c^2 k_j^2 + \omega_p^2)/\omega_j^2] = E_j^2/8\pi$ and in the longitudinal plasma wave is $W_p = \langle E_p^2/8\pi + \frac{1}{2} m n v_x^2 \rangle = (E_p^2/16\pi)(1 + \omega_p^2/\Delta\omega^2) = E_p^2/8\pi$. We define the dimensionless quiver or oscillation velocities by

$$a_j = \frac{eE_j}{m\omega_j c} \quad \text{and} \quad \mathcal{E} = \frac{eE_p}{m\omega_p c} = \frac{\omega_p \xi}{c} \quad (1)$$

where in the last equation we also introduce the amplitude of the Lagrangian displacement ξ of the thermal electrons in the plasma wave. The Poisson equation gives the exact linear relation between E_p and ξ through $E_p = 4\pi en\xi$ where $-en\xi$ is the surface charge density introduced by the displacement of ξ of the electrons at x_o from the ions by $x = x_o + \xi(x_o, t)$, unless the wave breaking takes place. The Lagrangian equation of motion for the plasma wave is linear

$\partial_t^2 \xi + \omega_p^2 \xi = 0$ but describes steepening of the plasma wave through the inversion of the Lagrangian equation for $x_o = x_o(x, t)$. For a sinusoidal oscillation $\xi \sin(k_p x)$ the inversion becomes multivalued when $dx/dx_o = 1 + k_p \xi \cos(k_p x) = 0$ which describes wave breaking at the limit $k_p \xi = 1$. For plasma waves with $v_p = \omega_p/k_p \simeq c$ the breaking limit gives $\mathcal{E} = E_p/m\omega_p c = \omega_p \xi/c = 1$.

The relativistic equation of motion² for $\xi(x_o, t)$ follows from calculating the rate of increase of $p_x = m\gamma(\dot{\xi})\dot{\xi}$ due to the resonant frequency part of the Lorentz force $-e[E_x + (v_y^{(1)} B_z^{(2)} + v_y^{(2)} B_z^{(1)})/c]$ to obtain

$$\gamma_\xi^3 \frac{\partial^2 \xi}{\partial t^2} + \omega_p^2 \xi = 2\Delta\omega c \lambda \sin[\Delta k(x_o + \xi) - \Delta\omega t + \Delta\varphi] \quad (2)$$

where

$$\lambda = \frac{a_1 a_2}{4\beta_p} \quad \text{and} \quad \Delta\varphi = \varphi_1 - \varphi_2.$$

For small λ the effective plasma frequency becomes $\omega_p [1 - \frac{3}{8}(\omega_p^2 \xi^2/c^2)]^{1/2}$ and the linear resonant growth $\mathcal{E} = \omega_p \xi/c = \lambda t \omega_p$ stops when $\mathcal{E}_{RL} = 4(\lambda/3)^{1/3}$ at $\omega_p t_s = 3.89\lambda^{-2/3}$.

To close the system we calculate the effect of the plasma wave on the transverse wave amplitude and phase $\hat{E}_j(x, t) = E_j \exp(i\phi_j)/2i$ where $E_j(x, t) = \text{Re} \hat{E}_j \exp(ik_j x - i\omega_j t)$. Reducing the $(\partial_t^2 - c^2 \partial_x^2) E_y = -4\pi \partial_t j_y$, we obtain

$$(\partial_t + v_j \partial_x) \hat{E}_j = -2\pi \langle j_y^{n\ell} e^{-ik_j x + i\omega_j t} \rangle \quad (3)$$

where $j_y^{n\ell} = -e\delta n_p v_y = (4\pi)^{-1} v_y \partial E_p / \partial x$. Introducing the envelope approximation in Eq. (2) with $E_p(x, t) = \text{Re} \hat{E}_p \exp(ik_p x - i\omega_p t)$ we obtain for the closed system of waves

$$(\partial_t + v_1 \partial_x) \hat{E}_1(x, t) = -\frac{ek_p}{2m\omega_2} \hat{E}_p \hat{E}_2 e^{-i\Delta\omega t} \quad (4)$$

$$(\partial_t + v_2 \partial_x) \hat{E}_2(x, t) = \frac{ek_p}{2m\omega_1} \hat{E}_1 \hat{E}_p^* e^{i\Delta\omega t} \quad (5)$$

$$(\partial_t - i\frac{3}{16}\omega_p |\epsilon|^2) \hat{E}_p(x, t) = \frac{e\omega_p \Delta k}{2m\omega_1 \omega_2} \hat{E}_1 \hat{E}_2^* e^{+i\Delta\omega t} \quad (6)$$

where $\Delta\omega = \omega_p + \omega_2 - \omega_1$, where only two original electromagnetic waves are retained. The rate of transfer of energy from the lasers to the plasma is determined by

$$\begin{aligned}
T &= \hat{E}_1^* \hat{E}_2 \hat{E}_p e^{-i\Delta\omega t} + \hat{E}_1 \hat{E}_2^* \hat{E}_p^* e^{i\Delta\omega t} \\
&= \frac{1}{4} E_1 E_2 E_p \sin(\psi - \Delta\omega t)
\end{aligned} \tag{7}$$

where $\psi = \phi_2 + \phi_p - \phi_1$ with rates given by

$$(\partial_t + v_1 \partial_x) W_1 = - \left(\frac{ek_p}{2m\omega_2} \right) T \tag{8}$$

$$(\partial_t + v_2 \partial_x) W_2 = \left(\frac{ek_p}{2m\omega_1} \right) T \tag{9}$$

$$\partial_t W_p = \left(\frac{e\omega_p \Delta k}{2m\omega_1 \omega_2} \right) T \tag{10}$$

From Eqs. (8)-(10) it follows that $\Delta(W_1/\omega_1) = -\Delta(W_2/\omega_2) = -\Delta(W_p/\omega_p)$ when $k_p = \Delta k$, a manifestation of the Manley-Rowe relation. For $v_1 \cong v_2 = v_p$ the transformation of laser energy to plasma wave energy is given by

$$(\partial_t + v_p \partial_x)(W_1 + W_2) = - \frac{ek_p \Delta\omega}{2m\omega_1 \omega_2} T \tag{11}$$

$$\partial_t W_p = \frac{e\Delta k \omega_p}{2m\omega_1 \omega_2} T. \tag{12}$$

We use the conservation of the total energy to calculate the depletion of the laser energy $W_\ell \equiv W_1 + W_2$ by the growth of the plasma wave

$$(\partial_t + v_p \partial_x) W_\ell(x, t) = - \frac{\partial W_p(x, t)}{\partial t}. \tag{13}$$

Since $W_p/W_\ell = (\omega_p^2/\omega^2)(\mathcal{E}^2)/(a_1^2 + a_2^2) \simeq (\omega_p^2/\omega^2)(\mathcal{E}^2/8\lambda) \ll 1$, in the first approximation the solution of Eq. (13) is $W_\ell = W_\ell(x - v_p t) f(t/\tau_D)$ with depletion time τ_D given by

$$\tau_D = W_\ell / (\partial W_p / \partial t) = \tau_\ell (\omega^2 / \omega_p^2) (4\lambda / \mathcal{E}_m^2)$$

where $c\tau_\ell$ is the laser pulse length. For $\mathcal{E}_m = \lambda\tau_\ell\omega_p$ the depletion length $L_D = c\tau_D = 4(c/\omega_p)(\omega^2/\omega_p^2)/\mathcal{E}_m$ and for $\tau_\ell\omega_p = 4\lambda^{-2/3}$ (if we set τ_ℓ for the relativistic detuning time) and $\mathcal{E}_m = 4(\lambda/3)^{1/3}$ the depletion length is

$$L_D = (c/\omega_p)(\omega^2/\omega_p^2)(3/\lambda)^{1/3}.$$

A similar estimate is given by T. Katsouleas et al.³ from energy conservation. The pump depletion length is only slightly larger than the dephasing length. Thus the pump depletion effect may limit the main attractiveness of the surfatron.⁴

Since the pump depletion time τ_D is long compared with growth period $1/\omega_p \lambda$ of the plasma wave, it is convenient to calculate the decay of the laser pulse in the wave frame where

$$\frac{\partial W_\ell}{\partial t'}(x', t') \cong v_p \frac{\partial W_p}{\partial x'}(x', \lambda(x', t')), \quad (14)$$

and the loss of total laser energy $E'_\ell = \int W_\ell d^3x'$ occurs from the flux of plasma waves $v_p W_p(x' = -L_\ell)$ out the rear of the laser pulse

$$\frac{d}{dt'} E'_\ell = \int \int_{-L_\ell}^0 \frac{d}{dt'} W_\ell da dx' = -v_p A_p W_p(x' = -L_p). \quad (15)$$

Equation (14) shows that the laser pulse decays most rapidly when $\partial W_p / \partial x'$ takes maximum.

We now calculate the growth of the plasma wave in terms of its amplitude $\mathcal{E}$ and phase ψ . The entire closed system has the three degrees of freedom Hamiltonian

$$\begin{aligned} H^{6D}(I_j, \phi_j) = & \frac{\omega_1}{\omega_p} I_1 + \frac{\omega_2}{\omega_p} I_2 + \left(1 - \frac{3}{32} \frac{e^2 I_p}{m^2 c^2 \omega_p}\right) I_p \\ & - \frac{ek_p(I_1 I_2 I_3)^{1/2}}{2m(\omega_1 \omega_2 \omega_p)^{1/2}} \cos(\phi_1 - \phi_2 - \phi_p) \end{aligned} \quad (16)$$

where $d\phi_j/dt\omega_p = \partial H^{6D}/\partial I_j$ and $dI_j/dt\omega_p = -\partial H^{6D}/\partial \phi_j$ reproduces the equations of motion for $(\hat{E}_j) = (\omega_j I_j)^{1/2}$ and $\phi_j = \arg(\hat{E}_j)$ with $j = 1, 2, p$. Taking into account the cyclic coordinates $\phi_1 + \phi_2$, and $\phi_1 + \phi_p$ at constant $\psi = \phi_2 + \phi_p - \phi_1$ the system reduces to a four dimensional phase space I_1, I_2, I_p, ψ . The rate of change of ψ is given by

$$\begin{aligned} \dot{\psi} = & \omega_2 + \omega_p \left(1 - \frac{3}{16} \mathcal{E}^2\right) - \omega_1 \\ & - \frac{ek_p \omega_p^{1/2} \cos \psi}{4m(\omega_1 \omega_2)^{1/2}} \left[I_p^{1/2} \left(\left(\frac{I_2}{I_1} \right)^{1/2} - \left(\frac{I_1}{I_2} \right)^{1/2} \right) - \frac{(I_1 I_2)^{1/2}}{I_p^{1/2}} \right] \end{aligned} \quad (17)$$

For large I_1, I_2 compared with I_p the phase rotation due to the change in the strength of the laser fields is small. The condition for negligible phase rotation due to pump depletion is

$$I_p \left[\left(\frac{I_1}{I_2} \right)^{1/2} - \left(\frac{I_2}{I_1} \right)^{1/2} \right] \ll (I_1 I_2)^{1/2}$$

or approximately

$$\left(\frac{\omega_p}{\omega} \right) \mathcal{E}^2 \left[\left(\frac{E_1}{E_2} \right)^{1/2} - \left(\frac{E_2}{E_1} \right)^{1/2} \right] \ll 4\lambda. \quad (18)$$

When condition (18) is satisfied the hamiltonian reduces to the usual fixed laser field hamiltonian $h(I_p, \psi)$ with one degree of freedom

$$h(I_p, \psi) = \left[\omega_1 - \omega_2 - \omega_p \left(1 - \frac{3}{32} \frac{e^2 I_p}{m^2 c^2 \omega_p} \right) \right] I_p - \frac{ek_p \omega_p^{1/2} E_1 E_2}{4m\omega_1 \omega_2} I_p^{1/2} \cos \psi. \quad (19)$$

The mean plasma density $\langle n \rangle = n$ is taken resonant $\omega_p = \omega_1 - \omega_2$, and we now consider the effect of fluctuations $\delta\omega_p = \omega_p(\delta n/2n)$ on the evolution of the beat wave given by Eq. (19)

$$\frac{d\mathcal{E}}{dt\omega_p} = \lambda \sin \psi \quad (20)$$

$$\frac{d\psi}{dt\omega_p} = -\frac{\delta n}{2n} + \frac{3}{16} \mathcal{E}^2 - \frac{\lambda}{\mathcal{E}} \cos \psi. \quad (21)$$

PLASMA NOISE

We define the dimensionless variance of the density fluctuations $\langle \delta n^2 \rangle = n^2 \sigma$ and the correlation time τ_c through $\langle \delta n(t_1) \delta n(t_2 = t_1 + \tau) \rangle = (2\pi)^{-1} \int_{-\infty}^{+\infty} d\omega N(\omega) \exp(-i\omega\tau)$ where $\int_{-\infty}^{+\infty} d\tau \langle \delta n_t \delta n_{t+\tau} \rangle = \tau_c \langle \delta n^2 \rangle = N(\omega = 0)$. A typical noise spectrum is the Lorentzian $N(\omega) = 2\delta\omega(\omega^2 + \delta\omega^2)^{-1}$, for which $\tau_c = 1/\delta\omega$.

Taking the ensemble average of Eq. (20) for noise near gaussian statistics gives

$$\frac{d\langle \mathcal{E} \rangle}{dt\omega_p} = \lambda \sin \langle \psi \rangle e^{-\nu t} \quad (22)$$

where

$$\nu = \frac{1}{2t} \int_0^t dt_1 \int_0^t dt_2 \langle \dot{\psi}(t_1) \dot{\psi}(t_2) \rangle. \quad (23)$$

When the noise dominates the nonlinear frequency shift in Eq. (21), the lowest order calculation of (23) gives

$$\nu = \frac{\omega_p^2}{4} \int_{-\infty}^{+\infty} \frac{\langle \delta n_t \delta n_{t+\tau} \rangle}{n^2} d\tau = \frac{\omega_p^2 \tau_c}{4} \sigma \quad (24)$$

valid for $\nu \tau_c = \omega_p^2 \tau_c^2 \sigma / 4 < 1$. In Ref. 5 we extend the calculation to $\sigma > \sigma_c = 4/\omega_p^2 \tau_c^2$ by using renormalized turbulence theory. In the noisy plasma the plasma wave growth is given by a renormalized propagator given in Eqs (16)-(18) in Ref. 5. For the Lorentzian noise spectrum the frequency integral in the renormalized propagator is performed exactly to give

$$\nu^2 + \nu \delta \omega - \omega_p^2 \sigma / 4 = 0$$

with

$$\nu = \frac{1}{2} \left[(\omega_p^2 \sigma + \delta \omega^2)^{1/2} - \delta \omega \right] = \begin{cases} \omega_p^2 \sigma / 4 \delta \omega & \text{for } \sigma < \sigma_c \\ \omega_p \sigma^{1/2} / 2 & \text{for } \sigma > \sigma_c \end{cases} \quad (25)$$

The effective growth time is now $1/\nu$ and the maximum amplitude from Eq. (22) is

$$\mathcal{E}_m = \omega_p \frac{\lambda}{\nu} \quad \text{for} \quad \frac{\nu}{\omega_p} > \frac{1}{4} \lambda^{2/3}$$

Numerous examples of the effect of noise on the beat wave growth rate are given in Ref. 5.

We conclude that limits on the amplitude of the accelerating plasma electric field may be set by either the pump depletion length $L_D = c\tau_D$ or the noise limited growth before reaching the ideal accelerator strength² under the condition derived in the present work.

ACKNOWLEDGEMENTS

*This work supported by the National Science Foundation and the United States Department of Energy.

REFERENCES

1. T. Tajima and J.M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
2. M.N. Rosenbluth and C.S. Liu, Phys. Rev. Lett. **29**, 701 (1972).
3. T. Katsouleas et al., *Proceedings of the Laser Acceleration Workshop*, Malibu, CA 1985.
4. T. Katsouleas and J.M. Dawson, Phys. Rev. Lett. **51**, 392 (1983).
5. W. Horton and T. Tajima, Phys. Rev. A, 1985 (to appear).